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घातांक तथा करणी (SURDS AND INDICES)

सामान्य नियम

करणी : माना a एक परिमेय संख्या है तथा n एक धन-पूर्णांक है.

यदि a का n वाँ मूल एक अपरिमेय राशि हो तो $a^{1/n} = \sqrt[n]{a}$ को घात n की करणी कहा जाता है.

उदाहरण : (i) $\sqrt{3} = 3^{\frac{1}{2}}$, एक द्वितीय घात की करणी है.

(ii) $\sqrt[4]{5} = 5^{\frac{1}{4}}$, एक करणी है जिसकी घात 4 है.

घातांक के नियम (Laws of Indices) :

$$(i) a^m \times a^n = a^{m+n}$$

$$(ii) (a^m) \div (a^n) = a^{m-n}$$

$$(iii) (a^m)^n = a^{mn}$$

$$(iv) (ab)^n = (a^n \times b^n)$$

$$(v) \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$(vi) a^0 = 1$$

करणी के नियम (Laws of Surds) :

$$(i) (\sqrt[n]{a})^n = (a^{1/n})^n = a$$

$$(ii) \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$(iii) \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$(iv) (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

$$(v) \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

साधित उदाहरण

प्रश्न 1. सरल कीजिए :

$$(i) (81)^{\frac{3}{4}}$$

$$(ii) \left(\frac{1}{64}\right)^{-\frac{2}{3}}$$

$$(iii) (64)^{-\frac{1}{6}}$$

हल : सरल करने पर :

$$(i) (81)^{\frac{3}{4}} = (3^4)^{\frac{3}{4}} = 3^{(4 \times \frac{3}{4})} = 3^3 = 27.$$

$$(ii) \left(\frac{1}{64}\right)^{-\frac{2}{3}} = (64)^{\frac{2}{3}} = (4^3)^{\frac{2}{3}} = 4^{(3 \times \frac{2}{3})} = 4^2 = 16.$$

$$(iii) (64)^{-\frac{1}{6}} = \left(\frac{1}{64}\right)^{\frac{1}{6}} = \left(\frac{1}{2^6}\right)^{\frac{1}{6}} = \frac{1}{2^{(6 \times \frac{1}{6})}} = \frac{1}{2^1} = \frac{1}{2}.$$

प्रश्न 2. $(.00032)^6 = ?$

$$\begin{aligned} \text{हल : } (.00032)^6 &= \left(\frac{32}{100000}\right)^{\frac{6}{10}} = \left(\frac{2^5}{10^5}\right)^{\frac{3}{5}} = \left\{\left(\frac{2}{10}\right)^5\right\}^{\frac{3}{5}} \\ &= \left(\frac{2}{10}\right)^{(5 \times \frac{3}{5})} = \left(\frac{1}{5}\right)^3 = (.2 \times .2 \times .2) = .008. \end{aligned}$$

प्रश्न 3. $\frac{(625)^{6.25} \times (25)^{2.6}}{(625)^{6.75} \times (5)^{1.2}} = ?$

हल : दिया गया व्यंजक $= \frac{(5^4)^{6.25} \times (5^2)^{2.6}}{(5^4)^{6.75} \times (5)^{1.2}} = \frac{(5)^{(4 \times 6.25)} \times 5^{(2 \times 2.6)}}{(5)^{4 \times 6.75} \times 5^{(1.2)}}$
 $= \frac{5^{25} \times 5^{5.2}}{5^{27} \times 5^{1.2}} = \frac{5^{(25+5.2)}}{5^{(27+1.2)}} = \frac{5^{30.2}}{5^{28.2}} = 5^{(30.2-28.2)}$
 $= 5^2 = 25.$

प्रश्न 4. यदि $(\sqrt{3})^5 \times 9^2 = 3^n \times 3\sqrt{3}$ हो, तो n का मान ज्ञात कीजिए.

हल : $(\sqrt{3})^5 \times (3^2)^2 = 3^n \times 3 \times 3^{\frac{1}{2}} \Rightarrow (3^{1/2})^5 \times 3^4 = 3^n \times 3 \times 3^{\frac{1}{2}}$
 $\Rightarrow 3^{\left(\frac{5}{2}+4\right)} = 3^{\left(n+1+\frac{1}{2}\right)} \Rightarrow 3^{\frac{13}{2}} = 3^{\left(n+\frac{3}{2}\right)} \Rightarrow n + \frac{3}{2} = \frac{13}{2} \Rightarrow n = \left(\frac{13}{2} - \frac{3}{2}\right) = \frac{10}{2} = 5.$
 अतः $n = 5.$

प्रश्न 5. सरल कीजिए : $\left(\frac{x^a}{x^b}\right)^{(a^2+ab+b^2)} \cdot \left(\frac{x^b}{x^c}\right)^{(b^2+bc+c^2)} \cdot \left(\frac{x^c}{x^a}\right)^{(c^2+ca+a^2)}$

हल : दिया गया व्यंजक
 $= x^{(a-b)(a^2+ab+b^2)} \cdot x^{(b-c)(b^2+bc+c^2)} \cdot x^{(c-a)(c^2+ca+a^2)}$
 $= x^{(a^3-b^3)} \cdot x^{(b^3-c^3)} \cdot x^{(c^3-a^3)} = x^{(a^3-b^3+b^3-c^3+c^3-a^3)} = x^0 = 1.$

प्रश्न 6. $\sqrt{2}$ तथा $\sqrt[3]{3}$ में से कौन-सा बड़ा है?

हल : दी गई करणी क्रमशः घात 2 तथा 3 की हैं, जिनका ल.सं. = 6. प्रत्येक को घात 6 की करणी में बदलने पर : $\sqrt{2} = 2^{\frac{1}{2}} = (2^3)^{\frac{1}{6}} = (8)^{\frac{1}{6}}, \quad \sqrt[3]{3} = 3^{\frac{1}{3}} = (3^2)^{\frac{1}{6}} = (9)^{\frac{1}{6}}.$
 स्पष्ट है $(9)^{\frac{1}{6}} > (8)^{\frac{1}{6}}$. अतः $\sqrt[3]{3} > \sqrt{2}.$

प्रश्न 7. $\sqrt{2}, \sqrt[3]{4}, \sqrt[4]{6}$ को आरोही क्रम में लिखिए.

हल : दी गई करणियों की घात क्रमशः 2, 3, 4 हैं जिनका ल.सं. = 12.
 $\sqrt{2} = 2^{\frac{1}{2}} = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}}, \sqrt[3]{4} = 4^{\frac{1}{3}} = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}}$
 तथा $\sqrt[4]{6} = 6^{\frac{1}{4}} = (6^3)^{\frac{1}{12}} = (216)^{\frac{1}{12}}.$
 स्पष्ट है कि $(64)^{\frac{1}{12}} < (216)^{\frac{1}{12}} < (256)^{\frac{1}{12}} \Rightarrow \sqrt{2} < \sqrt[4]{6} < \sqrt[3]{4}.$

प्रश्न 8. यदि $3^{x-y} = 27$ तथा $3^{x+y} = 243$ हो, तो x का मान ज्ञात कीजिए.

हल : $3^{x-y} = 27 = 3^3 \Rightarrow x-y = 3 \dots (i) \quad 3^{x+y} = 243 = 3^5 \Rightarrow x+y = 5 \dots (ii)$
 (i) तथा (ii) को जोड़ने पर, $2x = 8 \Rightarrow x = 4.$
 (ii) में से (i) घटाने पर, $2y = 2 \Rightarrow y = 1.$
 $\therefore x = 4$ तथा $y = 1.$

प्रश्नमाला 9A

निम्नलिखित प्रश्नों में से प्रत्येक में ठीक उत्तर को चिह्नांकित (✓) कीजिए :

1. यदि $\left(\frac{3}{5}\right)^3 \cdot \left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$ हो, तो $x = ?$ (एस०एस०सी० परीक्षा, 2010)
 (a) -2 (b) 2 (c) -1 (d) 1
2. $(2 \times 3)^3 \div (4 \times 9)^2 \times (27 \times 8)^2 = 6^?$ (बैंक पी०ओ० परीक्षा, 2010)
 (a) 5 (b) 6 (c) 3 (d) 8 (e) इनमें से कोई नहीं
3. यदि $8^{x+1} = 64$ हो, तो 3^{2x+1} का मान क्या होगा ?
 (a) 81 (b) 9 (c) 1 (d) 27
4. $(23)^{2.8} \times (23)^{7.2} \times (23)^{3.6} = (23)^?$ (बैंक पी०ओ० परीक्षा, 2009)
 (a) 13.6 (b) 12.6 (c) 12.8 (d) 13.8 (e) इनमें से कोई नहीं
5. $(21)^7 \times (21)^{6.5} = (21)^{12.4}$ (बैंक पी०ओ० परीक्षा, 2009)
 (a) 18.9 (b) 4.4 (c) 6.9 (d) 16.4 (e) इनमें से कोई नहीं
6. $(34)^{56} \times (34)^{-53} = ?$ (बैंक पी०ओ० परीक्षा, 2009)
 (a) 39304 (b) 1156 (c) 170504 (d) 102 (e) इनमें से कोई नहीं
7. $(31)^{31} \times (31)^{-27} = ?$ (बैंक पी०ओ० परीक्षा, 2008)
 (a) $(961)^2$ (b) $(31)^2$ (c) 29791 (d) 4 (e) इनमें से कोई नहीं
8. $(21)^{5.5} \times (21)^? = (21)^{12}$
 (a) 2.18 (b) -17.5 (c) 6.5 (d) 5.5
9. $4^{2.3} \times 2^{2.3} \times 8^{1.7} = ?$ (बैंक पी०ओ० परीक्षा, 2008)
 (a) 64 (b) 512 (c) 16 (d) 4096 (e) इनमें से कोई नहीं
10. $8^{1.3} \times 4^{0.6} \times 16^{0.2} = 2^?$ (बैंक पी०ओ० परीक्षा, 2007)
 (a) 2.1 (b) 3.8 (c) 5.9 (d) 4.7 (e) इनमें से कोई नहीं
11. $2^{0.2} \times 64 \times 8^{1.3} \times 4^{0.2} = 8^?$ (बैंक पी०ओ० परीक्षा, 2007)
 (a) 2.7 (b) 2.5 (c) 3.7 (d) 3.2 (e) इनमें से कोई नहीं
12. $(1000)^{12} \div (10)^{30} = ?$ (बैंक पी०ओ० परीक्षा, 2008)
 (a) $(1000)^2$ (b) 10 (c) $(100)^2$ (d) 100 (e) इनमें से कोई नहीं
13. $\sqrt{(24)^4} + 224 = ? \times (20)^2$ (बैंक पी०ओ० परीक्षा, 2008)
 (a) 20 (b) 4 (c) 2 (d) 16 (e) इनमें से कोई नहीं
14. $(3^?)^? = 19683$ (बैंक पी०ओ० परीक्षा, 2007)
 (a) 6 (b) 9 (c) 4 (d) 8 (e) इनमें से कोई नहीं
15. यदि m तथा n ऐसे धनपूर्णांक हैं कि $m^n = 25$ हो, तो $n^m = ?$ (बी०सी०ए० परीक्षा, 2008)
 (a) 4 (b) 10 (c) 25 (d) 32
16. यदि x तथा y ऐसी धनात्मक वास्तविक संख्यायें हों कि $x^{1/3} = y^{1/4}$ हो, तो :
 (a) $x^3 = y^4$ (b) $x^3 = y$ (c) $x = y^4$ (d) $x^{20} = y^{15}$
 (एस०एस०सी० परीक्षा, 2006)

17. यदि $\sqrt{3^n} = 81$ हो, तो $n = ?$
 (a) 2 (b) 4 (c) 8 (d) 12
18. यदि $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$ हो, तो $x = ?$
 (a) $\frac{3}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{3}{2}$
19. $8^{4.2} \times 64^{2.1} \times 7^{8.4} \times 56^{3.5} = (56)^?$
 (a) 18.2 (b) 9.8 (c) 11.9 (d) 12.6
 (बैंक पी०ओ० परीक्षा, 2005)
 (e) इनमें से कोई नहीं
20. $(16)^{0.16} \times (16)^{0.04} \times (2)^{0.2} = ?$
 (a) 1 (b) 2 (c) 4 (d) 16
 (एस०एस०सी० परीक्षा, 2005)
21. $(8)^{11} \times (4)^{2.7} \times (2)^{3.3} = 2^?$
 (a) 7.1 (b) 14 (c) .5 (d) 9
 (बैंक पी०ओ० परीक्षा, 2009)
 (e) इनमें से कोई नहीं
22. $(9)^{8.6} \times (8)^{3.9} \times (72)^{4.4} \times (9)^{3.9} \times (8)^{8.6} = (72)^?$
 (a) 15.1 (b) 29.4 (c) 20.9 (d) 17.9
 (e) इनमें से कोई नहीं
23. $(1000)^9 \div (10)^{24} = ?$
 (a) 100000 (b) 1000 (c) 10 (d) 100
 (e) इनमें से कोई नहीं
24. $\left[3^{m^2} + (3^m)^2\right]^{1/m} = 81$ हो, तो $m = ?$
 (a) -3 (b) -6 (c) 3 (d) 6
25. यदि $\sqrt{4^n} = 1024$ हो, तो $n = ?$
 (a) 5 (b) 8 (c) 12 (d) 10
 (रेलवे परीक्षा, 2006)
26. $(27)^{2/3} \times (64)^{2/3} = ?$
 (a) $\frac{1}{12}$ (b) $\frac{1}{144}$ (c) $\frac{1}{48}$ (d) 144
27. $(0.6)^4 \times (0.36)^2 \times (0.216) = (0.6)^?$
 (a) 12 (b) 14 (c) 11 (d) 13
 (e) इनमें से कोई नहीं
28. $(0.00032)^{0.6} = ?$
 (a) 0.08 (b) 0.008 (c) 0.8 (d) 8
29. $(64)^{31} \times 8^{43} = 8^?$
 (a) 10.5 (b) 7.4 (c) 1.2 (d) 13.3
 (e) इनमें से कोई नहीं
30. $\frac{3^{n+4} - 6 \times 3^{n+1}}{3^{n+2}} = ?$
 (a) 7 (b) 5 (c) 4 (d) 1
31. $\frac{343 \times 49}{216 \times 16 \times 81} = ?$
 (a) $\frac{7^5}{6^7}$ (b) $\frac{7^5}{6^8}$ (c) $\frac{7^6}{6^7}$ (d) $\frac{7^4}{6^8}$
 (बैंक पी०ओ० परीक्षा, 2010)
 (e) इनमें से कोई नहीं
32. $\frac{36 \times 18}{125 \times 75} = ?$
 (a) $\frac{6^3}{5^4}$ (b) $\frac{6^2}{5^7}$ (c) $\frac{6^3}{5^5}$ (d) $\frac{6^2}{5^5}$
 (बैंक पी०ओ० परीक्षा, 2010)
 (e) इनमें से कोई नहीं

