Chapter 7: Thermal Properties of Matter

EXERCISES [PAGE 140]

Exercises | Q 1. (i) | Page 140

Choose the correct option.
Range of temperature in a clinical thermometer, which measures the temperature of the human body, is

1. 70º C to 100º C
2. **34º C to 42º C**
3. 0º F to 100º F
4. 34º F to 80º F

SOLUTION

34º C to 42º C

Exercises | Q 1. (ii) | Page 140

Choose the correct option.
A glass bottle completely filled with water is kept in the freezer. Why does it crack?

1. Bottle gets contracted
2. Bottle is expanded
3. **Water expands on freezing**
4. Water contracts on freezing

SOLUTION

Water expands on freezing

Exercises | Q 1. (iii) | Page 140

Choose the correct option.
If two temperatures differ by 25° C on Celsius scale, the difference in temperature on Fahrenheit scale is

1. 65°
2. **45°**
3. 38°
4. 25°

SOLUTION

45°

Explanation:

100 divisions on celsius scale correspond to 180 divisions on fahrenheit scale.

$\therefore$ Temperature difference of 25° C, i.e., 25 divisions will correspond to, $\frac{25 \times 180}{100} = 45$ divisions on Fahrenheit scale, i.e., **45° F**.

Exercises | Q 1. (iv) | Page 140

Choose the correct option.
If α, β and γ are coefficients of linear, area l and volume expansion of a solid then

1. α : β : γ = 1 : 3 : 2
2. **α : β : γ = 1 : 2 : 3**
3. α : β : γ = 2 : 3 : 1
4. α : β : γ = 3 : 1 : 2

SOLUTION

α : β : γ = 1 : 2 : 3

Exercises | Q 1. (v) | Page 140

Choose the correct option.
Consider the following statements-
(I) The coefficient of linear expansion has dimension K^{-1}.
(II) The coefficient of volume expansion has dimension K^{-1}.

1. **I and II are both correct**
2. I is correct but II is wrong
3. II is correct but I is wrong
4. I and II are both wrong

SOLUTION

I and II are both correct.

Exercises | Q 1. (vi) | Page 140

Choose the correct option.
Water falls from a hight of 200 m. What is the difference in temperature between the water at the top and bottom of a waterfall given that the specific heat of water is 4200 J kg^{-1} °C-1?

1. 0.96° C
2. 1.02° C

3. **0.46° C**
4. 1.16° C

SOLUTION

0.46° C

Explanation:

The potential energy at height = kinetic energy at bottom Assuming all kinetic energy is converted into heat, Let ΔT be the rise in temperature of water observed at the bottom

$\therefore$ mgh = m s ΔT

$$\therefore \Delta T = \frac{gh}{s} = \frac{9.8 \times 200}{4200} = 0.467^{\circ}C$$

Exercises | Q 2. (i) | Page 140

Answer the following question.

Clearly, state the difference between heat and temperature?

SOLUTION

	Heat	Temperature
1.	Heat is energy in transit. When two bodies at different temperatures are brought in contact, they exchange heat. OR Heat is the form of energy transferred between two (or more) systems or a system and its surroundings by virtue of their temperature difference.	Temperature is a physical quantity that defines the thermodynamic state of a system. OR Heat transfer takes place between the body and the surrounding medium until the body and the surrounding medium are at the same temperature.
2.	Heat exchange can be measured with the help of a calorimeter.	Temperature is measured with the help of a thermometer
3.	Heat (being a form of energy) is a derived quantity.	Temperature is a fundamental quantity.
4.	S.I. unit: J (joule)	S.I. unit: K (kelvin)
5.	Dimension: $[L^2M^1T^{-2}K^0]$	Dimension: $[L^0M^0T^0K^1]$

Exercises | Q 2. (ii) | Page 140

Answer the following question.

How a thermometer is calibrated?

SOLUTION

1. For the calibration of a thermometer, a standard temperature interval is selected between two easily reproducible fixed temperatures.

2. The fact that substances change state from solid to liquid to a gas at fixed temperatures is used to define reference temperature called a fixed point.

3. The two fixed temperatures selected for this purpose are the melting point of ice or the freezing point of water and the boiling point of water.

4. This standard temperature interval is divided into sub-intervals by utilizing some physical property that changes with temperature.

