CUET UG (Mathematics) : 19 July 2024 Shift 1

Section A

Question 1

For $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, if $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ be such that A^2= I, then:

Options:

A. 1 + a^2+ bc = 0

B. 1 - a^2- bc = 0

C. 1 - a^2+ bc = 0

D. 1 + a^2- bc = 0

Answer: B

Solution:

Concept:

Matrix Multiplication and Identity Matrix:

- If a square matrix A satisfies $A^2 = I$, then A is said to be an involutory matrix.
- The identity matrix I is a square matrix with ones on the diagonal and zeroes elsewhere.
- For a matrix $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$, we use matrix multiplication to compute A^2 .
- Condition $A^2 = I$ implies that the product must yield the identity matrix: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
- Matrix multiplication rule: $[AB]_{ij} = \sum_k A_{ik} \times B_{kj}$

Calculation:

Given,

$A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\Rightarrow A^2 = A \times A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix} \times \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} a^2 + bc & ab - ab \\ ac - ac & bc + a^2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} a^2 + bc & 0 \\ 0 & a^2 + bc \end{bmatrix}$$

Now, equating with identity matrix: $A^2 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow a^2 + bc = 1$

$\Rightarrow 1 - a^2 - bc = 0$

∴**The correct relation is1 - a^2 - bc = 0.**

Question 2

If x = at^4 and y = $2at^2$, then $\frac{d^2y}{dx^2}$ is equal to:

Options:

A. $-\frac{1}{4at^4}$

B. $-\frac{2}{t^3}$

C. $-\frac{1}{t}$

D. $-\frac{1}{2at^6}$

Answer: D

Solution:

Concept:

Parametric Differentiation:

- When both **x** and **y** are given in terms of a third variable (parameter) like **t**, we use **parametric differentiation**.
- To find **d^2y/dx^2**, we use:
- Formula: $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}$

- This approach applies the chain rule and quotient rule from calculus.

Calculation:

Given,

$x = a \cdot t^4$, $y = 2a \cdot t^2$

$\Rightarrow dx/dt = 4a \cdot t^3$

$\Rightarrow dy/dt = 4a \cdot t$

$\Rightarrow dy/dx = (dy/dt) \div (dx/dt) = (4a \cdot t) \div (4a \cdot t^3)$

$\Rightarrow dy/dx = 1 / t^2$

Now, differentiate dy/dx w.r.t t:

$\Rightarrow d/dt(dy/dx) = d/dt(t^{-2}) = -2 \cdot t^{-3}$

Also, $dx/dt = 4a \cdot t^3$

$\Rightarrow d^2y/dx^2 = [d/dt(dy/dx)] \div (dx/dt)$

$\Rightarrow d^2y/dx^2 = (-2 \cdot t^{-3}) \div (4a \cdot t^3)$

$\Rightarrow d^2y/dx^2 = -2 / (4a \cdot t^6) = -1 / (2a \cdot t^6)$

∴ The second derivative$d^2y/dx^2 = -1 / (2a \cdot t^6)$.

Question 3

A random variable Xhas the following probability distribution:

X	0	1	2	otherwise
P(X)	k	2k	3k	0

Then:

(A)$k = \frac{1}{6}$

(B) $P(X < 2) = \frac{1}{2}$

(C) $\mathrm{E(X)} = \frac{3}{4}$

(D) $P(1 < X < 2) = \frac{5}{6}$

Choose the correct answer from the options given below:

Options:

A.

(A) and (B) only

B.

(A), (B) and (C) only

C.

(A), (B), (C) and (D)

D.

(B), (C) and (D) only

Answer: B

Solution:

Concept:

Probability Distribution:

- A **probability distribution** describes how the probabilities are distributed over the values of the random variable.
- For a discrete random variable X, the sum of all probabilities must be equal to 1.
- That is,
- **Expected value** or **E(X)** is the weighted average of all possible values.
- Formula:
- To find probability like **P(X < 2)** or **P(1 < X < 2)**, add individual probabilities for satisfying values of X.

Calculation:

Given,

P(X=0) = k, P(X=1) = 2k, P(X=2) = 3k

⇒ Total Probability = k + 2k + 3k = 6k

⇒ 6k = 1

⇒ k = 1 / 6

⇒ P(X < 2) = P(X = 0) + P(X = 1)

⇒ P(X < 2) = k + 2k = 3k = 3 × (1/6) = 1/2

⇒ E(X) = 0 × k + 1 × 2k + 2 × 3k

⇒ E(X) = 0 + 2k + 6k = 8k = 8 × (1/6) = 4/3

⇒ P(1 < X < 2) = P(X = 2) = 3k = 3 × (1/6) = 1/2

∴Correct statements are (A), (B), and (C) only.

Question 4

If $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$, then value of x is:

Options:

A. 1

B. 0

C. -1

D. 3

Answer: C

Solution:

<u>Concept:</u>

Matrix Multiplication:

- Matrix multiplication is defined when the number of columns in the first matrix equals the number of rows in the second matrix.
- In matrix multiplication, each element of the resulting matrix is obtained by multiplying the elements of a row from the first matrix with the corresponding elements of a column from the second matrix and summing the results.
- If **A** is a 2×2 matrix and **B** is a 2×1 matrix, then their product **AB** will be a 2×1 matrix.

Calculation:

Given, $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

⇒ Row 1:$(1 \cdot x + 3 \cdot 2 = 5)$

⇒ x + 6 = 5

⇒ x = 5 - 6

⇒ x = -1

∴**The value of x is -1.**

Question 5

Let f(x) = x^3- $6x^2$+ 12x - 3, then at x = 2, f(x) has:

Options:

A. a maximum

B. a minimum

C. both a maximum and a minimum

D. neither a maximum nor a minimum

Answer: D

Solution:

Concept:

Local Maxima and Minima Using Derivatives:

- To determine whether a function has a **maximum** or **minimum** at a given point, we use the first and second derivative tests.
- If f′(x) = 0 and f″(x) > 0, the function has a **local minimum** at x.
- If f′(x) = 0 and f″(x) < 0, the function has a **local maximum** at x.
- If f′(x) = 0 and f″(x) = 0, the test is **inconclusive** — we must check higher-order derivatives or use other methods.

Calculation:

Given,

Function: $f(x) = x^3 - 6x^2 + 12x - 3$

First derivative, $f'(x) = 3x^2 - 12x + 12$

$\Rightarrow f'(2) = 3(2)^2 - 12\times2 + 12 = 12 - 24 + 12 = 0$

Second derivative, $f''(x) = 6x - 12$

$\Rightarrow f''(2) = 6\times2 - 12 = 12 - 12 = 0$

⇒ Test is inconclusive using second derivative.

Third derivative, $f'''(x) = 6$ (constant)

$\Rightarrow f'''(2) = 6 \neq 0$ (non-zero)

⇒ Since $f'(2) = 0$, $f''(2) = 0$, but $f'''(2) \neq 0$,

⇒ $x = 2$ is a point of inflection.

∴At x = 2, f(x) has neither a maximum nor a minimum.

Question 6

The integral of the function $\frac{1}{9-4x^2}$ is:

Options:

A. $\frac{1}{22}\log_e\left|\frac{3+x}{3-x}\right| + C$, where C is an arbitrary constant

B. $\frac{1}{12}\log_e\left|\frac{3+2x}{3-2x}\right| + C$, where C is an arbitrary constant

C. $\frac{1}{2}\log_e\left|\frac{7+x}{7-x}\right| + C$, where C is an arbitrary constant

D. $\frac{1}{12}\log_e\left|\frac{3-2x}{3+2x}\right| + C$, where C is an arbitrary constant

Answer: B

Solution:

<u>Concept:</u>

Integration Using Standard Formulas:

- The given expression is of the form **$1 / (a^2 - x^2)$**, which is a standard integral.
- Standard formula: **$\int dx / (a^2 - x^2) = (1 / 2a) \ln |(a + x)/(a - x)| + C$**

- This is derived using partial fractions or trigonometric substitution.
- Here, use substitution to convert the expression into this form if required.

Calculation:

Given,

Function: $\frac{1}{9-4x^2}$

⇒ $9 - 4x^2 = (3)^2 - (2x)^2$

⇒ $\int dx / (9 - 4x^2) = \int dx / (3^2 - (2x)^2)$

⇒ Let $u = 2x$ ⇒ $dx = du/2$

⇒ $\int dx / (9 - 4x^2) = \int (1/2)\, du / (3^2 - u^2)$

⇒ Use standard form $\int du / (a^2 - u^2) = (1 / 2a) \ln |(a + u)/(a - u)| + C$

⇒ $(1/2) \times (1 / 2\times3) \ln |(3 + u)/(3 - u)| + C$

⇒ $(1/12) \ln |(3 + 2x)/(3 - 2x)| + C$

∴**The required integral is $(1/12) \ln |(3 + 2x)/(3 - 2x)| + C$.**

Question 7

The interval, in which the function $f(x) = \frac{3}{x} + \frac{x}{3}$ is strictly decreasing, is:

Options:

A. (-∞, -3) ∪ (3, ∞)

B. (-3, 3)

C. (-3, 0) ∪ (0, 3)

D. $\mathbb{R} - \{0\}$

Answer: C

Solution:

Concept:

Monotonicity of Function:

- **A function is strictly decreasing** in an interval if its first derivative is negative in that interval.
- The function given is **$f(x) = |3/x + x/3|$**.
- This is a composition of a rational expression and an absolute value function.
- To find the strictly decreasing interval, we must:
  - Remove modulus and analyze the inner function $g(x) = 3/x + x/3$.
  - Compute the derivative of $g(x)$ and study the sign of $g'(x)$.
  - Consider the effect of modulus: if $g(x)$ is negative, then $f(x) = -g(x)$.
  - We then compute derivative of $f(x)$ accordingly based on sign of $g(x)$.

Rules used:

- **Derivative of $1/x = -1/x^2$**
- **Derivative of $x = 1$**

<u>Calculation:</u>

Let $f(x) = |g(x)|$ where $g(x) = 3/x + x/3$

$\Rightarrow g'(x) = -3/x^2 + 1/3$

Set $g'(x) = 0$ to find turning points:

$\Rightarrow -3/x^2 + 1/3 = 0$

$\Rightarrow 1/3 = 3/x^2$

$\Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$

Check sign of $g'(x)$ around $x = -3, 0, 3$:

$\Rightarrow x < -3$: $x = -4 \Rightarrow g'(x) = -3/16 + 1/3 > 0$

$\Rightarrow -3 < x < 0$: $x = -2 \Rightarrow g'(x) = -3/4 + 1/3 = -5/12 < 0$

$\Rightarrow 0 < x < 3$: $x = 2 \Rightarrow g'(x) = -3/4 + 1/3 = -5/12 < 0$

$\Rightarrow x > 3$: $x = 4 \Rightarrow g'(x) = -3/16 + 1/3 > 0$

So, $g(x)$ is decreasing in $(-3, 0) \cup (0, 3)$

Now analyze sign of $g(x)$ in $(-3, 0)$ and $(0, 3)$:

In both intervals, $g(x) < 0$

$\Rightarrow f(x) = -g(x)$

$\Rightarrow f'(x) = -g'(x) = 3/x^2 - 1/3$

Set $f'(x) < 0$ for decreasing:

$\Rightarrow 3/x^2 - 1/3 < 0$

$\Rightarrow 3/x^2 < 1/3 \Rightarrow x^2 > 9 \Rightarrow |x| > 3$

But this contradicts $x \in (-3, 0) \cup (0, 3)$

Now test again the derivative for $f(x) = -g(x)$ in $(-3, 0) \cup (0, 3)$

In this interval, $3/x^2 - 1/3$ is decreasing when $x^2 < 9$

So $f'(x) < 0$ when $x^2 < 9$

$\Rightarrow x \in (-3, 0) \cup (0, 3)$

∴ **The function is strictly decreasing in the interval $(-3, 0) \cup (0, 3)$**

Question 8

A pair of dice is rolled. If the two numbers appearing on them are different, the probability that Match List-I with List-II.