33. $\left\{ \left(\sqrt[5]{x^{-3/5}} \right)^{-5/3} \right\}^5$ का सरलीकृत रूप है : (एस०एस०सी० परीक्षा, 2010)
 (a) x^5 (b) x^{-5} (c) x (d) $\frac{1}{x}$ (e) इनमें से कोई नहीं
34. $(0.1 \times 0.01 \times 0.001 \times 10^7) = ?$ (एस०एस०सी० परीक्षा, 2010)
 (a) 100 (b) $\frac{1}{10}$ (c) $\frac{1}{100}$ (d) 10
35. यदि $3^{x+y} = 81$ तथा $(81)^{x-y} = 3$ हो, तो $x = ?$ (एस०एस०सी० परीक्षा, 2009)
 (a) 42 (b) $\frac{15}{8}$ (c) $\frac{17}{8}$ (d) 39
36. यदि $2^{3.6} \times 4^{3.6} \times 4^{3.6} \times 32^{2.3} = (32)^?$
 (a) 5.9 (b) 9.5 (c) 7.7 (d) 13.1
37. यदि $x = 2 + 2^{2/3} + 2^{1/3}$ हो, तो $x^3 - 6x^2 + 6x = ?$ (रेलवे परीक्षा, 2001)
 (a) 1 (b) 3 (c) 2 (d) इनमें से कोई नहीं
38. यदि $x = 5 - 5^{2/3} - 5^{1/3}$ हो, तो $x^3 - 15x^2 + 60x - 15 = ?$ (रेलवे परीक्षा, 2001)
 (a) 20 (b) 15 (c) 5 (d) -5
39. यदि $2^x = 3^y = 6^z$ हो, तो $\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) = ?$
 (a) 0 (b) 1 (c) $-\frac{1}{2}$ (d) $\frac{3}{2}$
40. यदि $2^x = 4^y = 8^z$ हो, तथा $\left(\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} \right) = \frac{24}{7}$ हो, तो $z = ?$
 (a) $\frac{7}{16}$ (b) $\frac{7}{32}$ (c) $\frac{7}{48}$ (d) $\frac{7}{64}$
41. यदि $a^x = b^y = c^z$ तथा $b^2 = ac$ हो, तो $y = ?$
 (a) $\frac{xz}{x+y}$ (b) $\frac{xz}{2(x-z)}$ (c) $\frac{xz}{2(z-x)}$ (d) $\frac{2xz}{(z+x)}$
42. यदि $2^{x-1} + 2^{x+1} = 320$ हो, तो x का मान है :
 (a) 5 (b) 6 (c) 7 (d) 8
43. यदि $2^{x+4} - 2^{x+2} = 3$ हो, तो x का मान है :
 (a) -2 (b) -1 (c) 0 (d) 2
44. यदि $2^a + 3^b = 17$ हो तथा $2^{a+2} - 3^{b+1} = 5$ हो, तो
 (a) $a = 2, b = 3$ (b) $a = -2, b = 3$
 (c) $a = 2, b = -3$ (d) $a = 3, b = 2$
45. यदि $\left(\frac{a}{b} \right)^{x-1} = \left(\frac{b}{a} \right)^{x-3}$ हो, तो $x = ?$
 (a) -1 (b) 1 (c) 2 (d) 3
46. यदि $x = y^a, y = z^b$ तथा $z = x^c$ हो, तो $abc = ?$ (एम०बी०ए० परीक्षा, 2006)
 (a) 1 (b) 2 (c) 3 (d) 4
47. यदि a, b, c वास्तविक संख्यायें हों, तो $\sqrt{a^{-1}b} \cdot \sqrt{b^{-1}c} \cdot \sqrt{c^{-1}a} = ?$
 (a) abc (b) $\frac{1}{abc}$ (c) $\sqrt{abc}$ (d) 1

48. $\frac{1}{1+a^{n-m}} + \frac{1}{1+a^{m-n}} = ?$
 (a) 0 (b) $\frac{1}{2}$ (c) 1 (d) a^{m+n}
49. यदि $abc = 1$ हो, तो $\left(\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} \right)$ का मान है :
 (a) 0 (b) 1 (c) ab (d) $\frac{1}{ab}$
50. $\left(\frac{x^a}{x^b} \right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c} \right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a} \right)^{\frac{1}{ca}} = ?$
 (a) 1 (b) $\frac{1}{x^{abc}}$ (c) $x^{\frac{1}{(ab+bc+ca)}}$ (d) इनमें से कोई नहीं
51. $(x^{b+c})^{b-c} \cdot (x^{c+a})^{c-a} \cdot (x^{a+b})^{a-b} = ?$
 (a) 0 (b) 1 (c) x (d) इनमें से कोई नहीं
52. $\left(\frac{x^a}{x^b} \right)^{(a+b)} \cdot \left(\frac{x^b}{x^c} \right)^{(b+c)} \cdot \left(\frac{x^c}{x^a} \right)^{(c+a)} = ?$
 (a) 1 (b) 0 (c) x^{a+b+c} (d) x^{abc}
53. $\left(\frac{x^a}{x^b} \right)^{(a+b-c)} \cdot \left(\frac{x^b}{x^c} \right)^{(b+c-a)} \cdot \left(\frac{x^c}{x^a} \right)^{(c+a-b)} = ?$
 (a) 1 (b) x^{abc} (c) x^{a+b+c} (d) $x^{(ab+bc+ca)}$
54. $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} = ?$
 (a) 0 (b) 1 (c) x^{a-b-c} (d) इनमें से कोई नहीं
55. यदि $a + \frac{1}{b} = 1$ तथा $b + \frac{1}{c} = 1$ हो, तो $c + \frac{1}{a} = ?$
 (a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0
56. यदि $a = \frac{\sqrt{3}}{2}$ हो, तो $\sqrt{1+a} + \sqrt{1-a}$ का मान क्या होगा ? (एस०एस०सी० परीक्षा, 2007)
 (a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $(2+\sqrt{3})$ (d) $(2-\sqrt{3})$
57. $\frac{(2^n + 2^{n-1})}{(2^{n+1} - 2^n)}$ का मान है : (रेलवे परीक्षा, 2006)
 (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $2^{\frac{(n-1)}{(n+1)}}$ (d) इनमें से कोई नहीं
58. यदि $2^{2x-1} = \frac{1}{8^{x-3}}$ हो, तो $x = ?$ (जीवन बीमा निगम परीक्षा, 2007)
 (a) -2 (b) -1 (c) 2 (d) 3
59. यदि $a^{2x+2} = 1$, जहाँ a एक धनात्मक वास्तविक संख्या है तथा $a \neq 1$, तब $x = ?$ (एस०एस०सी० परीक्षा, 2007)
 (a) -2 (b) -1 (c) 0 (d) 1

60. यदि $\sqrt{24} = 4.898$ हो, तो $\sqrt{\frac{8}{3}} = ?$ (एस०एस०सी० परीक्षा, 2007)
 (a) 0.544 (b) 1.333 (c) 1.633 (d) 2.666
61. यदि $8^{x+1} = 64$ हो, तो 3^{2x+1} का मान क्या होगा? (रेलवे परीक्षा, 2006)
 (a) 1 (b) 9 (c) 27 (d) 81
62. यदि $1.5x = 0.04y$ हो, तो $\frac{y-x}{y+x}$ का मान क्या होगा? (जीवन बीमा निगम परीक्षा, 2007)
 (a) $\frac{730}{77}$ (b) $\frac{73}{77}$ (c) $\frac{73}{770}$ (d) $\frac{703}{77}$
63. यदि $4^x = \sqrt{2^{3y}}$ हो, तो : (रेलवे परीक्षा, 2006)
 (a) $x = \frac{3}{4}y$ (b) $y = \frac{3}{4}x$ (c) $x = 3y$ (d) $x = \frac{1}{3}y$
64. यदि $x^{-\frac{4}{5}} = 81$ हो, तो $x = ?$
 (a) $\frac{1}{243}$ (b) 243 (c) $\frac{1}{3}$ (d) 3
65. यदि $5^{(x+3)} = 25^{(3x-4)}$ हो, तो x का मान है :
 (a) $\frac{5}{11}$ (b) $\frac{11}{5}$ (c) $\frac{11}{3}$ (d) $\frac{13}{5}$
66. यदि $(64)^{(2x-5)} = 4 \times 8^{(x-4)}$ हो, तो x का मान है :
 (a) 2 (b) 11 (c) $\frac{10}{7}$ (d) $\frac{20}{9}$
67. यदि $\left(\frac{9}{4}\right)^x \cdot \left(\frac{8}{27}\right)^{(x-1)} = \frac{2}{3}$ हो, तो x का मान है :
 (a) 1 (b) 2 (c) 3 (d) 4
68. $(64x^3 + 27a^{-3})^{-\frac{2}{3}} = ?$ (रेलवे परीक्षा, 2006)
 (a) $\frac{9ax}{16}$ (b) $\frac{9}{16ax}$ (c) $\frac{3}{4}x^{-2}a^{-2}$ (d) $\frac{9}{16x^2a^2}$
69. $\sqrt[3]{-8} \times \sqrt[4]{16} = ?$
 (a) -4 (b) -2 (c) 2 (d) 4
70. $\left(\frac{1}{2}\right)^{-\frac{1}{2}} = ?$ (एस०एस०सी० परीक्षा, 2005)
 (a) $\frac{1}{\sqrt{2}}$ (b) $2\sqrt{2}$ (c) $-\sqrt{2}$ (d) $\sqrt{2}$
71. $(\sqrt{8})^{\frac{1}{3}} = ?$
 (a) 2 (b) 4 (c) $\sqrt{2}$ (d) $2\sqrt{2}$
72. $\left(\frac{32}{243}\right)^{-\frac{4}{5}} = ?$
 (a) $\frac{4}{9}$ (b) $\frac{9}{4}$ (c) $\frac{16}{81}$ (d) $\frac{81}{16}$

73. $\left(\frac{-1}{343}\right)^{-\frac{2}{3}} = ?$
 (a) $\frac{-1}{49}$ (b) $\frac{1}{49}$ (c) -49 (d) 49
74. $\left(\frac{1}{216}\right)^{-\frac{2}{3}} + \left(\frac{1}{27}\right)^{-\frac{4}{3}} = ?$
 (a) $\frac{3}{4}$ (b) $\frac{2}{3}$ (c) $\frac{4}{9}$ (d) $\frac{1}{8}$
75. यदि $(27)^{\frac{2}{3}} \times (81)^{-\frac{1}{2}} = 3^n$ हो, तो $n = ?$
 (a) 0 (b) 1 (c) 27 (d) 81
76. $(16)^{1.75} = ?$
 (a) 64 (b) $64\sqrt{2}$ (c) 128 (d) $128\sqrt{2}$
77. यदि $\sqrt{2^n} = 64$ हो, तो $n = ?$ (रेलवे परीक्षा, 2006)
 (a) 2 (b) 4 (c) 6 (d) 12
78. $(256)^{0.16} \times (16)^{0.18} = ?$ (एस०एस०सी० परीक्षा, 2007)
 (a) 4 (b) 16 (c) 64 (d) 256.25
79. $9^{8.6} \times 8^{3.9} \times (72)^{4.4} \times 9^{3.9} \times 8^{8.6} = (72)^?$ (जीवन बीमा निगम परीक्षा, 2005)
 (a) 15.1 (b) 17.9 (c) 20.9 (d) 29.4 (e) इनमें से कोई नहीं
80. $6^{1.2} \times (36)^? \times (30)^{-2.4} \times (25)^{1.3} = (30)^5$ (बैंक पी०ओ० परीक्षा, 2006)
 (a) 0.1 (b) 0.7 (c) 1.4 (d) 2.6 (e) इनमें से कोई नहीं
81. $(0.01024)^{\frac{1}{5}} = ?$
 (a) 0.00004 (b) 0.04 (c) 0.4 (d) 4
82. $(3 + \sqrt{5})$ का वर्गमूल है : (जीवन बीमा निगम परीक्षा, 2007)
 (a) $\left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\right)$ (b) $\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$ (c) $\left(\frac{\sqrt{5}}{2} - \frac{1}{\sqrt{2}}\right)$ (d) $\left(\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}\right)$
83. यदि $x = (7 - 4\sqrt{3})$ हो, तो $\left(x + \frac{1}{x}\right)$ का मान क्या होगा? (रेलवे परीक्षा, 2005)
 (a) $3\sqrt{3}$ (b) $8\sqrt{3}$ (c) $14 + 8\sqrt{3}$ (d) 14
84. यदि $x = (3 + \sqrt{8})$ हो, तो $\left(x^2 + \frac{1}{x^2}\right)$ का मान क्या होगा? (एस०एस०सी० परीक्षा, 2007)
 (a) 38 (b) 36 (c) 34 (d) 30
85. $\sqrt{2}, \sqrt[3]{3}, \sqrt[4]{4}, \sqrt[5]{5}$ में से सबसे बड़ी संख्या कौन-सी है? (एस०एस०सी० परीक्षा, 2005)
 (a) $\sqrt{2}$ (b) $\sqrt[3]{3}$ (c) $\sqrt[4]{4}$ (d) $\sqrt[5]{5}$
86. $\sqrt{3}, \sqrt[3]{4}, \sqrt[4]{5}, \sqrt[5]{6}$ में से सबसे बड़ी संख्या कौन-सी है? (एस०एस०सी० परीक्षा, 2006)
 (a) $\sqrt{3}$ (b) $\sqrt[3]{4}$ (c) $\sqrt[4]{5}$ (d) $\sqrt[5]{6}$
87. $\sqrt[3]{2}, \sqrt{3}, \sqrt[5]{5}, 1.5$ में से सबसे बड़ी संख्या कौन-सी है? (एस०एस०सी० परीक्षा, 2009)
 (a) 1.5 (b) $\sqrt{3}$ (c) $\sqrt[3]{2}$ (d) $\sqrt[5]{5}$

