CBSE Class 11 Mathematics
Sample Papers 07 (2019-20)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
iv. There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. Two finite sets have m and n elements. The number o elements in the power set of the first is 48 more than the total number of elements in the power set of the second. Then the values of m and n are

    a. 6, 4

    b. 6, 3

    c. 3, 7

    d. 7, 6

2. The number of ways in which n ties can be selected from a rack displaying 3n

different ties is

a. none of these

b. $3 \times n!$

c. $(3n)!$

d. $\frac{(3n)!}{n!(2n)!}$

3. If the coefficients of x^{-7} and x^{-8} in the expansion of $\left(2+\frac{1}{3x}\right)^n$ are equal then n =

a. 45

b. 55

c. 56

d. 15

4. A fair dice is rolled n times. The number of all the possible outcomes is

a. 6n

b. n^6

c. 6^n

d. none of these

5. If f : $R \to R$ and g: $R \to R$ are given by f (x0 = | x | and g (x) = [x] for each x ∈ R, then {x ∈ R} : g (f (x0) ∈f (g (x))] =

a. Z U (-∞, 0)

b. $(-\infty, 0)$

c. R

d. Z

6. If $49^n + 16n + \lambda$ is divisible by 64 for all n $\in$ N , then the least negative integral value of λ is

   a. -1

   b. -3

   c. -4

   d. -2

7. A coin is tossed once. If a head comes up, then it is tossed again and if a tail comes up, a dice is thrown. The number of points in the sample space of experiment is

   a. 4

   b. 12

   c. 8

   d. 24

8. A line making angles $45^0 and 60^0$ with the positive directions of the axis of x and y makes with the positive direction of Z-axis, an angle of

   a. $60^0 or 120^0$

   b. 60^0

   c. 120^0

   d. 45^0

9. From each of the four married couples, one of the partners is selected at random. The probability that those selected are of the same sex is

   a. $\frac{1}{8}$

   b. $\frac{1}{16}$

   c. $\frac{1}{2}$

d. $\frac{1}{4}$

10. The exponent of x occurring in the 7^{th} term of expansion of $\left(\frac{3x}{2} - \frac{8}{7x}\right)^9$ is

a. - 5

b. 3

c. 5

d. - 3

11. Fill in the blanks:

Let A and B be any two non-empty finite sets containing m and n elements respectively, then, the total number of subsets of (A $\times$ B) is ________.

12. Fill in the blanks:

The structure which is used to understand and remember the coefficients of variables in any expansion, look like a triangle with 1 at the top vertex and running down the two slanting sides is called ________.

13. Fill in the blanks:

The values of P(15, 3) is ________.

14. Fill in the blanks:

L is the foot of perpendicular drawn from the point P(3, 4, 5) on zx-planes. The coordinates of L are ________

OR

Fill in the blanks:

The equation x = b represents a plane parallel to ________ plane.

15. Fill in the blanks:

The derivative of cos x is ________.

OR

Fill in the blanks:

The derivative of x at x = 1 is ________.

16. Describe {x: x∈Z and $|x| \leq 2$} set in Roster form.
17. Find the number of chords that can be drawn through 16 points on a circle.
18. Express the complex numbers $i^9 + i^{19}$

OR

Show that $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \forall n \in N$.

19. If N = {1,2,3}, then find the relation
    R = {(x,y) : x ∈ N ,y ∈ N and 2x + y = 10} in N × N.
20. If ${}^nC_8 = {}^nC_2$. find ${}^nC_2$.
21. Let A and B be two sets. Prove that: (A - B) ∪ B = A if and only if B ⊂ A.

OR

Describe the following sets in Roster form:

i. The set of all vowels in the word 'EQUATION'
ii. The set of all-natural numbers less than 7.

22. A letter is chosen at random from the word ASSASSINATION find the probability that letter is
    (i) a vowel
    (ii) a consonant
23. Expand $\left(\frac{2x}{3} - \frac{3}{2x}\right)^4$
24. Find the equation of the perpendicular bisector of the line segment joining the points

(1,1) and (2, 3).

OR

Check whether the points (1, -1), (5, 2) and (9, 5) are collinear or not.

25. Check the validity of the statement:
    p: 100 is a multiple of 4 and 5.

26. Solve: 2 $\cos^2$ x + 3 sin x = 0

27. In a group of 400 people in USA, 250 can speak Spanish and 200 can speak English. How many people can speak both Spanish and English?

