22. Pythagoras Theorem

Let us Work Out 22

1 A. Question

If the followings are the lengths of the three sides of a triangle, then let us write by calculating, the cases where the triangles are right-angled triangles:

8cm., 15cm. and 17 cm.

Answer

Given: Three sides of a triangle are 8 cm, 15 cm and 17 cm.

To check if the triangle is a right-angled triangle, we verify by Pythagoras theorem.

Taking the longest side to be hypotenuse and other two sides as base and perpendicular, we have,

Hypotenuse, h = 17 cm

Perpendicular, p = 8 cm

Base, b = 15 cm

According to Pythagoras theorem,

$h^2 = p^2 + b^2$

$\Rightarrow 17^2 = 8^2 + 15^2$

$\Rightarrow 289 = 64 + 225$

$\Rightarrow 289 = 289$

Hence, verified that the triangle is a right- angled triangle.

1 B. Question

If the followings are the lengths of the three sides of a triangle, then let us write by calculating, the cases where the triangles are right-angled triangles:

9cm., 11cm. and 6cm.

Answer

Given: Three sides of the triangle are 9 cm, 11 cm and 6 cm.

To check if the triangle is a right-angled triangle, we verify by Pythagoras theorem.

Taking the longest side to be hypotenuse and other two sides as base and perpendicular, we have,

Hypotenuse, h = 11 cm

Perpendicular, p = 6 cm

Base, b = 9 cm

According to Pythagoras theorem,

$h^2 = p^2 + b^2$

$\Rightarrow 11^2 = 6^2 + 9^2$

$\Rightarrow 121 = 36 + 81$

$\Rightarrow 121 = 117$

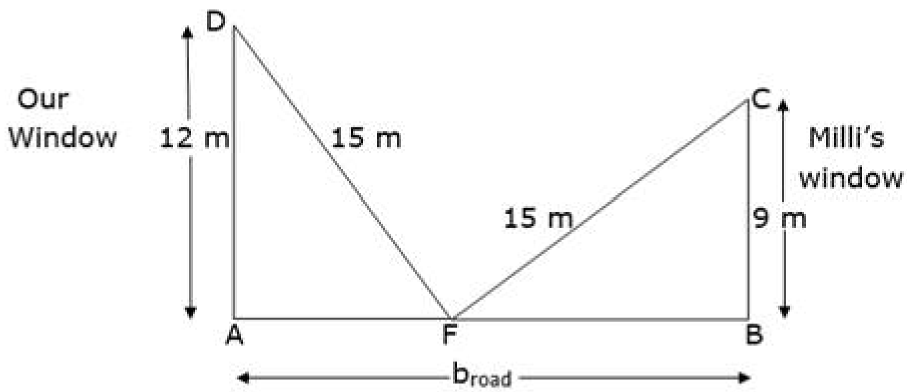
2. Question

In the road of our locality there is a ladder of 15m. length kept in such a way that it has touched Milli's window at a height of 9m. above the ground. Now keeping the foot of the ladder at the same point of that road. The ladder is rotated in such a way that it touched our window situated on the other side of the road. If our window is 12m. above the ground, then let us determine the breadth of that road in our locality.

Answer

Given: Length of ladder, l = 15 m

Now, the situation can be shown in a diagram as:



As the wall is always perpendicular to the floor or road, so ΔAFD and ΔBFC are right angled triangle.

Here b_{road} represents breadth of road.

To find b_{road} first we need to find the lengths AF and FB.

In right ΔAFD,

h = 15 m

p = 12 m

b = AF

By applying Pythagoras theorem, we have,

$h^2 = p^2 + b^2$

$\Rightarrow 15^2 = 12^2 + AF^2$

$\Rightarrow 225 = 144 + AF^2$

$\Rightarrow AF^2 = 225 - 144$

$\Rightarrow AF^2 = 121$

$\Rightarrow AF = \sqrt{121}$

$\Rightarrow$ AF = 11 m …………………… (1)

In right ΔBFC,

h = 15 m

p = 9 m

b = FB

By applying Pythagoras theorem, we have,

$h^2 = p^2 + b^2$

$\Rightarrow 15^2 = 9^2 + FB^2$

$\Rightarrow 225 = 81 + FB^2$

$\Rightarrow FB^2 = 225 - 81$

$\Rightarrow FB^2 = 144$

$\Rightarrow FB = \sqrt{144}$

$\Rightarrow$ FB = 12 m …………………… (2)

Now,

$b_{road} = AF + FB$

Substituting from eqn. (1) and (2), we have,

b_{road} = 11 + 12 m

b_{road} = 13 m

Therefore, the breath of road is 13 m.

3. Question

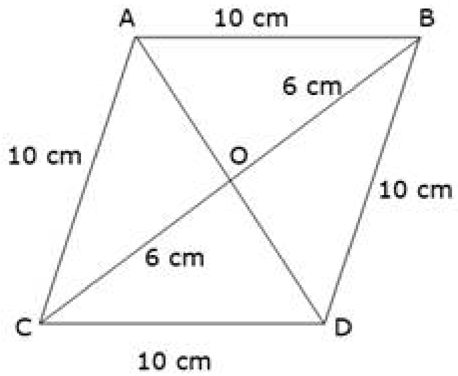
If the length of one diagonal of a rhombus having the side 10cm. length is 12cm., then let us write, by calculating the length of other diagonal.

Answer

Given: Side of rhombus = 10 cm

Length of one diagonal = 12 cm

The figure for the question is:



We know that diagonals a rhombus are perpendicular bisector to each other.

So, the ΔAOB is a right angled triangles with ∠AOB as right angle.

Now, in ΔAOB,

h = 10 cm

p = 6 cm

b = AO

By applying Pythagoras Theorem we have,

$h^2 = p^2 + b^2$

$10^2 = 6^2 + AO^2$

$\Rightarrow 100 = 36 + AO^2$

$\Rightarrow AO^2 = 100 - 36$

$\Rightarrow AO^2 = 64$

$\Rightarrow AO = \sqrt{64} = 8$ cm

Also,

$OD = AO = 1/2AD$ [∵ Diagonals of rhombus bisect each other]

$\Rightarrow AD = 2 \times AO$

$\Rightarrow AD = 2 \times 8$

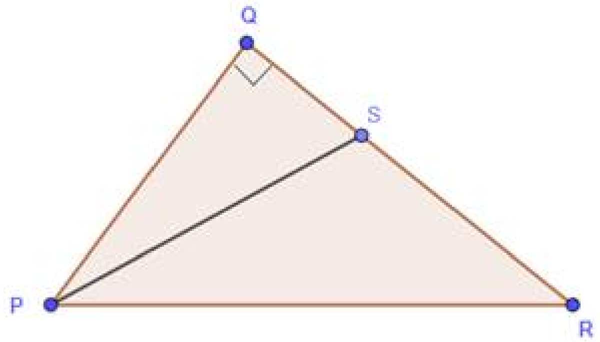
$\Rightarrow AD = 16$ cm

Thus the length of the other diagonal is 16 cm.

4. Question

I have drawn a triangle PQR whose ∠Q is right angle. If S is any point on QR, let us prove that, $PS^2 + QR^2 = PR^2 + QS^2$

Answer



Given: ∠Q is a right angle.

