

CHAPTER

1.8

CIRCUIT ANALYSIS IN THE S-DOMAIN

1. $Z(s) = ?$

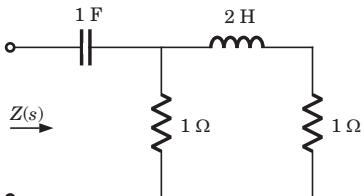


Fig. P1.8.1

- (A) $\frac{s^2 + 1.5s + 1}{s(s + 1)}$
 (B) $\frac{s^2 + 3s + 1}{s(s + 1)}$
 (C) $\frac{2s^2 + 3s + 2}{s(s + 1)}$
 (D) $\frac{2s^2 + 3s + 1}{2s(s + 1)}$

(A) $\frac{s^2 + 1}{s^2 + 2s + 1}$

(B) $\frac{2(s^2 + 1)}{(s + 1)^2}$
 (C) $\frac{2s^2 + 1}{s^2 + 2s + 2}$
 (D) $\frac{s^2 + 1}{3s + 2}$

2. $Z(s) = ?$

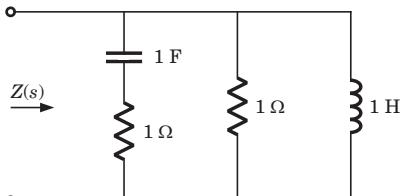


Fig. P1.8.2

- (A) $\frac{s^2 + s + 1}{s(s + 1)}$
 (B) $\frac{2s^2 + s + 1}{s(s + 1)}$
 (C) $\frac{s(s + 1)}{2s^2 + s + 1}$
 (D) $\frac{s(s + 1)}{s^2 + s + 1}$

(A) $\frac{3s^2 + 8s + 7}{s(5s + 6)}$

(B) $\frac{s(5s + 6)}{3s^2 + 8s + 7}$
 (C) $\frac{3s^2 + 7s + 6}{s(5s + 6)}$
 (D) $\frac{s(5s + 6)}{3s^2 + 7s + 6}$

3. $Z(s) = ?$

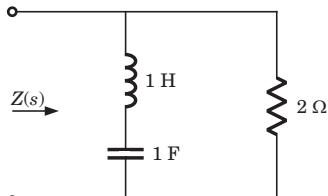


Fig. P1.8.3

4. $Z(s) = ?$

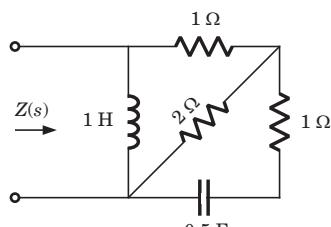


Fig. P1.8.4

(A) $\frac{3s^2 + 8s + 7}{s(5s + 6)}$

(B) $\frac{s(5s + 6)}{3s^2 + 8s + 7}$
 (C) $\frac{3s^2 + 7s + 6}{s(5s + 6)}$
 (D) $\frac{s(5s + 6)}{3s^2 + 7s + 6}$

5. The s -domain equivalent of the circuit of Fig. P1.8.5, is

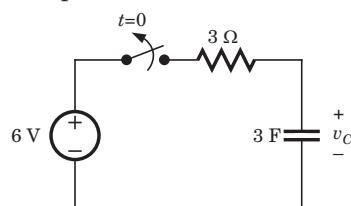
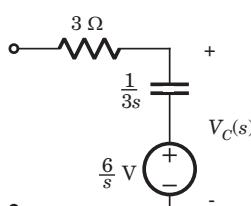
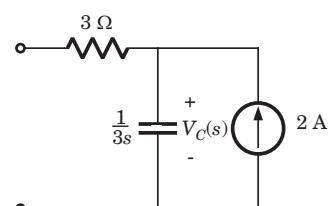


Fig. P1.8.5



(A)



(B)

(C) Both A and B

(D) None of these

6. The s -domain equivalent of the circuit shown in Fig. P1.8.6 is

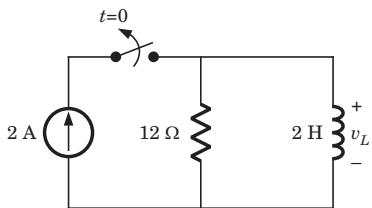


Fig. P1.8.6

- (A) (B) (C) Both A and B (D) None of these

Statement for Q.7-8:

The circuit is as shown in fig. P1.8.7–8. Solve the problem and choose correct option.

