CBSE SAMPLE PAPER - 08

Class 11 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. R is a relation from {11, 12, 13} to {8, 10, 12} defined by y = x - 3. Then, R^{-1} is **[1]**

   a) None of these
   b) {(10,13), (8,11), (12,10)}
   c) {(11,8), (13,10)}
   d) {(8,11), (10,13)}

2. If $|z_1| = |z_2| = |z_3| = 1$ and $z_1 + z_2 + z_3 = \sqrt{2} + i$, then $z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1$ is **[1]**

   a) purely imaginary
   b) purely real
   c) a positive real number
   d) a negative real number

3. Solve the system of inequalities x - 2 > 0, 3x < 18 **[1]**

   a) 2 < x < 6
   b) 1 < x < 3
   c) 3 < x < 18
   d) -6 < x < -2

4. 4 letters are chosen at random from the letters of the word **INFINITE**. The probability that there will be three like letters and one different is: **[1]**

   a) $\frac{4}{21}$
   b) $\frac{7}{22}$
   c) $\frac{1}{14}$
   d) $\frac{2}{11}$

5. If A and B are two given sets , then $A \cap (A \cap B)^c$ is equal to **[1]**

   a) B
   b) A
   c) $A \cap B^c$
   d) ϕ

6. $\lim\limits_{x \to 0} \frac{8}{x^8}\left\{1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4}\right\}$ is equal to **[1]**

a) $-\frac{1}{32}$ | b) $\frac{1}{32}$
c) $-\frac{1}{16}$ | d) $\frac{1}{16}$

7. If $\left(1-x+x^2\right)^n = a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$, then $a_0 + a_2 + a_4 + \ldots + a_{2n}$ equals. **[1]**

a) $3^n + \frac{1}{2}$
b) $\frac{3^n+1}{2}$
c) $\frac{3^n-1}{2}$
d) $\frac{1-3^n}{2}$

8. The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is **[1]**

a) 3.57
b) 2.57
c) 2.23
d) 3.23

9. In a G.P. (m + n)th and (m – n)th terms are respectively p and q. Its mth term is **[1]**

a) p q
b) $\frac{p+q}{2}$
c) $\sqrt{p\,q}$
d) None of these

10. The value of $\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\frac{5\pi}{8} + \sin^2\frac{7\pi}{8}$ is : **[1]**

a) $\frac{9}{8}$
b) $\frac{17}{8}$
c) 2
d) 1

11. The lines x + (k - 1)y + 1 = 0 and 2x + k^{2}y - 1 = 0 are at right angles if **[1]**

a) k > 1
b) k = 1
c) k = - 1
d) none of these

12. If $\sin x = \frac{t}{\sqrt{1+t^2}}, then \frac{dx}{dt}$ is equal to **[1]**

a) $\frac{1}{\sqrt{1-t^2}}$
b) $\frac{1}{1+t^2}$
c) $\cos x + t^2$
d) $\frac{1}{\left(1+t^2\right)^{3/2}}$

13. If z = (3i - 1)^2 then |z| = ? **[1]**

a) 10
b) None of these
c) 8
d) 4

14. Two finite sets have m and n elements. The number of elements in the power set of the first is 48 more than the total number of elements in the power set of the second. Then the values of m and n are **[1]**

a) 6, 4
b) 6, 3
c) 3, 7
d) 7, 6

15. Three vertices of a parallelogram taken in order are (-1, -6), (2, -5) and (7, 2). The fourth vertex is **[1]**

a) (4, 1)
b) (1, 4)
c) (4, 4)
d) (1, 1)

16. For $\forall n \in N$, $2^{3n+3} - 8$ is divisible by: **[1]**

a) 343
b) 7
c) 49
d) none of these

[1]

17. $\underset{h \to 0}{Lt} \frac{\sin\sqrt{x+h} - \sin\sqrt{x}}{h}$ is equal to

a) $\frac{1}{2\sin\sqrt{x}}$

b) $\frac{\cos\sqrt{x}}{3\sqrt{x}}$

c) $\frac{\cos\sqrt{x}}{2\sqrt{x}}$

d) $\sin\sqrt{x}$

18. Solve the system of inequalities $4x + 3 \geq 2x + 17$, $3x - 5 < -2$, for the values of x, then **[1]**

a) no solution

b) $(-\frac{3}{2}, \frac{2}{5})$

c) $(-4, 12)$

d) (-2 , 2)

19. **Assertion (A):** If sin x = $-\frac{1}{3}$, then cos x = $\frac{2\sqrt{2}}{3}$ **[1]**

Reason (R): If the value of cos x is -ve and sin x is -ve then x $\in \left[\frac{3\pi}{2}, 2\pi\right]$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** No. of Permutation of n dis-similar things taken n at a time is ^np_n. **[1]**

Reason (R): n(A) = n(B) = n, then total no. of functions from A to B are n!.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the angle between the minute hand and the hour hand of a clock when the time is 7:20 **[2]**

22. Write the set in roster form: D = {x : x is an integer, $x^2 \leq 9$} **[2]**

23. Three coins are tossed once. Find the probability of getting at least two heads. **[2]**

OR

Events E and F are such that P (not E or not F) = 0.25 state whether E and F are mutually exclusive.

24. Draw the lines x = -3, x = 2, y = -2, y = 3 and write the coordinates of the vertices of the square so formed. **[2]**

25. Let R = {(x, x + 5): x $\in$ {0, 1, 2, 3, 4, 5)} **[2]**

i. Write R in roster form.

ii. Find dom (R) and range (R)

Section C

26. Evaluate: $\sqrt{4i}$. **[3]**

27. Solve the inequation $\frac{2x+4}{x-3} \leqslant 4$. **[3]**

OR

To receive grade A in a course one must obtain an average of 90 marks or more in five papers, each of 100 marks. If Tanvy scored 89, 93, 95 and 91 marks in first four papers, find the minimum marks that she must score in the last paper to get grade A in the course.