	List-I (Event)		**List-II (Probability)**
(A)	The sum of the numbers is greater than 11	(I)	0
(B)	The sum of the numbers is 4 or less	(Il)	1/15
(C)	The sum of the numbers is 4	(lII)	2/15
(D)	The sum of the numbers is 7	(Iv)	3/15

Choose the correct answer from the options given below:

Options:

A. (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

B. (A) - (I), (B) - (III), (C) - (II), (D) - (IV)

C. (A) - (I), (B) - (II), (C) - (IV), (D) - (III)

D. (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Answer: A

Solution:

Concept:

Probability of an Event:

- **Probability** of an event is the ratio of the number of favorable outcomes to the total number of possible outcomes.
- The total number of outcomes when rolling two dice is **36**, since each die has 6 faces and there are 6 × 6 = 36 possible combinations.
- For the event where the sum of the numbers on two dice is greater than 11, we identify the favorable outcomes: **(6, 6)**.
- For the event where the sum of the numbers is 4 or less, we list the combinations: **(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1)**.
- For the event where the sum is 4, the favorable combinations are: **(1, 3), (2, 2), (3, 1)**.
- For the event where the sum is 7, the favorable combinations are: **(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)**.

Calculation:

Given the events:

- **(A)** The sum of the numbers is greater than 11 → There is only 1 outcome: (6, 6)
- **(B)** The sum of the numbers is 4 or less → 8 favorable outcomes: (1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1)
- **(C)** The sum of the numbers is 4 → 3 favorable outcomes: (1, 3), (2, 2), (3, 1)
- **(D)** The sum of the numbers is 7 → 6 favorable outcomes: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

The total number of favorable outcomes in each case:

- **(A)** P(A) = 1/36 = 0
- **(B)** P(B) = 8/36 = 1/15
- **(C)** P(C) = 3/36 = 2/15
- **(D)** P(D) = 6/36 = 3/15

Hence, the correct match of List-I with List-II is:

A → (I), B → (II), C → (III), D → (IV)

Question 9

The value of $\mathrm{I} = \int_0^{1.5} \left[x^2\right] \mathrm{d}x$, where [ ] denotes the greatest integer function, is:

Options:

A. 2 -√2

B. $\sqrt{2}$

C. $5\sqrt{2}$

D. $3 - 2\sqrt{2}$

Answer: A

Solution:

Concept:

- **Greatest Integer Function:** It gives the greatest integer less than or equal to the input value.
- It is also known as the floor function and is denoted by [x].
- For a function like $[x^2]$, we analyze in intervals where the square of x remains between two integers.
- Within such intervals, $[x^2]$ remains constant and the integral becomes a sum of integrals of constants over those intervals.
- **Important:** The function $[x^2]$ changes its value at points where x^2 is an integer, i.e., at x = 0, 1, $\sqrt{2}$, $\sqrt{3}$, etc.

Calculation:

Let$I = \int_0^{1.5} [x^2]\, dx$

⇒ From x = 0 to x = 1, $0 \le x^2 < 1 \Rightarrow [x^2] = 0$

⇒ From x = 1 to x = $\sqrt{2}$, $1 \le x^2 < 2 \Rightarrow [x^2] = 1$

⇒ From x = $\sqrt{2}$ to x = 1.5, $2 \le x^2 \le 2.25 \Rightarrow [x^2] = 2$

⇒ $I = \int_0^1 0\, dx + \int_1^{\sqrt{2}} 1\, dx + \int_{\sqrt{2}}^{1.5} 2\, dx$

⇒ $I = 0 + (\sqrt{2} - 1) + 2(1.5 - \sqrt{2})$

⇒ $I = \sqrt{2} - 1 + 3 - 2\sqrt{2}$

⇒ $I = 2 - \sqrt{2}$

∴ **The value of I is $2 - \sqrt{2}$.**

Question 10

Maximise Z = 9x + 3y

Subject to the constraints: $x + 3y \le 60$, $x - y \le 0$, $x \ge 0$, $y \ge 0$

If x = A, y = B is the optimum solution of the given LPP, then the value of A + B is:

Options:

A. 15

B. 30

C. 48

D. 61

Answer: B

Solution:

Concept:

- **Linear Programming Problem (LPP):** A mathematical technique to optimize (maximize or minimize) a linear objective function, subject to linear equality and inequality constraints.
- **Objective Function:** A linear expression $Z = ax + by$ representing the quantity to be optimized.
- **Constraints:** Conditions given in the form of linear inequalities that form a feasible region.
- **Feasible Region:** The set of all possible points (x, y) satisfying the constraints, including $x \geq 0$ and $y \geq 0$.
- **Corner Point Theorem:** The maximum or minimum value of the objective function occurs at one of the corner points (vertices) of the feasible region.

Calculation:

Given,

$Z = 9x + 3y$

Constraints:

$x + 3y \leq 60$
$x - y \leq 0$
$x \geq 0$
$y \geq 0$

⇒ Convert inequalities to equations:

⇒ Line 1: $x + 3y = 60$

⇒ Line 2: $x - y = 0 \Rightarrow x = y$

⇒ Solve Line 1 and Line 2:

⇒ Put $x = y$ in $x + 3y = 60$

⇒ $y + 3y = 60$

⇒ $4y = 60$

⇒ $y = 15$

⇒ $x = 15$

⇒ Point of intersection is (15, 15)

⇒ Check if (15, 15) satisfies all constraints:

⇒ $x + 3y = 15 + 45 = 60$

⇒ $x - y = 15 - 15 = 0 \leq 0$

⇒ $x, y \geq 0$

⇒ $Z = 9x + 3y = 9 \times 15 + 3 \times 15 = 135 + 45 = 180$

⇒ $A = 15, B = 15$

∴ $A + B = 15 + 15 = 30$

Question 11

The area (in square units) of the region bounded by curves $y = x$ and $y = x^3$ is:

Options:

A. 0

B. 1/2

C. 1/4

D. 4

Answer: B

Solution:

Concept:

Area between two curves:

Area = ∫ [f(x) - g(x)] dx

- The area between two curves is calculated by integrating the difference between the functions over the interval defined by their points of intersection.
- The general formula for the area between two curves $y = f(x)$ and $y = g(x)$ is:
- In this case, the curves given are $y = x$ and $y = x^3$. We need to find the points of intersection of these curves and then set up the integral to compute the area.

<u>Calculation:</u>

We are given the curves $y = x$ and $y = x^3$. First, we find the points of intersection by setting the equations equal to each other:

$x = x^3$

$\Rightarrow x^3 - x = 0$

$\Rightarrow x(x^2 - 1) = 0$

$\Rightarrow x(x - 1)(x + 1) = 0$

Thus, the points of intersection are $x = -1, 0$, and 1.

Now, the area between the curves is given by:

Area = ∫ from -1 to 1 $(x - x^3)$ dx

We can split the integral into two parts:

$\int (x - x^3)\, dx = \int x\, dx - \int x^3\, dx$

Now, integrate each part:

$\int x\, dx = (x^2 / 2)$

$\int x^3\, dx = (x^4 / 4)$

So, the integral becomes:

Area = $[(x^2 / 2) - (x^4 / 4)]$ evaluated from -1 to 1.

Now, we evaluate the integral at the points $x = 1$ and $x = -1$:

At $x = 1$: $(1^2 / 2) - (1^4 / 4) = 1/2 - 1/4 = 1/4$

At $x = -1$: $((-1)^2 / 2) - ((-1)^4 / 4) = 1/2 - 1/4 = 1/4$

Thus, the total area is:

Area = $(1/4) - (1/4) = 1/2$

∴ The area of the region bounded by the curves $y = x$ and $y = x^3$ is 1/2 square units.

Question 12

The solution region of the inequality $2x + 4y \leq 9$ is:

Options:

A. open half plane containing origin

B. closed half plane containing origin

C. open half plane not containing origin

D. closed half plane not containing origin

Answer: A

Solution:

Concept:

Solution Region of an Inequality:

- The solution region of an inequality is the set of all points that satisfy the inequality.
- For linear inequalities, the solution region is typically a half-plane or a region bounded by lines.
- The inequality given is: $\mathbf{2x + 4y \leq 9}$.
- We can rewrite the inequality as a line equation: $\mathbf{2x + 4y = 9}$ and plot it on the coordinate plane.
- The solution region consists of all points that satisfy the inequality, which is typically one side of the line.

Calculation:

Given the inequality: $\mathbf{2x + 4y \leq 9}$

First, rewrite the inequality as the equation of a line:

$\mathbf{2x + 4y = 9}$

Now, solve for y:

$4y = 9 - 2x$

$y = (9 - 2x) / 4$

$y = 9/4 - x/2$

The slope of the line is $\mathbf{-1/2}$ and the y-intercept is $\mathbf{9/4}$.

Plot the line $y = (9 - 2x)/4$ on the coordinate plane.

The solution region will be the area below this line since the inequality is $\leq$ (i.e., points that satisfy the inequality lie below or on the line).

Thus, the solution region is the half-plane below the line **2x + 4y = 9**, including the line itself.

∴ The solution region is the region below and on the line 2x + 4y = 9.

Question 13

If p, q, r are distinct, then value of $\begin{vmatrix} p & p^2 & 1+p^3 \\ q & q^2 & 1+q^3 \\ r & r^2 & 1+r^3 \end{vmatrix}$ is:

Options:

A. (1 + pqr) (q - p) (r - p) (r - q)

B. (1 - pqr) (q + p) (r + p) (r - q)

C. (1 + pqr) (q - p) (r + p) (r - q)

D. (1 - pqr) (q + p) (r - p) (r + q)

Answer: A

Solution:

Concept:

Determinant properties and simplification:

- Use of row operations can simplify a determinant without changing its value or by changing its sign in a known manner.
- If any two rows of a determinant are identical or proportional, its value is zero.
- We can apply transformations such as **R3 → R3 - R2** or **R2 → R2 - R1** etc. to simplify the structure.
- Here, we aim to simplify the determinant using suitable operations and observe patterns/factorization.

Calculation:

Given matrix:

$$\begin{vmatrix} p & p^2 & 1+p^3 \\ q & q^2 & 1+q^3 \\ r & r^2 & 1+r^3 \end{vmatrix}$$

Apply row operation: $R_3 \to R_3 - R_2$, and $R_2 \to R_2 - R_1$

New matrix becomes:

$$\begin{vmatrix} p & p^2 & 1+p^3 \\ q-p & q^2-p^2 & q^3-p^3 \\ r-q & r^2-q^2 & r^3-q^3 \end{vmatrix}$$

Use identity: $a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + + b^{n-1})$

Now factor R_2 and R_3:

$R_2 = (q - p) \times [1, q + p, q^2 + qp + p^2]$

$R_3 = (r - q) \times [1, r + q, r^2 + rq + q^2]$

Take out scalars (q - p) and (r - q) from R_2 and R_3:

Det = (q - p)(r - q) × determinant of:

$$\begin{vmatrix} p & p^2 & 1+p^3 \\ 1 & p+q & qp^2+pq+q^2 \\ 1 & q+r & q^2+qr+r^2 \end{vmatrix}$$

Now apply $C_1 \to C_1 - p \times C_2 + (p^2 - 1) \times C_3$, and simplify to get a factor (1 + pqr)

Final result becomes:

∴ **Required value = (1 + pqr)(q - p)(r - q)(r - p)**

Question 14

The degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{4/5} = 10\frac{dy}{dx} + 2$ are:

Options:

A. Degree 2, Order 5

B. Degree 5, Order 1

C. Degree 20, Order 2

D. Degree 4, Order 2

Answer: C

Solution:

Concept:

Degree and Order of a Differential Equation:

- **Degree** of a differential equation is the highest power of the highest order derivative in the equation, after the equation has been made free of fractions and radicals involving derivatives.
- **Order** of a differential equation is the order of the highest derivative in the equation.
- In the given differential equation, we are required to find both the degree and order.