To prove: $PS^2 + QR^2 = PR^2 + QS^2$

Proof:

It can be seen both the triangles ΔPQS and ΔPQR are right angled triangles.

Applying Pythagoras Theorem to ΔPQS gives,

$PS^2 = PQ^2 + QS^2$ [∵ $H^2 = P^2 + B^2$]

$\Rightarrow PQ^2 = PS^2 - QS^2$ ……………… (1)

Applying Pythagoras theorem to ΔPQR gives,

$PR^2 = PQ^2 + QR^2$ [∵ $H^2 = P^2 + B^2$]

Substituting PQ^2 from (1) gives,

$PR^2 = (PS^2 - QS^2) + QR^2$

$\Rightarrow PR^2 = PS^2 - QS^2 + QR^2$

$\Rightarrow PR^2 + QS^2 = PS^2 + QR^2$

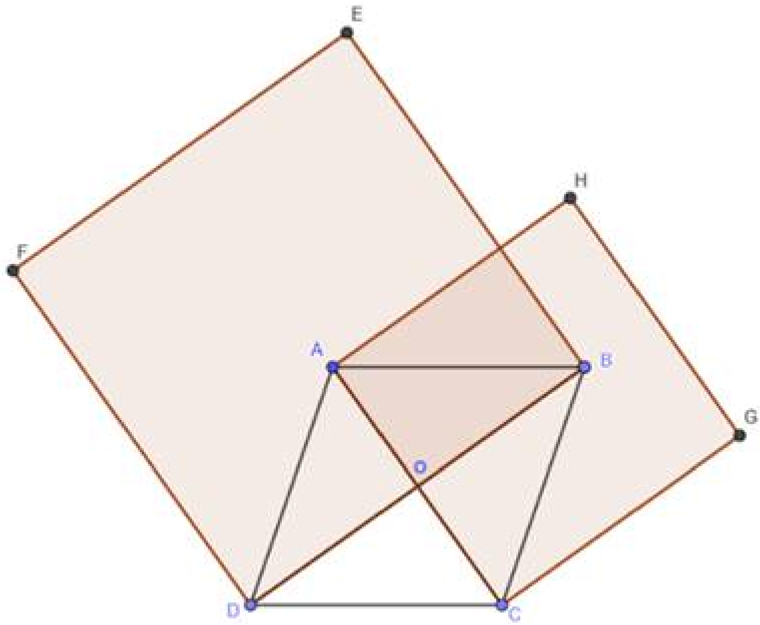
$\Rightarrow PS^2 + QR^2 = PR^2 + QS^2$

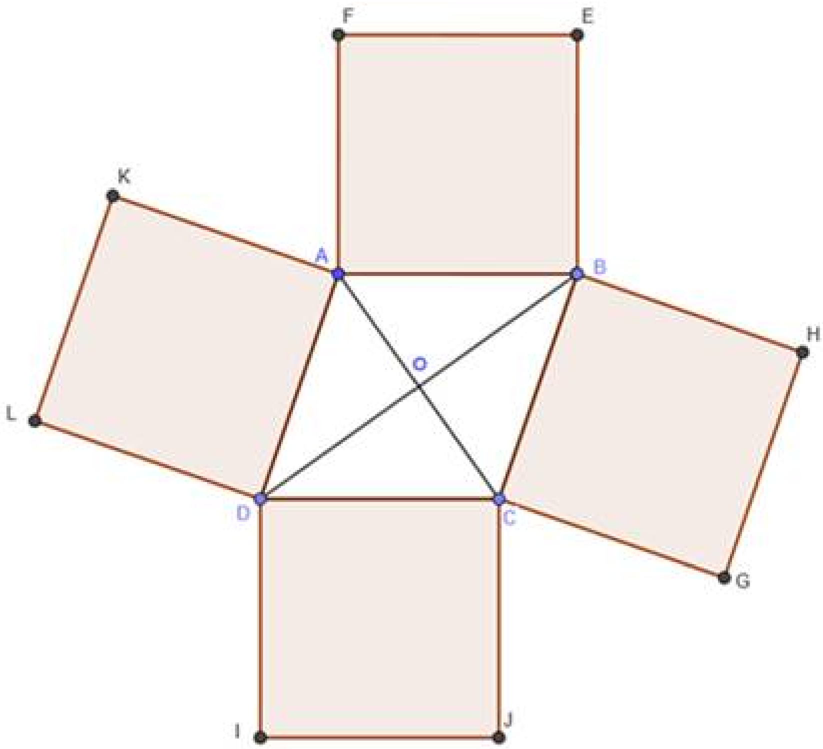
Hence proved.

5. Question

Let us prove that, the sum of squares drawn on the sides of a rhombus is equal to the sum of squares drawn on two diagonals.

Answer





Given: ABCD is a rhombus.

To prove: $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Proof:

We know that diagonals of a rhombus are perpendicular bisector to each other.

So,

AO = OC = 1/2AC

BO = OD = 1/2BD

$\angle AOB = \angle BOC = \angle COD = \angle AOD = 90°$

Also all sides of rhombus are equal,

AB = BC = CD = DA ………………… (1)

By applying Pythagoras Theorem to ΔAOB, we get,

$AB^2 = AO^2 + BO^2$ [$\because H^2 = P^2 + B^2$]

$\Rightarrow AB^2 = (1/2AC)^2 + (1/2BD)^2$

$\Rightarrow AB^2 = 1/4AC^2 + 1/4BD^2$

$\Rightarrow AB^2 = 1/4(AC^2 + BD^2)$

$\Rightarrow 4AB^2 = AC^2 + BD^2$

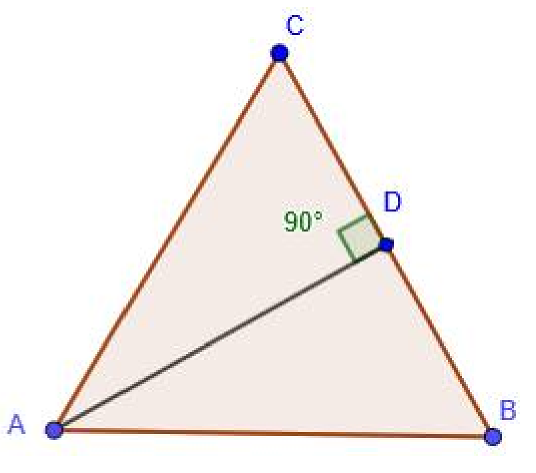
$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$ [By using eqn. (1)]

Hence proved.

6. Question

ABC is an equilateral triangle. AD is perpendicular on the side BC, let us prove that, $AB^2 + BC^2 + CA^2 = 4AD^2$.

Answer



Given: AB = BC = CA = x

$\angle ADC = \angle ADB = 90°$

To prove: $AB^2 + BC^2 + CA^2 = 4AD^2$

Proof:

We know that in an equilateral triangle perpendicular from any vertex on opposite side bisects it.