28. If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$ find the values of x and y.

OR

Let A = {1, 2} and B = {3, 4}write $A \times B$. How many sub sets will $A \times B$ have? List them

29. Evaluate $\lim_{x \rightarrow 0} \frac{(X+1)^5-1}{x}\left[\frac{0}{0} \text{form}\right]$

30. Solve $3x^2-4x+\frac{20}{3}=0$

31. Solve the following inequation: $\frac{28-3}{4}+19 \geqslant 13+\frac{4x}{3}$

OR

Solve the inequalities graphically in two-dimensional plane: $2x+y \geqslant 6$

32. Prove the following by using the principle of mathematical induction for all $n \in N \cdot 41^n-14^n$ is a multiple of 27.

33. Prove that: cos 40° cos 80° cos 160° = $-\frac{1}{8}$.

OR

If cos $(\alpha - \beta)$ + cos $(\beta - \gamma)$ + cos $(\gamma - \alpha)$ = $\frac{-3}{2}$, then prove that cosα + cosβ + cosγ = sinα + sinβ + sinγ = 0.

34. Find the sum of n terms of series $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \ldots$

35. Find the equation of the circle which is circumscribed about the triangle, whose vertices are (- 2, 3), (5, 2) and (6,-1).

OR

Find the equation of the circle which passes through the centre of the circle $x^2 + y^2 + 8x + 10y - 7 = 0$ and is concentric with the circle $2x^2 + 2y^2 - 8x - 12y - 9 = 0$

36. The measurements of the diameters (in mm) of the heads of 107 screws are given below:

Diameter (in mm)	33-35	36-38	39-41	42-44	45-47
No. of screws	17	19	23	21	27

Calculate the standard deviation.

Solution

Section A

1. (a) 6, 4

   Explanation: Let A has m elements and B gas n elements. Then, no. of elements in

   P(A) = 2^m and no. of elements in P(B) = 2^n.]

   By the question,

   $2^m = 2^n + 48$

   $\Rightarrow 2^m - 2^n = 48$

   This is possible, if $2^m = 64$, $2^n = 16$. (As 64 - 16 = 48)

   $\therefore 2^m = 64 \Rightarrow 2^m = 2^6$

   $\Rightarrow m = 6.$

   Also, $2^4 = 16 \Rightarrow 2^4 = 2^4$

   $\Rightarrow n = 4$

2. (d) $\frac{(3n)!}{n!(2n)!}$

   Explanation:

   The number of selections of r objects from the given n objects is denoted by ${}^nC_r$ and we have ${}^nC_r = \frac{n!}{r!(n-r)!}$

   Now n ties can be selected from a rack displaying 3n different ties in ${}^{3n}C_n = \frac{3n!}{n!(3n-n)!} = \frac{3n!}{n!(2n)!}$ different ways

3. (b) 55

Explanation: We have the general term in the expansion of $\left(2+\frac{1}{3x}\right)^n$ is given by

$T_{r+1} = ^n C_r \quad (2)^{(n-r)}\left(\frac{1}{3x}\right)^r$

Now $x^{-r} = x^{-7} \quad \Rightarrow r = 7$

and $x^{-r} = x^{-8} \quad \Rightarrow r = 8$

$\therefore T_8 = T_{7+1} = ^n C_7 \quad (2)^{(n-7)}\left(\frac{1}{3x}\right)^7$

$T_9 = T_{8+1} = ^n C_8 \quad (2)^{(n-8)}\left(\frac{1}{3x}\right)^8$

Given $\frac{^nC_7 \quad 2^{n-7}}{3^7} = \frac{^nC_8 \quad 2^{n-8}}{3^8}$

$\Rightarrow \frac{n!}{(n-7)!7!}\frac{2^{n-7}}{3^7} = \frac{n!}{(n-8)!\cdot 8!}\frac{2^{n-8}}{3^8}$

$\Rightarrow \frac{2^{n-7}}{n-7} = \frac{2^{n-8}}{8\times 3}$

$\Rightarrow \frac{2^{n-7}}{2^{n-8}} = \frac{(n-7)}{24}$

$\Rightarrow 24 \times 2 = n - 7$

$\Rightarrow$ 4 = 55

4. (c) 6^n

Explanation:

each time there are 6 possibilities, therefore for n times there are 6^n possibilities.

5. (c) R

Explanation:

We have, $f(x_0) = |x|$ and $g(x) = [x]$

now, $g(f(x_0)) \in f(g(x))$,for some $x \in R$

$\Rightarrow g(|x|) = f([x]) \Rightarrow [|x|] = f([x])$

$\Rightarrow f([x]) = [|x|]$

$\Rightarrow f([x]) = n$ Where n is a positive integer $\geqslant 0$

$\Rightarrow f(x) = n$

Domain of F = R

$\therefore\ \{x \in\ R : g(f(x_0)) \in\ f(g(x))\}\ =\ R$

6. (a) -1

   Explanation:

   When n = 1 we have the value of the expression as 65 . Given that the expression is divisible be 64. Hence the value is -1.

