# Class XII Session 2024-25
# Subject - Mathematics
# Sample Question Paper - 6

**Time Allowed: 3 hours** **Maximum Marks: 80**

**General Instructions:**

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

**Section A**

1. If A = $[a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, & \text{when } i \neq j \\ 0, & \text{when } i = j \end{cases}$, then $A^2$ is **[1]**

a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

2. Three points P(2x, x + 3), Q(0, x) and R(x + 3, x + 6) are collinear, then x is equal to: **[1]**

a) 2

b) 0

c) 3

d) 1

3. If A·(adj A) = $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of |A| + |adj A| is equal to: **[1]**

a) 12

b) 3

c) 27

d) 9

4. If A is a non singular matrix and A' denotes the transpose of A, then **[1]**

a) $|AA'| \neq |A^2|$

b) $|A| - |A'| \neq 0$

c) $|A| + |A'| \neq 0$

d) $|A| \neq |A'|$

5. If $(a_1, b_1, c_1)$ and $(a_2, b_2, c_2)$ be the direction ratios of two parallel lines then **[1]**

a) $a_1^2 + b_1^2 + c_1^2 = a_2^2 + b_2^2 + c_2^2$

b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

c) $a_{1,} = a_2, b_1 = b_2, c_1 = c_2$

d) $a_1a_2 + b_1b_2 + c_1c_2 = 0$

6. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is **[1]**

a) y = kx
b) log y - kx
c) cos x
d) tan x

7. The corner points of the feasible region for a Linear Programming problem are P(0, 5), Q(1, 5), R(4, 2) and S(12, 0). The minimum value of the objective function Z = 2x + 5y is at the point. **[1]**

a) Q
b) S
c) R
d) P

8. In $\triangle ABC$, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is mid-point of BC, then vector $\overrightarrow{AD}$ is equal to: **[1]**

a) $\hat{i} - \hat{j} + \hat{k}$
b) $2\hat{i} - 2\hat{j} + 2\hat{k}$
c) $4\hat{i} + 6\hat{k}$
d) $2\hat{i} + 3\hat{k}$

9. $\int e^{5 \log x}$ dx is equal to: **[1]**

a) $\frac{x^5}{5} + C$
b) $6x^5 + C$
c) $\frac{x^6}{6} + C$
d) $5x^4 + C$

10. If A = $\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and A = $A^T$, where $A^T$ is the transpose of the matrix A, then **[1]**

a) x = 0, y = 5
b) x = 5, y = 0
c) x = y
d) x + y = 5

11. Which of the following statements is correct? **[1]**

a. Every LPP admits an optimal selection.
b. A LPP admits unique optimal solution.
c. If a LPP admits two optimal solutions it has an infinite solution.
d. The set of all feasible solutions of a LPP is not a convex set.

a) Option (d)
b) Option (a)
c) Option (b)
d) Option (c)

12. If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then the value of $\lambda$ is: **[1]**

a) 0
b) 1
c) $\frac{-3}{2}$
d) $\frac{-2}{3}$

13. Adj.(KA) = _______ **[1]**

a) $K^{n-1}$ Adj. A
b) $K^{n+1}$ Adj. A
c) K Adj. A
d) $K^n$ Adj.A

14. X and Y are independent events such that $P(X \cap \bar{Y}) = \frac{2}{5}$ and $P(X) = \frac{3}{5}$. Then P(Y) is equal to: **[1]**

a) $\frac{2}{3}$
b) $\frac{1}{3}$
c) $\frac{1}{5}$
d) $\frac{2}{5}$

15. The general solution of a differential equation of the type $\frac{dx}{dy} + P_1 x = Q_1$ is **[1]**

a) $xe^{\int P_1 dy} = \int \left(Q_1 e^{\int P_1 dy}\right) dy + C$
b) $ye^{\int P_1 dy} = \int \left(Q_1 e^{\int P_1 dy}\right) dy + C$

c) $y \cdot e^{\int P\,dx} = \int \left(Q_1 e^{\int P_1 dx}\right) dx + C$

d) $xe^{\int P^1 dx} = \int \left(Q_1 e^{\int P_1 dx}\right) dx + C$

16. If $|\vec{a}| = 4$ and $-3 \leq \lambda \leq 2$, then the range of $|\lambda\vec{a}|$ is **[1]**

a) [0, 12]

b) [0, 8]

c) [8, 12]

d) [-12, 8]

17. If x = a sec $\theta$, y = b tan $\theta$ then $\frac{dy}{dx}$ = ? **[1]**

a) $\frac{b}{a}\sec\theta$

b) $\frac{b}{a}\tan\theta$

c) $\frac{b}{a}cosec\theta$

d) $\frac{b}{a}\cot\theta$

18. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular to each other then k = ? **[1]**

a) $\frac{-10}{7}$

b) $\frac{5}{7}$

c) $\frac{-5}{7}$

d) $\frac{10}{7}$

19. **Assertion (A):** If two positive numbers are such that sum is 16 and sum of their cubes is minimum, then numbers are 8, 8. **[1]**

**Reason (R):** If f be a function defined on an interval I and c ∈ l and let f be twice differentiable at c, then x = c is a point of local minima if f'(c) = 0 and f''(c) > 0 and f(c) is local minimum value of f.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Let A ={2, 4, 6} and B = {3, 5, 7, 9} and defined a function f = {(2, 3), (4, 5), (6, 7)} from A to B. Then, f is not onto. **[1]**

**Reason (R):** A function f : A → B is said to be onto, if every element of B is the image of some elements of A under f.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

## Section B

21. Write the value of $\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right)$ **[2]**

OR

Find the value of $\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right]$

22. A man is walking at the rate of 6.5 km/hr towards the foot of a tower 120 m high. At what rate is he approaching the top of the tower when he is 50 m away from the tower **[2]**

23. Find the intervals in which the function f given by f(x) = $2x^3 - 3x^2 - 36x + 7$ is decreasing. **[2]**

OR

The volume of a sphere is increasing at the rate of 8 $cm^3$/s. Find the rate at which its surface area is increasing when the radius of the sphere is 12 cm.