So, $BD = DC = 1/2BC$ ……………… (1)

By applying Pythagoras theorem in ΔABD, we get,

$AB^2 = AD^2 + BD^2$ [$\because H^2 = P^2 + B^2$]

By substituting BD from eqn. (1) we get,

$AB^2 = AD^2 + (1/2BC)^2$

$\Rightarrow AB^2 = AD^2 + 1/4BC^2$

$\Rightarrow 4AB^2 = 4AD^2 + BC^2$

$\Rightarrow 4AB^2 - BC^2 = 4AD^2$

$\Rightarrow 4AB^2 - AB^2 = 4AD^2$ [Given: AB = BC]

$\Rightarrow 3AB^2 = 4AD^2$

$\Rightarrow AB^2 + BC^2 + CA^2 = 4AD^2$ [$\because AB = BC = CA$]

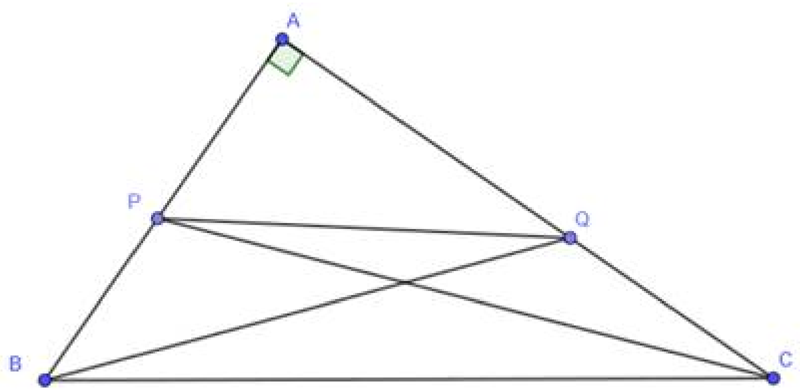
Hence proved.

7. Question

I have drawn a right angled triangle ABC whose ∠A is right angle. I took two points P and Q on the sides AB and AC respectively. By joining P,B,Q and C, P let us prove that, $BQ^2 + PC^2 = BC^2 + PQ^2$

Answer

Given: ABC is a right angled triangle with $\angle A = 90°$



To prove: $BQ^2 + PC^2 = BC^2 + PQ^2$

Proof:

By applying Pythagoras theorem in ΔAPQ, we get,

$PQ^2 = AP^2 + AQ^2$ ……………… (1)

By applying Pythagoras theorem in ΔABQ, we get,

$BQ^2 = AB^2 + AQ^2$ ……………… (2)

By applying Pythagoras theorem in ΔAPC, we get,

$PC^2 = AP^2 + AC^2$ ………………… (3)

By applying Pythagoras theorem in ΔABC, we get,

$BC^2 = AB^2 + AC^2$ ……………… (4)

By adding (1) and (2) we get,

$BQ^2 + PC^2 = AB^2 + AQ^2 + AP^2 + AC^2$

$\Rightarrow BQ^2 + PC^2 = AB^2 + AC^2 + AQ^2 + AP^2$

Substituting from (1) and (4) we get,

$BQ^2 + PC^2 = BC^2 + PQ^2$

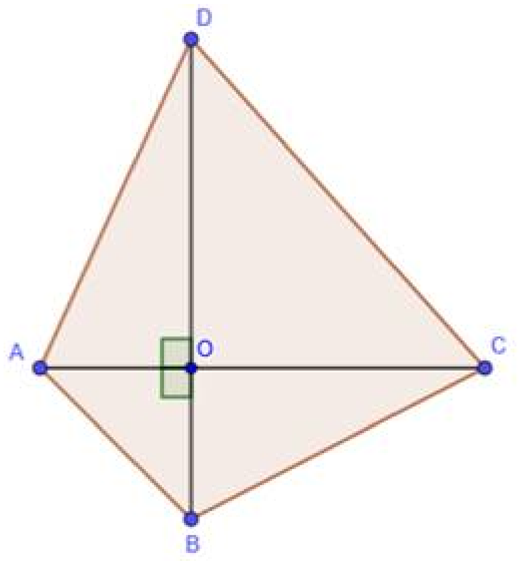
Hence proved.

8. Question

If two diagonals of a quadrilateral ABCD intersect each other perpendicularly, then let us prove that, $AB^2 + CD^2 = BC^2 + DA^2$

Answer

Given: $AC \perp BD$



To prove: $AB^2 + CD^2 = BC^2 + DA^2$

Proof:

By applying Pythagoras theorem in ΔAOB, we get,

$AB^2 = AO^2 + BO^2$ ……………… (1)

By applying Pythagoras theorem in ΔBOC, we get,

$BC^2 = BO^2 + CO^2$ ……………… (2)

By applying Pythagoras theorem in ΔCOD, we get,

$CD^2 = CO^2 + DO^2$ ……………… (3)

By applying Pythagoras theorem in ΔDOA, we get,

$DA^2 = AO^2 + DO^2$ ……………… (4)

By adding eqn. (1) and (3), we get,

$AB^2 + CD^2 = AO^2 + BO^2 + CO^2 + DO^2$

$\Rightarrow AB^2 + CD^2 = (BO^2 + CO^2) + (AO^2 + DO^2)$

Substituting from eqn. (2) and (4) gives,

$AB^2 + CD^2 = BC^2 + DA^2$

Hence proved.

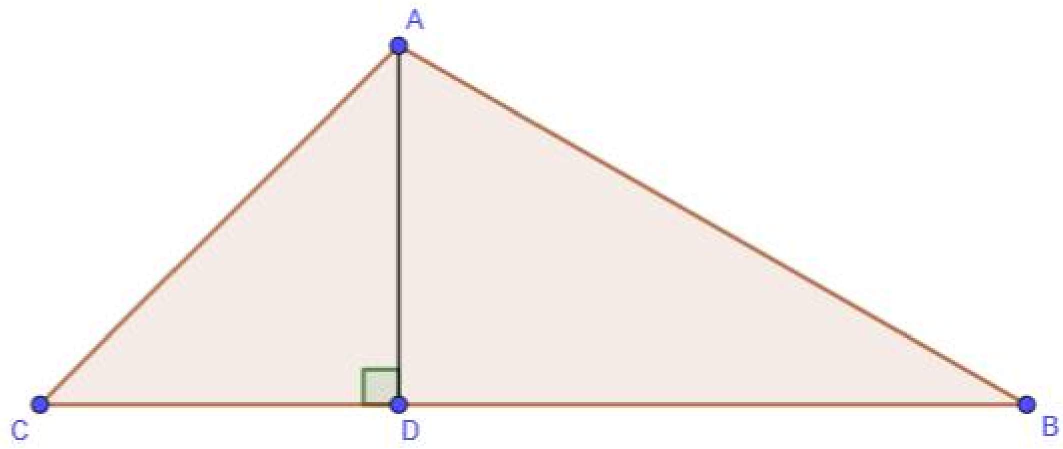
9. Question

I have drawn a triangle ABC whose height is AD. If AB>AC, let us prove that, AB^2-AC = BD^2-CD^2

Answer

Given: In ΔABC, AB>AC

Height, h = AD



To prove: $AB^2 - AC^2 = BD^2 - CD^2$

Proof:

By applying Pythagoras theorem in ΔACD, we have,

$AC^2 = AD^2 + CD^2$ ……………… (1)

By applying Pythagoras theorem in ΔABD, we have,

$AB^2 = AD^2 + BD^2$ ……………… (2)

Subtracting eqn. (1) from (2), we get,

$AB^2 - AC^2 = AD^2 + BD^2 - (AD^2 + CD^2)$

$\Rightarrow AB^2 - AC^2 = AD^2 + BD^2 - AD^2 - CD^2$

$\Rightarrow AB^2 - AC^2 = BD^2 - CD^2$

Hence proved.

