

Class 09 - Mathematics

Maximum Marks: 80

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

1. If $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$, then $x + y + xy =$ [1]
 - a) 5
 - b) 9
 - c) 17
 - d) 7
2. The positive solutions of the equation $ax + by + c = 0$ always lie in the [1]
 - a) 3rd quadrant
 - b) 4th quadrant
 - c) 2nd quadrant
 - d) 1st quadrant
3. If $(x, y) = (y, x)$, then [1]
 - a) $x - y = 0$
 - b) $x + y = 0$
 - c) $x \div y = 0$
 - d) $xy = 0$
4. In a histogram, which of the following is proportional to the frequency of the corresponding class? [1]
 - a) Width of the rectangle
 - b) Length of the rectangle
 - c) Perimeter of the rectangle
 - d) Area of the rectangle
5. A linear equation in two variables is of the form $ax + by + c = 0$, where [1]
 - a) $a \neq 0$ and $b = 0$
 - b) $a = 0$ and $b = 0$
 - c) $a \neq 0$ and $b \neq 0$
 - d) $a = 0$ and $b \neq 0$
6. Which of the following needs a proof? [1]

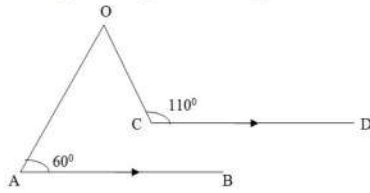
a) Postulate

b) Axiom

c) Theorem

d) Definition

7. In the given figure, $AB \parallel CD$. If $\angle BAO = 60^\circ$ and $\angle OCD = 110^\circ$, then $\angle AOC = ?$ [1]



a) 50°

b) 40°

c) 70°

d) 60°

8. If APB and CQD are two parallel lines, then the bisectors of the angles APQ, BPQ, CQP and PQD form [1]

a) any other parallelogram

b) a rhombus

c) a square

d) a rectangle

9. If $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = ky^2$, then $k =$ [1]

a) 8

b) 2

c) 1

d) 4

10. The graph of $y = 6$ is a line [1]

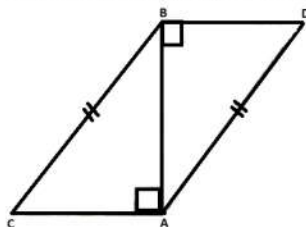
a) Parallel to x-axis at a distance 6 units from the origin

b) Making an intercept 6 on the x- axis.

c) Making an intercept 6 on both the axes.

d) Parallel to y-axis at a distance 6 units from the origin

11. In the adjoining figure, $BC = AD$, $CA \perp AB$ and $BD \perp AB$. The rule by which $\triangle ABC \cong \triangle BAD$ is [1]



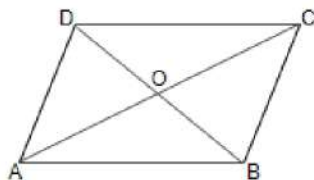
a) ASA

b) RHS

c) SSS

d) SAS

12. In the given figure, ABCD is a Rhombus. Then, [1]



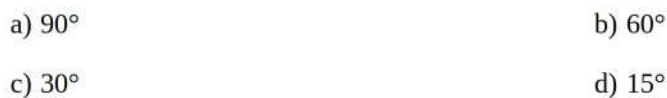
a) $(AC^2 + BD^2) = 3AB^2$

b) $AC^2 + BD^2 = 4AB^2$

c) $AC^2 + BD^2 = AB^2$

d) $AC^2 + BD^2 = 2AB^2$

13. In the given figure, O is the centre of a circle and $\angle ACB = 30^\circ$. Then, $\angle AOB = ?$ [1]



- a) $\frac{7}{11}$
c) $\frac{7}{10}$

- a) $x^4 + \frac{1}{x^4}$ b) $x^3 - \frac{1}{x^3} + 2$
c) $x^2 + \frac{1}{x^2} - 2$ d) $x^4 - \frac{1}{x^4}$

-

- a) 115°
c) 100°

- a) none of these b) 64
c) 76 d) 52

- a) 3 : 5 b) 2 : 5
c) 1 : 3 d) 3 : 1

- Reason (R):** All the sides of an equilateral triangle are equal.

- a) Both A and R are true and R is the correct explanation of A.
- b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false.
- d) A is false but R is true.

- Reason (R):** The linear equation $ax + b = 0$ can be expressed as a linear equation in two variables as $ax + y + b = 0$.

a) Both A and R are true and R is the correct explanation of A.

c) A is true but R is false.

b) Both A and R are true but R is not the correct explanation of A.

d) A is false but R is true.

Section B

21. Find the area of an isosceles triangle, whose equal sides are of length 15 cm each and third side is 12 cm. [2]

22. Factorise: $9x^2 + 4y^2 + 16z^2 + 12xy - 16yz - 24xz$ [2]

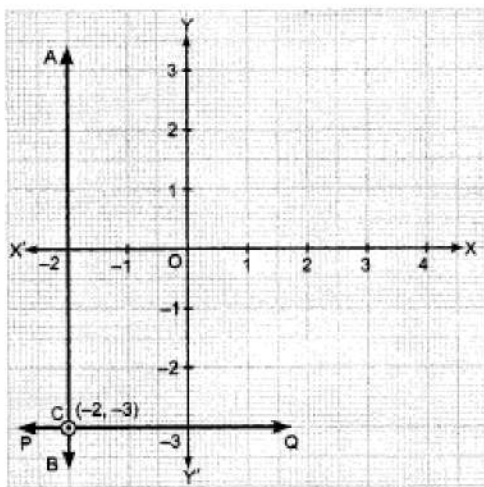
23. The surface areas of two spheres are in the ratio 1 : 4. Find the ratio of their volumes. [2]

24. By Remainder Theorem find the remainder, when $p(x)$ is divided by $g(x)$, where $p(x) = x^3 - 6x^2 + 2x - 4$, $g(x) = 1 - \frac{3}{2}x$ [2]

OR

Factorise: $\sqrt{2}x^2 + 9x + 4\sqrt{2}$.

25. Write the linear equation represented by line AB and PQ. Also find the co-ordinate of intersection of line AB and PQ. [2]



OR

Find whether the given equation have $x = 2$, $y = 1$ as a solution: $x + y + 4 = 0$.

