Inverse Trigonometric Functions

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. The value of $\sin^{-1}\left(\cos\frac{13\pi}{5}\right)$ is **[CBSE 2021-22 (Term-1)]**

   (a) $-\frac{3\pi}{5}$ (b) $-\frac{\pi}{10}$ (c) $\frac{3\pi}{5}$ (d) $\frac{\pi}{10}$

2. If $\sin^{-1}x > \cos^{-1}x$, then x should lie in the interval **[CBSE 2021-22 (Term-1)]**

   (a) $\left(-1, -\frac{1}{\sqrt{2}}\right)$ (b) $\left(0, -\frac{1}{\sqrt{2}}\right)$ (c) $\left(\frac{1}{\sqrt{2}}, 1\right)$ (d) $\left(\frac{1}{\sqrt{2}}, 0\right)$

3. $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to **[CBSE Sample Paper 2021-22 (Term-1)]**

   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) -1 (d) 1

4. The value of $\cot(\sin^{-1}x)$ is **[NCERT Exemplar]**

   (a) $\frac{\sqrt{1+x^2}}{x}$ (b) $\frac{x}{\sqrt{1+x^2}}$ (c) $\frac{1}{x}$ (d) $\frac{\sqrt{1-x^2}}{x}$

5. The value of $\sin^{-1}\left(\cos\frac{\pi}{9}\right)$ is **[NCERT Exemplar]**

   (a) $\frac{\pi}{9}$ (b) $\frac{5\pi}{9}$ (c) $\frac{-5\pi}{9}$ (d) $\frac{7\pi}{18}$

6. Let $\theta = \sin^{-1}(\sin(-600°))$, then value of θ is

   (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{-2\pi}{3}$

7. The value of the expression $2\sec^{-1}2 + \sin^{-1}\left(\frac{1}{2}\right)$ is **[NCERT Exemplar]**

   (a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{7\pi}{6}$ (d) 1

8. The value of $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right]$ is **[NCERT Exemplar]**

   (a) $\frac{25}{24}$ (b) $\frac{25}{7}$ (c) $\frac{24}{25}$ (d) $\frac{7}{24}$

9. The value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$ is

   (a) $\frac{3\pi}{5}$ (b) $\frac{-7\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $\frac{-\pi}{10}$

10. The value of $\cot\left[\frac{1}{2}\sin^{-1}\frac{\sqrt{3}}{2}\right]$ is

    (a) 1 (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 0

11. $\sin(\cot^{-1}x)$ is equal to

    (a) $\sqrt{1+x^2}$ (b) x (c) $(1+x^2)^{-3/2}$ (d) $(1+x^2)^{-1/2}$

12. If $\tan^{-1}x = y$, then **[CBSE Sample Paper 2021-22 (Term-1)]**

    (a) $-1 < y < 1$ (b) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$ (c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ (d) $y \in \left\{\frac{-\pi}{2}, \frac{\pi}{2}\right\}$

13. If $y = \cot^{-1} x, x < 0$, then [CBSE 2021-22 (Term-1)]

(a) $\frac{\pi}{2} < y \le \pi$ (b) $\frac{\pi}{2} < y < \pi$ (c) $-\frac{\pi}{2} < y < 0$ (d) $-\frac{\pi}{2} \le y < 0$

14. $\sin(\tan^{-1} x), |x| < 1$ is equal to

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

15. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then $\alpha(\beta+\gamma) + \beta(\gamma+\alpha) + \gamma(\alpha+\beta)$ equals [NCERT Exemplar]

(a) 0 (b) 1 (c) 6 (d) 12

16. Simplest form of $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right), \pi < x < \frac{3\pi}{2}$ is [CBSE Sample Paper 2021-22 (Term-1)]

(a) $\frac{\pi}{4} - \frac{x}{2}$ (b) $\frac{3\pi}{2} - \frac{x}{2}$ (c) $-\frac{x}{2}$ (d) $\pi - \frac{x}{2}$

17. The number of real solutions of the equation $\sqrt{1+\cos 2x} = \sqrt{2}\cos^{-1}(\cos x)$ in $\left[\frac{\pi}{2}, \pi\right]$ is

(a) 0 (b) 1 (c) 2 (d) ∞

18. The value of $\cos^{-1}(2x^2 - 1), 0 \le x \le 1$ is equal to

(a) $2\cos^{-1} x$ (b) $2\sin^{-1} x$ (c) $\pi - 2\cos^{-1} x$ (d) $\pi + 2\cos^{-1} x$

19. If $\cos\left(\sin^{-1}\frac{3}{5} + \cos^{-1}x\right) = 0$, then x is equal to

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) 0 (d) 1

20. Which of the following is the principal value branch of $\text{cosec}^{-1} x$?

(a) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (b) $(0, \pi) - \left\{\frac{\pi}{2}\right\}$ (c) $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$ (d) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

21. The domain of the function $\cos^{-1}(2x - 1)$ is

(a) $[0, 1]$ (b) $[-1, 1]$ (c) $(-1, 1)$ (d) $[0, \pi]$

22. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is [NCERT Exemplar]

(a) $[1, 2]$ (b) $[-1, 1]$ (c) $[0, 1]$ (d) none of these

23. $\sin(\tan^{-1} x)$, where $|x| < 1$, is equal to [CBSE Sample Paper 2021-22 (Term-1)]

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

24. $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to [CBSE 2023 (65/5/1)]

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

Answers

1. (b)	2. (c)	3. (d)	4. (d)	5. (d)	6. (a)	7. (b)
8. (d)	9. (d)	10. (c)	11. (d)	12. (c)	13. (b)	14. (d)
15. (c)	16. (a)	17. (a)	18. (a)	19. (b)	20. (d)	21. (a)
22. (a)	23. (d)	24. (a)				

Solutions of Selected Multiple Choice Questions

1. We have,

$$\sin^{-1}\left(\cos\frac{13\pi}{5}\right) = \sin^{-1}\left[\cos\left(3\pi - \frac{2\pi}{5}\right)\right]$$

$$\Rightarrow \quad \sin^{-1}\left[-\cos\frac{2\pi}{5}\right] = \sin^{-1}\left[-\sin\left(\frac{\pi}{2} - \frac{2\pi}{5}\right)\right]$$

$$\Rightarrow \quad \sin^{-1}\left[-\sin\frac{\pi}{10}\right] = \sin^{-1}\left[\sin\left(-\frac{\pi}{10}\right)\right] = \frac{-\pi}{10}$$

$\therefore$ Option (*b*) is correct.

2. We have,

$$\sin^{-1}x > \cos^{-1}x$$

$$\Rightarrow \quad \sin^{-1}x > \frac{\pi}{2} - \sin^{-1}x \quad \Rightarrow 2\sin^{-1}x > \frac{\pi}{2} \Rightarrow \sin^{-1}x > \frac{\pi}{4}$$

$$\Rightarrow \quad x > \sin\frac{\pi}{4} \quad \Rightarrow x > \frac{1}{\sqrt{2}}$$

As we know that $\sin^{-1}x$ and $\cos^{-1}x$ exist for $x \in [-1, 1]$.

$$\therefore \quad x \in \left(\frac{1}{\sqrt{2}}, 1\right)$$

$$\Rightarrow \quad \left(\frac{1}{\sqrt{2}}, 1\right) \subset \left(\frac{1}{2}, 1\right)$$

$\therefore$ Option (*c*) is correct.

3. We have, $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

$$= \sin\left[\frac{\pi}{3} - \sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)\right] = \sin\left[\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right]$$

$$= \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right] = \sin\frac{\pi}{2} = 1$$

$\therefore$ Option (*d*) is correct.

4. Let $\sin^{-1}x = \theta$, then $\sin\theta = x$.

$$\Rightarrow \quad \operatorname{cosec}\theta = \frac{1}{x} \quad \Rightarrow \quad \operatorname{cosec}^2\theta = \frac{1}{x^2}$$

$$\Rightarrow \quad 1 + \cot^2\theta = \frac{1}{x^2} \quad \Rightarrow \quad \cot\theta = \frac{\sqrt{1-x^2}}{x} \quad \Rightarrow \quad \cot(\sin^{-1}x) = \frac{\sqrt{1-x^2}}{x}$$

$\therefore$ Option (*d*) is correct.