Calculation:

The given differential equation is:

$(d^2y/dx^2)^{4/5} = 10\ (dy/dx) + 2$

Step 1: First, simplify the equation. The equation involves a fractional power of the second derivative, which needs to be cleared.

Taking both sides to the power of 5to remove the fractional power:

$$\left(\frac{d^2y}{dx^2}\right)^{20} = \left(10\frac{dy}{dx} + 2\right)^5$$

Now, observe the highest derivative. The highest order derivative is **d^2y/dx^2**, which means the **order** of the differential equation is 2.

Step 3: The highest power of the highest order derivative is **20**

Hence, the degree and order of the differential equation are:

- **Order = 2**
- **Degree = 20**

Question 15

The particular solution of the differential equation $(y - x^2y)dy = (1 - x^3)dx$ with $y(0) = 1$, is:

Options:

A. $y^2 = x^2 + 2\log_e|1 + x| + 1$

B. $y^2 = 1 + x^2 + 2\log_e\left|\frac{1+x}{2}\right|$

C. $y^2 = x^2 + 2x - 3$

D. $y^2 = x^2 + 2x + 1$

Answer: A

Solution:

Concept:

First Order Differential Equation:

- A differential equation involving the function y and its first derivative dy/dx is called a **first order differential equation**.
- **Separable differential equations** can be solved by separating the variables y and x on opposite sides of the equation.
- Once variables are separated, integrate both sides with respect to their own variable.
- Use initial condition to find the constant of integration and obtain the particular solution.

Logarithmic Function:

- The natural logarithm function is denoted as $\log_e$ or ln.
- Important identity: $\int(1/x)\,dx = \log_e|x| + C$

Calculation:

Given, $y(0) = 1$

Equation: $(y - x^2y)\,dy = (1 - x^3)\,dx$

$\Rightarrow y(1 - x^2)\,dy = (1 - x^3)\,dx$

$\Rightarrow y\,dy = [(1 - x^3)/(1 - x^2)]\,dx$

$\Rightarrow y\,dy = [(1 + x + x^2)/(1 + x)]\,dx$

$\Rightarrow y\,dy = x + (1/(1 + x))\,dx$

Integrate both sides,

$\Rightarrow \int y\,dy = \int (x + 1/(1 + x))\,dx$

$\Rightarrow y^2/2 = x^2/2 + \log_e|1 + x| + C$

$\Rightarrow y^2 = x^2 + 2\log_e|1 + x| + C'$

Apply initial condition: $x = 0$, $y = 1$

$\Rightarrow (1)^2 = 0 + 2\log_e(1) + C'$

$\Rightarrow 1 = 0 + 0 + C'$

$\Rightarrow C' = 1$

∴ Hence, the particular solution is $y^2 = x^2 + 2 \log_e|1 + x| + 1$

Mathematics

Question 16

Relation R on the set A = {1, 2, 3, ..., 13, 14} defined as R = {(x, y) : 3x - y = 0} is:

Options:

A. Reflexive, symmetric and transitive

B. Reflexive and transitive but not symmetric

C. Neither reflexive nor symmetric but transitive

D. Neither reflexive nor symmetric nor transitive

Answer: D

Solution:

Concept:

Properties of Relations:

- A relation on a set is defined by a set of ordered pairs.
- For a relation to be **reflexive**, each element in the set must be related to itself, i.e., for every element x in set A, (x, x) must belong to the relation R.
- For a relation to be **symmetric**, if (x, y) belongs to the relation, then (y, x) must also belong to the relation.
- For a relation to be **transitive**, if (x, y) and (y, z) belong to the relation, then (x, z) must also belong to the relation.

Calculation:

Given the relation R on the set A = {1, 2, 3, ..., 13, 14} defined as:

R = {(x, y) : 3x - y = 0}

This implies that for a pair (x, y), the relation holds if and only if y = 3x.

Now, let's check the properties:

Reflexivity: For reflexivity, we need (x, x) to belong to the relation.

This means that for every x, 3x = x must hold.

This is only true when x = 0, but since 0 is not an element of set A, the relation is **not reflexive**.

Symmetry: For symmetry, if (x, y) belongs to R, we also need (y, x) to belong to R. If (x, y) satisfies y = 3x, then we would need 3y = x.

This is not true in general, so the relation is **not symmetric**.

Transitivity: For transitivity, if (x, y) and (y, z) belong to R, we need (x, z) to belong to R.

If y = 3x and z = 3y, then z = 9x, which still satisfies the relation, meaning that the relation is **transitive**.

∴ The relation is **neither reflexive nor symmetric but transitive**.

Question 17

If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, then $B^{-1} A^{-1}$ is equal to:

Options:

A. $-\frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$

B. $\frac{1}{11}\begin{bmatrix} 15 & 11 \\ 1 & 0 \end{bmatrix}$

C. $\frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$

D. $-\frac{1}{11}\begin{bmatrix} 15 & 11 \\ 1 & 0 \end{bmatrix}$

Answer: C

Solution:

Explanation:

Step 1: Find the Inverse of Matrix A

Given matrix A :

$$A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$$

The inverse of a 2x2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by the formula:

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

For matrix A :

a = 2, b = 3, c = 1, d = -4

det(A) = (2)(-4) - (3)(1) = -8 - 3 = -11

Thus, the inverse of matrix A is:

$$A^{-1} = \frac{1}{-11} \begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ \frac{1}{11} & \frac{-2}{11} \end{bmatrix}$$

Step 2: Find the Inverse of Matrix B

Given matrix B :

$$B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

The inverse of matrix B is calculated using the same formula as for A . For matrix B :

a = 1, b = -2, c = -1, d = 3

The determinant {det}(B) is:

det(B) = (1)(3) - (-2)(-1) = 3 - 2 = 1

Thus, the inverse of matrix B is:

$$B^{-1} = \frac{1}{1} \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

Step 3: Compute $B^{-1}A^{-1}$

Now, we compute the product $B^{-1}A^{-1}$:

$$B^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ \frac{1}{11} & \frac{-2}{11} \end{bmatrix}$$

Performing the matrix multiplication:

$$B^{-1}A^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ \frac{1}{11} & \frac{-2}{11} \end{bmatrix}$$

The elements of the product matrix are:

1st row, 1st column:

$(3)(\frac{4}{11}) + (2)(\frac{1}{11}) = \frac{12}{11} + \frac{2}{11} = \frac{14}{11}$

1st row, 2nd column:

$(3)(\frac{3}{11}) + (2)(\frac{-2}{11}) = \frac{9}{11} - \frac{4}{11} = \frac{5}{11}$

2nd row, 1st column:

$(1)(\frac{4}{11}) + (1)(\frac{1}{11}) = \frac{4}{11} + \frac{1}{11} = \frac{5}{11}$

2nd row, 2nd column:

$(1)(\frac{3}{11}) + (1)(\frac{-2}{11}) = \frac{3}{11} - \frac{2}{11} = \frac{1}{11}$

Thus, the matrix $B^{-1}A^{-1}$ is:

$$B^{-1}A^{-1} = \begin{bmatrix} \frac{14}{11} & \frac{5}{11} \\ \frac{5}{11} & \frac{1}{11} \end{bmatrix}$$

Step 4: Match with the options

The matrix $B^{-1}A^{-1}$ matches with option (3):

$$\frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$$

Thus, the correct answer is Option (3).

Question 18

If (cos x)^{y}= (sin y)^{x} then $\frac{\mathrm{d}y}{\mathrm{d}x}$ is:

Options:

A. $\frac{\log_e \sin y - y \tan x}{\log_e \cos x + x \cot y}$

B. $\frac{\log_e \sin y + y \tan x}{\log_e \cos x + x \cos y}$

C. $\frac{\log_e \sin y + y \tan x}{\log_e \cos x - x \cot y}$

D. $\frac{\log_e \cos x - x \cos y}{\log_e \sin y + y \tan x}$

Answer: C

Solution:

Concept:

Logarithmic Differentiation:

- Logarithmic differentiation is a technique used to differentiate functions that are products or quotients of functions or involve exponents.
- It simplifies the differentiation of complex functions by taking the natural logarithm of both sides of the equation.
- The general steps for logarithmic differentiation are:
  - Take the natural logarithm (ln) of both sides of the equation.
  - Differentiate both sides using implicit differentiation.
  - Solve for the derivative.
- In this question, we will use logarithmic differentiation to find the derivative of the given equation.

Calculation:

Given:

$(\cos x)^y = (\sin y)^x$

We will take the natural logarithm of both sides:

$\ln((\cos x)^y) = \ln((\sin y)^x)$

Using the logarithmic identity $\ln(a^b) = b \ln(a)$, we get:

$y \ln(\cos x) = x \ln(\sin y)$

Now, differentiate both sides with respect to x:

For the left-hand side, we apply the product rule:

$d/dx[y \ln(\cos x)] = (dy/dx) \ln(\cos x) + y (-\tan x)$

For the right-hand side, we use the product rule again:

$d/dx[x \ln(\sin y)] = \ln(\sin y) + x (\cot y) (dy/dx)$

Now, equating both sides:

$(dy/dx) \ln(\cos x) - y \tan x = \ln(\sin y) + x \cot y (dy/dx)$

Rearrange the terms to isolate (dy/dx):

$(dy/dx) [\ln(\cos x) - x \cot y] = \ln(\sin y) + y \tan x$

Therefore,

$(dy/dx) = (\ln(\sin y) + y \tan x) / [\ln(\cos x) - x \cot y]$

∴ **The derivative is:**

(dy/dx) = (ln(sin y) + y tan x) / [ln(cos x) - x cot y]

The correct answer is option (3).

Question 19

A particle moves along the curve 6x = y^3+ 2. The points on the curve at which the x coordinate is changing 8 times as fast as y coordinate are:

Options:

A. $(11, 4), \left(-\frac{31}{3}, 4\right)$

B. $(-11, 4), \left(\frac{31}{3}, -4\right)$

C. $(11, -4), \left(-\frac{31}{3}, -4\right)$

D. $(11, 4), \left(-\frac{31}{3}, -4\right)$

Answer: D

Solution:

Concept:

Rate of Change and Logarithmic Differentiation:

- In the given problem, the particle moves along the curve, and we are asked to find the points on the curve where the x-coordinate is changing 8 times as fast as the y-coordinate.
- The rate of change of x with respect to time (dx/dt) is related to the rate of change of y with respect to time (dy/dt) as given in the problem.
- By using implicit differentiation, we differentiate the given equation with respect to time, considering the rates of change of both coordinates.
- This is an application of **logarithmic differentiation** to find how the variables change with respect to time.

Calculation:

Given, the equation of the curve is:

6x = y^3 + 2

The particle moves along this curve, and the rate of change of the x-coordinate is 8 times the rate of change of the y-coordinate. We are asked to find the points where this condition holds.