5. Each sub-interval is called as a degree of temperature. Thus, an empirical scale for temperature is set up.

Exercises | Q 2. (iii) | Page 140

Answer the following question.

What are different scales of temperature? What is the relation between them?

SOLUTION

Celsius scale:

1. The ice point (melting point of pure ice) is marked as 0° C (lower point) and steam point (boiling point of water) is marked as 100° C (higher point).

2. Both are taken at one atmospheric pressure.

3. The interval between these points is divided into 100 equal parts. Each of these parts is called as one-degree celsius and it is written as 1° C.

Fahrenheit scale:

1. The ice point (melting point of pure ice) is marked as 32° F and steam point (boiling point of water) is marked as 212° F.

2. The interval between these two reference points is divided into 180 equal parts. Each part is called a degree Fahrenheit and is written as 1° F.

Kelvin scale:

1. The temperature scale that has its zero at - 273.15° C and temperature intervals are the same as that on the Celsius scale is called as Kelvin scale or absolute scale.

2. The absolute temperature, T and Celsius temperature, T_C are related as, T = T_C + 273.15 eg.: when T_C = 27° C,
T = 27 + 273.15 K = 300.15 K

- **Relation between different scales of temperature:**

$$\frac{T_F - 32}{180} = \frac{T_C - 0}{100} = \frac{T_C - 273.15}{100}$$

where, T_F = temperature in fahrenheit scale,

T_C = temperature in celsius scale,

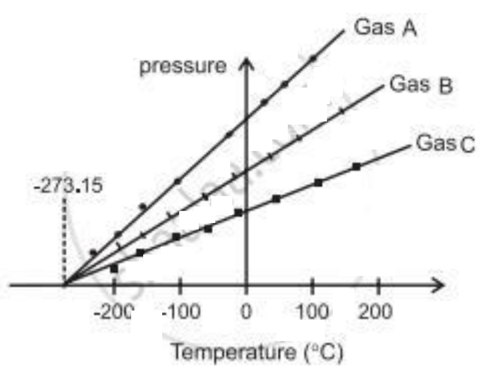
T_K = temperature in kelvin scale.

Exercises | Q 2. (iv) | Page 140

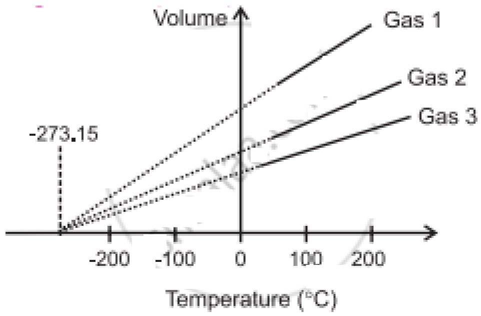
Answer the following question.

What is absolute zero?

SOLUTION



Graph of pressure versus temperature (in °C) at constant volume.



Graph of volume versus temperature (in °C) at constant pressure.

1. When the graph of pressure (P) against temperature T (°C) at constant volume for three ideal gases A, B, and C is plotted, in each case, P-T graph is a straight line indicating direct proportion between them. The slopes of these graphs are different.

2. The individual straight lines intersect the pressure axis at different values of pressure at 0 °C, but each line intersects the temperature axis at the same point, i.e., at absolute temperature (-273.15 °C).

3. Similarly, graph at constant pressure for three different ideal gases A, B and C extrapolate to the same temperature intercept -273.15 °C i.e., absolute zero temperature.

4. It is seen that all the lines for different gases cut the temperature axis at the same point at - 273.15 °C.

5. This point is termed as the absolute zero of temperature.

6. It is not possible to attain a temperature lower than this value. Even to achieve absolute zero temperature is not possible in practice.

Exercises | Q 2. (v) | Page 140

Answer the following question.
Derive the relation between three coefficients of thermal expansion.

