

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

Direct and Inverse Proportion

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

Learning objectives

- To recall the concept of ratio and proportion.
- To understand the concept of direct and inverse proportion.
- To be able to differentiate direct proportion and inverse proportion.
- To solve application problems using direct and inverse proportion.



Recap

Ratio and Proportion

In class VI we have learnt ratio and proportion. Let us recall them now.

Ratio is the comparison of two quantities of the same kind. If the quantity 'a' is compared with the quantity 'b', the ratio can be written in the form $a : b$ (read as a is to b)

When two ratios $a : b$ and $c : d$ are equal then we say that the ratios are in proportion. This is denoted as $a : b :: c : d$ and it is read as ' a is to b is as c is to d '. In $a : b :: c : d$, product of the means is equal to product of the extremes, that is $bc = ad$.



Try these

1. Find the ratio of the number of circles to number of squares.



2. Find the ratio (i) 555 g to 5 kg (ii) 21 km to 175 m
3. Find the value of 'x' in the following proportions.
(i) $110 : x :: 8 : 88$ (ii) $x : 26 :: 5 : 65$

4.1 Introduction

We come across many situations in day-to-day life where we see a change in one quantity brings a change in the other quantity. Let us learn more about the concept of variation that helps us to handle situations efficiently.

Now, let us consider such situations which we come across in every day life.

We consider the task of cleaning a school,

- (i) when **more** number of students are involved, the time taken to clean will be **less**.
- (ii) when the number of students are **more**, the work done will also be **more**.

In the first case, number of students is compared with time taken and in the second case, the number of students is compared with the amount of work. The quantities taken for comparison decides the type of variation.

Situation 1 Manimala prepares vegetable soup for her family of 4 members. She uses 2 cup of vegetables, 600 ml of water, 1 teaspoon of salt and $\frac{1}{2}$ teaspoon of pepper. Suddenly her aunt and uncle join them. What would be the change in quantities of the ingredients to prepare soup for all the six members?

Situation 2 In a military camp there are 200 soldiers. Grocery for them are available for 40 days. If 50 more soldiers join them, how long will these grocery last?

In the above situations a change in one quantity results in the corresponding change in other quantity. That is in situation 1, more the number of members, more will be the quantity of items. In situation 2, the more number of soldiers in the group, the grocery needed will be more and so it will last for less number of days.

In these situations when one quantity varies that brings a change in other quantity in two ways, that is,

- (i) both the quantities increase or decrease.
- (ii) increase in one quantity causes decrease in the other quantity or vice-versa.

If such quantities varies in constant ratio, they are said to be in proportion. Two different variations give rise to two types of proportion which will be discussed now.

MATHEMATICS ALIVE-DIRECT AND INVERSE PROPORTION IN REAL LIFE



Direct proportion



Inverse proportion

4.2 Direct Proportion

For example, if the cost of a shirt is ₹ 500, then the price of 3 shirts will be ₹ 1,500. The price of the shirts increases as the number of shirts increases. Proceeding the same way we can find the cost of any number of such shirts.

When observing the above situation, two quantities namely the number of shirts and their prices are related to each other. When the number of shirts increases, the price also increases in such a way that their ratio remains constant.

Let us denote the number of shirts as x and the price of shirts as y rupees. Now observe the following table

Number of shirts (x)	1	2	3	6	7	...
Price of shirts in ₹ (y)	500	1000	1500	3000	3500	...

From the table, we can observe that when the values of x increases the corresponding values of y also increases in such way that the ratio $\frac{x}{y}$ in each case has the same value

which is a constant (say k). Now let us find the ratio for each of the value from the table.

$\frac{x}{y} = \frac{1}{500} = \frac{2}{1000} = \frac{3}{1500} = \frac{6}{3000} = \frac{7}{3500}$ and so on. All the ratios are equivalent and its simplified form is $\frac{1}{500}$.

In general, $\frac{x}{y} = \frac{1}{500} = k$ (k is a constant)

When x and y are in direct proportion, we get $\frac{x}{y} = k$ or $x = ky$ (k is a constant)

Considering any two ratios given above, say, $\frac{2}{1000} = \frac{6}{3000}$, where 2 (x_1) and 6 (x_2) are the number of shirts (x) and 1000 (y_1) and 3000 (y_2) are their prices (y). So, when x and y are in direct proportion we can write $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ [where, y_1, y_2 are values of y corresponding to the values x_1, x_2 of x].