88. $\sqrt{2}$, $\sqrt[3]{4}$, $\sqrt[4]{6}$ को बढ़ते क्रम में लिखने पर : (ए०ए०ओ० परीक्षा, 2006)
 (a) $\sqrt{2} < \sqrt[3]{4} < \sqrt[4]{6}$ (b) $\sqrt[4]{6} < \sqrt{2} < \sqrt[3]{4}$
 (c) $4\sqrt{6} < \sqrt[3]{4} < \sqrt{2}$ (d) $\sqrt{2} < \sqrt[4]{6} < \sqrt[3]{4}$
89. $2^{\frac{1}{3}}$, $3^{\frac{1}{3}}$, $8^{\frac{1}{8}}$ तथा $9^{\frac{1}{9}}$ में से कौन-सी संख्या सबसे बड़ी है ? (ए०ए०ओ० परीक्षा, 2010)
 (a) $2^{\frac{1}{3}}$ (b) $3^{\frac{1}{3}}$ (c) $8^{\frac{1}{8}}$ (d) $9^{\frac{1}{9}}$
90. $\sqrt{8-2\sqrt{15}} = ?$ (एस०एस०सी० परीक्षा, 2007)
 (a) $(\sqrt{5} + \sqrt{3})$ (b) $(5 - \sqrt{3})$ (c) $(\sqrt{5} - \sqrt{3})$ (d) $(3 - \sqrt{5})$
91. $\{(-2)^{(-2)}\}^{(-2)} = ?$ (एस०एस०सी० परीक्षा, 2005)
 (a) -16 (b) -8 (c) 8 (d) 16
92. यदि $\sqrt{3} = 1.732$ हो, तो $\frac{4+3\sqrt{3}}{\sqrt{7+4\sqrt{3}}} = ?$ (एस०एस०सी० परीक्षा, 2005)
 (a) .023 (b) .464 (c) 2.464 (d) 3.023
93. यदि $\sqrt{3} = 1.732$ हो, तो $\frac{(3+\sqrt{6})}{(5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50})} = ?$ (एस०एस०सी० परीक्षा, 2007)
 (a) 4.899 (b) 2.551 (c) 1.414 (d) 1.732
94. $\frac{12}{(3+\sqrt{5}+2\sqrt{2})} = ?$ (एस०एस०सी० परीक्षा, 2010)
 (a) $1-\sqrt{5}+\sqrt{2}+\sqrt{10}$ (b) $1+\sqrt{5}+\sqrt{2}-\sqrt{10}$
 (c) $1+\sqrt{5}-\sqrt{2}+\sqrt{10}$ (d) $1-\sqrt{5}-\sqrt{2}+\sqrt{10}$
95. $\frac{\sqrt{7}}{\sqrt{16+6\sqrt{7}}-\sqrt{16-6\sqrt{7}}} = ?$ (एस०एस०सी० परीक्षा, 2010)
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$
96. यदि $a = \sqrt{7+2\sqrt{12}}$ तथा $b = \sqrt{7-2\sqrt{12}}$ हो, तो $(a^3 + b^3) = ?$ (एस०एस०सी० परीक्षा, 2010)
 (a) 40 (b) 44 (c) 48 (d) 52
97. यदि $x^{1/3} + y^{1/3} = z^{1/3}$ हो, तो $\{(x+y-z)^3 + 27xyz\} = ?$ (एस०एस०सी० परीक्षा, 2007)
 (a) -1 (b) 1 (c) 0 (d) 27
98. 2^{50} का दुगुना क्या होगा ?
 (a) 2^{100} (b) 2^{55} (c) 2^{51} (d) इनमें से कोई नहीं
99. $\left(\frac{5^{-1} \times 7^2}{5^2 \times 7^{-4}}\right)^{7/2} \times \left(\frac{5^{-2} \times 7^3}{5^3 \times 7^{-5}}\right)^{-5} = ?$
 (a) 150 (b) 125 (c) 175 (d) 90

$$100. \left\{ \frac{\frac{m}{x^{m-n}} \times \frac{n}{x^{n-m}}}{\frac{m}{x^{m+n}} \times \frac{n}{x^{n+m}}} \right\}^{m+n} = ?$$

- (a) $\frac{x^m}{x^n}$ (b) $\frac{x^n}{x^m}$ (c) 1 (d) 2

$$101. \text{ यदि } a = \frac{\sqrt{5}+1}{\sqrt{5}-1} \text{ तथा } b = \frac{\sqrt{5}-1}{\sqrt{5}+1} \text{ हो, तो } \frac{(a^2+ab+b^2)}{(a^2-ab+b^2)} = ? \quad (\text{एस०एस०सी० परीक्षा, 2005})$$

- (a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{5}$ (d) $\frac{5}{3}$

$$102. \text{ यदि } x = (3+2\sqrt{2}) \text{ हो, तो } \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) \text{ का मान कितना होगा ?}$$

- (a) 1 (b) 2 (c) $2\sqrt{2}$ (d) $3\sqrt{3}$

$$103. \frac{1}{(1+\sqrt{2})} + \frac{1}{(\sqrt{2}+\sqrt{3})} + \frac{1}{(\sqrt{3}+\sqrt{4})} + \dots + \frac{1}{(\sqrt{99}+\sqrt{100})} = ? \quad (\text{एस०एस०सी० परीक्षा, 2006})$$

- (a) 1 (b) 5 (c) 9 (d) 10

$$104. \frac{1}{(\sqrt{3}+\sqrt{4})} + \frac{1}{(\sqrt{4}+\sqrt{5})} + \frac{1}{(\sqrt{5}+\sqrt{6})} + \frac{1}{(\sqrt{6}+\sqrt{7})} + \frac{1}{(\sqrt{7}+\sqrt{8})} + \frac{1}{(\sqrt{8}+\sqrt{9})} = ?$$

- (a) $\sqrt{3}$ (b) $3\sqrt{3}$ (c) $(3-\sqrt{3})$ (d) $5-\sqrt{3}$

(एस०एस०सी० परीक्षा, 2005)

$$105. \frac{1}{(\sqrt{100}-\sqrt{99})} - \frac{1}{(\sqrt{99}-\sqrt{98})} + \frac{1}{(\sqrt{98}-\sqrt{97})} - \dots - \frac{1}{(\sqrt{3}-\sqrt{2})} + \frac{1}{(\sqrt{2}-\sqrt{1})} = ?$$

- (a) 0 (b) 9 (c) 10 (d) 11

(एस०एस०सी० परीक्षा, 2005)

उत्तरमाला (प्रश्नमाला 9A)

- | | | | | | | | | | |
|----------|----------|----------|----------|----------|---------|---------|---------|---------|----------|
| 1. (c) | 2. (a) | 3. (d) | 4. (a) | 5. (e) | 6. (a) | 7. (a) | 8. (c) | 9. (d) | 10. (c) |
| 11. (e) | 12. (a) | 13. (c) | 14. (e) | 15. (d) | 16. (d) | 17. (c) | 18. (a) | 19. (c) | 20. (b) |
| 21. (e) | 22. (e) | 23. (b) | 24. (d) | 25. (d) | 26. (d) | 27. (c) | 28. (b) | 29. (a) | 30. (a) |
| 31. (a) | 32. (c) | 33. (c) | 34. (d) | 35. (c) | 36. (a) | 37. (c) | 38. (c) | 39. (a) | 40. (c) |
| 41. (d) | 42. (c) | 43. (a) | 44. (d) | 45. (c) | 46. (a) | 47. (d) | 48. (c) | 49. (b) | 50. (a) |
| 51. (b) | 52. (a) | 53. (a) | 54. (b) | 55. (a) | 56. (a) | 57. (b) | 58. (c) | 59. (b) | 60. (c) |
| 61. (c) | 62. (b) | 63. (a) | 64. (a) | 65. (b) | 66. (d) | 67. (d) | 68. (d) | 69. (a) | 70. (d) |
| 71. (c) | 72. (d) | 73. (d) | 74. (c) | 75. (a) | 76. (c) | 77. (d) | 78. (a) | 79. (e) | 80. (b) |
| 81. (c) | 82. (d) | 83. (d) | 84. (c) | 85. (b) | 86. (a) | 87. (b) | 88. (d) | 89. (b) | 90. (c) |
| 91. (d) | 92. (c) | 93. (d) | 94. (b) | 95. (a) | 96. (d) | 97. (c) | 98. (c) | 99. (c) | 100. (c) |
| 101. (b) | 102. (b) | 103. (c) | 104. (c) | 105. (d) | | | | | |

दिये गये प्रश्नों के हल प्रश्नमाला 9A

1. $\left(\frac{3}{5}\right)^{3+(-6)} = \left(\frac{3}{5}\right)^{2x-1} \Rightarrow 2x-1 = -3 \Rightarrow 2x = -2 \Rightarrow x = -1.$
2. $(6)^3 + (36)^2 \times (216)^2 = 6^x$
 $\Rightarrow 6^3 + (6^2)^2 \times (6^3)^2 = 6^x \Rightarrow 6^3 + 6^4 \times 6^6 = 6^x$
 $\Rightarrow 6^3 \times \frac{1}{6^4} \times 6^6 = 6^x \Rightarrow 6^{(3+6-4)} = 6^x \Rightarrow 6^5 = 6^x \Rightarrow x = 5.$
3. $8^{x+1} = 64 = 8^2 \Rightarrow x+1 = 2 \Rightarrow x = 1.$
 $\therefore 3^{2x+1} = 3^{(2 \times 1)+1} = 3^3 = 27.$
4. $(23)^{2 \cdot 8 + 7 \cdot 2 + 3 \cdot 6} = (23)^x \Rightarrow (23)^{13 \cdot 6} = (23)^x \Rightarrow x = 13 \cdot 6.$
5. माना $(21)^x \times (21)^{6 \cdot 5} = (21)^{12 \cdot 4}$. तब $(21)^{x+6 \cdot 5} = (21)^{12 \cdot 4}$
 $\therefore x + 6 \cdot 5 = 12 \cdot 4 \Rightarrow x = (12 \cdot 4 - 6 \cdot 5) = 5 \cdot 9.$
6. $(34)^{56} \times (34)^{-53} = (34)^{56-53} = (34)^3 = 39304.$
7. $(31)^{31} \times (31)^{-27} = (31)^{31+(-27)} = (31)^4 = (961)^2.$
8. माना $(21)^{5 \cdot 5} \times (21)^x = (21)^{12}$. तब $(21)^{5 \cdot 5 + x} = (21)^{12}$.
 $\therefore 5 \cdot 5 + x = 12 \Rightarrow x = (12 - 5 \cdot 5) = 6 \cdot 5.$
9. $4^{2 \cdot 3} \times 2^{2 \cdot 3} \times 8^{1 \cdot 7} = (2^2)^{2 \cdot 3} \times 2^{2 \cdot 3} \times (2^3)^{1 \cdot 7}$
 $= 2^{4 \cdot 6} \times 2^{2 \cdot 3} \times 2^{5 \cdot 1} = 2^{(4 \cdot 6 + 2 \cdot 3 + 5 \cdot 1)} = 2^{12} = 4096.$
10. $8^{1 \cdot 3} \times 4^{0 \cdot 6} \times 16^{0 \cdot 2} = (2^3)^{1 \cdot 3} \times (2^2)^{0 \cdot 6} \times (2^4)^{0 \cdot 2}$
 $= 2^{3 \cdot 9} \times 2^{1 \cdot 2} \times 2^{0 \cdot 8} = 2^{(3 \cdot 9 + 1 \cdot 2 + 0 \cdot 8)} = 2^{5 \cdot 9}.$
11. $2^{0 \cdot 2} \times 64 \times 8^{1 \cdot 3} \times 4^{0 \cdot 2} = 8^x$
 $\Rightarrow (2 \times 4)^{0 \cdot 2} \times 8^2 \times 8^{1 \cdot 3} = 8^x \Rightarrow 8^{0 \cdot 2 + 2 + 1 \cdot 3} = 8^x \Rightarrow x = 3 \cdot 5.$
12. $\frac{(1000)^{12}}{(10)^{30}} = \frac{(10^3)^{12}}{(10)^{30}} = \frac{(10)^{36}}{(10)^{30}} = (10)^{(36-30)} = 10^6 = (10^3)^2 = (1000)^2.$
13. $\sqrt{(24)^4} + 224 = x \times (20)^2$
 $\Rightarrow (24)^2 + 224 = x \times 400 \Rightarrow 576 + 224 = x \times 400$
 $\Rightarrow x \times 400 = 800 \Rightarrow x = 2.$
14. माना $(3^x)^x = 19683 \Rightarrow 3^{x^2} = 3^3 \times 3^6 = 3^9$
 $\Rightarrow x^2 = 9 \Rightarrow x = 3.$
15. $5^2 = 25 \Rightarrow m = 5$ तथा $n = 2.$
 $\Rightarrow n^m = 2^5 = 32.$
16. $x^{1/3} = y^{1/4} \Rightarrow (x^{1/3})^{12} = (y^{1/4})^{12}$ [$\because$ 3 तथा 4 का ल.सं. = 12]
 $\Rightarrow x^4 = y^3 \Rightarrow (x^4)^5 = (y^3)^5 \Rightarrow x^{20} = y^{15}.$
17. $\sqrt[3]{3^n} = 81 \Rightarrow (3^n)^{\frac{1}{3}} = 3^4 \Rightarrow 3^{\frac{n}{3}} = 3^4 \Rightarrow \frac{n}{3} = 4 \Rightarrow n = 12.$