SOLUTION

1. Consider a square plate of side l_0 at 0° C and l_T at T °C.

$\therefore l_T = l_0 (1 + \alpha T)$

If area of plate at 0° C is A_0, $A_0 = l_0^2$

If area of plate at T °C is A_T,

$$A_T = l_1^2 = l_0^2(1+\alpha T)^2$$
or $A_T = A_0(1+\alpha T)^2$ ....(1)

Also,

$$A_T = A_0(1+\beta T) \quad ....(2) \quad \left[\because \beta = \frac{A_T - A_0}{A_0(T-T_0)}\right]$$

2. Using Equations (1) and (2),
   $$A_0(1+\alpha T)^2 = A_0(1+\beta T)$$
   $$\therefore 1 + 2\alpha T + \alpha^2 T^2 = 1 + \beta T$$
3. Since the values of α are very small, the term $\alpha^2 T^2$ is very small and may be neglected.
   $\therefore \beta = 2\alpha$
4. The result is general because any solid can be regarded as a collection of small squares.
5. Consider a cube of side l_0 at 0 °C and l_T at T °C.
   $\therefore l_T = l_0(1 + \alpha T)$
   If volume of the cube at 0 °C is V_0, $V_0 = l_0^3$
   If volume of the cube at T °C is
   $$V_T, V_T = l_T^3 = l_0^3(1+\alpha T)^3$$
   $$V_T = V_0(1+\alpha T)^3 \quad ....(1)$$
   Also,
   $$V_T = V_0(1+\gamma T) \quad ....(2) \quad ...\left[\therefore \gamma = \frac{V_T - V_0}{V_0(T-T_0)}\right]$$
6. Using Equations (1) and (2),
   $$V_0(1+\alpha T)^3 = V_0(1+\gamma T)$$
   $$\therefore 1 + 3\alpha T + 3\alpha^2 T^2 + \alpha^3 T^3 = 1 + \gamma T$$
7. Since the values of α are very small, the terms with higher powers of α may be neglected.
   $\therefore \gamma = 3\alpha$
8. The result is general because any solid can be regarded as a collection of small cubes.
9. Relation between α, β and γ is given by,
   $$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$$
   where, α = coefficient of linear expansion,
   β = coefficient of superficial expansion,
   γ = coefficient of cubical expansion.

Exercises | Q 2. (vi) | Page 140

Answer the following question.

State applications of thermal expansion.

SOLUTION

1. The steel wheel is heated to expand. This expanded wheel can easily fit over axle. The wheel is then cooled quickly. Upon cooling, wheel contracts and fits tightly upon the axle.

2. An electric light bulb gets hot quickly when in use. The wire leads to the filament are sealed into the glass. If the glass of the bulbs has significantly different thermal expansivity from the wire leads, the glass and the wire would separate, breaking down the vacuum. To prevent this, wires are made of platinum or some suitable alloy with the same expansivity as ordinary glass.

Exercises | Q 2. (vii) | Page 140

Answer the following question.

Why do we generally consider two specific heats of a gas?

SOLUTION

1. A slight change in temperature causes a considerable change in pressure as well as the volume of the gas.

2. Therefore, two principal specific heats are defined for a gas viz., specific heat capacity at constant volume (S_V), and specific heat capacity at constant pressure (S_P).

Exercises | Q 2. (viii) | Page 140

Answer the following question.

Are freezing point and melting point same with respect to change of state? Comment.

SOLUTION

Though the freezing point and melting point mark the same temperature (0° C or 32° F), state of change is different for the two points. At freezing point, liquid gets converted into solid, whereas at melting point solid gets converted into its liquid state.

Exercises | Q 2. (ix) | Page 140

Answer the following question.

Define Sublimation.

SOLUTION

The change from solid state to vapour state without passing through the liquid state is called sublimation and the substance is said to sublime

Exercises | Q 2. (x) | Page 140

Answer the following question.
Define Triple point.

SOLUTION

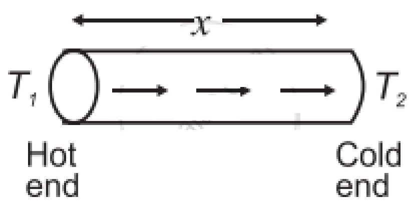
The triple point of water is that point where water in a solid, liquid and gas state co-exists in equilibrium and this occurs only at a unique temperature and a pressure.

Exercises | Q 2. (xi) | Page 140

Answer the following question.
Explain the term 'steady state'.

SOLUTION

1. When one end of a metal rod is heated, the heat flows by conduction from hot end to the cold end.



Section of a metal bar in the steady state.