Section C

26. State whether the following statements are true or false. Give reasons for your answers. [3]

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

27. Verify: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ [3]

28. The sides of a triangle are in the ratio of 13 : 14 : 15 and its perimeter is 84 cm. Find the area of the triangle. [3]

OR

From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle.

29. Write linear equation $3x + 2y = 18$ in the form of $ax + by + c = 0$. Also write the values of a , b and c . Are $(4, 3)$ and $(1, 2)$ solution of this equation? [3]

30. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that a triangle ABC is isosceles: [3]

OR

If two isosceles triangles have a common base, prove that the line joining their vertices bisects them at right angles.

31. Draw the graphs of $y = x$ and $y = -x$ in the same graph. Also find the co-ordinates of the point where the two [3]

lines intersect.

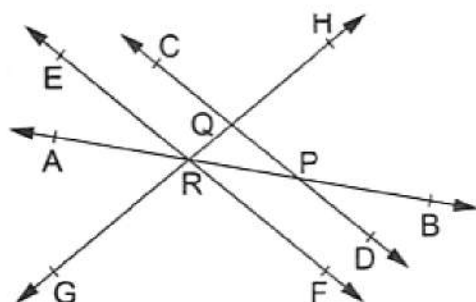
Section D

32. If x is a positive real number and exponents are rational numbers, simplify [5]
- $$\left(\frac{x^b}{x^c}\right)^{b+c-a} \cdot \left(\frac{x^c}{x^a}\right)^{c+a-b} \cdot \left(\frac{x^a}{x^b}\right)^{a+b-c}$$

OR

Represent each of the numbers $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ on the real line.

33. In the adjoining figure, name: [5]
- Two pairs of intersecting lines and their corresponding points of intersection
 - Three concurrent lines and their points of intersection
 - Three rays
 - Two line segments



34. If two parallel lines are intersected by a transversal, then prove that the bisectors of the interior angles form a rectangle. [5]

OR

Prove that if the arms of an angle are respectively perpendicular to the arms of another angle, then the angles are either equal or supplementary.

35. The heights of 75 students in a school are given below: [5]

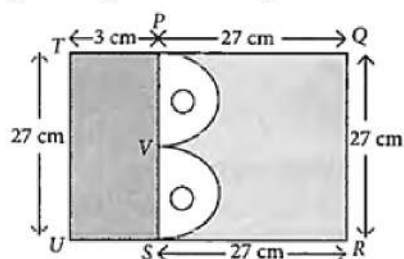
Height (in cm)	130-136	136-142	142-148	148-154	154-160	160-166
Number of students	9	12	18	23	10	3

Draw a histogram to represent the above data.

Section E

36. Read the text carefully and answer the questions: [4]

Mr. Vivekananda purchased a plot QRUT to build his house. He leaves space of two congruent semicircles for gardening and a rectangular area of breadth 3 cm for car parking.



- Find the total area of Garden.
- Find the area of rectangle left for car parking.
- Find the radius of semi-circle.

OR

Find the area of a semi-circle.

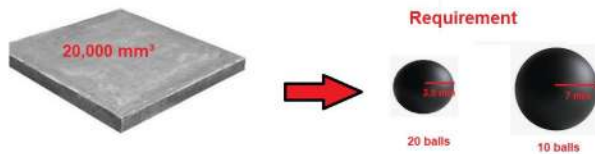
37. **Read the text carefully and answer the questions:**

[4]

In Agra in a grinding mill, there were installed 5 types of mills. These mills used steel balls of radius 5 mm, 7 mm, 10 mm, 14 mm and 16 mm respectively. All the balls were in the spherical shape.

For repairing purpose mills need 10 balls of 7 mm radius and 20 balls of 3.5 mm radius. The workshop was having 20000 mm^3 steel.

This 20000 mm^3 steel was melted and 10 balls of 7 mm radius and 20 balls of 3.5 mm radius were made and the remaining steel was stored for future use.



- (i) What was the volume of one ball of 3.5 mm radius?
- (ii) What was the surface area of one ball of 3.5 mm radius?

OR

How much steel was kept for future use?

- (iii) What was the volume of 10 balls of radius 7 mm?

38. **Read the text carefully and answer the questions:**

[4]

Harish makes a poster in the shape of a parallelogram on the topic SAVE ELECTRICITY for an inter-school competition as shown in the follow figure.



- (i) If $\angle A = (4x + 3)^\circ$ and $\angle D = (5x - 3)^\circ$, then find the measure of $\angle B$.
- (ii) If $\angle B = (2y)^\circ$ and $\angle D = (3y - 6)^\circ$, then find the value of y .

OR

If $AB = (2y - 3)$ and $CD = 5 \text{ cm}$ then what is the value of y ?

- (iii) If $\angle A = (2x - 3)^\circ$ and $\angle C = (4y + 2)^\circ$, then find how x and y relate.

Solution

CBSE SAMPLE PAPER - 02

Class 09 - Mathematics

Section A

1. (b) 9

Explanation: Given $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

Then,

$$\begin{aligned} x + y + xy &= \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} + \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \\ &= \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} + 1 \\ &= \frac{(\sqrt{5}+\sqrt{3})^2}{5-3} + \frac{(\sqrt{5}-\sqrt{3})^2}{5-3} + 1 \\ &= \frac{(\sqrt{5})^2 + (\sqrt{3})^2 + 2(\sqrt{5})(\sqrt{3})}{2} + \frac{(\sqrt{5})^2 + (\sqrt{3})^2 - 2(\sqrt{5})(\sqrt{3})}{2} + 1 \\ &= \frac{5+3+2\sqrt{15}}{2} + \frac{5+3-2\sqrt{15}}{2} + 1 \\ &= \frac{8+2\sqrt{15}}{2} + \frac{8-2\sqrt{15}}{2} + 1 \\ &= 4 + \sqrt{15} + 4 - \sqrt{15} + 1 \\ &= 8+1 \\ &= 9 \end{aligned}$$

2. (d) 1st quadrant

Explanation: The positive solutions of the equation $ax + by + c = 0$ always lie in the 1st quadrant
Because in 1st quadrant both x and y have positive value.

3. (a) $x - y = 0$

Explanation: If $(x, y) = (y, x)$,

It means abscissa = ordinate or, $x = y$

So,

$X - Y = 0$ {since $x = y$,}

4. (d) Area of the rectangle

Explanation: In Histogram each rectangle is drawn, where width equivalent to class interval and height equivalent to the frequency of the class.