28. Using g binomial theorem, expand $\left\{(x+y)^5 + (x-y)^5\right\}$ and hence find the value of $\left\{(\sqrt{2}+1)^5 + (\sqrt{2}-1)^5\right\}$ **[3]**

OR

Find the remainder, when 5^{99} is divided by 13.

29. If A and B be the points (3, 4, 5) and (-1, 3, -7), respectively, find the equation of the set of points P such that **[3]**

$PA^2 + PB^2 = k^2$, where k is a constant.

OR

Show that the points (a, b, c), (b, c, a) and (c, a, b) are the vertices of an equilateral triangle.

30. There are four men and, six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman? **[3]**

31. If $f(x) = x^2$ find $\frac{f(1.1)-f(1)}{(1.1-1)}$. **[3]**

Section D

32. Find the mean deviation about the median for the data: **[5]**

Class	0 -10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency	6	8	11	18	5	2

33. In a survey of 100 students, the number of students studying the various languages is found as: English only 18; English but not Hindi 23; English and Sanskrit 8; Sanskrit and Hindi 8; English 26; Sanskrit 48 and no language 24. Find **[5]**

i. how many students are studying Hindi,
ii. how many students are studying English and Hindi both.

OR

In a group of 100 people, 65 like to play Cricket, 40 like to play Tennis and 55 like to play Volleyball. All of them like to play at least one of the three games. If 25 like to play both Cricket and Tennis, 24 like to play both Tennis and Volleyball and 22 like to play both Cricket and Volleyball, then

i. how many like to play all the three games?
ii. how many like to play Cricket only?
iii. how many like to play Tennis only?

Represent the above information in a Venn diagram.

34. A visitor with sign board 'DO NOT LITTER' is moving on a circular path in an exhibition. During the movement he stops at points represented by (3, - 2) and (-2, 0). Also, centre of the circular path is on the line 2x - y = 3. What is the equation of the path? What message he wants to give to the public? **[5]**

OR

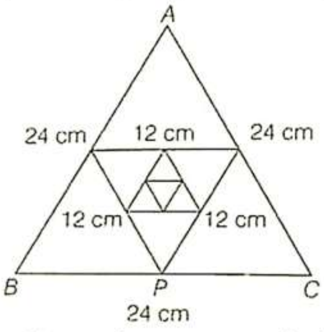
Find the equation of the circle which is circumscribed about the triangle, whose vertices are (- 2, 3), (5, 2) and (6,-1).

35. Find the differential coefficient of sec x, using first principle. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Each side of an equilateral triangle is 24 cm. The mid-point of its sides are joined to form another triangle. This process is going continuously infinite.



(i) The perimeter of 7th triangle is (in cm)

a) $\frac{3}{4}$ b) $\frac{5}{8}$

c) $\frac{9}{8}$ d) $\frac{7}{8}$

(ii) The sum of perimeter of all triangle is (in cm)

a) 144 b) 625

c) 400 d) 169

(iii) The area of all the triangle is (in sq cm)

a) 576 b) $144\sqrt{3}$

c) $169\sqrt{3}$ d) $192\sqrt{3}$

OR

The sum of perimeter of first 6 triangle is (in cm)

a) 120 b) $\frac{567}{4}$

c) $\frac{569}{4}$ d) 144

37. **Read the text carefully and answer the questions:** **[4]**

One evening, four friends decided to play a card game Rummy. Rummy is a card game that is played with decks of cards. To win the rummy game a player must make a valid declaration by picking and discarding cards from the two piles given. One pile is a closed deck, where a player is unable to see the card that he is picking, while the other is an open deck that is formed by the cards discarded by the players. To win at a rummy card game, the players have to group cards invalid sequences and sets.

In rummy, the cards rank low to high starting with Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. Ace, Jack, Queen, and King each have 10 points. The remaining cards have a value equal to their face value. For example, 5 cards will have 5 points, and so on.


Four cards are drawn from a pack of 52 playing cards, then:

(i) How many different ways can this is done

a) $\frac{48!4!}{52!}$ b) $\frac{52!}{4!48!}$

c) $\frac{52!4!}{48!}$ d) $\frac{48!}{4!52!}$

(ii) exactly one card of each suit

a) $(13)^2$ ways. b) $(13)^4$ ways

c) 13 ways d) $^{13}C_1$

(iii) all cards of the same suit

a) 2860 ways b) 2060 ways

c) 2000 ways d) 2800 ways

OR

The value of P(n, n – 1) is:

a) n

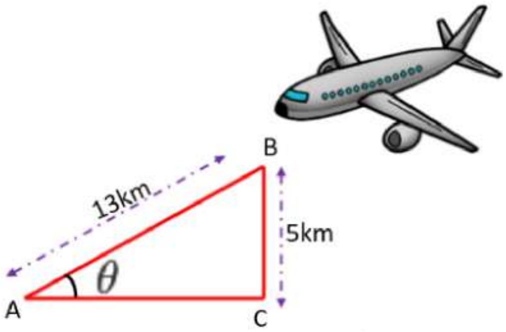
b) n!

c) 2n

d) 2n!

38. **Read the text carefully and answer the questions:** **[4]**

An airplane is observed to be approaching a point that is at a distance of 13 km from the point of observation and makes an angle of elevation of θ and the height of the airplane above the ground is 5km. Based on the above information answer the following questions.



(i) Find the value of sin 2θ.

(ii) Find the value of cos 2θ.

Solution

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Class 11 - Mathematics

Section A

1. **(d)** {(8,11), (10,13)}
Explanation: Since, y = x – 3;
Therefore, for x = 11, y = 8.
For x = 12, y = 9. [But the value y = 9 does not exist in the given set.]
For x = 13, y =10.
So, we have R = {(11, 8), (13, 10)}
Now, R^{-1} = {(8, 11), (10, 13)}.

