Sample Question Paper - 23
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

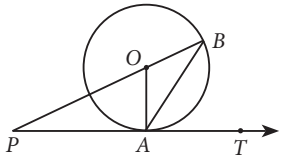
Time Allowed : 2 hours ***Maximum Marks : 40***

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

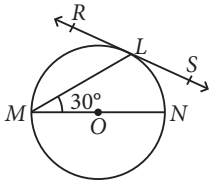
SECTION - A

1. In the given figure, if $\angle BAP = 150°$, then $\angle AOB = k$, then find the value of k.



OR

In the given figure, RS is the tangent to the circle at L and MN is a diameter. If $\angle NML = 30°$, determine $\angle RLM$.



2. Find the volume of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm.

3. $(x + 2)$, x and $(x - 1)$ are the frequencies of the numbers 12, 15 and 20 respectively. If the mean of the distribution is 14.5, then find the value of x.

OR

Mode for the following distribution is 17.5 and x is less than 6. Find x.

Class-interval	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25
Frequency	5	2	3	6	x

4. What is the common difference of an A.P. in which $a_{21} - a_7 = 84$?

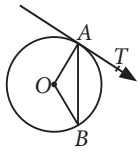
5. Solve : $4x^2 - \sqrt{3}x - 5 = 0$

6. Find the value of mode, using an empirical relation, when it is given that mean and median are 10.5 and 9.6 respectively.

SECTION - B

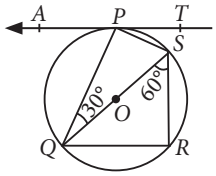
7. If p, q, r are in A.P., then find the value of $p^3 + r^3 - 8q^3$ in terms of pqr.

8. In the given figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If $\angle AOB = 110°$, then find $\angle BAT$.



OR

In the following figure, QS is the diameter and O is the centre of circle. APT is the tangent at P. Find $\angle APQ$.



9. A girl standing on the top of a 7 m high building observes that, the angle of elevation of the top of a tower is 60°and the angle of depression of the foot of the tower is 30°. Find the height of the tower.

10. Which term of the A.P. 4, 7, 10, 13, ..., is 49 ?

SECTION - C

11. From the following data find the mode and median age of 150 residents of a colony who took part in swachch bharat abhiyan :

Age (in yrs.) more than or equal to	0	10	20	30	40	50
Number of residents	50	46	40	20	10	3

OR

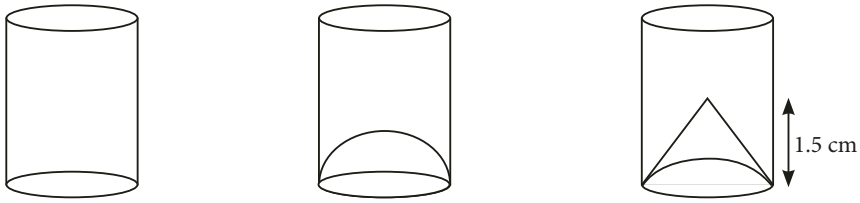
Find the mean and mode of the following frequency distribution :

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	4	7	10	12	8	5

12. Draw a circle of radius 3 cm. From a point P, 7 cm away from its centre, draw two tangents to the circle. Measure the length of each tangent.

Case Study - 1

13. Rohini went to a juice stall with her mother. While drinking juice she observed that shopkeeper has three types of glasses of inner diameter 5 cm to serve customers. The glass height is 10 cm and volume of type (A) glass is 196.43 cm^3. $\left(\text{Use } \pi = \frac{22}{7}\right)$



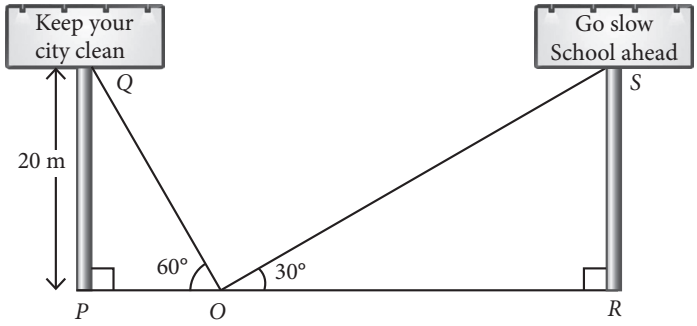
(A) (B) (C)

(i) Find the volume of type (B) glass.

(ii) Which glass has minimum capacity?

Case Study - 2

14. Two hoardings are put on two poles of equal heights standing on either side of the road. From a point between them on the road the angle of elevation of the top of poles are 60° and 30° respectively. Height of the each pole is 20 m.



Based on the above information, answer the following questions. (Take $\sqrt{3} = 1.73$).

(i) Find the length of *PO*.

(ii) Find the width of the road.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

1. $\angle OAP = 90°$ [$\because$ Tangent is perpendicular to the radius through the point of contact]

$\therefore$ $\angle OAB = \angle BAP - \angle OAP = 150° - 90° = 60°$

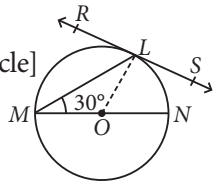
Now, $OA = OB \Rightarrow \angle OAB = \angle OBA = 60°$

$\therefore$ $\angle AOB = 180° - 2(60°) = 60° \Rightarrow k = 60$

OR

Join OL.

