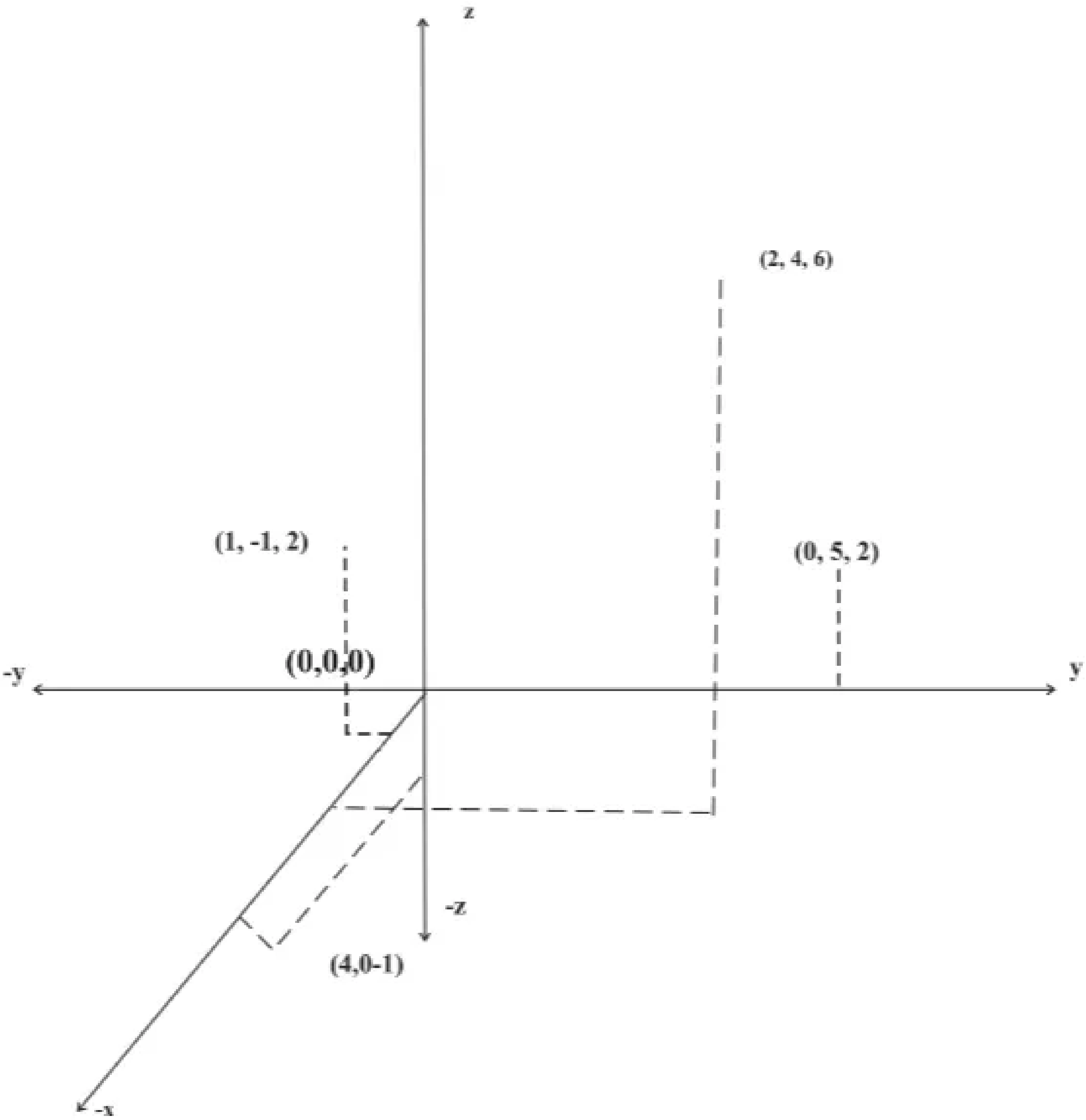
Exercise 12.1

Answer 1E.

Since we are moving along the $x-$axis a distance of 4 units in the positive direction, so $x-$coordinate of the point is 4.
and we are moving downward a distance of 3 units, so $z-$coordinate of the point is -3. Since the point lies on $xz-$plane, so $y-$coordinate of the point is zero.

Therefore required coordinates of the point are $\boxed{(4,0,-3)}$

Answer 2E.



Answer 3E.

Consider the following points:

$A(-4,0,-1)$, $B(3,1,-5)$ and $C(2,4,6)$.

The objective is to find which of the above points is closest to the *yz*-plane. Also find which point lies in the *xz*-plane.

The distance of a point $P(x,y,z)$ from the *yz*-plane is the absolute value of the *x*-coordinate of the point $P(x,y,z)$. Then, notice that,

- the absolute value of the *x*-coordinate of the point $A(-4,0,-1)$ is 4,
- the absolute value of the *x*-coordinate of the point $B(3,1,-5)$ is 3,
- the absolute value of the *x*-coordinate of the point $C(2,4,6)$ is 2.

Here, the absolute value of the *x*-coordinate of the point $C(2,4,6)$ is the smallest absolute value, so *C* is the point closest to the *yz*-plane.

Therefore, the point $\boxed{C(2,4,6)}$ is the point closest to the *yz*-plane.

The distance of a point $P(x,y,z)$ from the *xz*-plane is the absolute value of the *y*-coordinate of the point $P(x,y,z)$.

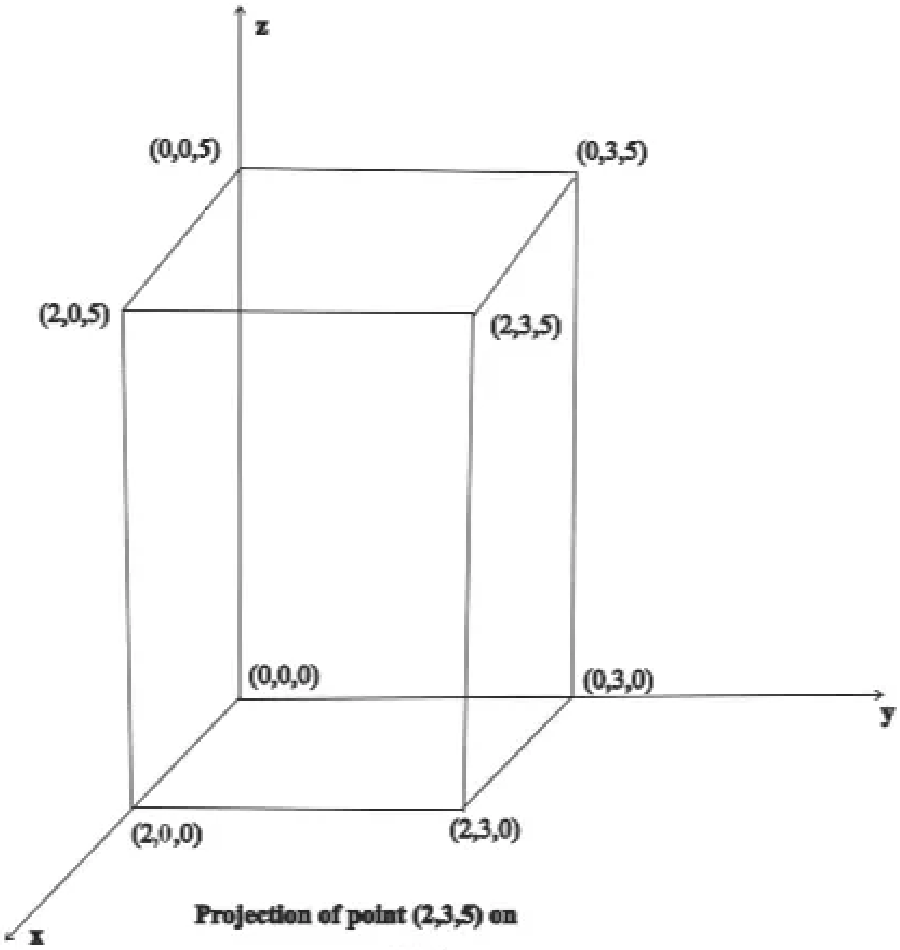
Hence, if a point lies on *xz*-plane, then the distance of that point from the *xz*-plane is zero, that is, the *y*-coordinate of that point will be 0.

Notice that, the absolute value of the *y*-coordinate of the point $A(-4,0,-1)$ is 0, so the distance from the point *A* to the *xz*-plane is 0.

Hence, the point *A* must lie in the *xz*-plane, because the distance from the point *A* to the *xz*-plane is 0.

Therefore, the point $\boxed{A(-4,0,-1)}$ lies in the *xz*-plane.

Answer 4E.



Projection of point (2,3,5) on

xy - plane is (2,3,0)
yz - plane is (0,3,5)
xz - plane is (2,0,5)

Length of the diagonal = distance between the points (0, 0, 0) and (2, 3, 5)

$$= \sqrt{(2-0)^2 + (3-0)^2 + (5-0)^2}$$

$$= \sqrt{4+9+25}$$

$$= \sqrt{38}$$

Answer 5E.

Consider the following equation,

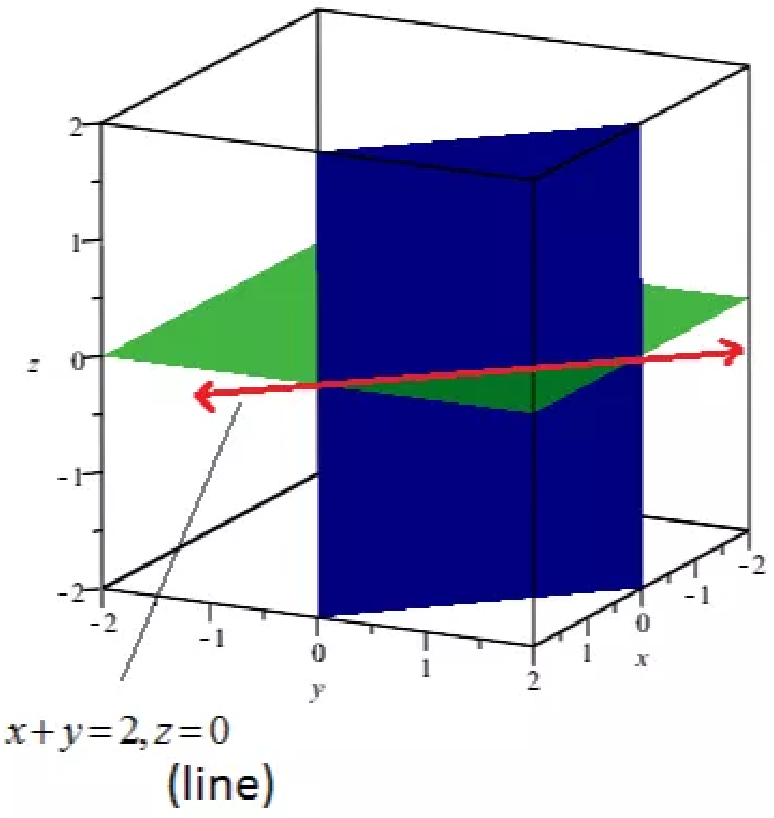
$x + y = 2$

The equation $x + y = 2$ represents the set of all points $\mathbb{R}^3$ whose z-coordinates are zero. That is $\{(x, y, 0) \mid y = 2 - x, x \in R\}$.

This represents a vertical plane that intersects *xy*-plane in the line $y = 2 - x, z = 0$.

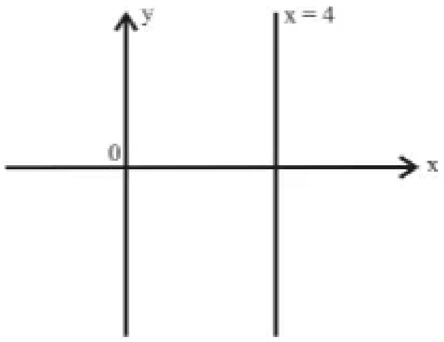
The equation $x + y = 2$ represents a vertical plane perpendicular to *xy*-plane and containing the line $x + y = 2$.

The sketch of the equation $x + y = 2$ is shown below:

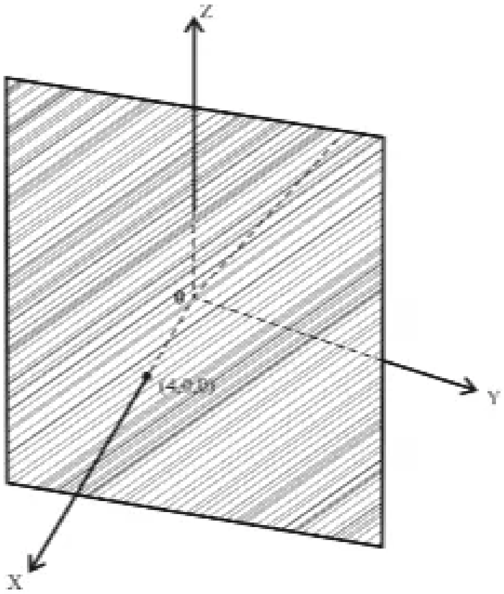


Answer 6E.