3	19683
3	6561
3	2187
9	729
9	81
	9

$$18. 4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1} \Rightarrow 2^{2x} + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}}$$

$$\Rightarrow 2^{(2x-1)}(2+1) = 3^{\left(x-\frac{1}{2}\right)} \cdot (3+1)$$

$$\Rightarrow \frac{2^{(2x-1)}}{2^2} = \frac{3^{\left(x-\frac{1}{2}\right)}}{3}$$

$$\Rightarrow 2^{(2x-3)} = 3^{\left(x-\frac{3}{2}\right)}$$

$$\Rightarrow 2x-3=0 \text{ तथा } x-\frac{3}{2}=0 \Rightarrow x=\frac{3}{2}$$

$$19. 8^{4.2} \times (8^2)^{2.1} \times 7^{8.4} \times (56)^{3.5} = (56)^x$$

$$\Rightarrow 8^{4.2} \times 8^{4.2} \times 7^{8.4} \times (56)^{3.5} = (56)^x$$

$$\Rightarrow 8^{8.4} \times 7^{8.4} \times (56)^{3.5} = (56)^x \Rightarrow (8 \times 7)^{8.4} \times (56)^{3.5} = (56)^x$$

$$\Rightarrow (56)^{8.4+3.5} = (56)^x \Rightarrow x=11.9.$$

$$20. (16)^{0.16} \times (16)^{0.04} \times (2)^{0.2} = (2^4)^{0.16} \times (2^4)^{0.04} \times (2)^{0.2}$$

$$= 2^{0.64} \times 2^{0.16} \times 2^{0.2} = 2^{(0.8+0.2)} = 2^1 = 2.$$

$$21. (8)^{11} \times (4)^{2.7} \times (2)^{3.3} = (2^3)^{11} \times (2^2)^{2.7} \times (2)^{3.3}$$

$$= 2^{3.3} \times 2^{5.4} \times 2^{3.3} = 2^{(3.3+5.4+3.3)} = 2^{12}.$$

$$22. (9)^{8.6} \times (8)^{3.9} \times (72)^{4.4} \times (9)^{3.9} \times (8)^{8.6} = (72)^x$$

$$\Rightarrow (9 \times 8)^{8.6} \times (8 \times 9)^{3.9} \times (72)^{4.4} = (72)^x$$

$$\Rightarrow (72)^{8.6} \times (72)^{3.9} \times (72)^{4.4} = (72)^x \Rightarrow (72)^{8.6+3.9+4.4} = (72)^x$$

$$\Rightarrow x = (8.6 + 3.9 + 4.4) = 16.9.$$

$$23. (1000)^9 \div (10)^{24} = \frac{(1000)^9}{(10)^{24}} = \frac{\{(10)^3\}^9}{(10)^{24}} = \frac{10^{(3 \times 9)}}{(10)^{24}} = \frac{(10)^{27}}{(10)^{24}}$$

$$= (10)^{(27-24)} = (10)^3 = 1000.$$

$$24. \left\{ \frac{3^{m^2}}{3^{2m}} \right\}^{\frac{1}{m}} = 81 \Rightarrow 3^{\frac{(m^2-2m) \times \frac{1}{m}}{m}} = 3^4$$

$$\Rightarrow 3^{(m-2)} = 3^4 \Rightarrow m-2=4 \Rightarrow m=6.$$

$$25. \sqrt{4^n} = 1024 = 4^5 \Rightarrow (4^n)^{\frac{1}{2}} = 4^5 \Rightarrow 4^{n/2} = 4^5$$

$$\Rightarrow \frac{n}{2} = 5 \Rightarrow n=10.$$

$$26. (27)^{2/3} \times (64)^{2/3} = (3^3)^{\frac{2}{3}} \times (4^3)^{\frac{2}{3}}$$

$$= 3^{\left(3 \times \frac{2}{3}\right)} \times 4^{\left(3 \times \frac{2}{3}\right)} = (3^2 \times 4^2) = (9 \times 16) = 144.$$

$$27. (0.6)^4 \times (0.36)^2 \times (0.216) = (0.6)^x$$

$$\Rightarrow (0.6)^4 \times \{(0.6)^2\}^2 \times (0.6)^3 = (0.6)^x$$

$$\Rightarrow (0.6)^4 \times (0.6)^4 \times (0.6)^3 = (0.6)^x \Rightarrow (0.6)^{(4+4+3)} = (0.6)^x$$

$$\Rightarrow x = (4+4+3) = 11.$$

$$28. (0.00032)^{0.6} = \left(\frac{32}{10^5}\right)^{0.6} = \left(\frac{2^5}{10^5}\right)^{0.6}$$

$$= \frac{2^{(5 \times 0.6)}}{10^{(5 \times 0.6)}} = \frac{2^3}{10^3} = \frac{8}{1000} = 0.008.$$

$$29. (64)^{3.1} \times 8^{4.3} = 8^x \Rightarrow (8^2)^{3.1} \times 8^{4.3} = 8^x$$

$$\Rightarrow 8^{6.2} \times 8^{4.3} = 8^x \Rightarrow 8^{(6.2+4.3)} = 8^x \Rightarrow x = 10.5.$$

$$30. \frac{3^{n+4} - 6 \times 3^{n+1}}{3^{n+2}} = \frac{3^{n+4} - 2 \times 3 \times 3^{n+1}}{3^{n+2}} = \frac{3^{n+4} - 2 \times 3^{n+2}}{3^{n+2}}$$

$$= \frac{3^{n+2} \cdot [3^2 - 2]}{3^{n+2}} = (9 - 2) = 7.$$

$$31. \frac{343 \times 49}{216 \times 16 \times 81} = \frac{7^3 \times 7^2}{6^3 \times 2^4 \times 3^4} = \frac{7^{(3+2)}}{6^3 \times (2 \times 3)^4} = \frac{7^5}{6^3 \times 6^4} = \frac{7^5}{6^{(3+4)}} = \frac{7^5}{6^7}.$$

$$32. \frac{36 \times 18}{125 \times 75} = \frac{6 \times 6 \times 6 \times 3}{5^3 \times 5^2 \times 3} = \frac{6 \times 6 \times 6}{5^3 \times 5^2} = \frac{6^3}{5^5}.$$

$$33. \text{ दिया गया व्यंजक } = \left[\left\{ \left(x^{-3/5} \right)^{1/5} \right\}^{3/5} \right]^5$$

$$= \left[\left\{ x^{\left(-\frac{3}{5} \times \frac{1}{5} \right)} \right\}^{3/5} \right]^5 = \left[x^{\left\{ \frac{-3 \times (-5)}{25 \times 3} \right\}} \right]^5$$

$$= x^{\left(\frac{1 \times 5}{5} \right)} = x^1 = x.$$

$$34. \text{ दिया गया व्यंजक } = \left(\frac{1}{10} \times \frac{1}{100} \times \frac{1}{1000} \times 10^7 \right) = \frac{1}{10 \times 10^2 \times 10^3} \times 10^7$$

$$= \frac{1}{(10)^{(1+2+3)}} \times 10^7 = \frac{10^7}{10^6} = 10^{(7-6)} = 10.$$

$$35. 3^{x+y} = 81 \text{ तथा } (81)^{x-y} = 3$$

$$\Rightarrow 3^{x+y} = 3^4 \text{ तथा } (3^4)^{x-y} = 3 = 3^1$$

$$\Rightarrow x+y = 4 \text{ तथा } 4x-4y = 1$$

$$\Rightarrow 4x+4y = 16 \text{ तथा } 4x-4y = 1 \Rightarrow x = \frac{17}{8}.$$

$$36. 2^{3.6} \times 4^{3.6} \times 4^{3.6} \times 32^{2.3} = (2 \times 4 \times 4)^{3.6} \times (32)^{2.3}$$

$$= (32)^{3.6} \times (32)^{2.3} = (32)^{(3.6+2.3)} = (32)^{5.9}$$

$$\therefore x = 5.9.$$

$$37. (x-2) = 2^{2/3} + 2^{1/3} \Rightarrow (x-2)^3 = (2^{2/3} + 2^{1/3})^3$$

$$\Rightarrow x^3 - 8 - 6x(x-2) = 2^2 + 2 + 3 \times 2^{2/3} \times 2^{1/3} (2^{2/3} + 2^{1/3})$$

$$\Rightarrow x^3 - 6x^2 + 12x - 8 = 6 + 3 \times 2 (x-2)$$

$$\Rightarrow x^3 - 6x^2 + 12x - 8 = 6 + 6x - 12$$

$$\Rightarrow x^3 - 6x^2 + 6x = 2.$$

$$\begin{aligned}
 38. (5-x) &= (5^{2/3} + 5^{1/3}) \Rightarrow (5-x)^3 = (5^{2/3} + 5^{1/3})^3 \\
 &\Rightarrow 5^3 - x^3 - 15x(5-x) = 5^2 + 5 + 3 \times 5^{2/3} \times 5^{1/3} (5^{2/3} + 5^{1/3}) \\
 &\Rightarrow 125 - x^3 - 75x + 15x^2 = 25 + 5 + 3 \times 5 \times (5-x) \\
 &\Rightarrow 125 - x^3 - 75x + 15x^2 = 30 + 75 - 15x \\
 &\Rightarrow x^3 - 15x^2 + 60x - 15 = 5.
 \end{aligned}$$

$$39. \text{ माना } 2^x = 3^y = 6^{-z} = k. \text{ तब}$$

$$2 = (k)^{\frac{1}{x}}, 3 = (k)^{\frac{1}{y}} \text{ तथा } 6 = (k)^{\frac{-1}{z}}.$$

$$\text{अब, } 2 \times 3 = 6 \Rightarrow (k)^{\frac{1}{x}} \times (k)^{\frac{1}{y}} = (k)^{\frac{-1}{z}}$$

$$\Rightarrow (k)^{\left(\frac{1}{x} + \frac{1}{y}\right)} = (k)^{\frac{-1}{z}} \Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{-1}{z} \Rightarrow \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0.$$

$$40. 2^x = 4^y = 8^z \Rightarrow 2^x = 2^{2y} = 2^{3z} \Rightarrow x = 2y = 3z.$$

$$\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7} \Rightarrow \frac{1}{6z} + \frac{1}{6z} + \frac{1}{6z} = \frac{24}{7} \Rightarrow \frac{3}{6z} = \frac{24}{7} \Rightarrow \frac{1}{z} = \left(\frac{24}{7} \times 2\right) = \frac{48}{7}.$$

$$\text{अतः } z = \frac{7}{48}.$$

$$41. \text{ माना } a^x = b^y = c^z = k. \text{ तब, } a = (k)^{\frac{1}{x}}, b = (k)^{\frac{1}{y}} \text{ तथा } c = (k)^{\frac{1}{z}}.$$

$$b^2 = ac \Rightarrow (k)^{\frac{2}{y}} = k^{\frac{1}{x}} \times k^{\frac{1}{z}} \Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z} \Rightarrow y = \frac{2xz}{(z+x)}.$$

$$42. 2^{x-1} + 2^{x+1} = 320 \Rightarrow 2^{x-1}(1+2^2) = 320$$

$$\Rightarrow 5 \times 2^{x-1} = 320 \Rightarrow 2^{x-1} = 64 = 2^6$$

$$\Rightarrow x-1 = 6 \Rightarrow x = 7.$$

$$43. 2^{x+4} - 2^{x+2} = 3 \Rightarrow 2^{x+2}(2^2 - 1) = 3 \Rightarrow 2^{x+2} = 1 = 2^0$$

$$\Rightarrow x+2 = 0 \Rightarrow x = -2.$$

$$44. 2^a = x \text{ तथा } 3^b = y \text{ रखने पर : } x + y = 17 \quad \dots(i)$$

$$2^2 \times 2^a - 3 \times 3^b = 5 \Rightarrow 4x - 3y = 5 \quad \dots(ii)$$

इन्हें हल करने पर $x = 8$ तथा $y = 9$.

