CBSE Class 10 Mathematics Standard

Sample Paper - 09 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. Explain why $(3 \times 5 \times 7) + 7$ is a composite number?

OR

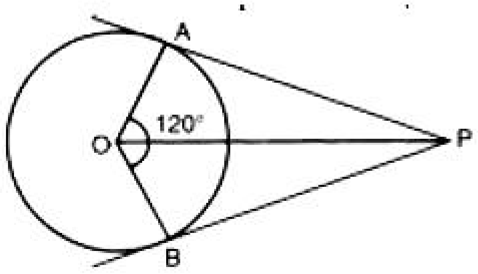
Without performing the long division, find whether $\frac{987}{10500}$ have terminating or non-terminating (repeating) decimal expansion. Give a reason for your answer.

2. What is the nature of roots of the quadratic equation $5x^2 - 2x - 3 = 0$?
3. For what value of k the following pair of the linear equation has a unique solution?

$kx + 3y = 3$

$12x + ky = 6$

4. In the figure, PA and PB are tangents to a circle with centre O. If $\angle AOB = 120°$, then find $\angle OPA$.



5. Find the value of x for which the numbers (5x+2), (4x-1) and (x+2) are in AP.

OR

If in an A.P, a = 15, d = -3 and $a_n = 0$, then find the value of n

6. How many three-digit numbers are divisible by 9?
7. Write the discriminant of the quadratic equation $4x^2 - px + 12 = 0$

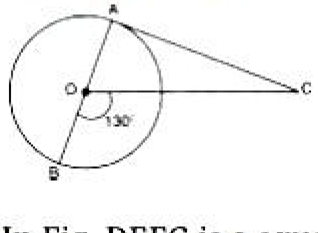
OR

Using quadratic formula solve the quadratic equation : $2x^2 - 11x + 9 = 0$

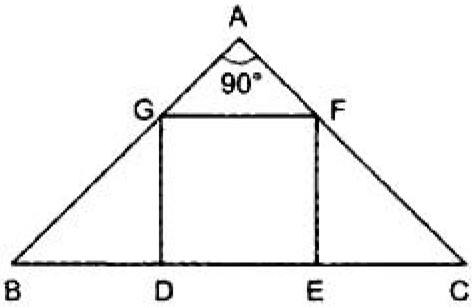
8. A line m is tangent to the circle with radius 5 cm. Find the distance between the centre and the line m.
9. What do you say about the line which is perpendicular to the radius of the circle through the point of contact?

OR

In the given figure, AOB is a diameter of the circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130°$, then find $\angle ACO$



10. In Fig. DEFG is a square and $\angle BAC = 90°$. Prove that $\Delta AGF \sim \Delta DBG$



11. The n^{th} term of an A.P. is (5n - 2). Find its First term.

12. Write the value of $3\cot^2\theta - 3\operatorname{cosec}^2\theta$

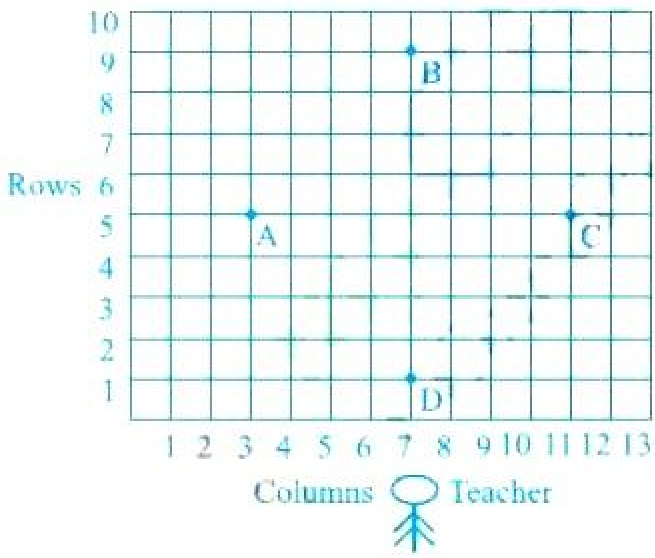
13. If $\cos\theta = \frac{2}{3}$, write the value of $\frac{(\sec\theta - 1)}{(\sec\theta + 1)}$.

14. A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. Find the volume of the cone.

15. If 2x, x + 10, 3x + 2 are in A.P., find the value of x.

16. Two dice are thrown simultaneously. Find the probability of getting a multiple of 2 on one dice and a multiple of 3 on the other.

17. Students of a school are standing/seating in rows and columns in their playground for Yoga practice. A, B, C and D are the positions of four students as shown in the figure.



i. The positions of A, B respectively are:

a. (3, 5), (8, 7)

b. (3, 5), (9, 7)

c. (3, 5), (7, 9)

d. (5, 3), (7, 9)

ii. The distance between A and B is:

a. $\sqrt{32}$ units

b. $\sqrt{23}$ units

c. $\sqrt{42}$ units

d. $\sqrt{35}$ units

iii. It is possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B, C and D then the Position of Jaspal is:

a. (3, 7)

b. (3, 5)

c. (5, 7)

d. (7, 5)

iv. The distance between A and C is

a. 8 units

b. 6 units

c. 4 units

d. $\sqrt{32}$ units

v. The positions of C and B respectively are:

a. (11, 5), (9, 7)

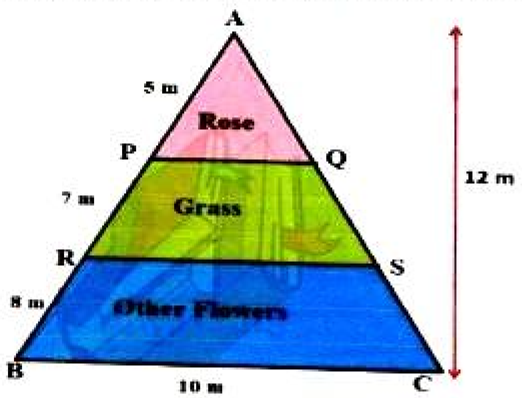
b. (11, 5), (7, 9)

c. (5, 11), (7, 9)

d. (11, 7), (5, 9)

18. Shankar is having a triangular open space in his plot. He divided the land into three parts by drawing boundaries PQ and RS which are parallel to BC.