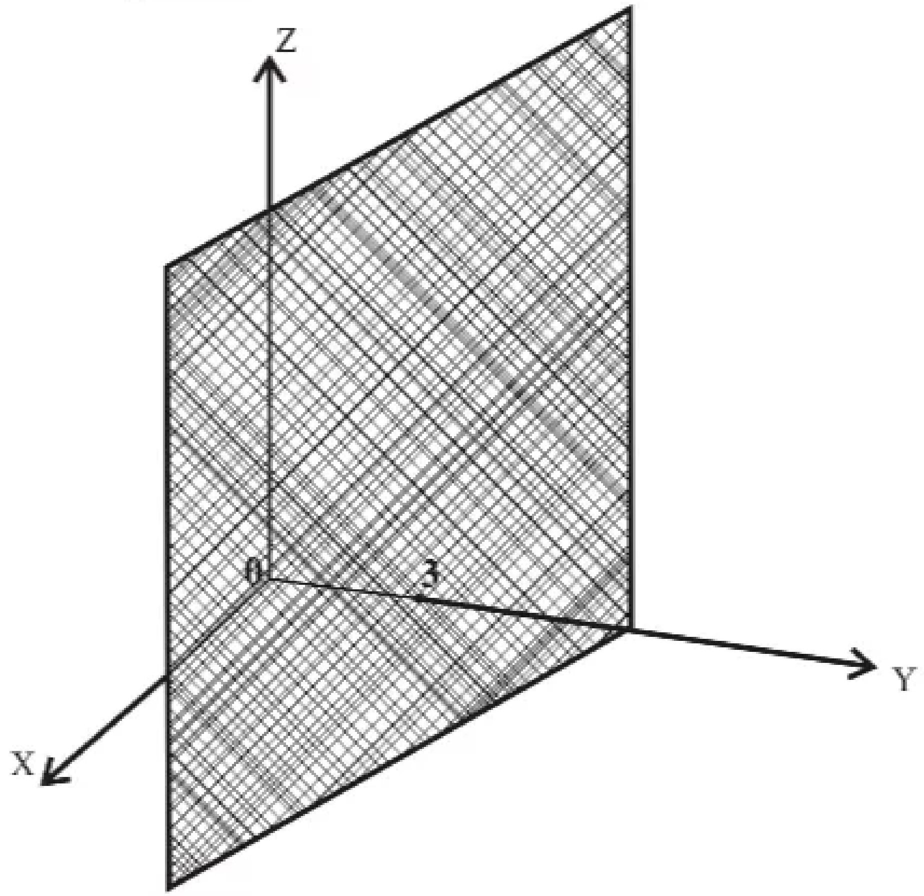
(A) In $\mathbb{R}^2$ the equation $x = 4$ represent a straight line parallel to y axis at a distance 4 units to the right of y axis.



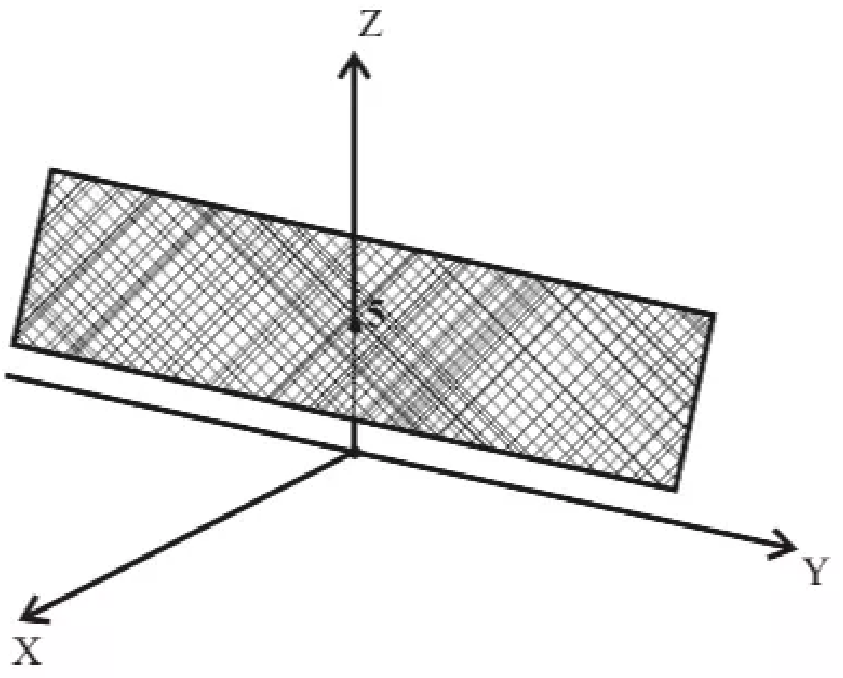
In $\mathbb{R}^3$ the equation $x = 4$ represents a plane parallel to yz plane at a distance 4 units towards positive direction of x axis.



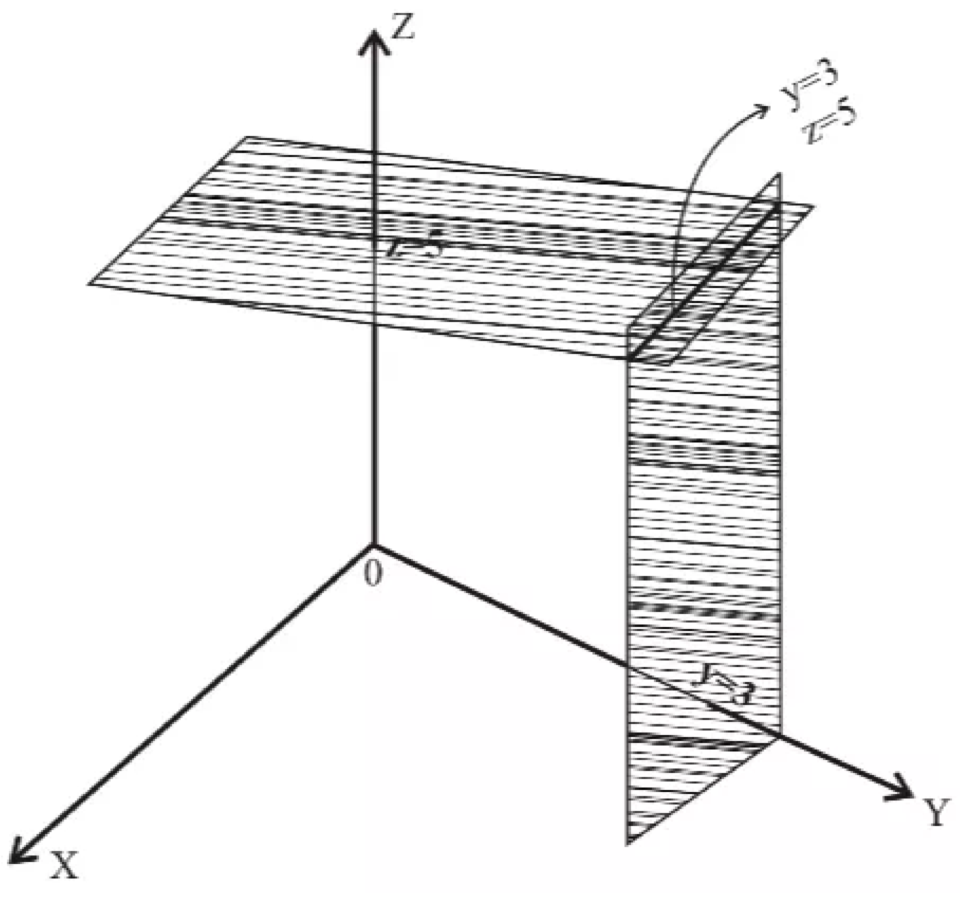
(B) The equation $y = 3$ in $\mathbb{R}^3$ represents a plane parallel to xz – plane at a distance 3 units to the right of it.



The equation $z = 5$ in $\mathbb{R}^3$ represents a plane parallel to xy plane at a distance five units above it.



The pair of equation $y = 3$, $z = 5$ represent a straight line which is the intersection of plane $y = 3$ and plane $z = 5$.



Answer 7E.

Given Points are $P(3, -2, -3), Q(7, 0, 1), R(1, 2, 1)$ *****

we know that the distance formula

$$\left|P_1 P_2\right| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

To find the distance of each of the segments of the triangle.

Start with PQ:

|PQ| = √[(3 - 7) 2 + (-2 - 0) 2 + (-3 - 1) 2]

= √36 = 6

|PR| = √[(3 - 1) 2 + (-2 - 2) 2 + (-3 - 1) 2]

= √36 = 6

|QR| = √[(7 - 1) 2 + (0 - 2) 2 + (1 - 1) 2]

= 2√10 ˜ 6.32

|PQ| = |PR|

∴ ΔPQR is an isosceles triangle

Answer 8E.

Consider the points are $P(2,-1,0)$, $Q(4,1,1)$, $R(4,-5,4)$

Recollect that:

The distance formula in three dimensions: The distance $|P_1P_2|$ between the points $P_1(x_1,y_1,z_1)$ and $P_2(x_2,y_2,z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The distance of the points $P(2,-1,0)$, $Q(4,1,1)$

$$|PQ| = \sqrt{4+4+1}$$
$$= \sqrt{9}$$
$$= 3$$

The distance of the points $Q(4,1,1)$, $R(4,-5,4)$

$$|QR| = \sqrt{0+36+9}$$
$$= \sqrt{45}$$
$$= 3\sqrt{5}$$

The distance of the points $P(2,-1,0)$, $R(4,-5,4)$

$$|PR| = \sqrt{4+16+16}$$
$$= \sqrt{36}$$
$$= 6$$

Hence, $|QR|^2 = |PQ|^2 + |PR|^2$

Therefore the triangle PQR is a right triangle.

Answer 9E.

(a)

Consider the following points:

$A(2,4,2)$, $B(3,7,-2)$, $C(1,3,3)$

Determine whether these three points lie on a straight line or not.

The distance formula in three dimensions:

The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Find the distance between the points $A(2,4,2)$ and $B(3,7,-2)$.

$$|AB| = \sqrt{(3-2)^2+(7-4)^2+(-2-2)^2}$$
$$= \sqrt{(1)^2+(3)^2+(-4)^2}$$
$$= \sqrt{1+9+16}$$
$$= \sqrt{26}$$

The distance between the points $B(3,7,-2)$ and $C(1,3,3)$ is,

$$|BC| = \sqrt{(1-3)^2+(3-7)^2+(3-(-2))^2}$$
$$= \sqrt{(-2)^2+(-4)^2+(5)^2}$$
$$= \sqrt{4+16+25}$$
$$= \sqrt{45}$$
$$= 3\sqrt{5}$$

The distance between the points $A(2,4,2)$ and $C(1,3,3)$ is,

$$|AC| = \sqrt{(1-2)^2+(3-4)^2+(3-2)^2}$$
$$= \sqrt{(-1)^2+(-1)^2+(1)^2}$$
$$= \sqrt{1+1+1}$$
$$= \sqrt{3}$$

Therefore, $|AB| \neq |BC| \neq |AC|$ also we cannot write any distance as the scalar multiple of the other.

Hence, conclude that the points $A(2,4,2)$, $B(3,7,-2)$, $C(1,3,3)$ are not lies on a straight line.

(b)

Consider the following points $D(0,-5,5)$, $E(1,-2,4), F(3,4,2)$.

Determine whether these three points lie on a straight line or not.

The distance between the points $D(0,-5,5)$ and $E(1,-2,4)$ is

$$\begin{aligned}|DE| &= \sqrt{(1-0)^2+(-2-(-5))^2+(4-5)^2} \\ &= \sqrt{(1)^2+(3)^2+(1)^2} \\ &= \sqrt{1+9+1} \\ &= \sqrt{11}\end{aligned}$$

The distance between the points $E(1,-2,4)$ and $F(3,4,2)$ is,

$$\begin{aligned}|EF| &= \sqrt{(3-1)^2+(4-(-2))^2+(2-4)^2} \\ &= \sqrt{(2)^2+(6)^2+(-2)^2} \\ &= \sqrt{4+36+4}\end{aligned}$$

$$= \sqrt{44}$$
$$= \sqrt{4\times 11}$$
$$= 2\sqrt{11}$$

The distance between the points $D(0,-5,5)$ and $F(3,4,2)$ is,

$$\begin{aligned}|DF| &= \sqrt{(3-0)^2+(4-(-5))^2+(2-5)^2} \\ &= \sqrt{(3)^2+(9)^2+(-3)^2} \\ &= \sqrt{9+81+9}\end{aligned}$$

$$= \sqrt{99}$$
$$= \sqrt{9\times 11}$$
$$= 3\sqrt{11}$$

Therefore, $|DE|+|EF|=|DF|$ also $|DF|=3|DE|$.