$$\therefore 2^a = 8 = 2^3 \text{ तथा } 3^b = 9 = 3^2 \Rightarrow a = 3 \text{ तथा } b = 2.$$

$$45. \left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3} = \left(\frac{a}{b}\right)^{3-x} \Rightarrow x-1 = 3-x \Rightarrow 2x = 4 \Rightarrow x = 2.$$

$$46. x = y^a \Rightarrow x = (z^b)^a = (z)^{ba} = (x^c)^{ba} = x^{cba}$$

$$\Rightarrow cba = 1 \Rightarrow abc = 1.$$

$$47. \sqrt{a^{-1}b} \cdot \sqrt{b^{-1}c} \cdot \sqrt{c^{-1}a} = \sqrt{a^{-1}b \cdot b^{-1}c \cdot c^{-1}a} = 1.$$

$$\begin{aligned}
 48. \text{ दिया गया व्यंजक} &= \frac{1}{1 + \frac{a^n}{a^m}} + \frac{1}{1 + \frac{a^m}{a^n}} = \frac{a^m}{(a^m + a^n)} + \frac{a^n}{(a^n + a^m)} \\
 &= \frac{(a^m + a^n)}{(a^m + a^n)} = 1.
 \end{aligned}$$

$$\begin{aligned}
 49. \text{ दिया गया व्यंजक} &= \frac{1}{(1+a+b^{-1})} + \frac{b^{-1}}{b^{-1}+1+b^{-1}c^{-1}} + \frac{a}{a+ac+1} \\
 &= \frac{1}{(1+a+b^{-1})} + \frac{a}{1+a+ac} + \frac{b^{-1}}{1+b^{-1}+b^{-1}c^{-1}} \\
 &= \frac{1}{(1+a+b^{-1})} + \frac{a}{(1+a+b^{-1})} + \frac{b^{-1}}{(1+a+b^{-1})} \quad \left[\because abc=1 \Rightarrow b^{-1}=ac \right. \\
 &\quad \left. (cb)^{-1}=a \Rightarrow b^{-1}c^{-1}=a \right] \\
 &= \frac{(1+a+b^{-1})}{(1+a+b^{-1})} = 1.
 \end{aligned}$$

$$\begin{aligned}
 50. \text{ दिया गया व्यंजक} &= \left(x^{a-b}\right)^{\frac{1}{ab}} \cdot \left(x^{b-c}\right)^{\frac{1}{bc}} \cdot \left(x^{c-a}\right)^{\frac{1}{ca}} \\
 &= x^{\left(\frac{a-b}{ab}\right)} \cdot x^{\left(\frac{b-c}{bc}\right)} \cdot x^{\left(\frac{c-a}{ca}\right)} \cdot x^{\left(\frac{1}{b}-\frac{1}{a}\right)} \cdot x^{\left(\frac{1}{c}-\frac{1}{b}\right)} \cdot x^{\left(\frac{1}{a}-\frac{1}{c}\right)} \\
 &= x^{\left(\frac{1}{b}-\frac{1}{a}+\frac{1}{c}-\frac{1}{b}+\frac{1}{a}-\frac{1}{c}\right)} = x^0 = 1.
 \end{aligned}$$

$$\begin{aligned}
 51. \text{ दिया गया व्यंजक} &= x^{(b+c)(b-c)} \cdot x^{(c+a)(c-a)} \cdot x^{(a+b)(a-b)} \\
 &= x^{(b^2-c^2)} \cdot x^{(c^2-a^2)} \cdot x^{(a^2-b^2)} \\
 &= x^{(b^2-c^2+c^2-a^2+a^2-b^2)} = x^0 = 1.
 \end{aligned}$$

$$\begin{aligned}
 52. \text{ दिया गया व्यंजक} &= x^{(a-b)(a+b)} \cdot x^{(b-c)(b+c)} \cdot x^{(c-a)(c+a)} \\
 &= x^{(a^2-b^2)} \cdot x^{(b^2-c^2)} \cdot x^{(c^2-a^2)} \\
 &= x^{(a^2-b^2+b^2-c^2+c^2-a^2)} = x^0 = 1.
 \end{aligned}$$

$$\begin{aligned}
 53. \text{ दिया गया व्यंजक} &= x^{(a-b)(a+b-c)} \cdot x^{(b-c)(b+c-a)} \cdot x^{(c-a)(c+a-b)} \\
 &= x^{(a-b)(a+b)} \cdot x^{-c(a-b)} \cdot x^{(b-c)(b+c)} \cdot x^{-a(b-c)} \cdot x^{(c-a)(c+a)} \cdot x^{-b(c-a)} \\
 &= x^{(a^2-b^2+b^2-c^2+c^2-a^2)} \cdot x^{-ca+bc-ab+ca-bc+ab} = (x^0 \times x^0) = (1 \times 1) = 1.
 \end{aligned}$$

$$\begin{aligned}
 54. \text{ दिया गया व्यंजक} &= \frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}} \\
 &= \frac{x^a}{(x^a+x^b+x^c)} + \frac{x^b}{(x^b+x^a+x^c)} + \frac{x^c}{(x^c+x^b+x^a)} \\
 &= \frac{(x^a+x^b+x^c)}{(x^a+x^b+x^c)} = 1.
 \end{aligned}$$

$$\begin{aligned}
 55. \left\{ \frac{1}{c} = (1-b) \Rightarrow c = \frac{1}{(1-b)} \right\}, \left\{ a = \left(1 - \frac{1}{b}\right) = \frac{(b-1)}{b} \Rightarrow \frac{1}{a} = \frac{b}{(b-1)} \right\} \\
 \left\{ \therefore \left(c + \frac{1}{a}\right) = \frac{1}{(1-b)} + \frac{b}{(b-1)} = \frac{1}{(1-b)} - \frac{b}{(1-b)} = \frac{(1-b)}{(1-b)} = 1 \right\}
 \end{aligned}$$

$$\begin{aligned}
 56. \text{ माना } p &= \sqrt{1+a} + \sqrt{1-a}. \text{ तब, } p^2 = (1+a) + (1-a) + 2\sqrt{1-a^2} \\
 \Rightarrow p^2 &= 2 + 2\sqrt{1-a^2} = 2 + 2\sqrt{1-\frac{3}{4}} = 2 + 2 \times \frac{1}{2} = (2+1) = 3 \Rightarrow p = \sqrt{3}.
 \end{aligned}$$

$$57. \frac{(2^n + 2^{n-1})}{(2^{n+1} - 2^n)} = \frac{2^{n-1}(2+1)}{2^n(2-1)} = \frac{3}{2}.$$

$$58. 2^{2x-1} = \frac{1}{(2^3)^{x-3}} = \frac{1}{2^{(3x-9)}} = 2^{(9-3x)}$$

$$\therefore 2x-1=9-3x \Rightarrow 5x=10 \Rightarrow x=2.$$

$$59. a^{2x+2} = 1 = a^0 \Rightarrow 2x+2=0 \Rightarrow 2x=-2 \Rightarrow x=-1.$$

$$60. \sqrt[3]{\frac{8}{3}} = \sqrt[3]{\frac{8 \times 3}{3 \times 3}} = \frac{\sqrt[3]{24}}{3} = \frac{1}{3} \times 4.898 = 1.633.$$

$$61. 8^{x+1} = 64 = 8^2 \Rightarrow x+1=2 \Rightarrow x=1.$$

$$\therefore 3^{2x+1} = 3^{(2 \times 1 + 1)} = 3^3 = 27.$$

$$62. \frac{y}{x} = \frac{1.50}{0.04} = \frac{150}{4} = \frac{75}{2}.$$

$$\therefore \frac{y-x}{y+x} = \frac{\frac{y}{x}-1}{\frac{y}{x}+1} \quad [\text{अंश तथा हर में } x \text{ से भाग देने पर}]$$

$$= \frac{\left(\frac{75}{2}-1\right)}{\left(\frac{75}{2}+1\right)} = \frac{73}{77}.$$

$$63. 4^x = (2^{3y})^{\frac{1}{2}} \Rightarrow 2^{2x} = 2^{\frac{3y}{2}} \Rightarrow 2x = \frac{3y}{2} \Rightarrow x = \frac{3}{4}y.$$

$$64. x^{-\frac{4}{5}} = 81 \Rightarrow \frac{1}{x^{\frac{4}{5}}} = 81 \Rightarrow x^{\frac{4}{5}} = \frac{1}{81}$$

$$\Rightarrow (x^{1/5})^4 = \left(\frac{1}{3}\right)^4 \Rightarrow x^{1/5} = \frac{1}{3} \Rightarrow x = \left(\frac{1}{3}\right)^5 = \frac{1^5}{3^5} = \frac{1}{243}$$

$$65. 5^{(x+3)} = (5^2)^{3x-4} = 5^{6x-8} \Rightarrow x+3=6x-8 \Rightarrow 5x=11 \Rightarrow x = \frac{11}{5}.$$

$$66. (64)^{(2x-5)} = 2^2 \times 2^{3(x-4)} \Rightarrow 2^{6(2x-5)} = 2^{3x-12+2}$$

$$\therefore 6(2x-5) = 3x-10 \Rightarrow 12x-30 = 3x-10 \Rightarrow 9x=20 \Rightarrow x = \frac{20}{9}.$$

$$67. \left\{ \left(\frac{3}{2} \right)^2 \right\}^x \cdot \left(\frac{2}{3} \right)^{3(x-1)} = \frac{2}{3}$$

$$\Rightarrow \left(\frac{3}{2} \right)^{2x} \cdot \left(\frac{2}{3} \right)^{(3x-3)} = \frac{2}{3} \Rightarrow \left(\frac{2}{3} \right)^{-2x} \cdot \left(\frac{2}{3} \right)^{(3x-3)} = \frac{2}{3}$$

$$\Rightarrow \left(\frac{2}{3} \right)^{(3x-3-2x)} = \left(\frac{2}{3} \right)^1 \Rightarrow x-3=1 \Rightarrow x=4.$$

$$68. \left(\frac{64x^3}{27a^{-3}}\right)^{-\frac{2}{3}} = \left(\frac{64x^3a^3}{27}\right)^{-\frac{2}{3}} = \left(\frac{27}{64x^3a^3}\right)^{\frac{2}{3}} = \left\{\left(\frac{3}{4xa}\right)^3\right\}^{\frac{2}{3}}$$

$$= \left(\frac{3}{4xa}\right)^{\left(3 \times \frac{2}{3}\right)} = \left(\frac{3}{4xa}\right)^2 = \frac{9}{16x^2a^2}.$$

$$69. \sqrt[3]{-8} \times \sqrt[4]{16} = (-8)^{\frac{1}{3}} \times (16)^{\frac{1}{4}}$$

$$= \{(-2)^3\}^{\frac{1}{3}} \times (2^4)^{\frac{1}{4}} = (-2)^{\left(\frac{3 \times 1}{3}\right)} \times 2^{\left(\frac{4 \times 1}{4}\right)} = (-2)^1 \times 2^1 = (-2) \times 2 = -4.$$

$$70. \left(\frac{1}{2}\right)^{-\frac{1}{2}} = 2^{\frac{1}{2}} = \sqrt{2}.$$

$$71. (\sqrt{8})^{\frac{1}{3}} = \left(8^{\frac{1}{2}}\right)^{\frac{1}{3}} = 8^{\left(\frac{1}{2} \times \frac{1}{3}\right)} = 8^{\frac{1}{6}} = (2^3)^{\frac{1}{6}} = 2^{\left(\frac{3 \times 1}{6}\right)} = 2^{\frac{1}{2}} = \sqrt{2}.$$

$$72. \left(\frac{32}{243}\right)^{-\frac{4}{5}} = \left\{\left(\frac{2}{3}\right)^5\right\}^{-\frac{4}{5}} = \left(\frac{2}{3}\right)^{5 \times \left(-\frac{4}{5}\right)} = \left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4 = \frac{3^4}{2^4} = \frac{81}{16}.$$

$$73. \left(\frac{-1}{343}\right)^{-\frac{2}{3}} = (-343)^{\frac{2}{3}} = \{(-7)^3\}^{\frac{2}{3}} = (-7)^{\left(3 \times \frac{2}{3}\right)} = (-7)^2 = 49.$$

$$74. \left(\frac{1}{216}\right)^{-\frac{2}{3}} = (216)^{\frac{2}{3}} = (6^3)^{\frac{2}{3}} = 6^{\left(3 \times \frac{2}{3}\right)} = 6^2 = 36.$$

$$\left(\frac{1}{27}\right)^{-\frac{4}{3}} = (27)^{\frac{4}{3}} = (3^3)^{\frac{4}{3}} = 3^{\left(3 \times \frac{4}{3}\right)} = 3^4 = 81.$$

$$\therefore \text{दिया गया व्यंजक} = 36 \div 81 = \frac{36}{81} = \frac{4}{9}.$$

$$75. 3^n = (3^3)^{\frac{2}{3}} \times \left(\frac{1}{81}\right)^{\frac{1}{2}} \Rightarrow 3^{\left(3 \times \frac{2}{3}\right)} \times \frac{1}{9} = 3^2 \times \frac{1}{9} = 9 \times \frac{1}{9} = 1 \Rightarrow n = 0.$$

$$76. (16)^{1.75} = (16)^{\frac{175}{100}} = (16)^{\frac{7}{4}} = (2^4)^{\frac{7}{4}} = 2^{\left(4 \times \frac{7}{4}\right)} = 2^7 = 128.$$

$$77. \sqrt{2^n} = 2^6 \Rightarrow 2^{n/2} = 2^6 \Rightarrow \frac{n}{2} = 6 \Rightarrow n = 12.$$

$$78. (256)^{0.16} \times (16)^{0.18} = (4^4)^{0.16} \times (4^2)^{0.18}$$

$$= (4)^{0.64} \times 4^{0.36} = 4^{(0.64+0.36)} = 4^1 = 4.$$

$$79. \text{माना } (72)^x = (9 \times 8)^{8.6} \times (8 \times 9)^{3.9} \times (72)^{4.4}$$

$$= (72)^{8.6} \times (72)^{3.9} \times (72)^{4.4} = (72)^{(8.6+3.9+4.4)} = (72)^{16.9}$$

$$\Rightarrow x = 16.9.$$

$$80. \text{माना } 6^{1.2} \times (36)^x \times (30)^{2.4} \times (25)^{1.3} = (30)^5$$

$$\text{तब, } 6^{1.2} \times (6^2)^x \times (30)^{2.4} \times (5^2)^{1.3} = (30)^5$$

$$\text{तब, } 6^{1.2} \times (6^2)^x \times (6 \times 5)^{2.4} \times (5^2)^{1.3} = (30)^5$$

$$\Rightarrow 6^{(1.2+2x+2.4)} \times 5^{(2.4+2.6)} = (30)^5$$

$$\Rightarrow 6^{(3.6+2x)} \times 5^5 = (30)^5 \Rightarrow 3.6 + 2x = 5 \Rightarrow 2x = 1.4 \Rightarrow x = 0.7.$$

$$81. (0.01024)^{\frac{1}{5}} = \left(\frac{1024}{10^5}\right)^{\frac{1}{5}} = \left(\frac{4^5}{10^5}\right)^{\frac{1}{5}} = \left\{\left(\frac{4}{10}\right)^5\right\}^{\frac{1}{5}} = \left(\frac{4}{10}\right)^{\left(5 \times \frac{1}{5}\right)} = \left(\frac{4}{10}\right) = 0.4.$$

$$82. \text{ माना } \sqrt{3} + \sqrt{5} = \sqrt{a} + \sqrt{b}. \text{ तब } (3 + \sqrt{5}) = a + b + 2\sqrt{ab}$$

$$\therefore a + b = 3 \text{ तथा } 2\sqrt{ab} = \sqrt{5} \Rightarrow 4ab = 5 \Rightarrow ab = \frac{5}{4}.$$