2. **(a)** purely imaginary
Explanation: $z_1 + z_2 + z_3 = \sqrt{2} + i$
$\Rightarrow |z_1 + z_2 + z_3|^2 = |\sqrt{2} + i|^2$
$\Rightarrow |z_1|^2 + |z_2|^2 + |z_3|^2 + 2\text{Re}\left(z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1\right) = (\sqrt{2})^2 + (1)^2$
$\Rightarrow (1)^2 + (1)^2 + (1)^2 + 2\text{Re}\left(z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1\right) = 3$
$\Rightarrow \text{Re}\left(z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1\right) = 0$
$\Rightarrow \left(z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1\right)$ is purely imaginary.

3. **(a)** 2 < x < 6
Explanation: x - 2 > 0
$\Rightarrow$ x > 2
$\Rightarrow x \in (2, \infty)$
Now 3x < 18
$\Rightarrow$ x < 6
$\Rightarrow x \in (-\infty, 6)$
So solution set is $(2, \infty) \cap (-\infty, 6)$ = (2, 6)
$\Rightarrow$ 2 < x < 6

4. **(c)** $\frac{1}{14}$
Explanation: $\frac{1}{14}$

5. **(c)** $A \cap B^c$
Explanation: $A \cap B^c$
A and B are two sets.
$A \cap B$ is the common region in both the sets.
$(A \cap B^c)$ is all the region in the universal set except $A \cap B$
Now, $A \cap (A \cap B)^c = A \cap B^c$

6. **(b)** $\frac{1}{32}$
Explanation: $\lim_{x \to 0} \frac{8}{x^8}\left[1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4}\right]$
$= \lim_{x \to 0} \frac{8}{x^8}\left[\left(1 - \cos\frac{x^2}{4}\right) - \cos\frac{x^2}{2}\left(1 - \cos\frac{x^2}{4}\right)\right]$
$= \lim_{x \to 0} \frac{8}{x^8}\left[\left(1 - \cos\frac{x^2}{4}\right)\left(1 - \cos\frac{x^2}{2}\right)\right]$
$= \lim_{x \to 0} \frac{8}{x^8}\left[\left(2\sin^2\frac{x^2}{8}\right)\left(2\sin^2\frac{x^2}{4}\right)\right]$
$= \lim_{x \to 0} 4 \times 8 \frac{\left(\sin^2\frac{x^2}{8}\right)}{\left(64 \times \frac{x^4}{64}\right)} \frac{\left(\sin^2\frac{x^2}{4}\right)}{16\left(\frac{x^4}{16}\right)}$
$= \frac{32}{64 \times 16}$
$= \frac{1}{32}$

7. **(b)** $\frac{3^n+1}{2}$

Explanation: $\left(1-x+x^2\right)^n=a_0+a_1x+a_2x^2+\ldots+a_{2n}x^{2n}$ ..(1)

Put x=1 in (1),we get

$1=a_0+a_1+a_2+a_3+\ldots+a_{2n}$ ..(2)

Put x=-1 in(1),we get

$3^n=a_0-a_1+a_2-a_3+\ldots+a_{2n}$ ..(3)

Adding(1) and(2),we get

$3^n+1=2\left(a_0+a_2+a_4+\ldots+a_{2n}\right)$

Thus, $a_0+a_2+a_4+\ldots+a_{2n}=\frac{3^n+1}{2}$

8. **(b)** 2.57

Explanation: M.D. $(\bar{x})=\frac{|x_i-\bar{x}|}{n}=\frac{4+3+3+3+0+3+2}{7}$ = 2.57

9. **(c)** $\sqrt{p\,q}$

Explanation: Let a be the first term and r be the common ratio of the G.P

Then since $T_n = a^{n-1}$, we get

$T_{m+n} = p \Rightarrow ar^{m+n-1} = p$ ....(i)

$T_{m-n} = q \Rightarrow ar^{m-n-1} = q$ ...(ii)

Now dividing equation (i) by (ii)

$\frac{ar^{m+n-1}}{ar^{m-n-1}}=\frac{p}{q}\Rightarrow$ $r^{(m + n - 1) - (m - n - 1)} = \frac{p}{q}$

$\Rightarrow r2n=\frac{p}{q}$

Taking square root on both sides we get $r^n = \sqrt{\frac{p}{q}}$ ....(iii)

Substituting equation (iii) in (i), we get

$a.\sqrt{\frac{p}{q}}.r^{m-1}$ = p

$\Rightarrow a.r^{m-1}=p.\sqrt{\frac{q}{p}}=\sqrt{pq}$

10. **(c)** 2

Explanation: 2

11. **(c)** k = - 1

Explanation: If the lines are at right angles to each other, then the product of their slopes = -1. Slope of any line = -(coefficient of x/coeffiecient of y)

Therefore the slope of line 1 = $-\frac{1}{k-1}$

The slope of line 2 = $\frac{-2}{k^2}$

Therefore $\frac{-1}{(k-1)}\times\frac{-2}{k^2}$ = - 1

That is $k^2 (k - 1) = -2$

i.e; $k^3 - k^2 + 2 = 0$

On factorizing we get $(k + 1)(k^2 - 2k - 2) = 0$

This imlpies k + 1 is a factor, hence k = -1

Hence they are at right angles if k = -1

12. **(b)** $\frac{1}{1+t^2}$

Explanation: $we\ have,$

$$x=sin^{-1}\frac{t}{\sqrt{1+t^2}}$$
$$let\ t=\tan\theta$$
$$\Rightarrow x=\theta\Rightarrow\frac{dx}{d\theta}=1$$
$$Also,\ \frac{dt}{d\theta}=\sec^2\theta$$
$$Now,\frac{dx}{dt}=\frac{1}{\sqrt{1+t^2}}$$

13. **(a)** 10

Explanation: $z = (3i - 1)^2 = (9i^2 + 1 - 6i) = (-9 + 1 - 6i) = (-8 - 6i)$

$\Rightarrow |z|^2 = \{(-8)^2 + (-6)^2\} = (64 + 36) = 100$

$\Rightarrow |z| = \sqrt{100} = 10$

14. **(a)** 6, 4

Explanation: Let A has m elements and B gas n elements. Then, no. of elements in

P(A) = 2^m and no. of elements in P(B) = 2^n.]

