Chapter 4

Rectilinear Motion

CHAPTER HIGHLIGHTS

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- *Types of motion*
- *Rectilinear motion*
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- *Vertical motion under gravity*
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- *Work and energy*
- *Ideal systems—conservation of energy*

Introduction

Dynamics

Dynamics is the branch of mechanics dealing with the motion of a particle or a system of particles under the action of a force. Dynamics is broadly divided into two categories:

1. Kinematics
2. Kinetics

Kinematics is the study of motion of a body without any reference to the forces or other factors which causes the motion. Kinematics relates displacement, velocity and acceleration of a particle of system of particles.

Kinetics studies the force which causes the motion. It relates the force and the mass of a body, and hence the motion of the body. So, the motion of a particle or body is largely covered and interpreted by Kinematics and Kinetics.

Types of Motion

The rate of change of position is motion. The type of motion is explained by the type of path traced by it. If the path traced is a straight line, the motion is said to be rectilinear motion or translation.

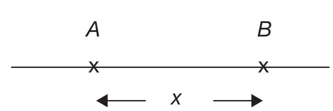
If the path traced by the motion (or path traversed by the particle) is a curve, it is known as curvilinear motion. When the curve becomes a circle, then it is known as circular motion.

The two types of motion, i.e., rectilinear and curvilinear motions, explained above, can be together termed as the general plane motion.

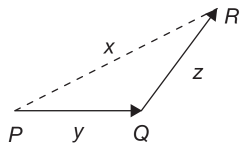
Rectilinear Motion

Displacement, Distance, Velocity and Acceleration

Displacement and Distance



Let the particle be at the position A at any point of time t. Let the position of the particle be at B at time $t + dt$ ($dt > 0$). Then the particle is said to move from A to B. The change in position is the displacement x. It is the shortest distance between A and B. Distance is the length of the path described by the particle from point A to point B.

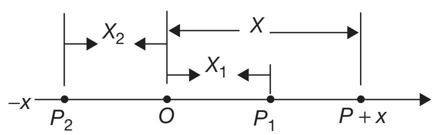


Let a body start from a point P and move towards a point Q, and then turn and reach at point R. During this course of motion, the total displacement is denoted by x. The distance traversed is given by $y + z$.

NOTE

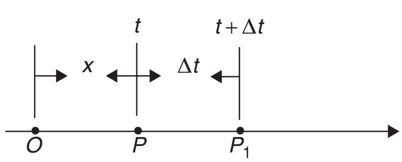
When the motion of a particle is considered along a line segment, both distance and displacement are the same in magnitudes.

Motion can also be defined as the change in the position of a body with respect to a given object. The position of a point P at any time t is expressed in terms of the distance x from a fixed origin O on the reference X-axis or Y-axis or Z-axis, and can be taken as positive or negative as per the usual sign convention.



Average Velocity

The average velocity v_{av} of a point P, in the time interval between $t + \Delta t$ and t, i.e., in the time interval Δt, during which its position changes from x to $x + \Delta x$ is defined by $v_{av} = \dfrac{\Delta x}{\Delta t}$.



Instantaneous Velocity and Speed

The instantaneous velocity v of a point P at time t is the limiting value of the average velocity as the increment of time approaches zero as a limit. Mathematically it can be expressed as:

$$v = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}.$$

The velocity v is positive if the displacement x is increasing and the particle is moving in a positive direction. The unit of velocity is metre per second (m/s).

If s is the distance covered by a moving particle at time t, then speed $= \dfrac{ds}{dt}$. The unit of speed is the same as that of the velocity.

Average Acceleration

The average acceleration a_{av} of a point P, in the time interval between $t + \Delta t$ and t, i.e., in the time interval Δt, during which its velocity changes from v to $v + \Delta v$ is defined by $a_{av} = \dfrac{\Delta v}{\Delta t}$.

Instantaneous Acceleration

The instantaneous acceleration of a point P is the limiting value of the average acceleration as the increment of time approaches zero. Mathematically it can be expressed as:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = \frac{dv}{dx} \times v.$$

Acceleration is positive when velocity is increasing. *A* positive acceleration means that the particle is either moving further in a positive direction or is slowing down in the negative direction.

Retardation or deceleration of a body in motion is the negative acceleration, i.e., retarding acceleration. Acceleration is the rate of increase in the velocity and deceleration is the rate of decrease in the velocity.

Uniform motion: When a particle moves with a constant velocity so that its acceleration is zero, then the motion is termed as uniform motion.

Uniformly accelerated motion: When a particle moves with a constant acceleration, then the motion is termed as a uniformly accelerated motion.

Motion at a Uniform Acceleration

Let the uniform acceleration be 'a'. Then,

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

$$s_n = u + a(n - \frac{1}{2})$$

Where

v = Velocity at any time instant t (seconds)

u = Initial velocity

s = Distance travelled during the time t (seconds)

s_n = Distance travelled at the nth second

NOTE

For motion under constant retardation or deceleration, assign negative sign for acceleration (a).

Vertical Motion under Gravity

A body in motion above the ground will be under influence of the gravitational force of attraction (g). If the body moves upwards, then it is subjected to gravitational retardation, i.e., $a = -g$. Then, the equations for the upward motion of a body under gravity will be:

$$v = u - gt$$
$$v^2 = u^2 - 2gs$$
$$s = ut - \frac{1}{2} gt^2$$
$$s_n = u - g\left(n - \frac{1}{2}\right)$$

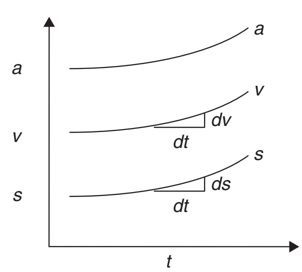
If the body moves downwards, then it is subjected to gravitational attraction, and hence an acceleration, i.e., $a = g$. Then, the equations for the downward motion of a body under gravity will be:

$$v = u + gt$$
$$v^2 = u^2 + 2gs$$
$$s = ut + \frac{1}{2} gt^2$$
$$s_n = u + g\left(n - \frac{1}{2}\right)$$

NOTES

1. For a body that is just dropped, $a = g$ and $u = 0$.
2. The final vertical velocity of a body thrown upwards as it reaches the maximum height, will be zero, i.e., $v = 0$.

Motion curves: These are the graphical representation of displacement, velocity and acceleration against time.



Considering the general case of acceleration not being a constant, the above graphical representation is made.

- The slope of the displacement-time curve—Velocity
- The slope of the velocity-time curve—Acceleration
- The area under the velocity-time curve—Displacement
- The area under the acceleration-time curve—Velocity

SOLVED EXAMPLES

Example 1

A particle has two velocities v_1 and v_2. Its resultant is v_1 in magnitude. When the velocity v_1 is doubled, the new resultant is

(A) perpendicular to v_2 (B) parallel to v_2
(C) equal to v_2 (D) equal to $2v_2$

Solution

Applying the principle of vector, the magnitude of the resultant between $|\vec{v}_1 + \vec{v}_2|$.

Given that $|\vec{v}_1 + \vec{v}_2| = \vec{v}_1$

$$|\vec{v}_1 + \vec{v}_2|^2 = \vec{v}_1^{\,2}$$
$$(\vec{v}_1 + \vec{v}_2)\cdot(\vec{v}_1 + \vec{v}_2) = \vec{v}_1 \cdot \vec{v}_1$$
$$\vec{v}_1 \cdot \vec{v}_1 + 2\,\vec{v}_1 \cdot \vec{v}_2 + \vec{v}_2 \cdot \vec{v}_2 = \vec{v}_1 \cdot \vec{v}_1$$
$$2\,\vec{v}_1 \cdot \vec{v}_2 + \vec{v}_2 \cdot \vec{v}_2 = 0$$
$$(2\vec{v}_1 + \vec{v}_2)\cdot \vec{v}_2 = 0.$$

Dot product zero means the new resultant between $2v_1$ and v_2 is at right angles to v_2.

Hence, the correct answer is option (A).

Example 2

If the two ends of a train, moving with a constant acceleration, pass a certain point with velocities u and v respectively, the velocity with which the middle point of the train passes through the same point is

(A) $\frac{u+v}{2}$ (B) $\sqrt{\frac{u^2+v^2}{u+v}}$

(C) $u - v$ (D) $\sqrt{\frac{u^2+v^2}{2}}$

Solution

We have the relation:

$$v^2 = u^2 + 2as \tag{1}$$

If V is the velocity with which the mid-point of the train crosses the point, we have:

$$V^2 = u^2 + 2a\,\frac{s}{2} \tag{2}$$

Eliminating s from Eqs. (1) and (2):

$$V^2 - u^2 = as$$
$$v^2 - u^2 = 2as$$
$$\frac{V^2 - u^2}{v^2 - u^2} = \frac{1}{2}$$
$$2V^2 - 2u^2 = v^2 - u^2$$
$$2V^2 = v^2 + u^2$$
$$V^2 = \frac{v^2 + u^2}{2}$$
$$\therefore V = \sqrt{\frac{v^2 + u^2}{2}}.$$

Hence, the correct answer is option (D).

Direction for solved examples 3 and 4:

The motion of a particle is defined as $s = 2t^3 - 6t^2 + 15$, where s is in metres and t is in seconds.

Example 3

The acceleration when the velocity is zero, is

(A) 12 m/s^2 (B) 8 m/s^2
(C) 6 m/s^2 (D) 4 m/s^2

Solution

$$s = 2t^3 - 6t^2 + 15$$
$$\frac{ds}{dt} = 6t^2 - 12t$$
$$a = \frac{ds^2}{dt^2} = 12t - 12$$

When velocity is zero,

$$6t^2 - 12t = 0, \therefore\ t = 2 \text{ seconds}$$

Then acceleration is, $a = 12 \times 2 - 12 = 12$ m/s^2

Hence, the correct answer is option (A).

Example 4

The minimum velocity is

(A) –2 m/s (B) 6 m/s
(C) –6 m/s (D) 2 m/s

Solution

Also, velocity is minimum when $\frac{dv}{dt} = 0$, i.e., when $12t - 12 = 0$,

$\therefore t = 1$ sec

$(\text{velocity})_{min} = 6t^2 - 12t = 6 - 12 = -6$ m/s.

Hence, the correct answer is option (C).

Example 5

The velocity of a particle along the X-axis is given by $v = 5x^{3/2}$, where x is in metres and v is in m/s.

The acceleration when $x = 2$ m is

(A) 300 m/s^2 (B) 200 m/s^2
(C) 180 m/s^2 (D) 150 m/s^2

Solution

Given, $v = 5x^{3/2}$, differentiating with respect to t, we have:

$$\frac{dv}{dt} = 5 \times \frac{3}{2} x^{3/2 - 1} \left(\frac{dx}{dt}\right)$$
$$= \frac{15}{2} x^{1/2} \frac{dx}{dt}, \text{ but } \frac{dx}{dt} = v$$
$$\therefore a = \frac{15}{2} x^{1/2} \times 5\, x^{3/2} = \frac{75}{2} x^2$$

When $x = 2$, $a = \frac{75}{2} \times 4 = 150$ m/s^2.

Hence, the correct answer is option (D).