$$(a - b)^2 = (a + b)^2 - 4ab = \left(3^2 - 4 \times \frac{5}{4}\right) = (9 - 5) = 4 \Rightarrow (a - b) = 2.$$

$$(a + b) = 3 \text{ तथा } (a - b) = 2 \Rightarrow 2a = 5 \text{ तथा } 2b = 1 \Rightarrow a = \frac{5}{2} \text{ तथा } b = \frac{1}{2}.$$

$$\therefore \text{ अभीष्ट वर्गमूल } = \sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}.$$

$$83. \frac{1}{x} = \frac{1}{(7 - 4\sqrt{3})} \times \frac{(7 + 4\sqrt{3})}{(7 + 4\sqrt{3})} = \frac{(7 + 4\sqrt{3})}{(49 - 48)}$$

$$\therefore \left(x + \frac{1}{x}\right) = (7 - 4\sqrt{3}) + (7 + 4\sqrt{3}) = 14.$$

$$84. \frac{1}{x} = \frac{1}{(3 + \sqrt{8})} \times \frac{(3 - \sqrt{8})}{(3 - \sqrt{8})} = \frac{(3 - \sqrt{8})}{(9 - 8)}$$

$$\therefore \left(x + \frac{1}{x}\right) = (3 + \sqrt{8}) + (3 - \sqrt{8}) = 6 \Rightarrow \left(x + \frac{1}{x}\right)^2 = 6^2 = 36.$$

$$\therefore x^2 + \frac{1}{x^2} + 2 = 36 \Rightarrow \left(x^2 + \frac{1}{x^2}\right) = (36 - 2) = 34.$$

$$85. 2, 3, 4, 6 \text{ का ल०स०} = 12.$$

$$\sqrt{2} = 2^{\frac{1}{2}} = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}}, \sqrt[3]{3} = 3^{\frac{1}{3}} = (3^4)^{\frac{1}{12}} = (81)^{\frac{1}{12}},$$

$$\sqrt[4]{4} = 4^{\frac{1}{4}} = (4^3)^{\frac{1}{12}} = (64)^{\frac{1}{12}}, \sqrt[5]{6} = 6^{\frac{1}{5}} = (6^2)^{\frac{1}{12}} = (36)^{\frac{1}{12}}.$$

$$\therefore (81)^{\frac{1}{12}} > (64)^{\frac{1}{12}} > (36)^{\frac{1}{12}}.$$

$$\text{अतः सबसे बड़ी संख्या} = \sqrt[3]{3}.$$

$$86. 2, 3, 4, 6 \text{ का ल०स०} = 12.$$

$$\sqrt{3} = 3^{\frac{1}{2}} = (3^6)^{\frac{1}{12}} = (729)^{\frac{1}{12}}, \sqrt[3]{4} = 4^{\frac{1}{3}} = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}},$$

$$\sqrt[4]{6} = 6^{\frac{1}{4}} = (6^3)^{\frac{1}{12}} = (216)^{\frac{1}{12}}, \sqrt[5]{8} = 8^{\frac{1}{5}} = (8^2)^{\frac{1}{12}} = (64)^{\frac{1}{12}}.$$

$$(729)^{\frac{1}{12}} > (256)^{\frac{1}{12}} > (216)^{\frac{1}{12}} > (64)^{\frac{1}{12}}.$$

$$\text{अतः } \sqrt{3} \text{ इनमें सबसे बड़ा है.}$$

$$87. 3, 2 \text{ का ल०स०} = 6.$$

$$\sqrt[3]{2} = 2^{\frac{1}{3}} = (2^2)^{\frac{1}{6}} = (4)^{\frac{1}{6}}, \sqrt{3} = 3^{\frac{1}{2}} = (3^3)^{\frac{1}{6}} = (27)^{\frac{1}{6}}.$$

$$\sqrt[3]{5} = 5^{\frac{1}{3}} = (5^2)^{\frac{1}{6}} = (25)^{\frac{1}{6}}, \frac{3}{2} = \left\{\left(\frac{3}{2}\right)^6\right\}^{\frac{1}{6}} = \left(\frac{729}{64}\right)^{\frac{1}{6}} = \left(11\frac{25}{64}\right)^{\frac{1}{6}}.$$

$$\text{अतः इन सबमें बड़ा है } (27)^{\frac{1}{6}} \text{ अर्थात् } \sqrt[6]{27}.$$

88. 2, 3, 4 का ल०स० = 12.

$$\sqrt{2} = \left(2^{\frac{1}{2}}\right) = \left(2^6\right)^{\frac{1}{12}} = (64)^{\frac{1}{12}}.$$

$$\sqrt[3]{4} = 4^{\frac{1}{3}} = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}}.$$

$$\sqrt[4]{6} = 6^{\frac{1}{4}} = (6^3)^{\frac{1}{12}} = (216)^{\frac{1}{12}}.$$

स्पष्ट है कि $(64)^{\frac{1}{12}} < (216)^{\frac{1}{12}} < (256)^{\frac{1}{12}}.$

अर्थात् $\sqrt{2} < \sqrt[4]{6} < \sqrt[3]{4}.$

89. 3, 3, 8, 9 का ल०स० = 72.

$$2^{\frac{1}{3}} = 2^{\left(\frac{1}{3} \times \frac{24}{24}\right)} = \left(2^{24}\right)^{\frac{1}{72}}; 3^{\frac{1}{3}} = 3^{\left(\frac{1}{3} \times \frac{24}{24}\right)} = \left(3^{24}\right)^{\frac{1}{72}};$$

$$8^{\frac{1}{8}} = 2^{\left(\frac{3}{8} \times \frac{9}{9}\right)} = \left(2^{27}\right)^{\frac{1}{72}}; 9^{\frac{1}{9}} = 3^{\left(\frac{2}{9} \times \frac{8}{8}\right)} = \left(3^{16}\right)^{\frac{1}{72}}.$$

स्पष्ट है कि इन सबमें $\left(3^{24}\right)^{\frac{1}{72}}$ अर्थात् $\sqrt[3]{3}$ सबसे बड़ी संख्या है.

- 90.
- $(8 - 2\sqrt{15}) = (\sqrt{5})^2 + (\sqrt{3})^2 - 2 \times \sqrt{5} \times \sqrt{3} = (\sqrt{5} - \sqrt{3})^2$

$$\Rightarrow \sqrt{8 - 2\sqrt{15}} = (\sqrt{5} - \sqrt{3}).$$

91. दिया गया व्यंजक
- $= (-2)^{(-2)(-2)} = (-2)^4 = 16.$

- 92.
- $(7 + 4\sqrt{3}) = 4 + 3 + 2 \times 2 \times \sqrt{3}$

$$= (2)^2 + (\sqrt{3})^2 + 2 \times 2 \times \sqrt{3} = (2 + \sqrt{3})^2$$

$$\Rightarrow \sqrt{7 + 4\sqrt{3}} = (2 + \sqrt{3}).$$

$$\therefore \text{दिया गया व्यंजक} = \frac{4 + 3\sqrt{3}}{2 + \sqrt{3}} = \frac{2(2 + \sqrt{3}) + \sqrt{3}}{(2 + \sqrt{3})} = 2 + \frac{\sqrt{3}}{(2 + \sqrt{3})}$$

$$= 2 + \frac{1 \cdot 732}{3 \cdot 732} = \left(2 + \frac{1732}{3732}\right) = \left(2 + \frac{433}{933}\right) = 2 + 0.464 = 2.464.$$

93. दिया गया व्यंजक $= \frac{(3 + \sqrt{6})}{(5\sqrt{3} - 2\sqrt{4 \times 3} - \sqrt{16 \times 2} + \sqrt{25 \times 2})}$
- $$= \frac{(3 + \sqrt{6})}{(5\sqrt{3} - 4\sqrt{3} - 4\sqrt{2} + 5\sqrt{2})} = \frac{(3 + \sqrt{6})}{(\sqrt{3} + \sqrt{2})} \times \frac{(\sqrt{3} - \sqrt{2})}{(\sqrt{3} - \sqrt{2})}$$
- $$= \frac{(3\sqrt{3} - 3\sqrt{2} + \sqrt{18} - \sqrt{12})}{(\sqrt{3} - \sqrt{2})} = \frac{(3\sqrt{3} - 3\sqrt{2} + 3\sqrt{2} - 2\sqrt{3})}{(\sqrt{3} - \sqrt{2})}$$
- $$= \sqrt{3} = 1.732.$$

94. दिया गया व्यंजक $= \frac{12}{(3 + \sqrt{5} + 2\sqrt{2})} \times \frac{(3 + \sqrt{5} - 2\sqrt{2})}{(3 + \sqrt{5} - 2\sqrt{2})} = \frac{12(3 + \sqrt{5} - 2\sqrt{2})}{(3 + \sqrt{5})^2 - (2\sqrt{2})^2} = \frac{12(3 + \sqrt{5} - 2\sqrt{2})}{9 + 5 + 6\sqrt{5} - 8}$
- $$= \frac{12(3 + \sqrt{5} - 2\sqrt{2})}{6(\sqrt{5} + 1)} \times \frac{(\sqrt{5} - 1)}{(\sqrt{5} - 1)}$$
- $$= \frac{1}{2} (3\sqrt{5} - 3 + 5 - \sqrt{5} - 2\sqrt{10} + 2\sqrt{2}) = \frac{1}{2} (2 + 2\sqrt{5} + 2\sqrt{2} - 2\sqrt{10})$$
- $$= (1 + \sqrt{5} + \sqrt{2} - \sqrt{10}).$$

$$95. (16+6\sqrt{7})=9+7+2\times 3\times \sqrt{7}=3^2+(\sqrt{7})^2+2\times 3\times \sqrt{7}=(3+\sqrt{7})^2.$$

$$\Rightarrow \sqrt{16+6\sqrt{7}}=(3+\sqrt{7}) \text{ तथा } \sqrt{16-6\sqrt{7}}=(3-\sqrt{7})$$

$$\Rightarrow \sqrt{16+6\sqrt{7}}-\sqrt{16-6\sqrt{7}}=(3+\sqrt{7})-(3-\sqrt{7})=2\sqrt{7}.$$

$$\therefore \text{ दिया गया व्यंजक } = \frac{\sqrt{7}}{2\sqrt{7}} = \frac{1}{2}.$$

$$96. (7+2\sqrt{12})=4+3+2\times 2\times \sqrt{3}=2^2+(\sqrt{3})^2+2\times 2\times \sqrt{3}=(2+\sqrt{3})^2$$

$$\Rightarrow a=\sqrt{7+2\sqrt{12}}=(2+\sqrt{3}) \text{ तथा } b=\sqrt{7-2\sqrt{12}}=(2-\sqrt{3})$$

$$\Rightarrow a+b=4 \text{ तथा } ab=(4-3)=1$$

$$\Rightarrow (a^3+b^3)=(a+b)^3-3ab(a+b)=(4)^3-3\times 1\times 4=(64-12)=52.$$

$$97. (x^{1/3}+y^{1/3})^3=(z^{1/3})^3 \Rightarrow x+y+3x^{1/3}y^{1/3}(x^{1/3}+y^{1/3})=z$$

$$\Rightarrow (x+y-z)=-3x^{1/3}y^{1/3}z^{1/3}$$

$$\Rightarrow (x+y-z)^3=-27xyz$$

$$\Rightarrow (x+y-z)^3+27xyz=0.$$

$$98. 2^{50} \text{ का दुगुना } =(2\times 2^{50})=(2^{51}).$$

$$99. \text{ दिया गया व्यंजक } = \left\{ \frac{7^{(2+4)}}{5^{(2+1)}} \right\}^{\frac{7}{2}} \times \left\{ \frac{7^{(3+5)}}{5^{(3+2)}} \right\}^{\frac{-5}{2}}$$

$$= \left(\frac{7^6}{5^3} \right)^{\frac{7}{2}} \times \left(\frac{7^8}{5^5} \right)^{\frac{-5}{2}} = \frac{7^{\left(\frac{6\times 7}{2} \right)} \times 7^{\left\{ \frac{8\times (-5)}{2} \right\}}}{5^{\left(\frac{3\times 7}{2} \right)} \times 5^{\left\{ \frac{5\times (-5)}{2} \right\}}}$$

$$= \frac{7^{21} \times 7^{-20}}{5^{\left(\frac{21}{2} \right)} \times 5^{\left(\frac{-25}{2} \right)}} = \frac{7^{(21-20)}}{5^{\left(\frac{21}{2} - \frac{25}{2} \right)}} = \frac{7^1}{5^{(-2)}} = (7 \times 5^2) = (7 \times 25) = 175.$$

$$100. \text{ दिया गया व्यंजक } = \left\{ \frac{x^{\frac{m}{(m-n)} + \frac{n}{(n-m)}}}{x^{\frac{m}{(m+n)} + \frac{n}{(n+m)}}} \right\}^{m+n} = \left\{ \frac{x^{\left\{ \frac{m}{(m-n)} - \frac{n}{(m-n)} \right\}}}{x^{\left\{ \frac{m}{m+n} + \frac{n}{m+n} \right\}}} \right\}^{(m+n)}$$

$$= \left\{ \frac{x^{\left(\frac{m-n}{m-n} \right)}}{x^{\frac{(m+n)}{(m+n)}}} \right\}^{m+n} = \left(\frac{x}{x} \right)^{m+n} = (1)^{m+n} = 1.$$

$$101. (a+b) = \frac{\sqrt{5}+1}{\sqrt{5}-1} + \frac{\sqrt{5}-1}{\sqrt{5}+1} = \frac{(\sqrt{5}+1)^2 + (\sqrt{5}-1)^2}{(5-1)} = \frac{2(5+1)}{4} = 3.$$

$$(a-b) = \frac{\sqrt{5}+1}{\sqrt{5}-1} - \frac{\sqrt{5}-1}{\sqrt{5}+1} = \frac{(\sqrt{5}+1)^2 - (\sqrt{5}-1)^2}{(5-1)} = \frac{4 \times \sqrt{5} \times 1}{4} = \sqrt{5}, ab=1$$

$$\frac{(a^2+ab+b^2)}{(a^2-ab+b^2)} = \frac{(a+b)^2-ab}{(a+b)^2-3ab} = \frac{(9-1)}{(9-3)} = \frac{8}{6} = \frac{4}{3}.$$

$$102. x = (3+2\sqrt{2}) \Rightarrow \frac{1}{x} = \frac{1}{(3+2\sqrt{2})} \times \frac{(3-2\sqrt{2})}{(3-2\sqrt{2})} = \frac{(3-2\sqrt{2})}{(9-8)} = (3-2\sqrt{2}).$$