24. Prove that: $\int_0^{\pi/2} \frac{dx}{(1+\sqrt{\tan x})} = \frac{\pi}{4}$ **[2]**

25. Show that f(x) = (x – 1) $e^x$ + 1 is an increasing function for all x > 0. **[2]**

## Section C

26. Evaluate $\int \frac{\sin(x-a)}{\sin(x+a)} dx$. **[3]**

27. A problem in mathematics is given to three students whose chances of solving it correctly are 1/ 2, 1/ 3 and 1/4 respectively. What is the probability that only one of them solves it correctly? **[3]**

28. Evaluate $\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 dx$. **[3]**

OR

Evaluate the integral: $\int \frac{1}{x\sqrt{1+x^n}} dx$

29. $(x^2 + y^2)$ dy = xydx. If y(1) = 1 and y $(x_0)$ = e, then find the value of $x_0$. **[3]**

OR

Find a particular solution of the differential equation $\frac{dy}{dx} + 2y \tan x = \sin x$, given that y = 0, when $x = \frac{\pi}{3}$.

30. Solve the following LPP graphically: **[3]**

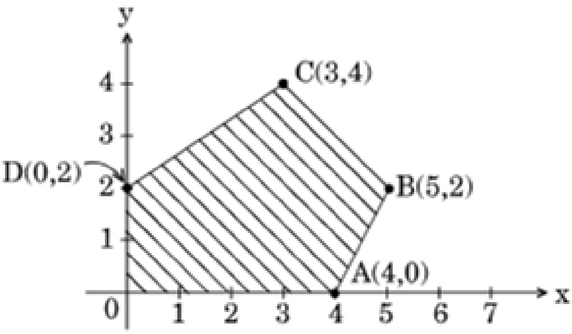
Minimise Z = 5x + 10y

subject to the constraints x + 2y $\geq$120

x + y $\geq$ 60, x - 2y $\geq$ 0 and x, y $\geq$ 0

OR

The corner points of the feasible region determined by the system of linear inequations are as shown below:




Answer each of the following:

i. Let z = 13x - 15y be the objective function. Find the maximum and minimum values of z and also the corresponding points at which the maximum and minimum values occur.

ii. Let z = kx + y be the objective function. Find k, if the value of z at A is same as the value of z at B.

31. If $e^x + e^y = e^{x+y}$, prove that $\frac{dy}{dx} + e^{y-x} = 0$. **[3]**

**Section D**

32. Using method of integration find the area of the triangle ABC, co-ordinates of whose vertices are A (2, 0), B (4, 5) and C (6, 3). **[5]**

33. Let R be relation defined on the set of natural number N as follows: **[5]**

R = {(x, y): x $\in$ N, y $\in$ N, 2x + y = 41}. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

OR

Show that the function f: $R_0 \to R_0$, defined as f (x) = $\frac{1}{x}$, is one-one onto, where $R_0$ is the set non-zero real numbers. Is the result true, if the domain $R_0$ is replaced by N with co-domain being same as $R_0$?

34. Given $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ find AB and use this result in solving the following system of equations. **[5]**

$x - y + z = 4$

$x - 2y - 2z = 9$

$2x + y + 3z = 1$

35. Find the shortest distance between the given lines. $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$, $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$ **[5]**

OR

Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$. Also, find the coordinates of the foot of the perpendicular and the equation of the perpendicular.

**Section E**

36. **Read the following text carefully and answer the questions that follow:** **[4]**

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

i. 960 of the total applications were the folk genre.
ii. 192 of the folk applications were for the below 18 category.
iii. 104 of the classical applications were for the 18 and above category.

**Questions:**

i. What is the probability that an application selected at random is for the 18 and above category provided it is under the classical genre? Show your work. (1)
ii. An application selected at random is found to be under the below 18 category. Find the probability that it is under the folk genre. Show your work. (1)
iii. If P(A) = 0.4, P(B) = 0.8 and P(B|A) = 0.6, then P(A ∪ B) is equal to. (2)

**OR**

iv. If A and B are two independent events with P(A) = $\frac{3}{5}$ and P(B) = $\frac{4}{9}$, then find P(A' ∩ B'). (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

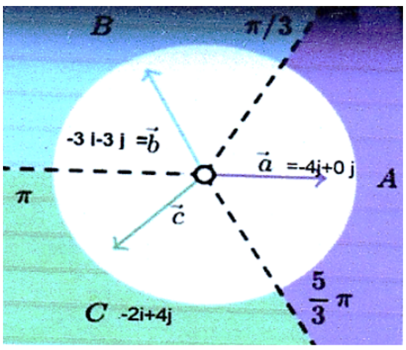
Three friends Ganesh, Dinesh and Ramesh went for playing a Tug of war game. Team A, B, and C belong to Ganesh, Dinesh and Ramesh respectively.

Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area (team areas shown below).

Team A pulls with $F_1 = 4\hat{i} + 0\hat{j}$ KN

Team B $\rightarrow$ $F_2 = -2\hat{i} + 4\hat{j}$ KN

Team C $\rightarrow$ $F_3 = -3\hat{i} - 3\hat{j}$ KN



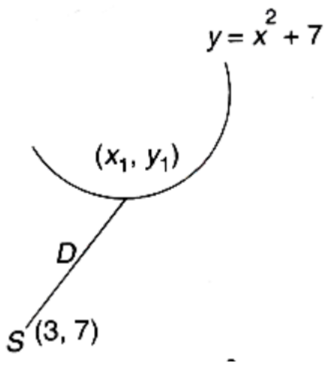

i. Which team will win the game? (1)
ii. What is the magnitude of the teams combine Force? (1)
iii. What is the magnitude of the force of Team B? (2)

**OR**

How many KN Force is applied by Team A? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

An Apache helicopter of the enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7) want to shoot down the helicopter when it is nearest to him.




i. If P $(x_1, y_1)$ be the position of a helicopter on curve $y = x^2 + 7$, then find distance D from P to soldier place at (3, 7). (1)
ii. Find the critical point such that distance is minimum. (1)
iii. Verify by second derivative test that distance is minimum at (1, 8). (2)

**OR**

Find the minimum distance between soldier and helicopter? (2)

# Solution

## Section A

1.

**(b)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**Explanation:** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2.

**(d)** 1

**Explanation:** As points are collinear

⇒ Area of triangle formed by 3 points is zero.

$\Rightarrow \frac{1}{2}\begin{vmatrix} (x_1 - x_2) & (x_2 - x_3) \\ (y_1 - y_2) & (y_2 - y_3) \end{vmatrix} = 0$

$\Rightarrow \frac{1}{2}\begin{vmatrix} (2x - 0) & \{0 - (x + 3)\} \\ (x + 3 - x) & \{x - (x + 6)\} \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 2x & -(x + 3) \\ 3 & -6 \end{vmatrix} = 0$

⇒ -12x + 3(x + 3) = 0

⇒ -12x + 3x + 9 = 0

⇒ -9x = -9

⇒ x = 1

3. **(a)** 12

**Explanation:** 12

Explaination:

A·(adj A) = $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

we know that A.(AdjA) = I.|A|

$\Longrightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ = |A| I

$\Longrightarrow 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = |A| I

$\Longrightarrow$ 3I = |A| I

$\Longrightarrow$ |A|=3 ---(1)

|Adj A| = $|A|^{3-1}$ [ Since order n=3]

|Adj A| = $(3)^2$=9

|adj(A)| = 9 -------(2)

Now,

|A| + |adj A| = 3 + 9 =12

4.

**(c)** |A| + |A'| $\neq$ 0

**Explanation:** Because, the determinant of a matrix and its transpose are always equal that is |A| = |A'|

5.

**(b)** $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

**Explanation:** We know that if there are two parallel lines then their direction ratios must have a relation
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

6. **(a)** y = kx

**Explanation:** We have,

$\frac{dy}{dx} = \frac{y}{x}$

$\Rightarrow \frac{dy}{y} = \frac{dx}{x}$

Integrating on both sides,

$\int \frac{dy}{y} = \int \frac{dx}{x}$

log |y| = log |x| + log k

$\Rightarrow$ log $\left(\frac{y}{x}\right)$ = log k

$\Rightarrow$ y = k x

7.

**(c)** R

**Explanation:**

| Corner points | Value of Z = 2x + 5y |
|---|---|
| P(0, 5) | Z = 2(0) + 5(5) = 25 |
| Q(1, 5) | Z = 2(1) + 5(5) = 27 |
| R(4, 2) | Z = 2(4) + 5(2) = 18 $\rightarrow$ Minimum |
| S(12, 0) | Z = 2(12) + 5(0) = 24 |

Thus, minimum value of Z occurs ar R(4, 2)

8.

**(d)** $2\hat{\mathrm{i}} + 3\hat{\mathrm{k}}$

**Explanation:** $2\hat{\mathrm{i}} + 3\hat{\mathrm{k}}$

9.

**(c)** $\frac{\mathrm{x}^6}{6} + \mathrm{C}$

**Explanation:** $\frac{\mathrm{x}^6}{6} + \mathrm{C}$

10.

**(c)** x = y

**Explanation:** A = $A^T$

$\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix} = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix}$

x = y

11.

**(d)** Option (c)

**Explanation:** If a LPP admits two optimal solutions it has an infinite solution.

12.

**(d)** $\frac{-2}{3}$

**Explanation:** $\frac{\vec{a} \cdot \vec{b}}{|b|}$ = 0

$\vec{a} \cdot \vec{b}$ = 0

$(\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (2\hat{\mathrm{i}} + \lambda\hat{\mathrm{k}})$ = 0

2 $\times$ 1 + 3$\lambda$ = 0

2 + 3$\lambda$ = 0

3$\lambda$ = -2

$\lambda = \frac{-2}{3}$

13. **(a)** $K^{n-1}$ Adj. A

**Explanation:** Adj. (KA) = $K^{n-1}$ Adj.A, where K is a scalar and A is a n $\times$ n matrix.

14.