Other measurements are as shown in the figure.



i. What is the area of this land?

a. $120m^2$

b. $60m^2$

c. $20m^2$

d. $30m^2$

ii. What is the length of PQ?

i. 2.5 m

ii. 5 m

iii. 6 m

iv. 8 m

iii. The length of RS is

a. 5 m

b. 6 m

c. 8 m

d. 4 m

iv. Area of $\triangle APQ$ is

a. $7.5\ m^2$

b. $10\ m^2$

c. $3.75\ m^2$

d. $5\ m^2$

v. What is the area of $\triangle ARS$?

a. $21.6\ m^2$

b. $10\ m^2$

c. $3.75\ m^2$

d. $6\ m^2$

19. Education with vocational training is helpful in making a student self-reliant and to help and serve the society. Keeping this in view, a teacher made the following table giving the frequency distribution of a student undergoing vocational training from the training institute.

Age (in years)	15-19	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-above
Frequency(no. of participants)	62	132	96	37	13	8	6	4	4	3

i. Median class of above data:

a. 20 - 24

b. 20.5 - 24.5

c. 19.5 - 24.5

d. 24.5 - 29.5

ii. Calculate the median.

a. 24.06

b. 30.07

c. 24.77

d. 42.07

iii. The empirical relationship between mean, median, mode:

a. Mode = 3 Median + 2 Mean

b. Mode = 3 Median - 2 Mean

c. Mode = 3 Mean + 2 Median

d. 3 Mode = Median - 2 Mean

iv. If mode = 80 and mean = 110, then find the median.

a. 200

b. 500

c. 190

d. 100

v. The mode is the:

a. middlemost frequent value

b. least frequent value

c. maximum frequent value

d. none of these

20. Mathematics teacher of a school took her 10th standard students to show Red fort. It was a part of their educational trip. The teacher had interest in history as well. She narrated the facts of Red fort to students. Then the teacher said in this monument one can find a combination of solid figures. There are 2 pillars which are cylindrical in shape. Also 2 domes at the corners which are hemispherical. 7 smaller domes at the centre. Flag hoisting ceremony on Independence Day takes place near these domes.

i. How much cloth material will be required to cover 2 big domes each of radius of 2.5 meters? (Take π = 22/7)

a. 75 m^2

b. 78.57 m^2

c. 87.47 m^2

d. 25.8 m^2

ii. Write the formula to find the volume of a cylindrical pillar.

a. $\pi r^2 h$

b. $\pi r l$

c. $\pi r(l + r)$

d. $2\pi r$

iii. Find the lateral surface area of two pillars if the height of the pillar is 7m and the radius of the base is 1.4m.

a. 112.3 cm^2

b. 123.2 m^2

c. 90 m^2

d. 345.2 cm^2

iv. How much is the volume of a hemisphere if the radius of the base is 3.5m?

a. 85.9 m^3

b. 80 m^3

c. 98 m^3

d. 89.83 m^3

v. What is the ratio of the sum of volumes of two hemispheres of radius 1cm each to the volume of a sphere of radius 2 cm?

a. 1:1

b. 1:8

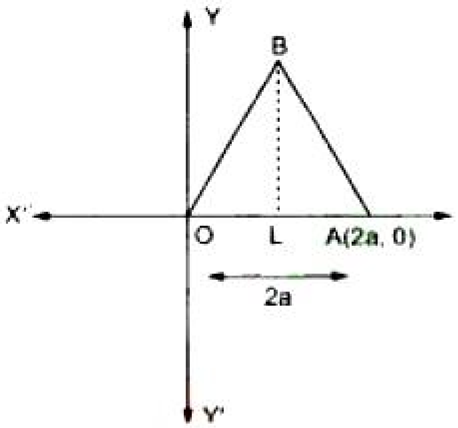
c. 8:1

d. 1:16

Part-B

21. Without actual division, show that a rational number $\frac{66}{180}$ is a nonterminating repeating decimal.

22. Find the coordinates of the vertices of an equilateral triangle of side 2a as shown in the figure.



OR

Find the co-ordinates of the points which divide the line segment joining the points (–4, 0) and (0, 6) in four equal parts.

23. If α and β are the zeroes of the polynomial $2y^2 + 7y + 5$, then find the value of $\alpha + \beta + \alpha\beta$

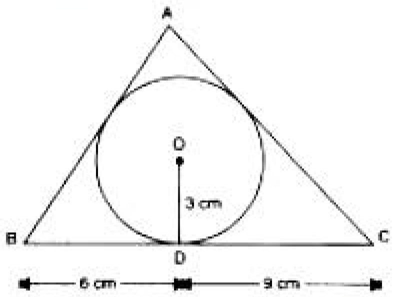
24. Draw a line segment of length 8 cm and divide it in the ratio 3 : 2. Measure the two parts.

25. Prove that $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$ where the angles involved are acute angles for which the expressions are defined.

OR

Find the value of x in $\tan 3x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$.

26. In the figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of $\triangle ABC$ is 54 square centimeter, then find the lengths of sides AB and AC.

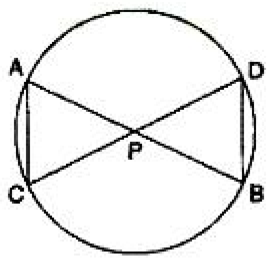


27. Prove that $2 + 5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.
28. If the list price of a toy is reduced by ₹ 2, a person can buy 2 toys more for ₹ 360. Find the original price of the toy.