Example 6

A particle is moving in a straight line starting from rest. Its acceleration is given by the expression $a = 50 - 36t^2$, where t is in seconds. The velocity of the particle when it has travelled 52 m can be

(A) 2.3 m/s (B) 4 m/s
(C) 6.7 m/s (D) 8 m/s

Solution

$$a = 50 - 36t^2$$
$$\frac{dv}{dt} = 50 - 36t^2$$
$$dv = 50dt - 36t^2\, dt$$

Integrating the above equation, we have: $v = 50t - 36 \frac{t3}{3} + C = 50t - 12t^3 + C$.

When $t = 0$, $v = 0$

$\therefore\ C = 0$

$\therefore\ v = 50t - 12t^3$

$\frac{ds}{dt} = 50t - 12t^3$

Integrating, $s = 50 \frac{t^2}{2} - 12 \frac{t^4}{4} + C_1$

$= 25t^2 - 3t^4 + C_1$

When $t = 0$, $s = 0$

$\therefore\ C_1 = 0$

$s = 25t^2 - 3t^4$

Here, we can find the time when $s = 52\ m$.

$\therefore\ 25t^2 - 3t^4 = 52$

Let $t^2 = u$, then $25u - 3u^2 = 52$

$3u^2 - 25u + 52 = 0$

$$u = \frac{25 \pm \sqrt{625 - 624}}{6}$$
$$u = \frac{25 \pm 1}{6} = \frac{26}{6} \text{ or } \frac{24}{6}$$

Case 1: When $t^2 = \frac{24}{6} = 4$

$\therefore\ t = 2$ seconds

$v = 50t - 12t^3$

$= 50 \times 2 - 12 \times 8$

$= 100 - 96 = 4$ m/s

Case 2: When $t^2 = \frac{26}{6} = 4.333$

$\therefore\ t = 2.08$ seconds

The value of the velocity calculated with this t value is not available in the options provided.

Hence, the correct answer is option (B).

Example 7

A body dropped from a certain height covers $\frac{5}{9}$ of the total height in the last second, the height from which the body is dropped is:

(A) 36.8 m (B) 40.3 m
(C) 44.1 m (D) 50.6 m

Solution

Let 'h' be the height and let 'n' be the time taken for the fall. Then,

$$s = u + a\left(n - \frac{1}{2}\right)$$

$$\frac{5}{9}h = 0 + g\left(n - \frac{1}{2}\right)$$

$$\frac{5}{9}h = g\left(n - \frac{1}{2}\right) \quad (1)$$

Also, $$h = un + \frac{1}{2}an^2$$

$$h = 0 + \frac{1}{2}gn^2 \quad (2)$$

Putting (2) in (1),

$$\frac{5}{9}\times\frac{1}{2}gn^2 = g\left(n - \frac{1}{2}\right)$$

$\therefore\ 5n^2 - 18n + 9 = 0$

$5n^2 - 15n - 3n + 9 = 0$

$5n(n - 3) - 3(n - 3) = 0$

$\therefore\ (5n - 3)(n - 3) = 0$

$\therefore\ n = \frac{3}{5}$ or $n = 3$, but $n > 1$

$\therefore\ n = 3$

$\therefore\ h = \frac{1}{2}gn^2 = \frac{1}{2}\times 9.81\times 9 = 44.1$ m.

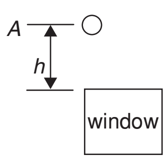
Hence, the correct answer is option (C).

Example 8

A stone falls past a window 2 m high in a time of 0.2 seconds. The height above the window from where the stone has been dropped is

(A) 4.15 m (B) 5.23 m
(C) 5.87 m (D) 6.32 m

Solution



The stone is dropped from A. Let the body reach the top of the window with a velocity of u m/s. Then,

$$u^2 = 0^2 + 2gh$$

$$u^2 = 2gh \quad (1)$$

Falling with an initial velocity u, it covers the window 2 m high in 0.5 seconds.

$$s = ut + \frac{1}{2}at^2$$

$$2 = u\times 0.2 + \frac{1}{2}\times 9.81\times 0.2^2$$

$$2 = 0.2u + \frac{1}{2}\times 9.81\times 0.04$$

$$2 = 0.2u + 9.81\times 0.02$$

$$u = 9.019 \text{ m/s};$$

From Eq. (1), $u^2 = 2gh$,

$$\therefore h = \frac{9.0192}{29.81} = 4.145 \text{ m}.$$

Hence, the correct answer is option (A).

Example 9

A ball is projected vertically upwards with a velocity of 49 m/s. If another ball is projected in the same manner after 2 seconds, and if both meet t seconds after the second ball is projected, then t is equal to:

(A) 3 seconds (B) 10 seconds
(C) 5 seconds (D) 6 seconds

Solution

Let both the balls meet T seconds after the first ball is projected. Therefore, when the balls meet,

for the first ball: $h = 49\times T - \frac{1}{2}gT^2$,

for the second ball: $h = 49\times(T - 2) - \frac{1}{2}g(T - 2)^2$

Equating $49T - \frac{1}{2}gT^2 = 49(T - 2)$

$$- \frac{1}{2}g(T - 2)^2$$

$\therefore\ T = 11.99$ sec

$\therefore\ t = T - 2 = 9.99$ sec ≈ 10 seconds

Hence, the correct answer is option (B).

Example 10

Two bodies are uniformly moving towards each other. The distance between them decreases at a rate of 6 m/s. If both the bodies move in the same direction at the same speeds, then the distance between them increases at a rate of 4 m/s. The respective speeds of the bodies are

(A) 3 m/s and 1 m/s (B) 5 m/s and 1 m/s
(C) 4 m/s and 2 m/s (D) 3 m/s and 5 m/s

Solution

Let u and v be the velocities of the bodies. From the statement of the problem,

$$u + v = 6$$

$$u - v = 4$$

$\therefore$ $u = 5$ m/s and $v = 1$ m/s.

Hence, the correct answer is option (B).

Example 11

Two cars are moving in the same direction each at a speed of 45 km/h. The distance separating them is 10 km. Another vehicle coming from the opposite direction meets these two cars in an interval of 6 minutes. The speed of the vehicle is

(A) 45 km/h (B) 50 km/h
(C) 55 km/h (D) 60 km/h

Solution

The distance between the cars moves with a velocity of 45 km/h. If the speed of the vehicle is u, then its velocity relative to the moving distance is $45 + u$ m/s.

It takes 6 minutes to cover the distance of 10 km.

$$\therefore\ (45 + u) \times \frac{6}{60} = 10$$

$$\therefore\ 45 + u = 100$$

$$u = 55 \text{ km/h.}$$

Hence, the correct answer is option (C).

Motion under Variable Acceleration

In practical conditions, a body may very often move with variable acceleration. The rate of change of velocity will not remain constant. We know that acceleration,

$$a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt}$$

or, $$a = v \cdot \frac{dv}{ds}$$

Also, when displacement can be expressed as a third degree or higher degree equation in time, the acceleration becomes a variable with respect to time.

For example, if $s = 4t^3 + 3t^2 + 5t + 1$

$$= 12t^2 + 6t + 5$$

$$= 24t + 6$$

The velocity and displacement are evaluated by integration.

Example 12

A body is starting from rest and moving along a straight line whose acceleration is given by $f = 10 - 0.006x^2$, where x is the displacement in m and f is the acceleration in m/s^2. The distance travelled by it when it comes to rest is

(A) 70.7 m (B) 68.3 m
(C) 62.6 m (D) 58.5 m

Solution

Given that $f = 10 - 0.006x^2$

$$\frac{dv}{dt} = 10 - 0.006x^2$$

$$\frac{dv}{dx} \cdot \frac{dx}{dt} = 10 - 0.006x^2$$

$$v \cdot \frac{dv}{dx} = 10 - 0.006x^2$$

$v\,dv = (10 - 0.006x^2)dx$ Integrating

$$\frac{v^2}{2} = 10x - 0.006\,\frac{x^3}{3} + C$$

when $x = 0$, $v = 0$

$\therefore$ C = 0

$$\frac{v^2}{2} = 10x - 0.006\,\frac{x^3}{3}$$

$v^2 = 20x - 0.004x^3$

when $v = 0$; $20x - 0.004x^3 = 0$

$\therefore$ $0.004\,x^2 = 20$ (note that the solution of $x = 0$ is also possible for the above equation, but the value of $x > 0$ is sought for)

$\therefore$ $x = 70.7$ m

Hence, the correct answer is option (A).

Direction for solved examples 13 and 14:

An electric train starting from rest has an acceleration f in m/s^2 which vary with time as shown in the following table.

t (secsnd)	0	6	12	18
f (m/s^2)	12	10	9.5	8

Example 13

The velocity at the end of the first 6 seconds is

(A) 18 m/s (B) 27 m/s
(C) 43 m/s (D) 66 m/s

Solution

During the first 6 seconds, the average acceleration $= \frac{12+10}{2} = 11\text{m/s}^2$.

$\therefore$ Increase in velocity during this interval of 6 seconds = average acceleration × 6 = 66 m/s.

$\therefore$ Velocity at the end of 6 second = 66 m/s.

Hence, the correct answer is option (D).

Example 14

The distance travelled during these 6 seconds is

(A) 242 m (B) 218 m
(C) 198 m (D) 124 m

Solution

Average velocity during this interval

$$= \frac{0+66}{2} = 33 \text{ m/s}$$

∴ Distance travelled during this interval

$$= 33 \times 6 = 198 \text{ m}$$

Hence, the correct answer is option (C).

Example 15

At any instant, the acceleration of a train starting from rest is given by $f = \frac{10}{u+1}$, where u is the velocity of the train in m/s. The distance at which the train will attain a velocity of 54 km/h, is:

(A) 123.7 m (B) 185.4 m
(C) 214.4 m (D) 228.2 m

Solution

It is given, $f = \frac{10}{u+1}$

$$u \cdot \frac{du}{dx} = \frac{10}{u+1}$$

$$u(u+1)du = 10dx$$

Integrating we have, $\frac{u^3}{3} + \frac{u^2}{2} = 10x + c$

when $x = 0$, $u = 0$. ∴ $c = 0$

$$\frac{u^3}{3} + \frac{u^2}{2} = 10x$$

when $u = 54$ km/h $= 54 \times 5/18 = 15$ m/s

$$\frac{15^3}{3} + \frac{15^2}{2} = 10x$$

$$1125 + 112.5 = 10x$$

∴ $x = 123.7$ m.

Hence, the correct answer is option (A).

Example 16

The motion of a particle is given by the equation $a = t^3 - 3t^2 + 5$, where 'a' is acceleration in m/s^2 and t is time in seconds. It is seen that the velocity and displacement of the particle at 't' = 1 sec are 6.25 m/s and 8.3 m, respectively. Then the displacement at time $t = 2$ seconds, is

(A) 17.3 m (B) 15.6 m
(C) 14.8 m (D) 12.6 m

Solution

Given $a = t^3 - 3t^2 + 5$

$$\frac{dv}{dt} = t^3 - 3t^2 + 5$$

Integrating, $v = \frac{t^4}{4} - 3\frac{t^3}{3} + 5t + c$ at

$t = 1$ second,

$v = 6.25$ m/s

i.e., $6.25 = \frac{1}{4} - 1 + 5 + c$

$= 4.25 + c$

∴ $c = 2$

∴ $v = \frac{t^4}{4} - t^3 + 5t + 2$

$$\frac{ds}{dt} = \frac{t^4}{4} - t^3 + 5t + 2$$

Integrating, $s = \frac{t^5}{20} - \frac{t^4}{4} + 5 \cdot \frac{t^2}{2} + 2t + c,$

at $t = 1$, $s = 8.3$ m

$$8.3 = \frac{1}{20} - \frac{1}{4} + \frac{5}{2} + 2 + c,$$

$$8.3 = \frac{1}{20} + 4.25 + c,$$

$c = 8.3 - 4.25 - 0.05 = 4.05 - 0.05 = 4$

$$s = \frac{t^5}{20} - \frac{t^4}{4} + 5 \cdot \frac{t^2}{2} + 2t + 4$$

s at $t = 2$ seconds is

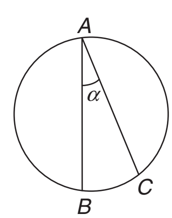
$$s = \frac{32}{20} - \frac{16}{4} + 10 + 4 + 4$$

$$= \frac{32}{20} + 14 = 15.6 \text{ m.}$$

Hence, the correct answer is option (B).