7. (c) 8

   Explanation:

   Sample Space is

   S = { HH, HT, T1, T2, T3, T4, T5, T6 }

   so number of outcomes in sample space is 8

8. (a) $60^0 or 120^0$

   Explanation:

   $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

   $\alpha = 45, \beta = 60$

   put the values in above equation

   $(1/\sqrt{2})^2 + (1/2)^2 + \cos^2\gamma = 1$

   $\cos\gamma = \pm 1/2$

9. (a) $\frac{1}{8}$

   Explanation:

   Here, s = {(M M M M), (F F F F), ...}

   Clearly, n (s) = 16

   ∴ Required proability = P [(M M M M) or (F F F F)]

= P [(M M M M) + (F F F F)]

$\frac{2}{16}+\frac{2}{16}=\frac{4}{16}=\frac{1}{8}$

10. (d) - 3

Explanation: We have the general term of $(x+a)^n$ is $T_{r+1}={}^n C_r \quad (x)^{n-r}a^r$

Now consider $\left(\frac{3x}{2}-\frac{8}{7x}\right)^9$

here n = 9 and r + 1 = 7 $\Rightarrow$ r = 6

Also $x=\frac{3x}{2}$ and $a=-\frac{8}{7x}$

$\therefore T_7=T_{6+1}={}^9 C_6 \quad \left(\frac{3x}{2}\right)^3\left(\frac{-8}{7x}\right)^6$

$={}^9 C_6 \quad \left(\frac{3}{2}\right)^3\left(\frac{-8}{7}\right)^6 x^{-3}$

Hence the exponent of x = -3

11. 2^{mn}

12. Pascal's triangle

13. 2730

14. (3, 0, 5)

OR

yz-plane

15. -sin x

OR

1

16. We find that x is an integer satisfying $|x| \leq 2$

and, $|x|$ = 0, 1, 2

$\Rightarrow$ x = 0, ± 1, ± 2

So, x can take values - 2, -1, 0, 1, 2.

{x :x∈Z and $|x| \leq 2$}= {-2, - 1, 0, 1, 2}

17. Since, the points lies on the circumference of the circle. So, no three of them are collinear.

    Thus, number of chords formed by 16 points by taking 2 at time = $^{16}C_2$

    $= \frac{16!}{2!14!} = \frac{16\times15}{2\times1}$ = 120

18. $i^9 + i^{19} = (i^2)^4 \cdot i + (i^2)^9 \cdot i$

    $= (-1)^4 \cdot i + (-1)^9 \cdot i$

    =i - i = 0

OR

Given, **LHS** $= i^n + i^{n+1} + i^{n+2} + i^{n+3}$

$= i^n + i^n \,.\, i + i^n \,.\, i^2 + i^n \,.\, i^3 = i^n \,(1 + i + i^2 + i^3)$

= i^n (1 + i - 1 - i) [$\because$ i^2 = - 1, i^3 = i^2 . i = - i]

= i^n (0) = 0 = RHS

Hence proved.

19. Here, R = {(x, y): x $\in$ N, y $\in$ V and 2x + y = 10} in

    N $\times$ N.

    R ={$(1,8),(2,6),(3,4),(4,2)$}

    Domain of = {$1,2,3,4$}

    Range of R = {$8,6,4,2$}

20. Here $^nC_8 = {^nC_2} \Rightarrow {^nC_8} = {^nC_{n-2}}$ $[\because {^nC_r} = {^nC_{n-r}}]$

    $\Rightarrow 8 = n - 2 [\because {^nC_x} = {^nC_y} \Rightarrow x = y]$

    $\Rightarrow n = 10 \therefore {^nC_2} = {^{10}C_2} = \frac{10!}{2!8!} = 45$

21. First, let us consider that, (A - B) $\cup$ B = A.

    Then, we have to prove that B $\subset$ A.

    We know that A - B refers to those elements of A which are not present in B, that is A - B = A $\cap$ B' .....(i)

    Now, (A - B) $\cup$ B = A

    $\Rightarrow \quad (A \cap B') \cup B = A$ ...... [from (i)]

    $\Rightarrow \quad (A \cup B) \cap (B' \cup B) = A$

$\Rightarrow \quad (A \cup B) \cap U = A$

$\Rightarrow \quad A \cup B = A$

The above condition is only possible when,

$\Rightarrow \quad B \subset A$

Conversely, let B ⊂ A. Then, we have to prove that (A - B) ∪ B = A.