10. Question

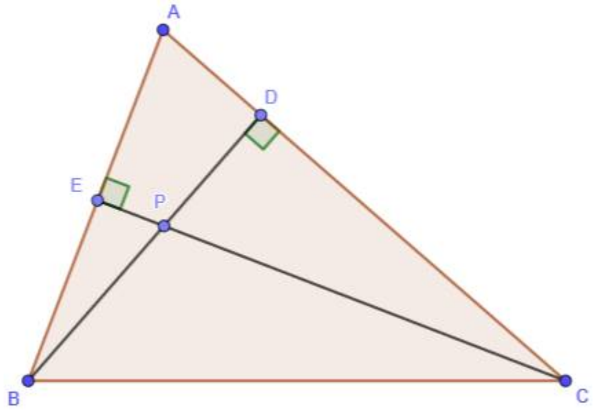
In ΔABC I have drawn two perpendiculars from two vertices B and C on AC and AB (AC>AB) which are intersected each other at the point P. Let us prove that, $AC^2 + BP^2 = AB^2 + CP^2$

Answer

Given: In ΔABC, AC>AB

$CE \perp AB$

$BD \perp AC$



To prove: $AC^2 + BP^2 = AB^2 + CP^2$

Proof:

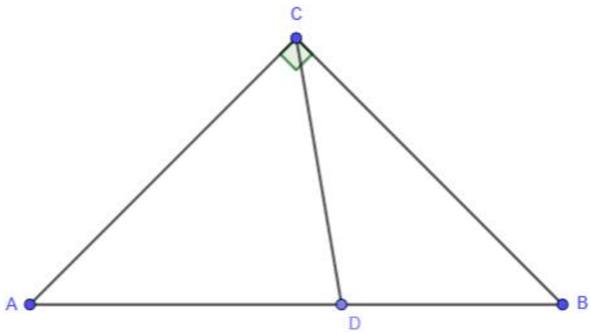
11. Question

ABC is and isosceles triangle whose ∠C is right angle. If D is mid point of AB, then let us prove that, $AD^2 + DB^2 = 2CD^2$

Answer

Given: ABC is an isosceles triangle. So, AC = BC

∠C = right angle



To prove: $AD^2 + DB^2 = 2CD^2$

Proof:By Pythagoras Theroem,

In ΔABC, We have

$AB^2 = AC^2 + BC^2$ [1]

In ΔADC and ΔADB, we have

AC = BC [Given]

CD = CD [Common]

AD = DB [D is the mid point]

⇒ ΔADC ≅ ΔADB [By SSS Congruency Criterion]

$\Rightarrow \angle ADC = \angle ADB$ [Corresponding parts of congruent triangles are equal]

Also, $\angle ADC + \angle ADB = 180°$ [Linear Pair]

$\Rightarrow \angle ADC + \angle ADC = 180°$

$\Rightarrow \angle ADC = \angle ADB = 90°$

Hence, ADC and ADB are right triangles, ∴ By Pythagoras theorem

$AC^2 = CD^2 + AD^2$ [2]

$BC^2 = CD^2 + BD^2$ [3]

Adding [2] and [3], we have

$AC^2 + BC^2 = AD^2 + BD^2 + 2CD^2$

$\Rightarrow AB^2 = AD^2 + BD^2 + 2CD^2$

$\Rightarrow (2AD)^2 = AD^2 + AD^2 + 2CD^2$

[∵ D is mid-point of AB, AD = BD and AB = 2AD = 2BD]

$\Rightarrow 4AD^2 = 2AD^2 + 2CD^2$

$\Rightarrow 2AD^2 = 2CD^2$

$\Rightarrow AD^2 + BD^2 = 2CD^2$ [∵ AD = BD]

Hence, Proved!

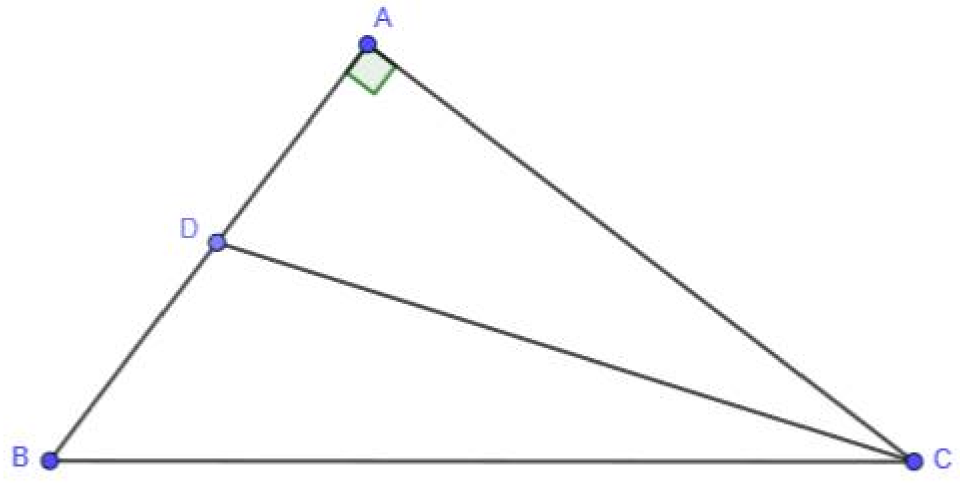
12. Question

In the triangle ABC, A is right angle, if CD is a median, let us prove that, $BC^2 = CD^2 + 3AD^2$

Answer

Given: $\angle A$ is right angle.

CD is median.



To prove: $BC^2 = CD^2 + 3AD^2$

Proof:

We know that median divides the side into two halves.

So,

$AB = 2AD = 2BD$ ………… (1)

By applying Pythagoras theorem in ΔADC, we have,

$CD^2 = AD^2 + AC^2$

$\Rightarrow AC^2 = CD^2 - AD^2$ ……… (2)

By applying Pythagoras theorem in ΔABC, we have,

$BC^2 = AB^2 + AC^2$ ………… (3)