2. As a result, the temperature of every section of the rod starts increasing.
3. Under this condition, the rod is said to be in a variable temperature state.
4. After some time, the temperature at each section of the rod becomes steady i.e., does not change.
5. The temperature of each cross-section of the rod now becomes constant though not the same. This is called a steady state condition.

Exercises | Q 2. (xii) | Page 140

Answer the following question.
Define coefficient of thermal conductivity.

SOLUTION

The coefficient of thermal conductivity of a material is defined as the quantity of heat that flows in one second between the opposite faces of a cube of side 1 m, the faces being kept at a temperature difference of 1°C (or 1 K).

Exercises | Q 2. (xiii) | Page 140

Answer the following question.

Give any four applications of thermal conductivity in everyday life.

SOLUTION

1. a. Thick walls are used in the construction of cold storage rooms.

   b. Brick being a bad conductor of heat is used to reduce the flow of heat from the surroundings to the rooms.

   c. Better heat insulation is obtained by using hollow bricks.

   d. Air being a poorer conductor than a brick, it further avoids the conduction of heat from outside.

2. Street vendors keep ice blocks packed in saw dust to prevent them from melting rapidly.

3. The handle of a cooking utensil is made of a bad conductor of heat, such as ebonite, to protect our hand from the hot utensil.

4. Two bedsheets used together to cover the body help retain body heat better than a single bedsheet of double the thickness. Trapped air being a bad conductor of heat, the layer of air between the two sheets reduces thermal conduction better than a sheet of double the thickness. Similarly, a blanket coupled with a bedsheet is a cheaper alternative to using two blankets.

Exercises | Q 2. (xiv) | Page 140

Answer the following question.

Explain the term thermal resistance. State its SI unit and dimensions.

SOLUTION

1. Consider expression for conduction rate,

$$P_{cond} = kA\frac{T_1 - T_2}{x}$$

$$\Rightarrow \frac{T_1 - T_2}{P_{cond}} = \frac{x}{kA} \quad .....(1)$$

2. Ratio $\frac{T_1 - T_2}{P_{cond}}$ is called as thermal resistance (R_T) of material.
3. The SI unit of thermal resistance is °C s/ kcal or °C s/J and its dimensional formula is $[L^{-2} M^{-1}T^3K^1]$.

Exercises | Q 2. (xv) | Page 140

Answer the following question.

How heat transfer occurs through radiation in absence of a medium?

SOLUTION

1. All objects possess thermal energy due to their temperature T(T > 0 K).
2. The rapidly moving molecules of a hot body emit EM waves travelling with the velocity of light. These are called thermal radiations.
3. These carry energy with them and transfer it to the low-speed molecules of a cold body on which they fall.
4. This results in an increase in the molecular motion of the cold body and its temperature rises.
5. Thus transfer of heat by radiation is a twofold process-the conversion of thermal energy into waves and reconversion of waves into thermal energy by the body on which they fall.

Exercises | Q 2. (xvi) | Page 140

Answer the following question.
What is thermal stress?

SOLUTION

1. Consider a metallic rod of length l_0 fixed between two rigid supports at T °C.
2. If the temperature of rod is increased by ΔT, length of the rod would become, $l = l_0 (1 + \alpha\Delta T)$ Where, α is the coefficient of linear expansion of the material of the rod.

3. But the supports prevent the expansion of the rod. As a result, rod exerts stress on the supports. Such stress is termed as thermal stress.

Exercises | Q 2. (xvii) | Page 140

Answer the following question.
Give an example of the disadvantages of thermal stress in practical use?

SOLUTION

Disadvantage: Thermal stress can lead to fracture or deformation in substance under certain conditions.

Example: Railway tracks are made up of metals which expand upon heating. If no gap is kept between tracks, in hot weather, expansion of metal tracks may exert thermal stress on track. This may lead to bending of tracks which would be dangerous. Hence, railway track is not a continuous piece but is made up of segments separated by gaps.

Exercises | Q 2. (xvii) | Page 140

Answer the following question.
Which materials can be used as thermal insulators and why?

SOLUTION

1. Substances such as glass, wood, rubber, plastic, etc. can be used as thermal insulators.

2. These substances do not have free electrons to conduct heat freely throughout the body. Hence, they are poor conductors of heat.