5. $\sin^{-1}\left(\cos\frac{\pi}{9}\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \frac{\pi}{9}\right)\right)$

$$= \sin^{-1}\left(\sin\frac{7\pi}{18}\right) = \frac{7\pi}{18}$$

$\therefore$ Option (*d*) is correct.

7. We have,

$$2\sec^{-1}(2) + \sin^{-1}\left(\frac{1}{2}\right)$$

$$= 2\sec^{-1}\left(\sec\left(\frac{\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right) = 2 \times \frac{\pi}{3} + \frac{\pi}{6} = \frac{2\pi}{3} + \frac{\pi}{6} = \frac{5\pi}{6}$$

$\therefore$ Option (*b*) is correct.

9. $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right] = \sin^{-1}\left[\cos\left(6\pi + \frac{3\pi}{5}\right)\right]$

$$= \sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)\right] = \sin^{-1}\left[\sin\left(-\frac{\pi}{10}\right)\right]$$

$$= \sin^{-1}\left[-\sin\left(\frac{\pi}{10}\right)\right] = -\sin^{-1}\left[\sin\left(\frac{\pi}{10}\right)\right]$$

$$= -\frac{\pi}{10} \qquad \left(\because \frac{\pi}{10} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right)$$

$\therefore$ Option (*d*) is correct.

10. $\cot\left[\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right] = \cot\left[\frac{1}{2} \times \frac{\pi}{3}\right] = \cot^{-1}\left(\frac{\pi}{6}\right) = \sqrt{3}$

$\therefore$ Option (*c*) is correct.

11. Let $\cot^{-1} x = \alpha \Rightarrow \cot\alpha = x = \frac{b}{p}$

$\Rightarrow \sin\alpha = \frac{p}{h} = \frac{1}{\sqrt{1+x^2}}$

$\Rightarrow \sin(\cot^{-1} x) = \sin(\alpha) = \frac{1}{\sqrt{1+x^2}} = (x^2+1)^{-1/2}$

$\therefore$ Option (*d*) is correct.

12. $\because \tan^{-1} x = y$ (Given) $\quad \Rightarrow \quad \frac{-\pi}{2} < y < \frac{\pi}{2}$

$\therefore$ Option (*c*) is correct.

13. We have, $\quad y = \cot^{-1} x, x < 0$

$\Rightarrow \qquad x = \cot y < 0$

As we know that cotangent (*i.e.* cot *y*) is negative in IInd and IVth quadrant, i.e. cot *y* lie in IInd or IVth quadrant.

$\Rightarrow \quad \frac{\pi}{2} < y < \pi$ or $\frac{3\pi}{2} < y < 2\pi$

$\therefore$ Option (*b*) is correct.

15. We have, $\quad \cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$

We know that, $\quad 0 \le \cos^{-1} x \le \pi$

$$\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$$

If and only if, $\quad \cos^{-1}\alpha = \cos^{-1}\beta = \cos^{-1}\gamma = \pi$

$\Rightarrow \qquad \cos\pi = \alpha = \beta = \gamma \qquad \Rightarrow \qquad -1 = \alpha = \beta = \gamma$

$\Rightarrow \qquad \alpha = \beta = \gamma = -1$

$\therefore \qquad \alpha(\beta+\gamma) + \beta(\gamma+\alpha) + \gamma(\alpha+\beta) = -1(-1-1) - 1(-1-1) - 1(-1-1)$

$$= 2 + 2 + 2 = 6$$

$\therefore$ Option (*c*) is correct.

16. Given, $\tan^{-1}\left(\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}}\right)$

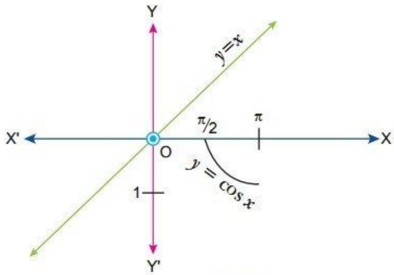
$$= \tan^{-1}\left(\frac{-\sqrt{2}\cos\frac{x}{2} + \sqrt{2}\sin\frac{x}{2}}{-\sqrt{2}\cos\frac{x}{2} - \sqrt{2}\sin\frac{x}{2}}\right), \text{ where } \pi < x < \frac{3\pi}{2}$$

$$= \tan^{-1}\left(\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right) = \tan^{-1}\left(\frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}\right)$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right] = \frac{\pi}{4} - \frac{x}{2}$$

$\therefore$ Option (*a*) is correct.

17. $\sqrt{1 + \cos 2x} = \sqrt{2}\cos^{-1}(\cos x)$



$\Rightarrow \quad \sqrt{2\cos^2 x} = \sqrt{2}x, x \in \left[\frac{\pi}{2}, \pi\right] \subset [0, \pi]$

$\Rightarrow \quad \sqrt{2}\cos x = \sqrt{2}x$

$\Rightarrow \quad \cos x = x \qquad \left(\because x \in \left[\frac{\pi}{2}, \pi\right]\right)$

As in the given figure graph of $y = x$ and $y = \cos x$ does not intersect in $\left[\frac{\pi}{2}, \pi\right]$ so the number of solutions is zero.

$\therefore$ Option (*a*) is correct.

18. $\cos^{-1}(2x^2 - 1)$

Put $x = \cos\alpha \Rightarrow \alpha = \cos^{-1}x$

$\therefore \quad \cos^{-1}(2x^2 - 1) = \cos^{-1}(2\cos^2\alpha - 1)$

$\qquad = \cos^{-1}(\cos 2\alpha) = 2\alpha = 2\cos^{-1}x$

$\therefore$ Option (*a*) is correct.

$\left[\begin{array}{l}\because 0 \le x \le 1 \\ \Rightarrow \cos\left(-\frac{\pi}{2}\right) \le \cos\alpha \le \cos 0 \\ \Rightarrow -\frac{\pi}{2} \le \alpha \le 0 \\ \Rightarrow -\pi \le 2\alpha \le 0\end{array}\right]$

19. $\cos\left(\sin^{-1}\frac{3}{5} + \cos^{-1}x\right) = 0$

$\Rightarrow \quad \sin^{-1}\frac{3}{5} + \cos^{-1}x = \cos^{-1}(0) = \frac{\pi}{2} \qquad \Rightarrow \qquad \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}\frac{3}{5} = \cos^{-1}\frac{3}{5}$

$\Rightarrow \quad x = \cos\left(\cos^{-1}\left(\frac{3}{5}\right)\right) = \frac{3}{5} \qquad \left[\because \frac{3}{5} < 1\right]$

$\Rightarrow \quad x = \frac{3}{5}$

$\therefore$ Option (*b*) is correct.

20. Principal value branch of $\text{cosec}^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

$\therefore$ Option (*d*) is correct.

21. Let $\alpha = \cos^{-1}(2x - 1) \Rightarrow \cos\alpha = 2x - 1$

$\because \quad -1 \le \cos\alpha \le 1 \Rightarrow -1 \le 2x - 1 \le 1$

$\Rightarrow \quad 0 \le 2x \le 2 \Rightarrow 0 \le x \le 1 \Rightarrow x \in [0, 1]$

Hence domain of $\cos^{-1}(2x - 1)$ is $[0, 1]$.

$\therefore$ Option (*a*) is correct.

22. $\because \quad f(x) = \sin^{-1}\sqrt{x-1}$

$\Rightarrow \quad 0 \le x-1 \le 1 \qquad [\because \sqrt{x-1} \ge 0 \text{ and } -1 \le \sqrt{x-1} \le 1]$

$\Rightarrow \quad 1 \le x \le 2$

$\therefore \quad x \in [1, 2]$

$\therefore$ Option (*a*) is correct.

23. Let $\tan^{-1}x = \theta \Rightarrow x = \tan\theta \Rightarrow \cot\theta = \frac{1}{x}$

$$\therefore \quad \sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{\sqrt{1+\cot^2\theta}} = \frac{1}{\sqrt{1+\frac{1}{x^2}}} = \frac{x}{\sqrt{1+x^2}}$$

$$\Rightarrow \quad \sin(\tan^{-1}x) = \frac{x}{\sqrt{1+x^2}}$$

$\therefore$ Option (*d*) is correct.

