CHAPTER 2

LINEAR EQUTIONS IN TWO VARIABLES

- **Linear Equations in Two Variables:**

  An equation of the form $ax + by + c = 0$ where a, b, c $\in$ R(real numbers) and $a \neq 0, b \neq 0$ and x, y are variables is called a linear equation in two variables.

  Examples : Each of the following equations is a linear equation :

(i) $4x + 7y = 13$

(ii) $2x - 5y = 36$

Inconsistent System : A system consisting of two simultaneous linear equations is said to be inconsistent, if it has no solution at all.

E.g.: Consider the system of equations: $x + y = 9$ & $3x + 3y = 5$.

Clearly, there are no values of x and y which may simulatneously satisfy the given equations. So, the system given above is inconsistent.

- The graphs of $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + C_2 = 0$ will be :

(i) Parallel, if the system has no Solution ;

(ii) Coincident, if the system has infinite number of solutions ;

(iii) Intersecting, if the system has a unique solution.

Exercise
LEVEL - 1

1. If $11x - 13 = -2x + 78$, then $x = ?$

   (a) 7 (b) 8
   (c) 6 (d) 4

2. If $2x + 3y = 29$ and $y = x + 3$, what is the value of x ?

   (a) 5 (b) 6
   (c) 4 (d) 7.

3. If $2x + 3y = 5$ and $x = -2$, then the value of y is :

   (a) $\frac{1}{3}$ (b) 3
   (c) 1 (d) 9

4. The value of $x + y$ in the solution of the equations $\frac{x}{4}+\frac{y}{3}=\frac{5}{12}$ and $\frac{x}{2}+y=1$

   (a) $\frac{1}{2}$ (b) 2
   (c) $\frac{5}{2}$ (d) $\frac{3}{2}$

5. If $2x + 3y = 12$ and $3x - 2y = 5$, then x and y must have the values :

   (a) 2 and 3 (b) 2 and -3
   (c) 3 and -2 (d) 3 and 2

6. The equations $ax + b = 0$ and $cx + d = 0$ are consistent, if :

   (a) $ad = bc$ (b) $ad + bc = 0$
   (c) $ab - cd = 0$ (d) $ab + cd = 0$

7. The equations $2x + y = 5$ and $x + 2y = 4$ are

   (a) consistent and have infinitely many solutions
   (b) consistent and have a unique solution.
   (c) inconsistent
   (d) none of these

Exercise
LEVEL - 2

1. The cost of 2 sarees and 4 shirts is ₹ 1600 while 1 saree and 6 shirts cost the same. The cost of 12 shirts is :

   (a) ₹ 12,000 (b) ₹ 24,000
   (c) ₹ 48,000
   (d) ₹ Can't be determined

2. The system of equations $kx - y = 2$ and $6x - 2y = 3$ has a unique solution when :

   (a) K = 0 (b) K ≠ 0
   (c) K = 3 (d) K ≠ 3

3. The value of y in the solution of the equation $2^{x+y} = 2^{x-y} = \sqrt{8}$ is :

   (a) 0 (b) $\frac{1}{4}$
   (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

4. The solutions of the equations $\frac{3x-y+1}{3}=\frac{2x+y+2}{5}=\frac{3x+2y+1}{6}$ is :

   (a) $x = 2, y = 1$ (b) $x = 1, y = 1$
   (c) $x = -1, y = -1$ (d) $x = 1, y = 2$

5. If $x + 2y \le 3$, $x > 0$ and $y > 0$, then one of the solutions is :
(a) $x = -1, y = 2$ (b) $x = 2, y = 1$
(c) $x = 1, y = 1$ (d) $x = 0, y = 0$

6. A purse contains 25 paise and 10 paise coins. The total amount in the purse is ₹ 8.25. If the number of 25 paise coins is one- third of the number of 10 paise coins in the purse, then the total number of coins in the purse :
(a) 30 (b) 40
(c) 45 (d) 60

7. The value of k for which the system of equations $x + 2y = 5$, $3x + ky + 15 = 0$ has no solution, is :
(a) 6 (b) -6
(c) 2 (d) 4

8. The equations $2x - 5y = 9$ and $8x - 20y = 36$ have :
(a) no common solution
(b) exactly one common solution
(c) exactly two common solutions
(d) more than two common solutions