**(b)** $\frac{1}{3}$

**Explanation:** $\frac{1}{3}$

15. **(a)** $xe^{\int \mathrm{P}_1 dy} = \int \left(\mathrm{Q}_1 e^{\int \mathrm{P}_1 dy}\right) dy + \mathrm{C}$

**Explanation:** The integrating factor of the given differential equation

$\frac{\mathrm{dx}}{\mathrm{dy}} + \mathrm{P}_1\mathrm{x} = \mathrm{Q}_1$ is $\mathrm{e}^{\int \mathrm{P}_1 \mathrm{dy}}$

Thus, the general solution of the differential equation is given by,

$\mathrm{x}(\mathrm{I.F.}) = \int(\mathrm{Q}_1 \times \mathrm{I.F.})\mathrm{dy} + \mathrm{C}$

$\Rightarrow \mathrm{x}.\mathrm{e}^{\int \mathrm{P}_1 \mathrm{dy}} = \int \left(\mathrm{Q}_1 \mathrm{e}^{\int \mathrm{P}_1 \mathrm{dy}}\right) \mathrm{dy} + \mathrm{C}$

16. **(a)** [0, 12]

**Explanation:** Given that, $|\vec{\mathrm{a}}| = 4$ and $-3 \leq \lambda \leq 2$

We know that, $|\lambda\vec{\mathrm{a}}| = |\lambda||\vec{\mathrm{a}}|$

$\Rightarrow |\lambda\vec{a}| = |-3||\vec{a}| = 3.4 = 12$ at $\lambda = -3$

$\Rightarrow |\lambda\vec{\mathrm{a}}| = |0||\vec{\mathrm{a}}| = 0.4 = 0$ at $\lambda = 0$

$\Rightarrow |\lambda\vec{\mathrm{a}}| = |2||\vec{\mathrm{a}}| = 2.4 = 8$ at $\lambda = 2$

Hence, the range of $|\lambda\vec{\mathrm{a}}|$ is [0, 12].

17.

**(c)** $\frac{b}{a} cosec\theta$

**Explanation:** x = a sec$\theta$ ,we get

$\therefore \frac{dx}{d\theta} = \mathrm{a}\sec\theta \cdot \tan\theta$

$\therefore \frac{d\theta}{dx} = \frac{1}{\mathrm{a}\sec\theta \cdot \tan\theta}$

y = b tan $\theta$ ,we get

$\therefore \frac{dy}{d\theta} = b \cdot \sec^2\theta$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$\Rightarrow \frac{dy}{dx} = b \cdot \sec^2\theta \times \frac{1}{\mathrm{a}\sec\theta \cdot \tan\theta}$

$\Rightarrow \frac{dy}{dx} = \frac{b\sec\theta}{\mathrm{a}\tan\theta}$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{b} \cdot \frac{1}{\cos\theta}}{\mathrm{a} \cdot \frac{\sin\theta}{\cos\theta}}$

$\Rightarrow \frac{dy}{dx} = \frac{b}{a} cosec\theta$

18. **(a)** $\frac{-10}{7}$

**Explanation:** If the lines are perpendicular to each other then the angle between these lines will be $\frac{\pi}{2}$, then the cosine will be 0

$\vec{a} = -3\hat{i} + 2k\hat{j} + 2\hat{k}$

$\Rightarrow |\vec{a}| = \sqrt{3^2 + (2k)^2 + 2^2}$

$= \sqrt{13 + 4k^2}$

$\overrightarrow{\mathrm{b}} = 3\mathrm{k}\hat{\mathrm{i}} + \hat{\mathrm{j}} - 5\hat{\mathrm{k}}$

$\Rightarrow |\overrightarrow{\mathrm{b}}| = \sqrt{(3\mathrm{k})^2 + 1 + 5^2}$

$= \sqrt{9k^2 + 26}$

$\cos\left(\frac{\pi}{2}\right) = \frac{(3\mathrm{k}\hat{\mathrm{i}} + \hat{\mathrm{j}} - 5\hat{\mathrm{k}}) \cdot (-3\hat{\mathrm{i}} + 2\mathrm{k}\hat{\mathrm{j}} + 2\hat{\mathrm{k}})}{\sqrt{13 + 4\mathrm{k}^2} \times \sqrt{9\mathrm{k}^2 + 26}}$

$0 = \frac{-9k + 2k - 10}{\sqrt{13 + 4k^2} \times \sqrt{9k^2 + 26}}$

$\Rightarrow k = -\frac{10}{7}$

19. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** Let one number be x, then the other number will be (16 - x).

Let the sum of the cubes of these numbers be denoted by S.

Then, S = $x^3 + (16 - x)^3$

On differentiating w.r.t. x, we get

$\frac{dS}{dx}$ = $3x^2 + 3(16 - x)^2(-1)$

= $3x^2 - 3(16 - x)^2$

$\Rightarrow \frac{d^2S}{dx^2}$ = 6x + 6(16 - x) = 96

For minima put $\frac{dS}{dx}$ = 0.

$\therefore 3x^2 - 3(16 - x)^2 = 0$

$\Rightarrow x^2 - (256 + x^2 - 32x) = 0$

$\Rightarrow$ 32x = 256

$\Rightarrow$ x = 8

At x = 8, $\left(\frac{d^2S}{dx^2}\right)_{x=8}$ = 96 > 0

By second derivative test, x = 8 is the point of local minima of S.