Assertion-Reason Questions

The following questions consist of two statements—Assertion(A) and Reason(R). Answer these questions selecting the appropriate option given below:

(a) Both A and R are true and R is the correct explanation for A.
(b) Both A and R are true but R is not the correct explanation for A.
(c) A is true but R is false.
(d) A is false but R is true.

1. **Assertion (A) :** Domain of $f(x) = \sin^{-1}x + \cos x$ is $[-1, 1]$.
   Reason (R) : Domain of a function is the set of all possible values for which function will be defined.
2. **Assertion (A) :** Function $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = \sin x$ is not a bijection.
   Reason (R) : A function $f: A \to B$ is said to be bijection if it is one-one and onto.
3. **Assertion (A) :** Principal value of $\tan^{-1}(-\sqrt{3})$ is $-\frac{\pi}{3}$.
   Reason (R) : $\tan^{-1}: \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ so for any $x \in \mathbb{R}$, $\tan^{-1}(x)$ represents an angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.
4. **Assertion (A) :** $\sin^{-1}(-x) = -\sin^{-1}x;\ x \in [-1, 1]$
   Reason (R) : $\sin^{-1}: [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is a bijection map.
5. **Assertion (A) :** A function $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = \cos x$ is a bijection.
   Reason (R) : A function $g: A \to B$ is a bijection then $\exists$ a function $h: B \to A$ such that $goh = I_B$ and $hog = I_A$.
6. **Assertion (A) :** Range of $[\sin^{-1}x + 2\cos^{-1}x]$ is $[0, \pi]$.
   Reason (R) : Principle value branch of $\sin^{-1}x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. *[CBSE 2023 (65/3/2)]*
7. **Assertion (A) :** All trigonometric functions have their inverses over their respective domains.
   Reason (R) : The inverse of $\tan^{-1}x$ exists for some $x \in \mathbb{R}$. *[CBSE 2023 (65/4/1)]*

8. **Assertion (A) :** The range of the function $f(x) = 2\sin^{-1} x + \frac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Reason (R) : The range of the principal value branch of $\sin^{-1}(x)$ is $[0, \pi]$. *[CBSE 2023 (65/1/1)]*

9. **Assertion (A) :** Maximum value of $(\cos^{-1} x)^2$ is π^2.

Reason (R) : Range of the principal value branch of $\cos^{-1}x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$. *[CBSE 2023 (65/2/1)]*

Answers

1. (*a*) 2. (*a*) 3. (*a*) 4. (*b*) 5. (*d*) 6. (*d*) 7. (*c*)
8. (*c*) 9. (*c*)

Solutions of Assertion-Reason Questions

1. The domain of $\sin^{-1} x$ is $[-1, 1]$ and that of $\cos x$ is $\mathbb{R}$.

$\therefore$ Domain of $f(x) = \sin^{-1} x + \cos x$ is $[-1, 1] \cap \mathbb{R} = [-1, 1]$.

Here, A and R are true and R gives correct explanation A.

$\therefore$ Option (*a*) is correct.

2. For $0, \pi \in \mathbb{R}$ such that $f(0) = 0 = f(\pi)$. But $0 \neq \pi$

So, f is not one-one.

Therefore f is not a bijection map.

Here A and R are true and R gives the correct explanation of A.

$\therefore$ Option (*a*) is correct.

3. $\because$ $\tan^{-1} : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\tan^{-1}(x) = \theta$, where $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ & $x \in \mathbb{R}$

$\therefore$ $\tan^{-1}(-\sqrt{3}) = \theta \Rightarrow \tan\theta = -\sqrt{3} = \tan\left(-\frac{\pi}{3}\right) \Rightarrow \theta = \frac{-\pi}{3}$

Here, A and R are correct and R gives correct explanation of statement A.

$\therefore$ Option (*a*) is correct.

4. Let $\sin^{-1}(-x) = y \Rightarrow -x = \sin y$

$\Rightarrow x = -\sin y = \sin(-y) \qquad \Rightarrow \qquad \sin^{-1} x = -y = -\sin^{-1}(-x)$

$\Rightarrow \sin^{-1}(-x) = -\sin^{-1} x$

A and R are correct but R does not give correct explanation of A.

$\therefore$ Option (*b*) is correct.

5. For $\frac{\pi}{2}, \frac{3\pi}{2} \in \mathbb{R}$

$$f\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0 = \cos\frac{3\pi}{2} = f\left(\frac{3\pi}{2}\right) \text{ but } \frac{\pi}{2} \neq \frac{3\pi}{2}$$

$\therefore$ f is not a one-one function. So, it is not a bijection.

Here, A is false and R is true.

$\therefore$ Option (*d*) is correct.

6. Let $f(x) = \sin^{-1} x + 2\cos^{-1} x = \frac{\pi}{2} - \cos^{-1} x + 2\cos^{-1} x$

$\Rightarrow \qquad f(x) = \frac{\pi}{2} + \cos^{-1} x$

As $0 \leq \cos^{-1} x \leq \pi$

$$0 + \frac{\pi}{2} \leq \frac{\pi}{2} + \cos^{-1}x \leq \pi + \frac{\pi}{2} \qquad \Rightarrow \qquad \frac{\pi}{2} \leq f(x) \leq \frac{3\pi}{2}$$

Range of $f(x) = \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

Here, Assertion (*A*) is false but Reason (*R*) is true.

$\therefore$ Option (*d*) is correct.

7. A is true but *R* is false as $\tan^{-1} x$ exists for $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

$\therefore$ Option (*c*) is correct.

8. We have, $f(x) = 2\sin^{-1}x + \frac{3\pi}{2}$

As $-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$ $\Rightarrow$ $-\frac{2\pi}{2} \le 2\sin^{-1}x \le \frac{2\pi}{2}$

$\Rightarrow$ $-\frac{2\pi}{2} + \frac{3\pi}{2} \le 2\sin^{-1}x + \frac{3\pi}{2} \le \frac{2\pi}{2} + \frac{3\pi}{2}$

$\Rightarrow$ $\frac{\pi}{2} \le f(x) \le \frac{5\pi}{2}$

$\Rightarrow$ Range of $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Here, Assertion (*A*) is true but Reason (*R*) is false.

$\therefore$ Option (*c*) is correct.

9. Let $\cos^{-1} x = \theta \Rightarrow x = \cos\theta$

$\Rightarrow$ $\theta \in [0, \pi] \Rightarrow 0 \le \theta \le \pi$

$\Rightarrow$ $0 \le \theta^2 \le \pi^2$

$\Rightarrow$ Maximum value of θ^2 is π^2.

$\Rightarrow$ Maximum value of $(\cos^{-1}x)^2$ is π^2.

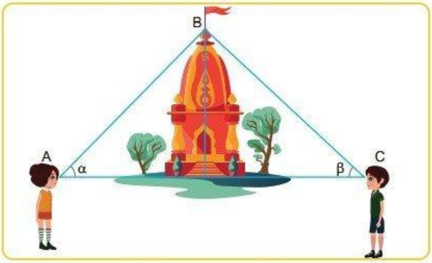
$\Rightarrow$ Here, Assertion (*A*) is true but Reason (*R*) is false.

$\therefore$ Option (*c*) is correct.

Case-based/Data-based Questions

Each of the following questions are of 4 marks.

1. **Read the following passage and answer the following questions.**



Two men on either side of a temple 30 meters high observe its top at the angles of elevation α and β respectively. (as shown in the figure above). The distance between the two men is $40\sqrt{3}$ metres and the distance between the first person A and the temple is $30\sqrt{3}$ metres.