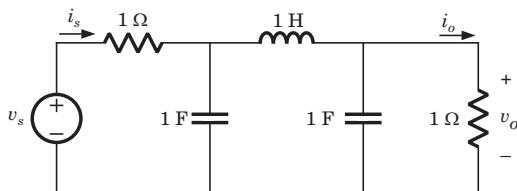


Fig. P1.8.7-8

7. $H_1(s) = \frac{V_o(s)}{V_s(s)} = ?$

- (A) $s(s^3 + 2s^2 + 3s + 1)^{-1}$
 (B) $(s^3 + 3s^2 + 2s + 1)^{-1}$
 (C) $(s^3 + 2s^2 + 3s + 2)^{-1}$
 (D) $s(s^3 + 3s^2 + 2s^2 + 2)^{-1}$

8. $H_2(s) = \frac{I_o(s)}{V_s(s)} = ?$

- (A) $\frac{-s}{(s^3 + 3s^2 + 2s + 1)}$
 (B) $-(s^3 + 3s^2 + 2s + 1)^{-1}$
 (C) $\frac{-s}{(s^3 + 2s^2 + 3s + 1)}$
 (D) $(s^3 + 2s^2 + 3s + 2)^{-1}$

9. For the network shown in fig. P1.8.9 voltage ratio transfer function G_{12} is

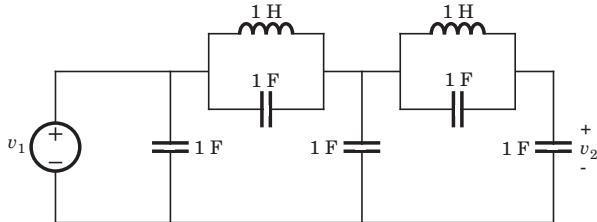


Fig. P1.8.9

(A) $\frac{(s^2 + 2)}{5s^4 + 5s^2 + 1}$

(C) $\frac{(s^2 + 2)^2}{5s^4 + 5s^2 + 1}$

(B) $\frac{s^2 + 1}{5s^4 + 5s^2 + 1}$

(D) $\frac{(s^2 + 1)^2}{5s^4 + 5s^2 + 1}$

10. For the network shown in fig. P1.8.10, the admittance transfer function is

$$Y_{12} = \frac{K(s+1)}{(s+2)(s+4)}$$

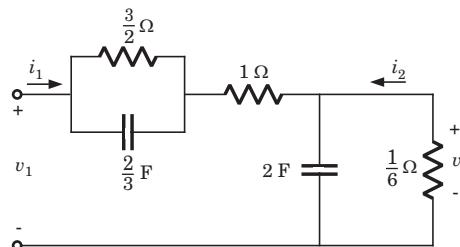


Fig. P1.8.10

The value of K is

- (A) -3 (B) 3
 (C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

11. In the circuit of fig. P1.8.11 the switch is in position 1 for a long time and thrown to position 2 at $t=0$. The equation for the loop currents $I_1(s)$ and $I_2(s)$ are

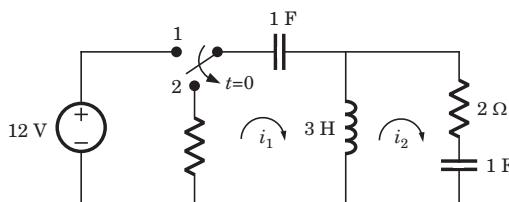


Fig. P1.8.11

(A)
$$\begin{bmatrix} 2 + 3s + \frac{1}{s} & -3s \\ -3s & 2 + \frac{1}{s} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{12}{s} \\ 0 \end{bmatrix}$$

- 18.** The driving point impedance $Z(s)$ of a network has the pole zero location as shown in Fig. P1.8.18. If $Z(0) = 3$, the $Z(s)$ is

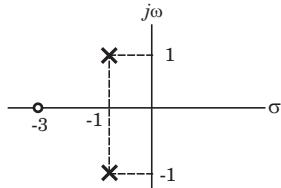


Fig. P1.8.18

- (A) $\frac{4(s+3)}{s^2+s+1}$ (B) $\frac{2(s+3)}{s^2+2s+2}$
 (C) $\frac{2(s+3)}{s^2+2s+2}$ (D) $\frac{4(s+3)}{s^2+s+2}$

Statement for Q.19-21:

The circuit is as shown in the fig. P1.8.19–21. All initial conditions are zero.