Hence, conclude that the points $D(0,-5,5)$, $E(1,-2,4)$ and $F(3,4,2)$ are lies on a straight line.

Answer 10E.

a)

Consider the following point:

$(4,-2,6)$.

The objective is to find the distance from the above point to the *xy*-plane.

The distance of a point $P(x,y,z)$ from the *xy*-plane is the absolute value of the *z*-coordinate of the point $P(x,y,z)$.

That is, the distance from $P(x,y,z)$ to the *xy*-plane is $|z|$.

Because, the *z*-coordinate of the point $(4,-2,6)$ is 6, so the distance from $(4,-2,6)$ to the *xy*-plane is,

$|z|=|6|$

= 6 units.

Therefore, the distance from the point $(4,-2,6)$ to the *xy*-plane is 6 units.

(b)

Consider the following point:

$(4,-2,6)$.

The objective is to find the distance from the above point to the *yz*-plane.

The distance of a point $P(x,y,z)$ from the *yz*-plane is the absolute value of the *x*-coordinate of the point $P(x,y,z)$.

That is, the distance from $P(x,y,z)$ to the *yz*-plane is $|x|$.

Because, the *x*-coordinate of the point $(4,-2,6)$ is 4, so the distance from $(4,-2,6)$ to the *yz*-plane is,

$|x|=|4|$

= 4 units.

Therefore, the distance from the point $(4,-2,6)$ to the *yz*-plane is 4 units.

(c)

Consider the following point:

$(4,-2,6)$.

The objective is to find the distance from the above point to the *xz*-plane.

The distance of a point $P(x,y,z)$ from the *xz*-plane is the absolute value of the *y*-coordinate of the point $P(x,y,z)$.

That is, the distance from $P(x,y,z)$ to the *xz*-plane is $|y|$.

Because, the *y*-coordinate of the point $(4,-2,6)$ is -2, so the distance from $(4,-2,6)$ to the *xz*-plane is,

$|y| = |-2|$

= 2 units.

Therefore, the distance from the point $(4,-2,6)$ to the *xz*-plane is 2 units.

(d)

Consider the following point:

$(4,-2,6)$.

The objective is to find the distance from the above point to the *x*-axis.

The distance of a point $P(x,y,z)$ from the *x-axis* is $\sqrt{y^2+z^2}$.

Now, for the point $(4,-2,6)$ is $y=-2, z=6$, so the distance from $(4,-2,6)$ to the *x*-axis is,

$$\begin{aligned}\sqrt{y^2+z^2} &= \sqrt{(-2)^2+6^2} \\ &= \sqrt{4+36} \\ &= \sqrt{40}\end{aligned}$$

$=2\sqrt{10}$ units.

Therefore, the distance from the point $(4,-2,6)$ to the *x*-axis is $2\sqrt{10}$ units.

(e)

Consider the following point:

$(4,-2,6)$.

The objective is to find the distance from the above point to the *y*-axis.

The distance of a point $P(x,y,z)$ from the *y-axis* is $\sqrt{x^2+z^2}$.

Now, for the point $(4,-2,6)$ is $x=4, z=6$, so the distance from $(4,-2,6)$ to the *y*-axis is,

$$\begin{aligned}\sqrt{x^2+z^2} &= \sqrt{4^2+6^2} \\ &= \sqrt{16+36} \\ &= \sqrt{52}\end{aligned}$$

$=2\sqrt{13}$ units.

Therefore, the distance from the point $(4,-2,6)$ to the *y*-axis is $2\sqrt{13}$ units.

(f)

Consider the following point:

$(4,-2,6)$.

The objective is to find the distance from the above point to the *z*-axis.

The distance of a point $P(x,y,z)$ from the *z-axis* is $\sqrt{x^2+y^2}$.

Now, for the point $(4,-2,6)$ is $x=4, y=-2$, so the distance from $(4,-2,6)$ to the *z*-axis is,

$$\sqrt{x^2+y^2}=\sqrt{4^2+(-2)^2}$$
$$=\sqrt{16+4}$$
$$=\sqrt{20}$$
$$=2\sqrt{5} \text{ units.}$$

Therefore, the distance from the point $(4,-2,6)$ to the *z*-axis is $2\sqrt{5}$ units.

Answer 11E.

A sphere has radius $r=4$ with center $(h,k,l)=(-3,2,5)$.

The objective is to find an equation of this above sphere. Also find the intersection of this sphere with the *yz*-plane.

Standard Equation of a Sphere:

"A standard equation of a sphere with center (h,k,l) and radius r is,

$(x-h)^2+(y-k)^2+(z-l)^2=r^2$." ......(1)

The sphere has radius $r=4$ with center $(h,k,l)=(-3,2,5)$.

Substitute $r=4$ and $(h,k,l)=(-3,2,5)$ into equation (1), and find the standard equation of the sphere with center $(-3,2,5)$ and radius 4 as follows:

$(x-h)^2+(y-k)^2+(z-l)^2=r^2$ Write standard equation (1)

$(x-(-3))^2+(y-2)^2+(z-5)^2=4^2$ Substitute $r=4$ and $(h,k,l)=(-3,2,5)$

$(x+3)^2+(y-2)^2+(z-5)^2=16$. Simplify

Therefore, a standard equation of the sphere with center $(-3,2,5)$ and radius 4 is,

$$\boxed{(x+3)^2+(y-2)^2+(z-5)^2=16}.$$

By above step, a standard equation of the sphere with center $(-3,2,5)$ and radius 4 is,

$$(x+3)^2+(y-2)^2+(z-5)^2=16 \text{. ....(1)}$$

Next, find the intersection of this sphere with the *yz*-plane.

The intersection of this sphere with the *yz*-plane is the set of the points on the sphere whose *x*-coordinate is 0. So, by substituting $x=0$ into the original equation of the sphere, the resulting equation will represent the intersection of the surface with the *yz*-plane.

Substitute $x=0$ into the equation (1), and simplify as follows:

$(x+3)^2+(y-2)^2+(z-5)^2=16$ Write original equation (1)

$$\begin{aligned}(0+3)^2+(y-2)^2+(z-5)^2&=16, \quad x=0\\ 9+(y-2)^2+(z-5)^2&=16, \quad x=0\\ (y-2)^2+(z-5)^2&=16-9, \quad x=0\\ (y-2)^2+(z-5)^2&=7, \quad x=0,\end{aligned}$$

which represent a circle in the *yz*-plane with center $(0,2,5)$ and radius $\sqrt{7}$.

Therefore, the intersection of this sphere with the *yz*-plane is **a circle in the *yz*-plane with center $(0,2,5)$ and radius $\sqrt{7}$**.

Answer 12E.

We are given that the center of the sphere is $(2, \ -6, 4)$ and the radius is 5.

So, the equation of the sphere is

$$(x-2)^2+(y+6)^2+(z-4)^2=25\text{..........}(1)$$

To find the intersection of this sphere (1) with xy-plane which is $z = 0$, so

$$(x-2)^2 + (y+6)^2 + (0-4)^2 = 25$$

$$\Rightarrow (x-2)^2 + (y+6)^2 = 25 - 16$$

$$\Rightarrow (x-2)^2 + (y+6)^2 = 9$$

To find the intersection of this sphere (1) with yz- plane which is $x = 0$, so

$$(0-2)^2 + (y+6)^2 + (z-4)^2 = 25$$

$$\Rightarrow (y+6)^2 + (z-4)^2 = 21$$

To find the intersection of this sphere (1) with xz-plane which is $y = 0$, so

$$(x-2)^2 + (z-4)^2 = -11$$

So, xz- plane not intersect with sphere (1).

Answer 13E.

The equation of sphere with centre $(3,8,1)$ and radius r (say) is

$$(x-3)^2 + (y-8)^2 + (z-1)^2 = r^2 \qquad ---(1)$$

Now this sphere passes through the point (4, 3,-1),
Then

$$(4-3)^2+(3-8)^2+(-1-1)^2=r^2$$
$$\Rightarrow 1+25+4=r^2$$
$$\Rightarrow r^2=30$$

Thus the required equation of sphere is

$$\boxed{(x-3)^2+(y-8)^2+(z-1)^2=30} \qquad [From\ (1)]$$

Answer 14E.

The equation of sphere with center $(1,2,3)$ and radius r (say) is

$$(x-1)^2+(y-2)^2+(z-3)^2=r^2$$

Now, this sphere passes through the origin (0, 0, 0),
Then

$$(-1)^2+(-2)^2+(-3)^2=r^2$$
or $1+4+9=r^2$
or $14=r^2$

Thus the required equation of sphere is

$$\boxed{(x-1)^2+(y-2)^2+(z-3)^2=14}$$

Answer 15E.

Consider the following equation:

$$x^2+y^2+z^2-2x-4y+8z=15 \text{ ......(1)}$$

The objective is to show that this equation represents a sphere, and also find its center and radius.

Standard Equation of a Sphere: The standard equation of a sphere with center (h,k,l) and radius r is,

$$(x-h)^2+(y-k)^2+(z-l)^2=r^2 \text{ ......(2)}$$

First rewrite the above equation (1) by completing the squares.

Write the original equation:

$$x^2+y^2+z^2-2x-4y+8z=15 \text{ ......(1)}$$

Group the terms with the same variable:

$$\left(x^2-2x\right)+\left(y^2-4y\right)+\left(z^2+8z\right)=15$$

Then add "the square of half the coefficient of each linear term" to each side of the equation.

So, to complete the square of $\left(x^2-2x\right)$, add $\left[\frac{1}{2}(-2)\right]^2=1$ to each side. To complete the square of $\left(y^2-4y\right)$, add $\left[\frac{1}{2}(-4)\right]^2=4$ to each side. To complete the square of $\left(z^2+8z\right)$, add $\left[\frac{1}{2}(8)\right]^2=16$ to each side. Hence,

$$\left(x^2-2x\right)+\left(y^2-4y\right)+\left(z^2+8z\right)=15$$

$$\left(x^2-2x+1\right)+\left(y^2-4y+4\right)+\left(z^2+8z+16\right)=15+1+4+16$$

$$(x-1)^2+(y-2)^2+(z+4)^2=36$$

$$(x-1)^2+(y-2)^2+(z-(-4))^2=6^2 \text{ ......(3)}$$

Hence the original equation (1) can be rewritten in the form of an equation (3).

Compare this equation (3),

$$(x-1)^2+(y-2)^2+(z-(-4))^2=6^2,$$

with the standard equation (2) of a sphere $(x-h)^2+(y-k)^2+(z-l)^2=r^2$.

So, it is obvious that the original equation (1) can be rewritten in the form of a standard equation of sphere.

Therefore, **yes**, the original equation (1) represents a sphere.

Now, compare this equation (3) of the original sphere,

$$(x-1)^2+(y-2)^2+(z-(-4))^2=6^2.$$

with the standard form, $(x-h)^2+(y-k)^2+(z-l)^2=r^2$, and identify the values of *r*, *h*, *k*, and *l* in (3).

Then, the required values are,

$$h=1,\ k=2,\ l=-4,\ r=6.$$

Hence, it is clear that,

- the center of the equation (3) of the original sphere is $(h,k,l)=\boxed{(1,2,-4)}$ and,
- the radius is $r=\boxed{6}$.

Therefore, the following equation:

$$x^2+y^2+z^2-2x-4y+8z=15,$$

represents a sphere with center $\boxed{(1,2,-4)}$, and radius $\boxed{6}$.

Answer 16E.

Given $x^2+y^2+z^2+8x-6y+2z+17=0$

we are suppose to find its center and radius.

so we do it by completing the square

$$\left(x^2+8x+16\right)+\left(y^2-6y+9\right)+\left(z^2+2z+1\right)= -17+16+9+1$$

$$(x+4)\frown 2+(y-3)\frown 2+(z+1)\frown 2=9$$

so the center is (-4, 3, -1) and the radius is 3

Answer 17E.

$$2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1$$

First, we set the equation equal to zero by moving everything to the left side which gives us

$$2x^2 + 2y^2 + 2z^2 - 8x + 24z - 1 = 0$$

We can now rewrite the above equation in the form of an equation of a sphere if we complete squares:

$$2\left(x^2 - 4x + 4\right) + 2(y-0)^2 + 2\left(z^2 + 12z + 36\right) = 1 + 8 + 72$$

Dividing both sides by 2 gives

$$\left(x-2\right)^2 + \left(y-0\right)^2 + \left(z+6\right)^2 = 81/2$$

Comparing this equation with the standard form $\left[(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2\right]$, we see that it is the equation of a sphere with center $(2,\ 0,\ -6)$ and radius $\sqrt{\frac{81}{2}} = \sqrt{81}/\sqrt{2} = \frac{9}{\sqrt{2}}$.

Answer 18E.