Now, (A - B) ∪ B = (A ∪ B') ∪ B

= (A ∪ B) ∩ (B' ∪ B)

$= (A \cup B) \cap U$

$= A \cup B$

Now as we know that B ⊂ A

= A $[\because B \in A \therefore A \cup B = A]$

OR

In the Roaster form all the elements of the set are listed inside "{}" brackets and are seperated by commas.

i. The vowels in the word 'EQUATION' are A, E, I, O, U
   So, the required set can be described as follows: {A, E, I, O, U}
ii. Natural numbers less than 7 are 1, 2, 3, 4, 5, 6.
   Hence, the required set can be described as follows: {1, 2, 3, 4,5, 6}.

22. There are 13 letters in the word ASSASSINATION of which 6 vowels and 7 consonants.
One letter is selected out of 13 letters in $^{13}C_1 = 13$ ways
(i) Out of 6 vowels, 1 vowel can be selected in 6 ways
$\therefore$ P(1 vowel selected)$= \frac{6}{13}$
(ii) Out of 7 consonants, 1 consonant can be selected in 7 ways.
$\therefore$ P(1 consonant selected) t $= \frac{7}{13}$

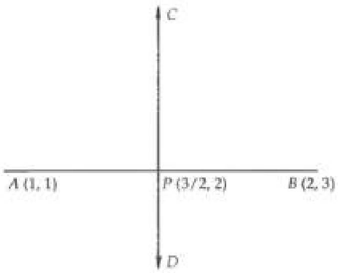
23. $\left(\frac{2x}{3} - \frac{3}{2x}\right)^4 = {}^4C_0 \left(\frac{2x}{3}\right)^4 + {}^4C_1 \left(\frac{2x}{3}\right)^3 \left(\frac{-3}{2x}\right) + {}^4C_2 \left(\frac{2x}{3}\right)^2 \left(\frac{-3}{2x}\right)^2 + {}^4C_3 \left(\frac{2x}{3}\right) \left(\frac{-3}{2x}\right)^3 + {}^4C_4 \left(\frac{-3}{2x}\right)^4$

$= 1 \times \frac{16x^4}{81} + 4 \times \frac{8x^3}{27} \left(\frac{-3}{2x}\right) + 6 \times \frac{4x^2}{9} \left(\frac{9}{4x^2}\right) + 4 \left(\frac{2x}{3}\right) \left(\frac{-27}{8x^3}\right) + 1 \times \left(\frac{81}{16x^4}\right)$

[using ${}^4C_0 = {}^4C_4 = 1, {}^4C_3 = {}^4C_1 = 4$ and ${}^4C_2 = \frac{4!}{2!2!} = \frac{4 \times 3 \times 2!}{2 \times 1 \times 2!} = 6$]

= $\frac{16}{81}x^4$ - $\frac{16}{9}x^2$ + 6 - $\frac{9}{x^2}$ + $\frac{81}{16x^4}$

24. Let P be the mid-point of the line segment joining points A (1,1) and B (2, 3). Then, the coordinates of P are $\left(\frac{3}{2}, 2\right)$.



Let m be the slope of the perpendicular bisector of AB.

Then,

m $\times$ Slope of AB = - 1

m $\times$ $\frac{3-1}{2-1}$ = - 1

$\Rightarrow$ m = $\frac{-1}{2}$

Clearly, the perpendicular bisector of AB passes through P $\left(\frac{3}{2}, 2\right)$ and has slope m = - $\frac{1}{2}$. So, its equation is

y - 2 = - $\frac{1}{2}\left(x - \frac{3}{2}\right)$ or, 2x + 4y - 11 = 0.

OR

Let A = (1, -1), B = (5, 2) and C = (9, 5)

Now, distance between A and B,

$$AB = \sqrt{(5-1)^2 + (2+1)^2} \left[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$

$$= \sqrt{(4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

Distance between B and C,

$$BC = \sqrt{(5-9)^2 + (2-5)^2} = \sqrt{(-4)^2 + (-3)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5$$

Distance between A and C,

$$AC = \sqrt{(1-9)^2 + ((-1-5)^2} = \sqrt{(-8)^2 + (-6)^2}$$

$$= \sqrt{64+36} = 10$$

Clearly, AC = AB + BC

Hence, A, B and C are collinear points.