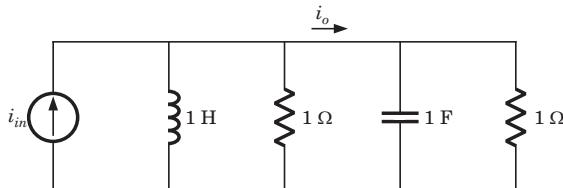


Fig. P1.8.19–21

- 19.** $\frac{I_o(s)}{I_{in}(s)} = ?$
 (A) $\frac{(s+1)}{2s}$ (B) $2s(s+1)^{-1}$
 (C) $(s+1)s^{-1}$ (D) $s(s+1)^{-1}$

- 20.** If $i_{in}(t) = 4\delta(t)$ then $i_o(t)$ will be
 (A) $4\delta(t) - e^{-t}u(t)$ A
 (B) $4\delta(t) - 4e^{-t}u(t)$ A
 (C) $4e^{-t}u(t) - 4\delta(t)$ A
 (D) $e^{-t}u(t) - \delta(t)$ A

- 21.** If $i_{in}(t) = tu(t)$ then $i_o(t)$ will be
 (A) $e^{-t}u(t)$ A (B) $(1 - e^{-t})u(t)$ A
 (C) $u(t)$ A (D) $(2 - e^{-t})u(t)$ A

- 22.** The voltage across $200 \mu\text{F}$ capacitor is given by

$$V_C(s) = \frac{2s+6}{s(s+3)}$$

- The steady state voltage across capacitor is
 (A) 6 V (B) 0 V
 (C) ∞ (D) 2 V

- 23.** The transformed voltage across the $60 \mu\text{F}$ capacitor is given by

$$V_C(s) = \frac{20s+6}{(10s+3)(s+4)}$$

The initial current through capacitor is

- (A) 0.12 mA (B) -0.12 mA
 (C) 0.48 mA (D) -0.48 mA

- 24.** The current through an 4 H inductor is given by

$$I_L(s) = \frac{10}{s(s+2)}$$

The initial voltage across inductor is

- (A) 40 V (B) 20 V
 (C) 10 V (D) 5 V

- 25.** The amplifier network shown in fig. P1.8.25 is stable if

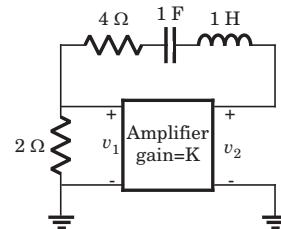


Fig.P1.8.25

- (A) $K \leq 3$ (B) $K \geq 3$
 (C) $K \leq \frac{1}{3}$ (D) $K \geq \frac{1}{3}$

- 26.** The network shown in fig. P1.8.26 is stable if

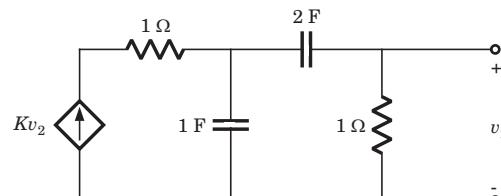


Fig.P1.8.26

- (A) $K \geq \frac{5}{2}$ (B) $K \leq \frac{5}{2}$
 (C) $K \geq \frac{2}{5}$ (D) $K \leq \frac{2}{5}$

27. A circuit has a transfer function with a pole $s = -4$ and a zero which may be adjusted in position as $s = -a$. The response of this system to a step input has a term of form Ke^{-4t} . The K will be (H= scale factor)

- (A) $H\left(1 - \frac{a}{4}\right)$ (B) $H\left(1 + \frac{a}{4}\right)$
 (C) $H\left(4 - \frac{a}{4}\right)$ (D) $H\left(4 + \frac{a}{4}\right)$