[CBSE Question Bank]

(*i*) **Find $\angle CAB = \alpha$ in terms of $\sin^{-1}$.**

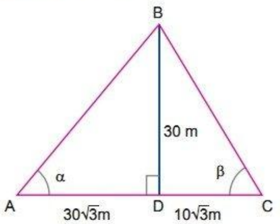
(*ii*) **Find $\angle CAB = \alpha$ in terms of $\cos^{-1}$.**

(*iii*) (*a*) **Find $\angle BCA = \beta$ in terms of $\tan^{-1}$.**

OR

(*iii*) (*b*) **Find the domain and range of $\cos^{-1} x$.**

Sol. We have,



(*i*) Now in ΔABD (right angled)

$$\tan\alpha = \frac{BD}{AD} = \frac{30}{30\sqrt{3}} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \tan\alpha = \frac{1}{\sqrt{3}} = \tan 30° \quad \Rightarrow \quad \alpha = 30°$$

$$\Rightarrow \quad \sin\alpha = \sin 30° = \frac{1}{2}$$

$$\Rightarrow \quad \alpha = \sin^{-1}\left(\frac{1}{2}\right)$$

(*ii*) We have from (*i*) $\alpha = 30°$ $\quad \Rightarrow \quad \cos\alpha = \cos 30° = \frac{\sqrt{3}}{2}$

$$\Rightarrow \quad \alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

(*iii*) (*a*) In right ΔBCD, we have

$$\tan\beta = \frac{BD}{DC} \quad \Rightarrow \quad \tan\beta = \frac{30}{10\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \quad \beta = \tan^{-1}(\sqrt{3})$$

OR

(*iii*) (*b*) Let $\cos^{-1} x = y \quad \Rightarrow \quad x = \cos y$

$-1 \le \cos y \le 1 \Rightarrow -1 \le x \le 1 \quad \Rightarrow \quad$ Domain = $[-1, 1]$

$0 \le y \le \pi \quad \Rightarrow \quad$ Range = $[0, \pi]$.

2. Read the following passage and answer the following questions.

In a school project Manish was asked to construct a triangle *ABC* in which two angles *B* and *C* are given by $\tan^{-1}\left(\frac{1}{2}\right)$ and $\tan^{-1}\left(\frac{1}{3}\right)$ respectively.

(*i*) **Find the value of $\sin B$.**

(*ii*) **Find the value of $\cos C$.**

(*iii*) (*a*) **Find the value of $B + C$.**

OR

(*iii*) (*b*) **Find the value of $\cos (B + C)$**

Sol. We have, $\tan^{-1}\left(\frac{1}{2}\right) = B \quad \Rightarrow \quad \frac{1}{2} = \tan B$

$$\tan^{-1}\left(\frac{1}{3}\right) = C \quad \Rightarrow \quad \frac{1}{3} = \tan C$$

(i) $\therefore \sin B = \frac{1}{\sqrt{1^2+2^2}} = \frac{1}{\sqrt{5}}$

(ii) $\cos C = \frac{3}{\sqrt{1^2+3^2}} = \frac{3}{\sqrt{10}}$

(iii) (a) $\because \tan B = \frac{1}{2}, \tan C = \frac{1}{3}$

$$\therefore \tan(B+C) = \frac{\tan B + \tan C}{1 - \tan B \tan C}$$

$$= \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}} = \frac{\frac{5}{6}}{\frac{6-1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$$\Rightarrow \quad B + C = \tan^{-1}(1) = \frac{\pi}{4}$$

OR

(iii) (b) $\because$ From (i) and (ii) we have

$$\sin B = \frac{1}{\sqrt{5}}, \cos C = \frac{3}{\sqrt{10}}$$

$$\cos B = \sqrt{1-\sin^2 B} = \sqrt{1-\frac{1}{5}} = \frac{2}{\sqrt{5}}$$

$$\sin C = \sqrt{1-\cos^2 C} = \sqrt{1-\frac{9}{10}} = \frac{1}{\sqrt{10}}$$

$$\therefore \quad \cos(B+C) = \cos B \cos C - \sin B \sin C$$

$$= \frac{2}{\sqrt{5}}\times\frac{3}{\sqrt{10}} - \frac{1}{\sqrt{5}}\times\frac{1}{\sqrt{10}} = \frac{6}{\sqrt{50}} - \frac{1}{\sqrt{50}}$$

$$= \frac{5}{\sqrt{50}} = \sqrt{\frac{25}{50}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

CONCEPTUAL QUESTIONS

1. Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$. *[CBSE (F) 2014]*

Sol. We have, $\cos^{-1}\left(\frac{1}{2}\right) = \cos^{-1}\left(\cos\frac{\pi}{3}\right) = \frac{\pi}{3}$ $\quad\left[\because \frac{\pi}{3} \in [0, \pi]\right]$

Also, $\sin^{-1}\left(\frac{1}{2}\right) = \sin^{-1}\left(\sin\frac{\pi}{6}\right) = \frac{\pi}{6}$ $\quad\left[\because \frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$$\therefore \quad \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + 2\left(\frac{\pi}{6}\right) = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

[**Note:** Principal value branches of $\sin^{-1} x$ and $\cos^{-1} x$ are $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$ respectively.]

2. Write the principal value of $\tan^{-1} 1 + \cos^{-1}\left(-\frac{1}{2}\right)$. *[CBSE Delhi 2013]*

Sol. $\tan^{-1} 1 + \cos^{-1}\left(-\frac{1}{2}\right) = \tan^{-1}\left(\tan\frac{\pi}{4}\right) + \cos^{-1}\left(\cos\left(\pi - \frac{\pi}{3}\right)\right)$

$$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) + \cos^{-1}\left(\cos\frac{2\pi}{3}\right)$$

$$= \frac{\pi}{4} + \frac{2\pi}{3} \qquad \left[\because \frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \text{ and } \frac{2\pi}{3} \in [0, \pi]\right]$$

$$= \frac{3\pi + 8\pi}{12} = \frac{11\pi}{12}$$

3. **Find the value of $\tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})$.** [CBSE 2018]

Sol.

$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$

$\tan^{-1}\sqrt{3} - (\pi - \cot^{-1}\sqrt{3})$

$\tan^{-1}\sqrt{3} - \pi + \cot^{-1}\sqrt{3}$

$\tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3} - \pi$

$\frac{\pi}{2} - \pi$ As we know that $\tan^{-1}x + \cot^{-1}x = \pi/2$

$\frac{\pi - 2\pi}{2} = -\frac{\pi}{2}$ Ans.

[*Topper's Answer* 2018]

4. **Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$.** [CBSE 2020 (65/5/1)]

Sol. $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right] = -\sin^{-1}\left(2\pi + \frac{\pi}{8}\right)$ ½

$= -\frac{\pi}{8}$ ½

[*CBSE Marking Scheme 2020 (65/5/1)*]

5. **Write the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.** [CBSE Delhi 2011; (AI) 2009]

Sol. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi - \frac{5\pi}{6}\right)\right)$

$= \cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \frac{5\pi}{6}$ $\left[\because \frac{5\pi}{6} \in [0, \pi]\right]$

6. **Write the principal value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$.** [CBSE (F) 2013]

Sol. $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \tan^{-1}\left(\tan\left(\pi + \frac{\pi}{6}\right)\right)$

$= \tan^{-1}\left(\tan\frac{\pi}{6}\right) = \frac{\pi}{6}$ $\left[\because \frac{\pi}{6} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$

7. **Find the value of $\sin^{-1}\left(\sin\frac{4\pi}{5}\right)$.** [CBSE (AI) 2010]

Sol. We are given $\sin^{-1}\left(\sin\frac{4\pi}{5}\right) = \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{5}\right)\right) = \sin^{-1}\left(\sin\frac{\pi}{5}\right) = \frac{\pi}{5}$ $\left[\because \frac{\pi}{5} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

8. **Find the value of $\sin^{-1}\left(\cos\left(\frac{43\pi}{5}\right)\right)$.** [NCERT Exemplar]

Sol. $\sin^{-1}\left(\cos\left(8\pi + \frac{3\pi}{5}\right)\right) = \sin^{-1}\left(\cos\frac{3\pi}{5}\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)\right)$

$= \sin^{-1}\left(\sin\left(-\frac{\pi}{10}\right)\right) = -\frac{\pi}{10}$ $\left[\because -\frac{\pi}{10} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

9. **Find the principal value of $\cos^{-1}[\cos(-680°)]$.** [NCERT Exemplar]

Sol. $\cos^{-1}[\cos(-680°)] = \cos^{-1}[\cos(680°)]$ $[\because \cos(-\theta) = \cos\theta]$