Exercises | Q 3. (i) | Page 140

Solve the following problem.
A glass flask has a volume 1×10^{-4} m^3. It is filled with a liquid at 30 ºC. If the temperature of the system is raised to 100 ºC, how much of the liquid will overflow. (Coefficient of volume expansion of glass is 1.2×10^{-5} $(ºC)^{-1}$ while that of the liquid is 75×10^{-5} $(ºC)^{-1}$).

SOLUTION

Given: $V_1 = 1 \times 10^{-4}$ $m^3 = 10^{-4}$ m^3, $T_1 = 30°$ C, $T_2 = 100$ ºC.

To find: Volume of liquid that overflows

Formula: $\gamma = \frac{V_2 - V_1}{V_1(T_2 - T_1)}$

Calculation: From formula,

Increase is volume = $V_2 - V_1 = \gamma = V_1(T_2 - T_1)$

Increase in volume of beaker

$= \gamma_{glass} \times V_1(T_2 - T_1)$

$= 1.2 \times 10^{-5} \times 10^{-4} \times (100 - 30)$

$= 1.2 \times 70 \times 10^{-9}$

$= 84 \times 10^{-9}\ m^3$

∴ Increase in volume of beaker = $84 \times 10^{-9}\ m^3$

Increase in volume of liquid = $\gamma_{liquid} \times V_1(T_2 - T_1)$

$= 75 \times 10^{-5} \times 10^{-4} \times (100 - 30)$

$= 75 \times 10^{-5} \times 10^{-4} \times (100 - 30)$

$= 75 \times 70 \times 10^{-9}$

$= 5250 \times 10-9\ m^3$

∴ Increase in volume of liquid = $5250 \times 10^{-9}\ m^3$

∴ Volume of liquid which overflows

$= (5250 - 84) \times 10^{-9}\ m^3$

$= 5166 \times 10^{-9}\ m^3$

$= 0.5166 \times 10^{-7}\ m^3$

Volume of liquid that overflows is **$0.5166 \times 10^{-7}\ m^3$**

Exercises | Q 3. (ii) | Page 140

Solve the following problem.

Which will require more energy, heating a 2.0 kg block of lead by 30 K or heating a 4.0 kg block of copper by 5 K?

(s_{lead} = 128 J kg^{-1} K^{-1}, s_{copper} = 387 J kg^{-1} K^{-1})

SOLUTION

Given: m_{lead} = 2 kg, ΔT_{lead} = 30 K,
s_{lead} = 128 J/kg K,
m_{Cu} = 4 kg, ΔT_{Cu} = 5 K,
s_{Cu} = 387 J/kg K

To find: Substance requiring more heat energy.

Formula: $Q = ms\,\Delta T$

Calculation: From formula,

For lead, $Q_{lead} = 2 \times 128 \times 30 = 7680$ J

For Copper, $Q_{Cu} = 4 \times 387 \times 5 = 7740$ J

$Q_{Cu} > Q_{lead}$, copper will require more heat energy.

Copper will require more heat energy.

Exercises | Q 3. (iii) | Page 140

Solve the following problem.
Specific latent heat of vaporization of water is 2.26 × 106 J/kg. Calculate the energy needed to change 5.0 g of water into steam at 100 ºC.

SOLUTION

Given: $L_{vap} = 2.26 \times 10^6$ J/kg, m = 5g = 5×10^{-3} kg
In this case, no temperature change takes place an only a change of state occurs.

To find: Heat required to convert water into steam.

Formula: Heat required = mL_{vap}

Calculation: From formula,

Heat required = $5 \times 10^{-3} \times 2.26 \times 10^6$

= 11300 J

= 1.13×10^4 J

Heat required to convert water into steam is **1.13×10^4 J.**

Exercises | Q 3. (iv) | Page 141

Solve the following problem.
A metal sphere cools at the rate of 0.05 ºC/s when its temperature is 70 ºC and at the rate of 0.025 ºC/s when its temperature is 50 ºC. Determine the temperature of the surroundings and find the rate of cooling when the temperature of the metal sphere is 40 ºC.

SOLUTION

Given: T_1 = 70 °C, $\left(\frac{dT}{dt}\right)_1$, 0.05 °C/s

T_2 = 50 °C, $\left(\frac{dT}{dt}\right)_2$, 0.025 °C/s, T_3 = 40 °C.