Thus, the sum of the cubes of the numbers is the minimum when the numbers are 8 and 16 - 8 = 8

Hence, the required numbers are 8 and 8.

20. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation: Assertion:** Given that,

A = {2, 4, 6},

R = {3, 5, 7, 9}

and R = {(2,3), (4, 5), (6, 7)}

Here, f(2) = 3, f(4) = 5 and f(6) = 7

It can be seen that the images of distinct elements of A under f are distinct.

Hence, function f is one-one but f is not onto as $9 \in B$ does not have a pre-image in A.

Hence, both Assertion and Reason are true, but Reason is not a correct explanation of Assertion.

**Section B**

21. Given $\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right)$

We know that $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$

$= \sin^{-1}\left(\frac{1}{3}\right) - \left[\pi - \cos^{-1}\left(\frac{1}{3}\right)\right]$

$= \sin^{-1}\left(\frac{1}{3}\right) - \pi + \cos^{-1}\left(\frac{1}{3}\right)$

$= \sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1}\left(\frac{1}{3}\right) - \pi$

$= \frac{\pi}{2} - \pi$

$= -\frac{\pi}{2}$

Therefore we have,

$\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right) = -\frac{\pi}{2}$

OR

Let $\cot^{-1}\left(\frac{-5}{12}\right) = y$

Then $\cot y = \frac{-5}{12}$

Now,

$\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right] = \sin 2y$

$= 2\sin y\cos y = 2\left(\frac{12}{13}\right)\left(\frac{-5}{13}\right)$ $\left[\text{since } \cot y < 0, \text{ so } y \in \left(\frac{\pi}{2}, \pi\right)\right]$

$= \frac{-120}{169}$

22. Let at any time t, the man be at distances of x and y metres from the foot and top of the tower respectively. Then,

$y^2 = x^2 + (120)^2$ ...(i)

$\Rightarrow \quad 2y\frac{dy}{dt} = 2x\frac{dx}{dt}$

$\Rightarrow \quad \frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$

given: $\frac{dx}{dt} = -6 \cdot 5\text{km/hr}$ negative sign due to decreasing,

therefore;

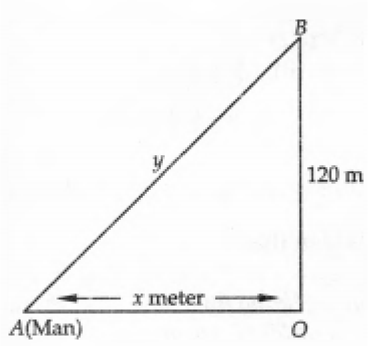
$\frac{dy}{dt} = -\frac{6 \cdot 5x}{y}$ ... (ii)

Putting x = 50 in (i) we get $y = \sqrt{50^2 + 120^2} = 130$

Putting x = 50, y = 130 in (ii), we get

$\frac{dy}{dt} = -\frac{6\cdot5\times50}{130} = -2\cdot5$

Thus, the man is approaching the top of the tower at the rate of 2.5 km/hr.




23. It is given that function f(x) = $2x^3 - 3x^2 - 36x + 7$

$\Rightarrow$ f'(x) = $6x^2 - 6x + 36$

$\Rightarrow$ f'(x) = $6(x^2 - x + 6)$

$\Rightarrow$ f'(x) = 6(x + 2)(x - 3)

If f'(x) = 0, then we get,

$\Rightarrow$ x = -2, 3

So, the point x = -2 and x = 3 divides the real line into two disjoint intervals, $(-\infty, 2), (-2, 3)$ and $(3, \infty)$


So, in interval (-2, 3)

f'(x) = 6(x + 2)(x - 3) < 0

Therefore, the given function (f) is strictly decreasing in interval (-2, 3).

OR

Let r be the radius, V be the volume and S be the surface area of sphere

Then, we have $\frac{dV}{dt}$ = 8 cm$^3$/ s

To find $\frac{dS}{dt}$, when r = 12 cm

Since, V $= \frac{4}{3}\pi r^3$

$\therefore \quad \frac{dV}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt} \Rightarrow 8 = 4\pi \times r^2 \frac{dr}{dt}$

$\Rightarrow \quad \frac{dr}{dt} = \frac{2}{\pi r^2}\mathrm{cm/s}$ .........(i)

Now, S = $4\pi r^2$

$\therefore \quad \frac{dS}{dt} = \frac{d}{dt}\left(4\pi r^2\right) = 4\pi \times 2r \cdot \frac{dr}{dt}$

$= 8\pi r \times \frac{2}{\pi r^2}$ [ From Eq(i)]

$= \frac{16}{r}$

$\Rightarrow \quad \left(\frac{dS}{dt}\right)_{r=12} = \frac{16}{12} = \frac{4}{3}\mathrm{cm}^2/\mathrm{s}$

24. Let y = $\int\limits_0^{\pi/2} \frac{dx}{(1+\sqrt{\tan x})}$

$y = \int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\frac{\sin x}{\cos x}}} dx$

$y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\sin x}+\sqrt{\cos x})}$ ........ (i)

Using theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$y = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\left(\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}\right)} dx$

$y = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\cos x}+\sqrt{\sin x})} dx$ ....... (ii)

Adding eq.(i) and eq.(ii), we get

$2y = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx + \int_0^{\pi/2} \frac{\sqrt{\sin x}}{(\sqrt{\cos x}+\sqrt{\sin x})} dx$

