Chapter 2

Algebraic Expression & Inequalities

VARIABLE

An unknown quantity used in any equation may be constant known as variable. Variables are generally denoted by the last English alphabet x, y, z etc.
An equation is a statement of equality of two algebraic expressions, which involve one or more variables.

LINEAR EQUATION

An equation in which the highest power of variables is one, is called a linear equation. These equations are called linear because the graph of such equations on the x–y cartesian plane is a straight line.

Linear Equation in one variable

A linear equation which contains only one variable is called **linear equation in one variable**.
The general form of such equations is $ax + b = c$, where a, b and c are constants and $a \neq 0$.
All the values of x which satisfy this equation are called its solution(s).

NOTE : *An equation satisfied by all values of the variable is called an identity. For example :* $2x + x = 3x$.

Linear equation in two variables

General equation of a linear equation in two variables is $ax + by + c = 0$, where a, $b \neq 0$ and c is a constant, and x and y are the two variables.
The sets of values of x and y satisfying any equation are called its solution(s).
Consider the equation $2x + y = 4$. Now, if we substitute $x = -2$ in the equation, we obtain $2.(-2) + y = 4$ or $-4 + y = 4$ or $y = 8$. Hence $(-2, 8)$ is a solution. If we substitute $x = 3$ in the equation, we obtain $2.3 + y = 4$ or $6 + y = 4$ or $y = -2$

Hence (3, –2) is a solution. The following table lists six possible values for x and the corresponding values for y, i.e. six solutions of the equation.

x	–2	–1	0	1	2	3
y	8	6	4	2	0	–2

Systems of Linear equation

Consistent System : A system (of 2 or 3 or more equations taken together) of linear equations is said to be consistent, if it has at least one solution.
Inconsistent System: A system of simultaneous linear equations is said to be inconsistent, if it has no solutions at all.

e.g. $X + Y = 9;$ $3X + 3Y = 8$

Clearly there are no values of X & Y which simultaneously satisfy the given equations. So the system is inconsistent.

REMEMBER

★ The system $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ has :

- a unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.
- Infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.
- No solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

★ The homogeneous system $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$ has the only solution $x = y = 0$ when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

★ The homogeneous system $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$ has a non-zero solution only when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, and in this case, the system has an infinite number of solutions.

QUADRATIC EQUATION

An equation of the degree two of one variable is called quadratic equation.
General form : $ax^2 + bx + c = 0$...........(1) where a, b and c are all real number and $a \neq 0$.

For Example :

$2x^2 - 5x + 3 = 0;$ $2x^2 - 5 = 0;$ $x^2 + 3x = 0$

If $b^2 - 4ac \geq 0$, then the quadratic equation gives two and only two values (either same or different) of the unknown variable and both these values are called the roots of the equation.

The roots of the quadratic equation (1) can be evaluated using the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad ...(2)$$

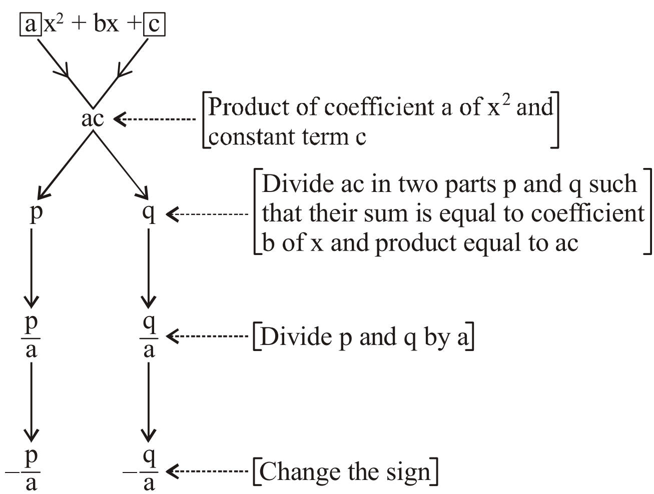
The above formula provides both the roots of the quadratic equation, which are generally denoted by α and β,

say $\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

The expression inside the square root $b^2 - 4ac$ is called the DISCRIMINANT of the quadratic equation and denoted by D. Thus, Discriminant (D) $= b^2 - 4ac$.

☞ *Shortcut Approach*

Shortcut Approach to solve Quadratic equation $ax^2 + bx + c = 0$, if $b^2 - 4ac \geq 0$,



Here $\dfrac{-p}{a}$ and $\dfrac{-q}{a}$ are two roots or solutions of quadratic equation

$ax^2 + bx + c = 0$ i.e. $x = -\dfrac{p}{a}$ or $-\dfrac{q}{a}$.

See Example : Refer ebook Solved Examples/Ch-2

Nature of Roots

The nature of roots of the equation depends upon the nature of its discriminant D.

1. If $D < 0$, then the roots are non-real complex, Such roots are always conjugate to one another. That is, if one root is $p + iq$ then other is $p - iq, q \neq 0$.
2. If $D = 0$, then the roots are real and equal. Each root of the equation becomes $-\frac{b}{2a}$. Equal roots are referred as repeated roots or double roots also.
3. If $D > 0$ then the roots are real and unequal.