To find: i. Temperature of surrounding (T_0)

ii. Rate of cooling $\left(\frac{dT}{dt}\right)_3$

Formula: $\frac{dT}{dt} = C(T - T_0)$

Calculation: From formula,

$$\left(\frac{dT}{dt}\right)_1 = C(T_1 - T_0) \text{ and } \left(\frac{dT}{dt}\right)_2 = C(T_2 - T_0)$$

$$\therefore \frac{0.05}{0.025} = \frac{C(70 - T_0)}{C(50 - T_0)}$$

∴ 2 (50 - T_0) = 70 - T_0

∴ T_0 = **30 πC**

Substituting value of T_o,

0.05 = C(70 - 30)

∴ C = $\frac{0.05}{40} = 0.00125$/s.

For T_3 = 40 °C

$$\left(\frac{dT}{dt}\right)_3 = C(T_3 - T_0) = 0.00125(40 - 30)$$

= 0.00125 × 10

= **0.0125 °C/s.**

i. Temperature of surroundings is **30 °C**.

ii. Rate of cooling at 40 °C is **0.0125 °C/s**.

Exercises | Q 3. (v) | Page 141

Solve the following problem.

The volume of a gas varied linearly with absolute temperature if its pressure is held constant. Suppose the gas does not liquefy even at very low temperatures, at what temperature the volume of the gas will be ideally zero?

SOLUTION

At temperature of –273.15 ºC, the volume of the gas will be ideally zero.

Exercises | Q 3. (vi) | Page 141

Solve the following problem.

In olden days, while laying the rails for trains, small gaps used to be left between the rail sections to allow for thermal expansion. Suppose the rails are laid at room temperature 27 °C. If maximum temperature in the region is 45 °C and the length of each rail section is 10 m, what should be the gap left given that $\alpha = 1.2 \times 10^{-5} K^{-1}$ for the material of the rail section?

SOLUTION

Given: T_1 = 27 °C, T_2 = 45 °C, L_1 = 10 m. $\alpha = 1.2 \times 10^{-5}$ / K

To find: Gap that should be left ($L_2 - L_1$)

Formula: $L_2 - L_1 = L_1 \alpha (T_2 - T_1)$

Calculation: From formula,

$L_2 - L_1 = 10 \times 1.2 \times 10^{-5} \times (45 - 27)$

$= 2.16 \times 10^{-3}$ m

= 2.16 mm

The gap that should be left between rail sections is **2.16 mm**.

Exercises | Q 3. (vii) | Page 141

Solve the following problem.

A blacksmith fixes iron ring on the rim of the wooden wheel of a bullock cart. The diameter of the wooden rim and the iron ring are 1.5 m and 1.47 m respectively at room temperature of 27 °C. To what temperature the iron ring should be heated so that it can fit the rim of the wheel? ($\alpha_{iron} = 1.2 \times 10^{-5} K^{-1}$).

SOLUTION

Given: $d_w = 1.5$ m, $d_i = 1.47$ m, $T_1 = 27$ °C.

$\alpha_i = 1.2 \times 10^{-5}$/ K

To find: Temperature (T_2)

Formula: $\alpha = \dfrac{d_w - d_i}{d_i \alpha} + T_1$

Calculation: From formula,

$$T_2 = \frac{d_w - d_i}{d_i \alpha} + T_1$$

$$= \frac{1.5 - 1.47}{1.47 \times 1.2 \times 10^{-5}} + 27$$

= 1700.7 + 27

= 1727.7 °C

Iron ring should be heated to temperature of **1727.7 °C**.

Exercises | Q 3. (viii) | Page 141

Solve the following problem.
In a random temperature scale X, water boils at 200 °X and freezes at 20 °X. Find the boiling point of a liquid in this scale if it boils at 62 °C.

SOLUTION

Here thermometric property P is temperature at some random scale X.

Using equation,

$$T = \frac{100(P_T - P_1)}{P_2 - P_1}$$

For $P_1 = 20$ °X,

$P_2 = 200$ °X,

T = 62 °C

$$\therefore 62 = \frac{100(P_T - 20)}{200 - 20}$$

$$\therefore P_T = \frac{62 \times (200 - 20)}{100} + 20$$

= 111.6 + 20

= 131.6 °X

The boiling point of a liquid in this scale is **131.6 °X**.

Exercises | Q 3. (ix) | Page 141

Solve the following problem.