Step 1: Differentiate both sides with respect to time (t):

d/dt[6x] = d/dt[y^3 + 2]

Using the chain rule, we get:

6(dx/dt) = $3y^2$ (dy/dt)

Step 2: We are given that dx/dt = 8(dy/dt). Substitute this into the equation:

6 × 8(dy/dt) = $3y^2$ (dy/dt)

Step 3: Cancel (dy/dt) from both sides (assuming dy/dt ≠ 0):

48 = $3y^2$

Step 4: Solve for y:

y^2 = 48 / 3 = 16

y = ±4

Step 5: Substitute the values of y = 4 and y = -4 into the original equation to find the corresponding x-coordinates:

For y = 4:

6x = 4^3 + 2 = 64 + 2 = 66

x = 66 / 6 = 11

For y = -4:

6x = $(-4)^3$ + 2 = -64 + 2 = -62

x = -62 / 6 = -31 / 3

Step 6: The points on the curve are:

(11, 4) and (-31/3, -4)

∴ The points on the curve are (11, 4) and (-31/3, -4), which corresponds to option (3).

Question 20

Area of the parallelogram, whose adjacent sides are given by the vectors $\overrightarrow{\mathrm{a}} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\overrightarrow{\mathrm{b}} = 2\hat{i} + \hat{j} + 2\hat{k}$, is:

Options:

A. $\sqrt{105}$

B. $\sqrt{101}$

C. $\sqrt{103}$

D. $\sqrt{102}$

Answer: B

Solution:

<u>Concept:</u>

Area of a Parallelogram Using Cross Product of Vectors:

$a \times b = i\,(a_2 b_3 - a_3 b_2) - j\,(a_1 b_3 - a_3 b_1) + k\,(a_1 b_2 - a_2 b_1)$

$|a \times b| = \sqrt{[(a_2 b_3 - a_3 b_2)^2 + (a_1 b_3 - a_3 b_1)^2 + (a_1 b_2 - a_2 b_1)^2]}$

- The area of a parallelogram formed by two adjacent sides represented by vectors **a** and **b** is given by the magnitude of the cross product of the two vectors:
    - Area = $|a \times b|$
    - The cross product of two vectors **a** = $a_1 i + a_2 j + a_3 k$ and **b** = $b_1 i + b_2 j + b_3 k$ is calculated as:
- The magnitude of the cross product is:

<u>Calculation:</u>

Given:

Vector $a = 2i - j + 5k$

Vector $b = 2i + j + 2k$

We will find the cross product of vectors a and b using the formula for the cross product:

$a \times b = i\,(a_2 b_3 - a_3 b_2) - j\,(a_1 b_3 - a_3 b_1) + k\,(a_1 b_2 - a_2 b_1)$

Substitute the values of a and b:

$a \times b = i\,[(-1)(2) - (5)(1)] - j\,[(2)(2) - (5)(2)] + k\,[(2)(1) - (-1)(2)]$

$a \times b = i\,[-2 - 5] - j\,[4 - 10] + k\,[2 + 2]$

$a \times b = -7i + 6j + 4k$

Now, calculate the magnitude of the cross product:

$|a \times b| = \sqrt{[(-7)^2 + 6^2 + 4^2]}$

$|a \times b| = \sqrt{[49 + 36 + 16]}$

$|a \times b| = \sqrt{101}$

∴ **The area of the parallelogram is √101.**

The correct answer is **Option (2)**

Question 21

The angle between the lines $\vec{r} = 3\hat{i} - 2\hat{j} + 1\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$ and $\overrightarrow{\mathrm{r}} = 7\hat{i} - 3\hat{j} + 9\hat{k} + \lambda(5\hat{i} + 8\hat{j} - 4\hat{k})$ is:

Options:

A. $\cos^{-1} \frac{10}{7\sqrt{105}}$

B. $\cos^{-1} \frac{5}{72}$

C. $\cos^{-1} \frac{2}{35}$

D. $\cos^{-1} \frac{7}{98}$

Answer: A

Solution:

Concept:

Angle Between Two Lines in 3D:

- To find the angle between two lines in space, we use the angle between their **direction vectors**.
- If direction vectors are **a** and **b**, then angle θ between them is given by:
- **cosθ = (a · b) / (|a| × |b|)**
- Here, **a · b** is the dot product of vectors, and **|a|** is the magnitude of vector a.

Dot Product:

- The dot product of vectors $a = a_1i + a_2j + a_3k$ and $b = b_1i + b_2j + b_3k$ is:
- $\mathbf{a \cdot b = a_1b_1 + a_2b_2 + a_3b_3}$

Calculation:

Given, direction vector of first line = 4i + 6j + 12k

Let a = 4i + 6j + 12k

Direction vector of second line = 5i + 8j − 4k

Let b = 5i + 8j − 4k

⇒ $a \cdot b = (4)(5) + (6)(8) + (12)(-4)$

⇒ $a \cdot b = 20 + 48 - 48 = 20$

⇒ $|a| = \sqrt{(4^2 + 6^2 + 12^2)} = \sqrt{(16 + 36 + 144)} = \sqrt{196} = 14$

⇒ $|b| = \sqrt{(5^2 + 8^2 + (-4)^2)} = \sqrt{(25 + 64 + 16)} = \sqrt{105}$

⇒ $\cos\theta = (a \cdot b) / (|a| \times |b|) = 20 / (14 \times \sqrt{105})$

⇒ $\cos\theta = 2 / (\sqrt{105} / 7)$

⇒ $\theta = \cos^{-1}(20 / (14\sqrt{105}))$

⇒ $\theta = \cos^{-1}(10 / (7\sqrt{105}))$

∴ **Hence, Option 1 is the correct answer.**

Question 22

If P(A) = 0.4, P(B) = 0.8 and P(A|B) = 0.6, then P(A ∪ B) is:

Options:

A. 0.96

B. 0.72

C. 0.36

D. 0.42

Answer: B

Solution:

Concept:

Probability of Union of Two Events:

- In probability theory, the probability of the union of two events A and B is given by the formula:
  - **$P(A \cup B) = P(A) + P(B) - P(A \cap B)$**
- Where:
  - **$P(A \cup B)$** is the probability of either event A or event B occurring.

- **P(A)** is the probability of event A occurring.
- **P(B)** is the probability of event B occurring.
- **P(A ∩ B)** is the probability of both events A and B occurring together (i.e., their intersection).

- The problem also involves conditional probability, where we are given the conditional probability P(A | B), which represents the probability of event A occurring given that event B has already occurred. The formula for conditional probability is:
  - **P(A | B) = P(A ∩ B) / P(B)**

Calculation:

Given:

P(A) = 0.4

P(B) = 0.8

P(A | B) = 0.6

Step 1: Calculate P(A ∩ B) using the formula for conditional probability:

P(A | B) = P(A ∩ B) / P(B)

Rearrange the formula to solve for P(A ∩ B):

P(A ∩ B) = P(A | B) × P(B)

P(A ∩ B) = 0.6 × 0.8 = 0.48

Step 2: Use the formula for the probability of the union of two events:

P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

P(A ∪ B) = 0.4 + 0.8 - 0.48 = 1.2 - 0.48 = 0.72

∴ **The probability P(A ∪ B) = 0.72.**

The correct answer is **Option (2): 0.72**

Question 23

Let f : $\mathbb{R} \rightarrow \mathbb{R}$ be defined as f(x) = 10 - x^2, then:

Options:

A.

f is one-one and onto.

B.

f is one-one but not onto.

C.

f is neither one-one nor onto.

D.

f is onto but not one-one.

Answer: C

Solution:

Concept:

One-to-One (Injective) and Onto (Surjective) Functions:

- A function is said to be one-to-one (or injective) if different inputs always produce different outputs. In other words, if $f(a) = f(b)$, then $a = b$.
- A function is said to be onto (or surjective) if every element in the codomain has a pre-image in the domain, i.e., for every y in the codomain, there exists an x in the domain such that $f(x) = y$.
- In the given problem, we are tasked with analyzing the function $f(x) = 10 - x^2$, which is a quadratic function, and determining whether it is one-to-one or onto.

Calculation:

We are given the function:

$f(x) = 10 - x^2$

Step 1: Check if the function is one-to-one (injective):

- To be one-to-one, for any two values x_1 and x_2, if $f(x_1) = f(x_2)$, then it must follow that $x_1 = x_2$.
- For the function $f(x) = 10 - x^2$, consider two points: $x_1 = 2$ and $x_2 = -2$. We get $f(2) = 10 - (2)^2 = 6$ and $f(-2) = 10 - (-2)^2 = 6$.
- Thus, $f(2) = f(-2)$, but $2 \neq -2$. Therefore, the function is not one-to-one because different x-values can give the same output.

Step 2: Check if the function is onto (surjective):

- To be onto, the function must cover all possible real values of y in the codomain. The maximum value of $f(x)$ occurs when $x = 0$, giving $f(0) = 10 - 0^2 = 10$.
- As x moves away from 0, the function $f(x)$ decreases and takes only values less than or equal to 10 (i.e., $f(x) \leq 10$).
- Therefore, the function does not cover all real values, particularly values greater than 10, meaning it is not onto.

∴ The function $f(x) = 10 - x^2$ is neither one-to-one nor onto.

The correct answer is Option (3): "f is neither one-one nor onto."

Question 24

The domain of f(x) = $\cos^{-1}7x$ is:

Options:

A. $\left[-\frac{1}{7}, \frac{1}{7}\right]$

B. [-7, 7]

C. [0, 7]

D. [-1, 1]

Answer: A

Solution:

Concept:

Domain of Inverse Trigonometric Functions:

- The function $f(x) = \cos^{-1}(x)$ is the inverse of the cosine function.
- The domain of $\cos^{-1}(x)$ is restricted to values where x lies within the range [-1, 1] because the cosine function has values between -1 and 1.
- The domain of $f(x) = \cos^{-1}(7x)$ is determined by ensuring that the expression 7x lies within the range [-1, 1]. Thus, $-1 \leq 7x \leq 1$.
- From this, we can solve for x to find the appropriate domain for $f(x) = \cos^{-1}(7x)$.

Calculation:

We are given the function:

f(x) = $\cos^{-1}(7x)$

The domain of $\cos^{-1}(x)$ is restricted to the interval [-1, 1].