Sign of Roots:

Let α, β are real roots of the quadratic equation $ax^2 + bx + c = 0$ that is $D = b^2 - 4ac \geq 0$. Then

1. Both the roots are positive if a and c have the same sign and the sign of b is opposite.
2. Both the roots are negative if a, b and c all have the same sign.
3. The Roots have opposite sign if sign of a and c are opposite.
4. The Roots are equal in magnitude and opposite in sign if $b = 0$ [that is its roots α and $-\alpha$]
5. The roots are reciprocal if $a = c$.

[that is the roots are α and $\frac{1}{\alpha}$]

Symmetric Functions of Roots :

An expression in α, β is called a symmetric function of α, β if the function is not affected by interchanging α and β. If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$ then,

Sum of roots : $\alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

and Product of roots : $\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

Formation of quadratic Equation with Given Roots:

➢ An equation whose roots are α and β can be written as $(x - \alpha)(x - \beta) = 0$ or $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ or $x^2 -$ (sum of the roots) $x +$ product of the roots $= 0$.

- Further if α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$, then
  $ax^2 + bx + c = a(x - \alpha)(x - \beta)$ is an identity.

INEQUATIONS :

A statement or equation which states that one thing is not equal to another, is called an inequation.

Symbols :

'$<$' means "is less than"

'$>$' means "is greater than"

'$\leq$' means "is less than or equal to"

'$\geq$' means "is greater than or equal to"

For example :

(a) $x < 3$ means x is less than 3.

(b) $y \geq 9$ means y is greater than or equal to 9.

Properties

1. Adding the same number to each side of an inequation does not effect the sign of inequality, i.e. if $x > y$ then, $x + a > y + a$.
2. Subtracting the same number to each side of an inequation does not effect the sign of inequaltiy, i.e., if $x < y$ then, $x - a < y - a$.
3. Multiplying each side of an inequality with same positive number does not effect the sign of inequality, i.e., if $x \leq y$ then $ax \leq ay$ (where, $a > 0$).
4. Multiplying each side of an inequality with a negative number reverse the sign of inequality i.e., if $x < y$ then $ax > ay$ (where $a < 0$).
5. Dividing each side of an inequation by a positive number does not effect the sign of inequality, i.e., if $x \leq y$ then $\frac{x}{a} \leq \frac{y}{a}$ (where $a > 0$).
6. Dividing each side of an inequation by a negative number reverses the sign of inequality, i.e., if $x > y$ then $\frac{x}{a} < \frac{y}{a}$ (where $a < 0$).

REMEMBER

★ If $a > b$ and a, b, n are positive, then $a^n > b^n$ but $a^{-n} < b^{-n}$.
For example $5 > 4$; then $5^3 > 4^3$ or $125 > 64$, but

$5^{-3} < 4^{-3}$ or $\frac{1}{125} < \frac{1}{64}$.

★ If $a > b$ and $c > d$, then $(a + c) > (b + d)$.

★ If $a > b > 0$ and $c > d > 0$, then $ac > bd$.

★ If the signs of all the terms of an inequality are changed, then the sign of the inequality will also be reversed.

MODULUS :

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

1. If a is positive real number, x and y be the fixed real numbers, then

(i) $|x - y| < a \Leftrightarrow y - a < x < y + a$

(ii) $|x - y| \leq a \Leftrightarrow y - a \leq x \leq y + a$

(iii) $|x - y| > a \Leftrightarrow x > y + a$ or $x < y - a$

(iv) $|x - y| \geq a \Leftrightarrow x \geq y + a$ or $x \leq y - a$

2. Triangle inequality :

(i) $|x + y| \leq |x| + |y|, \ \forall\, x, y \in R$

(ii) $|x - y| \geq |x| - |y|, \ \forall\, x, y \in R$

Applications Formulation of Equations/ Expressions:

A formula is an equation, which represents the relations between two or more quantities.

For example :

Area of parallelogram (A) is equal to the product of its base (b) and height (h), which is given by

$$A = b \times h$$

or $A = bh$.

Perimeter of triangle (P),

$P = a + b + c$, where a, b and c are length of three sides.

More Applications of Equations :

Problems on Ages can be solved by linear equations in one variable, linear equations in two variables, and quadratic equations.

☞ Shortcut Approach

- If present age of the father is F times the age of his son. T years hence, the father's age become Z times the age of son then present age of his son is given by $\frac{(Z-1)T}{(F-Z)}$

- If T_1 years earlier the age of the father was n times the age of his son, T_2 years hence, the age of the father becomes m times the age of his son then his son's age is given by

$$\text{Son's age} = \frac{T_2(n-1)+T_1(m-1)}{n-m}$$

- Present age of Father : Son = a : b
  After / Before T years = m : n
  Then son's age = $b \times \frac{T(m-n)}{an-bm}$
  and Father's age = $a \times \frac{T(m-n)}{an-bm}$

See Example : Refer ebook Solved Examples/Ch-2

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