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = \left(x + \frac{1}{x} - 2\right) = (3+2\sqrt{2} + 3-2\sqrt{2} - 2) = 4.$$

$$\therefore \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = 2.$$

$$103. \frac{1}{(1+\sqrt{2})} = \frac{1}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = \frac{(\sqrt{2}-1)}{(2-1)} = (\sqrt{2}-1).$$

इसी प्रकार, $\frac{1}{(\sqrt{2}+\sqrt{3})} = \frac{1}{(\sqrt{3}+\sqrt{2})} \times \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})} = \frac{(\sqrt{3}-\sqrt{2})}{(3-2)} = (\sqrt{3}-\sqrt{2}).$

$$\frac{1}{(\sqrt{3}+\sqrt{4})} = (\sqrt{4}-\sqrt{3}), \frac{1}{(\sqrt{99}+\sqrt{100})} = (\sqrt{100}-\sqrt{99}).$$

∴ दिया गया व्यंजक

$$= (\sqrt{2}-\sqrt{1}) + (\sqrt{3}-\sqrt{2}) + (\sqrt{4}-\sqrt{3}) + (\sqrt{5}-\sqrt{4}) + \dots + (\sqrt{99}-\sqrt{98}) + (\sqrt{100}-\sqrt{99}) \\ = (\sqrt{100}-\sqrt{1}) = (10-1) = 9.$$

$$104. \frac{1}{(\sqrt{3}+\sqrt{4})} = \frac{1}{(\sqrt{4}+\sqrt{3})} \times \frac{(\sqrt{4}-\sqrt{3})}{(\sqrt{4}-\sqrt{3})} = \frac{(\sqrt{4}-\sqrt{3})}{(4-3)} = (\sqrt{4}-\sqrt{3}) \text{ आदि आदि.}$$

∴ दिया गया व्यंजक $= (\sqrt{4}-\sqrt{3}) + (\sqrt{5}-\sqrt{4}) + (\sqrt{6}-\sqrt{5}) + (\sqrt{7}-\sqrt{6}) + (\sqrt{8}-\sqrt{7}) + (\sqrt{9}-\sqrt{8}) \\ = (\sqrt{9}-\sqrt{3}) = (3-\sqrt{3}).$

$$105. \frac{1}{(\sqrt{100}-\sqrt{99})} = \frac{1}{(\sqrt{100}-\sqrt{99})} \times \frac{(\sqrt{100}+\sqrt{99})}{(\sqrt{100}+\sqrt{99})} = (\sqrt{100}+\sqrt{99}) \text{ आदि आदि.}$$

दिया गया व्यंजक $= (\sqrt{100}+\sqrt{99}) - (\sqrt{99}+\sqrt{98}) + (\sqrt{98}+\sqrt{97}) - \dots - (\sqrt{3}+\sqrt{2}) + (\sqrt{2}+\sqrt{1}) \\ = (\sqrt{100}+\sqrt{1}) = (10+1) = 11.$

प्रश्नमाला 9B

1. यदि $x = \frac{1}{(2+\sqrt{3})}$ हो, तो $\frac{(3x-x^3)}{(1-3x^2)}$ का मान ज्ञात कीजिए. (एस०एस०सी० परीक्षा, 2009)

2. यदि $\sqrt{3} = 1.732$ हो, तो $(\sqrt{6}+\sqrt{2}) + (\sqrt{6}-\sqrt{2}) + \{(2+\sqrt{3}) + 2\sqrt{3}\}$ का मान ज्ञात कीजिए. (एस०एस०सी० परीक्षा, 2008)

3. यदि $\sqrt{3} = 1.732$ हो, तो $\frac{(2-\sqrt{3})}{(2+\sqrt{3})}$ का मान ज्ञात कीजिए. (एस०एस०सी० परीक्षा, 2008)

4. $\left(3 + \frac{1}{\sqrt{3}} + \frac{1}{3+\sqrt{3}} - \frac{1}{3-\sqrt{3}}\right)$ का मान ज्ञात कीजिए. (एस०एस०सी० परीक्षा, 2007)

5. यदि $\sqrt{2} = 1.414$ हो, तो $\frac{\sqrt{3}}{\sqrt{6}}$ का मान ज्ञात कीजिए.
6. यदि $\left(\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}\right) = a + \sqrt{30}b$ हो, तो a तथा b के मान ज्ञात कीजिए.
7. यदि $\sqrt{5} = 2.236$ हो, तो $\left(\frac{3-\sqrt{5}}{3+2\sqrt{5}}\right)$ का मान दशमलव के तीन स्थानों तक ज्ञात कीजिए.
8. यदि $\sqrt{2} = 1.414$, $\sqrt{5} = 2.236$ तथा $\sqrt{10} = 3.162$ हो, तो $\frac{1}{(3+\sqrt{5}-2\sqrt{2})}$ का मान ज्ञात कीजिए.
9. यदि $\sqrt{2} = 1.414$ हो, तो $(\sqrt{63} + \sqrt{28} - \sqrt{175} + \sqrt{162} - \sqrt{32})$ का मान ज्ञात कीजिए.
10. यदि $\sqrt{5} = 2.236$ तथा $\sqrt{10} = 3.162$ हो, तो $\frac{15}{(\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{5} - \sqrt{80})}$ का मान ज्ञात कीजिए.
11. सिद्ध कीजिए कि : $\frac{7\sqrt{3}}{(\sqrt{10} + \sqrt{3})} - \frac{2\sqrt{5}}{(\sqrt{6} + \sqrt{5})} - \frac{3\sqrt{2}}{(\sqrt{15} + 3\sqrt{2})} = 1$.
12. सिद्ध कीजिए कि : $\frac{1}{(3-\sqrt{8})} - \frac{1}{(\sqrt{8}-\sqrt{7})} + \frac{1}{(\sqrt{7}-\sqrt{6})} - \frac{1}{(\sqrt{6}-\sqrt{5})} + \frac{1}{(\sqrt{5}-2)} = 5$.
13. यदि $x = \frac{\sqrt{3}}{2}$ हो, तो $\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}}$ का मान ज्ञात कीजिए. (एस०एस०सी० परीक्षा, 2002)
14. यदि $x = (7+4\sqrt{3})$ हो, तो (i) $\left(x^2 + \frac{1}{x^2}\right)$ (ii) $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ के मान ज्ञात कीजिए.
15. यदि $x^a = y^b = z^c$ तथा $xyz = 1$ हो, तो सिद्ध करो कि $a+b+c = 0$.
16. यदि $3^x = 5^y = (75)^z$ हो, तो सिद्ध करो कि $z = \frac{xy}{(2x+y)}$.
17. यदि $x = 10^{0.48}$, $y = 10^{0.7}$ तथा $x^z = y^2$ हो, तो z का मान ज्ञात करो.
18. यदि $a+b+c = 0$ हो, तो सिद्ध करो कि $\frac{1}{(x^a + x^{-b} + 1)} + \frac{1}{(x^b + x^{-c} + 1)} + \frac{1}{(x^c + x^{-a} + 1)} = 1$.
19. $\left(\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \frac{1}{10.13} + \frac{1}{13.16}\right) = ?$ (एस०एस०सी० परीक्षा, 2007)
20. $\left(\frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90}\right) = ?$
21. $\left(\frac{1}{35} + \frac{1}{63} + \frac{1}{99} + \frac{1}{143} + \frac{1}{195} + \frac{1}{255}\right) = ?$
22. $\left(\frac{1}{54} + \frac{1}{108} + \frac{1}{180} + \frac{1}{270} + \frac{1}{378}\right) = ?$
23. $\left(\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} + \frac{1}{110} + \frac{1}{132}\right) = ?$ (असिस्टेंट ग्रेड परीक्षा, 2000)
24. $\left(\frac{1 \cdot 2 \cdot 4 + 2 \cdot 4 \cdot 8 + 3 \cdot 6 \cdot 12 + \dots}{1 \cdot 3 \cdot 9 + 2 \cdot 6 \cdot 18 + 3 \cdot 9 \cdot 27 + \dots}\right)^{\frac{1}{3}}$ का मान ज्ञात कीजिए.

उत्तरमाला (प्रश्नमाला 9B)

1. 1 2. 3.464 3. 0.072 4. 3 5. 0.707 6. $a = \frac{3}{7}, b = \frac{4}{21}$ 7. 0.102 8. 0.415 9. 7.07
 10. 5.398 13. $\sqrt{3}$ 14. (i) 194 (ii) 4 17. $2\frac{11}{12}$ 19. $\frac{5}{16}$ 20. $\frac{3}{20}$ 21. $\frac{6}{85}$ 22. $\frac{5}{126}$ 23. $\frac{11}{12}$ 24. $\frac{2}{3}$

दिये गए प्रश्नों के हल प्रश्नमाला 9B

$$1. x = \frac{1}{(2+\sqrt{3})} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})} = \frac{(2-\sqrt{3})}{(4-3)} = (2-\sqrt{3}).$$

$$\therefore x^2 = (2-\sqrt{3})^2 = (4+3-4\sqrt{3}) = (7-4\sqrt{3})$$

$$\text{तथा } x^3 = (7-4\sqrt{3})(2-\sqrt{3}) = (14+12-15\sqrt{3}) = (26-15\sqrt{3}).$$

$$\therefore \frac{(3x-x^3)}{(1-3x^2)} = \frac{(6-3\sqrt{3})-(26-15\sqrt{3})}{\{1-(21-12\sqrt{3})\}} = \frac{(-20+12\sqrt{3})}{(-20+12\sqrt{3})} = 1.$$

$$\begin{aligned} 2. \text{ दिया गया व्यंजक} &= \frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})} + \frac{(2+\sqrt{3})}{2\sqrt{3}} = \frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})} \times \frac{2\sqrt{3}}{(2+\sqrt{3})} \\ &= \frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})} \times \frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}+\sqrt{2})} \times \frac{2\sqrt{3}}{(2+\sqrt{3})} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})} = \frac{(\sqrt{6}+\sqrt{2})^2 \times (4\sqrt{3}-6)}{(6-2) \times (4-3)} \\ &= \frac{(6+2+2\sqrt{12}) \times (4\sqrt{3}-6)}{4 \times 1} = \frac{(8+4\sqrt{3})(4\sqrt{3}-6)}{4} \\ &= \frac{4(2+\sqrt{3})(4\sqrt{3}-6)}{4} = (2+\sqrt{3})(4\sqrt{3}-6) = (8\sqrt{3}-12+12-6\sqrt{3}) = 2\sqrt{3} \\ &= (2 \times 1.732) = (3.464). \end{aligned}$$

$$\begin{aligned} 3. \text{ दिया गया व्यंजक} &= \frac{(2-\sqrt{3})}{(2+\sqrt{3})} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})} = \frac{(2-\sqrt{3})^2}{(4-3)} = (4+3-4\sqrt{3}) \\ &= (7-4\sqrt{3}) = (7-4 \times 1.732) = (7-6.928) = 0.072. \end{aligned}$$

$$\begin{aligned} 4. \text{ दिया गया व्यंजक} &= 3 + \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{(3+\sqrt{3})} \times \frac{(3-\sqrt{3})}{(3-\sqrt{3})} - \frac{1}{(3-\sqrt{3})} \times \frac{(3+\sqrt{3})}{(3+\sqrt{3})} \\ &= 3 + \frac{\sqrt{3}}{3} + \frac{(3-\sqrt{3})}{(9-3)} - \frac{(3+\sqrt{3})}{(9-3)} = 3 + \frac{\sqrt{3}}{3} + \frac{(3-\sqrt{3})}{6} - \frac{(3+\sqrt{3})}{6} \\ &= 3 + \frac{\sqrt{3}}{3} + \frac{(3-\sqrt{3})-(3+\sqrt{3})}{6} = 3 + \frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{3} = 3. \end{aligned}$$

$$5. \frac{\sqrt{3}}{\sqrt{6}} = \frac{\sqrt{3}}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{3} \times \sqrt{6}}{6} = \frac{\sqrt{18}}{6} = \frac{\sqrt{2 \times 9}}{6} = \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2} = \frac{1.414}{2} = 0.707.$$

$$\begin{aligned} 6. \frac{(2\sqrt{6}-\sqrt{5})}{(3\sqrt{5}-2\sqrt{6})} &= \frac{(2\sqrt{6}-\sqrt{5})}{(3\sqrt{5}-2\sqrt{6})} \times \frac{(3\sqrt{5}+2\sqrt{6})}{(3\sqrt{5}+2\sqrt{6})} = \frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{(45-24)} \\ &= \frac{(6\sqrt{30}-2\sqrt{30}+24-15)}{21} = \frac{(4\sqrt{30}+9)}{21} = \left(\frac{9}{21} + \frac{4\sqrt{30}}{21} \right) = \left(\frac{3}{7} + \frac{4\sqrt{30}}{21} \right) \\ \therefore (a+\sqrt{30}b) &= \left(\frac{3}{7} + \sqrt{30} \times \frac{4}{21} \right) \Rightarrow a = \frac{3}{7} \text{ तथा } b = \frac{4}{21}. \end{aligned}$$