Since class interval are same across the distribution table, area of the rectangle is corresponding to frequency or height of the rectangle

5. (c) $a \neq 0$ and $b \neq 0$

Explanation: A linear equation in two variables is of the form $ax + by + c = 0$ as a and b are coefficient of x and y
so if $a = 0$ and $b = 0$ or either of one is zero in that case the equation will be one variable or there will be no equation respectively.

therefore when $a \neq 0$ and $b \neq 0$ then only the equation will be in two variable

6. (c) Theorem

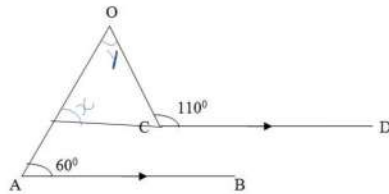
Explanation: Theorem — a mathematical statement that is proved using rigorous mathematical reasoning. In a mathematical paper, the term theorem is often reserved for the most important results.

Axiom/Postulate — a statement that is assumed to be true without proof. These are the basic building blocks from which all theorems are proved (Euclid's five postulates, Zermelo-Fraenkel axioms, Peano axioms).

Definition — a precise and unambiguous description of the meaning of a mathematical term. It characterizes the meaning of a word by giving all the properties and only those properties that must be true.

7. (a) 50°

Explanation:



Construction: Extend the line CD such that it intersect AO and is parallel to AB

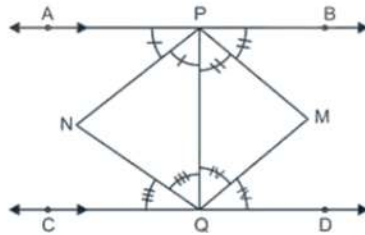
$x = 60^\circ$ (Corresponding angles)

$x + y + 180^\circ - 110^\circ = 180^\circ$ (Angle sum property)

$y = 110^\circ - 60^\circ = 50^\circ$

8. (d) a rectangle

Explanation:



PNQM is a rectangle.

9. (a) 8

Explanation: We have,

$$= (x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = ky^3$$

$$= (x + y - x + y)^3 + 3(x + y)(x - y)(x + y - x + y) - 6y(x^2 - y^2) = ky^3$$

$$= 2y^3 + 6y(x^2 - y^2) - 6y(x^2 - y^2) = ky^3$$

$$= 8y^3 = ky^3$$

$$= k = 8$$

10. (a) Parallel to x-axis at a distance 6 units from the origin

Explanation: As $y = a$ is an equation of a line parallel to x-axis at a distance of a units from the origin.

11. (b) RHS

Explanation: In $\triangle ABC$ and $\triangle BAD$, $\angle BAC = \angle ABD$

$\angle BAD$, we have (Right angles)

$BC = AD$ (Hypotenuses and Given)

$AB = AB$ (common in both)

Hence, $\triangle ABC \cong \triangle BAD$ by RHS criterion.

12. (b) $AC^2 + BD^2 = 4AB^2$

Explanation: ABCD is a rhombus.

$AB = BC = CD = DA$

In Rhombus, diagonals bisect each other at right angles.

So, $AO = CO$ and $BO = DO$

In triangle AOB, $AO^2 + BO^2 = AB^2$ (Pythagoras theorem)

$$(1/2 AC)^2 + (1/2 BD)^2 = AB^2$$

$$AC^2/4 + BD^2/4 = AB^2$$

$$AC^2 + BD^2 = 4 AB^2$$

13. (b) 60°

Explanation: We know that the angle at the centre of a circle is twice the angle at any point on the remaining part of the circumference. angles $\angle AOB$ and $\angle ACB$ are on the same arc AB.

Thus, $\angle AOB = (2 \times \angle ACB) = (2 \times 30^\circ) = 60^\circ$

14. (d) $\frac{7}{9}$

Explanation: $0.\overline{3} + 0.\overline{4}$

$$= 0.\overline{7} = \frac{7}{9}$$

15. (d) $x^4 - \frac{1}{x^4}$

Explanation: $\left(x - \frac{1}{x}\right) \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2}\right)$

$$= \left(x^2 - \frac{1}{x^2}\right) \left(x^2 + \frac{1}{x^2}\right) \text{ [Using identity } (a+b)(a-b) = a^2 - b^2]$$

$$= x^4 - \frac{1}{x^4} \text{ [Using identity } (a+b)(a-b) = a^2 - b^2]$$

16. (a) 115°

Explanation: In $\triangle ABC$ we have:

$$\angle A + \angle B + \angle C = 180^\circ \text{ [Sum of the angles of a triangle]}$$

$$\Rightarrow 50^\circ + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle B + \angle C = 130^\circ$$

$$\Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C = 65^\circ \dots(i)$$

In $\triangle OBC$, we have :

$$\angle OBC + \angle OCB + \angle BOC = 180^\circ$$

$$\Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C + \angle BOC = 180^\circ \text{ [Using (i)]}$$

$$\Rightarrow 65^\circ + \angle BOC = 180^\circ$$

$$\Rightarrow \angle BOC = 115^\circ$$

17. (d) 52

Explanation: $\left(x^4 + \frac{1}{x^4}\right) = 194$

$$\Rightarrow \left(x^2\right)^2 + \left(\frac{1}{x^2}\right)^2 + 2 \times x^2 \times \frac{1}{x^2} = 194 + 2 \times x^2 \times \frac{1}{x^2}$$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 196$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \sqrt{196} = 14$$

Now,

$$\Rightarrow \left(x^2\right) + \left(\frac{1}{x^2}\right) + 2 \times x \times \frac{1}{x} = 14 + 2 \times x \times \frac{1}{x}$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 16$$

$$\Rightarrow x + \frac{1}{x} = \sqrt{16} = 4$$

$$\text{Now, } \left(x + \frac{1}{x}\right)^3 = (4)^3$$

$$\Rightarrow \left(x\right)^3 + \left(\frac{1}{x}\right)^3 + 3 \times x \times \frac{1}{x} \left(x + \frac{1}{x}\right) = 64$$

$$\Rightarrow \left(x^3\right) + \left(\frac{1}{x^3}\right) + 3(4) = 64$$

$$\Rightarrow \left(x^3\right) + \left(\frac{1}{x^3}\right) = 64 - 12 = 52$$

18. (c) 1 : 3

Explanation: Let r be the radius of cylinder and cone and volumes are equal and h_1 and h_2 be their heights respectively

$$\therefore \text{Volume of cylinder} = \pi r h_1$$

$$\text{and volume of cone} = \frac{1}{3} \pi r^2 h_2$$

$$\therefore \pi r^2 h_1 = \frac{1}{3} \pi r^2 h_2$$

$$\Rightarrow h_1 = \frac{1}{3} h_2$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{1}{3}$$

$$\therefore h_1 : h_2 = 1 : 3$$

19. (d) A is false but R is true.