28. A circuit has input $v_{in}(t) = \cos 2t u(t)$ V and output $i_o(t) = 2 \sin 2t u(t)$ A. The circuit had no internal stored energy at $t = 0$. The admittance transfer function is

- (A) $\frac{2}{s}$ (B) $\frac{s}{2}$
 (C) s (D) $\frac{1}{s}$

29. A two terminal network consists of a coil having an inductance L and resistance R shunted by a capacitor C . The poles of the driving point impedance function Z of this network are at $-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$ and zero at -1 . If $Z(0) = 1$ the value of R, L, C are

- (A) $3 \Omega, 3 \text{ H}, \frac{1}{3} \text{ F}$ (B) $2 \Omega, 2 \text{ H}, \frac{1}{2} \text{ F}$
 (C) $1 \Omega, 2 \text{ H}, \frac{1}{2} \text{ F}$ (D) $1 \Omega, 1 \text{ H}, 1 \text{ F}$

30. The current response of a network to a unit step input is

$$I_o = \frac{10(s+2)}{s^2(s+11s+30)}$$

The response is

- (A) Under damped (B) Over damped
 (C) Critically damped (D) None of the above

Statement for Q.31-33:

The circuit is shown in fig. P1.8.31-33.

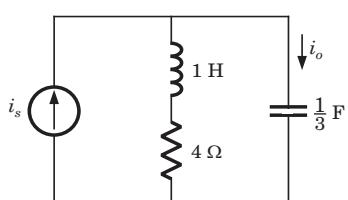


Fig. P1.8.31-33

31. The current ratio transfer function $\frac{I_o}{I_s}$ is

- (A) $\frac{s(s+4)}{s^2+3s+4}$ (B) $\frac{s(s+4)}{(s+1)(s+3)}$
 (C) $\frac{s^2+3s+4}{s(s+4)}$ (D) $\frac{(s+1)(s+3)}{s(s+4)}$

32. The response is

- (A) Over damped (B) Under damped
 (C) Critically damped (D) can't be determined

33. If input i_s is $2u(t)$ A, the output current i_o is

- (A) $(2e^{-t} - 3te^{-3t})u(t)$ A (B) $(3te^{-t} - e^{-3t})u(t)$ A
 (C) $(3e^{-t} - e^{-3t})u(t)$ A (D) $(e^{-3t} - 3e^{-t})u(t)$ A

34. In the network of Fig. P1.8.34, all initial condition are zero. The damping exhibited by the network is

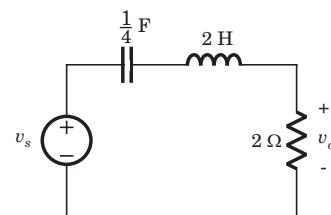


Fig. P1.8.34

- (A) Over damped (B) Under damped
 (C) Critically damped (D) value of voltage is requires

35. The voltage response of a network to a unit step input is

$$V_o(s) = \frac{10}{s(s^2 + 8s + 16)}$$

The response is

- (A) under damped (B) over damped
 (C) critically damped (D) can't be determined

36. The response of an initially relaxed circuit to a signal v_s is $e^{-2t}u(t)$. If the signal is changed to $\left(v_s + \frac{2dv_s}{dt}\right)$, the response would be

- (A) $5e^{-2t}u(t)$ (B) $-3e^{-2t}u(t)$
 (C) $4e^{-2t}u(t)$ (D) $-4e^{-2t}u(t)$

- 37.** Consider the following statements in the circuit shown in fig. P1.8.37

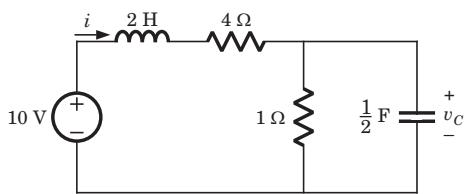


Fig. P1.8.37

1. It is a first order circuit with steady state value of $v_C = \frac{10}{3}$ V, $i = \frac{5}{3}$ A
2. It is a second order circuit with steady state of $v_C = 2$ V, $i = 2$ A
3. The network function $\frac{V(s)}{I(s)}$ has one pole.
4. The network function $\frac{V(s)}{I(s)}$ has two poles.