Example 17

In the figure shown, AB is the diameter 'd' of the circle and AC is the chord of the same circle making an angle α with AB. Two particles are dropped from rest one along AB and the other along AC. If t_1 is the time taken by the particle to slide along AB and t_2 is the time taken to slide along AC, then $t_1 : t_2$ is



(A) 1 : cos α (B) 1 : sec α
(C) 1 : 1 (D) 1 : 15

Solution

Let $AB = l$, $AC = l\cos\alpha$.

Consider sliding along AC,

acceleration is $g\cos\alpha$

$$s = ut + \frac{1}{2}at^2$$

$$l\cos\alpha = 0 + \frac{1}{2}g\cos\alpha\, t_2^{\,2}$$

$$\therefore\ t_2^2 = \frac{2l}{g} \quad \text{or} \quad t_2 = \sqrt{\frac{2l}{g}}$$

Consider sliding along AB,

$$l = 0 + \frac{1}{2}gt_1^2$$

$$t_1 = \sqrt{\frac{2l}{g}}$$

$\therefore\ t_1 : t_2 = 1 : 1$

Hence, the correct answer is option (C).

Relative Velocity

The motion of one body with respect to another moving body is known as relative motion.

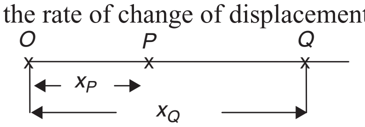
Take the case of two bodies P and Q moving along the same straight line. The position of the bodies is specified with reference to an origin O.

x_P and x_Q are measured from the origin O. The difference $x_Q - x_P$ defines the relative position of Q with respect to P. It is denoted as:

$$x_{Q/P} = x_Q - x_P$$

$$\therefore\ x_Q = x_P + x_{Q/P}.$$

Consider the rate of change of displacement, then:



$$v_{Q/P} = v_Q - v_P$$

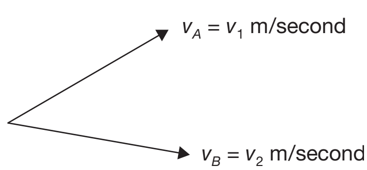
$$\therefore\ v_Q = v_P + v_{Q/P}$$

Similar relations hold good for acceleration also, i.e.,

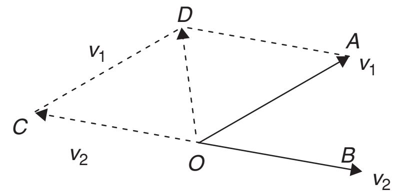
$$\therefore\ a_Q = a_P + a_{Q/P}$$

Working Rule

Let two particles A and B move with velocities v_1 m/s and v_2 m/s, respectively, in directions as shown in the following figure.



If we want to find out the velocity of A relative to B, the velocity of B is to be made zero. For that, we provide velocity v_2 in the reverse direction of OB and find the vector sum with $v_1 = OA$.



The vector $\overrightarrow{OD}$ gives both the magnitude and direction of the velocity of A relative to B.

Another method is to resolve their velocities into their components with sign. Then evaluate the relative velocity in the x-direction and in the y-direction. Find their resultant vector. This vector will be the relative velocity, both in magnitude and in direction.

Example 18

A train has a mass of 200 tonne. If the frictional resistance amounts to 55 N per tonne, what steady pull must the locomotive exert in order to increase the speed on a level road from 32 to 64 km/h within 1.5 minutes?

Solution

Considering the motion of the locomotive:

$$u = 32\text{ km/h} = \frac{32\times 5}{18} = \frac{80}{9}\text{ m/s}$$

$$v = 64\text{ km/h}$$

$$= \frac{64\times 5}{18} = \frac{160}{9}\text{ m/s}$$

$$t = 1.5\text{ min} = 90\text{ seconds}$$

Let a = acceleration of the locomotive

$v = u + at$

$$\Rightarrow \frac{160}{9} = \frac{80}{9} + 90a \Rightarrow a = \frac{8}{81}\text{ m/s}^2$$

Now, accelerating force,

$$F_a = \text{mass} \times \text{acceleration}$$

$$= \frac{200\times 1000}{1}\times\frac{8}{81}\text{ N} = 19753\text{ N}$$

Also frictional resistance to overcome;

$F_r = 200 \times 55 = 11000$ N

By D' Alembaert's principle for equilibrium.

Total pull of the locomotive,

$P = F_a + F_r = 19753 + 11000 = 30753$ N.

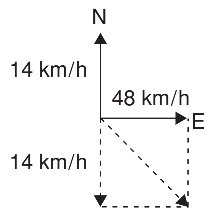
Example 19

A vessel which can steam in still water with a velocity of 48 km/h is steaming with its bow pointing due east. It is carried by a current which flows northward at a speed of 14 km/h. The distance it would travel in 12 minutes is

(A) 14 km (B) 12 km
(C) 10 km (D) 8 km

Solution

To find the velocity of the steamer relative to the flow, the flow velocity is reversed and vector sum is found.



$$\text{Relative velocity} = \sqrt{48^2 + 14^2}$$
$$= 50 \text{ km/h}$$

$$\text{Distance after 12 minutes} = 50 \times \frac{12}{60} = 10 \text{ km}$$

Hence, the correct answer is option (C).

Example 20

A man keeps his boat at right angles to the current and rows across a stream 0.25 km broad. He reaches the opposite bank 0.125 km below the point opposite to the starting point. If the speed of the boat in rowing alone is 6 km/h, the speed of the current is

(A) 5 km/h (B) 4 km/h
(C) 3 km/h (D) 2 km/h

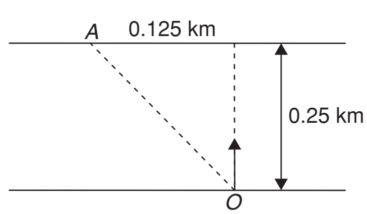
Solution

The speed required for reaching the opposite side is the rowing velocity of 6 km/h. Due to the velocity of the current by the time, the boat can cross the stream with its absolute velocity. It flows down 0.125 km due to the speed of the current.

$$\text{Time needed for crossing the stream} = \frac{0.25}{6} = 0.04166 \text{ hour.}$$

Let the stream velocity be v m/s.

$$\therefore \text{ Resultant speed} = \sqrt{v^2 + 6^2}$$



The distance covered by the boat within this time is:

$$OA = \sqrt{0.25^2 + 0.125^2}$$

$$\therefore\ 0.04166 \times \sqrt{v^2 + 6^2} = \sqrt{0.25^2 + 0.125^2}$$

$$\therefore\ v = 3 \text{ km/h}$$

Hence, the correct answer is option (C).

Example 21

A boat weighing 45 kg is initially at rest. *A* boy weighing 32 kg is standing on it. If he jumps horizontally at a speed of 2 m/s relative to the boat, the speed of the boat is:

(A) 2 m/s (B) 3.42 m/s
(C) 4.92 m/s (D) 5.36 m/s

Solution

Given $v_{A/B} = 2$ m/s

It is the relative velocity of the boy with respect to the boat.

$$v_{A/B} = v_A - v_B$$
$$2 = v_A - v_B$$
$$\therefore\ v_A = 2 + v_B$$

By conservation of momentum:

$$0 = 32(2 + v_B) - 45v_B = 64 - 13v_B$$
$$\therefore\ v_B = 4.92 \text{ m/s}$$

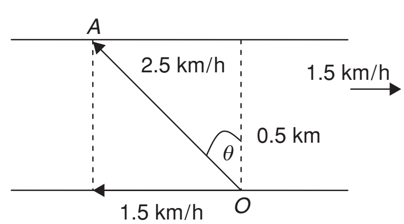
Hence, the correct answer is option (C).

Example 22

A stream of water flows with velocity of 1.5 km/h. A swimmer swims in still water with a velocity of 2.5 km/h. If the breadth of the stream is 0.5 km, the direction in which the swimmer should swim so that he can cross the stream perpendicularly is:

(A) 26° with the vertical
(B) 29.4° with the vertical
(C) 32.5° with the vertical
(D) 36.8° with the vertical

Solution



The swimmer must swim in the direction *OA* with velocity 2.5 m/s so that he can cross the stream at right angles.

From geometry, $2.5 \sin\theta = 1.5$

$$\therefore\ \sin\theta = \frac{1.5}{2.5} = 0.6$$
$$\theta = 36.8°.$$

Hence, the correct answer is option (D).

Example 23

An airplane is flying in a horizontal direction with a velocity of 1800 km/h. At a height of 1960 metres, when it is above a point A on the ground, a body is dropped from it. If the body strikes the ground at point B, then the distance AB is

(A) 18 km (B) 15 km
(C) 10 km (D) 8 km

Solution

The time taken by the body to fall down the distance 1960 m is:

$$h = \frac{1}{2}gt^2$$

$$1960 = \frac{1}{2}9.8\,t^2$$

$$\frac{2\times 1960}{9.8} = t^2$$

$$400 = t^2;\ t = 20 \text{ sec}$$

$$AB = v\times t = \frac{1800}{60\times 60}\times 20 = 10 \text{ km}$$

Hence, the correct answer is option (C).

Example 24

Two ships leave a port at the same time. The first ship 'A' steams north-west at 32 km/h, and the second ship 'B' 40° south of west at 24 km/h. The time after which they will be 160 km apart is

(A) 2.15 hours (B) 2.86 hours
(C) 3.46 hours (D) 4.19 hours

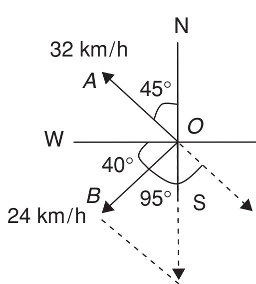
Solution

Let us find the velocity of the second ship relative to the first ship. For that, consider the velocity of the first ship in the reverse direction and evaluate the vector sum of the velocities.

Resultant or velocity of B relative to A is

$$= \sqrt{24^2 + 32^2 + 2\times 32\times 24\cos 95°}$$

$$= \sqrt{1466} = 38.3 \text{ km/h}$$



Time for two ships to be 160 km apart

$$= \frac{160}{38.3} = 4.19 \text{ hours.}$$

Hence, the correct answer is option (D).