$OL \perp RS$.



Also, $OL = OM$ [Radii of the same circle]

$\therefore$ $\angle OML = \angle OLM$

$\Rightarrow$ $\angle OLM = 30°$

$\Rightarrow$ $\angle RLM = 90° - 30° = 60°$

2. The radius of the greatest sphere that can be cut off from the cylinder = 1 cm

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi \text{ cm}^3$

3. We have, mean $= \frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow$ $\frac{12(x+2)+15(x)+20(x-1)}{(x+2)+(x)+(x-1)} = 14.5$

$\Rightarrow$ $2(47x + 4) = 29(3x + 1)$

$\Rightarrow$ $94x + 8 = 87x + 29 \Rightarrow 7x = 21 \Rightarrow x = 3$

OR

Given, mode = 17.5, which lies in the interval 15-20.

$\therefore$ 15-20 is the modal class.

So, $l = 15, f_0 = 3, f_1 = 6, f_2 = x$ *and* $h = 5$

$\therefore$ Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$\Rightarrow$ $17.5 = 15 + \left(\frac{6-3}{2\times 6 - 3 - x}\right) \times 5$

$\Rightarrow$ $2.5 = \left(\frac{3}{9-x}\right) \times 5$

$\Rightarrow$ $22.5 - 2.5x = 15 \Rightarrow 2.5x = 7.5 \Rightarrow x = 3$

Hence, the required value of x is 3.

4. Let a be 1st term and d be the common difference of the A.P.

According to the question, $a_{21} - a_7 = 84$

$\Rightarrow$ $a + (21 - 1)d - (a + (7 - 1)d) = 84$

$\Rightarrow$ $a + 20d - a - 6d = 84 \Rightarrow 14d = 84 \Rightarrow d = 6$

$\therefore$ Common difference is 6.

5. We have, $4x^2 - \sqrt{3}\,x - 5 = 0$

By quadratic formula, we have

$$x = \frac{-(-\sqrt{3}) \pm \sqrt{(-\sqrt{3})^2 - 4\times(-5)(4)}}{2\times 4} = \frac{\sqrt{3} \pm \sqrt{83}}{8}$$

6. We know that the empirical relationship is

Mode = 3 Median – 2 Mean

= 3(9.6) – 2(10.5) [$\because$ Median = 9.6 and Mean = 10.5]

= 28.8 – 21.0 = 7.8

7. Since p, q, r are in A.P.

$\therefore$ $q - p = r - q \Rightarrow 2q = p + r \Rightarrow p + r - 2q = 0$

$\Rightarrow$ $p^3 + r^3 + (-2q)^3 = 3 \times p \times r \times (-2q)$

[$\because$ If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$]

$\Rightarrow$ $p^3 + r^3 - 8q^3 = -6pqr$

8. In ΔOAB, $OA = OB$ [Radii of same circle]

$\therefore$ $\angle OAB = \angle OBA$ [$\because$ Angles opposite to equal sides are equal]

Let $\angle OAB = \angle OBA = x$

In ΔAOB, $\angle AOB + \angle OAB + \angle OBA = 180°$

$\Rightarrow$ $110° + x + x = 180°$ [$\because$ $\angle AOB = 120°$ (Given)]

$\Rightarrow$ $2x = 180° - 110° \Rightarrow x = 35°$

Now, $\angle OAT = \angle OAB + \angle BAT = 90°$

[$\because$ Tangent is perpendicular to radius at point of contact]

$\Rightarrow$ $35° + \angle BAT = 90° \Rightarrow \angle BAT = 55°$

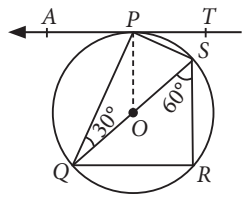
OR

Join OP

Now, $OP \perp AT$

$\Rightarrow$ $\angle APO = 90°$

[$\because$ APT is tangent]



In ΔQOP

$OQ = OP$ [Radii of same circle]

$\therefore$ $\angle OPQ = \angle OQP = 30°$

[$\because$ Angles opposite to equal sides of a triangle are equal]

$\therefore$ $\angle APQ = \angle APO - \angle OPQ = 90° - 30° = 60°$

9. Let $AB = 7$ m be the height of building and CD be the height of tower. Now, $AB = DE = 7$ m

Also, $BD = AE$

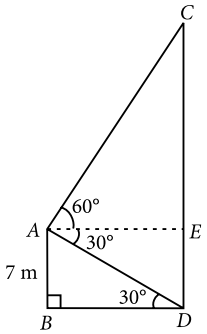
In ΔABD,

$\frac{AB}{BD} = \tan 30°$

$\Rightarrow \frac{7}{BD} = \frac{1}{\sqrt{3}}$

$\Rightarrow BD = 7\sqrt{3}$ m $= AE$

In ΔACE, $\frac{CE}{AE} = \tan 60°$



$\Rightarrow \frac{CE}{7\sqrt{3}} = \sqrt{3} \Rightarrow CE = \left(\sqrt{3} \times 7\sqrt{3}\right)$ m $= 21$ m

$\therefore$ $CD = CE + ED = (21 + 7)$ m $= 28$ m

Thus, the height of the tower is 28 m.