Therefore, we need to find when:

$-1 \leq 7x \leq 1$

Solve for x by dividing each part of the inequality by 7:

$-1/7 \leq x \leq 1/7$

Thus, the domain of the function f(x) = $\cos^{-1}(7x)$ is [-1/7, 1/7]

∴ **The domain of f(x) is [-1/7, 1/7].**

The correct answer is **Option (1): [-1/7, 1/7]**

Question 25

For a, b > 0, if $\mathrm{P} = \begin{bmatrix} 0 & -\mathrm{a} \\ 2\mathrm{a} & \mathrm{b} \end{bmatrix}$ **and** $\mathrm{Q} = \begin{bmatrix} \mathrm{b} & \mathrm{a} \\ -\mathrm{b} & 0 \end{bmatrix}$ **are two matrices such that** $PQ = \begin{bmatrix} 2 & 0 \\ 3 & 8 \end{bmatrix}$**, then the value of** $(\mathbf{a} + \mathbf{b})^{\mathbf{ab}}$ **is:**

Options:

A. 8

B. 9

C. 1/9

D. -1/27

Answer: B

Solution:

Concept:

Matrix Multiplication:

- To multiply two matrices A and B, compute the dot product of rows of A with columns of B.
- If $A = [a_{ij}]$ is m×n and $B = [b_{jk}]$ is n×p, then AB = C is m×p where:
- $\mathbf{c_{ik} = \Sigma\, a_{ij} \times b_{jk}}$

Calculation:

$$\mathrm{PQ} = \begin{bmatrix} 0{\times}b + (-a)(-b) & 0{\times}a + (-a){\times}0 \\ 2a{\times}b + b{\times}(-b) & 2a{\times}a + b{\times}0 \end{bmatrix}$$

$$PQ = \begin{bmatrix} ab & 0 \\ 2ab - b^2 & 2a^2 \end{bmatrix}$$

Given: $PQ = \begin{bmatrix} 2 & 0 \\ 3 & 8 \end{bmatrix}$

⇒ ab = 2

$\Rightarrow 2ab - b^2 = 3$

$\Rightarrow 2(2) - b^2 = 3$

$\Rightarrow 4 - b^2 = 3$

$\Rightarrow b^2 = 1$

$\Rightarrow b = 1$ (since $b > 0$)

$\Rightarrow ab = 2 \Rightarrow a = 2$

Now, $a + b = 2 + 1 = 3$

$\Rightarrow (a + b)^{ab} = 3^2 = 9$

$\therefore$ **Hence, the value of $(a + b)^{ab}$ is 9.**

Question 26

If $A = \begin{bmatrix} 2 & 4 \\ x & \frac{-1}{2} \end{bmatrix}$ and A is singular, then x is equal to:

Options:

A. $\frac{1}{4}$

B. $-\frac{1}{4}$

C. -7

D. 32

Answer: B

Solution:

Explanation:

We are given the matrix A as:

$$A = \begin{bmatrix} 2 & 4 \\ x & -\frac{1}{2} \end{bmatrix}$$

A matrix is singular if its determinant is zero. So, we need to calculate the determinant of the given 2x2 matrix.

The determinant of a 2 x 2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is det (A) = ad - bc

For the matrix $A = \begin{bmatrix} 2 & 4 \\ x & -\frac{1}{2} \end{bmatrix}$, the determinant is:

det(A) = $(2)\left(-\frac{1}{2}\right) - (4)(x)$

det (A) = -1 - 4x

Since A is singular, the determinant must be zero:

-1 - 4x = 0

-4x = 1

$x = -\frac{1}{4}$

Thus, the correct answer is Option (2)

Question 27

If $y = \frac{1}{\sqrt{1-4\sin^2 x \cos^2 x}}$, then $\frac{\mathrm{d}y}{\mathrm{d}x} =$

Options:

A. 2sec x tan x

B. sin 2x

C. 2sec 2x tan 2x

D. cos 2x

Answer: C

Solution:

Calculation:

We are given the function:

$y = \frac{1}{\sqrt{1-4\sin^2 x \cos^2 x}}$

Step 1: Simplify the expression inside the square root:

We know that$\sin^2 x \cos^2 x = \frac{1}{4}\sin^2 2x$ using the double angle identity for sine.

Thus, we have:

$y = 1/\sqrt{}(1 - 4\times(1/4)sin^2 2x) = 1/\sqrt{}(1 - sin^2 2x)$

Step 2: Simplify further:

y = 1 / cos 2x

Step 3: Differentiate y with respect to x:

dy/dx = d/dx(sec 2x) = sec 2x × tan 2x × 2

Step 4: Final answer:

dy/dx = 2 sec 2x tan 2x

∴ **The derivative of the function is 2 sec 2x tan 2x.**

The correct answer is **Option (3): 2 sec 2x tan 2x**

Question 28

The value of $\int_{-1}^{1} \left|\tan^{-1} x\right| dx$ is:

Options:

A. $\frac{\pi}{2} - \log_e 2$

B. $\frac{\pi}{2} + \log_e 2$

C. $\frac{\pi-1-\log_e 2}{2}$

D. $\frac{\pi-1+\log_e 2}{2}$

Answer: A

Solution:

Calculation:

We are given the function:

$I = \int_{-1}^{1} |\tan^{-1}(x)| \, dx$

Step 1: Split the integral based on the absolute value:

$I = \int_{-1}^{0} -\tan^{-1}(x) \, dx + \int_{0}^{1} \tan^{-1}(x) \, dx$

Step 2: Use the standard result for the integral of $\tan^{-1}(x)$:

$\int \tan^{-1}(x) \, dx = x \tan^{-1}(x) - 1/2 \ln(1 + x^2)$

Step 3: Compute each integral:

For $\int_{-1}^{0} -\tan^{-1}(x) \, dx$, we get:

$- [x \tan^{-1}(x) - 1/2 \ln(1 + x^2)]_{-1}^{0}$

At $x = 0$, the expression becomes 0.

At $x = -1$, we get:

$- [-1 \times (-\pi/4) - 1/2 \ln(1 + 1)] = \pi/4 - 1/2 \ln(2)$

Thus, the value of the first integral is:

$\pi/4 - 1/2 \ln(2)$

For $\int_{0}^{1} \tan^{-1}(x) \, dx$, we use the same result:

$[x \tan^{-1}(x) - 1/2 \ln(1 + x^2)]_{0}^{1}$

At $x = 1$, the expression becomes:

$1 \times (\pi/4) - 1/2 \ln(2) = \pi/4 - 1/2 \ln(2)$

At $x = 0$, the expression is 0.

Thus, the value of the second integral is:

$\pi/4 - 1/2 \ln(2)$

Step 4: Add the results of the two integrals:

$I = (\pi/4 - 1/2 \ln(2)) + (\pi/4 - 1/2 \ln(2)) = \pi/2 - \ln(2)$

∴ **The value of the integral is $\pi/2 - \ln(2)$.**

The correct answer is **Option (1): $\pi/2 - \ln(2)$**

Question 29

The area (in square units) bounded by the curve y = |x - 2| between x = 0, y = 0 and x = 5 is:

Options:

A. 8

B. 13

C. 6.5

D. 3.5

Answer: C

Solution:

Concept:

Area under the Curve:

- The problem asks us to find the area bounded by the curve $y = |x - 2|$ between the limits $x = 0$ and $x = 5$.
- The function $y = |x - 2|$ represents a V-shaped curve, which has a vertex at $x = 2$. We need to compute the area under this curve between $x = 0$ and $x = 5$.
- The function can be split into two parts depending on whether x is less than 2 or greater than 2:
    - For $x < 2$, $y = 2 - x$
    - For $x \geq 2$, $y = x - 2$
- To compute the area, we need to split the integral into two parts: one from $x = 0$ to $x = 2$, and the other from $x = 2$ to $x = 5$.

Calculation:

We are given the function:

$y = |x - 2|$

Step 1: Split the integral based on the behavior of the function:

$I = \int_0^2 (2 - x)\,dx + \int_2^5 (x - 2)\,dx$

Step 2: Compute the first integral from $x = 0$ to $x = 2$:

$\int_0^2 (2 - x)\,dx = [2x - (x^2/2)]_0^2$

At $x = 2$:

$2(2) - (2^2/2) = 4 - 2 = 2$

At $x = 0$:

$2(0) - (0^2/2) = 0$

So the value of the first integral is:

$2 - 0 = 2$

Step 3: Compute the second integral from $x = 2$ to $x = 5$:

$\int_2^5 (x - 2)\, dx = [(x^2/2) - 2x]_2^5$

At $x = 5$:

$(5^2/2) - 2(5) = (25/2) - 10 = 12.5 - 10 = 2.5$

At $x = 2$:

$(2^2/2) - 2(2) = (4/2) - 4 = 2 - 4 = -2$

So the value of the second integral is:

$2.5 - (-2) = 2.5 + 2 = 4.5$

Step 4: Add the results of the two integrals:

$I = 2 + 4.5 = 6.5$

∴ **The area under the curve is 6.5 square units.**

The correct answer is **Option (3)**

Question 30

The area (in square units) enclosed between the curve $x^2 = 4y$ and the line $x = y$ is:

Options:

A. 8

B. $\frac{16}{3}$

C. 16

D. $\frac{8}{3}$

Answer: D

Solution:

Concept:

Area Between a Parabola and a Line:

- We are given a curve $x^2 = 4y$ and a line x = y , and we are asked to find the area enclosed between the curve and the line.
- The parabola $x^2 = 4y$ represents a standard parabola that opens upwards with its vertex at the origin. The line x = y is a straight line passing through the origin with a slope of 1.
- To find the area enclosed between these two curves, we need to calculate the integral of the difference between the two functions, within the limits where the curves intersect.

Calculation:

We are given the curves:

1) Parabola: $x^2 = 4y$ which gives $y = \frac{x^2}{4}$

2) Line: x = y which gives y = x

Step 1: Find the points of intersection of the curves:

Set $\frac{x^2}{4} = x$ to find the points where the curves intersect:

Multiplying both sides by 4:

$x^2 = 4x$

x(x - 4) = 0

So, x = 0 and x = 4 .

Therefore, the curves intersect at x = 0 and x = 4 .

Step 2: Set up the integral to find the area:

The area between the curves from x = 0 to x = 4 is given by:

$A = \int_0^4 \left(x - \frac{x^2}{4}\right) dx$

Step 3: Compute the integral:

First, split the integral:

$A = \int_0^4 x\, dx - \int_0^4 \frac{x^2}{4}\, dx$

Compute the first integral:

$\int_0^4 x\, dx = \frac{x^2}{2}\Big|_0^4 = \frac{4^2}{2} - \frac{0^2}{2} = \frac{16}{2} = 8$

Compute the second integral:

$\int_0^4 \frac{x^2}{4}\,dx = \frac{1}{4}\int_0^4 x^2\,dx = \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$

$= \frac{1}{4}\left(\frac{4^3}{3} - \frac{0^3}{3}\right) = \frac{1}{4} \times \frac{64}{3} = \frac{64}{12} = \frac{16}{3}$

Step 4: Subtract the results of the two integrals:

$A = 8 - \frac{16}{3} = \frac{24}{3} - \frac{16}{3} = \frac{8}{3}$

∴ **The area enclosed between the curve $x^2 = 4y$ and the line x = y is $\frac{8}{3}$ square units.**

The correct answer is **Option (4)**

Question 31

If the solution of differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+5}$ represents a circle, then a is equal to:

Options:

A. 3

B. -2

C. -3

D. 5

Answer: C

Solution:

Concept:

Differential Equation Representing a Circle:

- The given differential equation represents the rate of change of the variable y with respect to x.
- The solution of this equation should describe the equation of a circle in standard form, which is generally represented as $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.
- The equation of a circle has equal coefficients for x^2 and y^2 and no x y term.
- To find the value of 'a' that will ensure the solution of the given equation represents a circle, we need to integrate and manipulate the terms to match the standard form of a circle's equation.

Calculation:

Given,

The differential equation is: dy/dx = (ax + 3) / (2y + 5)

Rearrange the equation:

(2y + 5) dy = (ax + 3) dx

Now integrate both sides:

∫ (2y + 5) dy = ∫ (ax + 3) dx

On integrating:

$y^2 + 5y = (a/2) x^2 + 3x + C$

For the equation to represent a circle, the equation should be of the form $(x - h)^2 + (y - k)^2 = r^2$,

where the coefficients of x^2 and y^2 must be equal, and there should be no mixed term (x y).

Comparing the equation with the standard form,

we find that the value of 'a' should be -3 for the equation to represent a circle.