Example 25

A particle is accelerated from (1, 2, 3), where it is at rest, according to the equation $a = 6t\,\vec{i} - 24t^2\,\vec{j} + 10\,\vec{k}$ m/s^2, where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are unit vectors along the X, Y and Z axes. The magnitude of the displacement after the lapse of 1 second is

(A) 5 m (B) $\sqrt{30}$ m
(C) 6 m (D) $\sqrt{47}$ m

Solution

It is given that $a = 6t\vec{i} - 24t^2\vec{j} + 10\vec{k}$

$\therefore\ v = 3t^2\vec{i} - 8t^3\vec{j} + 10t\vec{k} + c$

when $t = 0$, $v = 0$ $\therefore\ c = 0$

$\therefore\ v = 3t^2\vec{i} - 8t^3\vec{j} + 10t\vec{k}$

$$\frac{dx}{dt} = 3t^2\vec{i} - 8t^3\vec{j} + 10t\vec{k}$$

$$x = 3\frac{t^3}{3}\vec{i} - 8\frac{t^4}{4}\vec{j} + 10\frac{t^2}{2}\vec{k} + C$$

$$x = t^3\vec{i} - 2t^4\vec{j} + 5t^2\vec{k} + C$$

when $t = 0$, position of the particle is at (1, 2, 3) i.e., at $t = 0$, $x = 1\vec{i} + 2\vec{j} + 3\vec{k}$

$$\therefore\ C = 1\vec{i} + 2\vec{j} + 3\vec{k}$$

$$\therefore\ x = t^3\,\vec{i} - 2t^4\,\vec{j} + 5t^2\,\vec{k} + 1\,\vec{i} + 2\,\vec{j} + 3\,\vec{k}$$

$$= (t^3 + 1)\,\vec{i} - (2 - 2t^4)\,\vec{j} + (3 + 5t^2)\,\vec{k}$$

When $t = 1$,

$$x = 2\,\vec{i} + 8\,\vec{k}$$

$\therefore$ Displacement vector

$$= 2\,\vec{i} + 8\,\vec{k} - (1\vec{i} + 2\vec{j} + 3\vec{k})$$

$$= 1\vec{i} - 2\vec{j} + 5\vec{k}$$

Magnitude of the displacement vector

$$= \sqrt{1+4+25} = \sqrt{30} \text{ m.}$$

Hence, the correct answer is option (B).

Example 26

If a particle, moving with uniform acceleration, travels the distances of 8 and 9 cm in the 5th second and 9th second respectively, then its acceleration will be

(A) 1 cm/s^2 (B) 5 cm/s^2
(C) 25 cm/s^2 (D) 0.5 cm/s^2

Solution

$$s \text{ in the } n\text{th sec} = u + \frac{a}{2}(2n-1)$$

$$8 = u + \frac{a}{2}(2\times 5 - 1) = u + 4.5a \qquad (1)$$

$$9 = u + \frac{a}{2}(2\times9-1) = u + 8.5\,a \quad (2)$$

Subtracting Eq. (1) from Eq. (2),

$$1 = 4a \text{ or } a = 0.25 \text{ cm/s}^2.$$

Hence, the correct answer is option (C).

Example 27

The acceleration due to gravity on a planet is 200 cm/s^2. If it is safe to jump from a height of 2 m on earth, then the corresponding safe height on the planet is:

(A) 2 m (B) 9.8 m
(C) 10 m (D) 8 m

Solution

Let h_{se} and h_{sp} denote the safe heights on the earth and the planet.

On the earth, $v^2 = 2gh_{se} = 2 \times 9.8 \times 2$

$= 39.2 \text{ m}^2/\text{s}^2$.

On the planet, $v^2 = 2 \times 2 \times h_{sp}$.

For a safe jump, the final velocity (v) should be same on earth and the planet. Hence, $2 \times 2 \times h_{sp} = 39.2$.

$\therefore\ h_{sp} = 9.8$ m.

Hence, the correct answer is option (B).

Example 28

A ball weighing 500 gm is thrown vertically upwards with a velocity of 980 cm/s. The time that the ball will take to return to earth would be:

(A) 1 second (B) 2 seconds
(C) 3 seconds (D) 4 seconds

Solution

For the upward journey, $u = u_0 - gt$

$$0 = 980 \times 10^{-2} - 9.8\,t$$

$$\Rightarrow t = 1 \text{ s}$$

$$v^2 - u^2 = 2gs \Rightarrow 0 - 9.8^2 = -2 \times 9.8\,s$$

$$s = 4.9 \text{ m}$$

For the downward journey,

$$s = ut + \frac{1}{2}gt^2$$

$$4.9 = 0 + \frac{1}{2} \times 9.8\,t^2$$

$$t = 1 \text{ second}$$

Total time taken to return to earth

$= 1 + 1 = 2$ seconds.

Hence, the correct answer is option (B).

Kinetics of a Particle

Kinetics can be used to predict a particle's motion, given a set of forces (acting upon the particle) or to determine the forces necessary to produce a particular motion of the particle. Kinetics of the rectilinear motion of a particle are governed mainly by Newton's three laws of motion.

Newton's first law: *Every body continues in its state of rest, or uniform motion in a straight-line, unless compelled to change that state by forces impressed upon it.*

This law is sometimes called the law of inertia.

From Newton's first law, it follows that any change in the velocity of a particle is the result of a force. The question, of the relationship between this change in the velocity of the particle and the force that produces it, is answered by the second law of motion which is as follows.

Newton's second law: *The acceleration of a given particle is proportional to the force applied to it and takes place in the direction of the straight line in which the force acts.*

Newton's third law: *To every action there is always an equal and opposite reaction, or the forces of two bodies on each other are always equal and directed in opposite directions.*

General Equation of Motion for a Particle

From Newton's second law, the relationship between the acceleration 'a' produced in a body of mass 'm' (mass is always assumed to be invariant with time) by a resultant, 'F', of all the forces acting on the body can be derived as follows: $f = ma$, which is the general equation of motion for a particle.

For a stationary body lying on a surface (body with no motion), there is a force (F) exerted by the body on the surface which is equal to the weight of the body (W), i.e., $f = W = mg$, where 'm' is the mass of the body and 'g' is the acceleration due to gravity. There is an equal and opposite force exerted by the surface on the body (consequence of Newton's third law). Note that the weight of a body is obtained by multiplying the mass of the body by the acceleration causes due to gravity.

Differential Equation of Rectilinear Motion

The general equation of motion for a particle can be applied directly to the case of the rectilinear translation of a rigid body, since all the particles of the rigid body have the same velocity and acceleration (same motion) where the particles move in parallel straight lines. Here, the rigid body is considered as a particle concentrated at the center of gravity of the rigid body.

Whenever such a body or particle moves under the action of a force applied at its centre of gravity and having a fixed line of action, acceleration of the body is produced in the same direction, and if any initial velocity of the body is also directed along this line, then the motion corresponding to this case is known as rectilinear translation.

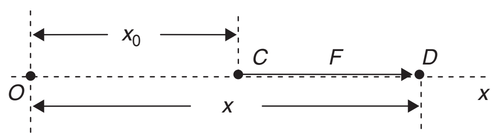
If the line of motion of a particle is taken to be along the X-axis (i.e., displacement at a time t is denoted by x),

$\ddot{x} = \frac{d^2x}{dt^2}$ represents the acceleration and f represents the resultant force acting, then the differential equation of the rectilinear motion of the particle is given by $F = m\ddot{x}$.

Two types of problems that can be solved by the above equation are: (a) *Determination of the force necessary to produce a given motion of the particle where the displacement x is given as a function of time t*, and (b) *Determination of the motion of a particle given a force f acting on the particle, i.e., to determine a function relating x and t, such that the above equation is satisfied.*

Motion of a Particle Acted Upon by a Constant Force

A particle, acted upon by a force of constant magnitude and direction, will move rectilinearly in the direction of the force subjected to a constant acceleration. Let us consider a particle moving along the X-axis (see figure below) where the initial (at $t = 0$) displacement and velocity of the particle are x_0 and $\dot{x}_0$, respectively.



If f is the magnitude of the constant force acting on the particle, then from the differential equation of rectilinear motion, $\ddot{x} = \frac{F}{m} = a$, where a is the constant acceleration produced in the particle due to the constant force. The equation, $\ddot{x} = a$ can be written as $\frac{d(\dot{x})}{dt} = a$. Integration of the above equation with the initial value condition, at $t = 0$, $\dot{x} = \dot{x}_0$, gives:

$$\dot{x} = \dot{x}_0 + at \tag{1}$$

which is the general velocity-time equation for the rectilinear motion of a particle under the action of a constant force 'F' producing the constant acceleration 'a' in the particle. With $\dot{x} = \frac{dx}{dt}$, Eq. (1) can be rewritten as follows:

$$\frac{dx}{dt} = \dot{x}_0 + at.$$

Integration of the above equation with the initial value condition, at $t = 0$ $x = x_0$, gives: $x = x_0 + \dot{x}_0 t + \frac{1}{2}at^2$, which is the general displacement-time equation for the rectilinear motion of a particle under the action of a constant force 'F' producing the constant acceleration 'a' in the particle.

Free-falling Object

The force acting on a free-falling object is the weight of the object (assuming no friction in the motion) and, therefore the acceleration produced in the object is the acceleration caused due to gravity, that is, $f = W = mg$, so $a = g$.

Hence, the velocity-time and displacement-time equations for a free-falling object are as follows:

$$\boxed{\dot{x} = \dot{x}_0 + gt}$$

$$\boxed{x = x_0 + \dot{x}_0 t + \frac{1}{2}gt^2}$$

If the free-falling object starts to fall from a resting position, i.e., it has an initial velocity of zero $(\dot{x}(0) = 0)$, and if the origin of displacement of the body is taken to coincide with the initial position of the body (i.e., it has an initial displacement of zero $(x_0 = 0)$), then the above equations reduce to:

$$\boxed{\dot{x} = gt}$$

$$\boxed{x = \frac{1}{2}gt^2}$$

Force as a Function of Time

If the force acting on the particle is a function of time t, (i.e., the acting force $= F(t)$), then the acceleration $a(t)$, velocity $\dot{x}(t)$ and displacement $x(t)$ of the particle at time t (with initial time, $t = 0$) is given by the following equations.

$$a(t) = \frac{F(t)}{m}$$

$$\dot{x}(t) = \int_0^t a(t)dt$$

$$x(t) = \int_0^t \dot{x}(t)dt$$

Dynamics of a Particle

D'Alembert's Principle

Let, ΣF_i, where F_i denotes the ith force, be the resultant of a set of forces acting on a particle in the X-axis direction. From the differential equation of the rectilinear motion of a particle, we have

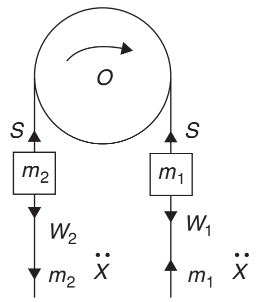
$$\sum F_i - m\ddot{x} = 0$$

or

$$\sum F_i + (-m\ddot{x}) = 0$$

From the above equation, it can be observed that if a fictitious force $(-m\ddot{x})$ is added to the system of forces acting on the particle, then an equation resembling equilibrium is obtained. The force $(-m\ddot{x})$ which has the same magnitude as $m\ddot{x}$, but opposite in direction is called 'the inertia force'. Hence, it can be observed that if an inertia force is added to the system of forces acting on a particle, then the particle is brought into an equilibrium state called 'dynamic equilibrium'. This is known as the D'Alembert's principle. The above equation thus represents the equation of dynamic equilibrium for the rectilinear translation of a rigid body.