By the question,

$2^m = 2^n + 48$

$\Rightarrow 2^m - 2^n = 48$

This is possible, if $2^m = 64$, $2^n = 16$. (As 64 - 16 = 48)

$\therefore 2m = 64 \Rightarrow 2^m = 2^6$

$\Rightarrow m = 6$

Also, $2^4 = 16 \Rightarrow 2^4 = 24$

$\Rightarrow n = 4$

15. **(a)** (4, 1)

Explanation: Let A(-1, -6), B(2, -5) and C(7, 2) be the given vertex. Let D(h, k) be the fourth vertex

The midpoints of AC and BD are (3, -2) and $\left(\frac{2+h}{2}, \frac{-5+k}{2}\right)$ respectively.

We know that the diagonals of a parallelogram bisect each other , using this result we get ,

$\therefore 3 = \frac{2+h}{2}$ and $-2 = \frac{-5+k}{2}$

$\Rightarrow$ h = 4 and k = 1

16. **(c)** 49

Explanation: 49

17. **(c)** $\frac{\cos\sqrt{x}}{2\sqrt{x}}$

Explanation: $\underset{h\to 0}{Lt} \frac{2\cos(\frac{\sqrt{x+h}+\sqrt{x}}{2})\sin(\frac{\sqrt{x+h}-\sqrt{x}}{2})}{h}$

$using\ L'Hospital$

$\Rightarrow \frac{\cos\sqrt{x}}{2\sqrt{x}}$

18. **(a)** no solution

Explanation:

We have given: $4x + 3 \geq 2x + 17$

$\Rightarrow 4x - 2x \geq 17 - 3 \Rightarrow 2x \geq 1$

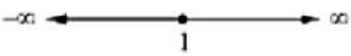
$\Rightarrow \quad x \geq \frac{14}{2}$ [Dividing by 2 on both sides]

$\Rightarrow x \geq 7$ ..... (i)

Also we have 3x - 5 < -2

$\Rightarrow \quad 3x < -2 + 5 \Rightarrow 3x < 3$

$\Rightarrow \quad x < 1$



On combining (i) and (ii), we see that solution is not possible because nothing is common between these two solutions.(i.e., x < 1, x $\geq$ 7)

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Assertion

sin x = $\frac{-1}{3}$

So, x $\in$ 3rd or 4th quadrant.

cos x = $\pm\sqrt{1 - \sin^2 x}$ [$\because \sin^2 x + \cos^2 x = 1$]

$= \pm\sqrt{1 - \left(-\frac{1}{3}\right)^2}$

$= \pm\sqrt{1 - \frac{1}{9}}$

$= \pm\sqrt{\frac{8}{9}}$

= cos x = $\pm\frac{2\sqrt{2}}{3}$

sin x = -ve, cos x = +ve, possible only when x $\in \pi$ quad. i.e. x $\in \left(\frac{3\pi}{2}, 2\pi\right)$

20. **(c)** A is true but R is false.

Explanation: $^{n}P_{n}$ = n!

Assertion: true, but the no. of functions from A to B is n^n.

Section B

21. Angle traced by the hour hand in 12 hours = 360 degrees

Angle traced by it in 7 h 20 min, i.e., in $\frac{22}{3}$ hours $= \left(\frac{360}{12} \times \frac{22}{3}\right)^{\circ} = 220^{\circ}$

Angle traced by the minute hand in 60 min = 360°

Angle traced by the minute hand in 20 min = $\left(\frac{360}{60} \times 20\right)^{\circ} = 120^{\circ}$

Hence, the required angle between the two hands is given below:

θ = (220° - 120°) =100°

22. We have,the set of integers = {…, -4, -3, -2, -1, 0, 1, 2, 3, 4, …}

x = -4, $x^2 = (-4)^2 = 16 > 9$

x = -3, $x^2 = (-3)^2 = 9$

x = -2, $x^2 = (-2)^2 = 4$

x = -1, $x^2 = (-1)^2 = 1$

x = 0, $x^2 = (0)^2 = 0$

x = 1, $x^2 = (1)^2 = 1$

x = 2, $x^2 = (2)^2 = 4$

x = 3, $x^2 = (3)^2 = 9$

x = 4, $x^2 = (4)^2 = 16$

The elements of this set are -3, -2, -1, 0, 1, 2, 3

Therefore, D = {-3, -2, -1, 0, 1, 2, 3}

23. Let S be the sample space associated with the random experiment of tossing three coins. Then sample space, S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

Clearly, there are 8 elements in S

Therefore Total number of elementary events = 8.

At least two heads can be obtained if we obtain one of the following elementary events as an outcome: HHH, HHT, HTH, THH

Favourable number of elementary events = 4

Hence, required probability $=\frac{4}{8}=\frac{1}{2}$

OR

Given, P (not E or not F) = 0.25

$\Rightarrow P\left(\overline{E} \cup \overline{F}\right) = 0.25$

$\Rightarrow P\left(\overline{E \cap F}\right) = 0.25 \ [\because (\overline{A} \cup \overline{B}) = (\overline{A \cap B}) \text{ by De Morgan's law}]$

$\Rightarrow 1 - P(E \cap F) = 0.25$

$\therefore P(E \cap F) = 1 - 0.25 = 0.75 \neq \phi$

Hence, E and F are not mutually exclusive events.