∴ **The value of 'a' is -3.**

Question 32

Match List-I with List-II:

	List-I		List-II
(A)	$4\hat{i}-2\hat{j}-4\hat{k}$	(I)	A vector perpendicular to both $\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+2\hat{j}+3\hat{k}$
(B)	$-4\hat{i}-4\hat{j}+2\hat{k}$	(lI)	Direction ratios are-2, 1, 2
(C)	$2\hat{i}-4\hat{j}+4\hat{k}$	(Ill)	Angle with the vector $\hat{i}-2\hat{j}-\hat{k}$ is $\cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$
(D)	$4\hat{i}-\hat{j}-2\hat{k}$	(Iv)	Dot product with $-2\hat{i}+\hat{j}+3\hat{k}$ is 10

Choose the correct answer from the options given below:

Options:

A. (A) - (I), (B) - (IV), (C) - (II), (D) - (III)

B. (A) - (II), (B) - (IV), (C) - (III), (D) - (I)

C. (A) - (II), (B) - (III), (C) - (IV), (D) - (I)

D. (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Answer: B

Solution:

Explanation:

D. $4\hat{i} - \hat{j} - 2\hat{k}$

We will find the vector perpendicular to both $\hat{i} + 2\hat{j} + \hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$ by taking the cross product of these two vectors:

$\vec{v}_1 = \hat{i} + 2\hat{j} + \hat{k}, \quad \vec{v}_2 = 2\hat{i} + 2\hat{j} + 3\hat{k}$

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{vmatrix}$$

Calculating the cross product:

$\vec{v}_1 \times \vec{v}_2 = \hat{i}(2 \cdot 3 - 1 \cdot 2) - \hat{j}(1 \cdot 3 - 1 \cdot 2) + \hat{k}(1 \cdot 2 - 2 \cdot 2)$

$=4\hat{i} - \hat{j} - 2\hat{k}$

Now, vector Dis a scalar multiple of this result, so Dcorresponds to (I).

B. Dot Product of$-4\hat{i} - 4\hat{j} + 2\hat{k}$ and$-2\hat{i} + \hat{j} + 3\hat{k}$ = 8- 4+ 6 = 10

B→ IV

C.$2\hat{i} - 4\hat{j} + 4\hat{k}$Direction ratios are-2, 1, 2

D.$4\hat{i} - 2\hat{j} - 4\hat{k}$Angle with the vector$\hat{i} - 2\hat{j} - \hat{k}is\cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$

Question 33

A die is thrown three times. Events A and B are defined as below

A: 6 on the third throw

B: 4 on the first and 5 on the second throw

The probability of Agiven that Bhas already occurred, is:

Options:

A. 1/6

B. 2/3

C. 3/4

D. 1/2

Answer: A

Solution:

Concept:

Conditional Probability:

- Conditional probability is the probability of an event occurring given that another event has already occurred. It is denoted as P(A | B), which represents the probability of event A occurring given that event B has already occurred.
- The formula for conditional probability is given by:
  - $P(A | B) = P(A \cap B) / P(B)$
- Where:
  - $P(A \cap B)$ is the probability of both events A and B occurring.
  - P(B) is the probability of event B occurring.
- In this problem, we are given the following events:
  - A: A 6 on the third throw.
  - B: A 4 on the first throw and 5 on the second throw.
- We need to find the probability of event A occurring given that event B has already occurred.

Calculation:

Step 1: Formula for Conditional Probability

We use the formula for conditional probability:

$P(A | B) = P(A \cap B) / P(B)$

Step 2: Find P(B)

Event B occurs when:

- A 4 appears on the first throw.
- A 5 appears on the second throw.

The probability of each event happening is:

- The probability of getting a 4 on the first throw = 1/6
- The probability of getting a 5 on the second throw = 1/6

Since the throws are independent, the probability of event B is:

P(B) = (1/6) × (1/6) = 1/36

Step 3: Find P(A ∩ B)

Event A ∩ B occurs when:

- A 4 appears on the first throw (for event B).
- A 5 appears on the second throw (for event B).
- A 6 appears on the third throw (for event A).

The probability of each event happening is:

- The probability of getting a 4 on the first throw = 1/6
- The probability of getting a 5 on the second throw = 1/6
- The probability of getting a 6 on the third throw = 1/6

Since the events are independent, the probability of A ∩ B is:

P(A ∩ B) = (1/6) × (1/6) × (1/6) = 1/216

Step 4: Calculate P(A | B)

Now, we can calculate the conditional probability:

P(A | B) = P(A ∩ B) / P(B) = (1/216) / (1/36) = 1/6

Step 5: Conclusion

The probability of A given that B has already occurred is:

P(A | B) = 1/6

∴ The correct answer is Option (1): 1/6

Question 34

Let A be a matrix such that A^2= I, where I is an identity matrix, then (I + A)^4- 8A is equal to:

Options:

A. 5I

B. 8I

C. 8(l +A)

D. 5(l +A)

Answer: B

Solution:

Concept:

Matrix Properties and Operations:

- A matrix is a rectangular array of numbers or functions arranged in rows and columns. It is denoted by A, where each element is specified by the row and column indices.
- In the given problem, the matrix A satisfies the condition $A^2 = I$, where I is the identity matrix.
- The identity matrix, denoted by I, is a square matrix with 1's on the diagonal and 0's elsewhere. The identity matrix behaves like 1 in matrix multiplication, such that $A \times I = A$ for any matrix A.
- We are tasked with simplifying the expression $(I + A)^4 - 8A$, using the property $A^2 = I$.

Calculation:

Given:

- Matrix A such that $A^2 = I$, where I is the identity matrix.
- We need to calculate $(I + A)^4 - 8A$.

Step 1: Use the given condition $A^2 = I$.

We know from the given condition that $A^2 = I$. This means that:

- $A^3 = A$ (since $A \times A^2 = A \times I = A$)
- $A^4 = I$ (since $A \times A^3 = A \times A = A^2 = I$)

Step 2: Simplify the expression $(I + A)^4$.

We can expand $(I + A)^4$ using the binomial expansion. However, a simpler approach is to recognize that:

- $(I + A)^2 = I + 2A + A^2 = I + 2A + I = 2I + 2A$
- $(I + A)^4 = (2I + 2A)^2 = 4I + 8A + 4I = 8I + 8A$

Step 3: Subtract 8A from $(I + A)^4$.

$(I + A)^4 - 8A = (8I + 8A) - 8A = 8I$

Step 4: Conclusion

The result is 8I

∴ **The correct answer is Option (2)**

Question 35

For $f(x) = \int \frac{e^x}{\sqrt{4-e^{2x}}}\, dx$, if the point $\left(0, \frac{\pi}{2}\right)$satisfies y = f(x), then the constant of integration of the given integral is:

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. 0

Answer: B

Solution:

Concept:

Integration and Constant of Integration:

- The problem involves finding the constant of integration for a given integral.
- The integral is of the form:
  - $f(x) = \int (e^x)/\sqrt{}(4 - e^{(2x)})dx$
- To solve this, we use the substitution method to simplify the integral.
- The substitution used is:
  - $u = e^x$, and hence $du = e^x dx$
- The integral then reduces to a standard form:
  - ∫ 1 / √(4 - u^2) du = $sin^{-1}(u/2) + C$
- We substitute back $u = e^x$ to get the final result:
  - $f(x) = sin^{-1}(e^x/2) + C$
- To find the constant C, we use the initial condition that f(0) = π/2.

Calculation:

Given,

$f(x) = \int (e^x)/\sqrt{}(4 - e^{(2x)})dx$

Substitute$u = e^x$, so that $du = e^x dx$.

The integral becomes $f(x) = sin^{-1}(e^x/2) + C$.

Use the initial condition:

f(0) = π/2

Substituting x = 0 into the equation:

$f(0) = sin^{-1}(e^0/2) + C = sin^{-1}(1/2) + C = \pi/6 + C$

Set this equal to π/2:

π/6 + C = π/2

Solve for C:

C = π/2 - π/6 = 3π/6 - π/6 = 2π/6 = π/3

Hence, the constant of integration is: C = π/3

Question 36

The equation of line passing through origin and parallel to the line $\overrightarrow{\mathbf{r}} = 3\hat{i} + 4\hat{j} - 5\hat{k} + \mathrm{t}(2\hat{i} - \hat{j} + 7\hat{k})$, where t is a parameter, is:

(A) $\frac{x}{2} = \frac{y}{-1} = \frac{z}{7}$

(B) $\overrightarrow{\mathrm{r}} = \mathrm{m}(12\hat{i} - 6\hat{j} + 42\hat{k})$**; where m is the parameter**

(C) $\overrightarrow{\mathbf{r}} = (12\hat{i} - 6\hat{j} + 42\hat{k}) + \mathrm{s}(0\hat{i} - 0\hat{j} + 0\hat{k})$**; where s is the parameter**

(D) $\frac{x-3}{0} = \frac{y-4}{0} = \frac{z+5}{0}$

(E) $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$

Choose the correct answer from the options given below:

Options:

A. (A) and (B) only

B. (A), (B) and (C) only

C. (C), (D) and (E) only

D. (A) only

Answer: A

Solution:

Given:

The equation of the line is:

r = 3i + 4j - 5k + t(2i - j + 7k)

Required:

The equation of the line passing through the origin and parallel to the given line.

Solution:

The direction vector for the line is (2, -1, 7) , and the line passes through the origin. Thus, the parametric equation of the line is:

r = s(2i - j + 7k), where s is the parameter.

Options Analysis:

- **Option (A):** Correct parametric form with direction ratios (2, -1, 7). **Correct**
- **Option (B):** A scalar multiple of the direction vector. **Correct**
- **Option (C):** Includes unnecessary zero terms. **Incorrect**
- **Option (D):** Does not match the direction vector (2, -1, 7). **Incorrect**
- **Option (E):** Does not match the direction vector (2, -1, 7). **Incorrect**

Final Answer:

The correct option is **(1)**: **(A) and (B) only**

Question 37

$$A = \begin{bmatrix} 0 & \alpha & \beta \\ -\alpha & 0 & \gamma \\ -\beta & -\gamma & 0 \end{bmatrix}$$ **is a**

(A) square matrix

(B) diagonal matrix

(C) symmetric matrix

(D) skew-symmetric matrix

Choose the correct answer from the options given below:

Options:

A. (A) and (D) only

B. (A) and (C) only

C. (A), (B) and (D) only

D. (A), (B) and (C) only

Answer: A

Solution:

Concept:

Square Matrix:

- A matrix is square if the number of rows equals the number of columns.
- In this case, the matrix has 3 rows and 3 columns, so it is a square matrix.

Diagonal Matrix:

- A matrix is diagonal if all off-diagonal elements are zero.
- In the given matrix, the off-diagonal elements are non-zero (α, β, γ), so it is not a diagonal matrix.

Symmetric Matrix:

- A matrix is symmetric if $A = A^T$ (the matrix is equal to its transpose).
- The transpose of the given matrix is not equal to the matrix itself, so it is not symmetric.

Skew-Symmetric Matrix:

- A matrix is skew-symmetric if $A = -A^T$ (the matrix is equal to the negative of its transpose).
- For the given matrix, $A = -A^T$, so it is a skew-symmetric matrix.

Calculation:

Given the matrix:

$$A = \begin{bmatrix} 0 & \alpha & \beta \\ -\alpha & 0 & \gamma \\ -\beta & -\gamma & 0 \end{bmatrix}$$

Step 1: Check if it is a square matrix: It is a square matrix because it has 3 rows and 3 columns.

Step 2: Check if it is diagonal: It is not diagonal because the off-diagonal elements are non-zero.

Step 3: Check if it is symmetric: It is not symmetric because $A \neq A^T$.