$$\text{Explanation: } s = \frac{6+6+6}{2} = \frac{18}{2} = 9 \text{ cm}$$

$$\text{Area} = \sqrt{9(9-6)(9-6)(9-6)}$$

$$= \sqrt{9 \times 3 \times 3 \times 3} = 9\sqrt{3} \text{ cm}^2$$

20. (c) A is true but R is false.

Explanation: $\left(-\frac{3}{2}, k\right)$ is a solution of $2x + 3 = 0$

$$2 \times \left(-\frac{3}{2}\right) + 3 = -3 + 3 = 0$$

$\left(-\frac{3}{2}, k\right)$ is the solution of $2x + 3 = 0$ for all values of k .

Also $ax + b = 0$ can be expressed as a linear equation in two variables as $ax + 0 \cdot y + b = 0$.

Section B

21. Length of equal sides of isosceles triangle = $b = 15$ cm

And the length of remaining side = $a = 12$ cm

$$\begin{aligned} \text{Area of isosceles triangle} &= \frac{a}{4} \sqrt{4 \times b^2 - a^2} = \frac{12}{4} \sqrt{4 \times 15^2 - 12^2} = \frac{12}{4} \sqrt{900 - 144} \\ &= 3\sqrt{756} = 3 \times 6\sqrt{21} = 18\sqrt{21} \text{ cm}^2 \end{aligned}$$

Therefore area of isosceles triangle is $18\sqrt{21} \text{ cm}^2$.

22. $9x^2 + 4y^2 + 16z^2 + 12xy - 16yz - 24xz$

$$= (3x)^2 + (2y)^2 + (-4z)^2 + 2(3x)(2y) + 2(2y)(-4z) + 2(-4z)(3x)$$

$$= \{3x + 2y + (-4z)\}^2 [\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2]$$

$$= (3x + 2y - 4z)^2 = (3x + 2y - 4z)(3x + 2y - 4z)$$

23. Suppose that the radii of the spheres are r and R .

We have:

$$\frac{4\pi r^2}{4\pi R^2} = \frac{1}{4}$$

$$\Rightarrow \frac{r}{R} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\text{Now, ratio of the volumes} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \left(\frac{r}{R}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Therefore, The ratio of the volumes of the spheres is $1 : 8$.

24. $g(x) = 0 \Rightarrow 1 - \frac{3}{2}x = 0; x = \frac{2}{3}$

$$\begin{aligned} \text{Remainder} &= p\left(\frac{2}{3}\right) = \frac{8}{27} - \frac{24}{9} + \frac{4}{3} - 4 \\ &= \frac{8 - 72 + 36 - 108}{27} = \frac{-136}{27} \end{aligned}$$

OR

The given expression is $\sqrt{2}x^2 + 9x + 4\sqrt{2}$

Here, $\sqrt{2} \times 4\sqrt{2} = 8$

We split 9 into two parts whose sum is 9 and product 8.

Clearly, $(8 + 1) = 9$ and $(8 \times 1) = 8$

$$\therefore \sqrt{2}x^2 + 9x + 4\sqrt{2} = \sqrt{2}x^2 + 8x + x + 4\sqrt{2}$$

$$= \sqrt{2}x(x + 4\sqrt{2}) + (x + 4\sqrt{2})$$

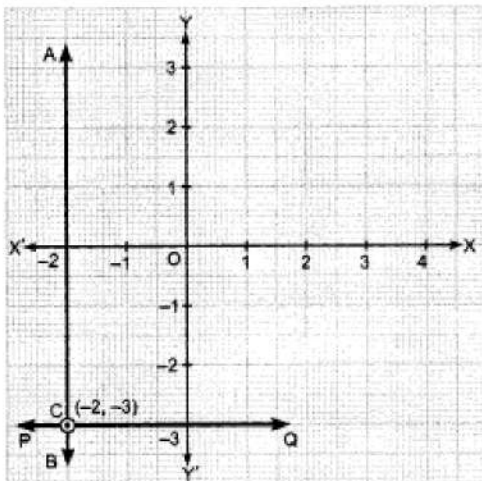
$$= (x + 4\sqrt{2})(\sqrt{2}x + 1)$$

$$\text{Hence, } (\sqrt{2}x^2 + 9x + 4\sqrt{2}) = (x + 4\sqrt{2})(\sqrt{2}x + 1)$$

This is the required factorisation.

25. $AB \Rightarrow x = -2$

$$PQ \Rightarrow y = -3$$



Point of intersection of AB and PQ is $C(-2, -3)$.

OR

For $x = 2, y = 1$

$$x + y + 4 = 0$$

$$\text{L.H.S.} = x + y + 4$$

$$= 2 + 1 + 4 = 7$$

$$\neq \text{R.H.S.}$$

$\therefore x = 2, y = 1$ is not a solution of $x + y + 4 = 0$.

Section C

26. (i) Consider the whole numbers and natural numbers separately.

We know that whole number series is $0, 1, 2, 3, 4, 5, \dots$

We know that natural number series is $1, 2, 3, 4, 5, \dots$

So, we can conclude that every number of the natural number series lie in the whole number series.

Therefore, we conclude that, yes every natural number is a whole number.

(ii) Consider the integers and whole numbers separately.

We know that integers are those numbers that can be written in the form of $\frac{p}{q}$, where $q = 1$

Now, considering the series of integers, we have $\dots -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots$

We know that whole number series is $0, 1, 2, 3, 4, 5, \dots$

We can conclude that all the numbers of whole number series lie in the series of integers. But every number of series of integers does not appear in the whole number series.

Therefore, we conclude that every integer is not a whole number.

(iii) Consider the rational numbers and whole numbers separately.