OR

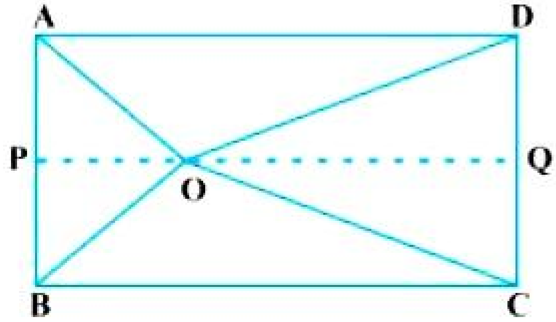
The difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$, then find the two numbers.

29. If the sum and product of the zeroes of the polynomial $ax^2 - 5x + c$, are equal to 10 each, find the value of 'a' and 'c'.
30. In the figure, two chords AB and CD intersect each other at the point P. Prove that:
    i. $\triangle APC \sim \triangle DPB$
    ii. AP.PB = CP.DP

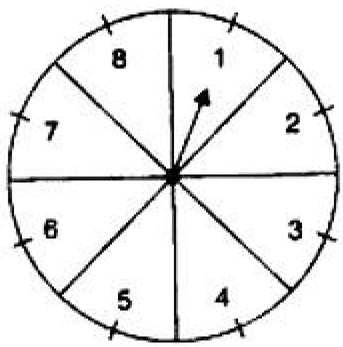


OR

O is any point inside a rectangle ABCD see Fig. Prove that $OB^2 + OD^2 = OA^2 + OC^2$.

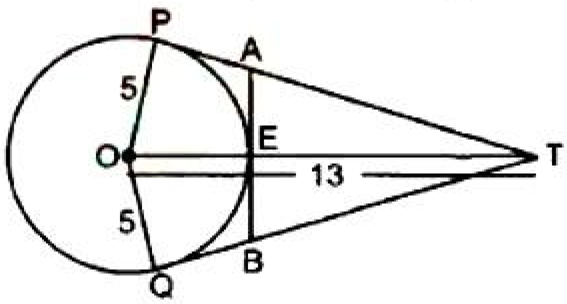


31. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see figure), and these are equally likely outcomes. What is the probability that it will point at



i. 8?
ii. an odd number?
iii. a number greater than 2?
iv. a number less than 9?

32. In the figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects the circle at E. If AB is a tangent to the circle at E, find the length of AB. where TP and TQ are two tangents to the circle.



33. The table below shows the daily expenditure on the food of 25 households in a locality.

Daily expenditure(in ₹)	100-150	150-200	200-250	250-300	300-250

Number of households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

34. The inner perimeter of a racing track is 400 m and the outer perimeter is 488 m. The length of each straight portion is 90 m and the end are semicircles. Find the cost of developing the track at the rate of Rs 12.50/m^2.

35. Find the value of k for which the system of the equation has infinitely many solutions:
$kx + 3y = 2k + 1$
$2(k + 1)x + 9y = 7k + 1$

36. From the top of a building AB, 60 m high, the angles of depression of the top and bottom of a vertical lamp-post CD are observed to be 30° and 60° respectively. Find the horizontal distance between AB and CD.

CBSE Class 10 Mathematics Standard

Sample Paper - 09 (2020-21)

Solution

Part-A

1. We have, $(3 \times 5 \times 7) + 7 = 105 + 7 = 112$

   Prime factors of $112 = 2 \times 2 \times 2 \times 2 \times 7 = 2^4 \times 7$

   So, it is the product of prime factors 2 and 7, i.e. it has factors other than 1 and itself.

   Hence, it is a composite number.

OR

Yes, The denominator can be expressed as $5^3 2^2$ and this is of the type $2^m . 5^n$ so, this is terminating decimal.

$\left(\because \frac{987}{10500} = \frac{47}{500} = \frac{47}{5^3 2^2}\right)$

2. On comparing equation with standard form of equation i.e, $ax^2 + bx + c = 0$, we get

   $a = 5, b = -2, c = -3$

   Now, $D = b^2 - 4ac = (-2)^2 - 4 \times 5 \times (-3)$

   $= 4 + 60 = 64$

   Therefore, D = 64

   We know, For D > 0 , the roots of equation are real and distinct.

   Therefore , $5x^2 - 2x - 3 = 0$ has real and distinct roots.

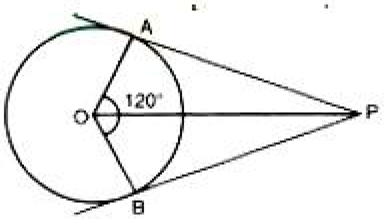
3. Given pair of equations

   $kx + 3y = 3, 12x + ky = 6$

   For unique solutions $\frac{k}{12} \neq \frac{3}{k}$

   $\Rightarrow k^2 \neq 36$

   $\Rightarrow k \neq \pm 6.$

4. 

In $\triangle$POA and $\triangle$POB

OA = OB (Radius of the circle)

AP = BP (tangents from one point)

$\angle \mathrm{OAP} = \angle \mathrm{OBP} = 90^\circ$

Hence $\triangle \mathrm{POA} \cong \triangle \mathrm{POB}$

So $\angle \mathrm{AOP} = \angle \mathrm{BOP}$

$\angle \mathrm{AOP} = \frac{1}{2} \angle \mathrm{AOB} = \frac{120}{2} = 60^\circ$

Now in $\triangle OAP$,

$\angle \mathrm{OPA} + \angle \mathrm{AOP} + \angle \mathrm{OAP} = 180^\circ$

$\angle \mathrm{OPA} + 60^\circ + 90^\circ = 180^\circ$

$\angle \mathrm{OPA} = 180^\circ - 150^\circ = 30^\circ$

$\therefore \angle \mathrm{OPA} = 30^\circ$

5. Since $(5x+2), (4x-1)\ and\ (x+2)$ are in AP, we have

$(4x-1) - (5x+2) = (x+2) - (4x-1)$

$\Rightarrow 4x - 1 - 5x - 2 = x + 2 - 4x + 1$

$\Rightarrow -x - 3 = -3x + 3$

$\Rightarrow$ 2x = 6

$\Rightarrow$ x = 3.