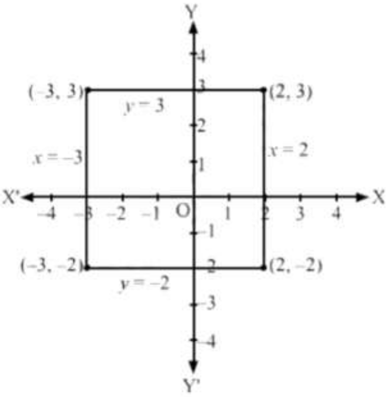
24. Clearly, lines x = -3 and x = 2 are parallel to the y -axis.

They pass through (-3, 0) and (2, 0), respectively.

Similarly, the lines y = -2, y = 3 are parallel to the x -axis.

They pass through (0, -2) and (0, 3), respectively.

The lines x = -3, x = 2, y = -2 and y = 3 are drawn in the figure below.



Therefore, the coordinates of the square that is formed are (2, 3), (-3, 3), (-3, -2) and (2, -2).

25. Here we are given that, R = {(x, x + 5} : x ∈ {9, 1, 2, 3, 4, 5)}

    i. Foster form of R

       R = {(9, 14), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}

    ii. Domain of R = {1, 2, 3, 4, 5, 9}

       Range of R = {6, 7, 8, 9, 10}

Section C

26. Let, $(a + ib)^2 = 0 + 4i$

    $\Rightarrow a^2 + (bi)^2 + 2abi = 0 + 4i$ $[(a + b)^2 = a^2 + b^2 + 2ab]$

    $\Rightarrow a^2 - b^2 + 2abi = 0 + 4i$ $[i^2 = -1]$

    Now, separating real and complex parts, we get

    $\Rightarrow a^2 - b^2 = 0$ …………..(1)

    $\Rightarrow 2ab = 4$

    $\Rightarrow a = \frac{2}{b}$ …….. (2)

    Now, using the value of a in (1), we get

    $\left(\frac{2}{b}\right)^2 - b^2 = 0$

    $\Rightarrow 4 - b^4 = 0$

    $\Rightarrow b^4 = 4$

    Simplify and get the value of b^2 , we get,

    $\Rightarrow b^2 = -2$ or $b^2 = 2$

    As b is real no. so, $b^2 = 2$

    b= $\sqrt{2}$ or b= $-\sqrt{2}$

    put value of b in equation (2) ==> a= $\sqrt{2}$ or a= $-\sqrt{2}$

    Hence the square root of the complex no. is $\sqrt{2}+\sqrt{2}i$ and $-\sqrt{2}-\sqrt{2}i$.

27. Here $\frac{2x+4}{x-3} \leqslant 4, x \neq 3$

    $\Rightarrow \frac{2x+4}{x-3} - 4 \leqslant 0$

    $\Rightarrow \frac{2x+4-4x+12}{x-3} \leqslant 0$

    $\Rightarrow \frac{-2x+16}{x-3} \leqslant 0$

    $\Rightarrow -2x+16 \leqslant 0$

    $\Rightarrow -2x \leqslant -16$

    Dividing both sides by -2

    $\Rightarrow x \geqslant 8$

    the solution set of given in equation is $[8, \infty)$.

    OR

    Let x marks be scored by Tanvy in her last paper.

    It is given that Tanvy scored 89, 93, 95 and 91 marks in the first 4 papers.

    To receive grade A, she must obtain an average of 90 marks or more.

    Therefore, the average of there marks must more than equal to 90

$\frac{89+93+95+91+x}{5} \geq 90$

Multiplying both the sides by 5 in the above equation

$\Rightarrow \left(\frac{89+93+95+91+x}{5}\right)(5) \geq 90(5)$

$\Rightarrow 368 + x \geq 450$

Subtracting 368 from both the sides in the above equation

$\Rightarrow 368 + x - 368 \geq 450 - 368$

$\Rightarrow x \geq 82$

Therefore, Tanvy should score a minimum of 82 marks in her last paper to get grade A in the course.

28. $We\ have$

$(x+y)^5 + (x-y)^5 = 2\left[{}^5C_0\, x^5 + {}^5C_2\, x^3y^2 + {}^5C_4\, x^1y^4\right]$

$= 2\left(x^5 + 10x^3y^2 + 5xy^4\right)$

$Putting\ x = \sqrt{2}\ and\ y = 1,\ we\ get$

$(\sqrt{2}+1)^5 + (\sqrt{2}-1)^5 = 2\left[\left(\sqrt{2}\right)^5 + 10\left(\sqrt{2}\right)^3 + 5\sqrt{2}\right]$

$= 2\left[4\sqrt{2} + 20\sqrt{2} + 5\sqrt{2}\right]$

$= 58\sqrt{2}$

OR

We know that, $5^4 = 625 = 13 \times 48 + 1$

$\Rightarrow 5^4 = 13\lambda + 1$, where λ is a postitive interger.

$\Rightarrow (5^4)^{24} = (13\lambda + 1)^{24}$

$= {}^{24}C_0\,(13\lambda)^{24} + {}^{24}C_1\,(13\lambda)^{23} + {}^{24}C_2\,(13\lambda)^{22} + \ldots + {}^{24}C_{23}\,(13\lambda) + {}^{24}C_{24}$ [by binomial theorem]

$\Rightarrow 5^{96} = 13\,[{}^{24}C_0\, 13^{23}\lambda^{24} + {}^{24}C_1 13^{22}\lambda^{23} + \ldots + {}^{24}C_{23}\lambda] + 1$ = (a multiple of 13) + 1

On multiplying both sides by 5^3, we get

$5^{96} \times 5^3$ = [(a multiple of 13) + 1] $[13 \times 9 + 8]$ $[\because 5^3 = 125 = 13 \times 9 + 8]$

$\Rightarrow 5^{99} = 13 \times 9$ (a multiple of 13) + $13 \times 9 + 8$ (a multiple of 13) + 8

Hence, the required remainder is 8.