$2y = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{(\sqrt{\sin x}+\sqrt{\cos x})} dx$
$2y = \int_0^{\pi/2} 1 dx$
$2y = (x)_0^{\frac{\pi}{2}}$
$y = \frac{\pi}{4}$

25. Given:- f(x) = (x – 1) $e^x$ + 1
$\Rightarrow \mathrm{f}'(\mathrm{x}) = \frac{\mathrm{d}}{\mathrm{dx}}((\mathrm{x}-1)\mathrm{e}^{\mathrm{x}}+1)$
= f'(x) = $e^x$ + (x – 1) $e^x$
= f'(x) = $e^x$(1+ x – 1)
= f'(x) = $xe^x$
as given
x > 0
= $e^x$ > 0
= $xe^x$ > 0
= f'(x) > 0
Hence, the condition for f(x) to be increasing
Thus, f(x) is increasing for all x > 0

**Section C**

26. According to the question, $I = \int \frac{\sin(x-a)}{\sin(x+a)} dx$
Put $x + a = t \Rightarrow$ dx = dt
$\therefore I = \int \frac{\sin(t-a-a)}{\sin t} dt = \int \frac{\sin(t-2a)}{\sin t} dt$
$= \int \frac{\sin t \cos 2a - \cos t \sin 2a}{\sin t} dt$
$[\because sin(A-B) = sinAcosB - cosAsinB]$
$= \int \cos 2a dt - \int \sin 2a \cdot \cot t dt$
= $cos2a[t] - sin2a[log|sint|] + C_1$
= $(x+a)cos2a - sin2alog|sin(x+a)| + C_1$
[put t = x + a)
= $xcos2a - sin2alog|sin(x+a)| + C1$

27. Let A, B, C be the given students and let $E_1$, $E_2$ and $E_3$ be the events that the problem is solved by A, B, C respectively. Then, $\bar{E}_1$, $\bar{E}_2$ and $\bar{E}_3$ are the events that the given problem is not solved by A, B, C respectively.
Therefore, we have,
P($E_1$) = $\frac{1}{2}$; P($E_2$) = $\frac{1}{3}$; P($E_3$) = $\frac{1}{4}$;
$P(\bar{E}_1) = (1 - \frac{1}{2}) = \frac{1}{2}; P(\bar{E}_2) = \left(1 - \frac{1}{3}\right) = \frac{2}{3}$ and $P(\bar{E}_3) = \left(1 - \frac{1}{4}\right) = \frac{3}{4}$
P(exactly one of them solves the problem)
$= P\left[\left(E_1 \cap \bar{E}_2 \cap \bar{E}_3\right) \text{ or } \left(\bar{E}_1 \cap E_2 \cap \bar{E}_3\right) \text{ or } \left(\bar{E}_1 \cap \bar{E}_2 \cap E_3\right)\right]$
$= P\left(E_1 \cap \bar{E}_2 \cap \bar{E}_3\right) + P\left(\bar{E}_1 \cap E_2 \cap \bar{E}_3\right) + P\left(\bar{E}_1 \cap \bar{E}_2 \cap E_3\right)$
$= \left\{P(E_1) \times P(\bar{E}_2) \times P(\bar{E}_3)\right\} + \left[P(\bar{E}_1) \times P(E_2) \times P(\bar{E}_3)\right] + \left[P(\bar{E}_1) \times P(\bar{E}_2) \times P(E_3)\right]$
$= \left(\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{1}{3} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}\right)$
$= \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{12}\right) = \frac{11}{24}$

28. Given, I = $\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 dx$
$= \int_{-\pi}^{\pi} \left(\cos^2 ax + \sin^2 bx - 2\cos ax \sin bx\right) dx$
$= \int_{-\pi}^{\pi} \left(\cos^2 ax + \sin^2 bx\right) dx - 2\int_{-\pi}^{\pi} \cos ax \sin bx dx$
=$I_1 - I_2$
Let,
$I_1$ = $\int_{-\pi}^{\pi} \left(\cos^2 ax + \sin^2 bx\right) dx$ [ be an even function]
$= 2\int_0^{\pi} \left(\cos^2 ax + \sin^2 bx\right) dx$
$= 2\int_0^{\pi} \left(\frac{1+\cos 2ax}{2} + \frac{1-\cos 2bx}{2}\right) dx$
$= \int_0^{\pi} (1 + \cos 2ax + 1 - \cos 2bx) dx$
$= \int_0^{\pi} (2 + \cos 2ax - \cos 2bx) dx$

$= \left(2x + \frac{\sin 2ax}{2a} - \frac{\sin 2bx}{2b}\right)_0^\pi$

$= \left(2\pi + \frac{\sin 2a\pi}{2a} - \frac{\sin 2b\pi}{2b}\right) - 0$

$= 2\pi + \frac{\sin 2a\pi}{2a} - \frac{\sin 2b\pi}{2b}$

$I_2 = 2\int_{-\pi}^{\pi}(\cos ax \sin bx)dx$ [ be a odd function]

= 0 $\left[\because \int_{-a}^{a} f(x) = 2\int_0^a f(x)dx,\ \text{if } f(x) \text{ is even} \atop 0\ \ ,\ \text{if } f(x) \text{ is odd}\right]$

$\therefore I = I_1 - I_2 = 2\pi + \frac{\sin 2a\pi}{2a} - \frac{\sin 2b\pi}{2b}$