OR

Given that

a = 15, d = -3, a_n = 0

We know that

a_n = a + (n - 1)d

0 = 15 + (n - 1)-3

0 = 15 - 3n + 3

0 = 18 - 3n

3n = 18

n = 6.

6. The three-digit numbers divisible by 9 start from 108,117,126,135,...,999

Here,

$a = 108$

$d = 9$

$a_n = a + (n-1)d$

$\Rightarrow 999 = 108 + (n-1)(9)$

$\Rightarrow 999 = 108 + 9n - 9$

$\Rightarrow 900 = 9n$

$\Rightarrow$ n=100

Thus, 100 three-digit numbers are divisible by 9.

7. The given equation is : $4x^2 - px + 12 = 0$

   Here, a = 4, b = -p and c = 12

   $\therefore D = b^2 - 4ac = (-p)^2 - 4(4)(12)$

   $= p^2 - 192$

   Thus, the discriminant of the given equation is $p^2 - 192$.

OR

$2x^2 - 11x + 9 = 0$

Here, $a = 2, b = -11, c = 9,$

D = $(-11)^2 - 4 \times 2 \times 9 = 121 - 72 = 49 > 0$

$\therefore x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-11) \pm \sqrt{49}}{2 \times 2}$

$= \frac{11+7}{4}, \frac{11-7}{4} = \frac{9}{2}, 1$

8. A line m is tangent to the circle with radius 5 cm.

   $\therefore$ Distance between the centre and the line m= Radius of the circle = 5 cm.
9. The line which is perpendicular to the radius of the circle through the point of contact will be tangent to the circle. A line which intersects a circle at any one point is called the tangent.

OR

In Fig, OA is the radius and AC is the tangent from the external point C.

Therefore, $\angle OAC = 90^\circ$ (Theorem: Radius and tangent are always perpendicular to each other at the point of contact)

$\angle BOC = \angle OAC + \angle ACO$ (Exterior angle property)

$\Rightarrow 130^\circ = 90^\circ + \angle ACO$

$\Rightarrow \angle ACO = 130^\circ - 90^\circ = 40^\circ$

10. In ΔAGF and ΔDBG, we have

$\angle$GAF = $\angle$BDG [Each equal to 90°]

and, $\angle$AGF = $\angle$DBG [Corresponding angles]

$\therefore$ ΔAGF ~ ΔDBG [By AA-criterion of similarity]

11. $T_n = (5n - 2)$ (given)

$\Rightarrow T_1 = (5 \times 1) - 2 = 3$

12. $3\cot^2\theta - 3\operatorname{cosec}^2\theta$

$= 3(\cot^2\theta - \operatorname{cosec}^2\theta)$

$= 3 \times (-1)$

$= -3$

13. Given, $\cos\theta = \frac{2}{3}$

$\Rightarrow \sec\theta = \frac{1}{\cos\theta} = \frac{3}{2}$

Now,

$\frac{\sec\theta-1}{\sec\theta+1} = \frac{\frac{3}{2}-1}{\frac{3}{2}+1} = \frac{\frac{3-2}{2}}{\frac{3+2}{2}} = \frac{1}{5}$

14. h = 24cm, r = 6cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(6)^2(24)$

$= \frac{1}{3} \times 36 \times 24\pi = 288\pi cm^3$

15. If 2x, x + 10, 3x + 2 are in A.P., we have to find the value of x.

Since, 2x, x + 10, 3x + 2 are in A.P.

Therefore 2 (x + 10) = 2x + 3x + 2

$\Rightarrow$ 2x + 20 = 5x + 2

$\Rightarrow$ 3x = 18

$\Rightarrow$ x = 6

16. Two dice are thrown simultaneously.We have to find the probability of getting a multiple of 2 on one dice and a multiple of 3 on the other.

Let A be the event of getting a multiple of 2 on one die and a multiple of 3 on the other.

Then, the elementary events favourable to A are:

(2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6), (3, 2), (3, 4), (3, 6), (6, 2), (6, 4).

Favourable number of elementary events = 11

Hence, the required probability $= \frac{11}{36}$

17. i. (c) The positions of the students are A (3, 5), B(7, 9), C(11, 5) and D(7, 1).

ii. (a) To find the distance between them, we use the distance formula.

So,

$AB = \sqrt{(7-3)^2 + (9-5)^2} = \sqrt{(4)^2 + (4)^2} = \sqrt{16+16} = \sqrt{32}$
$BC = \sqrt{(11-7)^2 + (5-9)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$
$\mathrm{CD} = \sqrt{(7-11)^2 + (1-5)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$
And $DA = \sqrt{(3-7)^2 + (5-1)^2} = \sqrt{(-4)^2 + (4)^2} = \sqrt{16+16} = \sqrt{32}$

iii. (d) We see that, AB = BC = CD = DA i.e., all sides are equal.

Now, we find the length of both diagonals;

$AC = \sqrt{(11-3)^2 + (5-5)^2} = \sqrt{(8)^2 + 0} = 8$
and $BD = \sqrt{(7-7)^2 + (1-9)^2} = \sqrt{0 + (-8)^2} = 8$

Here, AC = BD

Since AB = BC = CD = DA and AC = BD, so we can say that ABCD is a square.

Thus, it is possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B, C and D.

As we also know that diagonals of a square bisect each other, so, let P be a position of Jaspal in which he is equidistant from each of the four students A, B, C and D.

