PERMUTATION & COMBINATION

Learning Objectives

- Factorial
- Permutation
- Combination

Factorial

The factorial, symbolized by an exclamation mark (!), is a quantity defined for all integers greater than or equal to 0. Mathematically, the formula for the factorial is as follows.

If n is an integer greater than or equal to 1, then
n! = n (n – 1) (n – 2) (n – 3) …. (3)(2)(10).

Example:

$1! = 1,\ 2! = 2,\ 3! = 6,\ 4! = 4.3.2.1 = 24,\ 5! = 5\times4\times3\times2\times1 = 120$

$61 = 6\times5\times4\times3\times2\times1 = 720,\ 7! =$ 5040 *and* $8! = 40320$ *etc.*

The special case 0! is defined to have value 0! = 1.

Permutation

The different arrangements which can be made by taking some or all of the given things or objects at a time is called Permutation.
All permutations (arrangements] made with the letters a, b, c by taking two at a time will be (ab, be, ca, ba, ac, cb).

Number of Permutations: Number of all permutations of n things, taking r at a time is:

$$^{n}P_{r} = \frac{n!}{n-r!} = n(n-1)(n-2)(n-3)\ldots\ldots(n-r+1)$$

Note: This is valid only when repetition is not allowed.

- Permutation of n different things taken rat a time. When repetition is Allowed: $n\times n\times n$ .....*r times* $= n^{r}$ ways
- Permutation of n things taking all n things at a time = n!
- Out of n objects n_1 are alike one type, n_2 are alike another type, n_3 are alike third type, nr nr are alike another type such that $(n_1 + n_2 + n_3 \ldots\ldots nr) = n$

  Number of permutations of these n things are $= \frac{n!}{n_1!n_2!\ldots\ldots n_r!}$

Combination

Each of the different selections or groups which are made by taking some or all of a number of things or objects at a time is called combination.
The number of combinations of n dissimilar things taken r at a time is denoted by $^{n}C_{r}$ *or* $C(n,r)$.

$$^{n}C_{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2).....(n-r+1)}{1.2.3.....r}$$

Also $^{n}C_{0} = 1;\ ^{n}C_{n} = 1;$

Note : (i) $^{n}C_{r} + {}^{n}C_{r-1} = {}^{(n+1)}C_{r}$

Important Formula

- In a group of n-members if each member offers a shake hand to the remaining members then the total number of handshakes ${}^{n}C_{2} = \frac{n(n-1)}{1.2} = \frac{n(n-1)}{2}$
- The number of diagonals in a regular polygon of 'n' sides is $\frac{n(n-3)}{2}$
- From a group of m-men and n-women, if a committee of remembers $(r \le m+n)$ to be formed, then the number of ways it can be done is equal to ${}^{(m+n)}C_{r}$.
- The number of ways a group of r-boys (men) and s-girls (women) can be made out of m boys (men) and n-girls (women) is equal to $({}^{m}C_{r} \times {}^{n}C_{s})$.
- From a group of m-boys and n-girls the number of different ways that a committee of remembers can be formed so that the committee will have at least one girl is $C_{r} - {}^{m}C_{r}$

Commonly Asked Questions

- If a die is cast and then a coin is tossed, find the number of all possible outcomes.
  (a) 11 (b) 12
  (c) 10 (d) 15
  (e) None of these

  Answer: (b)
  Explanation: A die can fall in 6 different ways showing six different points 1, 2, 3, 4, 5, 6,... and a coin can fall in 2 different ways showing head (H) or tail (T).
  $\therefore$ The number of all possible outcomes from a die and a coin $= 6 \times 2 = 12.$

- There are 6 trains running between indore and Bhopal. In how many ways can a man go from indore to Bhopla and by a different train?
  (a) 25 (b) 35
  (c) 30 (d) 20
  (e) None of these

  Answer (c)
  Explanation: A man can go from Indore to Bhopal in 6 ways by any one of the fc trains available. Then he can return from Bhopal to Indore in 5 ways by the remaining 3 trains, since he cannot return by the same train by which he goes to Bhopal from Indore.
  Thus, the required number of ways $= 6 \times 5 = 30.$

- Find the value of 9?
  (a) 504 (b) 309
  (c) 405 (d) 600
  (e) None of these

  Answer (a)
  Explanation:
  $${}^{9}P_{30} = \frac{9!}{6!} \qquad \left(\therefore {}^{n}P_{r} = \frac{n!}{(n-r)!} \right)$$
  $$= \frac{9 \times 8 \times 7 \times 6!}{6!} = 9 \times 8 \times 7 = 504$$

- In how many different ways can the letters of the word 'stress' be arranged?
  (a) 120 (b) 420
  (c) 840 (d) 240
  (e) None of these

Answer: (a)
Explanation: The word 'STRESS' has a total of six Setters (n = 6) out of which a group of three letters are same (a = 3)

$\therefore$ The letters can be arranged in $\frac{n!}{a!} = \frac{6!}{3!} = \frac{720}{6} = 120$.

- Find the value of ${}^{8}C_{3}$
  (a) 56 (b) 8!
  (c) 65 (d) 3^8
  (e) None of these

Answer: (a)
Explanation: ${}^{8}C_{3} = \frac{8!}{3!.5!} = \frac{8 \times 7 \times 6}{6} = 56$

Directions: Study the given information carefully and answer the questions that follow:

A committee of five members is to be formed out of 3 trainees, 4 professors and 6 research associates. In how many different ways can this be done if–

- The committee should have all 4 professors and 1 research associate or all 3 trainees and 2 professors?
  (a) 13 (b) 12
  (c) 24 (d) 35
  (e) None of these

Answer: (b)
Explanation: Five member team with 4 professors and 1 research associate can be selected in ${}^{4}C_{4} \times {}^{6}C_{1} = 1 \times 6 = 6$ ways. Five member team with 3 trainees and 2 professors can be selected in ${}^{3}C_{3} \times {}^{4}C_{1} = 1 \times 6 = 6$ ways.
$\therefore$ Total number of ways of selecting the committee = 6 + 6 = 12.

- The committee should have 2 trainees and 3 research associates?
  (a) 15 (b) 45
  (c) 60 (d) 75
  (e) None of these

Answer: (c)
Explanation: 2 trainees and 3 research associates can be selected in ${}^{3}C_{2} \times {}^{6}C_{3} = 3 \times 20 = 60$ ways.