10. The given A.P. is 4, 7, 10, 13, …

Here, $a = 4$, $d = 7 - 4 = 3$

Let the n^{th} term of the A.P. be 49.

Then, $a_n = a + (n - 1)d \Rightarrow 49 = 4 + (n - 1)(3)$

$\Rightarrow 45 = 3(n - 1) \Rightarrow n - 1 = 15 \Rightarrow n = 16$

Hence, 16^{th} term of the A.P. is 49.

11. The frequency distribution table for the given data can be drawn as :

Class	Cumulative frequency (*c.f.*)	Frequency (f_i)
0-10	50	4
10-20	46	6
20-30	40	20
30-40	20	10
40-50	10	7
50-60	3	3
Total		50

Here $\frac{N}{2} = \frac{50}{2} = 25$

$\therefore$ Median class is 20-30.

$\therefore$ Median $= 20 + \left(\frac{25 - 20}{20}\right) \times 10 = 20 + 2.5 = 22.5$

Now, maximum frequency is 20.

$\therefore$ Modal class is 20-30

$\therefore$ Mode $= 20 + \left[\frac{20 - 6}{2(20) - 6 - 10}\right] \times 10$

$= 20 + \left[\frac{14}{24}\right] \times 10 = 25.83$

OR

The frequency distribution table for the given data can be drawn as :

Class	(x_i)	(f_i)	$f_i x_i$
0-10	5	4	20
10-20	15	4	60
20-30	25	7	175
30-40	35	10	350
40-50	45	12	540
50-60	55	8	440
60-70	65	5	325
		$\Sigma f_i = 50$	$\Sigma f_i x_i = 1910$

$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1910}{50} = 38.2$

$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

The maximum frequency is 12.

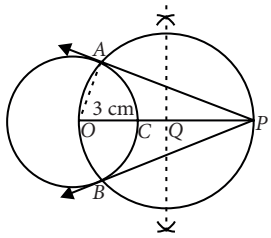
$\therefore$ Modal class is 40-50.

$\therefore$ Mode $= 40 + \left[\frac{12 - 10}{2(12) - 10 - 8}\right] \times 10$

$= 40 + \left[\frac{2}{24 - 18}\right] \times 10 = 40 + \frac{10}{3} = \frac{130}{3} = 43.3$

12. Steps of construction :

Step-I : Draw a circle of radius 3 cm, taking O as centre and OC be its radius.



Step-II : Produce OC to P such that $OP = 7$ cm.

Step-III : Draw perpendicular bisector of OP that meets OP at Q.

Step-IV : Taking Q as centre and radius QP draw a circle which intersect previous circle at points A and B.

Step-V : Join P to A and P to B.

Thus, PA and PB are the required tangents.

Now, join OA.

In ΔAOP, $\angle OAP = 90°$ [Angle in semicircle]

$\therefore$ $AP^2 = OP^2 - OA^2$ [By Pythagoras theorem]

$= 7^2 - 3^2 = 40$

$\Rightarrow$ $AP = 6.32$ cm

Similarly, $BP = 6.32$ cm

Thus, length of each tangent = 6.32 cm

13. Diameter of each glass = 5 cm

$\therefore$ Radius of each glass = 2.5 cm

Height of each glass = 10 cm

(i) Volume of type (B) glass

= Volume of type (A) glass – Volume of hemisphere

$$= 196.43 - \frac{2}{3}\pi r^3 = 196.43 - \frac{2}{3} \times \frac{22}{7} \times 2.5 \times 2.5 \times 2.5$$

$= 196.43 - 32.74 = 163.7 \text{ cm}^3$

(ii) Volume of type (C) glass = Volume of type (A) glass – Volume of cone

$$= 196.43 - \frac{1}{3}\pi r^2 h = 196.43 - \frac{1}{3} \times \frac{22}{7} \times 2.5 \times 2.5 \times 1.5$$

$= 196.43 - 9.82 = 186.61 \text{ cm}^3$

Glass (B) has minimum capacity.

14. (i) In ΔOPQ, we have

$$\tan 60° = \frac{PQ}{PO}$$

$$\Rightarrow \sqrt{3} = \frac{20}{PO} \quad \Rightarrow \quad PO = \frac{20}{\sqrt{3}} \text{ m}$$

(ii) In ΔORS, we have

$$\tan 30° = \frac{RS}{OR} \Rightarrow \frac{1}{\sqrt{3}} = \frac{20}{OR} \Rightarrow OR = 20\sqrt{3} \text{ m}$$

Clearly, width of the road = PR

$$= PO + OR = \left(\frac{20}{\sqrt{3}} + 20\sqrt{3}\right) \text{m}$$

$$= 20\left(\frac{4}{\sqrt{3}}\right) \text{m} = \frac{80}{\sqrt{3}} \text{ m} = 46.24 \text{ m}$$