25. The statement is:
"100 is multiple of 4 and 5".
We know that 100 is a multiple of 4 as well as 5. Thus, p is a true statement.
Hence, the statement is true i.e. the statement "p" is a valid statement.

26. $2cos^2x + 3sinx = 0$
$\Rightarrow 2(1 - sin^2x) + 3sinx = 0$
$\Rightarrow 2sin^2x - 3sinx - 2 = 0$
$\Rightarrow 2sin^2\ x - 4sin\ x + sin\ x - 2 = 0$
$\Rightarrow$ 2 sin x (sin x - 2) +1 (sin x - 2) = 0
$\Rightarrow$ (sin x - 2) (2 sin x +1) = 0
$\Rightarrow$ 2sin x + 1 = 0 $[\because \sin x \neq 2 \quad \therefore \sin x - 2 \neq 0]$
$\Rightarrow \quad \sin x = -\frac{1}{2}$
$\Rightarrow \quad \sin x = \sin\left(-\frac{\pi}{6}\right) \Rightarrow x = n\pi + (-1)^n\left(-\frac{\pi}{6}\right), n \in Z \Rightarrow x = n\pi +$
$(-1)^{n+1}\frac{\pi}{6}, \quad n \in Z.$

27. Let S be the set of people who speak Spanish, and E be the set of people who speak English
∴ *n*(S ∪ E) = 400, *n*(S) = 250, *n*(E) = 200
n(S ∩ E) = ?
We know that:
n(S ∪ E) = *n*(S) + *n*(E) – *n*(S ∩ E)
∴ 400 = 250 + 200 – *n*(S ∩ E)
⇒ 400 = 450 – *n*(S ∩ E)
⇒ *n*(S ∩ E) = 450 – 400
∴ *n*(S ∩ E) = 50
Thus, 50 people can speak both Spanish and English.

28. Here $\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$
$\therefore \frac{x}{3} + 1 = \frac{5}{3}$ and $y - \frac{2}{3} = \frac{1}{3}$

$\Rightarrow \frac{x}{3} = \frac{5}{3} - 1$ and $y = \frac{1}{3} + \frac{2}{3}$
$\Rightarrow \frac{x}{3} = \frac{2}{3}$ and $y = \frac{3}{3}$
$\Rightarrow$ x = 2 and y = 1

OR

Here A = {1, 2} and B = {3, 4}

$\therefore A \times B = (1, 2) \times (3, 4\}$

= {(1, 3}, (1, 4), (2, 3), (2, 4)}

Number of elements in $A \times B = 4$

Number of subsets of $A \times B = 2^4 = 16$

The subset are:

ϕ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)}, {(1, 3), (1, 4)}, {(1, 3), (2, 3)}, {(1, 3), (2, 4)}, {(1,4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3)}, {(1, 3), (1, 4), (2, 4)}, {(1, 3), (2, 3), (2, 4)} {(1, 4), (2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3), (2, 4)}

29. Here $\lim\limits_{x \to 0} \frac{(X+1)^5 - 1}{x} \left[\frac{0}{0} \text{form}\right]$

$= \lim\limits_{x \to 0} \frac{(X+1)^5 - 1}{(x+1) - 1}$

Putting x + 1 = y, as $x \to 0,\ y \to 1$

$\therefore \lim\limits_{y \to 0} \frac{y^5 - 1}{y - 1} = 5.(1)^{5-1}$

$= 5 \times 1 = 5 \left[\because \lim\limits_{x \to a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}\right]$

30. Here $3x^2 - 4x + \frac{20}{3} = 0$

Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we have

a = 3, b = -4 and $c = \frac{20}{3}$

$\therefore x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 3 \times \frac{20}{3}}}{2 \times 3} = \frac{4 \pm \sqrt{16 - 80}}{6}$

$= \frac{4 \pm \sqrt{-64}}{6} = \frac{4 \pm 8\sqrt{-1}}{6} = \frac{4 \pm 8i}{6} = \frac{2 \pm 4i}{3}$