Think

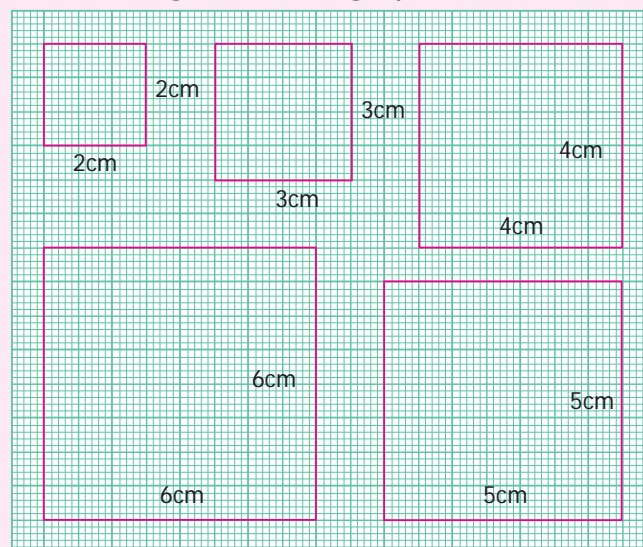
The number of chocolates to be distributed to the number of children. Is this statement in direct proportion?



Try this

Observe the following 5 squares of different sides given in the graph sheet

The measures of the sides are recorded in the table given below. Find the corresponding perimeter and the ratios of each of these with the sides given and complete the table.



Side of the square (x) in cm	2	3	4	5	6
Perimeter of the square (y) in cm					
$\frac{x}{y}$					

From the information so obtained state whether the side of a square is in direct proportion to the perimeter of the square



Think

When a fixed amount is deposited for a fixed rate of interest, the simple interest changes proportionally with the number of years it is being deposited. Can you find any other examples of such kind.

Example 4.1

If 6 children shared 24 pencils equally, then how many pencils are required for 18 children?

Solution

Let x be the number of pencils required for 18 children. As the number of children increases, number of pencils also increases.

In the case of direct proportion we take $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

Number of children	6	18
Number of pencils	24	x

$$\frac{6}{24} = \frac{18}{x}$$

$$\frac{6 \times x}{24} = 18$$

$$x = \frac{18 \times 24}{6} = 72$$

Hence, 72 pencils are required for 18 children.

PENCIL KIDS



Example 4.2

If 15 chart papers together weigh 50 grams, how many of the same type will be there in a pack of $2\frac{1}{2}$ kilogram?

Solution

Let x be the required number of charts.

Number of chart papers	15	x
Weight in grams	50	2500

As weight increases, the number of charts also increases. So the quantities are in direct proportion.

$$\text{Hence } \frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{15}{50} = \frac{x}{2500}$$

$$15 \times 2500 = x \times 50$$

$$x \times 50 = 15 \times 2500$$

$$x = \frac{15 \times 2500}{50} = 750$$

Therefore, 750 charts will weigh $2\frac{1}{2}$ kilogram.



Unitary Method

We know the concept of unitary method studied in the previous class.

First, the value of one unit will be found. It will be useful to find the value of the required number of units. We will try to solve more problems using unitary method.

Example 4.3

Anbu bought 2 notebooks for ₹ 24. How much money will be needed to buy 9 such notebooks?

Solution Using unitary method we can solve this as follows:

$$\text{The cost of 2 notebooks} = ₹ 24$$

$$\text{The cost of 1 notebook} = \frac{24}{2} = ₹ 12$$

$$\begin{aligned}\text{Therefore, the cost of 9 notebooks} &= 9 \times ₹ 12 \\ &= ₹ 108\end{aligned}$$

Hence, Anbu has to pay ₹108 for 9 notebooks.

Example 4.4

A car travels 90km in 2hours 30minutes. How much time is required to cover 210km?

Solution Time taken to cover 90 km = 2hrs 30mins
= 150 minutes

$$\begin{aligned}1 \text{ hour} &= 60 \text{ minutes} \\ 2 \text{ hours} &= 120 \text{ minutes}\end{aligned}$$

$$\text{Time taken to cover 1 km} = \frac{150}{90} \text{ minutes.}$$

$$\begin{aligned}\text{Time taken to cover 210 km} &= \frac{150}{90} \times 210 \text{ minutes} \\ &= 350 \text{ minutes} \\ &= 5 \text{ hours } 50 \text{ minutes}\end{aligned}$$

Thus, the time taken to travel 210 km is 5 hours 50 minutes

Exercise 4.1

1. Fill in the blanks

- If the cost of 8 apples is ₹ 56 then the cost of 12 apples is _____.
- If the weight of one fruit box is $3\frac{1}{2}$ kg, then the weight of 6 such boxes is _____.
- A car travels 60 km with 3 liters of petrol. If the car has to cover the distance of 200 km, it requires _____ liters of petrol.
- If the cost of 7 m cloth is ₹ 294, then the cost of 5 m of cloth is _____.
- If a machine in a cool drinks factory fills 600 bottles in 5 hrs, then it will fill _____ bottles in 3 hours.