We know that rational numbers are the numbers that can be written in the form $\frac{p}{q}$ where $q \neq 0$

We know that whole number series is $0, 1, 2, 3, 4, 5, \dots$

We know that every number of whole number series can be written in the form of $\frac{p}{q}$ as

$$\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}, \dots$$

We conclude that every number of the whole number series is a rational number. But, every rational number does not appear in the whole number series. like $\frac{2}{3}, \frac{5}{6}$

Therefore, we conclude that every rational number is not a whole number.

$$27. (i) x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$\text{We know that } (x + y)^3 = x^3 + y^3 + 3xy(x + y)$$

$$\Rightarrow x^3 + y^3 = (x + y)^3 - 3xy(x + y)$$

$$= (x + y) \left[(x + y)^2 - 3xy \right]$$

$$\therefore \text{ We know that } (x + y)^2 = x^2 + 2xy + y^2$$

$$\therefore x^3 + y^3 = (x + y)(x^2 + 2xy + y^2 - 3xy)$$

$$= (x + y)(x^2 - xy + y^2)$$

Therefore, the desired result has been verified.

28. Perimeter = 84 cm.

$$\text{Ratio of sides} = 13 : 14 : 15$$

$$\text{Sum of the ratios} = 13 + 14 + 15 = 42$$

$$\therefore \text{ One side (a)} = \frac{13}{42} \times 84 = 26 \text{ cm.}$$

$$\text{Second side (b)} = \frac{14}{42} \times 84 = 28 \text{ cm.}$$

$$\text{Third side (c)} = \frac{15}{42} \times 84 = 30 \text{ cm}$$

$$\therefore s = \frac{a+b+c}{2}$$

$$= \frac{26+28+30}{2} = \frac{84}{2} = 42 \text{ cm}$$

\therefore Area of the triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{42(42-26)(42-28)(42-30)}$$

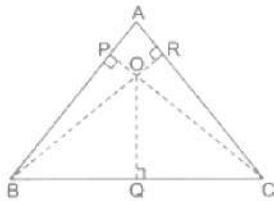
$$= \sqrt{42(16)(14)(12)}$$

$$= \sqrt{42(16)(14)(4 \times 3)}$$

$$= (42)(4)(2) = 336 \text{ cm}^2$$

OR

Let ABC be an equilateral triangle, O be the interior point and OQ, OR and OC are the perpendicular drawn from points O. Let the sides of an equilateral triangle be a m.



$$\text{Area of } \triangle AOB = \frac{1}{2} \times AB \times OP$$

$$[\because \text{Area of a triangle} = \frac{1}{2} \times (\text{base} \times \text{height})]$$

$$= \frac{1}{2} \times a \times 14 = 7a \text{ cm}^2 \dots(1)$$

$$\text{Area of } \triangle OBC = \frac{1}{2} \times BC \times OQ = \frac{1}{2} \times a \times 10$$

$$= 5a \text{ cm}^2 \dots(2)$$

$$\text{Area of } \triangle OAC = \frac{1}{2} \times AC \times OR = \frac{1}{2} \times a \times 6$$

$$= 3a \text{ cm}^2 \dots(3)$$

$$\therefore \text{Area of an equilateral } \triangle ABC$$

$$= \text{Area of } (\triangle OAB + \triangle OBC + \triangle OAC)$$

$$= (7a + 5a + 3a) \text{ cm}^2$$

$$= 15a \text{ cm}^2 \dots(4)$$

$$\text{We have, semi-perimeter } s = \frac{a+a+a}{2}$$

$$\Rightarrow s = \frac{3a}{2} \text{ cm}$$

$$\therefore \text{Area of an equilateral } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)} \quad [\text{By Heron's formula}]$$

$$= \sqrt{\frac{3a}{2} \left(\frac{3a}{2} - a \right) \left(\frac{3a}{2} - a \right) \left(\frac{3a}{2} - a \right)}$$

$$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}}$$

$$= \frac{\sqrt{3}}{4} a^2 \dots(5)$$

From equations (4) and (5), we get

$$\frac{\sqrt{3}}{4} a^2 = 15a$$

$$\Rightarrow a = \frac{15 \times 4}{\sqrt{3}} = \frac{60}{\sqrt{3}}$$

$$\Rightarrow a = \frac{60}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3} \text{ cm}$$

On putting $a = 20\sqrt{3}$ in equation (5), we get

$$\text{Area of } \triangle ABC = \frac{\sqrt{3}}{4} (20\sqrt{3})^2 = \frac{\sqrt{3}}{4} \times 400 \times 3 = 300\sqrt{3} \text{ cm}^2$$

Hence, the area of an equilateral triangle is $300\sqrt{3} \text{ cm}^2$.

29. We have the equation as $3x + 2y = 18$

In standard form

$$3x + 2y - 18 = 0$$

$$\text{Or } 3x + 2y + (-18) = 0$$

But standard linear equation is

$$ax + by + c = 0$$

On comparison we get, $a = 3$, $b = 2$, $c = -18$

If (4, 3) lie on the line, i.e., solution of the equation $\text{LHS} = \text{RHS}$

$$\therefore 3(4) + 2(3) = 18$$

$$12 + 6 = 18$$

$$18 = 18$$

As $\text{LHS} = \text{RHS}$, Hence (4, 3) is the solution of given equation.

Again for (1,2)

$$3x + 2y = 18$$

$$\therefore 3(1) + 2(2) = 18$$

$$3 + 4 = 18$$

$$7 = 18$$

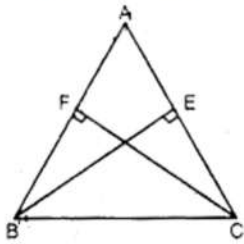
LHS \neq RHS

Hence (1, 2) is not the solution of given equation.

Therefore (4,3) is the point where the equation of the line $3x + 2y = 18$ passes through where as the line for the equation $3x + 2y = 18$ does not pass through the point (1,2).