Substituting from eqn. (1) and (2) into (3) gives,

$BC^2 = (2AD)^2 + (CD^2 - AD^2)$

$\Rightarrow BC^2 = 4AD^2 + CD^2 - AD^2$

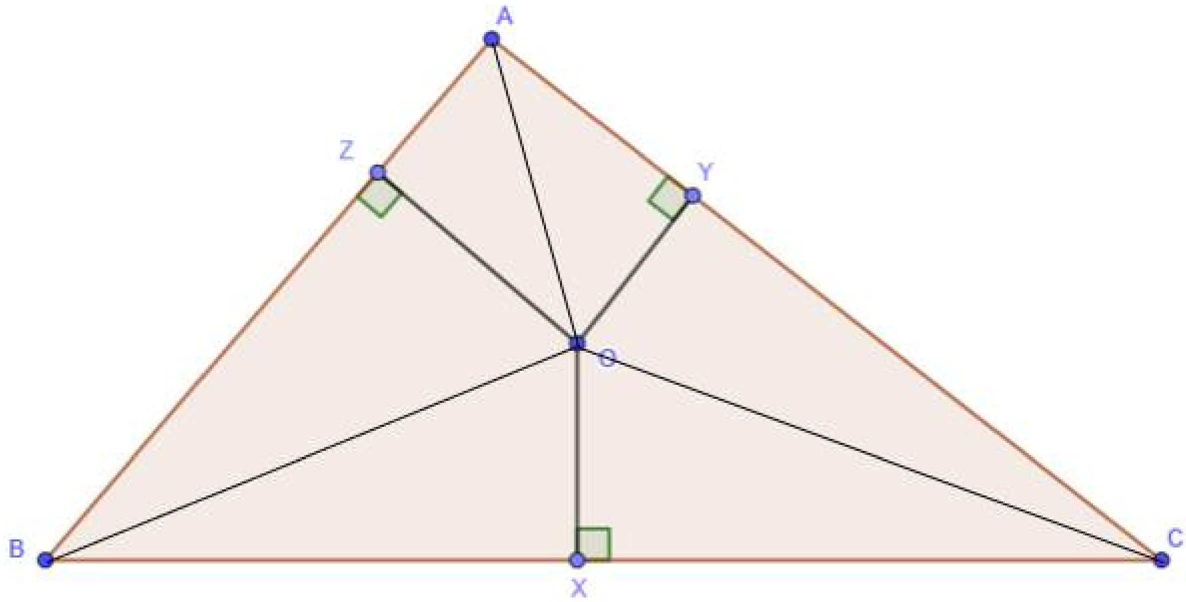
$\Rightarrow BC^2 = CD^2 + 3AD^2$

Hence proved.

13. Question

From a point O within a tringle ABC, I have drawn the perpendiculars OX, OY and OZ on BC, CA and AB respectively. Let us prove that, $AZ^2 + BX^2 + CY^2 = AY^2 + CX^2 + BZ^2$

Answer



By applying Pythagoras theorem in ΔAOZ, we get,

$AO^2 = AZ^2 + ZO^2$

$\Rightarrow AZ^2 = AO^2 - ZO^2$ ……………… (1)

By applying Pythagoras theorem in ΔBOX, we get,

$BO^2 = BX^2 + XO^2$

$\Rightarrow BX^2 = BO^2 - XO^2$ ………………… (2)

By applying Pythagoras theorem in ΔCOY, we get,

$CO^2 = CY^2 + YO^2$

$\Rightarrow CY^2 = CO^2 - YO^2$ ………………… (3)

By adding eqn. (1), (2) and (3), we have,

$AZ^2 + BX^2 + CY^2 = AO^2 - ZO^2 + BO^2 - XO^2 + CO^2 - YO^2$

$\Rightarrow AZ^2 + BX^2 + CY^2 = AO^2 + BO^2 + CO^2 - XO^2 - YO^2 - ZO^2$ …… (i)

By applying Pythagoras theorem in ΔAOY, we get,

$AO^2 = AY^2 + YO^2$

$\Rightarrow AY^2 = AO^2 - YO^2$ ………… (4)

By applying Pythagoras theorem in ΔBOZ, we get,

$BO^2 = BZ^2 + ZO^2$

$\Rightarrow BZ^2 = BO^2 - ZO^2$ …………… (5)

By applying Pythagoras theorem in ΔCOX, we get,

$CO^2 = CX^2 + XO^2$

$\Rightarrow CX^2 = CO^2 - XO^2$ …………… (6)

By adding eqn. (4), (5) and (6), we have,

$AY^2 + BZ^2 + CX^2 = AO^2 - YO^2 + BO^2 - ZO^2 + CO^2 - XO^2$

$\Rightarrow AY^2 + CX^2 + BZ^2 = AO^2 + BO^2 + CO^2 - XO^2 - YO^2 - ZO^2$ …… (ii)

By comparing eqn. (i) and (ii) we get,

$AZ^2 + BX^2 + CY^2 = AY^2 + CX^2 + BZ^2$

Hence proved.

14. Question

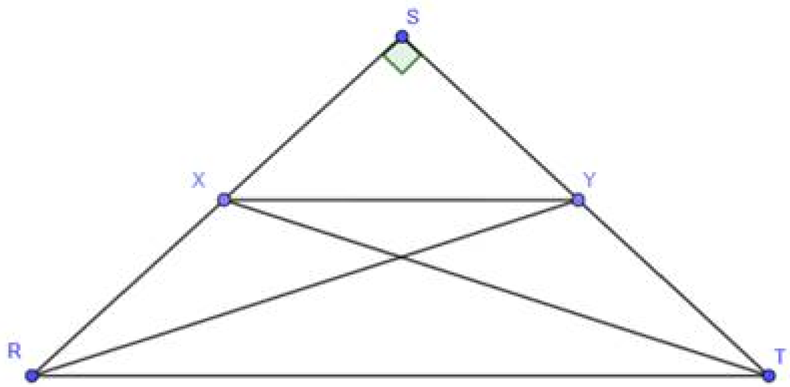
In ΔRST, ∠S is right angle. The mid- points of tow sides RS and ST are X and Y respectively; let us prove that, $RY^2 + XT^2 = 5XY^2$

Answer

Given: ΔRST is a right angled triangle with ∠S as right angle.

X is midpoint of RS.

Y is midpoint of ST.



To prove: $RY^2 + XT^2 = 5XY^2$

Proof:

In ΔRSY,

$RY^2 = RS^2 + SY^2$ [$\because H^2 = P^2 + B^2$]

$\Rightarrow RY^2 = (2XS)^2 + SY^2$ [∵ X is midpoint of RS]

$\Rightarrow RY^2 = 4XS^2 + SY^2$ ……………… (1)

In ΔXST,

$XT^2 = XS^2 + ST^2$ [$\because H^2 = P^2 + B^2$]

$\Rightarrow XT^2 = XS^2 + (2SY)^2$ [$\because$ Y is midpoint of ST]

$\Rightarrow XT^2 = XS^2 + 4SY^2$ ……………… (2)

In ΔXSY,

$XY^2 = XS^2 + SY^2$ …………… (3) [$\because H^2 = P^2 + B^2$]

Now, adding eqn. (1) and (2) gives,

$RY^2 + XT^2 = 4XS^2 + SY^2 + XS^2 + 4SY^2$

$\Rightarrow RY^2 + XT^2 = 5XS^2 + 5SY^2$

$\Rightarrow RY^2 + XT^2 = 5(XS^2 + SY^2)$

Substituting from (3) gives,

$RY^2 + XT^2 = 5XY^2$

Hence proved.