2. Say True or False

- Distance travelled by a bus and time taken are in direct proportion.
- Expenditure of a family to number of members of the family are in direct proportion.
- Number of students in a hostel and consumption of food are not in direct proportion.



- (iv) If Mallika walks 1 km in 20 minutes, then she can cover 3 km in 1 hour.
(v) If 12 men can dig a pond in 8 days, then 18 men can dig it in 6 days.

3. A dozen bananas costs ₹ 20. What is the price of 48 bananas?
4. A group of 21 students paid ₹ 840 as the entry fee for a magic show. How many students entered the magic show if the total amount paid was ₹ 1,680?
5. A birthday party is arranged in third floor of a hotel. 120 people take 8 trips in a lift to go to the party hall. If 12 trips were made how many people would have attended the party?
6. The shadow of a pole with the height of 8 m is 6m. If the shadow of another pole measured at the same time is 30m, find the height of the pole?
7. A postman can sort out 738 letters in 6 hours. How many letters can be sorted in 9 hours?
8. If half a meter of cloth costs ₹ 15. Find the cost of $8\frac{1}{3}$ meters of the same cloth.
9. The weight of 72 books is 9kg. what is the weight of 40 such books? (using unitary method)
10. Thamarai pays ₹ 7500 as rent for 3 months. With the same rate how much does she have to pay for 1 year? (using unitary method).
11. If 30 men can reap a field in 15 days, then in how many days can 20 men reap the same field? (using unitary method)
12. Valli buys 10 pens for ₹ 180 and Kamala buys 8 pens for ₹ 96. Can you say who bought the pen cheaper? (using unitary method)
13. A motorbike requires 2 litres of petrol to cover 100 kilometers. How many litres of petrol will be required to cover 250 kilometers? (using unitary method)



Objective type questions

14. If the cost of 3 books is ₹ 90, then find the cost of 12 books.
(i) ₹ 300 (ii) ₹ 320 (iii) ₹ 360 (iv) ₹ 400
15. If Mani buys 5kg of potatoes for ₹ 75 then he can buy _____kg of potatoes for ₹ 105.
(i) 6 (ii) 7 (iii) 8 (iv) 5
16. 35 cycles were produced in 5 days by a company then _____ cycles will be produced in 21 days.
(i) 150 (ii) 70 (iii) 100 (iv) 147
17. An aircraft can accommodate 280 people in 2 trips. It can take _____trips to take 1400 people.
(i) 8 (ii) 10 (iii) 9 (iv) 12
18. Suppose 3 kg. of sugar is used to prepare sweets for 50 members, then ____ kg. of sugar is required for 150 members.
(i) 9 (ii) 10 (iii) 15 (iv) 6

4.3 Inverse Proportion

Let us consider the following situation.

Situation The following table shows the number of workers and the number of days to finish the construction of water tank in school.

Number of workers (x)	2	4	5	10
Number of days (y)	40	20	16	8



In this situation, the two quantities namely number of workers and the number of days are related to each other. We observe that when the number of workers increases, the number of days to finish the job decreases. Consider the number of workers as x and the number of days as y then we note that the product is always the same. That is,

$$x \times y = 2 \times 40 = 4 \times 20 = 5 \times 16 = 10 \times 8$$

Consider each of the value of x and the corresponding value of y . Their products are all equal say $xy = 80 = k$ (k is a constant) and it can be expressed as $xy = k$ (k is a constant)

If y_1 and y_2 are the values of y corresponding to the values of x_1 and x_2 of x respectively then $x_1 y_1 = x_2 y_2 (=k)$ $\frac{x_1}{y_1} = \frac{x_2}{y_2}$. We say that x and y are in inverse proportion.



Think

Think of an example in real life where two variables are inversely proportional.