First rewrite the above equation (1) by completing the squares.

Write the original equation:

$$3x^2+3y^2+3z^2=10+6y+12z \quad \text{......(1)}$$

Divide all terms of both sides by 3:

$$\frac{3x^2+3y^2+3z^2}{3}=\frac{10+6y+12z}{3}, \text{ or}$$

$$x^2+y^2+z^2=\frac{10}{3}+2y+4z.$$

Add $-2y-4z$ to both sides and simplify:

$$x^2+y^2+z^2-2y-4z=\frac{10}{3}+2y+4z-2y-4z$$

$$x^2+y^2+z^2-2y-4z=\frac{10}{3}.$$

Group the terms with the same variable:

$$x^2+\left(y^2-2y\right)+\left(z^2-4z\right)=\frac{10}{3}.$$

Then add "the square of half the coefficient of each linear term" to each side of the equation.

So, to complete the square of $\left(y^2-2y\right)$, add $\left[\frac{1}{2}(-2)\right]^2=1$ to each side. To complete the square of $\left(z^2-4z\right)$, add $\left[\frac{1}{2}(-4)\right]^2=4$ to each side. Hence,

$$x^2+\left(y^2-2y\right)+\left(z^2-4z\right)=\frac{10}{3}$$

$$x^2+\left(y^2-2y+1\right)+\left(z^2-4z+4\right)=\frac{10}{3}+1+4$$

$$x^2+(y-1)^2+(z-2)^2=\frac{25}{3}$$

$$x^2+(y-1)^2+(z-2)^2=\left(\sqrt{\frac{25}{3}}\right)^2$$

$$(x-0)^2+(y-1)^2+(z-2)^2=\left(\frac{5}{3}\sqrt{3}\right)^2 \quad \text{.......(3)}$$

Hence the original equation (1) can be rewritten in the form of an equation (3).

Compare this equation (3),

$$(x-0)^2+(y-1)^2+(z-2)^2=\left(\frac{5}{3}\sqrt{3}\right)^2,$$

with the standard equation (2) of a sphere $(x-h)^2+(y-k)^2+(z-l)^2=r^2$.

So, it is obvious that the original equation (1) can be rewritten in the form of a standard equation of sphere.

Therefore, **yes**, the original equation (1) represents a sphere.

Now, compare this equation (3) of the original sphere,

$$(x-0)^2+(y-1)^2+(z-2)^2=\left(\frac{5}{3}\sqrt{3}\right)^2,$$

with the standard form, $(x-h)^2+(y-k)^2+(z-l)^2=r^2$, and identify the values of *r*, *h*, *k*, and *l* in (3).

Then, the required values are,

$$h=0,\ k=1,\ l=2,\ r=\frac{5}{3}\sqrt{3}.$$

Hence, it is clear that,

- the center of the equation (3) of the original sphere is $(h,k,l)=\boxed{(0,1,2)}$ and,
- the radius is $r=\boxed{\frac{5}{3}\sqrt{3}}$.

Therefore, the following equation:

$$3x^2+3y^2+3z^2=10+6y+12z,$$

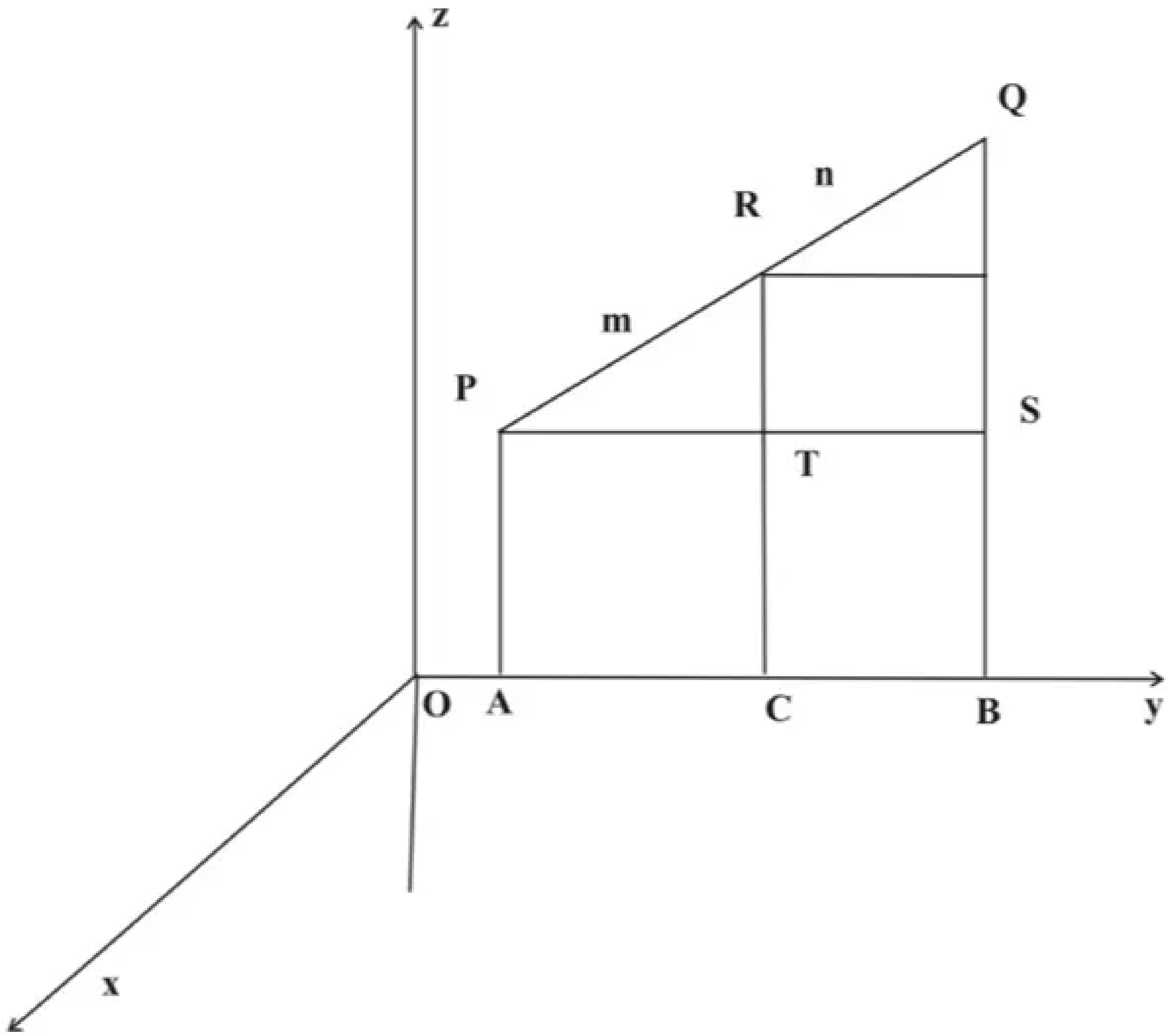
represents a sphere with center $\boxed{(0,1,2)}$, and radius $\boxed{\frac{5}{3}\sqrt{3}}$.

Answer 19E.

(a)

Let PQ be the projection of the line joining the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ in space.

Then the co-ordinates of the points P corresponding to the points P1 would be (y_1, z_1) and those of Q corresponding to the point P2, would be (y_2, z_2).



Let R be the point corresponding to the point which divides the line segment P1P2 in the ratio m:n let co-ordinates of the point R, which is in yz plane be $(\bar{y}, \bar{z})$.

Let us draw perpendiculars from points P and Q on the y-axis meeting it at A and B respectively. Let C be the point where perpendicular from R meets y-axis.

Then from similar triangles PRT and PQS,

$$\frac{PS}{PT} = \frac{PQ}{PR} = \frac{QS}{RT}$$

$$\frac{y_2 - y_1}{\overline{y} - y_1} = \frac{m+n}{m} = \frac{z_2 - z_1}{\overline{z} - z_1}$$

$$my_2 - my_1 = (m+n)\overline{y} - my_1 - ny_1$$

$$(m+n)\overline{y} = my_2 + ny_1$$

$$\overline{y} = \frac{my_2 + ny_1}{m+n}$$

and $(m+n)\overline{z} - mz_1 - nz_1 = mz_2 - mz_1$

$$(m+n)\overline{z} = mz_2 + nz_1$$

$$\overline{z} = \frac{mz_2 + nz_1}{m+n}$$

In a similar manner if we take projection of the line segment P1P2 in say xy plane, we will get

$$\overline{x} = \frac{mx_2 + nx_1}{m+n}, \overline{y} = \frac{my_2 + ny_1}{m+n}$$

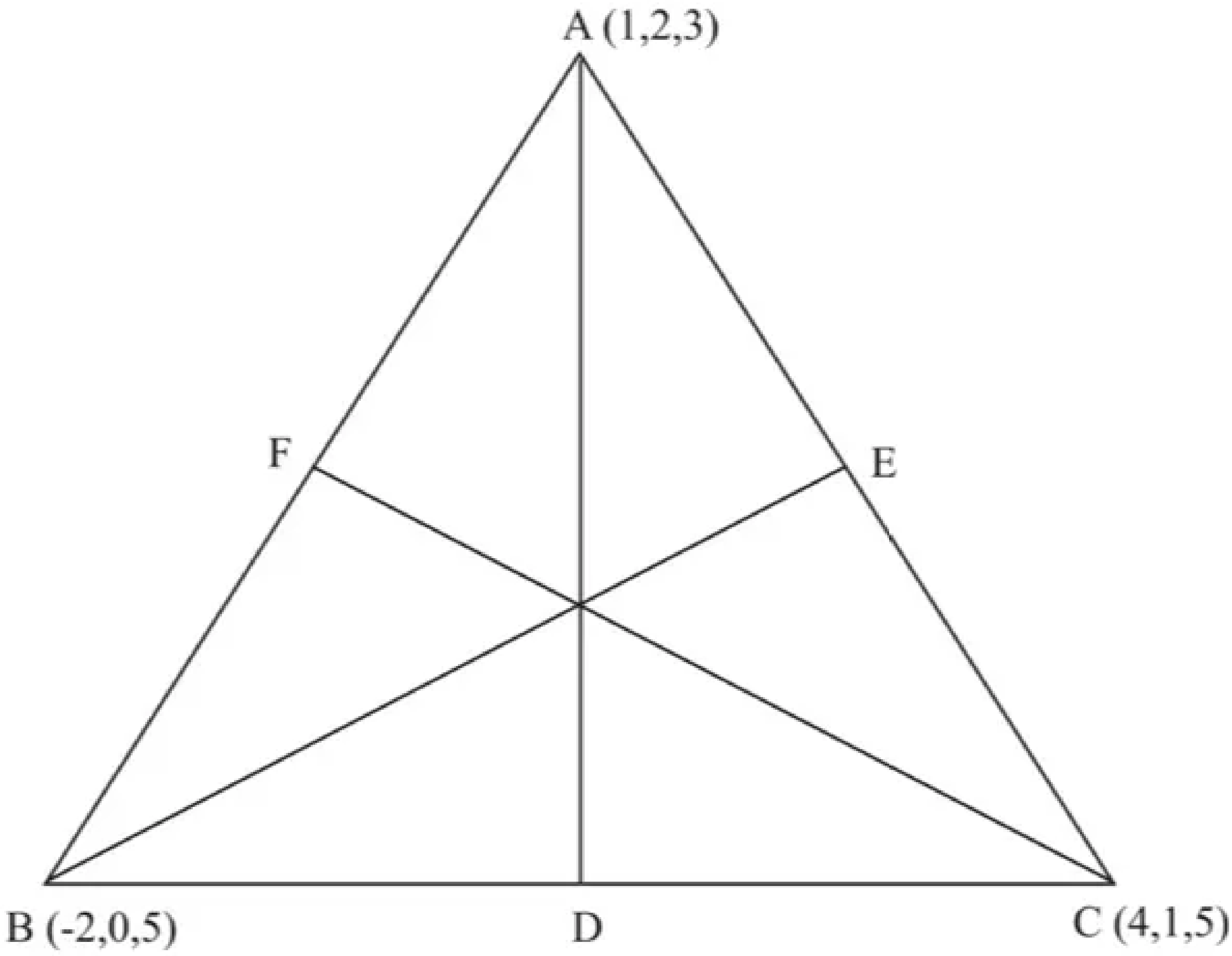
So the co-ordinates of the point R which divides the line segment P1 P2 having co-ordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio m:n is given by

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

In case R is the mid point, m = n. Then the mid point co-ordinates will be

$$\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}$$

Consider the vertices $A(1,2,3), B(-2,0,5), C(4,1,5)$



Let D,E,F be the mid-points of sides BC, AC and AB of the triangle ABC.