30. In $\triangle BEC$ and $\triangle CFB$,

$\angle BEC = \angle CFB$ [Each 90°]



$BC = BC$ [Common]

$BE = CF$ [Given]

$\therefore \triangle BEC \cong \triangle CFB$ [RHS congruency]

$\Rightarrow EC = FB$ [By C.P.C.T.] ... (i)

Now In $\triangle AEB$ and $\triangle AFC$

$\angle AEB = \angle AFC$ [Each 90°]

$\angle A = \angle A$ [Common]

$BE = CF$ [Given]

$\therefore \triangle AEB \cong \triangle AFC$ [ASA congruency]

$\Rightarrow AE = AF$ [By C.P.C.T.] ... (ii)

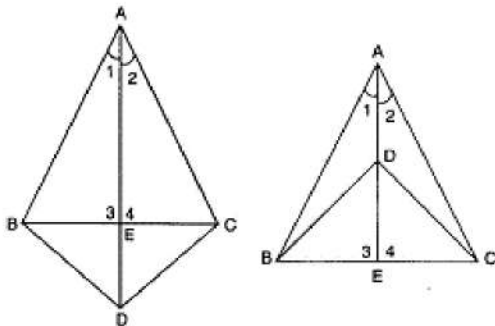
Adding eq. (i) and (ii), we get,

$EC + AE = FB + AF \Rightarrow AB = AC$

$\Rightarrow ABC$ is an isosceles triangle.

OR

In $\triangle ABD$ and $\triangle ACD$



$AB = AC, BD = CD$... [Given]

$AD = AD$... [Common]

$\therefore \triangle ABD \cong \triangle ACD$... [SSS axiom]

$\therefore \angle 1 = \angle 2$... [c.p.c.t.]

In $\triangle ABE$ and $\triangle ACE$,

$AB = AC$... [Given]

$AE = AE$... [Common]

$\angle 1 = \angle 2$... [As proved above]

$\therefore \triangle ABE \cong \triangle ACE$... [SAS axiom]

$\therefore BE = CE$... [c.p.c.t.]

and $\angle 3 = \angle 4$... [c.p.c.t.]

But $\angle 3 + \angle 4 = 180^\circ$... [Linear pair axiom]

$\therefore \angle 3 = \angle 4 = 90^\circ$

Hence, AD bisects BC at right angles.

31. $y = x$

We have, $y = x$

$$\text{Let } x = 1 : y = 1$$

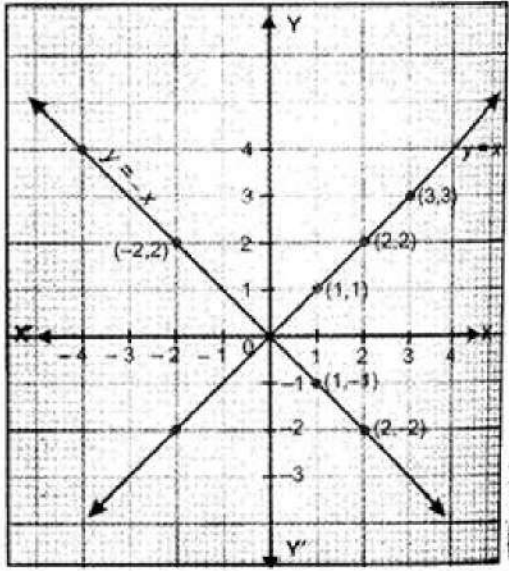
$$\text{Let } x = 2 : y = 2$$

$$\text{Let } x = 3 : y = 3$$

Thus, we have the following table :

x	1	2	3
y	1	2	3

By plotting the points (1, 1), (2, 2) and (3, 3) on the graph paper and joining them by a line, we obtain the graph of $y = x$.



$$y = -x$$

We have, $y = -x$

$$\text{Let } x = 1 : y = -1$$

$$\text{Let } x = 2 : y = -2$$

$$\text{Let } x = -2 : y = -(-2) = 2$$

Thus, we have the following table exhibiting the abscissa and ordinates of the points of the line represented by the equation $y = -x$.

x	1	2	-2
y	-1	-2	2

Now, plot the points (1, -1), (2, -2) and (-2, 2) and join them by a line to obtain the line represented by the equation $y = -x$.

The graphs of the lines $y = x$ and $y = -x$ are shown in figure.

Two lines intersect at O (0, 0).

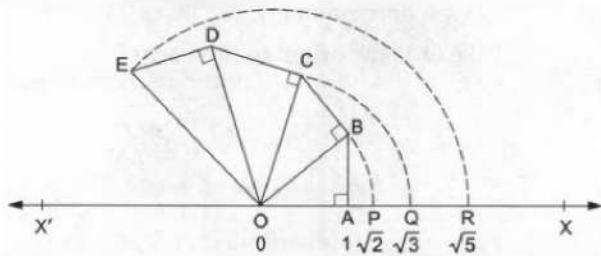
Section D

32. Given.

$$\begin{aligned}
 & \left(\frac{x^b}{x^c} \right)^{b+c-a} \cdot \left(\frac{x^c}{x^a} \right)^{c+a-b} \cdot \left(\frac{x^a}{x^b} \right)^{a+b-c} \\
 &= \left(\frac{x^{b^2+bc-ab}}{x^{bc+c^2-ac}} \right) \cdot \left(\frac{x^{c^2+ac-bc}}{x^{ac+a^2-ab}} \right) \cdot \left(\frac{x^{a^2+ab-ac}}{x^{ab+b^2-bc}} \right) \\
 &= \left(x^{b^2+bc-ab-bc-c^2+ac} \right) \left(x^{c^2+ac-bc-ac-a^2+ab} \right) \left(x^{a^2+ab-ac-ab-b^2+bc} \right) \\
 &= \left(x^{b^2-ab-c^2+ac} \right) \left(x^{c^2-bc-a^2+ab} \right) \left(x^{a^2-ac-b^2+bc} \right) \\
 &= x^{b^2-ab-c^2+ac+c^2-bc-a^2+ab+a^2-ac-b^2+bc} \\
 &= x^0 \\
 &= 1
 \end{aligned}$$

OR

Let $X'OX$ be a horizontal line, taken as the x-axis and let O be the origin. Let O represent 0.



Take $OA = 1$ unit and draw $AB \perp OA$ such that $AB = 1$ unit.

Join OB . Then, by Pythagoras Theorem

$$OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2 + 1^2} = \sqrt{2} \text{ units}$$

With O as centre and OB as radius, draw an arc, meeting OX at P .

Then, $OP = OB = \sqrt{2}$ units

Thus, the point P represents $\sqrt{2}$ on the real line.

Now, draw $BC \perp OB$ such that $BC = 1$ unit.