29. The equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant

Given: The points A (3, 4, 5) and B (-1, 3, -7)

$\Rightarrow x_1 = 3, y_1 = 4, z_1 = 5; x_2 = -1, y_2 = 3, z_2 = -7;$

$PA^2 + PB^2 = k^2$ …(i)

Let the point be P (x, y, z).

Now, by Distance Formula, we know that the distance between two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is given by

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$

So, $PA = \sqrt{(3-x)^2 + (4-y)^2 + (5-z)^2}$

And $PB = \sqrt{(-1-x)^2 + (3-y)^2 + (-7-z)^2}$

Now, substituting these values in (i), we have

$[(3-x)^2 + (4-y)^2 + (5-z)^2] + [(-1-x)^2 + (3-y)^2 + (-7-z)^2] = k^2$

$\Rightarrow [(9 + x^2 - 6x) + (16 + y^2 - 8y) + (25 + z^2 - 10z)] + [(1 + x^2 + 2x) + (9 + y^2 - 6y) + (49 + z^2 + 14z)] = k^2$

$\Rightarrow 9 + x^2 - 6x + 16 + y^2 - 8y + 25 + z^2 - 10z + 1 + x^2 + 2x + 9 + y^2 - 6y + 49 + z^2 + 14z = k^2$

$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$

$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = k^2 - 109$

$\Rightarrow 2\,(x^2 + y^2 + z^2 - 2x - 7y + 2z) = k^2 - 109$

$\Rightarrow x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$

OR

Let A (a, b, c), B (b, c, a), and C (c, a, b) be the vertices of $\triangle$ABC. Then,

AB = $\sqrt{(b-a)^2+(c-b)^2+(a-c)^2}$

$=\sqrt{b^2-2ab+a^2+c^2-2bc+b^2+a^2-2ca+c^2}$

$=\sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}$

AB $=\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}$

BC = $\sqrt{(c-b)^2+(a-c)^2+(b-a)^2}$

$=\sqrt{c^2-2bc+b^2+a^2-2ca+c^2+b^2-2ab+a^2}$

$=\sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}$

BC $=\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}$

CA = $\sqrt{(a-c)^2+(b-a)^2+(c-b)^2}$

$=\sqrt{a^2-2ca+c^2+b^2-2ab+a^2+c^2-2bc+b^2}$

$=\sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}$

CA $=\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}$

$\therefore$ AB = BC = CA

Therefore, $\triangle$ABC is an equilateral triangle.

30. Here total members in the council = 4 + 6 = 10

One member is selected out of 10 members

$\therefore\ n(S)={}^{10}C_1=10$

Let A be the event that the member is a woman.

$n(A)={}^{6}C_1=6$

Thus $P(A)=\frac{n(A)}{n(S)}=\frac{6}{10}=\frac{3}{5}$

31. Here f (x) = x^2

At x = 1.1

f(1.1) = $(1.1)^2$ = 1.21

f (1) = $(1)^2$ = 1

$\therefore\ \frac{f(1.1)-f(1)}{(1.1-1)}=\frac{1.21-1}{0.1}=\frac{0.21}{0.1}=2.1$

Section D

32. Given data:

Class	**Frequency f_i**
0 - 10	6
10 - 20	8
20 - 30	11
30 - 40	18
40 - 50	5
50 - 60	2

we get following table from the given data by adding some more columns as below.

Class	**Frequency f_i**	**Cumulative frequency e.f**	**Mid-points x_i**
0 - 10	6	6	5
10 - 20	8	14	15
20 - 30	11	25	25
30 - 40	18	43	35
40 - 50	5	48	45
50 - 60	2	50	55
	50		

The Class interval containing $\frac{N^{th}}{2}$ or 25th item is 20 - 30. Therefore 20 - 30 is the median class.

As we know that

$\text{Median} = 1 + \frac{\frac{N}{2} - C}{f} \times h$

Here, I = 20, C = 14, f = 11, h = 10 and N = 50

Therefore,

$\text{Median} = 20 + \frac{\frac{50}{2} - 14}{11} \times 10$ = 20 + 10 = 30

Class	Frequency f_i	Cumulative frequency e.f	Mid-points x_i	$[x_i - M]$	$f_i[x_i - M]$
0 - 10	6	6	5	25	150
10 - 20	8	14	15	15	120
20 - 30	11	25	25	5	55
30 - 40	18	43	35	5	90
40 - 50	5	48	45	15	75
50 - 60	2	50	55	25	50
	50				540

From above table, we get,

$\sum_{i=1}^{6} f_1 = 50$ and $\sum_{i=1}^{6} f_1 [x_i - M] = 540$

Therefore, Mean Deviation (M) $= \frac{\sum_{i=1}^{6} f_1[x_i - M]}{\sum_{i=1}^{6} f_1} = \frac{540}{50}$ = 10.80

33. Given: Total number of students = 100

Number of students studying English(E) only = 18

Number of students learning English but not Hindi(H) = 23

Number of students learning English and Sanskrit(S) = 8

Number of students learning Sanskrit and Hindi = 8

Number of students learning English = 26

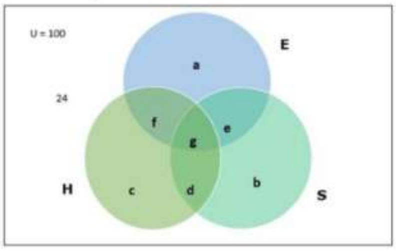
Number of students learning Sanskrit = 48

Number of students learning no language = 24

To Find:

i. Number of students studying Hindi

Venn diagram:



From the above Venn diagram,

a = Number of students who study only English = 18

b = Number of students who study only Sanskrit

c = Number of students who study only Hindi

d = Number of students learning Hindi and Sanskrit but not English

e = Number of students learning English and Sanskrit but not Hindi

f = Number of students learning Hindi and English but not Sanskrit

g = Number of students learning all the three languages

e + g = Number of students learning English and Sanskrit = 8

$= n(E \cap S)$

g + d = Number of students learning Hindi and Sanskrit = 8

$= n(H \cap S)$

E = a + e + f + g = Number of students learning English

26 = 18 + 8 + f

f = 26 -26 = 0

Therefore, f = 0

Now,

Number of students learning English but not Hindi = a + e = 23

23 = 18 + e

Therefore, e = 5

Now, e + g = 8

5 + g = 8

Therefore, g = 3

S = b + e + d + g = Number of students studying Sanskrit

48 = b + 5 + 8 (Because, d + g = 8)

b = 48 - 13

Therefore, b = 35 (Number of students studying Sanskrit only)

Also, d + g = 8

d + 3 = 8

Therefore, d = 5

Now,

Number of students studying Hindi only = c

c = 100 - (a + e + b + d + f + g) - 24

= 100 - (18 + 5 + 35 + 5 + 0 + 3) -24

= 100 - 66 - 24

= 100 - 90 = 10

Number of students studying Hindi = c + f + g + d

= 10 + 0 + 3 + 5 = 18

Therefore, number of students studying Hindi = 18

ii. Number of students studying English and Hindi both

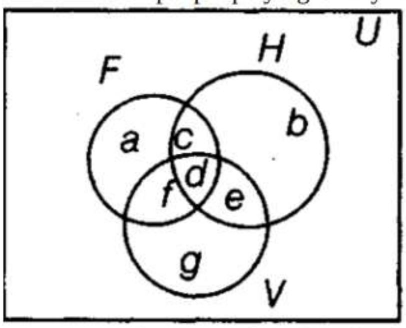
Number of students studying English and Hindi both = f + g

= 0 + 3 = 3

Therefore, the number of students studying English and Hindi both = 3

OR

Let n(C) represent the number of people playing Cricket, n(T) represents the number of people playing Tennis and n(V) represents the number of people playing Volleyball.



Let in Venn diagram a, b, c, d, e, f and g denote the number of elements in respective regions.

Now, from the Venn diagram, we have

n$(C \cup T \cup V)$ = a + b + c + d + e + f + g = 100 ...(i)

$n(C) = a + c + d + f = 65$ ....(ii)

$n(T) = c + b + d + e = 40$ ....(iii)

$n(V) = e + d + f + g = 55$ .... (iv)

$n(C \cap T) = c + d = 25$ ....(v)

$n(T \cap V) = d + e = 24$ ....(vi)

$n(C \cap V) = f + d = 22$ ....(vii)

and n(C ∩ T ∩V) = d

Using the identity,

$n(C \cup T \cup V) = [n(C) + n(T) + n(V)] - [n(C \cap T) + n(T \cap V) + n(C \cap V)] + n(C \cap T \cap V)$

$\Rightarrow n(C \cap T \cap V) = n(C \cup T \cup V)$ -[n(C) + n(T) + n(V)] + $[n(C \cap T) + n(T \cap V) + n(C \cap V)]$

$= 100 - (65 + 40 + 55) + (25 + 24 + 22)$

$= 100 - 160 + 71 = 11$

Thus, n($C \cap T \cap V$) = d = 11

From Eq. (v), we get

$c = 25 - 11 = 14$

From Eq. (vi), we get,

$e = 24 - 11 = 13$

From Eq. (vii), we get,

$f = 22 - 11 = 11$

From Eq. (iv), we get,

$g = 55 - (13 + 11 + 11) = 20$

From Eq. (iii), we get,

$b = 40 - (14 + 11 + 13) = 2$

From Eq. (ii), we get,

$a = 65 - (14 + 11 + 11) = 29$

Hence,

i. the number of people who like to play all three games, $d = 11$
ii. the number of people who like to play Cricket only, $a = 29$
iii. the number of people who like to play Tennis only, $b = 2$

34. Let the equation of circle whose centre (- g, - f) be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

Since, is passes through points (3, - 2) and (- 2, 0)

$\therefore (3)^2 + (-2)^2 + 2g(3) + 2f(-2) + c = 0$

and $(-2)^2 + (0)^2 + 2g(-2) + 2f(0) + c = 0$

$\Rightarrow 9 + 4 + 6g - 4f + c = 0$

and $4 + 0 - 4g + 0 + c = 0$

$\Rightarrow 6g - 4f + c = -13$

and $c = 4g - 4$ ...(ii)

$\therefore 6g - 4f + (4g - 4) = -13$

$\Rightarrow 10g - 4f = -9$ ...(iii)

Also, centre (- g, - f) lies on the line 2x - y = 3

$\therefore -2g + f = 3$ ...(iv)

On solving Eqs. (iii) and (iv), we get

$g = \frac{3}{2}$ and $f = 6$

On putting the values of g and f in Eq. (ii), we get

$c = 4\left(\frac{3}{2}\right) - 4 = 6 - 4 = 2$

On putting the values of g, f and c in Eq. (i), we get

$x^2 + y^2 + 2\left(\frac{3}{2}\right)x + 2(6)x + 2 = 0$

$\Rightarrow x^2 + y^2 + 3x + 12x + 2 = 0$

which is the required equation of the path

The message which he wants to give to the public is 'Keep your place clean'.

OR

The circle which is circumscribed about the triangle, whose vertices are (- 2, 3), (5, 2) and (6, - 1) means the circle passes through these three points.