Step 4: Check if it is skew-symmetric: It is skew-symmetric because $A = -A^T$.

Hence, the correct answer is option (1): (A) and (D) only.

Question 38

The integrating factor of the differential equation $(y \log_{\mathbf{e}} y)\frac{dx}{dy} + x = 2\log_{\mathbf{e}} y$ is:

Options:

A. y

B. $\frac{1}{y}$

C. $\log_e y$

D. $\log_e(\log_e y)$

Answer: C

Solution:

Concept:

Integrating Factor:

- The integrating factor is used to solve linear first-order differential equations.
- For a linear differential equation of the form: **dy/dx + P(x)y = Q(x)**, the integrating factor is given by:
- **Integrating Factor = $e^{\int P(x)dx}$**

Calculation:

Given the differential equation:

$y \log_e(y)\ dx/dy + x = 2 \log_e(y)$

Rewriting the equation:

$y \log_e(y)\ dx/dy = 2 \log_e(y) - x$

Dividing both sides by $y \log_e(y)$:

$dx/dy = 2/y - x/(y \log_e(y))$

This is in the standard form of a linear first-order differential equation:

$dx/dy + P(y)\ x = Q(y)$

Identifying **$P(y) = 1/(y \log_e(y))$** and **$Q(y) = 2/y$**, we find the integrating factor:

$\mu(y) = e^{\int P(y)dy} = e^{\int 1/(y \log_e(y))dy}$

The integral of $1/(y \log_e(y))$ is $\log_e(\log_e(y))$, so:

$\mu(y) = \log_e(y)$

Hence, the integrating factor is: $\log_e(y)$

Question 39

Optimise $Z = 3x + 9y$ subject to the constraints:

$x + 3y \leq 60,\ x + y \geq 10,\ x \leq y,\ x \geq 0,\ y \geq 0$, then

Options:

A. Maximum value ofZoccurs at the point(15, 15)only.

B. Maximum value ofZoccurs at the point(0, 20)only.

C. Maximum value ofZoccurs exactly at two points(15, 15)and(0, 20).

D. Maximum value ofZoccurs at all the points on the line segment joining(15, 15)and(0, 20).

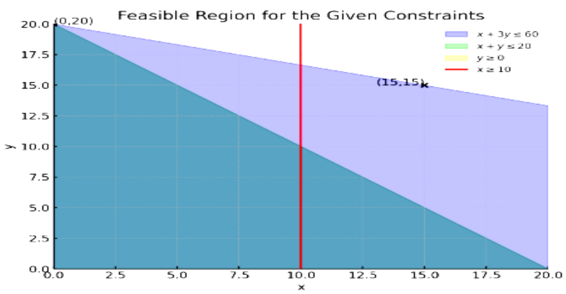
Answer: C

Solution:

Calculation:

We first graph the constraints:

- **x + 3y ≤ 60** represents a line with slope -1/3 and the region is below this line.
- **x ≥ 10** represents a vertical line at x = 10 and the region is to the right of this line.
- **x ≤ y** is a line where the values of x are less than or equal to y, meaning the region lies below or on the line y = x.
- **x ≥ 0** is the positive side of the x-axis.
- **y ≥ 0** is the region above the x-axis.



The vertices of the feasible region are:

- (15, 15)
- (0, 20)

Now, we evaluate Z = 3x + 9y at each vertex:

At (15, 15):

Z = 3(15) + 9(15) = 45 + 135 = 180

At (0, 20):

Z = 3(0) + 9(20) = 0 + 180 = 180

Hence, the maximum value of Z occurs exactly at two points (15, 15) and (0, 20).

Hence Option 3 is the correct answer.

Question 40

Minimise Z = -50x + 20y

subject to the constraints: 2x - y ≥ -5, 3x + y ≥ 3, 2x - 3y ≤ 12, x ≥ 0, y ≥ 0.

Then which of the following is/are true:

(A) Feasible region is unbounded.

(B) Z has no minimum value.

(C) The minimum value of Z is 100.

(D) The minimum value of Z is -300.

Choose the correct answer from the options given below:

Options:

A.

(A) and (D) only

B.

(C) and (D) only

C.

(A) and (C) only

D.

(A), (C) and (D) only

Answer: A

Question 41

If the system of linear equations $x + y + z = 2$, $2x + y - z = 3$ and $3x + 2y + kz = 4$ has a unique solution, then:

Options:

A. $k = 0$

B. $-1 < k < 1$

C. $k \neq 0$

D. -3 < k < 3

Answer: C

Solution:

Concept:

System of Linear Equations:

- A system of linear equations is a set of equations in which each equation is linear, meaning the variables appear only to the first power and are not multiplied by each other.
- For a system to have a unique solution, the determinant of the coefficient matrix must be non-zero.
- If the determinant of the coefficient matrix is zero, the system either has no solution or infinitely many solutions, depending on the consistency of the system.
- A unique solution exists if and only if the system of equations is consistent and the determinant of the coefficient matrix is non-zero.

Calculation:

Given,

The system of equations is:

1) x + y + z = 2

2) 2x + y - z = 3

3) 3x + 2y + kz = 4

The augmented matrix of the system is:

$$\begin{vmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -1 & 3 \\ 3 & 2 & k & 4 \end{vmatrix}$$

The coefficient matrix is:

$$A = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix}$$

The determinant of the coefficient matrix is:

det(A) = 1 × [(1 × k) - (-2)] - 1 × [(2 × k) - (-3)] + 1 × [(2 × 2) - (3 × 1)]

det(A) = 1 × (k + 2) - 1 × (2k + 3) + 1 × (4 - 3)

det(A) = k + 2 - 2k - 3 + 1

det(A) = -k + 0

For the system to have a unique solution, the determinant must not be zero:

$-k \neq 0$

Hence, $k \neq 0$

∴ The correct answer is option (3) $k \neq 0$.

Question 42

The function $f(x) = ||x| + 1 - x|$ is:

Options:

A. continuous and differentiable at $x = 0$ only

B. continuous at $x = 0$ but nowhere differentiable

C. continuous everywhere and differentiable at all points except at $x = 0$

D. continuous but not differentiable at $x = 1$

Answer: C

Solution:

Concept:

Continuity and Differentiability of Functions:

- To determine the continuity and differentiability of a function, we need to analyze the behavior of the function at different points.
- A function is continuous at a point if the following three conditions are met:
  - The function is defined at the point.
  - The limit of the function exists at the point.
  - The limit of the function equals the function's value at the point.
- A function is differentiable at a point if the derivative exists at that point. This implies that the function must be continuous at the point.
- In this case, we are given the function:
  - $f(x) = | |x| + 1 - |x| |$
- We need to determine the continuity and differentiability of this function at various points, including at $x = 0$ and $x = 1$.

Calculation:

Given function:

$f(x) = | |x| + 1 - |x| |$

Step 1: Simplify the function.

Notice that:

|x| + 1 - |x| = 1 for all values of x (since the terms |x| and -|x| cancel each other out). Therefore, the function simplifies to:

f(x) = |1| = 1

Step 2: Analyze continuity and differentiability.

Since f(x) is a constant function (f(x) = 1), it is continuous and differentiable everywhere, including at x = 0 and x = 1.

Conclusion:

Since the function is constant, it is continuous and differentiable at all points.

∴ The correct answer is Option (3):

continuous everywhere and differentiable at all points except at x = 0.

Question 43

Match List-I with List-II.

	List-I (Function)		List-II (Interval in which function is increasing)
(A)	$\frac{x}{\log_e x}$	(I)	$(-\infty, -2) \cup (2, \infty)$
(B)	$\frac{x}{2} + \frac{2}{x}, x \neq 0$	(II)	$\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$
(C)	x^x, x > 0	(III)	$\left(\frac{1}{e}, \infty\right)$
(D)	sinx - cosx	(IV)	(e, ∞)

Choose the correct answer from the options given below:

Options:

A. (A) - (II), (B) - (I), (C) - (III), (D) -(IV)

B. (A) - (I), (B) - (III), (C) - (II), (D) - (IV)

C. (A) - (IV), (B) - (I), (C) - (III), (D) - (II)

D. (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Answer: C

Solution:

Explanation:

	List-I (Function)		List-II (Interval in which function is increasing)
(A)	$\frac{x}{\log_e x}$	(I)	$(-\infty, -2) \cup (2, \infty)$
(B)	$\frac{x}{2} + \frac{2}{x}, x \neq 0$	(II)	$\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$
(C)	x^x, $x > 0$	(III)	$\left(\frac{1}{e}, \infty\right)$
(D)	sinx - cosx	(IV)	(e, ∞)

A. $\frac{x}{\log_e x}$

This function is defined for x > 0, as the logarithm function is only defined for positive values of x.

To find the interval in which the function is increasing, take the derivative of $\frac{x}{\log_e x}$ with respect to x.

After solving, it can be found that the function is increasing when x > e.

So, the correct interval is (IV).

B. $\frac{x}{2} + \frac{2}{x}$, $x \neq 0$

This function is defined for $x \neq 0$.

By finding the derivative and setting it to zero,

it can be shown that the function is increasing for x > 2 and x < -2.

So, the correct interval is (I).

C.x^x, $x > 0$

This function is increasing for x > 0.

So, the correct interval is (III).

D.$\sin x - \cos x$

The derivative of $\sin x - \cos x$ is $\cos x + \sin x$.

Setting this derivative greater than zero gives the interval $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$, so the correct interval is (II).

Final Answer:

The correct matching is:

(A) → (IV)
(B) → (I)
(C) → (III)
(D) → (II)

The correct option is (3).

Question 44

The value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\mathrm{d}x}{1+\tan^{18} x}$ is:

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{9}$

D. $\frac{\pi}{12}$

Answer: D

Solution:

Concept:

Integral of Trigonometric Function:

- The given integral is of the form: $\int(1/(1+tan^{18}(x)))dx$
- This type of integral has a standard result for specific limits, especially when the bounds are multiples of π/6 or π/3.
- The function involves symmetry properties related to the trigonometric function tan(x).
- For integrals of the form $\int(1/(1+tan^{n}(x)))dx$, where n is a large integer, specific results from integral tables or standard integral formulas can be applied.
- The result can be simplified for standard bounds like π/6 to π/3.

Calculation:

Given,

The integral to evaluate is:

I = $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\mathrm{d}x}{1+\tan^{18} x}$

This is a standard integral involving the function tan(x) raised to a high power.

By using a known result for integrals of this type, we can simplify the evaluation:

$\int_0^{\frac{\pi}{2}}(1/(1+tan^{n}(x)))dx = \pi/12$ for n = 18

Thus, the value of the integral is:

Hence, the value of the integral is: π / 12.

The correct answer is option (4)

Question 45

If the angle between $\overrightarrow{\mathrm{a}} = 2y^2\hat{i} + 4y\hat{j} + \hat{\mathrm{k}}$ and $\overrightarrow{\mathrm{b}} = 7\hat{i} - 2\hat{j} + y\hat{\mathrm{k}}$ is obtuse, then:

Options:

A.

$-1/2 < y < 0$

B.

$-1 < y < -1/2$

C.

$1/2 < y < 1$

D.

$0 < y < 1/2$

Answer: C

Solution:

<u>**Concept:**</u>

Dot Product and Angle Between Two Vectors:

- The dot product of two vectors is given by the formula:
  - $a \cdot b = |a|\,|b| \cos \theta$
- Where θ is the angle between the two vectors. For the vectors to be perpendicular (i.e., the angle between them is 90°), the dot product must be zero:
  - $a \cdot b = 0$
- In this question, we are given two vectors:
  - $a = 2y^2\, i + 4y\, j + k$
  - $b = 7\, i - 2\, j + y\, k$

- We need to find the value of y such that the angle between the two vectors is 90°.