A gas at 900 °C is cooled until both its pressure and volume are halved. Calculate its final temperature.

SOLUTION

Given: T_1 = 900 °C = 900 + 273.15 = 1173.15 K

$$V_2 = \frac{V_1}{2}, P_2 = \frac{P_1}{2}$$

To find: Final temperature (T_2)

Formula: $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

Calculation: From formula,

$$\frac{P_1V_1}{1173.15} = \frac{P_1V_1}{4T_2}$$

$$\therefore T_2 = \frac{1173.15}{4} = 293.29\text{K}$$

Final temperature of gas is **293.29 K**.

Exercises | Q 3. (x) | Page 141

Solve the following problem.
An aluminium rod and iron rod show 1.5 m difference in their lengths when heated at all temperature. What are their lengths at 0 °C if coefficient of linear expansion for aluminium is 24.5×10^{-6}/°C and for iron is 11.9×10^{-6}/°C?

SOLUTION

Given: $(L_T)_i - (L_T)_{al} = 1.5$ m, $T_0 = 0$ °C,
$\alpha_{al} = 24.5 \times 10^{-6}$/°C
$\alpha_i = 11.9 \times 10^{-6}$/°C

To find: Lengths of aluminium and iron rod $(L_0)_{al}$ and $(L_0)_i$

Formula: $L_T = L_0 [(1 + \alpha (T - T_0)]$

Calculation: For $T_0 = 0$ °C
From formula, $L_T = L_0 (1+ \alpha T)$
For aluminium,
$(L_T)_{al} = (L_0)_{al} (1+ \alpha_{al} T)$ ....(1)
For iron,
$(L_T)_i = (L_0)_i (1 + \alpha_i T)$ ....(2)

Subtracting equation (2) by (1),

$(L_T)_i = (L_T)_{al} = [(L_0)_i + (L_0)_i \alpha_i T] - [(L_0)_{al} + (L_0)_{al} \alpha_{al} T]$

$= (L_0)_i - (L_0)_{al} + [(L_0)_i \alpha_i - (L_0)_{al} \alpha_{al}]T$

$\therefore 1.5 = 1.5 + [(L_0)_i \alpha_i - (L_0)_{al} \alpha_{al}] T$

$\Rightarrow [(L_0)_i \alpha_i - (L_0)_{al} \alpha_{al}] T = 0$

$\therefore (L_0)_{al} \alpha_{al} = (L_0)_i \alpha_i$

$\therefore (L_0)_{al} = (L_0)_i \alpha_i$

$$\therefore (L_0)_{al} = (L_0)_i \frac{\alpha_i}{\alpha_{al}}$$

$$= (L_0)_i \times \frac{11.9 \times 10^{-6}}{24.5 \times 10^{-6}}$$

$$= (L_0)_i \times \frac{17}{35}$$

$$(L_0)_{al} = [(L_0)_{al} + 1.5] \times \frac{17}{35} \text{ ..... [Given:} (L_T)_i - (L_T)_{sl} = 1.5\text{m]}$$

$$\therefore 35(L_0)_{al} = 17(L_0)_{al} + 1.5 \times 17$$

$$\therefore 35(L_0)_{al} - 17(L_0)_{al} = 1.5 \times 17$$

$$\therefore 18(L_0)_{al} = 1.5 \times 17$$

$$\therefore (L_0)_{al} = \frac{1.5 \times 17}{18} = 1.417 \text{ m}$$

$$\therefore (L_0)_i = 1.5 + (L_0)_{al}$$

= 1.5 + 1.417

= 2.917 m

Length of aluminium rod at 0 °C is **1.417 m** and that of iron rod is **2.917 m**.

Exercises | Q 3. (xi) | Page 141

Solve the following problem.
What is the specific heat of metal if 50 cal of heat is needed to raise 6 kg of the metal from 20°C to 62 °C?

SOLUTION

Given: Q = 50 cal, m = 6 kg, ΔT = 62 - 20 = 42 °C.

To find: Specific heat (s)

Formula: Q = ms ΔT

Calculation: From formula,

$$s = \frac{Q}{m\Delta T} = \frac{50}{6 \times 42} = 0.198 \text{ cal/kg °C}$$

Specific heat of metal is copper **0.198 cal/kg °C**.