Thus $x = \frac{2+4i}{3}$ and $x = \frac{2-4i}{3}$

31. Here $\frac{28-3}{4} + 19 \geqslant 13 + \frac{4x}{3}$

$\Rightarrow \frac{2x-3}{4} - \frac{4x}{3} \geqslant 13 - 19$

$\Rightarrow \frac{6x - 9 - 16x}{12} \geqslant -6$

$\Rightarrow \frac{-10x - 9}{12} \geqslant -6$

Multiplying both sides by 12

$\therefore -10x - 9 \geqslant 6 \times 12$

$\Rightarrow -10x - 9 \geqslant -72$

$\Rightarrow -10x \geqslant -72+9$

$\Rightarrow -10x \geqslant -63$

Dividing both sides by -10

$\therefore \frac{-10x}{-10} \leqslant \frac{-63}{-10}$

$\therefore x \leqslant \frac{63}{10}$

Thus solution set of given in equation is $\left(-\infty, \frac{63}{10}\right]$

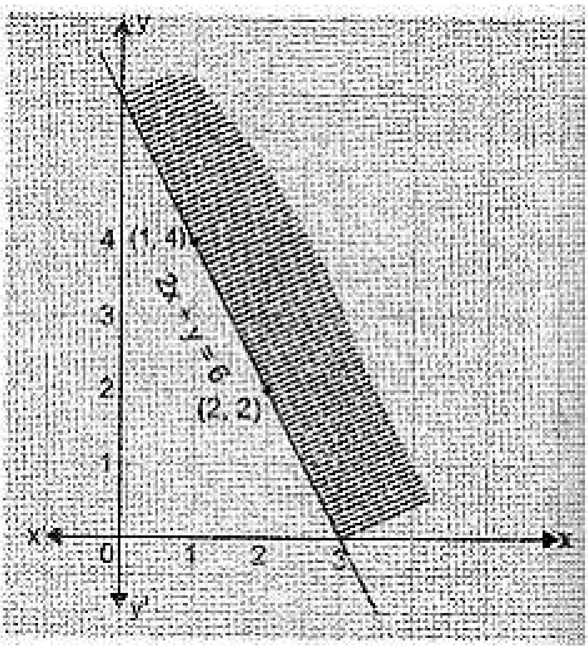
OR

The given inequality is $2x+y \geqslant 6$

Draw the graph of the line 2x + y = 6

Table of values satisfying the equation 2x + y = 6

X	1	2
Y	4	2



Putting (0, 0) in the given in equation, we have

$2\times 0+0 \geqslant 6 \Rightarrow 0 \geqslant 6$ which is false.

$\therefore$ Half plane of $2x+y \geqslant 6$ is always from origin

32. Let P(n) = 41^n - 14^n is a multiple of 27

For n = 1

P(1) = 41^1 - 14^1 is a multiple of 27 $\Rightarrow$ 27 is a multiple of 27

$\therefore$ P(1) is true

Let P(n) be true for n = k

$\therefore P(k) = 41^k - 14^k$ is a multiple of 27 $\Rightarrow 41^k - 14^k = 27\lambda$.... (i)

For n = k + 1

P(k + 1) 41^{k+1} - 14^{k+1} is a multiple of 27

Now $41^{k+1} - 14^{k+1} = 41^{k+1} - 41^k \cdot 14 + 41^k \cdot 14 - 14^{k+1}$

$= 41^k(41 - 14) + 14(41^k - 14^k) = 41^k \times 27 + 14 \times 27\lambda$ [Using (i)]

$= 27(41^k + 14\lambda)$

$\Rightarrow 41^{k+1} - 14^{k+1}$ is a multiple of 27

$\therefore$ P(k +1) is true

Thus P(k) is true $\Rightarrow$ P(k + 1) is true

Hence by principle of mathematical induction, P(n) is true for all $n \in N$.

33. cos 40° cos 80° cos 160° = $-\frac{1}{8}$

LHS = cos 40° cos 80° cos 160°

= cos 80° cos 40° cos 160°

Multiplying and dividing by 2

$= \frac{1}{2}$ {cos 80° $\times$ (2 cos 40° cos 160°)}

Because 2 cos A cos B = cos (A + B) + cos (A - B)

$= \frac{1}{2}$ cos 80° [cos (40° + 160°) + cos (40° - 160°)]

$= \frac{1}{2}$ cos 80° [cos 200 + cos (-120)]

$= \frac{1}{2}$ cos 80° [cos 200 + cos 120]

$= \frac{1}{2}$ cos 80° {cos (180° + 20°) + cos (180° - 60°)}

$= \frac{1}{2}$ cos 80° (-cos 20° - cos 60°)

$= -\frac{1}{2}$ cos 80° cos 20° - $\frac{1}{2}$ cos 80° cos 60°

$= -\frac{1}{4}$ (2 cos 80° cos 20°) - $\frac{1}{4}$ cos 80°

$= -\frac{1}{4}$ [2 cos 80° cos 20° + cos 80°]

$= -\frac{1}{4}$ [cos (80° + 20°) + cos(80° - 20°) + cos 80°]