The true statements are

- | | |
|-------------|-------------|
| (A) 1 and 3 | (B) 1 and 4 |
| (C) 2 and 3 | (D) 2 and 4 |

- 38.** The network function $\frac{s^2 + 10s + 24}{s^2 + 8s + 15}$ represent a

- | | |
|-------------------|-----------------------|
| (A) RC admittance | (B) RL impedance |
| (C) LC impedance | (D) None of the above |

- 39.** The network function $\frac{s(s+4)}{(s+1)(s+2)(s+3)}$ represents an

- | | |
|------------------|-------------------|
| (A) RC impedance | (B) RL impedance |
| (C) LC impedance | (D) None of these |

- 40.** The network function $\frac{s(3s+8)}{(s+1)(s+3)}$ represents an

- | | |
|-------------------|-----------------------|
| (A) RL admittance | (B) RC impedance |
| (C) RC admittance | (D) None of the above |

- 41.** The network function $\frac{(s+1)(s+4)}{s(s+2)(s+5)}$ is a

- | |
|---------------------------|
| (A) RL impedance function |
| (B) RC impedance function |
| (C) LC impedance function |
| (D) Above all |

- 42.** The network function $\frac{s^2 + 7s + 6}{s + 2}$ is a

- | | |
|---------------------------|-------------------|
| (A) RL impedance function | (B) RL admittance |
| (C) LC impedance function | (D) LC admittance |

- 43.** A valid immittance function is

- | | |
|--------------------------------------|--------------------------------------|
| (A) $\frac{(s+4)(s+8)}{(s+2)(s-5)}$ | (B) $\frac{s(s+1)}{(s+2)(s+5)}$ |
| (C) $\frac{s(s+2)(s+3)}{(s+1)(s+4)}$ | (D) $\frac{s(s+2)(s+6)}{(s+1)(s+4)}$ |

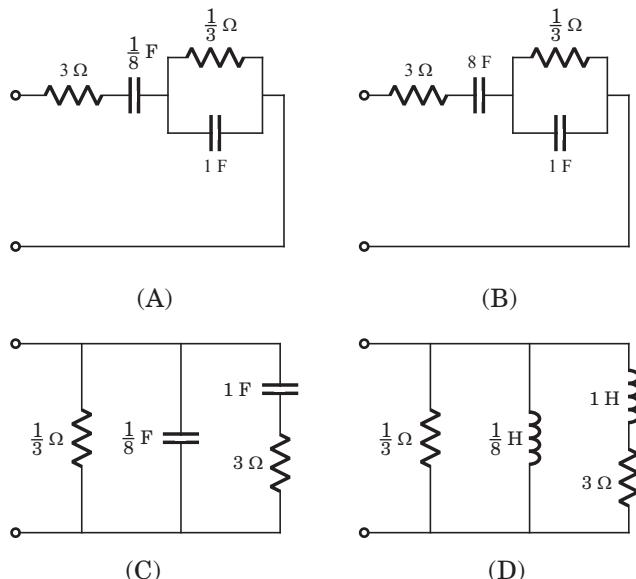
- 44.** The network function $\frac{s^2 + 8s + 15}{s^2 + 6s + 8}$ is a

- | | |
|-------------------|-------------------|
| (A) RL admittance | (B) RC admittance |
| (C) LC admittance | (D) Above all |

- 45.** A impedance function is given as

$$Z(s) = \frac{3(s+2)(s+4)}{s(s+3)}$$

The network for this function is



18. (B) $Z(s) = \frac{K(s+3)}{(s-(-1+j))(s-(1-j))} = \frac{K(s+3)}{s^2 + 2s + 2}$

$$Z(0) = \frac{3K}{2} = 3 \Rightarrow K = 2$$

19. (D) $\frac{I_o(s)}{I_{in}(s)} = \frac{\frac{s}{s+1}}{\frac{s}{s+1} + \frac{1}{s+1}} = \frac{s}{s+1}$

20. (B) $I_{in}(s) = 4$

$$I_o(s) = \frac{4s}{s+1} = 4 - \frac{4}{s+1} \Rightarrow i_o(t) = 4\delta(t) - 4e^{-t}u(t)$$