Let us consider, any system of particles connected between them and so constrained that each particle can have only a rectilinear motion. To exemplify such a system, the case of two weights, W_1 and W_2, attached to the ends of a flexible, but inextensible string overhanging a pulley (figure below) is considered.



The inertia of the pulley and the friction on its axle are assumed to be negligible. If the motion of the system is assumed to be in the direction as shown by the arrow on the pulley, an upward acceleration $\ddot{x}$ of the weight W_2 and a downward acceleration $\ddot{x}$ of the weight W_1 is obtained. The inertia forces acting on the corresponding weights are shown in the above figure.

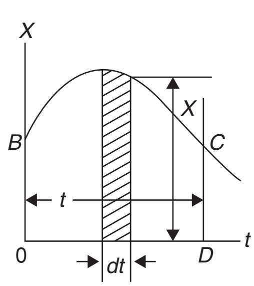
By adding the inertia forces to the real forces, (such as W_1 and W_2, and the string reactions S), a system of forces in equilibrium is obtained for each particle. Hence, the entire system of forces can be considered to be in equilibrium. An equation of equilibrium can be written for the entire system (instead of separate equilibrium equations for the individual weights) by equating to zero, the algebraic sum of moments of all the forces (including the inertia forces) with respect to the axis of the pulley or by using the principle of virtual work. In either case, the internal forces 'S' of the system need not be considered and the following equation of equilibrium can be obtained for the entire system.

$$W_2 + m_2\ddot{x} = W_1 - m_1\ddot{x} \text{ or } \ddot{x} = \left(\frac{W_1 - W_2}{W_1 + W_2}\right)g$$

Momentum and Impulse

The differential equation of the rectilinear motion of a particle can be written as:

$$m\frac{d\dot{x}}{dt} = F, \text{ or } d(m\dot{x}) = Fdt \qquad (1)$$



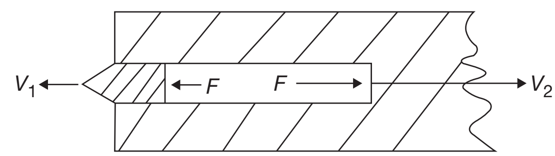
It is assumed that the force 'F' is known as a function of time. It is given by the force-time diagram as shown in the above figure. The right-hand side of Eq. (1) is then represented by the area of the shaded elemental strip of height 'F' and width 'dt' in the force-time diagram. This quantity is called the impulse of the force F in the time interval dt. The expression $m\dot{x}$ on the left-hand side of the equation is called the momentum of the particle. The equation states that the differential change of the momentum of the particle during the time interval dt is equal to the impulse of the acting force during the same time interval. Impulse and momentum have the same dimensions of the product of mass and velocity.

Integrating Eq. (1), we get:

$$m\dot{x} - m\dot{x}_0 = \int_0^t F\,dt,$$

where $\dot{x}_0$ is the velocity of the particle at time $t = 0$

Thus, the total change in the momentum of a particle, during a finite time interval, is equal to the impulse of the acting force during the same time interval. This impulse is represented by the area $OBCD$ of the force-time diagram. The equation of momentum-impulse is particularly useful when dealing with a system of particles, since in such cases the calculation of the impulse can often be eliminated. As a specific example, consider the case of a gun and shell as shown in the following figure, which may be considered



as a system of two particles. During the extremely short interval of explosion, the forces 'F' acting on the shell and gun and representing the gas pressure in the barrel are varying in an unknown manner. A calculation of the impulses of these forces would be extremely difficult.

However, the relation between the velocity of the shell and velocity of recoil of the gun can be obtained without calculation of the impulse. Since the forces 'F' are in the nature of action and reaction between the shell and gun, they must at all times be equal and opposite. Hence, their impulses for the interval of explosion are equal and opposite since the forces act exactly for the same time 't'.

Let m_1 and m_2 be the masses of the shell and gun. If the initial velocities of the shell and gun are assumed to be zero, and if the external forces are neglected, then:

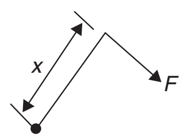
$$m_1v_1 = m_2v_2 \text{ , i.e., } \frac{v_2}{v_1} = \frac{m_1}{m_2}$$

The velocities of the shell and gun, after discharge, are in opposite directions, and inversely proportional to the corresponding masses. Internal forces in a system of particles always appear as pairs of equal and opposite forces and need not be considered when applying the equation of momentum and impulse. Thus, it may be stated that for a system of particles on which no external forces are applied, the momentum of the system remains unchanged, since the total impulse is zero. This is sometimes called the principle of conservation of momentum.

Moment and Couple

Moment or moment of a force is the turning effect caused by the force. It is the force acting at a perpendicular distance '*d*'.

Moment of a force = Force × Perpendicular distance.

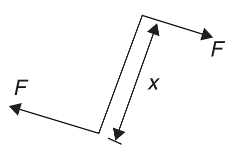


Moment = $F \times x$

Couple

Two equal and opposite forces with separate lines of action present in a system of forces constitute a couple. Both forces create their own moment of force. The net moment of the couple is independent of the location of the point considered.

Moment of couple = Force × Perpendicular distance between the forces.



Moment of couple = $F \cdot x$

- Moment is the measure of the turning effect produced by a force about a point. Couple consists of two forces, equal and opposite, acting in two different, but parallel lines of action.
- Moment of a couple is independent of the location of the pivot or point considered.

Work and Energy

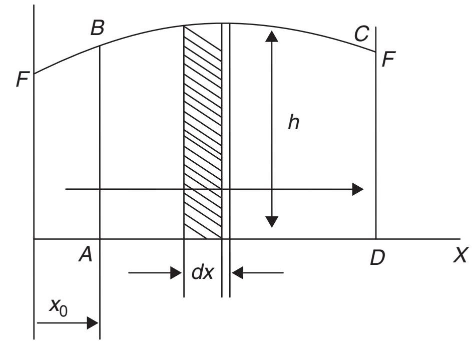
The differential equation of the rectilinear motion of a particle can be written in the following form:

$$m\frac{d\dot{x}}{dt} = F$$

Multiplying both sides of the above equation by $\dot{x}$ and with suitable modifications, the above equation can be written as follows:

$$d\left(\frac{m\dot{x}^2}{2}\right) = F\,dx \qquad (2)$$

It is assumed that the force '*F*' is known as a function of the displacement x of the particle. It is represented by the following force-displacement diagram.



The right-hand side of Eq. (2) is represented by the area of the elemental strip of the height '*h*' and width dx in the above figure. This quantity represents the work done by the force '*F*' on the infinitesimal displacement dx. The expression in the parenthesis on the left-hand side of Eq. (2) is called the kinetic energy of the particle. Eq. (2) thus states that the differential change in the kinetic energy of a moving particle is equal to the work done by the acting force on the corresponding infinitesimal displacement dx. Work and kinetic energy have the same dimensions of the product of force and length. They are usually expressed in the unit of Joules (J).

Integrating Eq. (2) with the assumption that the velocity of the particle is $\dot{x}_0$ when the displacement is x_0, we have:

$$\frac{m\dot{x}^2}{2} - \frac{m\dot{x}_0^2}{2} = \int_{x_0}^{x} F\,dx \qquad (3)$$

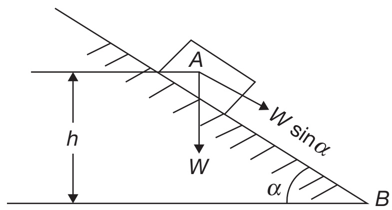
The definite integral on the right-hand side of Eq. (3) is represented by the area *ABCD* of the force-displacement diagram. This is the total work of the force '*F*' on the finite displacement of the particle from x_0 to x. The work of a force is considered positive if the force acts in the direction of the displacement. It is negative if acts in the opposite direction. The total change in the kinetic energy of a particle during a displacement from x_0 to x is equal to the work of the acting force on the displacement.

The equation of work and energy is especially useful in cases where the acting force is a function of displacement and where the velocity of the particle as a function of displacement is of interest. For example, the velocity with which a weight '*W*' falling from a height h strikes the ground is to be determined. In this case, the acting force $F = W$ and the total work is Wh. Thus, if the body starts from rest, the initial velocity $\dot{x}_0 = 0$, and hence Eq. (3) becomes:

$$\frac{m\dot{x}^2}{2} = Wh \qquad (4)$$

which yields $\boxed{\dot{x} = v = \sqrt{2gh}}$.

Let the same body slide, without friction, along an inclined plane AB starting from an elevation h above point B as shown in the following figure.



The equation of work and energy can be used to determine the velocity of the body when it reaches point B. Here, only the component $W\sin\alpha$ of the gravity force does work on the displacement. The component perpendicular to the inclined plane is at all times balanced by the reaction of the plane. In short, the resultant of all the forces acting on the body is $F = W\sin\alpha$ in the direction of motion, and this force acts through the distance $\left(\dfrac{h}{\sin\alpha}\right)$. The work of the force acting on the body is $= W\sin\alpha \times \dfrac{h}{\sin\alpha} = Wh$, and hence velocity at the point B (derived from Eq. (4)), $v = \sqrt{2gh}$. Hence, the velocity is the same as that gained in a free fall through the height h.

If μ is the coefficient of friction between the block and the inclined plane, then the work of friction has to be considered in Eq. (3).

In such a case, the resultant acting force in the direction of motion:

$$F = W\sin\alpha - \mu W\cos\alpha.$$

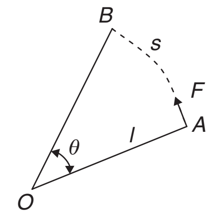
Then, through the displacement $\dfrac{h}{\sin\alpha}$ between points A and B, the work done is $= Wh - \mu Wh\cot\alpha$. Eq. (3) would then yield:

$$\boxed{v = \sqrt{2gh(1-\mu\cot(\alpha))}}$$

When $\alpha = \dfrac{\pi}{2}$, the above equation agrees with the velocity equation derived for a free-falling body. When $\mu = 0$, the above equation agrees with the velocity equation derived for the inclined plane motion of the body with no friction. Also, from the above equation, it can be noted that to obtain a real value for the velocity, $\mu < \tan\alpha$ otherwise, the block would not slide down.

Work done by Torque

Consider a light rod of length l pin joined at one end and is turned by an angle θ by force 'F' from position A to B. Work done by the constant torque is the product of the torque and the angle turned by the rod.