Then using the result obtained in part (a)

$$D \equiv \left(\frac{4-2}{2}, \frac{1+0}{2}, \frac{5+5}{2}\right)$$

$$D \equiv \left(1, \frac{1}{2}, 5\right)$$

$$E \equiv \left(\frac{4+1}{2}, \frac{1+2}{2}, \frac{5+3}{2}\right)$$

$$E \equiv \left(\frac{5}{2}, \frac{3}{2}, 4\right)$$

$$F \equiv \left(\frac{1-2}{2}, \frac{2+0}{2}, \frac{3+5}{2}\right)$$

$$F \equiv \left(\frac{-1}{2}, 1, 4\right)$$

Use distance formula, the length of median AD is

$$|AD| = \sqrt{(1-1)^2 + \left(2-\frac{1}{2}\right)^2 + (3-5)^2}$$

$$= \sqrt{0 + \frac{9}{4} + 4}$$

$$= \sqrt{\frac{25}{4}}$$

$$= \boxed{\frac{5}{2}}.$$

The length of median BE is

$$|BE| = \sqrt{\left(-2-\frac{5}{2}\right)^2 + \left(0-\frac{3}{2}\right)^2 + (5-4)^2}$$

$$= \sqrt{\frac{81}{4} + \frac{9}{4} + 1}$$

$$= \sqrt{\frac{94}{4}}$$

$$= \boxed{\frac{\sqrt{94}}{2}}.$$

The length of median CF is,

$$|CF| = \sqrt{\left(4+\frac{1}{2}\right)^2 + (1-1)^2 + (5-4)^2}$$

$$= \sqrt{\frac{81}{4} + 0 + 1}$$

$$= \sqrt{\frac{85}{4}}$$

$$= \boxed{\frac{1}{2}\sqrt{85}}.$$

Answer 20E.

The end points of diameter of the sphere are $(2,1,4)$ *and* $(4,3,10)$

The centre of the sphere is $\left(\frac{2+4}{2},\frac{1+3}{2},\frac{4+10}{2}\right)$

i.e. $(3,2,7)$

The equation of sphere with this centre and radius r (say) is

$$(x-3)^2+(y-2)^2+(z-7)^2=r^2$$

This sphere passes through the end point of diameter

i.e. $(2,1,4)$

$$(2-3)^2+(1-2)^2+(4-7)^2=r^2$$

$$\Rightarrow 1+1+9=r^2$$

$$\Rightarrow r^2=11$$

Then the required equation of sphere is

$$\boxed{(x-3)^2+(y-2)^2+(z-7)^2=11}.$$

Answer 21E.

Consider the center of the sphere is $C(2,-3,6)$

(a)

The sphere touches the *xy*-plane at the point $P(2,-3,0)$.

Then the radius of the sphere is the distance from the center *C* and the point *P*.

That is

$$CP=\sqrt{(2-2)^2+(-3+3)^2+(6-0)^2}$$

$$=\sqrt{0+0+6^2}$$

$$=6$$

Note that, the equation of the sphere with center $C(h,k,l)$ and radius *r* is

$$(x-h)^2+(y-k)^2+(z-l)^2=r^2$$

Hence the equation of the sphere is

$$(x-2)^2+(y+3)^2+(z-6)^2=6^2$$

$$\boxed{(x-2)^2+(y+3)^2+(z-6)^2=36}$$

(b)

The sphere touches the *yz*-plane at the point $Q(0,-3,6)$.

Then the radius of sphere is the distance from the center *C* and the point *Q*.

That is

$$\begin{aligned} CQ &= \sqrt{(2-0)^2+(-3+3)^2+(6-6)^2} \\ &= \sqrt{2^2+0+0} \\ &= 2 \end{aligned}$$

Hence the equation of the sphere is

$$(x-2)^2+(y+3)^2+(z-6)^2=2^2$$

$$\boxed{(x-2)^2+(y+3)^2+(z-6)^2=4}$$

(c)

The sphere touches the *xz*-plane at the point $R(2,0,6)$.

Then the radius of sphere is the distance from the center *C* and the point *R*.

That is

$$\begin{aligned} CR &= \sqrt{(2-2)^2+(-3-0)^2+(6-6)^2} \\ &= \sqrt{0+3^2+0} \\ &= 3 \end{aligned}$$

Hence the equation of the sphere is

$$(x-2)^2+(y+3)^2+(z-6)^2=3^2$$

$$\boxed{(x-2)^2+(y+3)^2+(z-6)^2=9}$$

Answer 22E.

The centre of the sphere is (5, 4, 9)
The projection of (5,4,9) on xy-plane is (5,4,0) and then the distance of (5,4,9) from xy-plane is 9.

The projection of (5,4,9) on yz-plane is (0,4,9) and then the distance of (5,4,9) from yz-plane is 5.

And the projection of (5,4,9) on xz-plane is (5,0,9) and then the distance of (5,4,9) from xz-plane is 4.

In order to have the largest sphere with centre (5, 4, 9) that is contained in first octant we must choose the shortest distance among the above calculated distance from the co-ordinate planes as the radius of sphere i.e., take the radius of sphere to be 4.

Then the equation of required sphere is

$$(x-5)^2+(y-4)^2+(z-9)^2=4^2$$

i.e. $$\boxed{(x-5)^2+(y-4)^2+(z-9)^2=16}$$

Answer 23E.

Consider the following equation:

$x=5$.

The objective is to find the representation of this equation in $\mathbb{R}^3$.

In three-dimensional analytical geometry, an equation in *x*, *y* and *z* represents a surface in $\mathbb{R}^3$.

The equation $x=5$ represents the set $\{(x,y,z)\mid x=5\}$, which is the set of all points in $\mathbb{R}^3$ whose *x*-coordinate is 5. This is the vertical plane parallel to the *yz*-plane and 5 units in front of it.

Therefore, in $\mathbb{R}^3$, the equation $x=5$ represents **a vertical plane parallel to the *yz*-plane and 5 units in front of it**.

Answer 24E.

Consider the following equation:

$y=-2$.

The objective is to find the representation of this equation in $\mathbb{R}^3$.

In three-dimensional geometry, an equation in *x*, *y* and *z* represents a surface in $\mathbb{R}^3$.

The equation $y=-2$ represents the set $\{(x,y,z)\mid y=-2\}$, which is the set of all points in $\mathbb{R}^3$ whose *y*-coordinate is -2. This is the vertical plane parallel to the *xz*-plane and 2 units to the left of it.

Therefore, in $\mathbb{R}^3$, the equation $y=-2$ represents **a vertical plane parallel to the *xz*-plane and 2 units to the left of it**.

Answer 25E.

Consider the following equation:

$y < 8$.

The objective is to find the representation of this equation in $\mathbb{R}^3$.

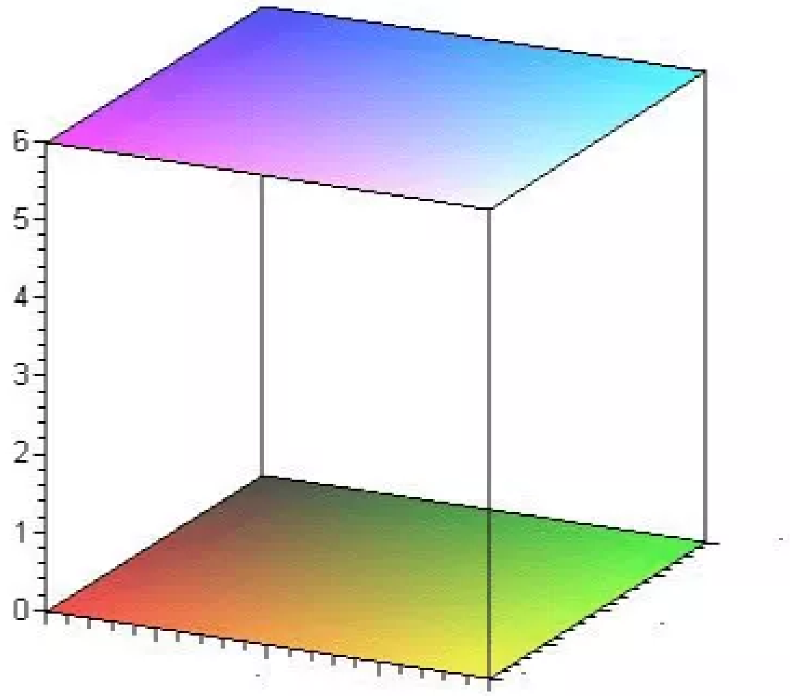
The inequality $y < 8$ represents the set $\{(x, y, z) | y < 8\}$, which is the set of all points in $\mathbb{R}^3$ whose y-coordinate is less than 8.

Because, the equation $y = 8$ represents a vertical plane parallel to the xz-plane, so the inequality $y < 8$ represents a half-space consisting of all points to the left of the plane $y = 8$.

Therefore, the inequality $y < 8$ represents **a half-space consisting of all points to the left of the plane** $y = 8$.

Answer 27E.

$0 \le z \le 6$ means all the points on or between the horizontal planes $z = 0$ and $z = 6$.



Answer 28E.

Consider the equation $z^2 = 1$

The above equation is equivalent to $z^2 - 1 = 0$

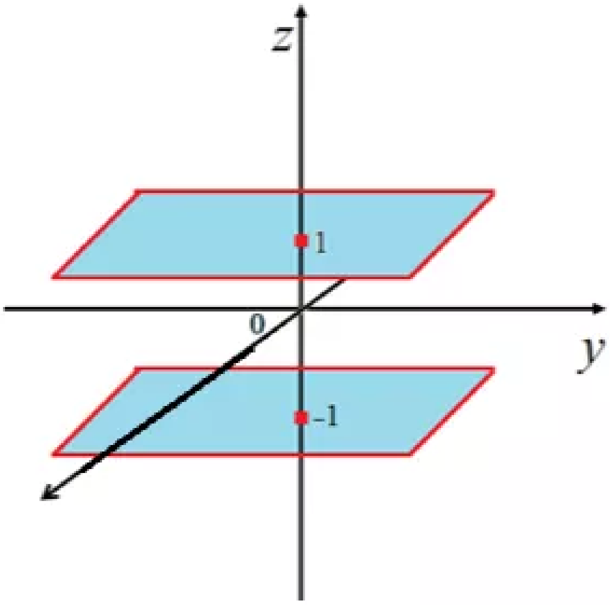
$$z^2 - 1 = 0$$

$$(z-1)(z+1) = 0$$

$(z-1) = 0$ and $(z+1) = 0$

The equations $z=-1$ and $z=1$ are parallel to the $xy-$plane and the intersecting the $z-$axis at $(0,0,1)$ and $(0,0,-1)$.

One unit above and one unit below it as shown in the below figure.



Answer 29E.

Consider the following equation:

$x^2+y^2=4,\ z=-1$.

The objective is to find the representation of this equation in $\mathbb{R}^3$.

In three-dimensional geometry, an equation in *x*, *y* and *z* represents a surface in $\mathbb{R}^3$.

The equation $z=-1$ represents the set $\{(x,y,z)\,|\,z=-1\}$, which is the set of all points in $\mathbb{R}^3$ whose *z*-coordinate is -1. This is the horizontal plane $z=-1$ parallel to the *xy*-plane.

Because $z=-1$, all points in the region must lie in the horizontal plane $z=-1$.

The equation $x^2+y^2=4$ represents the set $\{(x,y,z)\,|\,x^2+y^2=4\}$, which is the set of all points lie on a circle with radius 2 and center on the *z*-axis.

Therefore, in $\mathbb{R}^3$, the equation $x^2+y^2=4,\ z=-1$ represents **the region consists of all points that lies on a circle with radius 2 and center on the *z*-axis that is contained in the plane** $z=-1$.

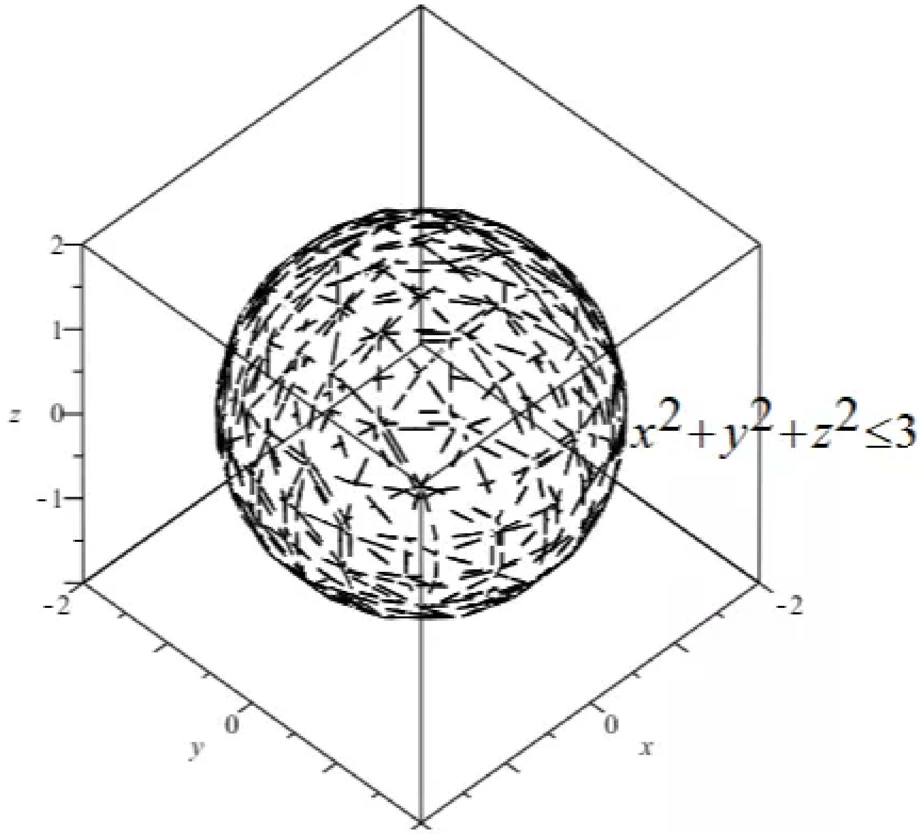
Answer 31E.