Exercises | Q 3. (xii) | Page 141

Solve the following problem.
The rate of flow of heat through a copper rod with temperature difference 30 °C is 1500 cal/s. Find the thermal resistance of copper rod.

SOLUTION

Given: ΔT = 30 °C, P_{cond} = 1500 cal/s

To find: Thermal resistance (R_T)

Formula: $R_T = \frac{\Delta T}{P_{cond}}$

Calculation: From formula,

$R_T = \frac{30}{1500}$ = 0.02 °C s/cal.

Thermal resistance of copper rod is **0.02 °C s/cal**.

Exercises | Q 3. (xiii) | Page 141

Solve the following problem.
An electric kettle takes 20 minutes to heat a certain quantity of water from 0 °C to boiling point. It requires 90 minutes to turn all the water at 100 °C into steam. Find the latent heat of vaporization. (Specific heat of water = 1 cal/g °C)

SOLUTION

Let heat supplied by the kettle in 20 minutes be Q_1 and that in 90 min. be Q_2.

Using heat Q_1, the temperature of the water is raised from 0 °C to 100 °C.

If the mass of water in the kettle is 'm' then,

$Q_1 = ms_{water}\Delta T = m \times 1 \times (100 - 0)$ = 100 m .....(1) .....$(\because S_{water} = 1 cal/g^\circ C)$

Similarly using heat Q_2 water is converted from liquid to gas,

$\therefore Q_2 = m L_{vap}$ ....(ii)

Given that heat Q_1, Q_2 are supplied to water in 20 min. (t_1) and 90 min (t_2) respectively.

Kettle being same its conduction rate (P_{cond}) is same.

Using, $P_{cond} = \frac{Q_1}{t_1} = \frac{Q_2}{t_2}$ ...(iii)

From (i), (ii) and (iii),

$$\frac{100\text{ m}}{20} = \frac{\text{mL}_{\text{vap}}}{90}$$

$$\therefore \text{L}_{\text{vap}} = 5 \times 90 = 450\text{ cal/g}$$

Latent heat of vaporisation for water is **450 cal/ g**.

Exercises | Q 3. (xiv) | Page 141

Solve the following problem.

Find the temperature difference between two sides of a steel plate 4 cm thick, when the heat is transmitted through the plate at the rate of 400 k cal per minute per square meter at a steady state. Thermal conductivity of steel is 0.026 kcal/m s K.

SOLUTION

Given: $\frac{\text{Q}}{\text{A t}}$ = 400 kcal/min m^2 = $\frac{400}{60}$ kcal/s m^2

x = 4 cm = 4×10^{-2} m, k = 0.026 kcal/m s k

To find: Temperature difference (T_1 - T_2).

Formula: $Q = \frac{\text{kA}(\text{T}_1 - \text{T}_2)}{\text{x}}$

Calculation: From formula,

$$\text{T}_1 - \text{T}_2 = \frac{\text{Q}}{\text{At}} \times \frac{\text{x}}{\text{k}}$$

$$\therefore \text{T}_1 - \text{T}_2 = \frac{400}{60} \times \frac{4 \times 10^{-2}}{0.026} = 10.26\text{K}$$

The temperature difference between the two sides is **10.26 K**.

Exercises | Q 3. (xv) | Page 141

Solve the following problem.

A metal sphere cools from 80 °C to 60 °C in 6 min. How much time with it take to cool from 60 °C to 40 °C if the room temperature is 30 °C?

SOLUTION

Given: T_1 = 80 °C, T_2 = 60 °C, T_3 = 40 °C, T_0 = 30 °C, (dt)$_1$ = 6 min.

To find: Time taken in cooling $(dt)_2$

Formula: $\frac{dT}{dt} = C(T - T_0)$

Calculation: From formula,

$$\left(\frac{dT}{dt}\right)_1 = C(T_1 - T_0)$$

$\therefore \frac{80 - 60}{6}$ = C(80 - 30)

$\therefore$ C = $\frac{20}{6 \times 50} = \frac{1}{15}$/min

Now, $\left(\frac{dT}{dt}\right)_2 = C(T_2 - T_0)$

$\therefore \frac{60 - 40}{dt^2} = \frac{1}{15}(60 - 30)$

$\therefore dt^2 = \frac{60 - 40}{30/15}$ = 10 min

Time taken in cooling is **10 min**.