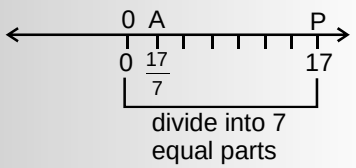
Rational Numbers

A number that can be expressed in the form $\frac{p}{q}$ (p: integer, q: integer) but $q \neq 0$

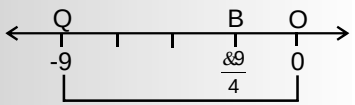
Rational number on number line

(i) To represent $\frac{17}{7}$



divide into 7 equal parts

(ii) $\frac{-9}{4}$



divide into 4 equal parts

(i) Positive rational number :

p and **q** both are positive or negative

eg : $\frac{3}{5}$; $\frac{4}{5}$

(ii) Negative rational number :

p and **q** both are have opposite sign

eg : $\frac{-3}{5}$ or $\frac{4}{-5}$

(iii) If $p = 0$ the $\frac{p}{q} = 0$ eg $\frac{0}{8}, \frac{0}{-6}$

* Natural numbers + whole numbers + integers + fractions are rational no.s eg : $\frac{1}{1}, \frac{0}{1}, \frac{5}{6}, \frac{-9}{1000}$ e.t.c.

* A fraction is a rational number, but a rational number may or may not be a fraction

eg :- $\frac{3}{5} \rightarrow$ rational no. + fraction ; $-\frac{3}{5} \rightarrow$ rational no but not a fraction

* There are infinite rational no's between two rational no's :

eg : Rational No. b/w $\frac{-3}{5}$ and $\frac{-1}{3}$; $-\frac{3}{5} < -\frac{9}{15}$; $-\frac{1}{3} < -\frac{5}{15} < -\frac{9}{15} < -\frac{7}{15} < -\frac{6}{15} < -\frac{5}{15}$

Comparison

*Two positive rational numbers can be compared as fractions

*(i) Compare the rational no.s ignoring their negative signs

(ii) Then reverse the order

eg : $\frac{7}{5} > \frac{5}{3} \Rightarrow \frac{-7}{5} < \frac{-5}{3}$

* A negative rational no. is always less then positive rational no.

eg : $\frac{-2}{7} > \frac{1}{2}$

Addition

Convert each of them into number with a positive denominator.

(i) $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ [when denominator are same]

eg : $\frac{-11}{5} + \frac{7}{5} = \frac{-11+7}{5} = \frac{-4}{5}$

(ii) $\frac{a}{b} + \frac{c}{a}$ [Find L. C. M. of b and d]

eg : $\frac{-3}{8} + \frac{5}{-12} = \frac{-3}{8} + \frac{(-5)}{12}$

L. C. M. of 8 and 12 = 24

$\frac{(-3)\times 3 + (-5)\times(2)}{24} = \frac{-19}{24}$

Subtraction

Add the additive inverse

ie $\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \left(\frac{-c}{d}\right)$

eg : Subtract $\frac{-11}{24}$ from $\frac{-5}{36}$

$= \frac{-5}{36} - \left(\frac{-11}{24}\right) = \frac{-5}{36} + \frac{11}{24}$

$= \frac{-10+33}{72} = \frac{23}{72}$

Multiplication

Product of Rational No.

$= \frac{\text{product of numerators}}{\text{product of denominators}}$

eg : $\frac{-3}{5} \times \frac{2}{7} \times \frac{(-5)}{8}$

$= \frac{(-3)\times 2\times(-5)}{8\times 7\times 8} = \frac{3}{28}$

Division

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ such that $\frac{c}{d} \neq 0$

eg : $3\frac{1}{17} \div \frac{33}{-34} = \frac{52}{17} \times \frac{-34}{33}$

$= \frac{-104}{33} = -3\frac{5}{33}$

Multiplicative inverse of

$\frac{p}{q}$ is $\frac{q}{p}$ where $p, q \neq 0$

eg : Reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$

Additive Inverse of

$\frac{p}{q}$ is $\frac{-p}{q}$ [ie $\frac{p}{q} + \frac{(-p)}{q} = 0$]

eg : Additive inverse of

(i) $\frac{-15}{2}$ is $\frac{15}{2}$ (ii) $\frac{4}{9}$ is $\frac{-4}{9}$

Simplest (Lowest) or standard form

If denominator is a positive integer and denominator and numerator have no common factor other then 1

eg : Standard form of $\frac{6}{-15}$ is $\frac{6-(-3)}{-15-(-3)} = \frac{2}{-5}$

divide by H.C.F of 6 and 15

Equivalent Rational no.

$\frac{p}{q} = \frac{p\times m}{q\times m}$ also $\frac{p}{q} = \frac{p-m}{q-m}$ where m = non-zero integer

eg $\underbrace{\frac{2}{3} = \frac{2\times 5}{3\times 5} = \frac{10}{15}}_{\text{equivalent Rational no.}}$; $\underbrace{\frac{5}{9} = \frac{5\times -2}{9\times -2} = \frac{-10}{-18}}_{\text{equivalent Rational no.}}$

eg: $\frac{-3}{11} = \frac{9}{x}$; $(-3)\times x = 9\times 11$; $x = \frac{9\times 11}{(-3)}$; $x = -33$