$= \cos^{-1}[\cos(720° - 40°)]$

$= \cos^{-1}[\cos(2 \times 360° - 40°)] = \cos^{-1}(\cos 40°)$

$= 40°$ or $\frac{2\pi}{9}$ $\left[\because 40° = \frac{2\pi}{9} \in [0, \pi]\right]$

Very Short Answer Questions

1. **Find the value of $\sin^{-1}\left[\sin\frac{13\pi}{7}\right]$** [*CBSE Sample Paper 2023*]

Sol. $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right]=\sin^{-1}\left[\sin\left(2\pi-\frac{\pi}{7}\right)\right]$ 1

$=\sin^{-1}\left[\sin\left(-\frac{\pi}{7}\right)\right]=-\frac{\pi}{7}$ 1

[*CBSE Marking Scheme Sample Paper 2023*]

2. **Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$.** [*CBSE (AI) 2013*]

Sol. $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]=\tan^{-1}\left(2\sin\left(2\times\frac{\pi}{6}\right)\right)$ $\left[\because \cos^{-1}\frac{\sqrt{3}}{2}=\frac{\pi}{6}\right]$

$=\tan^{-1}\left(2\sin\frac{\pi}{3}\right)=\tan^{-1}\left(2\times\frac{\sqrt{3}}{2}\right)=\tan^{-1}(\sqrt{3})=\frac{\pi}{3}$

3. **What is the principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?** [*CBSE (AI) 2011*]

Sol. $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\left(\pi-\frac{\pi}{3}\right)\right)$ $\left[\because \frac{2\pi}{3}\notin\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right]$

$=\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{\pi}{3}\right)=\frac{2\pi}{3}+\frac{\pi}{3}$

$=\frac{3\pi}{3}=\pi$ $\left[\because \sin^{-1}(\sin x)=x \text{ if } x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right] \text{ and } \cos^{-1}(\cos x)=x \text{ if } x\in[0,\pi]\right]$

4. **Find the value of $\sin^{-1}\left(\cos\left(\frac{43\pi}{5}\right)\right)$.** [*NCERT Exemplar*]

Sol. $\sin^{-1}\left(\cos\left(8\pi+\frac{3\pi}{5}\right)\right)=\sin^{-1}\left(\cos\frac{3\pi}{5}\right)=\sin^{-1}\left(\sin\left(\frac{\pi}{2}-\frac{3\pi}{5}\right)\right)$

$=\sin^{-1}\left(\sin\left(-\frac{\pi}{10}\right)\right)=-\frac{\pi}{10}$ $\left[\because -\frac{\pi}{10}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\right]$

5. **Evaluate: $\sin^{-1}\left[\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$** [*NCERT Exemplar*]

Sol. We have,

$\sin^{-1}\left[\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]=\sin^{-1}\left[\cos\left(\sin^{-1}\left(\sin\frac{\pi}{3}\right)\right)\right]$

$=\sin^{-1}\left[\cos\frac{\pi}{3}\right]=\sin^{-1}\left[\sin\left(\frac{\pi}{2}-\frac{\pi}{3}\right)\right]$

$=\sin^{-1}\left[\sin\frac{\pi}{6}\right]=\frac{\pi}{6}$ $\left[\because \frac{\pi}{6}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\right]$

6. **Evaluate : $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\cos^{-1}(0)$** [*CBSE 2023 (65/3/2)*]

Sol. We have, $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\cos^{-1}(0)$

$$= 3\sin^{-1}\left(\sin\frac{\pi}{4}\right) + 2\cos^{-1}\left(\cos\frac{\pi}{6}\right) + \cos^{-1}\left(\frac{\pi}{2}\right)$$

$$= 3 \times \frac{\pi}{4} + 2 \times \frac{\pi}{6} + \frac{\pi}{2} = \frac{3\pi}{4} + \frac{\pi}{3} + \frac{\pi}{2}$$

$$= \frac{9\pi + 4\pi + 6\pi}{12} = \frac{19\pi}{12}$$

7. Evaluate: $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}(\cos\pi) + \tan^{-1}(1)$. *[CBSE 2023 (65/2/1)]*

Sol. We have, $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}(\cos\pi) + \tan^{-1}(1)$

$$= \sin^{-1}\left\{\sin\left(\pi - \frac{\pi}{4}\right)\right\} + \pi + \tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$= \sin^{-1}\left(\sin\frac{\pi}{4}\right) + \pi + \frac{\pi}{4} = \frac{\pi}{4} + \pi + \frac{\pi}{4} = \frac{\pi + 4\pi + \pi}{4}$$

$$= \frac{6\pi}{4} = \frac{3\pi}{2}$$

8. Evaluate : $\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$ *[CBSE 2023 (65/4/1)]*

Sol. We have, $\cos^{-1}\left[\cos\left(\frac{-7\pi}{3}\right)\right] = \cos^{-1}\left[\cos\left(-2\pi - \frac{\pi}{3}\right)\right]$

$$= \cos^{-1}\left(\cos\frac{\pi}{3}\right) = \frac{\pi}{3}$$

9. Evaluate: $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}(1)$ *[CBSE 2023 (65/1/1)]*

Sol. We have, $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}(1)$

$$= \sin^{-1}\left\{\sin\left(\pi - \frac{\pi}{4}\right)\right\} + \cos^{-1}\left\{\left(\cos\frac{3\pi}{4}\right)\right\} + \tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$= \sin^{-1}\left(\sin\frac{\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$= \frac{\pi}{4} + \frac{3\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$$

10. Find the domain of $y = \sin^{-1}(x^2 - 4)$. *[CBSE 2023 (65/4/1)]*

Sol. Given, $y = \sin^{-1}(x^2 - 4)$

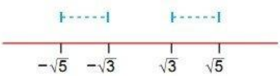
$\Rightarrow \quad -1 \le x^2 - 4 \le 1$

$\Rightarrow \quad 3 \le x^2 \le 5$

$\Rightarrow \quad x^2 \ge 3$ and $x^2 \le 5$

$\Rightarrow \quad x \le -\sqrt{3}, x \ge \sqrt{3}$ and $-\sqrt{5} \le x \le \sqrt{5}$

$\Rightarrow \quad x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$



11. Write the domain and range (principle value branch) of the following functions:

$f(x) = \tan^{-1} x$ *[CBSE 2023 (65/5/1)]*

Sol. $\because \quad \tan : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \longrightarrow \mathbb{R}$ is one-one and onto.

So its inverse exists and is given by $\tan^{-1} : \mathbb{R} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Domain = $\mathbb{R}$, range = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

12. Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range. *[CBSE 2023 (65/2/1)]*

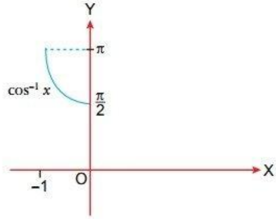
Sol. Given function,

$$f(x) = \cos^{-1}x,\ x \in [-1, 0]$$

When $x = -1 \Rightarrow f(-1) = \cos^{-1}(-1) = \cos^{-1}(\cos \pi) = \pi$

When $x = 0 \Rightarrow f(0) = \cos^{-1}(0) = \cos^{-1}\left(\cos \frac{\pi}{2}\right) = \frac{\pi}{2}$

$\therefore$ Graph of $f(x) = \cos^{-1}x$ shown alongside.



Its range is $\left[\frac{\pi}{2}, \pi\right]$.

$\therefore$ Range of $\cos^{-1}x$ is $\left[\frac{\pi}{2}, \pi\right]$.

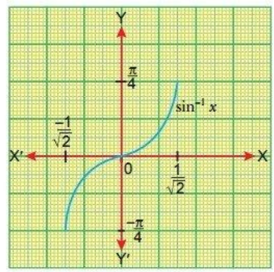
13. Draw the graph of $f(x) = \sin^{-1}x,\ x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of $f(x)$. *[CBSE 2023 (65/3/2)]*

Sol. Given function, $f(x) = \sin^{-1} x,\ x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

$$\Rightarrow \quad f\left(-\frac{1}{\sqrt{2}}\right) = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \sin^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right) = -\frac{\pi}{4}$$

and, $$f\left(\frac{1}{\sqrt{2}}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \sin^{-1}\left(\sin\frac{\pi}{4}\right) = \frac{\pi}{4}$$

Graph of $f(x) = \sin^{-1}x,\ x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$.