Coordinates of P = Midpoint of AC

$= \left(\frac{3+11}{2}, \frac{5+5}{2}\right) = \left(\frac{14}{2}, \frac{10}{2}\right) = (7, 5)$

Hence, the required position of Jaspal is (7, 5).

iv. (a) 8 units

v. (b) (11, 5), (7, 9)

18. i. (b) 60 m^2

ii. (a) 2.5 m

iii. (b) 6 m

iv. (c) 3.75 m^2

v. (a) 21.6 m^2

19. i. (c) 19.5 - 24.5

ii. (a) 24.06

iii. (b) Mode = 3 Median - 2 Mean

iv. (d) 100

v. (c) maximum frequent value

20. i. (b) Cloth material required = 2 $\times$ S A of hemispherical dome

$= 2 \times 2\pi r^2$

$= 2 \times 2 \times \frac{22}{7} \times (2.5)^2 m^2$

= 78.57 m^2

ii. (a) Volume of a cylindrical pillar = $\pi r^2 h$

iii. (b) Lateral surface area $= 2 \times 2\pi rh$

$= 4 \times \frac{22}{7} \times 1.4 \times 7m^2$

= 123.2 m^2

iv. (d) Volume of the hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\frac{22}{7}(3.5)^3 m^3$

= 89.83 m^3

v. (b) Sum of volumes of two hemispheres of radius 1cm each $= 2 \times \frac{2}{3}\pi 1^3$

Volume of the sphere of radius 2cm $= \frac{4}{3}\pi 2^3$

So, the required ratio is $\frac{2 \times \frac{2}{3}\pi 1^3}{\frac{4}{3}\pi 2^3} = 1 : 8$

Part-B

21. Given number is $\frac{66}{180}$ and HCF (66, 180) = 6.

$\therefore \quad \frac{66}{180} = \frac{66 \div 6}{180 \div 6} = \frac{11}{30}$

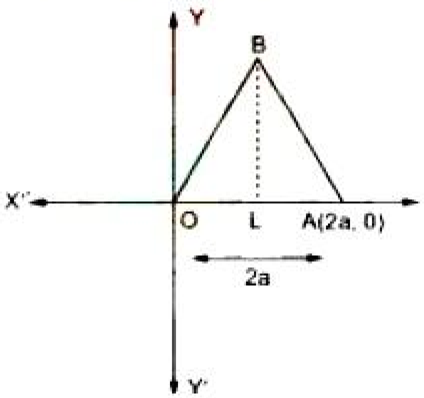
Now, $30 = (2 \times 3 \times 5)$ and none of 2, 3, 5 is a factor of 11.

$\therefore \quad \frac{11}{30}$ is in its simplest form.

Also, $30 = (2 \times 3 \times 5) \neq (2^m \times 5^n)$.

$\therefore \quad \frac{11}{30}$ and hence $\frac{66}{180}$ is a non-terminating repeating decimal.

22. We have to find the coordinates of the vertices of an equilateral triangle of side 2a as shown in the figure.



Since, OAB is an equilateral triangle of side 2a. Therefore,

OA = AB = OB = 2a

Let BL perpendicular from B on OA.Then

OL = LA = a

In $\triangle$ OLB, we have

$OB^2 = OL^2 + LB^2$

$\Rightarrow (2a)^2 = a^2 + LB^2$

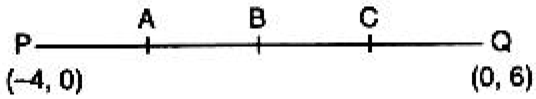
$\Rightarrow LB^2 = 3a^2$

$\Rightarrow \quad LB = \sqrt{3}a$

Clearly, coordinates of O are (0,0) and that of A are (2a, 0). Since, OL = a and LB = $\sqrt{3}$a. So, the coordinates of B are (a, $\sqrt{3}$a)

OR

Let the given points be denoted by P and Q.



Co-ordinate of B(mid-point of PQ) are: $\left(\frac{-4+0}{2}, \frac{0+6}{2}\right)$ i.e. (−2, 3)

Co-ordinates of A (mid-point of PB) are: $\left(\frac{-4-2}{2}, \frac{0+3}{2}\right)$ i.e. $\left(-3, \frac{3}{2}\right)$

Co-ordinates of C (mid-point of BQ) are: $\left(\frac{-2+0}{2}, \frac{6+3}{2}\right)$ i.e. $\left(-1, \frac{9}{2}\right)$.

Hence, the co-ordinates of the required mid-points are $\left(-1, \frac{9}{2}\right)$, (−2, 3) and $\left(-3, \frac{3}{2}\right)$

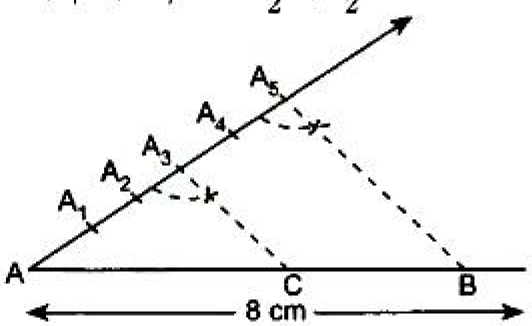
23. Given, polynomial $2y^2 + 7y + 5$,

Sum of zeroes = $\alpha + \beta = -\frac{7}{2}$

Product of zeroes = $\alpha\beta = \frac{5}{2}$

$\alpha + \beta + \alpha\beta = -\frac{7}{2} + \frac{5}{2} = -1$

24.