$= -\frac{1}{4}$ [cos 100° + cos 60° + cos 80°]

$= -\frac{1}{4}$ [cos (180° - 80°) + cos 60° + cos 80°]

$= -\frac{1}{4}$ [-cos 80° + cos 60° + cos 80°]

$= -\frac{1}{4}$ cos 60°

$= -\frac{1}{4} \times \frac{1}{2}$
$= -\frac{1}{8}$ = RHS

OR

Given,
$cos\ (\alpha - \beta)\ + cos\ (\beta - \gamma)$+ cos$(\gamma - \alpha)$ = $\frac{-3}{2}$
$\Rightarrow$ 2 [cos $(\alpha - \beta)$ + cos $(\beta - \gamma)$ + cos $(\gamma - \alpha)$] = - 3
$\Rightarrow$
$2[cos\alpha\ \times\ cos\beta\ + sin\alpha\ \times\ sin\ \beta + cos\ \beta \times\ \ cos\gamma\ + sin\beta\ \times\ sin\gamma\ + cos\gamma \times$
$[\because cos(A - B) = cosAcosB + sinAsinB]$
$\Rightarrow$ $[2cos\,\alpha\ \times cos\beta$ + 2 cos$\beta \times$ cosγ + 2 cos$\gamma \times$ cosα] + [2 sin$\alpha \times$ sinβ + 2 sin$\beta \times$ sin γ + 2 sin$\gamma \times$ sinα] + 3 = 0
$\Rightarrow$ [2 cos$\alpha \times$ cosβ + 2 cos$\beta \times$ cosγ + 2 cos$\gamma \times$ cosα] + [2 sin$\alpha \times$ sinβ + 2 sin$\beta \times$ sin γ + 2 sin$\gamma \times$ sinα] + (cos$^2\,\alpha$ + sin$^2\,\alpha$) + (cos$^2\,\beta$ + sin$^2\,\beta$) + (cos$^2\,\gamma$ + sin$^2\,\gamma$) = 0
$[\because cos^2x + sin^2x = 1]$
$\Rightarrow$ [cos$^2\,\alpha$ + cos$^2\,\beta$ + cos$^2\gamma$ + 2 cos$\alpha \times$ cosβ + 2 cos$\beta \times$ cosγ + 2 cos$\gamma \times$ cosα] + [sin^2 α + sin$^2\,\beta$ + sin$^2\,\gamma$ + 2 sin$\alpha \times$ sinβ + 2 sin$\beta \times$ sinγ + 2 sin$\gamma \times$ sinα] = 0
$\Rightarrow (cos\alpha + cos\beta\ + cos\gamma)^2 + (sin\alpha\ + sin\beta\ + sin\gamma)^2 = 0$
We know that, sum of two positive terms will be zero, if both are equal to zero.
$\therefore cos\alpha\ + cos\beta\ + cos\gamma\ = 0$
and $sin\alpha\ + sin\beta\ + sin\gamma\ = 0$
Hence proved.

34. Let the given series be S = $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \ldots$
Then, nth term T_n = $\frac{1}{n(n+1)}$
Now, we will split the denominator of the nth term into two parts or we will write T_n as the difference of two terms.
$\therefore T_n$ = $\frac{1}{n(n+1)} = \frac{(n+1)-n}{n(n+1)}$
= $\frac{n+1}{n(n+1)} - \frac{n}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$
On putting n = 1, 2, 3, 4,... successively, we get
T_1 = $\frac{1}{1} - \frac{1}{2}$

$T_2 = \frac{1}{2} - \frac{1}{3}$

$T_3 = \frac{1}{3} - \frac{1}{4}$................

$T_n = \frac{1}{n} - \frac{1}{n+1}$

On adding all these terms, we get

$S = T_1 + T_2 + T_3 + ...+ T_n$

$= \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \ldots + \left(\frac{1}{n} + \frac{1}{n+1}\right)$

$= 1 - \frac{1}{n+1} = \frac{1}{1} - \frac{1}{n+1} = \frac{n+1-1}{n+1}$

$\Rightarrow S = \frac{n}{n+1}$

35. The circle which is circumscribed about the triangle, whose vertices are (- 2, 3), (5, 2) and (6, - 1) means the circle passes through these three points.