Range of $f(x) = \sin^{-1}x,\ x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ is $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$.

14. Express $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right), -\frac{3\pi}{2} < x < \frac{\pi}{2}$ in the simplest forms.

[CBSE Sample Paper 2021, CBSE 2020 (65/3/1)]

Sol. $$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right) = \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2} - x\right)}{1 - \cos\left(\frac{\pi}{2} - x\right)}\right]$$ 1

$$= \tan^{-1}\left[\cot\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$= = \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \frac{\pi}{4} + \frac{x}{2}\right)\right] = \frac{\pi}{4} + \frac{x}{2}$$ 1

[CBSE Marking Scheme 2020 (65/3/1)]

Detailed solution:

$$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right) = \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2} - x\right)}{1 - \cos\left(\frac{\pi}{2} - x\right)}\right]$$

$$= \tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4} - \frac{x}{2}\right) \times \cos\left(\frac{\pi}{4} - \frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}\right]$$

$$= \tan^{-1}\left[\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right] = \tan^{-1}\left[\tan\left\{\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right] = \frac{\pi}{4}+\frac{x}{2}$$

15. Prove that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x, \frac{1}{\sqrt{2}} \le x \le 1$. [CBSE 2020 (65/1/1)]

Sol.

let $y = \sin^{-1}(2x\sqrt{1-x^2})$

let $x = \cos\theta$

$\Rightarrow \theta = \cos^{-1}x \quad 0 \le \theta \le \pi/4$

$\Rightarrow y = \sin^{-1}(2\cos\theta\sqrt{1-\sin^2\theta})$

$= \sin^{-1}\sin 2\theta$

$= 2\theta \quad [0 \le \theta \le \pi/4 \Rightarrow 0 \le 2\theta \le \pi/2]$

$= 2\cos^{-1}x$

Hence Proved

[*Topper's Answer* 2020]

16. Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$, $|x| > 1$ in simplest form. [CBSE (F) 2013]

Sol. $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$

Let $x = \sec\theta \Rightarrow \theta = \sec^{-1}x$

Now, $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \cot^{-1}\left(\frac{1}{\sqrt{\sec^2\theta-1}}\right)$

$$= \cot^{-1}\left(\frac{1}{\tan\theta}\right) = \cot^{-1}(\cot\theta) = \theta = \sec^{-1}x$$

17. Express $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$, where $-\frac{\pi}{4} < x < \frac{\pi}{4}$, in the simplest form.

Sol. We have, $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$

$$= \sin^{-1}\left(\frac{\sin x}{\sqrt{2}}+\frac{\cos x}{\sqrt{2}}\right)$$

$$= \sin^{-1}\left(\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4}\right)$$

$$= \sin^{-1}\left(\sin\left(x+\frac{\pi}{4}\right)\right)$$

$$= \left(x+\frac{\pi}{4}\right)$$

$$\left[\begin{aligned} &\because -\frac{\pi}{4} < x < \frac{\pi}{4} \\ &\Rightarrow -\frac{\pi}{4}+\frac{\pi}{4} < x+\frac{\pi}{4} < \frac{\pi}{4}+\frac{\pi}{4} \\ &\Rightarrow 0 < \left(x+\frac{\pi}{4}\right) < \frac{\pi}{2} \\ &\Rightarrow \left(x+\frac{\pi}{4}\right) \in \left(0, \frac{\pi}{2}\right) \subseteq \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] \end{aligned}\right]$$

18. **Prove that : $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$, $x \in \left[\frac{-1}{2}, \frac{1}{2}\right]$** *[CBSE 2018]*

Sol.

Ans 5.

$3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$

R.H.S. $\sin^{-1}(3x - 4x^3)$

Put $x = \sin\theta$

$\sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$\sin^{-1}(\sin 3\theta)$ — $\frac{1}{2}$

3θ

$3\sin^{-1}x$ — $\frac{1}{2}$

RHS = LHS

Hence Proved

$\frac{-1}{2} \le x \le \frac{1}{2}$

$\frac{-1}{2} \le \sin\theta \le \frac{1}{2}$

$\sin^{-1}\left(\frac{-1}{2}\right) \le \theta \le \sin^{-1}\left(\frac{1}{2}\right)$

$-\frac{\pi}{3} \le \theta \le \frac{\pi}{3}$

$\left[\frac{-\pi}{3}, \frac{\pi}{3}\right] \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

[Topper's Answer 2018]

19. **Solve for x: $\sin^{-1}4x + \sin^{-1}3x = -\frac{\pi}{2}$** *[CBSE 2020 (65/3/1)]*

Sol. $\sin^{-1}4x + \sin^{-1}3x = -\frac{\pi}{2}$

$\Rightarrow \quad \sin^{-1}(4x) = -\frac{\pi}{2} - \sin^{-1}(3x)$

$\Rightarrow \quad 4x = -\sin\left(\frac{\pi}{2} + \sin^{-1}3x\right)$

$= -\cos(\sin^{-1}3x)$ **1**

$\Rightarrow \quad -4x = \sqrt{1 - 9x^2}$ **½**

$\Rightarrow \quad 16x^2 = 1 - 9x^2$

$\Rightarrow \quad 25x^2 = 1$

$\Rightarrow \quad x^2 = \frac{1}{25} \quad \Rightarrow \quad x = \pm\frac{1}{5}$

As $\quad \sin^{-1}4x + \sin^{-1}3x < 0, \; x \neq \frac{1}{5}$ **½**

So, $\quad x = -\frac{1}{5}$

[CBSE Marking Scheme 2020 (65/3/1)]

Short Answer Questions

1. **Prove that: $\sin^{-1}\left(\frac{8}{17}\right) + \cos^{-1}\left(\frac{4}{5}\right) = \cot^{-1}\frac{36}{77}$** *[CBSE 2019 (65/4/2)]*

Sol. Let $\sin^{-1}\left(\frac{8}{17}\right) = \alpha \quad \Rightarrow \quad \sin\alpha = \frac{8}{17}$

$\Rightarrow \quad \cos\alpha = \sqrt{1 - \left(\frac{8}{17}\right)^2} = \frac{15}{17}$

$\cos^{-1}\left(\frac{4}{5}\right) = \beta \quad \Rightarrow \quad \cos\beta = \frac{4}{5}$

$\Rightarrow \quad \sin\beta = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}$

$\Rightarrow \quad \cot\alpha = \frac{15}{8}, \cot\beta = \frac{4}{3}$

Now, $\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$

$$= \frac{\frac{15}{8}\times\frac{4}{3} - 1}{\frac{15}{8}+\frac{4}{3}} = \frac{\frac{60}{24}-1}{\frac{45+32}{24}} = \frac{60-24}{45+32} = \frac{36}{77}$$

$\therefore \quad \alpha+\beta = \cot^{-1}\left(\frac{36}{77}\right)$

$\Rightarrow \quad \sin^{-1}\left(\frac{8}{17}\right) + \cos^{-1}\left(\frac{4}{5}\right) = \cot^{-1}\left(\frac{36}{77}\right)$ Hence proved.