AC : BC = 3 :2

AC $\simeq$ 4.8 cm

BC $\simeq$ 3.2 cm

25. To prove: $(\sin A + \text{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

taking L.H.S

Using the formula $(a + b)^2 = a^2 + b^2 + 2ab$ to get,

$= (\sin^2 A + \text{cosec}^2 A + 2\sin A\,\text{cosec} A) + (\cos^2 A + \sec^2 A + 2\cos A \sec A)$

Since $\sin\theta = \frac{1}{cosec\theta}$ and $\cos\theta = \frac{1}{\sec\theta}$

$= \left(\sin^2 A + \csc^2 A + 2\sin A \frac{1}{\sin A}\right) + \left(\cos^2 A + \sec^2 A + 2\cos A \frac{1}{\cos A}\right)$

$= \sin^2 A + \text{cosec}^2 A + 2 + \cos^2 A + \sec^2 A + 2$

$= (\sin^2 A + \cos^2 A) + \text{cosec}^2 A + \sec^2 A + 2 + 2$

Using the identities $\sin^2 A + \cos^2 A = 1$, $\sec^2 A = 1 + \tan^2 A$ and $\text{cosec}^2 A = 1 + \cot^2 A$ to get

$= 1 + 1 + \tan^2 A + 1 + \cot^2 A + 2 + 2$

$= 1 + 2 + 2 + 2 + \tan^2 A + \cot^2 A$

$= 7 + \tan^2 A + \cot^2 A$

= R.H.S.

Hence proved

OR

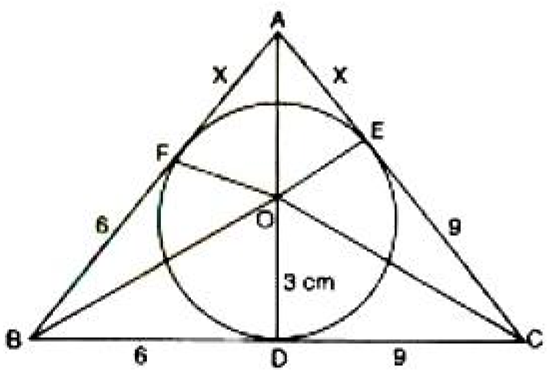
Given,

$\tan 3x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

$\Rightarrow \quad \tan 3x = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2}$

$\Rightarrow \quad \tan 3x = \frac{1}{2} + \frac{1}{2}$

$\Rightarrow \quad \tan 3x = 1 \Rightarrow \tan 3x = \tan 45^\circ \Rightarrow 3x = 45^\circ \Rightarrow x = 15^\circ$

26. 

Let, AF = AE = x

ar $\triangle$ABC = ar$\triangle$AOB + ar$\triangle$BOC + ar$\triangle$AOC ar $\triangle$ABC = $\frac{1}{2}(15)(3)+\frac{1}{2}(6+x)(3)+\frac{1}{2}(9+x)(3)$

$\frac{1}{2}[15+6+x+9+x].3=54$

45 + 3x - 54

x = 3

$\therefore$ AB = 9 cm, AC = 12 cm

and BC = 15 cm.

27. If possible, let us suppose that 2 + 5$\sqrt{3}$ is a rational number

Then, we can write

$2+5\sqrt{3}=\frac{p}{q}$ (Where p and q are coprime)

$\implies 5\sqrt{3}=\frac{p}{q}-2$

$\implies 5\sqrt{3}=\frac{p-2q}{q}$

$\implies \sqrt{3}=\frac{p-2q}{5q}$

$\implies \sqrt{3}=\frac{integer}{integer}$ (Since p and q are integers)

$\implies \sqrt{3}$ is rational number

which is a contradiction to the given fact that $\sqrt{3}$ is irrational.

$\therefore$ 2 + 5$\sqrt{3}$ cannot be rational

Hence, 2 + 5$\sqrt{3}$ is irrational.

28. Let the original list price of the toy be Rs. x.

$\therefore$ Number of toys can be bought for Rs 360 = $\frac{360}{x}$ toys

Now, the Reduced list price of the toy = Rs(x - 2)

$\therefore$ Number of toys can be bought with a new reduced list price for Rs 360 = $\frac{360}{x-2}$ toys

According to the question:

$\frac{360}{x-2}=2+\frac{360}{x}$ (2 extra toys can be bought if price reduces by 2 rupees)

$\therefore \frac{360}{x-2}-\frac{360}{x}$

$\Rightarrow \frac{360x-360(x-2)}{x(x-2)}$

$\Rightarrow \frac{360x-360x+720}{x^2-2x}$

$\Rightarrow$ 720 = 2(x^2 - 2x)

$\Rightarrow x^2-2x=\frac{720}{2}$

$\Rightarrow$ x^2 - 2x = 360

$\Rightarrow$ x^2 - 2x - 360 = 0

$\Rightarrow$ x^2 - 20x + 18x - 360 = 0

$\Rightarrow$ x(x - 20) + 18(x - 20) = 0

$\Rightarrow$ (x - 20)(x + 18) = 0

$\Rightarrow$ x - 20 = 0 [Since, Cost cannot be negative. $\therefore$, x + 18 $\neq$ 0]

$\Rightarrow$ x = 20

Hence, the original list price of the toy is x = Rs 20.

OR

Let the first number be x.