Calculation:

The given vectors are:

$a = 2y^2 i + 4y j + k$

$b = 7 i - 2 j + y k$

Step 1: Compute the dot product $a \cdot b$:

$a \cdot b = (2y^2)(7) + (4y)(-2) + (1)(y)$

$a \cdot b = 14y^2 - 8y + y$

$a \cdot b = 14y^2 - 7y$

Step 2: For the vectors to be perpendicular, set the dot product equal to zero:

$14y^2 - 7y = 0$

Step 3: Factor the equation:

$7y(2y - 1) = 0$

Step 4: Solve for y:

- From $7y = 0$, we get $y = 0$
- From $2y - 1 = 0$, we get $y = 1/2$

Thus, the two possible values for y are $y = 0$ and $y = 1/2$.

Step 5: Conclusion

The correct value of y that makes the vectors perpendicular (i.e., angle between them is 90°) is $y = 1/2$.

∴ The correct answer is Option (3): $1/2 < y < 1$

Question 46

Two events X and Y are such that $P(X) = \frac{1}{3}, P(Y) = n$ and the probability of occurrence of at least one event is 0.8. If the events are independent, then the value of n is:

Options:

A. $\frac{3}{10}$

B. $\frac{1}{15}$

C. $\frac{7}{10}$

D. $\frac{11}{15}$

Answer: C

Solution:

Concept:

Probability of Independent Events:

- The probability of two independent events X and Y occurring can be calculated using the formula for the union of two independent events:
  - $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$
- If the events X and Y are independent, then:
  - $P(X \cap Y) = P(X) \times P(Y)$
- In this case, we are given that the probability of at least one event occurring is 0.8.
- Using this formula, we can find the value of n, the probability of event Y.

Calculation:

Given,

$P(X) = 1/3$

$P(Y) = n$

$P(X \cup Y) = 0.8$

For independent events, the formula for $P(X \cup Y)$ becomes:

$P(X \cup Y) = P(X) + P(Y) - P(X) \times P(Y)$

Substituting the known values:

$0.8 = 1/3 + n - (1/3) \times n$

Multiply through by 3 to eliminate fractions:

$2.4 = 1 + 3n - n$

Simplify the equation:

$2.4 = 1 + 2n$

Subtract 1 from both sides:

$1.4 = 2n$

Divide by 2:

n = 1.4 / 2 = 0.7

Hence, the value of n is 0.7, which is equal to 7/10.

The correct answer is option (3) .

Question 47

The shortest distance (in units) between the lines $\frac{1-x}{1} = \frac{2y-10}{2} = \frac{z+1}{1}$ and $\frac{x-3}{-1} = \frac{y-5}{1} = \frac{z-0}{1}$ is:

Options:

A. $\frac{\sqrt{11}}{\sqrt{3}}$

B. $\frac{11}{3}$

C. $\frac{14}{3}$

D. $\sqrt{\frac{14}{3}}$

Answer: D

Solution:

Concept:

Shortest Distance Between Skew Lines:

- The shortest distance between two skew lines can be found using the formula:
  - d = |(b - a) × v| / |v|
- Here, d is the shortest distance, a and b are points on the respective lines, and v is the direction vector of the line formed by the cross product of the two direction vectors.
- The formula uses the cross product of the direction vectors of the two lines to determine the perpendicular distance between them.
- If the direction vectors of the two lines are $v_1 = (a_1, b_1, c_1)$ and $v_2 = (a_2, b_2, c_2)$, then the cross product is $v_1 \times v_2 = (b_1c_2 - b_2c_1, a_2c_1 - a_1c_2, a_1b_2 - a_2b_1)$.
- Then, the shortest distance is calculated using the above formula.

Calculation:

Given,

The first line is given by:

x - 3 / -1 = y - 5 / 1 = z / 1

So, the direction vector of the first line is $v_1 = (-1, 1, 1)$, and a point on the line is a = (3, 5, 0).

The second line is given by:

1 - x / 1 = 2y - 10 / 2 = z + 1 / 1

So, the direction vector of the second line is $v_2 = (1, 2, 1)$, and a point on the line is b = (1, 5, -1).

Now, we calculate the vector b - a = (1 - 3, 5 - 5, -1 - 0) = (-2, 0, -1).

The cross product of $v_1 \times v_2$ is:

$v_1 \times v_2 = (1\times1 - 2\times1, 1\times(-1) - (-1)\times1, (-1)\times2 - 1\times1)$

$v_1 \times v_2 = (-1, 0, -3)$

The magnitude of the cross product is:

$|v_1 \times v_2| = \sqrt{((-1)^2 + 0^2 + (-3)^2)} = \sqrt{(1 + 9)} = \sqrt{10}$

The magnitude of the direction vector v_1 is:

$|v_1| = \sqrt{((-1)^2 + 1^2 + 1^2)} = \sqrt{3}$

The shortest distance between the lines is:

$d = |(-2, 0, -1) \times (-1, 0, -3)| / \sqrt{3} = \sqrt{10} / \sqrt{3} = \sqrt{(10/3)}$

∴ **The shortest distance between the lines is $\sqrt{(14/3)}$.**

The correct answer is option (4) $\sqrt{(14/3)}$.

Question 48

The value of λ for which the lines $\frac{2-x}{3} = \frac{3-4y}{5} = \frac{z-2}{3}$ and $\frac{x-2}{-3} = \frac{2y-4}{3} = \frac{2-z}{\lambda}$ are perpendicular is:

Options:

A. -2

B. 2

C. $\frac{8}{19}$

D. $\frac{19}{8}$

Answer: D

Solution:

Explanation:

Line 1:

$\frac{2-x}{3} = \frac{3-4y}{5} = \frac{z-2}{3}$

Let parameter t

$\Rightarrow x = 2 - 3t,\ y = \frac{3-5t}{4},\ z = 2 + 3t$

$\Rightarrow$Direction ratios: $(-3, -\frac{5}{4}, 3)$

To eliminate fraction: Multiply by 4 → (-12, -5, 12)

Line 2:

$\frac{x-2}{-3} = \frac{2y-4}{3} = \frac{2-z}{\lambda}$

$\Rightarrow x = 2 - 3s,\ y = 2 + \frac{3s}{2},\ z = 2 - \lambda s$

$\Rightarrow$Direction ratios:$(-3, \frac{3}{2}, -\lambda)$

Now compute dot product:

$(-12)(-3) + (-5)\left(\frac{3}{2}\right) + (12)(-\lambda) = 36 - \frac{15}{2} - 12\lambda = 0$

$\Rightarrow \frac{72-15-24\lambda}{2} = 0$

$\Rightarrow \frac{57-24\lambda}{2} = 0$

$\Rightarrow 57 - 24\lambda = 0$

$\Rightarrow \lambda = \frac{57}{24} = \frac{19}{8}$

Hence Option 4 is the correct answer.

Question 49

Let Aand Bare two independent events such that $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$.

Match List-I with List-II

	List-I		List-II
(A)	$P(A \cap B)$	(I)	$\frac{2}{5}$
(B)	$P(A \mid B)$	(Il)	$\frac{4}{15}$
(C)	$P(A' \mid B)$	(Ill)	$\frac{3}{5}$
(D)	$P(A' \cap B')$	(IV)	$\frac{2}{9}$

Choose the correct answer from the options given below:

Options:

A. (A) - (III), (B) - (II), (C) - (I), (D) - (IV)

B. (A) - (II), (B) - (III), (C) - (I), (D) - (IV)

C. (A) - (II), (B) - (III), (C) - (IV), (D) - (I)

D. (A) - (II), (B) - (IV), (C) - (I), (D) - (III)

Answer: B

Solution:

Concept:

- Events A and B are said to be **independent** if the occurrence of one does not affect the occurrence of the other.
- For independent events, the probability of intersection is given by: $P(A \cap B) = P(A) \times P(B)$
- **Conditional Probability**: $P(A \mid B) = P(A \cap B) / P(B)$
- **Complement of Event**: If P(A) is the probability of A, then $P(A') = 1 - P(A)$
- Using the definition of conditional probability, $P(A' \mid B) = P(A' \cap B) / P(B)$
- The formula for union: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- From that, $P(A' \cap B') = 1 - P(A \cup B)$

Calculation:

Given,

$P(A) = 3/5$, $P(B) = 4/9$

Since A and B are independent:

$P(A \cap B) = P(A) \times P(B)$

$\Rightarrow P(A \cap B) = (3/5) \times (4/9) = 12/45 = 4/15$

⇒ P(A | B) = P(A ∩ B) / P(B) = (4/15) ÷ (4/9) = (4/15) × (9/4) = 9/15 = 3/5

⇒ P(A' | B) = 1 − P(A | B) = 1 − 3/5 = 2/5

⇒ P(A ∪ B) = P(A) + P(B) − P(A ∩ B) = 3/5 + 4/9 − 4/15

⇒ P(A ∪ B) = (27 + 20 − 12)/45 = 35/45

⇒ P(A' ∩ B') = 1 − P(A ∪ B) = 1 − 35/45 = 10/45 = 2/9

∴ **Final Matching:**

(A) → 4/15

(B) → 3/5

(C) → 2/5

(D) → 2/9

∴ **Correct option is (2): (A)-(II), (B)-(III), (C)-(I), (D)-(IV)**

Question 50

If $P = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation P^2- 3P - 7l = 0, where I is an identity matrix of order 2, then P^{-1} is:

Options:

A. $\frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

C. $\frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -1 \end{bmatrix}$

D. $\frac{1}{7}\begin{bmatrix} 2 & 5 \\ -1 & -1 \end{bmatrix}$

Answer: A

Solution:

Explanation:

$$P = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

The equation to satisfy is:

$$P^2 - 3P - 7I = 0$$

where I is the identity matrix of order 2:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

To findP^2, multiply the matrix P by itself:

$$P^2 = P \times P = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} \times \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

Multiplying the matrices:

$$P^2 = \begin{bmatrix} 5 \times 5 + 3 \times (-1) & 5 \times 3 + 3 \times (-2) \\ (-1) \times 5 + (-2) \times (-1) & (-1) \times 3 + (-2) \times (-2) \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 25 - 3 & 15 - 6 \\ -5 + 2 & -3 + 4 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

Now, multiply the matrix P by 3:

$$3P = 3 \times \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

$$3P = \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix}$$

Now, multiply the identity matrix by 7:

$$7I = 7 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

Now substitute the values of P^2, 3P, and 7I into the original equation:

$$\begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Simplifying:

$$\begin{bmatrix} 22 - 15 - 7 & 9 - 9 - 0 \\ -3 - (-3) - 0 & 1 - (-6) - 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

This confirms that the matrix P satisfies the equation.

To find the inverse of matrix P, use the formula for the inverse of a 2x2 matrix:

$$P^{-1} = \frac{1}{det(P)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

For matrix $P = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, we have a = 5, b = 3, c = -1, and d = -2.

First, calculate the determinant of P:

det(P) = (5)(-2) - (3)(-1) = -10 + 3 = -7

Now, find the inverse:

$$P^{-1} = \frac{1}{-7} \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} \\ -\frac{1}{7} & -\frac{5}{7} \end{bmatrix}$$

Thus, the inverse of P is:

$$P^{-1} = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} \\ -\frac{1}{7} & -\frac{5}{7} \end{bmatrix}$$

Hence Option 1 is the correct answer.