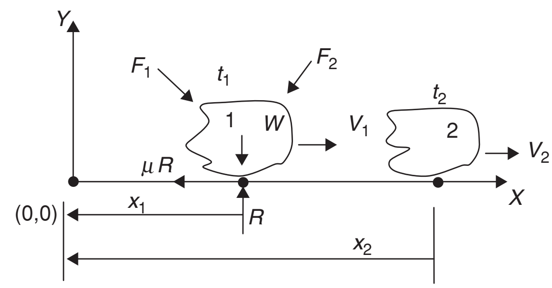


$$\therefore \text{Work done} = F\cdot s$$
$$= F\cdot r\cdot\theta = T\cdot\theta.$$

Work Energy Formulations

- Kinetic energy of a body/particle in translation $= \dfrac{1}{2}\, mv^2$.
- Kinetic energy of a body/particle in rotation and rotating about a point $= \dfrac{1}{2} I\, W^2$.
- Work-energy principle for a body/particle in translation. Work done on body/particle between points 1 and 2 is

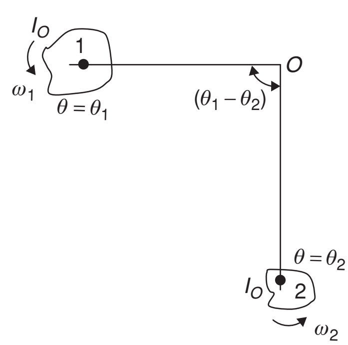
$$W_{1-2} = \int_{x_1}^{x_2} \Sigma F_x\, dx.$$



Change in kinetic energy from position 1 to 2 is $(\Delta\,\text{KE})_{1-2}$ $= \dfrac{1}{2} m\left(v_2^2 - v_1^2\right)$

$$\therefore\ W_{1-2} = \int_{x_1}^{x_2} \Sigma F_x\, dx = \frac{1}{2} m\left(v_2^2 - v_1^2\right)$$

Work-energy principle for a body/particle in rotation.



Work done from position 1 to 2 is given by:

$$W_{1-2} = \int_{\theta_1}^{\theta_2} \Sigma M_o\, d\theta$$

Change in kinetic energy from position 1 to 2 is:

$$KE_{1-2} = \frac{1}{2} I_O \left(\omega_1^2 - \omega_2^2\right)$$

$$\therefore \text{ Work done, } W_{1-2} = \int_{\theta_1}^{\theta_2} \Sigma M_o \, d\theta = \frac{1}{2} I_O \left(\omega_1^2 - \omega_2^2\right).$$

NOTES

1. Work done by a force is zero if displacement is zero or the force acts normal to the displacement. For example, gravity force does not work when a body moves horizontally.
2. Work done by a force is positive if the direction of force and the direction of displacement are same. For example, work done by force of gravity is positive when a body moves from a higher elevation to a lower elevation. A positive work can be described as the work done by a force. On the other hand, a negative work is the work done against a force.
3. Work is a scalar quantity. It has magnitude, but no direction.
4. Work done by a force depends on the path over which the force moves except in the case of conservative forces. Forces due to gravity, spring force are conservative forces, whereas friction force is a non-conservative force.

Example 29

If a bucket of water weighing 15 N is pulled up from a well of 25 m depth with a rope weighing 1.5 N/m, then the work done is

(A) 843.75 Nm (B) 500 Nm
(C) 575 Nm (D) 600 Nm

Solution

The work done to pull the rope

$= \int_0^{25} 1.5 \times (25 - h) \, dh$ (h is the tip of the rope from the bottom of the well)

$$= 1.5 \times \frac{252}{2} = 468.75 \text{ Nm}$$

Total work done = Work done to pull the bucket + Work done to pull the rope

= 15 *x* 25 + 468.75 = 843.75 Nm.

Hence, the correct answer is option (A).

Example 30

A uniform chain of length 10 m and mass 100 kg is lying on a smooth table such that one-third of its length is hanging vertically down over the edge of the table. If 'g' is the acceleration due to gravity, then the work required to pull the hanging part of the chain is

(A) $50g$ (B) $55.55g$
(C) $100g$ (D) $150g$

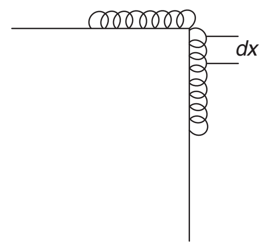
Solution

Work done = potential energy change in the raising of the centre of mass over the distance $\frac{L}{6}$.

$$= \frac{m}{3} g \frac{L}{6} = \frac{100 \times g \times 10}{18} = \frac{1000g}{18} = 55.55g$$

Hence, the correct answer is option (B).

Alternate Method

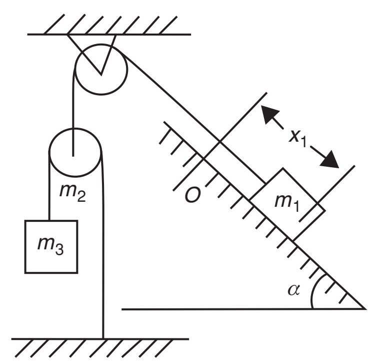


$$W = \int_0^{L/3} \frac{mg}{L} x \, dx = \frac{mg}{L} \left(\frac{x^2}{2}\right)_0^{L/3}$$

$$= \frac{mg}{L} \times \frac{L^2}{18} = mg \times \frac{L}{18}.$$

Ideal Systems—Conservation of Energy

The method of work and energy for a single particle can be extended to apply to a system of connected particles as shown in the following figure. In doing so, it is to be noted that the attention is limited to ideal systems with one degree of freedom. It is assumed that the system has frictionless constraints and inextensible connections and its



configuration can be completely specified by one coordinate, such as x_1 in the above figure. In the case shown in the above figure, for example, the assumptions involve a smooth inclined plane, frictionless bearings, inextensible strings and neglecting entirely the rotational inertia of the pulleys. Then the system may be regarded simply as three particles, m_1, m_2, and m_3, each of which performs a rectilinear motion. From kinematics, the displacements and velocities of all the three masses can be expressed in terms of one variable, say the coordinate x_1 of the particle m_1.

During motion of the system, an infinitesimal interval of time dt is considered during which the system changes its configuration slightly and each particle is displaced by a length of dx_i, along its line of motion. If F_i is the resultant force acting on any particle m_i, then the total increment of work of all the forces during such a displacement:

$$\boxed{dU = \Sigma F_i dx_i} \qquad (5)$$

For the system of particles, it can be shown as:

$$\boxed{dT = dU} \qquad (6)$$

where $T = \frac{1}{2}\Sigma\left(m_i \dot{x}_i^{\,2}\right)$, T is the total kinetic energy of the system of particles with the mass and velocity of the i^{th} particle being m_i and $\dot{x}_i$. Eq. (6) states that the differential change in the total kinetic energy of the system when it slightly changes its configuration is equal to the corresponding increment of work of all the forces.

Consider any two configurations of the system denoted by the subscripts A and B, then from Eq. (6), we have:

$$\boxed{\int_{T_A}^{T_B} dT = \int_{x_A}^{x_B} dU \text{ or}}$$

$$\boxed{T_B - T_A = \int_{x_A}^{x_B} dU} \qquad (7)$$

This is the equation of work and energy for a system of particles. It states that the total change in the kinetic energy of the system when it moves from configuration A to configuration B is equal to the corresponding work of all the forces acting upon it. In the case of an ideal system, the reactive forces will produce no work and work of all the internal forces which occur in equal and opposite pairs will cancel each other. Thus, for such systems, only the work of active external forces is to be considered on the right-hand side of Eq. (7).

The potential energy of a system in any configuration (A or B) is defined as the work which will be done by the acting forces if the system moves from that configuration (A or B) back to a certain base or reference configuration (O). If V_A and V_B are the potential energies of the system in configurations A and B, then

$$\boxed{V_A = \int_A^0 dU \text{ and } V_B = \int_B^0 dU.}$$

NOTE

If a particle of weight 'w' is at an elevation 'x' above a chosen datum plane, then the potential energy of the particle, $V = mx$. Similarly, for a system of particles at an elevation, the potential energy of the system, $V = \sum w_i x_i = W x_c$.

Where w_i and x_i are the weight and elevation above a chosen datum plane for the ith particle, W is the total weight of the system and x_c is the elevation of the center of gravity of the system above the chosen datum plane.

For the system of particles moving from the configuration A to the configuration B, it can be shown that $T_B + V_B = T_A + V_A$.

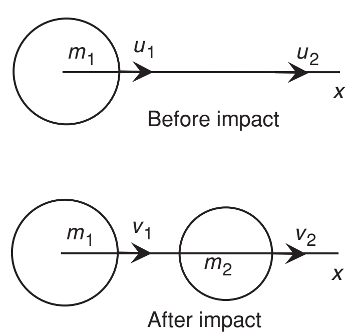
Law of Conservation of Energy

As the system moves from one configuration to another, the total energy (kinetic + potential) remains constant. Kinetic energy may be transformed into potential energy and vice versa, but the system as a whole can neither gain nor lose energy. This is the *law of conservation of energy* as it applies to a system of particles with ideal constraints. Such systems are sometimes called '*conservative systems*'.

Impact

The impact between two moving bodies refers to the collision of the two bodies that occur in a very small interval, and during which the bodies exert a very large force (active and reactive force) on each other. The magnitudes of the forces and the duration of impact depend on the shapes of the bodies, their velocities, and elastic properties.

Consider the impact of two spheres of masses m_1 and m_2 as shown in the following figure. Let the spheres have the respective velocities of u_1 and u_2, where $u_1 > u_2$ before impact, and the respective velocities of v_1 and v_2 after impact.



It is assumed that these velocities are directed along the line joining the centres of the two spheres and, are considered to be positive if they are in the positive direction of the X-axis. This is called the case of direct central impact. Two equal and opposite forces, i.e., action and reaction, are produced at the point of contact during impact. According to the law of conservation of momentum, such forces cannot change the momentum of the system of two balls, hence:

$$\boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2} \quad (8)$$

Elastic Impact

In an elastic impact, the momentum and kinetic energy is conserved. If the kinetic energy is conserved during impact, then:

$$\boxed{\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2} \quad (9)$$

Since momentum is conserved, Eq. (8) is also applicable in this type of impact. From Eqs. (8) and (9), it can be shown that:

$$\boxed{v_1 - v_2 = -(u_1 - u_2)} \quad (10)$$

This equation represents a combination of the law of conservation of momentum and conservation of energy. It states that for an elastic impact the relative velocity after impact has the same magnitude as was before impact, but with reversed sign.

For two bodies of equal masses undergoing an elastic impact, from Eqs. (8) and (10) it can be shown that they will exchange their velocities, i.e., $v_1 = u_2$ and $v_2 = u_1$. If the second body was at rest before the impact, i.e., $u_2 = 0$, then it would be noted that the striking body stops, i.e., $v_1 = 0$, after having imparted its velocity to the other ball. This phenomenon can be observed in the case of a moving billiard ball which squarely strikes one that was at rest. Again, if the two balls were moving toward each other with equal speeds before impact, an exchange of velocities will simply mean that they rebound from one another with the same speed with which they collided. As another special case, we assume that $m_2 = \infty$ while m_1 remains finite and further $u_2 = 0$. This will represent the case of an elastic impact of a ball against a flat immovable obstruction, such as the dropping of a ball on a cement floor. In this case, it is obtained that $v_1 = -u_1$, i.e., the striking ball rebounds with the same speed with which it hits the obstruction.

Plastic or Inelastic Impact

In a plastic or inelastic impact, the momentum is conserved but the kinetic energy is not (part of the kinetic energy is converted to a different form of energy). In a perfectly plastic impact, the colliding bodies will stick to each other after collision and will move with a common velocity. If v is the common velocity of two colliding bodies after a perfectly plastic impact, then from Eq. (8), we have:

$$\boxed{v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}}$$

Newton's Experimental Law of Colliding Bodies

Newton proposed an experimental law that describes how the impact of moving bodies was related to their velocities and found that:

$$\frac{\text{Speed of separation}}{\text{Speed of approach}} = e$$

e = Coefficient of restitution
e satisfies the condition $0 \le e \le 1$.
If $e = 1 \Rightarrow$ the collision is perfectly elastic
If $e = 0 \Rightarrow$ the collision is inelastic
If $0 < e < 1 \Rightarrow$ the collision is said to be elastic.