21. (B) $I_{in}(s) = \frac{1}{s^2},$

$$I_o(s) = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$

$$i_o(t) = u(t) - e^{-t}u(t) = (1 - e^{-t})u(t)$$

22. (D) $v_C(\infty) = \lim_{s \rightarrow 0} sV_C(s) = \lim_{s \rightarrow 0} \frac{2s+6}{s+3} = 2 \text{ V}$

23. (D) $v_C(0^+) = \lim_{s \rightarrow \infty} sV_C(s) = \frac{s(20s+6)}{(10s+3)(s+4)} = 2 \text{ V}$

$$\begin{aligned} I_C &= \frac{Cd v_C}{dt} \Rightarrow I_C(s) = C[sV_C(s) - v_C(0^+)] \\ &= 60 \times 10^{-6} \left(\frac{s(20s+6)}{(10s+3)(s+4)} - 2 \right) = \frac{-480 \times 10^{-6}(10s+3)}{10s^2 + 43s + 12} \end{aligned}$$

$$i_C(0^+) = \lim_{s \rightarrow \infty} sI_C(s) = -480 \times 10^{-6} = -0.48 \text{ mA}$$

24. (A) $v_L = L \frac{d i_L}{dt} \Rightarrow V_L(s) = L[sI_L(s) - i_L(0^+)]$

$$i_L(0^+) = \lim_{s \rightarrow \infty} sI_L(s) = \frac{10}{s+2} = 0$$

$$V_L(s) = \frac{40s}{s(s+2)} = \frac{40}{s+2}$$

$$v_L(0^+) = \lim_{s \rightarrow \infty} sV_L(s) = \frac{s40}{s+2} = 40$$

25. (A) $V_2(s) = KV_1(s)$

$$\Rightarrow \frac{V_1(s)}{2} + \frac{V_1(s) - KV_1(s)}{4+s+\frac{1}{s}} = 0$$

$$4+s+\frac{1}{s}+2-2K=0$$

$$\Rightarrow s^2 + (6-2K)s + 1 = 0$$

$$(6-2K) > 0 \Rightarrow K < 3$$

26. (B) Let v_1 be the node voltage of middle node

$$V_1(s) = \frac{KV_2(s) + 2sV_2(s)}{1+2s+s}$$

$$\Rightarrow (3s+1)V_1(s) = (2s+K)V_2(s)$$

$$\Rightarrow V_2(s) = \frac{2sV_1(s)}{2s+1}$$

$$\Rightarrow (2s+1)V_2(s) = 2sV_1(s)$$

$$\Rightarrow (3s+1)(2s+1) = 2s(2s+K)$$

$$2s^2 + (5-2K)s + 1 = 0,$$

$$5-2K > 0, K < \frac{5}{2}$$

27. (A) $H(s) = \frac{H(s+a)}{s+4}$

$$R(s) = \frac{H(s+a)}{s(s+4)} = \frac{Ha}{4s} + \frac{H\left(1-\frac{a}{4}\right)}{s+4}$$

$$r(t) = \frac{Ha}{4} u(t) + H\left(1-\frac{a}{4}\right)e^{-4t}$$

28. (A) $V_{in}(s) = \frac{s}{s^2 + 1}, I_o(s) = \frac{2}{s^2 + 1}, \frac{I_o(s)}{V_{in}(s)} = \frac{2}{s}$

29. (D) $Z(s) = \frac{(sL+R)\frac{1}{sC}}{sL+R+\frac{1}{sC}} = \frac{\frac{1}{C}\left(s+\frac{R}{L}\right)}{s^2 + \frac{R}{L} + \frac{1}{LC}}$

$$Z(s) = \frac{K(s+1)}{\left(s+\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\left(s+\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)} = \frac{K(s+1)}{(s^2 + s + 1)}$$

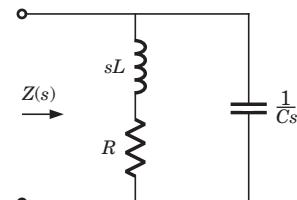


Fig. S1.8.29

Since $Z(0) = 1$, thus $K = 1$

$$\frac{1}{C} = 1, \frac{R}{L} = 1, \frac{1}{LC} = 1$$

$$\Rightarrow C = 1, L = 1, R = 1$$

30. (B) The characteristic equation is

$$s^2(s^2 + 11s + 30) = 0 \Rightarrow s^2(s+6)(s+5) = 0$$

$s = -6, -5$, Being real and unequal, it is overdamped.