Let the equation of circle be

$(x - h)^2 + (y - k)^2 = r^2$ ...(i)

Since, equation (i) passes through the points (- 2, 3),

$\therefore (-2 - h)^2 + (3 - k)^2 = r^2$

$\Rightarrow h^2 + 4h + 4 + k^2 - 6k + 9 = r^2$ ...(ii) also equation (i) passes through the point (5,2)

$\Rightarrow (5 - h)^2 + (2 - k)^2 = r^2$

$\Rightarrow h^2 - 10h + 25 + k^2 - 4k + 4 = r^2$ ...(iii)

again equation (i) passes through the point (6,-1) $\Rightarrow (6 - h)^2 + (-1 - k)^2 = r^2$

$\Rightarrow h^2 - 12h + 36 + k^2 + 2k + 1 = r^2$ ...(iv)

Now subtracting equation(iii) from equation (ii), we get

$14h - 21 - 2k + 5 = 0$

i.e., $14h - 2k = 16$

$\Rightarrow 7h - k = 8$ ...(v)

again subtracting equation (iv) from equation(iii), we get

$2h - 11 - 6k + 3 = 0$

i.e., $2h - 6k = 8$

$\Rightarrow h - 3k = 4$...(vi)

On solving equation (v) and equation (vi), we get

$h = 1$ and $k = -1$

On putting the values of $h = 1$ and $k = -1$ in equation (ii), we get

$1 + 4 + 4 + 1 + 6 + 9 = r^2$

$\Rightarrow r^2 = 25$

$\Rightarrow r = 5$

Now putting h=1, k=-1 and r=5 in equation (i) we get

$(x - 1)^2 + (y + 1)^2 = 25$

$\Rightarrow x^2 + y^2 - 2x + 2y - 23 = 0$

which is the required equation of the circle.

35. We have, $f(x) = \sec x$

By using first principle of derivative,

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{\sec(x+h)-\sec x}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{\cos(x+h)} - \frac{1}{\cos x}}{h}$

$= \lim_{h\to 0} \frac{\cos x - \cos(x+h)}{h \times \cos x \cdot \cos(x+h)}$

$= \lim_{h\to 0} \left[\frac{-2\sin\left(\frac{x+x+h}{2}\right)\cdot \sin\frac{(x-x-h)}{2}}{h\cdot \cos x \cdot \cos(x+h)}\right]$

$\left[\because \cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$

$= \lim_{h\to 0} \left[\frac{-2\sin\left(x+\frac{h}{2}\right)\cdot\left(-\sin\frac{h}{2}\right)}{h\cdot \cos x \cos(x+h)}\right]$

$= \lim_{h\to 0} \frac{\sin\left(x+\frac{h}{2}\right)}{\cos(x+h)\cdot \cos x} \cdot \lim_{h\to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}}$

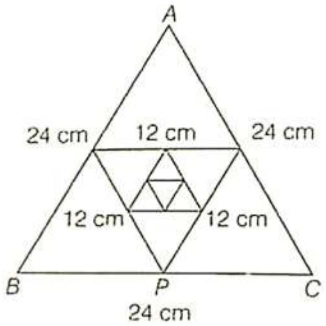
$= \frac{\sin x}{\cos^2 x} \times (1) = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$

$= \tan x \times \sec x$

Section E

36. **Read the text carefully and answer the questions:**

Each side of an equilateral triangle is 24 cm. The mid-point of its sides are joined to form another triangle. This process is going continuously infinite.



(i) **(c)** $\frac{9}{8}$
Explanation: $\frac{9}{8}$

(ii) **(a)** 144
Explanation: 144

(iii) **(d)** $192\sqrt{3}$
Explanation: $192\sqrt{3}$

OR

(b) $\frac{567}{4}$
Explanation: $\frac{567}{4}$

37. **Read the text carefully and answer the questions:**
One evening, four friends decided to play a card game Rummy. Rummy is a card game that is played with decks of cards. To win the rummy game a player must make a valid declaration by picking and discarding cards from the two piles given. One pile is a closed deck, where a player is unable to see the card that he is picking, while the other is an open deck that is formed by the cards discarded by the players. To win at a rummy card game, the players have to group cards invalid sequences and sets.
In rummy, the cards rank low to high starting with Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. Ace, Jack, Queen, and King each have 10 points. The remaining cards have a value equal to their face value. For example, 5 cards will have 5 points, and so on.

Four cards are drawn from a pack of 52 playing cards, then:

(i) **(b)** $\frac{52!}{4!48!}$
Explanation: $\frac{52!}{4!48!}$

(ii) **(b)** $(13)^4$ ways
Explanation: $(13)^4$ ways

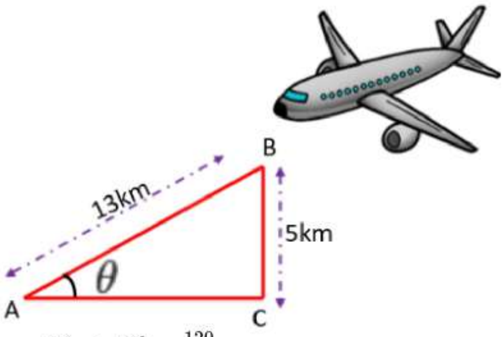
(iii) **(a)** 2860 ways
Explanation: 2860 ways

OR

(b) n!
Explanation: n!

38. **Read the text carefully and answer the questions:**
An airplane is observed to be approaching a point that is at a distance of 13 km from the point of observation and makes an angle of elevation of θ and the height of the airplane above the ground is 5km. Based on the above information answer the following

questions.



(i) $\sin 2\theta = \frac{120}{169}$

From diagram $\tan\theta = \frac{5}{12}$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\left(\frac{5}{12}\right)}{1+\left(\frac{5}{12}\right)^2} = \frac{\frac{10}{12}}{\frac{169}{144}}$$

$\Rightarrow \sin 2\theta = \frac{120}{169}$

(ii) $\cos 2\theta = \frac{119}{169}$

From diagram $\tan\theta = \frac{5}{12}$

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\left(\frac{5}{12}\right)^2}{1+\left(\frac{5}{12}\right)^2} = \frac{\frac{119}{144}}{\frac{169}{144}}$$

$\Rightarrow \cos 2\theta = \frac{119}{169}$