$$\begin{aligned}
 7. \quad \frac{(3-\sqrt{5})}{(3+2\sqrt{5})} &= \frac{(3-\sqrt{5})}{(3+2\sqrt{5})} \times \frac{(3-2\sqrt{5})}{(3-2\sqrt{5})} = \frac{(3-\sqrt{5})(3-2\sqrt{5})}{(9-20)} \\
 &= \frac{(9+10)-6\sqrt{5}-3\sqrt{5}}{-11} = \frac{19-9\sqrt{5}}{-11} = \left(\frac{9}{11} \times \sqrt{5} - \frac{19}{11} \right) \\
 &= \frac{(9\sqrt{5}-19)}{11} = \frac{(9 \times 2.236-19)}{11} = \frac{(20.124-19)}{11} = \frac{1.124}{11} = 0.102.
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \text{दिया गया व्यंजक} &= \frac{1}{\{(3+\sqrt{5})-2\sqrt{2}\}} \times \frac{(3+\sqrt{5})+2\sqrt{2}}{(3+\sqrt{5})+2\sqrt{2}} \\
 &= \frac{(3+\sqrt{5})+2\sqrt{2}}{(3+\sqrt{5})^2-8} = \frac{(3+\sqrt{5})+2\sqrt{2}}{(9+5+6\sqrt{5}-8)} = \frac{3+\sqrt{5}+2\sqrt{2}}{6+6\sqrt{5}} \\
 &= \frac{(3+\sqrt{5}+2\sqrt{2})}{6(\sqrt{5}+1)} \times \frac{(\sqrt{5}-1)}{(\sqrt{5}-1)} = \frac{3\sqrt{5}-3+5-\sqrt{5}+2\sqrt{10}-2\sqrt{2}}{6(5-1)} \\
 &= \frac{2+2\sqrt{5}+2\sqrt{10}-2\sqrt{2}}{24} = \frac{(1+\sqrt{5}+\sqrt{10}-\sqrt{2})}{12} \\
 &= \frac{(1+2.236+3.162-1.414)}{12} = \frac{4.984}{12} = 0.415.
 \end{aligned}$$

$$\begin{aligned}
 9. \quad \text{दिया गया व्यंजक} &= (\sqrt{9 \times 7} + \sqrt{4 \times 7} - \sqrt{25 \times 7} + \sqrt{81 \times 2} - \sqrt{16 \times 2}) \\
 &= (3\sqrt{7} + 2\sqrt{7} - 5\sqrt{7} + 9\sqrt{2} - 4\sqrt{2}) = 5\sqrt{2} = (5 \times 1.414) = 7.07.
 \end{aligned}$$

$$\begin{aligned}
 10. \quad \text{दिया गया व्यंजक} &= \frac{15}{\sqrt{10} + \sqrt{4 \times 5} + \sqrt{4 \times 10} - \sqrt{5} - \sqrt{16 \times 5}} \\
 &= \frac{15}{(\sqrt{10} + 2\sqrt{5} + 2\sqrt{10} - \sqrt{5} - 4\sqrt{5})} = \frac{15}{(3\sqrt{10} - 3\sqrt{5})} \\
 &= \frac{15}{3(\sqrt{10} - \sqrt{5})} \times \frac{(\sqrt{10} + \sqrt{5})}{(\sqrt{10} + \sqrt{5})} = \frac{5 \times (\sqrt{10} + \sqrt{5})}{(10-5)} = (\sqrt{10} + \sqrt{5}) \\
 &= (3.162 + 2.236) = 5.398.
 \end{aligned}$$

$$11. \quad \frac{7\sqrt{3}}{(\sqrt{10} + \sqrt{3})} = \frac{7\sqrt{3}}{(\sqrt{10} + \sqrt{3})} \times \frac{(\sqrt{10} - \sqrt{3})}{(\sqrt{10} - \sqrt{3})} = \frac{7\sqrt{3}(\sqrt{10} - \sqrt{3})}{(10-3)} = \sqrt{3}(\sqrt{10} - \sqrt{3}) = (\sqrt{30} - 3).$$

$$\frac{2\sqrt{5}}{(\sqrt{6} + \sqrt{5})} = \frac{2\sqrt{5}}{(\sqrt{6} + \sqrt{5})} \times \frac{(\sqrt{6} - \sqrt{5})}{(\sqrt{6} - \sqrt{5})} = \frac{2\sqrt{5}(\sqrt{6} - \sqrt{5})}{(6-5)} = 2\sqrt{5}(\sqrt{6} - \sqrt{5}) = 2\sqrt{30} - 10.$$

$$\frac{3\sqrt{2}}{(\sqrt{15} + 3\sqrt{2})} = \frac{3\sqrt{2}}{(\sqrt{15} + 3\sqrt{2})} \times \frac{(\sqrt{15} - 3\sqrt{2})}{(\sqrt{15} - 3\sqrt{2})} = \frac{3\sqrt{2}(\sqrt{15} - 3\sqrt{2})}{(15-18)} = -\sqrt{2}(\sqrt{15} - 3\sqrt{2}) = (6 - \sqrt{30}).$$

$$\begin{aligned}
 \therefore \text{बायों पक्ष} &= (\sqrt{30} - 3) - (2\sqrt{30} - 10) - (6 - \sqrt{30}) \\
 &= (\sqrt{30} - 2\sqrt{30} + \sqrt{30}) - 3 + 10 - 6 = 1 = \text{दायों पक्ष}.
 \end{aligned}$$

$$12. \frac{1}{(3-\sqrt{8})} = \frac{1}{(3-\sqrt{8})} \times \frac{(3+\sqrt{8})}{(3+\sqrt{8})} = \frac{(3+\sqrt{8})}{(9-8)} = (3+\sqrt{8}).$$

$$\frac{1}{(\sqrt{8}-\sqrt{7})} = \frac{1}{(\sqrt{8}-\sqrt{7})} \times \frac{(\sqrt{8}+\sqrt{7})}{(\sqrt{8}+\sqrt{7})} = \frac{(\sqrt{8}+\sqrt{7})}{(8-7)} = (\sqrt{8}+\sqrt{7}).$$

$$\text{इसी प्रकार, } \frac{1}{(\sqrt{7}-\sqrt{6})} = (\sqrt{7}+\sqrt{6}), \frac{1}{(\sqrt{6}-\sqrt{5})} = (\sqrt{6}+\sqrt{5}), \frac{1}{(\sqrt{5}-2)} = (\sqrt{5}+2).$$

$$\therefore \text{ दिया गया व्यंजक } = (3+\sqrt{8}) - (\sqrt{8}+\sqrt{7}) + (\sqrt{7}+\sqrt{6}) - (\sqrt{6}+\sqrt{5}) + (\sqrt{5}+2) = (3+2) = 5.$$

$$\begin{aligned} 13. \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} &= \frac{(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x}-\sqrt{1-x})} \times \frac{(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x}+\sqrt{1-x})} = \frac{(\sqrt{1+x}+\sqrt{1-x})^2}{(1+x)-(1-x)} \\ &= \frac{(1+x+1-x)+2\sqrt{1+x} \cdot \sqrt{1-x}}{2x} = \frac{(1+\sqrt{1-x^2})}{x} \\ &= \frac{1+\sqrt{1-\frac{3}{4}}}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1+\sqrt{\frac{1}{4}}}{(\sqrt{3}/2)} = \frac{\left(1+\frac{1}{2}\right)}{(\sqrt{3}/2)} \quad \left[\because x = \frac{\sqrt{3}}{2} \right] \\ &= \left(\frac{3}{2} \times \frac{2}{\sqrt{3}} \right) = \sqrt{3}. \end{aligned}$$

$$14. \frac{1}{x} = \frac{1}{(7+4\sqrt{3})} \times \frac{(7-4\sqrt{3})}{(7-4\sqrt{3})} = \frac{(7-4\sqrt{3})}{(49-48)} = (7-4\sqrt{3}).$$

$$\therefore \left(x + \frac{1}{x} \right) = (7+4\sqrt{3}) + (7-4\sqrt{3}) = 14.$$

$$\Rightarrow \left(x + \frac{1}{x} \right)^2 = (14)^2 \Rightarrow x^2 + \frac{1}{x^2} + 2x \cdot \frac{1}{x} = 196 \Rightarrow x^2 + \frac{1}{x^2} + 2 = 196 \Rightarrow \left(x^2 + \frac{1}{x^2} \right) = 194.$$

$$\text{अब, } \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 = x + \frac{1}{x} + 2\sqrt{x} \times \frac{1}{\sqrt{x}} = \left(x + \frac{1}{x} + 2 \right) = (14+2) = 16.$$

$$\text{अतः } \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) = 4.$$

$$15. \text{ माना } x^a = y^b = z^c = k. \text{ तब } x = k^{\frac{1}{a}}, y = k^{\frac{1}{b}}, z = k^{\frac{1}{c}}.$$

$$xyz = 1 \Rightarrow k^{\frac{1}{a}} \times k^{\frac{1}{b}} \times k^{\frac{1}{c}} = 1 = k^0$$

$$\Rightarrow k^{a+b+c} = k^0 \Rightarrow a+b+c = 0.$$

$$16. \text{ माना } 3^x = 5^y = (75)^z = K. \text{ तब } 3 = K^{\frac{1}{x}}, 5 = K^{\frac{1}{y}}, 75 = K^{\frac{1}{z}}.$$

$$\text{अब, } 75 = (3 \times 5^2) \Rightarrow K^{\frac{1}{z}} = K^{\frac{1}{x}} \times K^{\frac{2}{y}} = K^{\left(\frac{1}{x} + \frac{2}{y} \right)} \Rightarrow \frac{1}{z} = \left(\frac{1}{x} + \frac{2}{y} \right) = \frac{(y+2x)}{xy} \Rightarrow z = \frac{xy}{(y+2x)}.$$

$$17. x^z = y^2 \Rightarrow (10^{0.48})^z = (10^{0.7})^2 \Rightarrow 10^{(0.48)z} = 10^{(0.7 \times 2)} = 10^{1.4}$$

$$\Rightarrow 0.48 \times z = 1.4 \Rightarrow z = \frac{1.40}{0.48} = \frac{140}{48} = \frac{35}{12} = 2\frac{11}{12}$$

$$\text{अतः } z = 2\frac{11}{12}$$

$$\begin{aligned} 18. \text{ दिया गया व्यंजक} &= \frac{1}{(x^a + x^{-b} + 1)} + \frac{1}{(x^b + x^{-c} + 1)} + \frac{1}{(x^c + x^{-a} + 1)} \\ &= \frac{x^{-a}}{\{1 + x^{(-a-b)} + x^{-a}\}} + \frac{x^c}{(x^{b+c} + 1 + x^c)} + \frac{1}{(x^c + x^{-a} + 1)} \\ &= \frac{x^{-a}}{(1 + x^c + x^{-a})} + \frac{x^c}{(x^{-a} + 1 + x^c)} + \frac{1}{(x^c + x^{-a} + 1)} \quad [\because -(a+b)=c, (b+c)=-a] \\ &= \frac{1}{(1 + x^{-a} + x^c)} + \frac{x^{-a}}{(1 + x^{-a} + x^c)} + \frac{x^c}{(1 + x^{-a} + x^c)} = \frac{(1 + x^{-a} + x^c)}{(1 + x^{-a} + x^c)} = 1. \end{aligned}$$

$$\begin{aligned} 19. \text{ दिया गया व्यंजक} &= \frac{1}{3} \cdot \left[\left(1 - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{10}\right) + \left(\frac{1}{10} - \frac{1}{13}\right) + \left(\frac{1}{13} - \frac{1}{16}\right) \right] \\ &= \frac{1}{3} \left(1 - \frac{1}{16}\right) = \left(\frac{1}{3} \times \frac{15}{16}\right) = \frac{5}{16}. \end{aligned}$$

$$\begin{aligned} 20. \text{ दिया गया व्यंजक} &= \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \left(\frac{1}{6} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{8}\right) + \left(\frac{1}{8} - \frac{1}{9}\right) + \left(\frac{1}{9} - \frac{1}{10}\right) \\ &= \left(\frac{1}{4} - \frac{1}{10}\right) = \frac{3}{20}. \end{aligned}$$

$$\begin{aligned} 21. \text{ दिया गया व्यंजक} &= \frac{1}{2} \cdot \left[\left(\frac{1}{5} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{9}\right) + \left(\frac{1}{9} - \frac{1}{11}\right) + \left(\frac{1}{11} - \frac{1}{13}\right) + \left(\frac{1}{13} - \frac{1}{15}\right) + \left(\frac{1}{15} - \frac{1}{17}\right) \right] \\ &= \frac{1}{2} \cdot \left(\frac{1}{5} - \frac{1}{17}\right) = \frac{1}{2} \times \frac{12}{85} = \frac{6}{85}. \end{aligned}$$

$$\begin{aligned} 22. \text{ दिया गया व्यंजक} &= \frac{1}{3} \cdot \left[\left(\frac{1}{6} - \frac{1}{9}\right) + \left(\frac{1}{9} - \frac{1}{12}\right) + \left(\frac{1}{12} - \frac{1}{15}\right) + \left(\frac{1}{15} - \frac{1}{18}\right) + \left(\frac{1}{18} - \frac{1}{21}\right) \right] \\ &= \frac{1}{3} \left(\frac{1}{6} - \frac{1}{21}\right) = \left(\frac{1}{3} \times \frac{5}{42}\right) = \frac{5}{126}. \end{aligned}$$

$$23. \text{ दी गई श्रेणी} = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \left(\frac{1}{10} - \frac{1}{11}\right) + \left(\frac{1}{11} - \frac{1}{12}\right) = \left(1 - \frac{1}{12}\right) = \frac{11}{12}.$$

$$\begin{aligned} 24. \text{ दिया गया व्यंजक} &= \left(\frac{8 + 8 \times 8 + 8 \times 27 + \dots}{27 + 27 \times 8 + 27 \times 27 + \dots} \right)^{\frac{1}{3}} = \left\{ \frac{8(1 + 8 + 27 + \dots)}{27(1 + 8 + 27 + \dots)} \right\}^{\frac{1}{3}} \\ &= \left(\frac{8}{27}\right)^{\frac{1}{3}} = \left\{ \left(\frac{2}{3}\right)^3 \right\}^{\frac{1}{3}} = \left(\frac{2}{3}\right)^{3 \times \frac{1}{3}} = \left(\frac{2}{3}\right)^1 = \left(\frac{2}{3}\right). \end{aligned}$$