OR

Let the given integral be,

$I = \int \frac{dx}{x\sqrt{1+x^n}}$

$= \int \frac{x^{n-1}dx}{x^{n-1}x^1\sqrt{1+x^n}}$

$= \int \frac{x^{n-1}dx}{x^n\sqrt{1+x^n}}$

Putting $x^n = t$

$\Rightarrow n x^{n-1}dx = dt$

$\Rightarrow x^{n-1}dx = \frac{dt}{n}$

$\therefore I = \frac{1}{n}\int \frac{dt}{t\sqrt{1+t}}$

let $1 + t = p^2$

$\Rightarrow$ dp = 2p dp

$\therefore I = \frac{1}{n}\int \frac{2pdp}{(p^2-1)p}$

$= \frac{2}{n}\int \frac{dp}{p^2-1^2}$

$= \frac{2}{n} \times \frac{1}{2}\log\left|\frac{p-1}{p+1}\right| + C$

$= \frac{1}{n}\log\left|\frac{\sqrt{1+t}-1}{\sqrt{1+t}+1}\right| + C$

$= \frac{1}{n}\log\left|\frac{\sqrt{1+x^n}-1}{\sqrt{1+x^n}+1}\right| + C$

29. $\left(x^2 + y^2\right) dy = xydx$

$\Rightarrow \int \frac{x}{y}dy + \int \frac{y}{x}dy = \int dx$

$\Rightarrow x\log y + \frac{y^2}{2x} = x + c$

Now, at x = 1; y = e

$x\log y + \frac{y^2}{2x} = x + c \Rightarrow x + \frac{e^2}{2} = x + c \Rightarrow c = \frac{e^2}{2}$

Now at $x = x_0$; y = e

$x_0\log e + \frac{e^2}{2x_0} = x_0 + \frac{e^2}{2} \Rightarrow \frac{e^2}{2x_0} = \frac{e^2}{2} \Rightarrow x_0 = 1$

OR

We have,

$\frac{dy}{dx} + 2y\tan x = \sin x$

which is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$.

Here, P = 2 tanx and Q =sin x

$\therefore$ IF $= e^{\int Pdx} = e^{2\int \tan x dx} = e^{2\log|\sec x|}$

$= e^{\log\sec^2 x}$ $[\because m\log n = \log n^m]$

$= \sec^2 x$ $[\because e^{\log x} = x]$

The general solution is given by

$y \times IF = \int(Q \times IF)dx + C$ ...(i)

$\Rightarrow y\sec^2 x = \int\left(\sin x \cdot \sec^2 x\right)dx + C$

$\Rightarrow y\sec^2 x = \int \sin x \cdot \frac{1}{\cos^2 x}dx + C$

$\Rightarrow y\sec^2 x = \int \tan x \sec x dx + C$

$\Rightarrow y\sec^2 x = \sec x + C$ .............(ii)

Also, given that y = 0, when $x = \frac{\pi}{3}$.

On putting y = 0 and $x = \frac{\pi}{3}$ in Eq. (ii), we get

$0 \times \sec^2 \frac{\pi}{3} = \sec \frac{\pi}{3} + C$

$\Rightarrow \quad 0 = 2 + C \Rightarrow C = -2$

On putting the value of C in Eq. (ii), we get

$y \sec^2 x = \sec x - 2$

$\therefore \quad y = \cos x - 2\cos^2 x$

which is the required particular solution of the given differential equation

30. Our problem is to minimise the objective function Z = 5x + 10 y ...(i)

Subject to constraints

x + 2y $\leq$120......(ii)

x + y $\geq$ 60........(iii)

x -2y $\geq$ 0..........(iv)

x $\geq$ 0, y $\geq$ 0 ( which is the non negative constraint which will restrict the feasible region to the first quadrant only)

Table of values for line ( ii) x + 2y = 120 are given below.

| | | |
|---|---|---|
| **x** | 0 | 120 |
| **y** | 60 | 0 |

Replace O (0, 0) in the inequality x + 2y $\leq$120, we get

0 + 2 $\times$ 0 $\leq$120

$\Rightarrow \quad 0 \leq 120$ (which is true)

So, the half plane for the inequality of the line ( ii) is towards the origin which means that the origin O(0,0) is a point in the feasible region of the inequality of the line ( ii).

Secondly, draw the graph of the line x + y = 60.Hence the table of values of the line ( iii) is given as follows.

| | | |
|---|---|---|
| **x** | 0 | 60 |
| **y** | 60 | 0 |

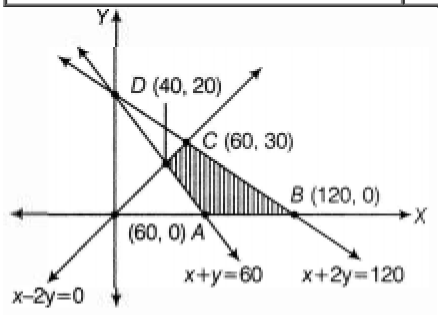
On replacing O(0, 0) in the inequality x + y$\geq$ 60, we get

$0 + 0 \geq 60 \Rightarrow 0 \geq 60$ (which is false)

So, the half plane for the inequality of the line ( iii) is away from the origin, which means that the origin is not a point on the feasible region .