15 A1. Question

A person goes 24m. west from a place and then he goes 10m. north. The distance of the person from starting point is

A. 34m.

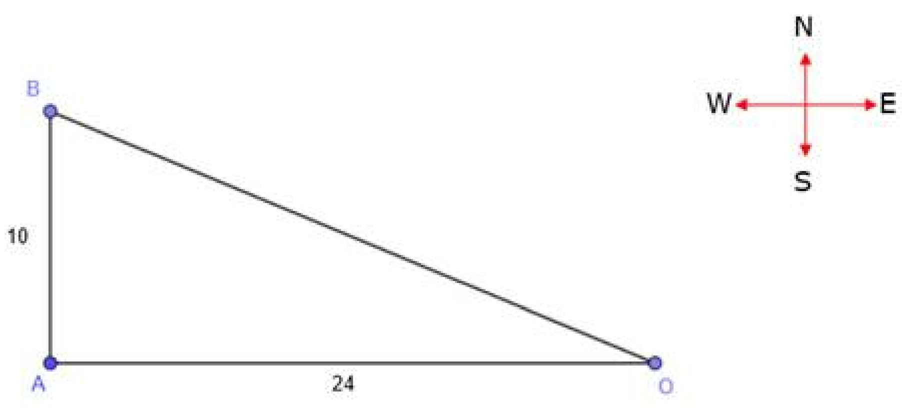
B. 17m.

C. 26m.

D. 25m.

Answer

Let the person start his journey from a point O.



The figure given above traverses his journey. Length of BO gives his distance from starting point.

It can be seen that ΔAOB is a right angled triangle because the direction axis are always perpendicular to each other.

By applying Pythagoras theorem to ΔAOB we get,

$h^2 = p^2 + b^2$

$\Rightarrow BO^2 = 10^2 + 24^2$

$\Rightarrow BO^2 = 100 + 576$

$\Rightarrow BO^2 = 676$

$\Rightarrow BO = \sqrt{676} = 26$ m

Thus, he is 26 m away from starting point.

15 A2. Question

If ABC is an equilateral triangle and AD ⊥ BC, then $AD^2 =$

A. $\frac{3}{2}DC^2$

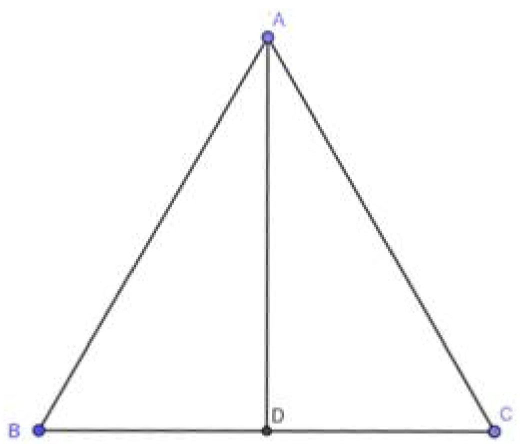
B. $2.DC^2$

C. $3DC^2$

D. $4DC^2$

Answer

The given triangle is shown below:



By applying Pythagoras theorem in right angled triangle ADC, we get,

$AC^2 = AD^2 + DC^2$ [$\because h^2 = p^2 + b^2$]

$\Rightarrow AD^2 = AC^2 – DC^2$

$\Rightarrow AD^2 = BC^2 – DC^2$ [AC = BC, sides of equilateral Δ]

$\Rightarrow AD^2 = (2DC)^2 - DC^2$ [BC = 2DC, AD bisects BC]

$\Rightarrow AD^2 = 4DC^2 - DC^2$

$\Rightarrow AD^2 = 3DC^2$

15 A3. Question

In an isosceles triangle ABC, if AC = BC and $AB^2 = 2AC^2$, then the measure of C is

A. 30°

B. 90°

C. 45°

D. 60°

Answer

Given: $AB^2 = 2AC^2$

$\Rightarrow AB^2 = AC^2 + AC^2$

$\Rightarrow AB^2 = AC^2 + BC^2$ [∵ Given: AC = BC]

The expression given above is equivalent to expression of Pythagoras theorem, $h^2 = p^2 + b^2$, which corresponds to a right angled triangle.

Also, h = AB and angle opposite to hypotenuse is right angle.

Thus ∠C = 90°.

15 A4. Question

Two rods of 13m. length and 7m. length are situated perpendicularly on the ground and the distance between their foots is 8m. The distance between their top parts is

A. 9m.

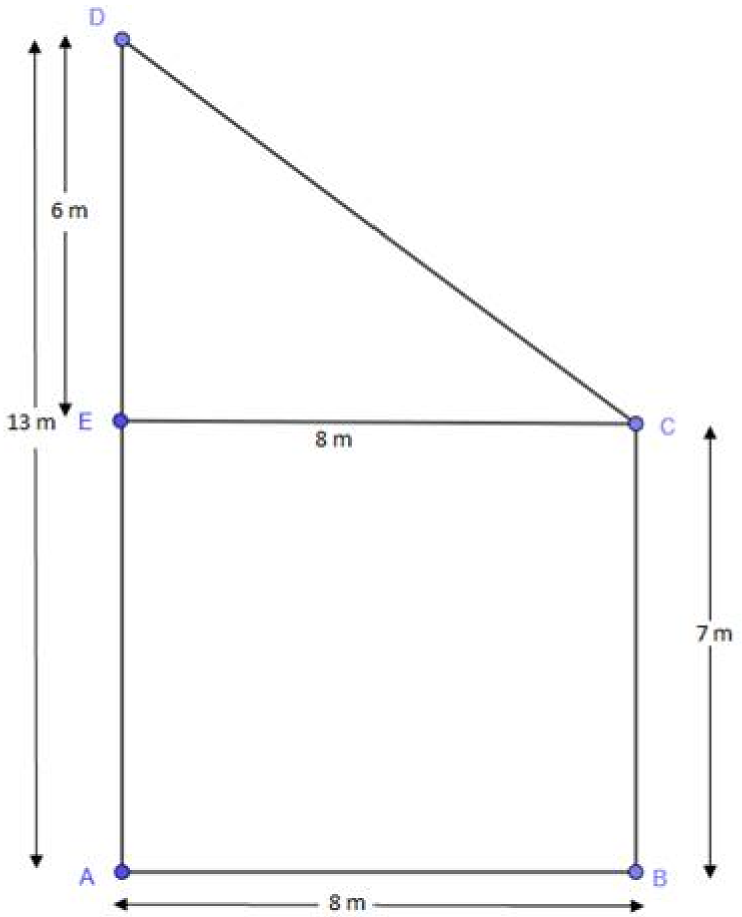
B. 10m.

C. 11m.

C. 12m.

Answer

The situation can be depicted by figure given below:



Here AD and BC are two rods.

Distance between top CD can be given by Pythagoras theorem.

By applying Pythagoras theorem in ΔCED, we get,

$h^2 = p^2 + b^2$

$\Rightarrow CD^2 = 6^2 + 8^2$

$\Rightarrow CD^2 = 36 + 64$

$\Rightarrow CD^2 = 100$

$\Rightarrow CD = \sqrt{100} = 10$ cm

15 A5. Question

If the lengths of two diagonals of a rhombus are 24cm. and 10cm, the perimeter of the rhombus is

A. 13cm.

B. 26cm.

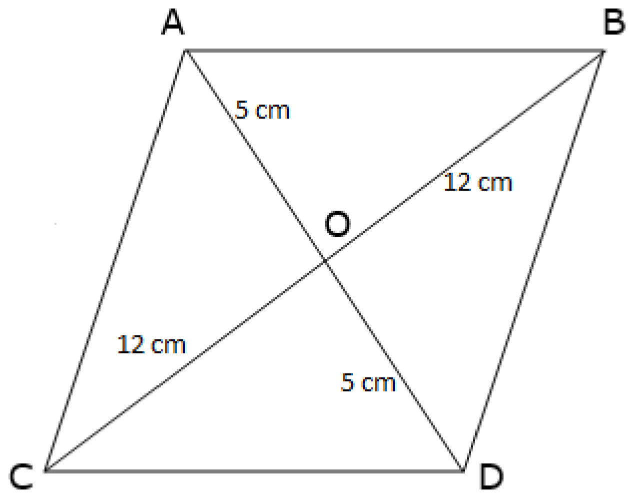
C. 52cm.