Then, second number = x + 4

According to the question,

$\frac{1}{x} - \frac{1}{x+4} = \frac{4}{21}$

$\frac{x + 4 - x}{x(x+4)} = \frac{4}{21}$

$\frac{4}{x^2 + 4x} = \frac{4}{21}$

$4(x^2 + 4x) = 84$

$\Rightarrow 4x^2 + 16x - 84 = 0$

$\Rightarrow 4(x^2 + 4x - 21) = 0$

$\Rightarrow (x^2 + 4x - 21) = 0$

$\Rightarrow$ (x +7) (x -3) = 0

$\Rightarrow$ x +7 = 0 or x - 3 = 0

$\Rightarrow$ x = -7 or x = 3

Therefore, the two numbers are 3 and 7 or -7 and -3

29. Given, polynomial, $f(x) = ax^2 - 5x + c$

Coefficient of x^2 = a, Coefficient of x = -5, Constant term = c

Sum of zeroes= $-\frac{coeff.of x}{coeff.of x^2} = -\left(\frac{-5}{a}\right)$

Product of zeroes= $\frac{constant\ term}{coeff.of x^2} = \frac{c}{a}$

Let the zeroes of f(x) be α and β, then according to the question

Sum of zeroes, $(\alpha + \beta)$ = Product of zeroes, $(\alpha\beta)$ = 10

Since, $\alpha + \beta = \frac{5}{a}$

So, $10 = \frac{+5}{a}$

$\therefore \quad a = \frac{1}{2}$

$and\ \alpha\beta = \frac{c}{a} = 10$

$\Rightarrow \quad \frac{c}{1/2} = 10$

2c = 10

$\therefore \quad c = 5$

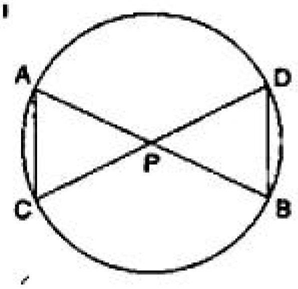
$\therefore$ a = $\frac{1}{2}$ and c = 5

30. Given: In the figure, two-chord AB and CD intersect each other at point P.

To prove:

i. $\triangle$ APC $\sim$ $\triangle$ DPB

ii. AP.PB = CP.DP



Proof:

i. $\triangle$ APC $\sim$ $\triangle$ DPB

$\angle$ APC= $\angle$ DPB ....Vert. opp. $\angle$ s

$\angle$ CDP= $\angle$ BDB ....Angles in the same segment

$\therefore$ $\triangle$APC$\sim$ $\triangle$ DPB ....AA similarity criterion

ii. $\therefore$ $\triangle$ APC$\sim$ $\triangle$DPB ....Proved above in (1)

$\therefore \frac{AP}{DP} = \frac{CP}{BP}$

$\because$ corresponding sides of two similar triangles are proportional

$\Rightarrow$ AP.BP = CP.DP

$\Rightarrow$ AP.PB = CP.DP

OR

Through O, draw PQ || BC so that P lies on AB and Q lies on DC.

Now, PQ || BC

Therefore, PQ $\perp$ AB and PQ$\perp$ DC ($\angle$B = 90° and $\angle$C = 90°)

So, $\angle$BPQ = 90° and $\angle$CQP = 90°

Therefore, BPQC and APQD are both rectangles.

Now, from $\triangle$OPB

$OB^2 = BP^2 + OP^2$ ...(1)

Similarly, from $\triangle$OQD,

$OD^2 = OQ^2 + DQ^2$ ...(2)

From Δ OQC, we have

$OC^2 = OQ^2 + CQ^2$ ...(3)

and from $\triangle$OAP, we have

$OA^2 = AP^2 + OP^2$ ...(4)

Adding (1) and (2),

$OB^2 + OD^2 = BP^2 + OP^2 + OQ^2 + DQ^2$

$= CQ^2 + OP^2 + OQ^2 + AP^2$ (As BP = CQ and DQ = AP)

$= CQ^2 + OQ^2 + OP^2 + AP^2$

$OB^2 + OD^2 = OC^2 + OA^2$ [From (3) and (4)]

Hence proved.

31. Total numbers = 8

$\therefore$ Number of all possible outcomes = 8

i. Number of outcomes favourable to
the event that the arrow will point at 8 = 1

$\therefore$ Probability that the arrow will point at 8

Probability of the event $= \frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{1}{8}$

ii. Number of outcomes favourable to the event that the arrow will point at an odd number (1, 3, 5, 7) = 4

$\therefore$ Probability that the arrow will point at an odd number

Probability of the event $= \frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{4}{8} = \frac{1}{2}$

iii. Number of outcomes favourable to the event that the arrow will point at a number greater than 2.(3, 4, 5, 6, 7, 8) = 6

$\therefore$ Probability that the arrow will point at a number greater than 2

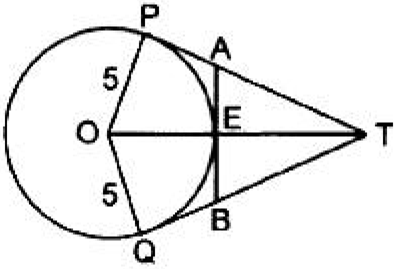
Probability of the event $= \frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{6}{8} = \frac{3}{4}$

iv. Number of outcomes favourable to the event that the arrow will point at a number less than 9.(1, 2, 3, 4, 5, 6, 7, 8) = 8

$\therefore$ Probability that the arrow will point at a number less than 9 = Probability of the event = $\frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{8}{8} = 1$

32. According to the question,

O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects the circle at E.



$\because$ OP $\perp$ TP [Radius from point of contact of the tangent]

$\therefore \angle OPT = 90°$

In right $\triangle$OPT *

$OT^2 = OP^2 + PT^2$

$\Rightarrow (13)^2 = (5)^2 + PT^2 \Rightarrow PT = 12$ cm

Let AP = x cm AE = AP $\Rightarrow$ AE = x cm

and AT = (12 - x)cm

TE = OT - OE = 13 - 5 = 8 cm

$\because$OE $\perp$ AB [Radius from the point of contact]

$\therefore \angle AEO = 90° \Rightarrow \angle AET = 90°$

In right $\triangle$AET,

$AT^2 = AE^2 + ET^2$

$(12 - x)^2 = x^2 + 8^2$

$\Rightarrow 144 + x^2 - 24x = x^2 + 64$

$\Rightarrow \quad 24x = 80 \Rightarrow x = \frac{80}{24} = \frac{10}{3}$ cm

Also BE = AE = $\frac{10}{3}$ cm

$\Rightarrow \quad AB = \frac{10}{3} + \frac{10}{3} = \frac{20}{3}$ cm

33. Take a = 225, h = 50

Daily expenditure (in ₹)	**Number of households (f_i)**	**Class (x_i)**	$d_i = x_i - 225$	$u_i = \frac{x_i - 225}{50}$	f_iu_i
100-150	4	125	–100	–2	–8
150-200	5	175	–50	–1	–5

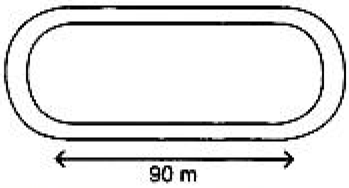
200-250	12	225	0	0	0
250-300	2	275	50	1	2
300-350	2	325	100	2	4
Total	$\sum f_i = 25$				$\sum f_i u_i = -7$

Using the step deviation method,

$\overline{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h = 225 + \left(\frac{-7}{25}\right) \times 50$

= 225 – 14 = ₹ 211

Hence, the mean daily expenditure on food is ₹ 211.