31. (B) $\frac{I_o}{I_s} = \frac{s+4}{s+4+\frac{3}{s}} = \frac{s(s+4)}{(s+1)(s+3)}$

32. (A) The characteristic equation is $(s+1)(s+3)=0$.
Being real and unequal root, it is overdamped response.

33. (C) $i_s = 2u(t) \Rightarrow I_s(s) = \frac{2}{s}$

$$I_o(s) = \frac{2(s+4)}{(s+1)(s+3)} = \frac{3}{s+1} - \frac{1}{s+3}$$

$$i_o = (3e^{-t} - e^{-3t})u(t)$$

34. (B) $\frac{V_o(s)}{V_s(s)} = \frac{2}{\frac{4}{s} + 2s + 2} = \frac{1}{s^2 + s + 2}$

The roots are imaginary so network is underdamped.

35. (C) The characteristic equation is

$$s(s^2 + 8s + 16) = 0, (s+4)^2 = 0, s = -4, -4$$

Being real and repeated root, it is critically damped.

36. (B) $v_o = e^{-2t}u(t) \Rightarrow V_o(s) = H(s)V_s(s) = \frac{1}{s+2}$

$$v'_s = v_s + \frac{2dv_s}{dt} \Rightarrow V'_s(s) = (1+2s)V_s(s)$$

$$V'_o(s) = H(s)V'_s(s) = (1+2s)V_s(s)H(s)$$

$$V'_o(s) = \frac{1+2s}{s+2} = 2 - \frac{3}{s+2} \Rightarrow v'_o = 2\delta(s) - 3e^{-2t}u(t)$$

37. (C) It is a second order circuit. In steady state

$$i = \frac{10}{4+1} = 2 \text{ A}, v = 2 \times 1 = 2 \text{ V}$$

$$I(s) = \frac{10}{2s+4 + \frac{1}{1+\frac{1}{2}s}} = \frac{5(s+2)}{(s+2)^2 + 1}$$

$$V(s) = \frac{\frac{10}{1+\frac{1}{2}s}}{(2s+4) + \frac{1}{1+\frac{1}{2}s}} = \frac{10}{(s+2)^2 + 1}$$

$$\frac{V(s)}{I(s)} = \frac{2}{s+2}, \text{ It has one pole at } s = -2$$

38. (D) $\frac{s^2 + 10s + 24}{s^2 + 8s + 15} = \frac{(s+4)(s+6)}{(s+3)(s+5)}$

The singularity near to origin is pole. So it may be *RC* impedance or *RL* admittance function.

39. (D) Poles and zero does not interlace on negative real axis so it is not a immittance function.

40. (C) The singularity nearest to origin is a zero. So it may be *RL* impedance or *RC* admittance function. Because of (D) option it is required to check that it is a valid *RC* admittance function. The poles and zeros interlace along the negative real axis. The residues of $\frac{Y_{RC}(s)}{s}$ are real and positive.

41. (B) The singularity nearest to origin is a pole. So it may be *RC* impedance or *RL* admittance function.

42. (A) $\frac{s^2 + 7s + 6}{s+2} = \frac{(s+1)(s+6)}{(s+2)}$

The singularity nearest to origin is at zero. So it may be *RC* admittance or *RL* impedance function.

43. (D)

- (A) pole lie on positive real axis
- (B) poles and zero does not interlace on axis.
- (C) poles and zero does not interlace on axis.
- (D) is a valid immittance function.

44. (A) $\frac{s^2 + 8s + 15}{s^2 + 6s + 8} = \frac{(s+3)(s+5)}{(s+2)(s+4)}$

The singularity nearest to origin is a pole. So it may be a *RL* admittance or *RC* impedance function.

45. (A) The singularity nearest to origin is a pole. So this is *RC* impedance function.

$$Z(s) = 3 + \frac{8}{s} + \frac{1}{s+3} = 3 + \frac{8}{s} + \frac{1/3}{1 + \frac{s}{3}}$$