Join OC . Then by Pythagoras Theorem

$$OC = \sqrt{OB^2 + BC^2} = \sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3} \text{ units}$$

With O as centre and OC as radius, draw an arc, meeting OX at Q . Then,

$$OQ = OC = \sqrt{3} \text{ units}$$

Thus, the point Q represents $\sqrt{3}$ on the real line

Now, draw $CD \perp OC$ such that $CD = 1$ unit.

Join OD . Then, by Pythagoras Theorem

$$OD = \sqrt{OC^2 + CD^2} = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2 \text{ units}$$

Now, draw $DE \perp OD$ such that $DE = 1$ unit.

Join OE . Then,

$$OE = \sqrt{OD^2 + DE^2} = \sqrt{2^2 + 1^2} = \sqrt{5} \text{ units.}$$

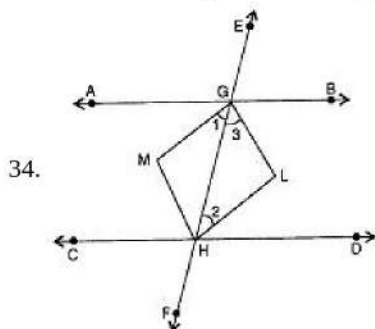
With O as centre and OE as radius, draw an arc, meeting OX

at R . Then, $OR = OE = \sqrt{5}$ units.

Thus, the point R represents $\sqrt{5}$ on the real line.

Hence, the points P, Q, R represent the numbers $\sqrt{2}, \sqrt{3}$ and $\sqrt{5}$ respectively.

33. i. $\overleftrightarrow{EF}, \overleftrightarrow{GH}$ and their corresponding point of intersection is R .
 $\overleftrightarrow{AB}, \overleftrightarrow{CD}$ and their corresponding point of intersection is P .
 ii. $\overleftrightarrow{AB}, \overleftrightarrow{EF}, \overleftrightarrow{GH}$ and their point of intersection is R .
 iii. Three rays are: $\overrightarrow{RB}, \overrightarrow{RH}, \overrightarrow{RG}$
 iv. Two line segments are: $\overline{RQ}, \overline{RP}$.



34.

as, $AB \parallel CD$ and EF cuts them

$\therefore \angle AGH = \angle GHD$ (Alternate Angles)

$$\Rightarrow \frac{1}{2} \angle AGH = \frac{1}{2} \angle GHD$$

$$\Rightarrow \angle 1 = \angle 2 \dots\dots (1)$$

But these angles form a pair of equal alternate angles for lines GM and HL and transversal GH .

$$\therefore GM \parallel HL \dots\dots (2)$$

Similarly, we can prove that

HM \parallel GL (3)

In view of (2) and (3),

GLHM is a parallelogram

AB \parallel CD and EF cuts them

$$\therefore \angle BGH + \angle GHD = 180^\circ$$

(The sum of the interior angles on the same side of a transversal is 180°)

$$\Rightarrow \frac{1}{2} \angle BGH + \frac{1}{2} \angle GHD = 90^\circ$$

$$\Rightarrow \angle 3 + \angle 2 = 90^\circ$$

In $\triangle GHL$,

$$\angle 3 + \angle 2 + \angle GLH = 180^\circ$$

(The sum of the three angles of a triangle is 180°)

$$\Rightarrow 90^\circ + \angle GLH = 180^\circ \text{ From (4)}$$

$$\Rightarrow \angle GLH = 180^\circ - 90^\circ = 90^\circ$$

\Rightarrow One angle of parallelogram GLHM is a right angle.

\Rightarrow Parallelogram GLHM is a rectangle.

OR

i. In $\triangle BOD$,

$$\angle OBD + \angle BOD + \angle ODB = 180^\circ$$

(The sum of the three angles of a triangle is 180°)

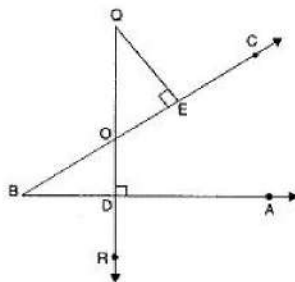
$$\Rightarrow \angle OBD + \angle BOD + 90^\circ = 180^\circ$$

$$\Rightarrow \angle OBD + \angle BOD = 90^\circ \text{ (1)}$$

In $\triangle OEQ$,

$$\angle EQO + \angle QOE + \angle OEQ = 180^\circ \text{ (2)}$$

(The sum of the three angles of a triangle is 180°)



$$\Rightarrow \angle EQO + \angle QOE + 90^\circ = 180^\circ$$

$$\Rightarrow \angle EQO + \angle QOE = 90^\circ \text{ (2)}$$

From (1) and (2), we get

$$\angle OBD + \angle BOD = \angle EQO + \angle QOE$$

But $\angle BOD = \angle QOE$ (Vertically Opposite Angles)

$$\therefore \angle OBD = \angle EQO$$

ii. Join BQ

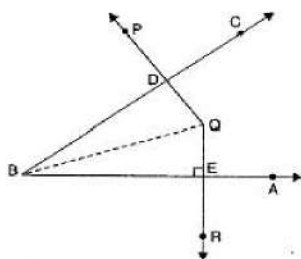
In $\triangle BDQ$,

$$\angle DBQ + \angle BQD + \angle QDB = 180^\circ$$

(The sum of the three angles of a triangle is 180°)

$$\Rightarrow \angle DBQ + \angle BQD + 90^\circ = 180^\circ$$

$$\Rightarrow \angle DBQ + \angle BQD = 90^\circ \text{ (1)}$$



In $\triangle BQE$,

$$\angle EBQ + \angle BQE + \angle BEQ = 180^\circ$$

(The sum of the three angles of a triangle is 180°)

$$\Rightarrow \angle EBQ + \angle BQE + 90^\circ = 180^\circ$$

$$\Rightarrow \angle EBQ + \angle BQE = 90^\circ$$

Adding (1) and (2), we get

$$(\angle DBQ + \angle EBQ) + (\angle BQD + \angle BQE) = 90^\circ + 90^\circ = 180^\circ$$

$$\Rightarrow \angle DBE + \angle EQD = 180^\circ$$

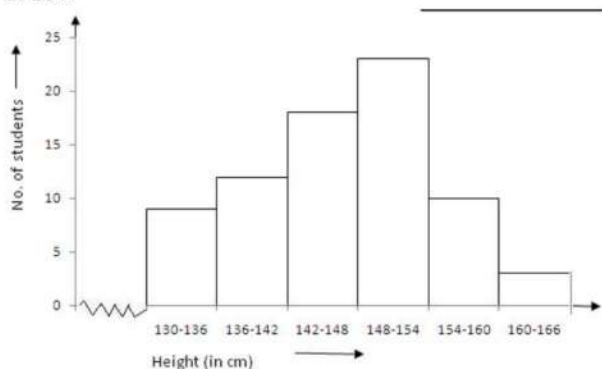
$\Rightarrow \angle DBE$ and $\angle EQD$ are supplementary.