Energy Loss Due to Impact

The energy lost in impact when $e \ne 1$, i.e., when the collision is not perfectly elastic is given by:

$$\text{Loss in kinetic energy} = \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}(u_1 - u_2)^2(1 - e^2).$$

$\therefore$ When $e = 1$ the loss is zero.

Coefficient of Restitution

It is defined as the ratio of the relative velocity of the impacting bodies after impact to their relative velocity before an impact. The coefficient of restitution 'e' is given by the following equation:

$$\boxed{e = \frac{(v_2 - v_1)}{(u_1 - u_2)}}$$

Example 31

A bullet travelling with a velocity of 800 m/s and weighing 0.25 N strikes a wooden block of weight 50 N resting on a horizontal floor. The coefficient of friction between floor and the block is 0.5. Determine the distance through which the block is displaced from its initial position.

Solution

Velocity of the bullet before impact, $v_a = 800$ m/s
Velocity of the block before impact, $v_b = 0$ m/s

Mass of the bullet, $m_a = \frac{0.25}{g}$ kg

Mass of the block, $m_b = \frac{50}{g}$ kg

The bullet after striking the block remains buried in the block and both move with a common velocity v.
Applying the principle of conservation of momentum:

$$m_av_a + m_bv_b = (m_a + m_b)v$$

$$\frac{0.25}{g} \times 800 + \frac{50}{g} \times 0 = \left(\frac{0.25}{g} + \frac{50}{g}\right)v$$

$$v = 3.98 \text{ m/s}$$

To find the distance travelled by the block, apply the priniciple of work and energy. Kinetic energy lost by the block with the bullet buried = work done to overcome the frictional force.

If s is the distance travelled by the block, then:

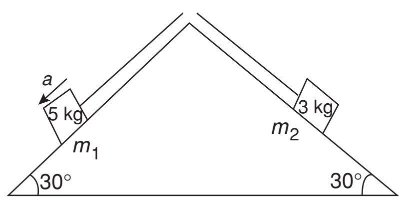
$$\frac{1}{2}(m_a + m_b)v^2 = \mu R\, s$$

$$= \mu g(m_a + m_b)\, s (\because\ R = g(m_a + m_b))$$

$$\therefore\ s = \frac{3.98^2}{2 \times 9.81 \times 0.5} = 1.61 \text{ m.}$$

Example 32

Two bodies of masses of 5 kg and 3 kg resting on two inclined planes each of elevation 30° and are connected by a string passing over the common apex. After two seconds, the body with 5 kg is removed. How far up the plane will be 3 kg body continue to move? (Neglecting the frictional force.)



Solution

$$a = \frac{m_1 \sin\theta_1 - m_2 \sin\theta_2}{m_1 + m_2} \cdot g \ = \frac{5\sin\theta_1 - m_2 \sin\theta_2}{m_1 + m_2} \times 9.81$$

$= 1.23$ m/s^2

Now, considering the motion of the 3 kg body when the 5 kg body is not removed.

Initial velocity = u = 0

Acceleration = a = 1.23 m/s^2

Time taken = t = 2 seconds

Let v = final velocity = $u + at = 0 + 2 \times 1.23 = 2.46$ m/s

Now, after 2 seconds, when the 5 kg body is removed:

Let u_1 = initial velocity = 2.46 m/s

V_1 = final velocity = 0

a_1 = acceleration = $-g\sin 30°$

$$= \frac{9.81}{2} = -\,4.905 \text{ m/s}^2$$

(Downward motion has resistance due to gravity because after cutting the string, mass 3 kg will tend to move further up, but gravity will pull it down).

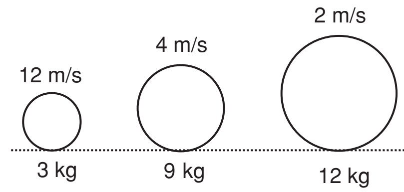
Using the relationship $v^2 - u^2 = 2as$

$\Rightarrow 0 - (2.46)^2 = 2 \times (-4.905)s$

$\Rightarrow s = 0.617$ m.

Example 33

Three spherical balls of masses 3 kg, 9 kg and 12 kg are moving in the same direction with velocities 12 m/s, 4 m/s, and 2 m/s. If the ball of mass 3 kg impinges with the ball of mass 9 kg which, in turn, impinges with the ball of mass 12 kg. Prove that the balls of masses 3 kg and 9 kg will be brought to rest by the impacts. Assume the balls to be perfectly elastic.



Solution

For perfectly elastic balls, $e = 1$

$m_a = 3$ kg, $m_b = 9$ kg, $m_c = 12$ kg

Impact of balls A and B

Conservation of momentum gives,

$$m_a v_a + m_b v_b = m_a v'_a + m_b v'_b$$

$$3 \times 12 + 9 \times 4 = 3v'_a + 9v'_b \qquad (1)$$

$$e = -\frac{v'_b - v'_a}{v_b - v_a}$$

$$v'_b - v'_a = e(v_a - v_b) = 1 \times (12 - 4) = 8 \qquad (2)$$

Solving Eqs. (1) and (2), we get $v'_b = 8$ m/s and $v'_a = 0$ m/s, i.e., the ball of mass 3 kg is brought to rest.

Impact of Balls *B* and *C*

Now, consider the impact of ball B of mass 9 kg and moving with the initial velocity of 8 m/s with the ball C of mass 12 kg and moving with the velocity of 2 m/s.

Conservation of momentum gives:

$$m_b v_b + m_c v_c = m_b v'_b + m_c v'_c$$

$$9 \times 8 + 12 \times 2 = 9v'_b + 12v'_c \qquad (3)$$

$$e = -\frac{v'_c - v'_b}{v_c - v_b}$$

$$v'_c - v'_b = e(v_b - v_c)$$

$$= 1 \times (8 - 2) = 6 \qquad (4)$$

Solving Eqs. (3) and (4), we get $v'_c = 6$ m/s and $v'_b = 0$ m/s, i.e., the ball of mass 9 kg is brought to rest.

Direction for solve examples 34 and 35:

The blocks 1 and 2 having a weight of 1 kg each and velocities of 10 m/s and 4 m/s undergo a perfect inelastic collision.

Example 34

The final velocity of the blocks is

(A) 7 m/s (B) 6 m/s
(C) 3 m/s (D) 4 m/s

Solution

$$V = \frac{M_1V_1 + M_2V_2}{m_1 + m_2} = \frac{1\times10+4\times1}{1+1}$$
$$= 7 \text{ m/s}$$

Hence, the correct answer is Option (A).

Example 35

The energy converted into heat as a result of the collision is

(A) 40 J (B) 9 J
(C) 50 J (D) 54 J

Solution

The original kinetic energy was:

$$K_1 = \frac{1}{2}\times1\times100 + \frac{1}{2}\times1\times16 = 58 \text{ J}$$

The final kinetic energy is:

$$K_2 = \frac{1}{2}\times2\times49 = 49 \text{ J}$$

Loss of kinetic energy = 58 – 49 = 9 J (converted to heat energy).
Hence, the correct answer is option (B).

Example 36

An elastic ball of mass 'm' is projected vertically upwards from a point on the horizontal plane with velocity u. If 'e' be the coefficient of elasticity, then find:

(i) The of the heights attained by the ball after each rebound, until it finally comes to rest.
(ii) The time that elapses to the instant of nth rebound. What is its kinetic energy after the nth rebound?

Solution

(i) When a body is projected upwards with a velocity u, it goes up a height of $\frac{u^2}{2g}$ The velocity with which a body having velocity u, rebounds from the floor = eu, where 'e' is the coefficient of restitution (or elasticity) between the ball and the floor.
Distance covered before the rebound

$$= \frac{u^2}{2g} \qquad (1)$$

Distance covered after the rebound

$$= \frac{(eu^2)}{2g}$$

∴ Total space described by the body

$$= \frac{u^2}{2g} + \frac{(eu)^2}{2g} + \frac{(e^2u)^2}{2g} + \frac{(e^nu)^2}{2g}$$

$$= \frac{u^2}{2g}[1 + e^2 + e^4 + e^6 + \ldots e^{2n}]$$

$$= \frac{u^2}{2g}\left(\frac{1-e^{2n}}{1-e^2}\right).$$

(ii) From the fact that a body projected upward with a velocity u m/s takes $\frac{2u}{g}$ seconds reaching the ground, for the present case:
Time taken by the body in rebound

$$= \frac{2u}{g}$$

Time taken after the first rebound

$$= \frac{2(eu)}{g}$$

Hence, after n rebounds, time taken after nth rebound to reach the ground = $\frac{2(e^nu)}{g}$

Summing up, time taken

$$= \frac{2u}{g} + \frac{2(eu)}{g} + \frac{2(e^2u)}{g} + \cdots + \frac{2(e^nu)}{g}$$

$$= \frac{2u}{g}(1 + e + e^2 + \ldots e^n)$$

$$= \frac{2u}{g}\left(\frac{1-e^n}{1-e}\right)$$

Also, at the end of the nth rebound, velocity of body $= e^nu$

$$\therefore \text{ KE of the body} = \frac{1}{2}m(e^nu)^2$$

$$= \frac{me^2nu^2}{2}$$

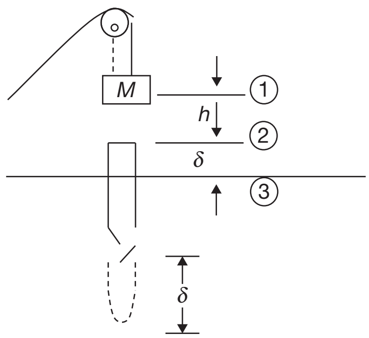
When the body with velocity 'u', falls form a height H, then using

$$H = \frac{u^2}{2g}.$$

Exercises

Direction for questions 1 and 2:

A pile of mass 400 kg is driven by a distance of d into the ground by the blow of a hammer of mass 800 kg through a height of h onto the top of the pile. Assume the impact between the hammer and pile to be plastic.



Given $M = 800$ kg, $m = 400$ kg, $h = 1.2$ m, $\delta = 10$ cm.

1. The work done is
(A) 5.28 kJ (B) 6.278 kJ
(C) 7.126 kJ (D) 6.8 kJ

2. The kinetic energy of the whole system in the position 3 is
(A) 0 J (B) 10 J
(C) 100 J (D) 20 J

3. A particle starts with velocity 2 m/s and accelerates at a rate of 3 m/s^2 for 15 seconds and then retards at 6 m/s^2 until it stops. The total distance covered is
(A) 184.08 m (B) 551.58 m
(C) 367.5 m (D) None of these

4. A point 'P' moves along a straight line as per the law $x = 4t^2 + 12t + 1$, the velocity of the point after 3rd and 4th seconds are respectively.
(A) 36 m/s and 48 m/s
(B) 36 m/s and 44 m/s
(C) 34 m/s and 44 m/s
(D) 34 m/s and 46 m/s

5. A particle moving in space with velocity $J = 3t^2i + 4tj - 7t^3k$. The acceleration of the particle at $t = 1$ will be
(A) $3i + 8j - 7k$ (B) $6i + 4j + 21k$
(C) $6i + 4j - 21k$ (D) zero

6. A ball of mass 5 kg moving with a velocity of 6 m/s makes impact with another ball of mass 3 kg moving in the same direction with a velocity of 4 m/s. If coefficient of restitution is 0.5, velocities of the balls after impact are
(A) 4.875 m/s, 5.875 m/s
(B) 4.962 m/s, 6.125 m/s
(C) 5.125 m/s, 6.536 m/s
(D) 5.565 m/s, 6.926 m/s

Direction for questions 7 and 8:

A pile of mass 500 kg is driven by a mass 350 kg falling on it vertically through a distance of 1 m. After impact, the falling mass and pile remain in contact and move together. The pile is moved 150 mm at each blow.