Let the equation of circle be

$(x - h)^2 + (y - k)^2 = r^2$ ...(i)

Since, equation (i) passes through the points (- 2, 3),

$\therefore (- 2 - h)^2 + (3 - k)^2 = r^2$

$\Rightarrow h^2 + 4h + 4 + k^2 - 6k + 9 = r^2$ ...(ii)

also equation (i) passes through the point (5,2)

$\Rightarrow (5 - h)^2 + (2 - k)^2 = r^2$

$\Rightarrow h^2 - 10h + 25 + k^2 - 4k + 4 = r^2$ ...(iii)

again equation (i) passes through the point (6,-1) $\Rightarrow (6 - h)^2 + (- 1 - k)^2 = r^2$

$\Rightarrow h^2 - 12h + 36 + k^2 + 2k + 1 = r^2$ ...(iv)

Now subtracting equation(iii) from equation (ii), we get

14h - 21 - 2k + 5 = 0

i.e., 14h - 2k = 16

=>7h - k = 8 ...(v)

again subtracting equation (iv) from equation(iii), we get

2h - 11 - 6k + 3 = 0

i.e., 2h - 6k = 8

=> h - 3k = 4...(vi)

On solving equation (v) and equation (vi), we get

h = 1 and k = - 1

On putting the values of h = 1 and k = -1 in equation (ii), we get

$1 + 4 + 4 + 1 + 6 + 9 = r^2$

$\Rightarrow r^2 = 25$

$\Rightarrow r = 5$

Now putting h=1, k=-1 and r=5 in equation (i) we get

$(x - 1)^2 + (y + 1)^2 = 25$

$\Rightarrow x^2 + y^2 - 2x + 2y - 23 = 0$

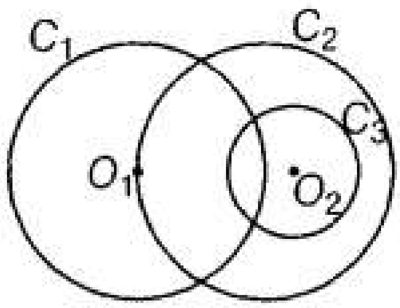
which is the required equation of the circle.

OR

We have to find the equation of circle (C_2) which passes through the centre of circle (C_1) and is concentric with circle (C_3).

We have, equation of circle (C_1),

$x^2 + y^2 + 8x + 10y - 7 = 0$ ...(i)



On comparing it with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

g = 4, f = 5 and c = - 7

$\therefore$ Centre of C_1 is $O_1 = (-g, -f)$

$O_1 = (-4, -5)$

Now, equation of circle (C_2) which is concentric with given circle (C_3) having equation

$2x^2 + 2y^2 - 8x - 12y - 9 = 0$ is

$2x^2 + 2y^2 - 8x - 12y + k = 0$ ...(ii)

Since, circle (C_2) passes through O_1 (- 4, - 5)

$\therefore$ 2 (- 4)^2 + 2 (- 5)^2 - 8 (- 4) - 12 (- 5) + k = 0

$\Rightarrow$ 32 + 50 + 32 + 60 + k = 0

$\Rightarrow$ k = - 174

On putting the value of k in Eq. (ii), we get

$2x^2 + 2y^2 - 8x - 12y - 174 = 0$

$\Rightarrow x^2 + y^2 - 4x - 6y - 87 = 0$ [dividing both sides by 2]

which is required equation of circle (C_2).

36. Here the class intervals are formed by the inclusive method. But, the mid-points of class-intervals remain the same whether they are formed by the inclusive method or exclusive method. So there is no need to convert them into an exclusive series.

Calculation of Standard Deviation

Diameter (in mm)	**Mid-values, x_i**	**No. of screws, f_i**	**$u_i = \frac{x_i - 40}{3}$**	**$f_i u_i$**	**$f_i u_i^2$**
33-35	34	17	-2	-34	68
36-38	37	19	-1	-19	19
39-41	40	23	0	0	0
42-44	43	21	1	21	21
45-47	46	27	2	54	108
		$\Sigma f_i = 107$		$\Sigma f_i u_i = 22$	$\Sigma f_i u_i^2 = 216$

Here $N = \Sigma f_i = 107$, $\Sigma f_i u_i = 22$, $\Sigma f_i u_i^2 = 216$, A = 40 and, h = 3

$\therefore$ Var (X) = $h^2 \left\{ \left(\frac{1}{N} \Sigma f_i u_i^2 \right) - \left(\frac{1}{N} \Sigma f_i u_i \right)^2 \right\} = 9 \left\{ \frac{216}{107} - \left(\frac{22}{107} \right)^2 \right\}$

$\Rightarrow$ Var (X) = 9 (2.0187 - 0.0420) = 9 $\times$ 1.9767 = 17.7903

$\therefore$ S.D. = $\sqrt{17.7903}$ = 4.2178