Consider the equation, $x^2+y^2+z^2 \le 3$

The inequality $x^2+y^2+z^2 \le 3$ is equivalent $\sqrt{x^2+y^2+z^2} \le \sqrt{3}$.

So the region contains of those points whose distance from the origin is less than or equal to 3.

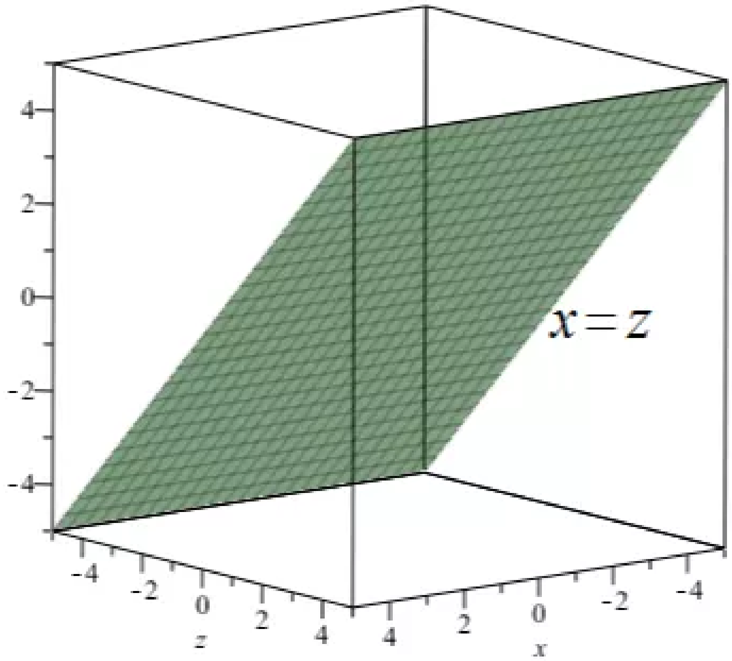
This is the set of all points inside the sphere with radius $\sqrt{3}$ and the center $(0,0,0)$ as shown in the figure as below.



Answer 32E.

Consider the equation, $x = z$

The $x-$coordinate and $z-$coordinates are in this satisfy $x = z$ as shown in the below figure.



The region is the plane crossed by the line $x = z, y = 0$ in the $xz-$plane and the $y-$axis.

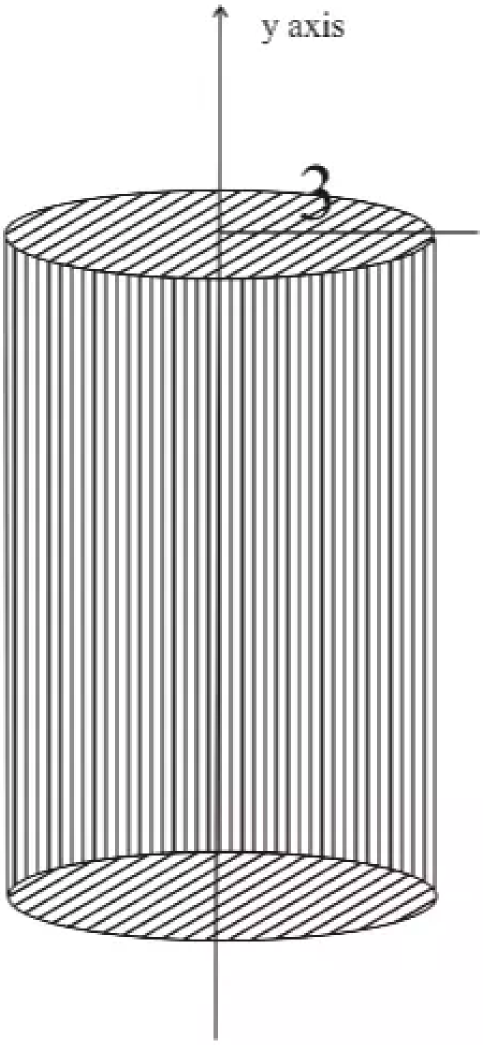
It's a plane containing of all the points that are halfway from the x and z axes.

Answer 33E.

The given inequality is

$$x^2 + z^2 \le 9$$

It represents all the points on or inside a circular cylinder of radius 3 with axis the y-axis.



Answer 34E.

Consider the following equation:

$$x^2 + y^2 + z^2 > 2z$$

Rearrange and simplify as follows:

$$\begin{aligned} x^2 + y^2 + z^2 &> 2z \\ x^2 + y^2 + z^2 - 2z &> 0 \\ x^2 + y^2 + z^2 - 2z + 1 - 1 &> 0 \\ x^2 + y^2 + (z-1)^2 - 1 &> 0 \\ x^2 + y^2 + (z-1)^2 &> 1 \end{aligned}$$

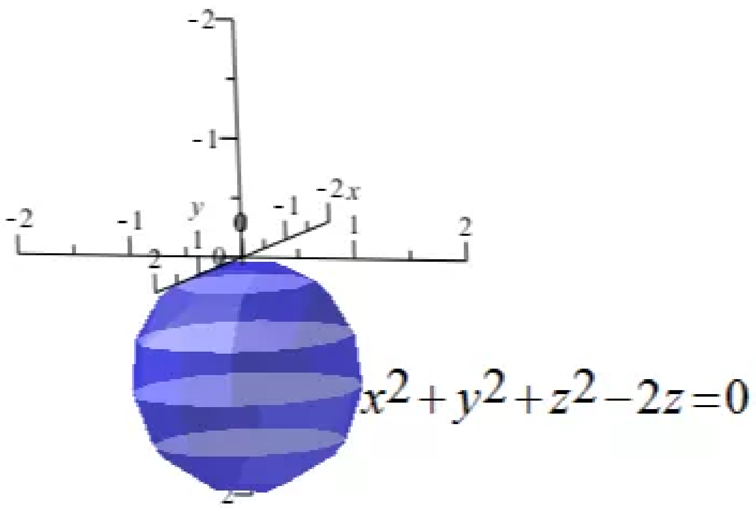
So they represent the point (x, y, z) whose distance from the origin is at least 1.

The points lie on or below the $xy-$plane.

Thus the given inequality represents the region that lies between the sphere

$x^2 + y^2 + (z-1)^2 = 1$

So, the region is the outside a sphere of radius is one and centered at $(0,0,1)$ as shown in the figure below.



Answer 35E.

Consider the following equality:

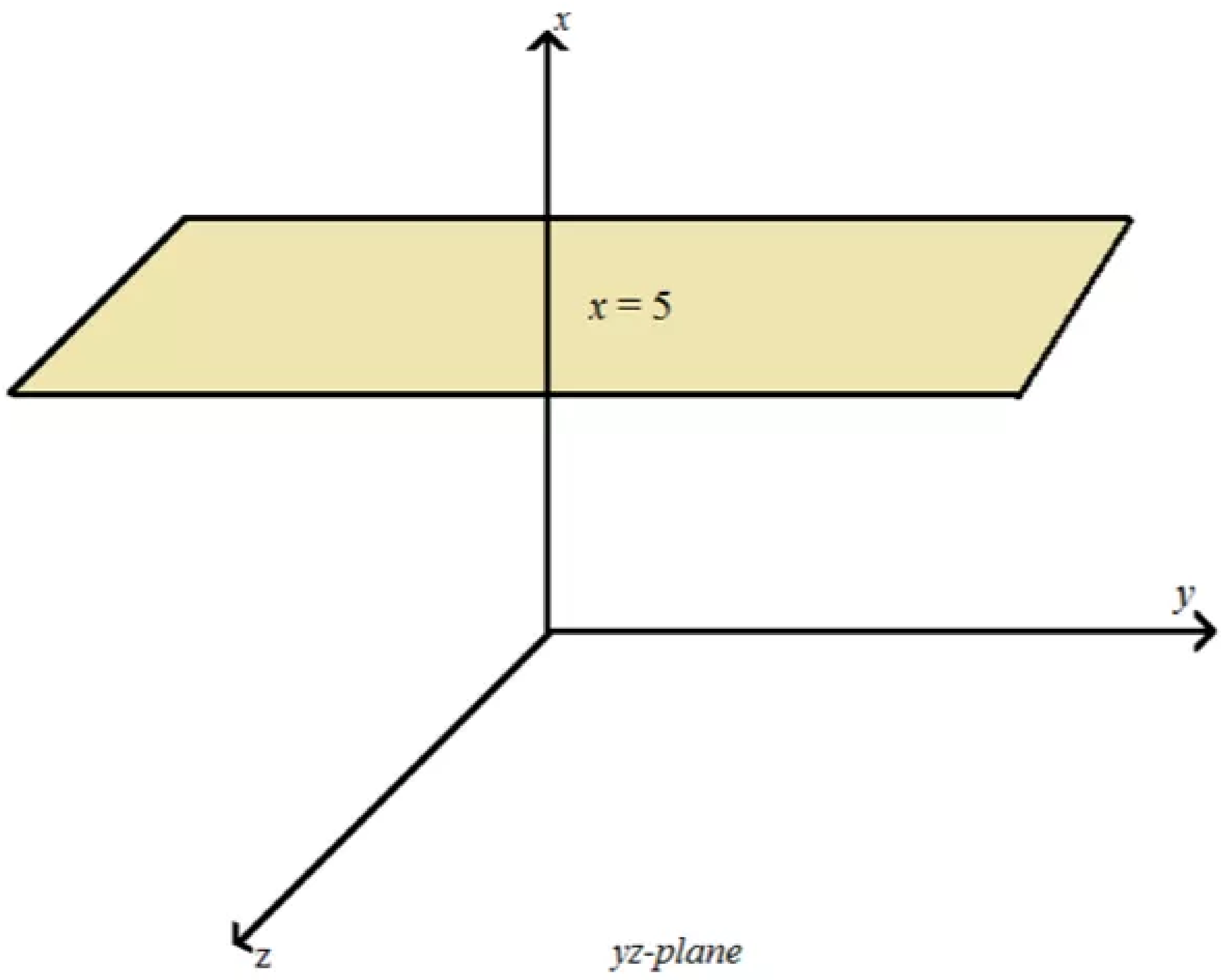
$x = 5$

In $\mathbb{R}^3$, if *k* is a constant then $x = k$ represents the planes parallel to $yz-$plane. And if *k* is positive, then the plane is a *k* unit above $yz-$plane.

On comparing the equation $x = 5$ with $x = k$, $k = 5.$

Therefore, in $\mathbb{R}^3$, the plane $x = 5$ represents a plane parallel to the $yz-$plane and it is 5 units above the *yz*-plane (as $k = 5$ is positive).

To support this view, the plane given by the graph as follows:



Answer 36E.

The solid disk with radius 2 and centre at origin is $x^2+y^2 \le 4$.

The region that lies on or below the plane $z=8$ is represented by the inequality $z \le 8$.