35. Height (in cm)	130-136	136-142	142-148	148-154	154-160	160-166
Number of students	9	12	18	23	10	3

Clearly, the given frequency distribution is in the exclusive form.

We take class intervals, i.e. height (in cm) along x-axis and frequencies i.e. number of students along y-axis. So we get the required histogram.

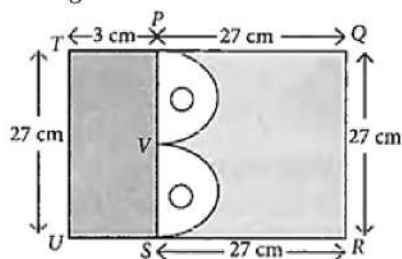
Since the scale on X-axis starts at 130, a kink(break) is indicated near the origin to show that the graph is drawn to scale beginning at 130.



Section E

36. Read the text carefully and answer the questions:

Mr. Vivekananda purchased a plot QRUT to build his house. He leaves space of two congruent semicircles for gardening and a rectangular area of breadth 3 cm for car parking.



(i) Area of Garden is $= 2 \times$ semicircles

$$\text{Area of a semi-circle} = 2 \times \frac{1}{2} \pi r^2$$

$$= \frac{22}{7} \times 6.75 \times 6.75 = 144.43 \text{ cm}^2$$

(ii) Area of rectangle left for car parking is area of region PSUT $= 27 \times 3 = 81 \text{ cm}^2$

(iii) Diameter of semi-circle $= PV = \frac{PS}{2} = \frac{27}{2} = 13.5 \text{ cm}$

$$\therefore \text{Radius of semi-circle} = \frac{13.5}{2} = 6.75 \text{ cm}$$

OR

$$\text{Diameter of semi-circle} = PV = \frac{PS}{2} = \frac{27}{2} = 13.5 \text{ cm}$$

$$\therefore \text{Radius of semi-circle} = \frac{13.5}{2} = 6.75 \text{ cm}$$

$$\text{Area of a semi-circle} = \frac{1}{2} \pi r^2$$

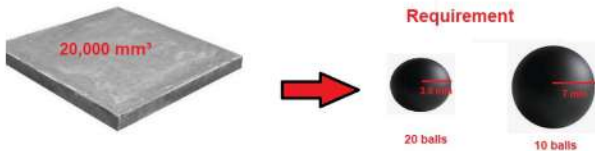
$$= \frac{1}{2} \times \frac{22}{7} \times 6.75 \times 6.75 = 71.59 \text{ cm}^2$$

37. Read the text carefully and answer the questions:

In Agra in a grinding mill, there were installed 5 types of mills. These mills used steel balls of radius 5 mm, 7 mm, 10 mm, 14 mm and 16 mm respectively. All the balls were in the spherical shape.

For repairing purpose mills need 10 balls of 7 mm radius and 20 balls of 3.5 mm radius. The workshop was having 20000 mm³ steel.

This 20000 mm³ steel was melted and 10 balls of 7 mm radius and 20 balls of 3.5 mm radius were made and the remaining steel was stored for future use.



- (i) The radius of the ball = 3.5 mm

Volume of the ball

$$= \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5$$

$$= 179.66 \text{ mm}^3$$

- (ii) Radius of one ball = 3.5 cm

The surface area of one ball

$$= 4\pi r^2$$

$$= 4 \times \frac{22}{7} \times 3.5 \times 3.5$$

$$= 154 \text{ mm}^2$$

OR

$$\text{Volume of 10 balls of 7 mm} = 14373.3 \text{ mm}^3$$

$$\text{Volume of 1 ball of 3.5 mm} = 179.66 \text{ mm}^3$$

$$\text{Volume of 20 balls of 3.5 mm} = 179.66 \times 20 = 3593.33 \text{ mm}^3$$

$$\text{Total steel required to be melted} = 14373.3 + 3593.33 = 17966 \text{ mm}^3 (\text{Approx})$$

$$\text{Thus steel left over} = 20,000 - 17966 = 2034 \text{ mm}^3$$

- (iii) Radius of one ball = 7 cm

Thus volume of 10 balls of radius 7 mm

$$= 10 \times \frac{4}{3} \pi r^3$$

$$= 10 \times \frac{4}{3} \times \frac{22}{7} \times 7^3$$

$$= 14373.3 \text{ mm}^3$$

38. Read the text carefully and answer the questions:

Harish makes a poster in the shape of a parallelogram on the topic SAVE ELECTRICITY for an inter-school competition as shown in the follow figure.



(i) Since, ABCD is a parallelogram.

$\angle A + \angle D = 180^\circ$ (adjacent angles of a quadrilateral are equal)

$$(4x + 3)^\circ + (5x + 3)^\circ = 180^\circ$$

$$9x = 180^\circ$$

$$x = 20$$

$$\angle D = (5x - 3)^\circ = 97^\circ$$

$\angle D = \angle B$ (opposite angles of a parallelogram are equal)

$$\text{Thus, } \angle B = 97^\circ$$

(ii) $\angle B = \angle D$ (opposite angles of a parallelogram are equal)

$$\Rightarrow 2y = 3y - 6$$

$$\Rightarrow 2y - 3y = -6$$

$$\Rightarrow -y = -6$$

$$\Rightarrow y = 6$$

OR

$$AB = CD$$

$$\Rightarrow 2y - 3 = 5$$

$$\Rightarrow 2y = 8$$

$$\Rightarrow y = 4$$

(iii) $\angle A = \angle C$ (opposite angles of a parallelogram are equal)

$$\Rightarrow 2x - 3 = 4y + 2$$

$$\Rightarrow 2x = 4y + 5$$

$$\Rightarrow x = 2y + \frac{5}{2}$$