D. 25cm.

Answer

We know that the diagonals of a rhombus are perpendicular bisector to each other.

It can be solved using figure given below:



By applying Pythagoras theorem to ΔAOB, we get,

$AB^2 = 5^2 + 12^2$

$\Rightarrow AB^2 = 25 + 144$

$\Rightarrow AB^2 = 169$

$\Rightarrow AB = \sqrt{169} = 13$ cm

Perimeter of rhombus = 4 × side

= 4 × AB

= 4 × 13 cm

= 52 cm

15 B. Question

Let us write whether the following statements are true or false:

(i) If the ratio of the lengths of three sides of a triangle is 3:4:5, then the triangle will always be a right angled triangle.

(ii) If in a circle of radius 10cm. length, a chord subtends right angle at the centre, then the length of the chord will be 5cm.

Answer

(i) True

Let the sides of triangle be 3x, 4x and 5x.

By applying Pythagoras theorem to this triangle, we get,

$(5x)^2 = (3x)^2 + (4x)^2$

$\Rightarrow 25x^2 = 9x^2 + 16x^2$

$\Rightarrow 25x^2 = (9 + 16)x^2$

$\Rightarrow 25x^2 = 25x^2$

Thus, all triangles having sides in ratio 3:4:5 will form right angled triangle.

(ii) False

Given: Length of chord = 10 cm

Since, the chord subtends right angle at the centre, then the triangle is a right angled triangle.

Let the radius of circle be x.

By applying Pythagoras theorem to this triangle we get,

$10^2 = x^2 + x^2$

$\Rightarrow 100 = 2x^2$

$\Rightarrow 2x^2 = 100$

$\Rightarrow x^2 = 50$

$\Rightarrow x = \sqrt{50} = 2\sqrt{5}$ cm

It is found that the radius of the circle must be $2\sqrt{5}$ cm which mismatch from the radius given in question as 5 cm. Hence, it is false statement.

15 C. Question

Let us fill in blanks:

(i) In a right angled triangle, the area of square drawn on the hypotenuse is equal to the ______of the areas of the squares drawn on other two sides.

(ii) In an isosceles right-angled triangle if the length of each of two equal sides is 42cm., then the length of the hypotenuse will be ______cm.

(iii) In a rectangular figure ABCD, the two diagonals AC and BD intersect each other at the point O, if AB = 12 cm., AO = 6.5 cm., then the length of BC is ______cm.

Answer

(i) In a right angled triangle, the area of square drawn on the hypotenuse is equal to the <u>SUM</u> of the areas of the squares drawn on the other two sides.

Explanation:

It is inferred from expression of Pythagoras theorem i.e.

$H^2 = P^2 + B^2$

(ii) In an isosceles right-angled triangle if the length of each of two equal sides is 42 cm, then the length of the hypotenuse will be <u>$42\sqrt{2}$</u> cm.

Explanation:

By Pythagoras theorem we get,

$h^2 = 42^2 + 42^2$

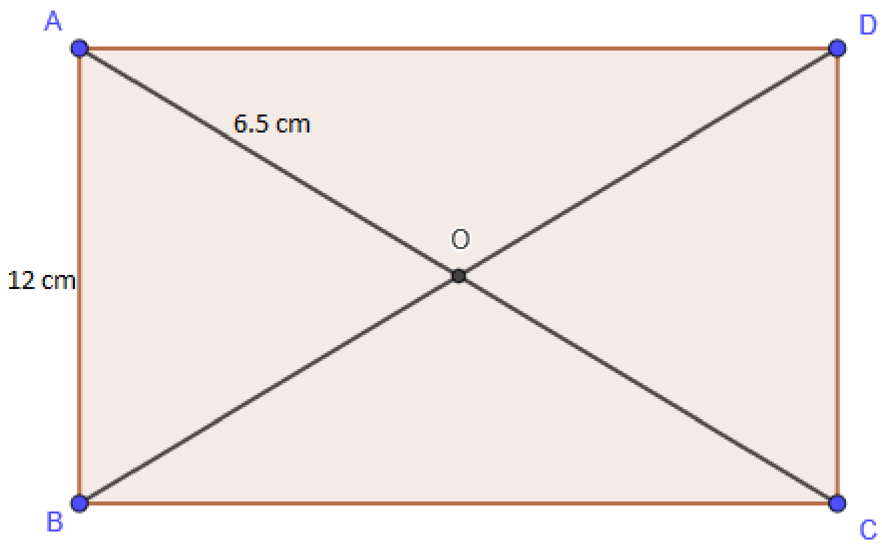
$\Rightarrow h^2 = 1764 + 1764$

$\Rightarrow h^2 = 3528$

$\Rightarrow h = \sqrt{3528} = 42\sqrt{2}$ cm

(iii) In a rectangular figure ABCD, the two diagonals AC and BD intersect each other at point O, if AB = 12 cm, AO = 6.5 cm, then the length of BC is <u>5</u> cm.

Explanation:



We know that, diagonals of a rectangle bisect each other.

So, AC = 2 × AO

$\Rightarrow$ AC = 2 × 6.5

$\Rightarrow$ AC = 13 cm

Now, applying Pythagoras theorem to ΔABC gives,

$13^2 = 12^2 + BC^2$

$\Rightarrow 169 = 144 + BC^2$

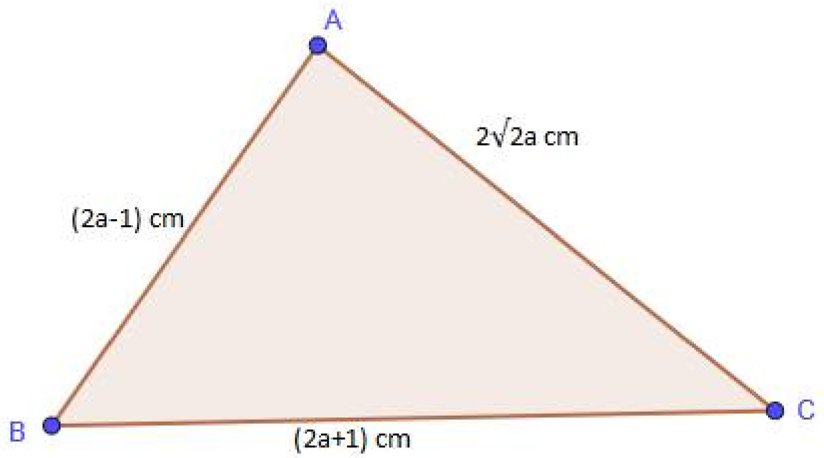
$\Rightarrow BC^2 = 169-144 = 25$

$\Rightarrow BC = \sqrt{25} = 5$ cm

16 A. Question

In ΔABC, if AB = (2a-1)cm. AC = $2\sqrt{2a}$cm. And BC = (2a + 1)cm., then let us write the value of ∠BAC.