Try these

- Complete the table given below and find the type of proportion

No. of chocolates	1	___	3	4	5	___	___ proportion
Price in rupees (₹)	5	10	___	20	___	___	

No. of workers	1	2	4	5	___	___	___	___ proportion
Time in hours	20	___	5	___	2	1	___	

- Read the following examples and group them in two categories

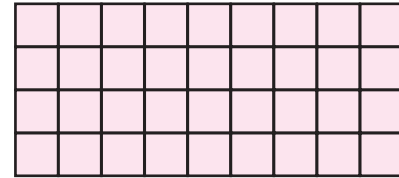
S. No.	Quantities	Direct proportion	Inverse proportion
1.	Number of note books purchased and its cost.		
2.	Food shared by number of students to the fixed quantity of food.		
3.	Number of boxes of same size and their weight		
4.	Number of uniforms to the number of students.		
5.	Speed of a vehicle and the time taken to cover the fixed distance		



Activity

Form all possible rectangles with area 36 sq.cm by completing the following table.

Length	36	18		9
Breadth	1		3	



Observe and answer the following

- (i) When length decreases, the breadth _____
- (ii) When breadth increases, the length _____
- (iii) If the length is 8 cm what will be the breadth? -Discuss.

Extend this activity and try the same with area 24 and 48 sq.units.

Example 4.5

60 workers can spin a bale of cotton in 7 days. In how many days will 42 workers spin it?

Solution

Let x be the required number of days. The decrease in number of workers lead to the increase in number of days. (Therefore, both are in inverse proportion)

Number of workers	60	42
Number of days	7	x

For inverse proportion $x_1y_1 = x_2y_2$

$$\text{Hence } 60 \times 7 = 42 \times x$$

$$42 \times x = 60 \times 7$$

$$x = \frac{60 \times 7}{42}$$

$$x = 10$$

In 10 days 42 workers can spin a bale of cotton.

Example 4.6

The cost of 1 box of tomato is ₹ 200. Vendan had money to buy 13 boxes. If the cost of the box is increased to ₹ 260 then how many boxes will he buy with the same amount?

Solution

The cost of one box = ₹ 200

Increased cost of one box = ₹ 260

Let x be the number of boxes bought by Vendan.

As cost of boxes increases the number of boxes decreases.

This is in inverse proportion. Therefore, $x_1y_1 = x_2y_2$

Number of boxes	13	x
Cost in ₹	200	260

$$13 \times 200 = x \times 260$$

$$x \times 260 = 13 \times 200$$

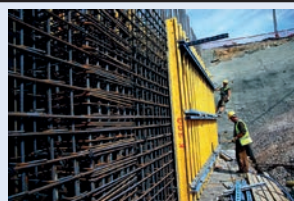
$$x = \frac{13 \times 200}{260}$$



$$x = 10$$

Therefore, he can buy 10 boxes for the same amount.

Direct and indirect proportion is very much useful in project scheduling. A project can be any work that is time bound like building a house, construction of bridges, etc.



Exercise 4.2

- Fill in the blanks.
 - 16 taps can fill a petrol tank in 18 minutes. The time taken for 9 taps to fill the same tank will be _____ minutes.
 - If 40 workers can do a project work in 8 days, then _____ workers can do it in 4 days.
- 6 pumps are required to fill a water sump in 1 hr 30 minutes. What will be the time taken to fill the sump if one pump is switched off?
- A farmer has enough food for 144 ducks for 28 days. If he sells 32 ducks, how long will the food last?
- It takes 60 days for 10 machines to dig a hole. Assuming that all machines work at the same speed, how long will it take 30 machines to dig the same hole?
- Forty students stay in a hostel. They had food stock for 30 days. If the students are doubled then for how many days the stock will last?
- Meena had enough money to send 8 parcels each weighing 500 grams through a courier service. What would be the weight of each parcel, if she has to send 40 parcels for the same money?
- It takes 120 minutes to weed a garden with 6 gardeners. If the same work is to be done in 30 minutes, how many more gardeners are needed?
- Neelaveni goes by bi-cycle to her school every day. Her average speed is 12km/hr and she reaches school in 20 minutes. What is the increase in speed, if she reaches the school in 15 minutes?
- A toy company requires 36 machines to produce car toys in 54 days. How many machines would be required to produce the same number of car toys in 81 days?

Objective type questions

- 12 cows can graze a field for 10 days. 20 cows can graze the same field for _____ days.

(i) 15	(ii) 18	(iii) 6	(iv) 8
--------	---------	---------	--------
- 4 typists are employed to complete a work in 12 days. If two more typists are added, they will finish the same work in _____ days.