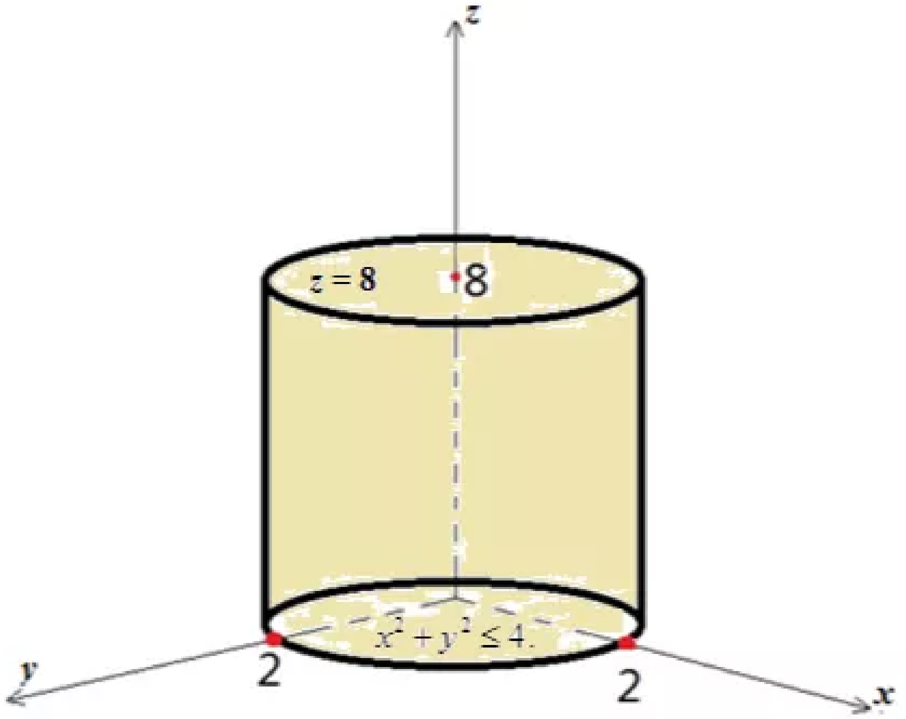
The region that lies on or above the xy-plane $(z=0)$ is represented by $z \ge 0$.

Thus, write the combined inequality $0 \le z \le 8$.

Therefore the solid cylinder lies on or below the plane $z=8$ and on or above the disk in xy-plane with centre at origin and radius 2 is given by the inequality,

$\boxed{x^2+y^2 \le 4, 0 \le z \le 8}$.

The graph of the cylinder is as shown below.



Answer 37E.

The region consisting of all points between (but not on) the spheres of radius r and R centered at the origin, where $r < R$ is represented by inequality:

$\boxed{r^2 < x^2+y^2+z^2 < R^2}$

Answer 38E.

Consider the solid upper hemisphere of the sphere of radius 2 centered at the origin.

Write the inequality to describe the region.

So, the general form of sphere is given as follows:

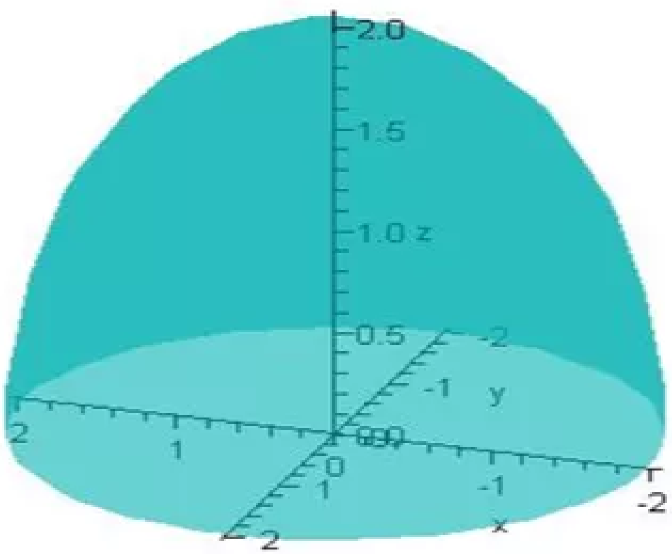
$$(x-0)^2+(y-0)^2+(z-0)^2=2^2$$
$$x^2+y^2+z^2=4.$$

But, the given problem requires upper hemisphere.

The solid upper hemisphere of sphere of radius 2, centered at the origin is represented as under:

$$\boxed{x^2+y^2+z^2\le 4, z\ge 0}$$

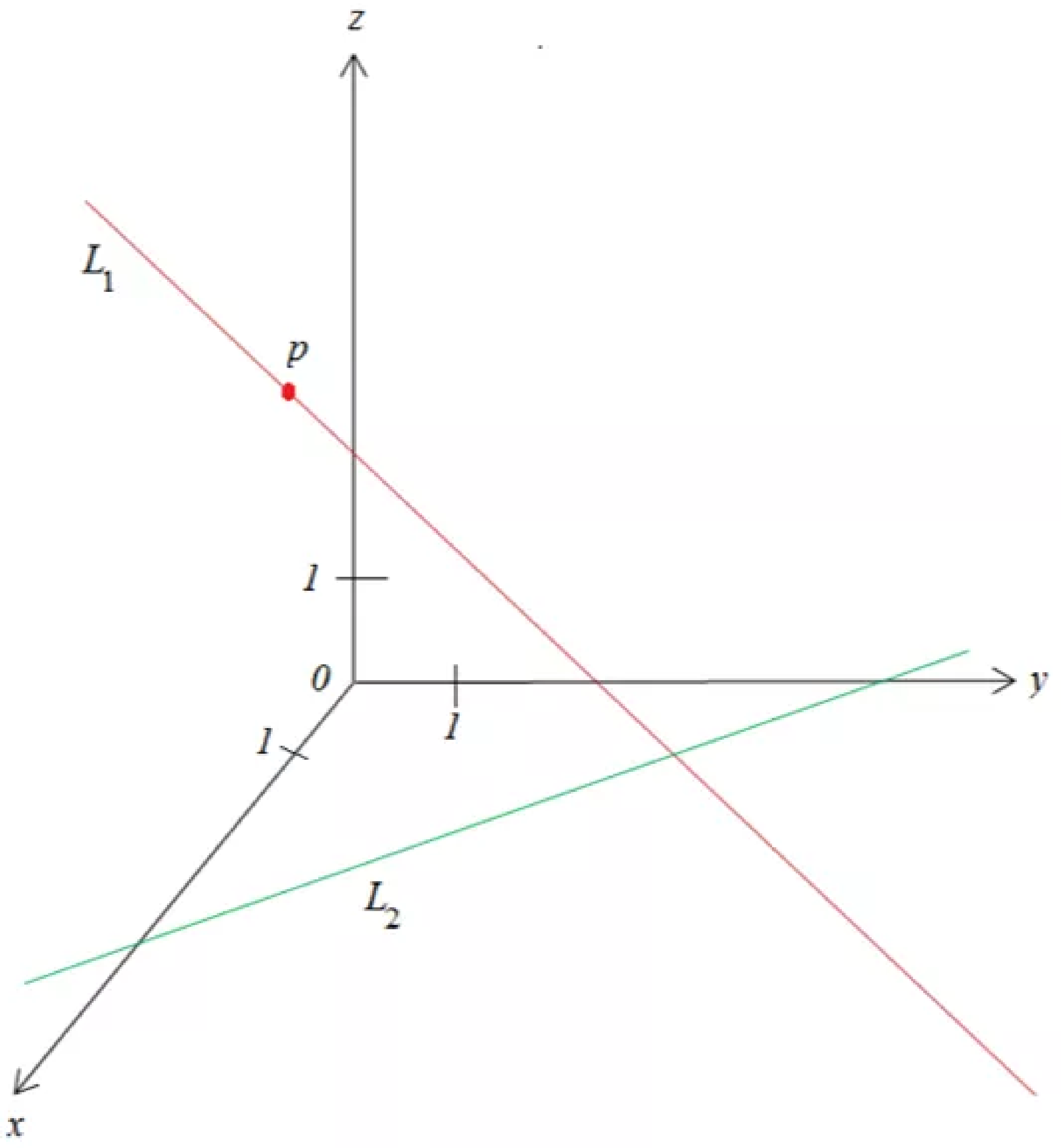
Graph of the solid of upper hemisphere is as follows:



Answer 39E.

(a)

Consider the following the diagram:



In the above diagram the line L_2 is a projection of the line L_1 on $xy-$ plane.

(a)

The objective is to find the coordinates of the point P on the line L_1.

To find $x-$and $y-$coordinates of the point $P,$ project it onto L_2.

The projection of the point P on the line L_1 is the point Q online L_2.

To find the $z-$coordinate, project the point P onto either $xz-$plane or $yz-$plane and then project the resulting point onto the $z-$axis.

Draw a line parallel to QO from the point P to the $z-$axis.

The sketch of the graph is shown in the following figure 1:

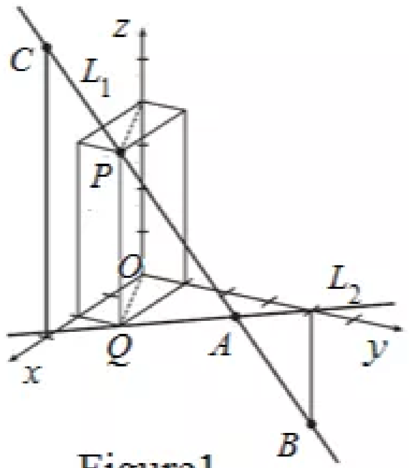


Figure1

From the figure1, the point P is situated at a vertical distance $z=4,$ horizontal distance $y=1,$ and the coordinates of x is 2.

Therefore, the coordinates of the point P are, $\boxed{P(2,1,4)}$.

(b)

The objective is to identify the points $A, B,$ and C which are intersection points of the line L_1 with $xy-$ plane, $yz-$ plane, and $xz-$ plane respectively.

From figure1, the point A is the intersection point of the line L_1 and $xy-$ plane.

The point B is directly below the $y-$intercept of L_2.

That is, the point B is intersection point of the line $L_1,$ and $yz-$ plane.

The point C is directly above the $x-$intercept of L_2.

That is, the point C is intersection point of the line $L_1,$ and $xz-$ plane.

Answer 40.

Let $P=(x,y,z)$

It is given that $|AP|=2|BP|$

i.e. $\sqrt{(x+1)^2+(y-5)^2+(z-3)^2}=2\sqrt{(x-6)^2+(y-2)^2+(z+2)^2}$

Squaring both sides,

$$(x+1)^2+(y-5)^2+(z-3)^2=4\left[(x-6)^2+(y-2)^2+(z+2)^2\right]$$

i.e. $x^2+2x+1+y^2-10y+25+z^2-6z+9$

$=4\left[x^2-12x+36+y^2-4y+4+z^2+4z+4\right]$

i.e. $x^2+y^2+z^2+2x-10y-6z+35=4x^2+4y^2+4z^2-48x-16y+16z+176$

i.e. $3x^2+3y^2+3z^2-50x-6y+22z+141=0$

or $x^2+y^2+z^2-\frac{50}{3}x-2y+\frac{22}{3}z+\frac{141}{3}=0$

or $\left(x-\frac{25}{3}\right)^2+(y-1)^2+\left(z+\frac{11}{3}\right)^2=\frac{332}{9}$

which is the equation of a sphere with centre $\boxed{\left(\frac{25}{3},1,\frac{-11}{3}\right)}$ and radius $\boxed{\frac{\sqrt{332}}{3}}$

Answer 41E.

Let $C(x,y,z)$ be the point equidistant from point $A(-1,5,3)$ and $B(6,2,-2)$

Then using distance formula,

$$\sqrt{(x+1)^2+(y-5)^2+(z-3)^2}=\sqrt{(x-6)^2+(y-2)^2+(z+2)^2}$$

Squaring both sides,

$$(x+1)^2+(y-5)^2+(z-3)^2=(x-6)^2+(y-2)^2+(z+2)^2$$

or $x^2+2x+1+y^2+25-10y+z^2+9-6z=x^2+36-12x+y^2+4-4y+z^2+4+4z$

or $2x-10y-6z+35=-12x-4y+4z+44$

or $\boxed{14x-6y-10z=9}$.

Answer 42E.

The equations of spheres are as follows:

$x^2+y^2+z^2+4x-2y+4z+5=0$ ...... (1)

$x^2+y^2+z^2=2^2$ ...... (2)

The equation (1) can be written as follows:

$$x^2+4x+y^2-2y+z^2+4z+5=0 \quad \text{Rearrange the terms}$$
$$x^2+4x+4+y^2-2y+1+z^2+4z+4+5=4+1+4 \quad \text{Add 4+1+4 on both sides}$$
$$(x+2)^2+(y-1)^2+(z+2)^2=2^2 \quad \text{Simplify}$$

Compare this with standard equation of sphere.

$(x-h)^2+(y-k)^2+(z-l)^2=r^2$, Where (h,k,l) is center and r is radius of sphere.