2. Find the value of $\sin\left(\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right)$. *[CBSE 2019 (65/2/2)]*

Sol. Let $\cos^{-1}\left(\frac{4}{5}\right) = \alpha \Rightarrow \cos\alpha = \frac{4}{5}, \sin\alpha = \frac{3}{5}$

and $\tan^{-1}\left(\frac{2}{3}\right) = \beta \Rightarrow \tan\beta = \frac{2}{3}, \sin\beta = \frac{2}{\sqrt{13}}, \cos\beta = \frac{3}{\sqrt{13}}$

$\therefore \quad \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$$= \frac{3}{5}\times\frac{3}{\sqrt{13}} + \frac{4}{5}\times\frac{2}{\sqrt{13}} = \frac{9}{5\sqrt{13}} + \frac{8}{5\sqrt{13}} = \frac{17}{5\sqrt{13}}$$

3. Prove that: $\sin^{-1}\left(\frac{63}{65}\right) = \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$ *[CBSE (F) 2012]*

Sol. Let $\sin^{-1}\left(\frac{5}{13}\right) = \alpha, \cos^{-1}\left(\frac{3}{5}\right) = \beta \quad \Rightarrow \quad \sin\alpha = \frac{5}{13}, \cos\beta = \frac{3}{5}$

$\Rightarrow \quad \cos\alpha = \sqrt{1-\left(\frac{5}{13}\right)^2}, \ \sin\beta = \sqrt{1-\left(\frac{3}{5}\right)^2} \quad \Rightarrow \quad \cos\alpha = \frac{12}{13}, \ \sin\beta = \frac{4}{5}$

Now, $\sin(\alpha+\beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$

$$= \frac{5}{13}.\frac{3}{5} + \frac{12}{13}.\frac{4}{5} = \frac{15}{65} + \frac{48}{65} = \frac{63}{65} \Rightarrow \alpha+\beta = \sin^{-1}\left(\frac{63}{65}\right)$$

Putting the value of α and β, we get

$$\sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5} = \sin^{-1}\left(\frac{63}{65}\right)$$

4. Prove that: $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$ *[CBSE (AI) 2012]*

Sol. Let $\cos^{-1}\frac{4}{5} = x, \cos^{-1}\frac{12}{13} = y \quad [x, y \in [0, \pi]]$

$\Rightarrow \quad \cos x = \frac{4}{5}, \cos y = \frac{12}{13}$

$\therefore \quad \sin x = \sqrt{1-\left(\frac{4}{5}\right)^2}, \sin y = \sqrt{1-\left(\frac{12}{13}\right)^2} \quad [\because x, y \in [0,\pi] \Rightarrow \sin x \text{ and } \sin y \text{ are } +\text{ve}]$

$\Rightarrow \quad \sin x = \frac{3}{5}, \sin y = \frac{5}{13}$

Now, $\cos(x+y) = \cos x . \cos y - \sin x . \sin y$

$$= \frac{4}{5}\times\frac{12}{13} - \frac{3}{5}\times\frac{5}{13} \quad \Rightarrow \quad \cos(x+y) = \frac{33}{65}$$

$\Rightarrow \quad x + y = \cos^{-1}\left(\frac{33}{65}\right) \quad \left[\because \frac{33}{65} \in [-1, 1]\right]$

Putting the value of x and y, we get

$$\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\left(\frac{33}{65}\right) = \text{RHS}$$

5. Solve for x: *[CBSE Panchkula 2015, CBSE 2020 (65/5/1)]*

$$\sin^{-1}(1-x) - 2\sin^{-1}(x) = \frac{\pi}{2}$$

Sol.

$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

$\sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x$

$(1-x) = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right)$

$(1-x) = \cos(2\sin^{-1}x)$

$\sin^{-1}x = \theta$

$x = \sin\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\cos 2\theta = 1 - 2x^2$

$\Rightarrow$ $1 - x = \cos 2\theta$

$1 - x = 1 - 2x^2$ $\Rightarrow$ $2x^2 - x = 0$

$x(2x - 1) = 0$

$x = 0$ or $x = \frac{1}{2}$

put $x = \frac{1}{2}$ in equation

$\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{2}$

$= \frac{\pi}{6} - 2\times\frac{\pi}{6}$

$\neq \frac{\pi}{2}$

So $x \neq \frac{1}{2}$

So $\boxed{x = 0}$

[Topper's Answer 2020]

6. Solve: $\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$ *[CBSE Delhi 2017; (AI) 2013; (F) 2014; (South) 2016]*

Sol. Given $\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$

$\Rightarrow \cos(\tan^{-1}x) = \cos\left(\frac{\pi}{2} - \cot^{-1}\frac{3}{4}\right) \Rightarrow \tan^{-1}x = \frac{\pi}{2} - \cot^{-1}\frac{3}{4}$ $\left[\because \sin\theta = \cos\left(\frac{\pi}{2} - \theta\right) \text{ and } \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$

$\Rightarrow \frac{\pi}{2} - \cot^{-1}x = \frac{\pi}{2} - \cot^{-1}\frac{3}{4} \Rightarrow \cot^{-1}x = \cot^{-1}\frac{3}{4}$

$\Rightarrow x = \frac{3}{4}$

7. **Show that:** $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$ *[CBSE (AI) 2013]*

Sol. Let $\sin^{-1}\frac{3}{4} = \theta \quad \Rightarrow \quad \sin\theta = \frac{3}{4} \qquad \left[\text{As } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$

$$\Rightarrow \quad \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{3}{4} \qquad \left[\because \sin 2x = \frac{2\tan x}{1+\tan^2 x}\right]$$

$$\Rightarrow \quad 3 + 3\tan^2\frac{\theta}{2} = 8\tan\frac{\theta}{2} \quad \Rightarrow \quad 3\tan^2\frac{\theta}{2} - 8\tan\frac{\theta}{2} + 3 = 0$$

$$\Rightarrow \quad \tan\frac{\theta}{2} = \frac{8 \pm \sqrt{64-36}}{6} \quad \Rightarrow \quad \tan\frac{\theta}{2} = \frac{8 \pm \sqrt{28}}{6}$$

$$\Rightarrow \quad \tan\frac{\theta}{2} = \frac{8 \pm 2\sqrt{7}}{6} \quad \Rightarrow \quad \tan\frac{\theta}{2} = \frac{4 \pm \sqrt{7}}{3}$$

$$\Rightarrow \quad \tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3} \qquad \left[\because \theta = \sin^{-1}\frac{3}{4}\right]$$

8. **Prove the following:** $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$ *[CBSE (AI) 2010]*

Sol. LHS $= \cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]$

Let $\cot^{-1}x = \theta \Rightarrow x = \cot\theta$

$$= \cos[\tan^{-1}\{\sin\theta\}] = \cos\left[\tan^{-1}\left\{\frac{1}{\operatorname{cosec}\theta}\right\}\right]$$

$$= \cos\left[\tan^{-1}\frac{1}{\sqrt{1+\cot^2\theta}}\right] = \cos\left[\tan^{-1}\frac{1}{\sqrt{1+x^2}}\right]$$

Let $\tan^{-1}\frac{1}{\sqrt{1+x^2}} = \alpha$

$$\Rightarrow \quad \frac{1}{\sqrt{1+x^2}} = \tan\alpha \quad \Rightarrow \quad \frac{1}{1+x^2} = \tan^2\alpha$$

$$\Rightarrow \quad \frac{1}{1+x^2} = \frac{\sin^2\alpha}{\cos^2\alpha} \quad \Rightarrow \quad \frac{1}{1+x^2} + 1 = \frac{\sin^2\alpha}{\cos^2\alpha} + 1$$

$$\Rightarrow \quad \frac{2+x^2}{1+x^2} = \frac{1}{\cos^2\alpha} \quad \Rightarrow \quad \cos\alpha = \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$$

$$\Rightarrow \quad \alpha = \cos^{-1}\left(\sqrt{\frac{1+x^2}{2+x^2}}\right) \Rightarrow \quad \cos\alpha = \cos\left(\cos^{-1}\sqrt{\frac{1+x^2}{2+x^2}}\right) = \sqrt{\frac{1+x^2}{2+x^2}} = \text{RHS}$$

9. **Prove that:** $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \frac{\pi}{4} - \frac{x}{2}, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ *[CBSE Delhi 2012, 2018]*

Sol. LHS $= \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

$$= \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}\right] = \tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$$

$$= \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)}{\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right] = \tan^{-1}\left(\frac{\pi}{4}-\frac{x}{2}\right) = \frac{\pi}{4} - \frac{x}{2} = \text{RHS}$$

10. Prove that: $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$ *[CBSE Delhi 2017; (AI) 2008; (F) 2010]*

Sol. LHS $= \tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)$

$= \tan\left(\frac{\pi}{4}+x\right)+\tan\left(\frac{\pi}{4}-x\right)$, where $x=\frac{1}{2}\cos^{-1}\frac{a}{b}$

$= \frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}.\tan x}+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}.\tan x}=\frac{1+\tan x}{1-\tan x}+\frac{1-\tan x}{1+\tan x}$