(i) 7	(ii) 8	(iii) 9	(iv) 10
-------	--------	---------	---------

Exercise 4.3

Miscellaneous Practice problems



1. If the cost of 7kg of onions is ₹ 84 find the following
 - (i) Weight of the onions bought for ₹ 180
 - (ii) The cost of 3 kg of onions
2. If $C = kd$, (i) what is the relation between C and d ?
(ii) find k when $C = 30$ and $d = 6$ (iii) find C , when $d = 10$
3. Every 3 months Tamilselvan deposits ₹ 5000 as savings in his bank account. In how many years he can save ₹ 1,50,000.
4. A printer, prints a book of 300 pages at the rate of 30 pages per minute. Then, how long will it take to print the same book if the speed of the printer is 25 pages per minute?
5. If the cost of 6 cans of juice is ₹ 210, then what will be the cost of 4 cans of juice?
6. x varies inversely as twice of y . Given that when $y = 6$, the value of x is 4. Find the value of x when $y = 8$.
7. A truck requires 108 liters of diesel for covering a distance of 594km. How much diesel will be required to cover a distance of 1650km?



Challenge Problems

8. If the cost of a dozen soaps is ₹ 396, what will be the cost of 35 such soaps?
9. In a school, there is 7 period a day each of 45 minutes duration. How long each period is, if the school has 9 periods a day assuming the number of hours to be the same?
10. Cost of 105 notebooks is ₹ 2415. How many notebooks can be bought for ₹ 1863?
11. 10 farmers can plough a field in 21 days. Find the number of days reduced if 14 farmers ploughed the same field?
12. A flood relief camp has food stock by which 80 people can be benefited for 60 days. After 10days 20 more people have joined the camp. Calculate the number of days of food shortage due to the addition of 20 more people?
13. Six men can complete a work in 12 days. Two days later, 6 more men joined them. How many days will they take to complete the remaining work?

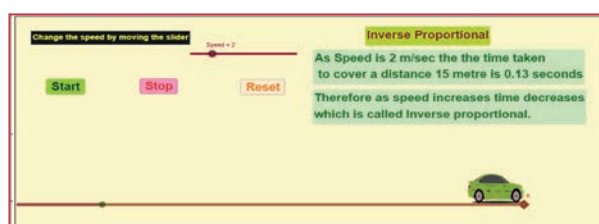
Summary

- A change in one quantity results a corresponding change in other quantity is called proportion.
- Two quantities x and y are said to be in direct proportion with each other if they increase or decrease together in such a way that $\frac{x}{y}$ remains constant.
- Two quantities x and y are said to be in inverse proportion if an increase in x leads to a decrease in y (or vice versa) in such a way that, the product xy remains constant.



ICT Corner

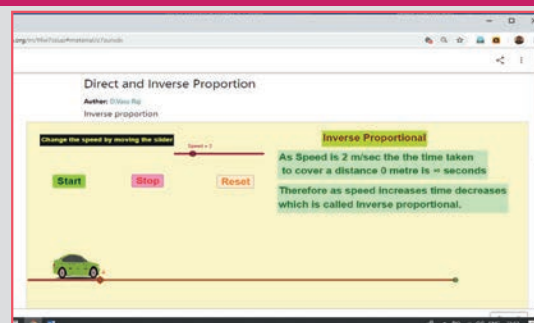
Expected Result is shown in this picture



Step 1

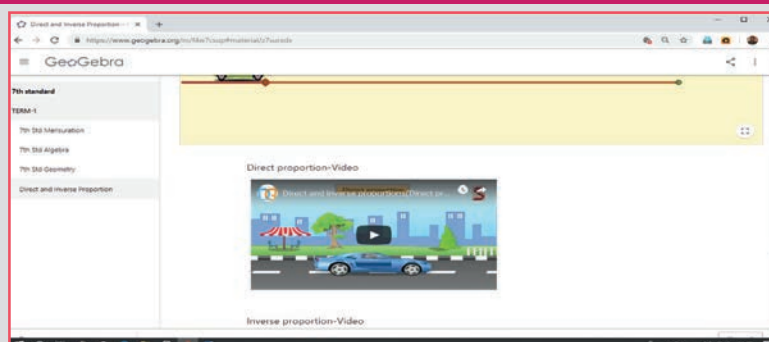
Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Direct and Inverse Proportion" will open. There is one activity for Inverse proportion and two videos.

In the activity "Inverse proportion", Move the slider to change the speed of the car. Click Start button to move the car at desired speed. Observe the time taken when the speed is increased.



Step 2

In the second and third page two videos for Direct and inverse proportion are given. Go through the video and compare what you learned from book.



Browse in the link

Direct and Inverse proportion <https://ggbm.at/z7surxdx>
or Scan the QR Code.