7. Energy lost in each blow is
(A) 1676 Nm (B) 1762 Nm
(C) 1915 Nm (D) 2020 Nm

8. Average resistance against the pile is
(A) 17.765 kN (B) 18.625 kN
(C) 20.516 kN (D) 22.835 kN

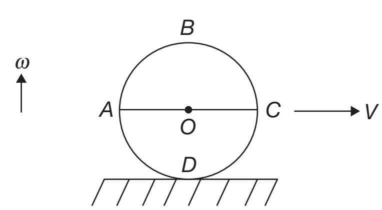
9. A body of mass 5 kg falls from a height of 50 m and penetrates into the ground by 90 cm. Average resistance to penetration is
(A) 2668 N (B) 2774 N
(C) 2814 N (D) 2892 N

10. Which of the following relation represents motion under variable acceleration?
(A) $v = a\dfrac{dv}{ds}$ (B) $a = v\dfrac{dv}{ds}$
(C) $v = \dfrac{1}{a}\dfrac{dv}{ds}$ (D) None of these

11. The value of coefficient of restitution is one for
(A) perfectly elastic collision
(B) perfectly inelastic collision
(C) neither plastic nor elastic collision
(D) None of these

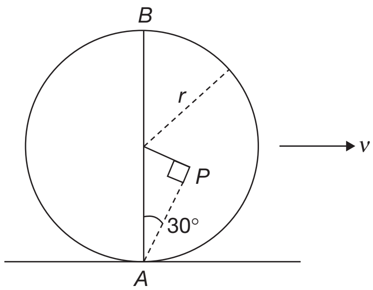
12. A rigid body has a combined translational and rotational motion. Its mass is 'M' and linear and angular velocities are 'V' and 'ω' respectively. If 'I' is its moment of inertia, the total energy of the rigid body is
(A) $\dfrac{1}{2}mV^2/I\omega^2$ (B) $\dfrac{1}{2}mV^2 + \dfrac{1}{2}I\omega^2$
(C) $mV^2 + \dfrac{1}{2}I\omega^2$ (D) $\dfrac{1}{4}(mV^2 + I\omega^2)$

13. A wheel is rolling on a straight road as shown below. For this wheel the acceleration of the center 'O' and its instantaneous center are



(A) $\omega^2 r$ and O (B) $\omega^2 r$ and D
(C) V^2/r and D (D) zero and O

14. A sphere A impinges directly with another sphere B of same mass at rest. Coefficient of restitution is 0.6. Ratio of their velocities (V_A:V_B) after impact is
(A) 1 : 2 (B) 1 : 3
(C) 1 : 4 (D) 1 : 5

15.


A circular disk of radius r rolls without slipping at a velocity v as shown in the figure. Magnitude of the resultant velocity at point P is
(A) $v\sqrt{3}$ (B) $v\dfrac{\sqrt{3}}{2}$
(C) $\dfrac{v}{2}$ (D) $\dfrac{2v}{3}$

16. A sphere moving with a uniform velocity impinges directly up on another identical sphere at rest. After impact the first sphere comes to rest and the other moves. During the collision, 36% of kinetic energy gets dissipated. Coefficient of restitution is
(A) 0.8 (B) 0.7
(C) 0.6 (D) 0.5

17. A bullet moving with a speed of 450 m/s penetrates 10 cm into a fixed wooden block. The average force exerted by the wooden block on the bullet is 20.25 kN. Then mass of the bullet is
(A) 0.015 kg (B) 0.018 kg
(C) 0.02 kg (D) 0.026 kg

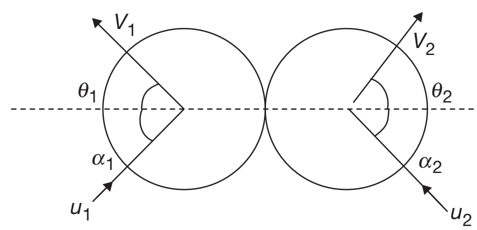
18. A batsman strikes a cricket ball of mass 100 gm bowled towards him. Before striking, the ball was moving horizontally and had a velocity of 20 m/s. After striking, the ball moved with a velocity of 35 m/s at an angle 45° with horizontal. If the impact of the bat on the ball lasted for 0.02 second, the average impulsive force exerted was
(A) 255.68 N (B) 268.32 N
(C) 277.46 N (D) 288.45 N

19. A car of weight 150 kN is climbing a slope of 1 in 40. The road resistance is 3600 N. Power required to run the car at a speed of 20 km/h is
(A) 34.33 kW (B) 36.44 kW
(C) 38.62 kW (D) 40.83 kW

20. Two metallic balls having potential energy in the ratio 3 : 5 are made to slide down a frictionless inclined plane with zero position. What will be the ratio of their kinetic energy when they reach at bottom of inclined plane?
(A) 5 : 3 (B) 3 : 5
(C) 1 : 1 (D) 2 : 3

21. Acceleration of a body moving along straight line varies with time and is given by $a = 2 - 3t$ 5 minutes after from start of the observation, the velocity was 20 m/s. Time after start in which the velocity becomes zero is
(A) 4.48 seconds (B) 5.22 seconds
(C) 6.33 seconds (D) 6.92 seconds

22. Two smooth balls of mass 1 kg each collide such that the line of impact is horizontal. The balls with initial velocities 20 m/s and 30 m/s respectively were moving at 30° and 60° to horizontal as shown in the figure. After collision the velocities of the balls were 16.7 m/s and 30.4 m/s respectively. Inclination of the velocities to horizontal are θ_1 and θ_2 are respectively.



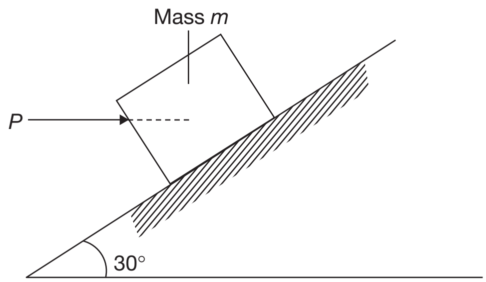
(A) 41° and 61° (B) 61° and 41°
(C) 34.8° and 57.8° (D) 36.8° and 58.7°

23. Angular displacement of a body is given by, $\theta = 6t^2 + 3t + 10$. Where t is in seconds. Angular velocity and angular acceleration of the body when $t = 10$ seconds are
(A) 123 rad/s, 12 rad/s^2
(B) 135 rad/s, 14 rad/s^2
(C) 142 rad/s, 16 rad/s^2
(D) 153 rad/s, 18 rad/s^2

24. Acceleration of a particle is given by, $a = t^3 - 3t^2 + 5$. Where t = time in seconds and a = acceleration in m/s^2. Velocity of particle when $t = 2$ s is 8 m/s. Velocity of the particle when $t = 4$ s is
(A) 22 m/s (B) 25 m/s
(C) 28 m/s (D) 32 m/s

25. A car starting from rest attains a speed of 64 km/hour over a distance of 480 m. Assuming uniform acceleration, time taken to cover the distance is
(A) 36 seconds (B) 42 seconds
(C) 48 seconds (D) 54 seconds

26.



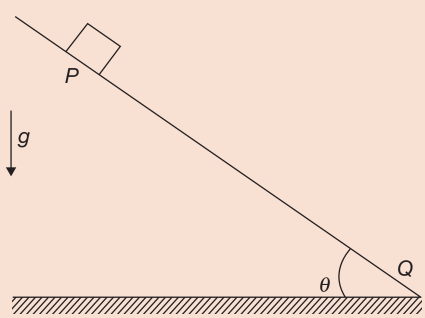
A block of mass m is in equilibrium on an inclined plane of 30° by the action of an horizontal force P as shown in the figure. The value of the force is

(A) $mg\sqrt{3}$ (B) $mg\sqrt{2}$

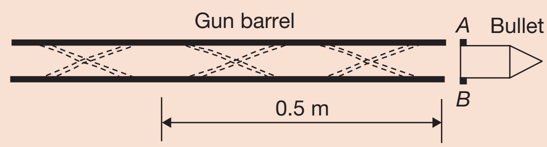
(C) $\frac{mg}{\sqrt{2}}$ (D) $\frac{mg}{\sqrt{3}}$

Previous Years' Questions

1. During inelastic collision of two particles, which one of the following is conserved? **[GATE 2007]**
   (A) Total linear momentum only.
   (B) Total kinetic energy only.
   (C) Both linear momentum and kinetic energy.
   (D) Neither linear momentum nor kinetic energy.
2. A block of mass M is released from point P on rough inclined plane with inclination angle θ, shown in the figure below. The coefficient of friction is μ. If $\mu < \tan\theta$, then the time taken by the block to reach another point Q on the inclined plane, where $PQ = s$, is **[GATE, 2007]**



   (A) $\sqrt{\frac{2s}{g\cos\theta(\tan\theta - \mu)}}$
   (B) $\sqrt{\frac{2s}{g\cos\theta(\tan\theta + \mu)}}$
   (C) $\sqrt{\frac{2s}{g\sin\theta(\tan\theta - \mu)}}$
   (D) $\sqrt{\frac{2s}{g\sin\theta(\tan\theta + \mu)}}$
3. The initial velocity of an object is 40 m/s. The acceleration a of the object is given by the following expression:
$$a = -0.1v$$
   where, v is the instantaneous velocity of the object. The velocity of the object after 3 seconds will be _____. **[GATE, 2015]**
4. A bullet spins as the shot is fired from a gun. For this purpose, two helical slots as shown in the figure are cut in the barrel. Projections A and B on the bullet engage in each of the slots.



   Helical slots are such that one turn of helix is completed over a distance of 0.5 m. If velocity of bullet when it exists the barrel is 20 m/s, it spinning speed in rad/s is _____. **[GATE, 2015]**
5. A ball of mass 1 kg, initially at rest, is dropped from a height of 1 m. Ball hits the ground and bounces off the ground. Upon impact with the ground, the velocity reduces by 20%. The height (in m) to which the ball will rise is _____. **[GATE, 2015]**

Answer Keys

Exercises

1. B	**2.** A	**3.** B	**4.** B	**5.** C	**6.** A	**7.** D	**8.** A	**9.** B	**10.** B
11. A	**12.** B	**13.** D	**14.** C	**15.** B	**16.** A	**17.** C	**18.** A	**19.** D	**20.** B
21. C	**22.** D	**23.** A	**24.** A	**25.** D	**26.** D				

Previous Years' Questions

1. A **2.** A **3.** 29.5 to 29.7 **4.** 251 to 252 **5.** 0.64