34. 

Perimeter of 2 inner semicircles = (400 - 2 × 90) m = (400 - 180) m = 220 m

Radius of each semicircle = $\frac{220}{2\pi} = \frac{220\times7}{2\times22}$ = 35 m

Perimeter of 2 outer semicircles = (488 - 180) m = 308 m

Radius of each outer semicircle = $\frac{308}{2\pi} = \frac{308\times7}{2\times22}$ = 49 m

Width of the track = outer radius - inner radius = 49 - 35 = 14 m

Area of rectangular tracks = 2 x area of rectangle

$= 2 \times l \times b = 2 \times 90 \times 14$

$= 28 \times 90$ = 2520 m^2

Area of two semicircular rings = area of one circular ring

$= \pi\left(R^2 - r^2\right) = \frac{22}{7}\left(49^2 - 35^2\right) m^2$

$= \frac{22}{7} \times (49 - 35)(49 + 35) m^2$

$= \frac{22}{7} \times 14 \times 84 m^2 = 44 \times 84$

= 3969 m^2

Total area of track = (2520 + 3696)m^2 = 6216 m^2

Cost of developing the track at the rate of Rs 12.50/ m^2 = Rs 6216 × 12.50 = Rs 77,700.

35. Given equations are

kx + 3y = 2k +1

So, kx + 3y - (2k + 1) = 0 .... (i)

And 2(k + 1)x + 9y =7k + 1

So, $2(k + 1)x + 9y - (7k + 1) = 0$ ....(ii)

(i) and (ii) are of the form

$a_1x + b_1y + c_1 = 0$,

$a_2x + b_2y + c_2 = 0$

where,

$a_1 = k$,$b_1 = 3$, $c_1 = -(2k + 1)$,

$a_2 = 2(k +1)$,$b_2 = 9$,$c_2 = -(7k + 1)$

For infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

This holds only when

$$\frac{k}{2(k+1)} = \frac{3}{9} = \frac{-(2k+1)}{-(7k+1)}$$

$$\Rightarrow \frac{k}{2(k+1)} = \frac{1}{3} = \frac{2k+1}{7k+1}$$

Now, the following cases arises:

Case 1:

$$\frac{k}{2(k+1)} = \frac{1}{3}$$

$\Rightarrow 2(k + 1) = 3k$

$\Rightarrow 2k + 2 = 3k$

$\Rightarrow 3k - 2k = 2$

$\Rightarrow k = 2$

Case 2:

$$\frac{1}{3} = \frac{2k+1}{7k+1}$$

$\Rightarrow 7k + 1 = 6k + 3$

$\Rightarrow 7k - 6k= 3 - 1$

$\Rightarrow k = 2$

Case 3:

$$\frac{k}{2(k+1)} = \frac{2k+1}{7k+1}$$

$\Rightarrow k(7k + 1) = 2(2k + 1)(k + 1)$

$\Rightarrow 7k^2 + k = 2(2k^2 + 2k + k + 1)$

$\Rightarrow 7k^2 + k = 2(2k^2 + 3k + 1)$

$\Rightarrow 7k^2 + k = 4k^2 + 6k + 2 = 0$

$\Rightarrow 7k^2 - 4k^2 + k - 6k - 2= 0$

$\Rightarrow 3k^2 - 5k - 2 = 0$

$\Rightarrow 3k^2 - (6k - 1k) - 2 = 0$

$\Rightarrow 3k(k - 2) + 1(k - 2) = 0$

$\Rightarrow (k - 2)(3k + 1) = 0$

k = 2 or $k = \frac{-1}{3}$

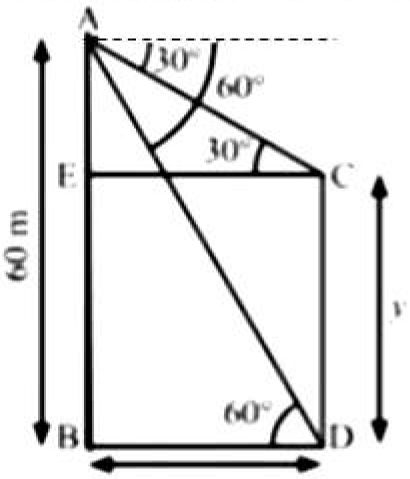
Thus, k = 2, is the common value for which there are infinitely many solutions.

36. Let CD be the tower and AB be the building. Let the height of the tower be y.

Let BD = CE = x and CD = BE = y

$\Rightarrow$ AE = AB - BE = 60 - y

It is given that the angle of depression of the top C and bottom D of the tower, observed from the top of the building be 30^0 and 60^0 respectively.



In right ΔABD,

$\tan 60° = \frac{AB}{BD}$

$\Rightarrow \sqrt{3} = \frac{60}{x}$

$\Rightarrow x = \frac{60}{\sqrt{3}}$

On rationalising we get,

$x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow x = \frac{60\sqrt{3}}{3}$

$\Rightarrow x = 20\sqrt{3}$

$\Rightarrow$ x = 20 × 1.732 = 34.64 m

Thus, the horizontal distance between AB and CD is 34.64 m.