9. The difference between two numbers is 5 and the difference between their squares is 65. The larger number is :
(a) 9 (b) 10
(c) 11 (d) 12

10. The number of solutions of the equations $x+\frac{1}{y}=2$ and $2xy - 3y = -2$ is :
(a) 0 (b) 1
(c) 2
(d) None of these

Exercise
LEVEL - 3

1. If $2^a + 3^b = 17$ and $2^{a+2} - 3^{b+1} = 5$, then:
(a) a = 2, b = 3 (b) a = -2, b = 3
(c) a = 2, b = -3 (d) a = 3, b = 2

2. The solution to the system of equations $|x + y| = 1$ and $x - y = 0$ is given by :
(a) $x=y=\frac{1}{2}$ (b) $x=y=-\frac{1}{2}$
(c) $x=y=\frac{1}{2}$ or $x=y=-\frac{1}{2}$
(d) $x = 1, y = 0$

Hints and Solutions :

LEVEL - 1

1.(a) $11x - 13 = -2x + 78$

$\Rightarrow 11x + 2x = 78 + 13$

$\Rightarrow 13x = 91$

$\Rightarrow x = \frac{91}{13} = 7$

2.(c) Putting $y = x + 3$ in $2x + 3y = 29$, we get,

$2x + 3(x + 3) = 29 \Rightarrow 2x + 3x + 9 = 29$

$\Rightarrow 5x = 29 - 9 = 20 \Rightarrow x = \frac{20}{5} = 4$

3.(b) Putting $x = -2$ in $2x + 3y = 5$, we get ;

$-4 + 3y = 5 \Rightarrow 3y = 5 + 4 = 9$

$\Rightarrow y = \frac{9}{3} = 3$

4.(d) Given equations are :

$3x + 4y = 5$ _______(i) and

$x + 2y = 2$ _______(ii)

(i) - 2× (ii): $x = 5 - 4 = 1$

$\therefore$ from (ii) $2y = 2 - x = 2 - 1 = 1 \Rightarrow y = \frac{1}{2}$

$\therefore x + y = 1 + \frac{1}{2} = \frac{3}{2}$

5.(d) $2x + 3y = 12$ _______(i)

$3x - 2y = 5$ _______(ii)

(i)× 2 + (ii)× 3, we get ; $x = 3$

putting $x = 3$ in (i), we get $2 \times 3 + 3y = 12$

$\Rightarrow 3y = 6 \Rightarrow y = 2$

$\therefore$ $x = 3$ and $y = 2$

6.(a) The equations are consistent if

$\frac{a}{c} = \frac{b}{d}$

i.e. ad = bc

7.(b) $2x + y = 5$ _______(i)

$x + 2y = 4$_______(ii)

On solving we get, $x = 2$, $y = 1$

Thus (b) is true

LEVEL - 2

1.(b) Let cost of 1 saree = ₹ x &

cost of 1 shirt = ₹ y

$\therefore$ $2x + 4y = 16000$ ....(i)

and $x + 6y = 16000$ ....(ii)

Multiplying (ii) by 2 and substracting (i) from it, we get,

$8y = 16000 \Rightarrow y = 2000$

$\therefore$ cost of 12 shirts = (₹ 2000× 12)

= ₹ 24000

2.(d) For a unique solution, we must have

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{k}{6} \neq \frac{-1}{-2} \Rightarrow k \neq \left(6 \times \frac{1}{2}\right) \Rightarrow k \neq 3$

3.(a) $2^{x+y} = 2^{x-y} = \sqrt{8} = 2^{3/2}$

$\Rightarrow x + y = \frac{3}{2}$ ....(i)

and $x - y = \frac{3}{2}$ .... (ii)

(i) - (ii)

$2y = 0 \Rightarrow y = 0$

4.(b) $\frac{3x - y + 1}{3} = \frac{2x + y + 2}{5}$

$\Rightarrow 5(3x - y + 1) = 3(2x + y + 2)$

$\Rightarrow 9x - 8y = 1$ ....(i)

and $\frac{3x - y + 1}{3} = \frac{3x + 2y + 1}{6}$

$= 2(3x - y + 1) = (3x + 2y + 1)$

$\Rightarrow 3x - 4y = -1$ ....(ii)

(i) - 2 × (ii):-

$(9 - 6)x - 8y + 8y = 1 - (-2) \Rightarrow x = 1$

putting $x = 1$ in (i) we get, $9 \times 1 - 8y = 1$

$\Rightarrow 8y = 8 \Rightarrow y = 1$

$\therefore x = 1, y = 1.$

5.(c) Here we will go through options.

in option (a) $x < 0$ and

in option (d) $x = 0$

hence (a) and (d) can't be the required answer because both does not satisfy the given condition i.e. $x > 0$.