$= \frac{(1+\tan x)^2+(1-\tan x)^2}{1-\tan^2 x}=\frac{1+\tan^2 x+2\tan x+1+\tan^2 x-2\tan x}{1-\tan^2 x}=\frac{2(1+\tan^2 x)}{1-\tan^2 x}$

$= \frac{2}{\cos 2x}=\frac{2}{\cos 2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}=\frac{2}{\cos\left(\cos^{-1}\frac{a}{b}\right)}$ $\left[\begin{array}{l}\because \cos(\cos^{-1}x)=x \text{ if } x\in[-1,1]\\ \text{Here } \frac{a}{b}\in[-1,1]\end{array}\right]$

$= \frac{2}{\frac{a}{b}}=\frac{2b}{a}=$ RHS

11. Prove that: $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ -\frac{1}{\sqrt{2}}\le x\le 1$ *[CBSE (AI) 2011, 2014]*

Sol. LHS $= \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\times\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right)$ [Rationalize]

$= \tan^{-1}\left(\frac{2-2\sqrt{1-x^2}}{1+x-1+x}\right)=\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x}\right)$

Put $x=\sin\theta \Rightarrow \theta=\sin^{-1}x$

$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)=\tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}\right)$

$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)=\frac{\theta}{2}=\frac{1}{2}\sin^{-1}x=\frac{1}{2}\left(\frac{\pi}{2}-\cos^{-1}x\right)$

$= \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x=$ RHS

$\left[\begin{array}{ll}\because & -\frac{1}{\sqrt{2}}\le x\le 1\\ \Rightarrow & \sin\left(-\frac{\pi}{4}\right)\le\sin\theta\le\sin\frac{\pi}{2}\\ \Rightarrow & -\frac{\pi}{4}\le\theta\le\frac{\pi}{2}\\ \Rightarrow & -\frac{\pi}{8}\le\frac{\theta}{2}\le\frac{\pi}{4}\\ \Rightarrow & \frac{\theta}{2}\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\\ \because & \cos^{-1}x+\sin^{-1}x=\frac{\pi}{2}\end{array}\right]$

12. Prove that: $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0,\frac{\pi}{4}\right)$

[CBSE Delhi 2011, 2014; (AI) 2009; (F) 2016]

Sol. LHS $= \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right),\ x\in\left(0,\frac{\pi}{4}\right)$ (Given)

$= \cot^{-1}\left(\frac{\sqrt{(\cos x/2+\sin x/2)^2}+\sqrt{(\cos x/2-\sin x/2)^2}}{\sqrt{(\cos x/2+\sin x/2)^2}-\sqrt{(\cos x/2-\sin x/2)^2}}\right)$

$= \cot^{-1}\left\{\frac{\left|\cos\frac{x}{2}+\sin\frac{x}{2}\right|+\left|\cos\frac{x}{2}-\sin\frac{x}{2}\right|}{\left|\cos\frac{x}{2}+\sin\frac{x}{2}\right|-\left|\cos\frac{x}{2}-\sin\frac{x}{2}\right|}\right\}$

$$= \cot^{-1}\left(\frac{(\cos x/2 + \sin x/2) + (\cos x/2 - \sin x/2)}{(\cos x/2 + \sin x/2) - (\cos x/2 - \sin x/2)}\right)$$

$$= \cot^{-1}\left(\frac{\cos x/2}{\sin x/2}\right) = \cot^{-1}(\cot x/2) = \frac{x}{2} = \text{RHS}$$

Questions for Practice

Objective Type Questions

1. **Choose and write the correct option in each of the following questions.**

(*i*) Which of the following corresponds to the principal value branch of $\tan^{-1} x$?

(*a*) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (*b*) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (*c*) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$ (*d*) $(0, \pi)$

(*ii*) The value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ corresponding to principal branches is

(*a*) $\frac{3\pi}{4}$ (*b*) $\frac{\pi}{4}$ (*c*) $-\frac{\pi}{4}$ (*d*) $-\frac{3\pi}{4}$

(*iii*) The principal value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is

(*a*) $-\frac{2\pi}{3}$ (*b*) $-\frac{\pi}{3}$ (*c*) $\frac{4\pi}{3}$ (*d*) $\frac{5\pi}{3}$

(*iv*) The value of $\tan(\sin^{-1} x)$ is

(*a*) $\frac{x}{\sqrt{1+x^2}}$ (*b*) $\frac{x}{\sqrt{1-x^2}}$ (*c*) $\frac{\sqrt{1-x^2}}{x}$ (*d*) $\frac{\sqrt{1+x^2}}{x}$

(*v*) Principal value branch of the function $\sin^{-1} x$ is

(*a*) $[0, \pi]$ (*b*) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (*c*) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (*d*) $(0, \pi)$

(*vi*) Domain of $\cot^{-1} x$ is

(*a*) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (*b*) $(-\infty, \infty)$ (*c*) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (*d*) $(0, \pi)$

(*vii*) Which of the following is the principal value branch of $\operatorname{cosec}^{-1} x$?

(*a*) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (*b*) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ (*c*) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (*d*) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

(*viii*) The value of $\tan^{-1}\left(2\sin\frac{\pi}{3}\right)$ is

(*a*) $\frac{\pi}{6}$ (*b*) $\frac{\pi}{3}$ (*c*) $\frac{\pi}{2}$ (*d*) π

Conceptual Questions

2. What is the domain of the function $\sin^{-1} x$? [CBSE (F) 2010]
3. Write the principal value of $\cot^{-1}(-\sqrt{3})$. [CBSE (AI) 2010]
4. If $4\cos^{-1} x + \sin^{-1} x = \pi$, then find the value of x.
5. Evaluate: $\tan(\tan^{-1}(-4))$. [NCERT Exemplar]
6. Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$. [CBSE Delhi 2012]
7. Write the value of $\sin(\cot^{-1} x)$.

Very Short Answer Questions

8. Find the value of $\sin(2\sin^{-1}(0.6))$.

9. Show that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x, -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$.

10. Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ corresponding to principal branches.

11. Write the simplest form of $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right), |x| < a$.

12. Write the principal value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$. [CBSE (AI) 2013, CBSE (Delhi) 2018]

Short Answer Questions

13. Prove that: $\cos\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right) = \frac{6}{5\sqrt{13}}$ [CBSE (AI) 2012]

14. Prove that: $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right) = \frac{\pi}{4} - \frac{x}{2}$, where $\pi < x < \frac{3\pi}{2}$ [CBSE Sample Paper 2016]

15. Prove that: $2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$ [CBSE (East) 2016]

16. Find the value of x, if $\tan\left[\sec^{-1}\left(\frac{1}{x}\right)\right] = \sin(\tan^{-1}2), x > 0$. [CBSE 2019 (65/3/3)]

17. Write the principal values of $\cos^{-1}(\cos(680°))$.

18. Prove that $\sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}5) = 31$.

19. Solve $\cot^{-1}x + \tan^{-1}8 = \frac{\pi}{2}$ for x.

20. Find the value of $2\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}(-1)$.

21. Find the value of $\sin\left(2\tan^{-1}\frac{2}{3}\right) + \cos(\tan^{-1}\sqrt{3})$.

22. Find the simplified form of $\cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right)$, where $x \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$.

23. Write the simplest form of $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right), |x| < a$.

24. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, then prove that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$.

25. If $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$, then prove that $a + b + c = abc$.

Answers

1. (i) (a) (ii) (a) (iii) (b) (iv) (b) (v) (b) (vi) (b) (vii) (d) (viii) (b)

2. $-1 \le x \le 1$ 3. $\frac{5\pi}{6}$ 4. $\frac{\sqrt{3}}{2}$ 5. -4 6. $\frac{2\pi}{3}$ 7. $\frac{1}{\sqrt{1+x^2}}$

8. 0.96 10. $\frac{3\pi}{4}$ 11. $\sin^{-1}\frac{x}{a}$ 12. $-\frac{\pi}{2}$ 16. $\frac{\sqrt{5}}{3}$ 17. 40°

19. 8 20. 0 21. $\frac{37}{26}$ 22. $\tan^{-1}\left(\frac{4}{3}\right) - x$ 23. $\sin^{-1}\left(\frac{x}{a}\right)$