Then center of sphere (1) is $A=(-2,1,-2)$ and radius is $r_1=2$

Also, the center of sphere (2) is $B=(0,0,0)$ and radius is $r_2=2$

The distance between the centers of spheres is calculated as follows:

$$AB=\sqrt{(-2-0)^2+(1-0)^2+(-2-0)^2}$$
$$=3$$

Let $d=3$

Sum of the radii is as follows:

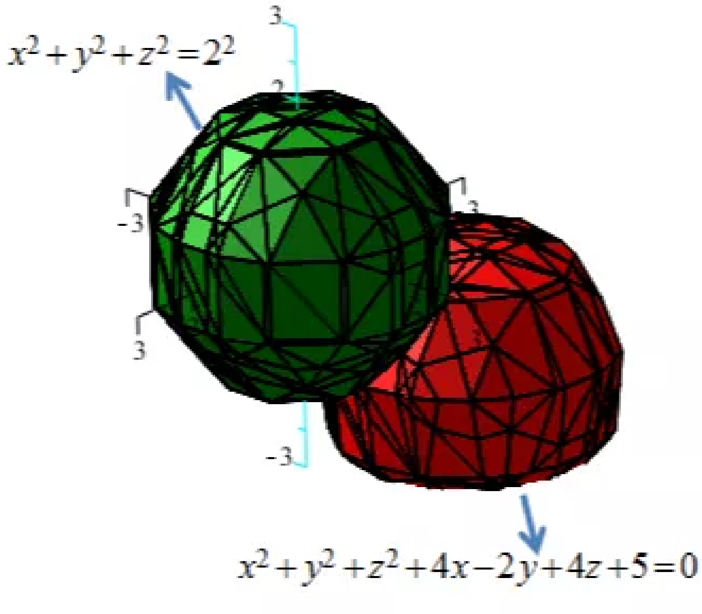
$$r_1+r_2=2+2$$
$$=4$$

Let $r=2$

That is, the sum of the radii of spheres is greater than distance between centers.

So, the spheres intersect each other.

The graph of intersecting sphere is shown below:



The volume of solid inside both the spheres when they intersect each other is given by the following equation:

$$V = \frac{1}{12}\pi[4r+d][2r-d]^2$$
$$= \frac{1}{12}\pi[4(2)+3][2(2)-3]^2 \qquad \text{Since } r=2, d=3$$
$$= \frac{1}{12}\pi(11)$$
$$= \frac{11\pi}{12}$$

Therefore, the volume of solid is given as under:

$$\boxed{V = \frac{11\pi}{12} \quad \text{cubic units}}$$

Answer 43E.

Consider the following two spheres:

$x^2+y^2+z^2=4$, and ......(i)

$x^2+y^2+z^2=4x+4y+4z-11$. ......(ii)

The objective is to find the distance between the above spheres.

The visual representation of the problem is as follows:

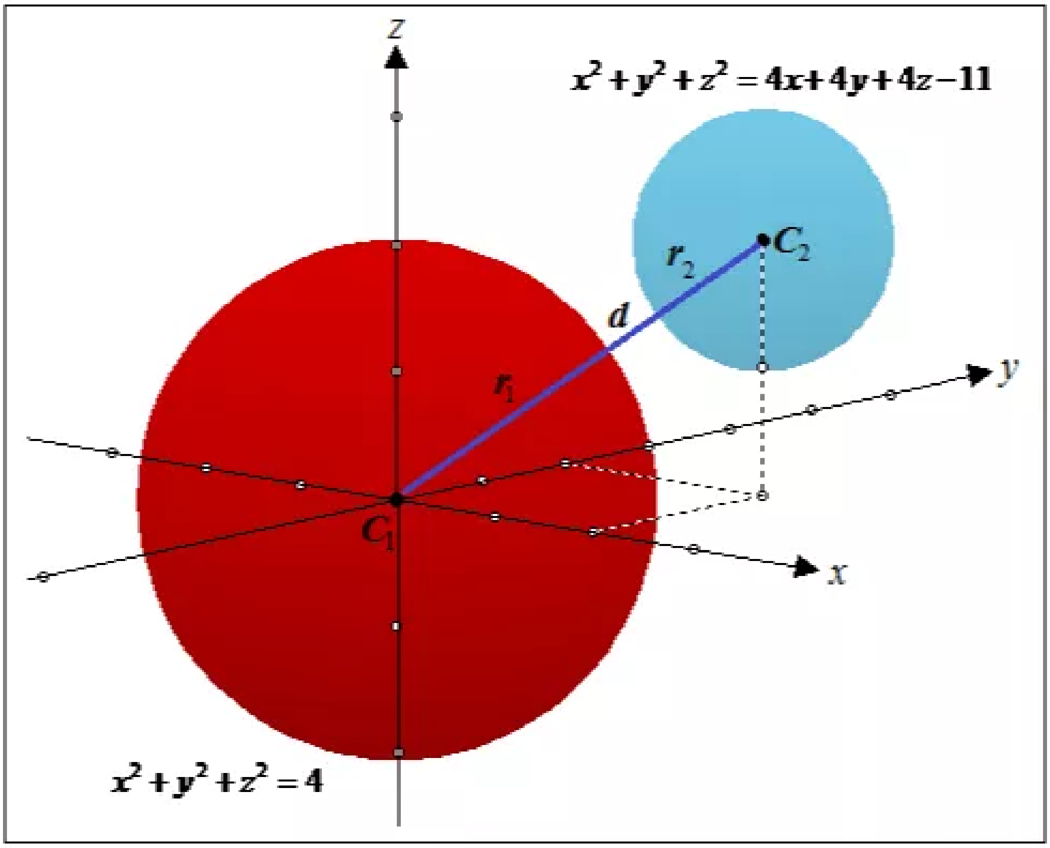


Figure-1

In this figure, the distance between the two spheres is represented by *d*, so the objective is to find the value of *d*.

First find the centers and radii of both spheres:

Standard Equation of a Sphere:

The standard equation of a sphere with center $(0,0,0)$ and radius r is,

$x^2+y^2+z^2=r^2$. ......(1)

And, the standard equation of a sphere with center (h,k,l) and radius r is,

$(x-h)^2+(y-k)^2+(z-l)^2=r^2$. ......(2)

Obviously, the first sphere (i) with equation $x^2+y^2+z^2=4$ has center $C_1(0,0,0)$ and radius $r_1=2$.

Now, find the center and radius of the second sphere (ii) with equation $x^2+y^2+z^2=4x+4y+4z-11$ by completing the squares.

Write the original equation of the second sphere (ii):

$x^2+y^2+z^2=4x+4y+4z-11$

$x^2+y^2+z^2-4x-4y-4z=-11$ Add $-4x-4y-4z$ to both sides

$(x^2-4x)+(y^2-4y)+(z^2-4z)=-11$ Group the terms with the same variables

Then add "the square of half the coefficient of each linear term" to each side of the equation.

$(x^2-4x+4)+(y^2-4y+4)+(z^2-4z+4)=-11+4+4+4$

$(x-2)^2+(y-2)^2+(z-2)^2=1$. Simplify

Hence the original equation $x^2+y^2+z^2=4x+4y+4z-11$ can be rewritten in the standard form of equation (2).

Compare this equation with the standard form (2), $(x-h)^2+(y-k)^2+(z-l)^2=r^2$, and identify the values of *r*, *h*, *k*, and *l* in the last equation. Then, the required values are,

$h=2,\ k=2,\ l=2,\ r=1$.

Therefore, it is clear that, the second sphere (ii) with equation $x^2+y^2+z^2=4x+4y+4z-11$ has center $C_2(2,2,2)$ and radius $r_2=1$.

By above steps, the centers of both spheres are $C_1(0,0,0)$ and $C_2(2,2,2)$, and radii of both spheres are $r_1 = 2$ and $r_2 = 1$.

The (shortest) distance between the spheres is measured along the line segment connecting their centers. To find the distance between the centers $C_1(0,0,0)$ and $C_2(2,2,2)$ of the spheres, use distance formula.

Distance Formula: The distance between the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is,

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

By the distance formula with $(x_1, y_1, z_1) = (0,0,0)$ and $(x_2, y_2, z_2) = (2,2,2)$, the distance between the centers C_1 and C_2 of the spheres is,

$|C_1C_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ Write distance formula

$= \sqrt{(2-0)^2 + (2-0)^2 + (2-0)^2}$ Substitute values

$= \sqrt{4+4+4}$ Simplify

$= \sqrt{12}$ Simplify

$= 2\sqrt{3}$.

Subtract the radius of each sphere to the distance between the spheres. From Figure-1, the distance between the spheres will be,

$$\begin{aligned} d &= |C_1C_2| - r_1 - r_2 \\ &= 2\sqrt{3} - 2 - 1 \\ &= 2\sqrt{3} - 3. \end{aligned}$$

Therefore, the distance between the spheres (i) and (ii) will be $\boxed{2\sqrt{3} - 3}$.

Answer 44E.

We construct a shape with the given constraints. Note that there may be other solutions.

If the shadow cast by rays parallel to the z-axis—i.e., the shadow cast on the xy-plane—is a circle, we assume that along the z-axis, the shape has a cross-section that is a circle at its widest point.

If the shadow cast by rays parallel to the x-axis is an isosceles triangle, which we will assume is oriented with the base on the bottom and the congruent sides rising to the apex, and then the widest part of that shadow, the base, must correspond with the circular cross section at the widest point when moving in the z-direction. We now have a shape that might look vaguely like a cone—it has a circular base but must raise to a point at the top so the shadow cast by rays along the x-axis remains a triangle.

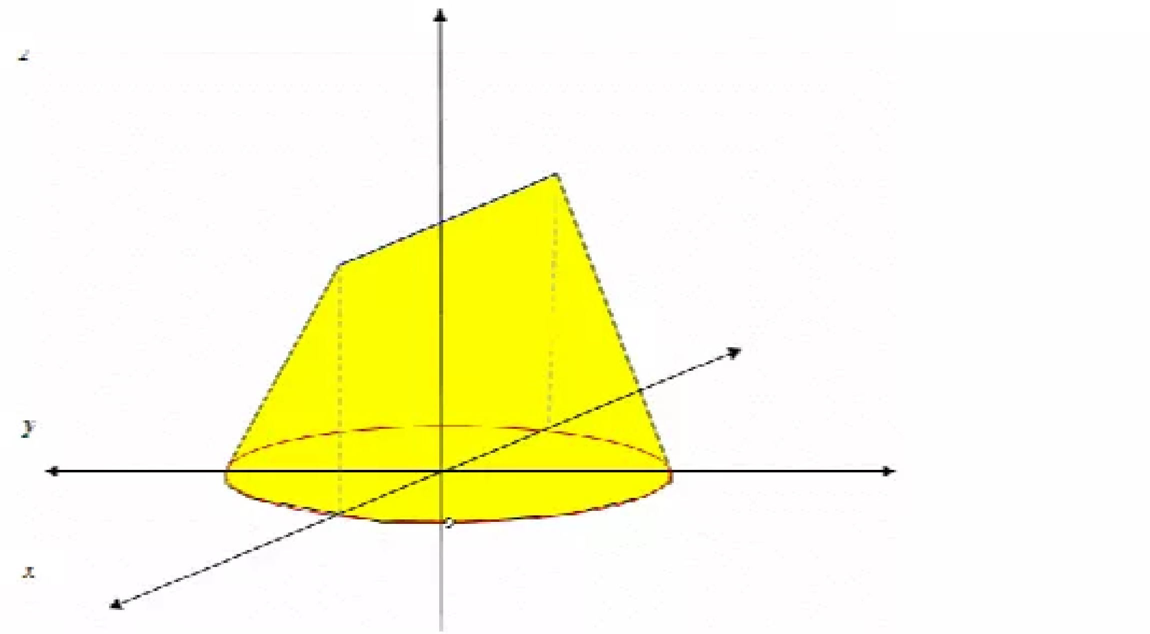
However, the shape cannot be an actual cone, because then the shadow cast by the rays along the y-axis would then also be a triangle, and it is a square. The apex of the cone-like shape could therefore be a line segment instead of a point, and run parallel to the x-axis: when looked at down the x-axis, its cross-section is a point, giving the apex of the isosceles triangle shadow. But when looking at it from the y-axis, it appears as the full line segment, forming the top of the square shadow.

This line segment should be parallel to the diameter of the circular base of the solid and be directly above the diameter that is parallel to the x-axis; this ensures the apex of the triangular shadow is above the midpoint of the base of the triangle and makes the triangle isosceles. The line segment forming the apex must also and have the same length as that diameter of the circular base, and the height of the object should be this length as well—this ensures the shadow of the square has equal sides and is, in fact, a square.

The entire solid is a solid with circular base. Call its diameter d. The solid has an apex that is a line segment of length d at height d above the circular base. The lateral sides of the solid slope smoothly up from the circumference of the circle to the line segment at the apex—think of drawing straight line segments from each point on the circumference of the circular base to the closest point on the apex line segment. When every point on the circumference is connected to the line segment in such a way, the sides of the solid have been formed, and the solid is complete.

In practical terms, this solid is what would happen if a tent were constructed by draping a piece of cloth over an elevated bar and stretching that cloth tightly to a circular ring on the ground (if the bar had length equal to the diameter of the ring and was centered above it).

Here is a sketch of the solid:



The red circle shows the circular base of the solid that the rays in the z-direction will hit. The blue line segment is the apex of the shape; the rays parallel to the x-axis will see it as the point at the top of an isosceles triangle with the diameter of the base being the base of the triangle. The gray dashed lines show that the blue line segment is directly above the diameter of the circle along the x-axis; this ensures that in the y-direction, the rays will cast the shadow of a square.