Answer



Let's consider that the triangle is a right angled triangle,

By applying Pythagoras theorem we get,

$(2a + 1)^2 = (2\sqrt{2a})^2 + (2a-1)^2$

$\Rightarrow (2a)^2 + 1^2 + 2\times2a\times1 = 4\times2a + (2a)^2 + 1^2 - 2\times2a\times1$

$\Rightarrow 4a^2 + 1 + 4a = 8a + 4a^2 + 1 - 4a$

$\Rightarrow 4a^2 + 1 + 4a = 4a^2 + 1 + 4a$

Since, the sides of the triangle satisfy Pythagoras theorem, thus the triangle is a right angled triangle with angle opposite to hypotenuse as right angle.

Here,

H = 2a + 1

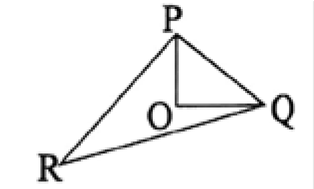
P = 2a-1

B = $2\sqrt{2a}$

Thus, ∠BAC is a right angle with measure 90°.

16 B. Question

In the adjoining figure, the point O is situated within the triangle PQR in such a way that ∠POQ = 90°, OP = 6cm. and OQ = 8cm. If PR = 24cm. and ∠QPR = 90°, then let us write the length of QR.



Answer

Given: PQR and POQ are right angled triangles.

$\angle QPR = 90°$

$\angle POQ = 90°$

OP = 6 cm

OQ = 8 cm

PR = 24 cm

By applying Pythagoras theorem to ΔPOQ, we get,

$PQ^2 = OP^2 + OQ^2$

$\Rightarrow PQ^2 = 6^2 + 8^2$

$\Rightarrow PQ^2 = 36 + 64$

$\Rightarrow PQ^2 = 100 = 10^2$

$\Rightarrow PQ = 10$ cm

Now, by applying Pythagoras Theorem to ΔPQR, we get,

$QR^2 = PQ^2 + PR^2$

$\Rightarrow QR^2 = 10^2 + 24^2$

$\Rightarrow QR^2 = 100 + 576$

$\Rightarrow QR^2 = 676 = 26^2$

$\Rightarrow QR = 26$ cm

Thus, QR is 26 cm long.

16 C. Question

The point O is situated within the rectangular figure ABCD in such a way that OB = 6 cm., OD = 8cm. and OA = 5cm. Let us determine the length of OC.

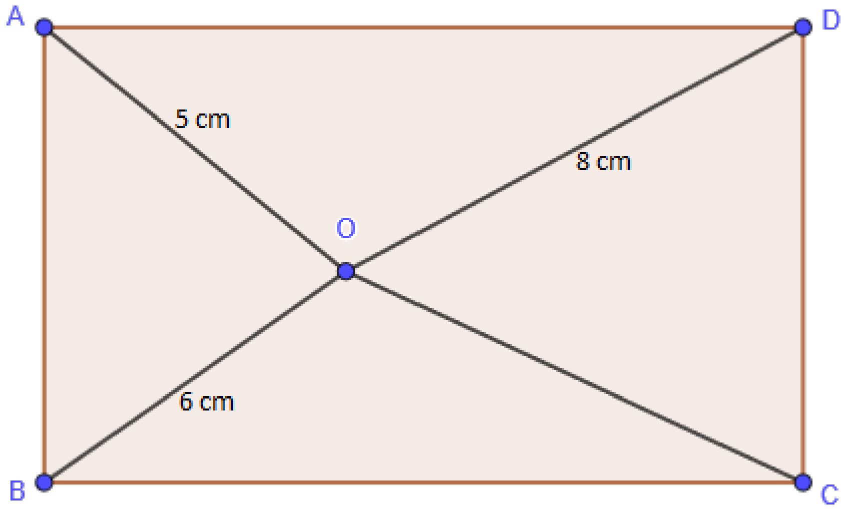
Answer

Given: In a rectangle ABCD with an interior point O,

$OA = 5$ cm

$OB = 6$ cm

$OD = 8$ cm



We know in a rectangle ABCD with an interior point O,

$OA^2 + OC^2 = OB^2 + OD^2$

$\Rightarrow 5^2 + OC^2 = 6^2 + 8^2$

$\Rightarrow 25 + OC^2 = 36 + 64$

$\Rightarrow OC^2 = 100\text{-}25 = 75$

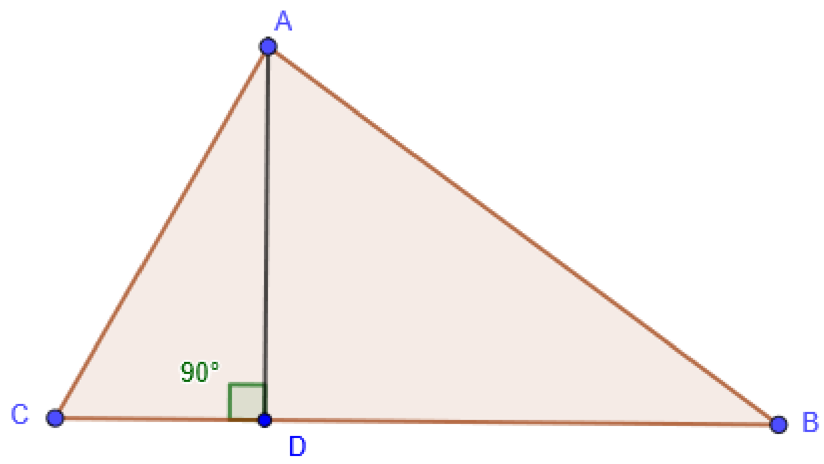
$\Rightarrow OC = \sqrt{75} = 5\sqrt{3}$ cm

Thus OC is $5\sqrt{3}$ cm long.

16 D. Question

In the triangle ABC the perpendicular AD from the point A on the side BC meets the side BC at the point D. If BD = 8cm, DC = 2cm. and AD = 4cm., then let us write the measure of $\angle BAC$.

Answer



Given: BD = 8 cm

DC = 2 cm

AD = 4 cm

By applying Pythagoras theorem to ΔACD we get,

$AC^2 = AD^2 + CD^2$

$AC^2 = 4^2 + 2^2$

$AC^2 = 16 + 4$

$AC^2 = 20$ ……………………… (1)

By applying Pythagoras theorem to ΔABD we get,

$AB^2 = AD^2 + BD^2$

$AB^2 = 4^2 + 8^2$

$AB^2 = 16 + 64$

$AB^2 = 80$ ……………………… (2)

Now,

BC = BD + DC

BC = 8 + 2

BC = 10

$BC^2 = 10^2 = 100$ ……………… (3)

By adding eqns. (1) and (2) and equating with (3) we get,

$BC^2 = AB^2 + AC^2$

$\Rightarrow h^2 = p^2 + b^2$

OR,

∠BAC is a right angle.

16 E. Question

In a right angled triangle ∠ABC, ABC = 90°, AB = 3cm., BC = 4cm. and the perpendicular BD on the side AC from the point B which meets the side AC at the point D. Let us determine the length of BD.

Answer