Now option (b) $x = 2, y = 1$, then

$x + 2y = 2 + 2(1) = 4$ which is > 3

clearly, values of option (b) do not satisfy $x + 2y \leq 3$

option (c) $x = 1, y = 1$, then $x + 2y = 1 + 2 = 3 \leq 3$

So $x = 1$, y = 1 is one of the solutions.

6.(d) Let the number of 25 paise coins be x & that of 10 paise coins be y, then:

$$\frac{25}{100}x + \frac{10}{100}y = 8.25$$

$\Rightarrow 5x + 2y = 165$ ______(i)

and $x = \frac{1}{3}y \Rightarrow y = 3x$ ______(ii)

putting $y = 3x$ in (i), we get :

$5x + 6x = 165 \Rightarrow 11x = 165 \Rightarrow x = 15$

$\therefore$ from (ii), $y = 3x = 3 \times 15 = 45$

$\therefore$ Total number of coins in the purse $= x + y = 15 + 45 = 60$

7.(a) $a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$ will have no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{1}{3} = \frac{2}{k} \Rightarrow k = 6$

8.(d) The given equations are $2x - 5y = 9$ and $8x - 20y = 36 \Rightarrow 2x - 5y = 9$

Thus, there is one equation in two variables. So, the given equations have an infinite number of solutions.

9.(a) Let the numbers be x and y. Then, $x - y = 5$ and $x^2 - y^2 = 65$

$$\therefore \frac{x^2 - y^2}{x - y} = \frac{65}{5} \Rightarrow x + y = 13$$

solving $x - y = 5$ and $x + y = 13$, we get;

$x = 9$ and y = 4

$\therefore$ larger number = 9

10.(d) $x + \frac{1}{y} = 2 \Rightarrow \frac{1}{y} = 2 - x \Rightarrow y = \frac{1}{2 - x}$ ....(i)

and $2xy - 3y = -2$

$\Rightarrow y(2x - 3) = -2$ ......(ii)

putting $y = \frac{1}{2 - x}$ in (ii)

$\frac{2x - 3}{2 - x} = -2 \Rightarrow 2x - 3 = -4 + 2x$. this gives $1 = 0$

This is impossible So, there is no solution.

LEVEL - 3

1.(b) $2^a + 3^b = 17$ and

$2^{a+2} - 3^{b+1} = 5 \Rightarrow 2^2.2^a - 3.3^b = 5$

$\Rightarrow 4.2^a - 3.3^b = 5$

let $2^a = x$ & $3^b = y$ then

$x + y = 17$ ....(i)

$4x - 3y = 5$ ....(ii)

3 × (i) + (ii), we get

$7x = 56 \Rightarrow x = 8 \Rightarrow 2^a = 8 = 2^3 \Rightarrow$ $\boxed{a = 3}$

putting $x = 8$ in (i), we get

$y = 17 - 8 = 9 \Rightarrow 3^b = 9 = 3^2 \Rightarrow$ $\boxed{b = 2}$

$\therefore$ a = 3 and b = 2.

2.(c) Note that $|a| = 1$ means a = 1 or a = -1

So, $|x+y|=1 \Rightarrow x+y=1$ or $-(x+y)=1$

$\Rightarrow (x+y) = -1$

solving $x + y = 1$, $x - y = 0$, we get $x = \frac{1}{2}$ and $y = \frac{1}{2}$

solving $x + y = -1$, $x - y = 0$, we get $x = -1/2$ and $y = -1/2$

$\therefore x = y = \pm\frac{1}{2}$

Answer-Key

LEVEL - 1

1. (a)	2. (c)	3. (b)
4. (d)	5. (d)	6. (a)
7. (b)		

LEVEL - 2

1. (b)	2. (d)	3. (a)
4. (b)	5. (c)	6. (d)
7. (a)	8. (d)	
9. (a)	10. (d)	

LEVEL - 3

1. (d)	2. (c)