Thirdly, draw the graph of the line x - 2y = 0 and the table of values for ( iv) is given as follows.

| | | |
|---|---|---|
| **x** | 0 | 10 |
| **y** | 0 | 5 |




On solving equations x - 2y = 0 and x + y = 60, we get D(40,20) and on solving equations x -2y = 0 and x + 2y = 120, we get C (60, 30)

Feasible region is ABCDA, which is a bounded feasible region, the coordinates of the corner points of the feasible region are given as A (60, 0), B ( 120, 0), C ( 60, 30) and D (40, 20).

| **Corner points** | **Z = 5x + 10y** |
|---|---|
| A(60,0) | Z = 300(minimum) |
| B(120,0) | Z= 600 |
| C(60,0) | Z= 600 |
| D(40,20) | Z= 400 |

The values of Z at these points are as follows So, the minimum value of Z is obtained as 300 , which occurs at the point (60, 0).

OR

i. $z(A) = 13(4) - 15(0) = 52$
   $z(B) = 13(5) - 15(2) = 35$
   $z(C) = 13(3) - 15(4) = -21$
   $z(D) = 13(0) - 15(2) = -30$
   $z(0) = 0$
   $\therefore$ Max (z) = 52 at A(4, 0), Min(z) = -30 at (0, 2)

ii. $z(A) = z(B) \Rightarrow 4k + 0 = 5k + 2 \Rightarrow k = -2$

31. Given $e^{x} + e^{y} = e^{x + y}$ ...(i)

On dividing Eq(i) by $e^{x+y}$, we get,

$e^{-y} + e^{-x} = 1$ ...(ii)

Therefore,on differentiating both sides of Eq(ii) w.r.t x, we get,

$e^{-y} \cdot \left(\frac{-dy}{dx}\right) + e^{-x}(-1) = 0$

$\Rightarrow -e^{-y}\frac{dy}{dx} + e^{-x}(-1) = 0$

$\Rightarrow -e^{-y}\frac{dy}{dx} = e^{-x}$

$\Rightarrow \frac{dy}{dx} = -\frac{e^{-x}}{e^{-y}}$

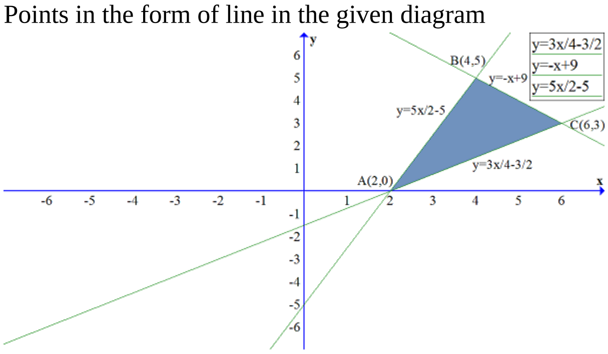
$\Rightarrow \frac{dy}{dx} = -e^{(y-x)}$

$\therefore \frac{dy}{dx} + e^{(y-x)} = 0$

Hence Proved.

**Section D**

32. Points in the form of line in the given diagram




The equation of side AB is,

$y - 0 = \frac{5-0}{4-2}(x-2)$

$\Rightarrow y = \frac{5}{2}(x-2)$

The equation of side BC is,

$y - 3 = \frac{5-3}{4-6}(x-6)$

$\Rightarrow y - 3 = \frac{2}{-2}(x-6)$

$\Rightarrow y - 3 = -1(x-6)$

$\Rightarrow y = -x + 9$

The equation of side AC is,

$y - 0 = \frac{3-0}{6-2}(x-2)$

$\Rightarrow y = \frac{3}{4}(x-2)$

Area $= \frac{5}{2}\int_{2}^{4}(x-2)dx + \int_{4}^{6} -(x-9)dx - \frac{3}{4}\int_{2}^{6}(x-2)dx$

$A = \int_{2}^{4}\frac{5}{2}(x-2)dx + \int_{0}^{1} -(x+9)dx + \int_{6}^{2}\frac{3}{4}(x-2)dx$

On integrating we get,

$A = \frac{5}{2}\left[\frac{x^2}{2} - 2x\right]_{2}^{4} + \left[\frac{-x^2}{2} + 9x\right]_{1}^{0} - \frac{3}{4}\left[\frac{x^2}{2} - 2x\right]_{2}^{6}$

On applying limits we get,

$A = \frac{5}{2}[8 - 8 - 2 + 4] + [-18 + 54 + 8 - 36] - \frac{3}{4}[18 - 12 - 2 + 4]$

$A = 5 - 8 - \frac{3}{4}(8)$

= 13 - 6 = 7 sq. units.

Hence the required area is 7 sq. units.

33. Given that,

R = {(1, 39), (2, 37), (3, 35) .... (19, 3), (20, 1)}

Domain = {1,2,3,.......,20}

Range = {1,3,5,7......,39}

R is not reflexive as (2, 2) $\notin$ R as

$2 \times 2 + 2 \neq 41$

R is not symmetric

as (1, 39) $\in$ R but (39, 1) $\notin$ R

R is not transitive

as (11, 19) $\in$ R, (19, 3) $\in$ R

But (11, 3) $\notin$ R

Hence, R is neither reflexive, nor symmetric and nor transitive.

OR

We observe the following properties of f.

Injectivity: Let x, y $\in R_0$ such that f(x) = f(y). Then,

$f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$

So, f : $R_0 \rightarrow R_0$ is one-one.

Surjectivity: Let y be an arbitrary element of $R_0$ (co-domain) such that f(x) = y. Then,

$f(x) = y \Rightarrow \frac{1}{x} = y \Rightarrow x = \frac{1}{y}$

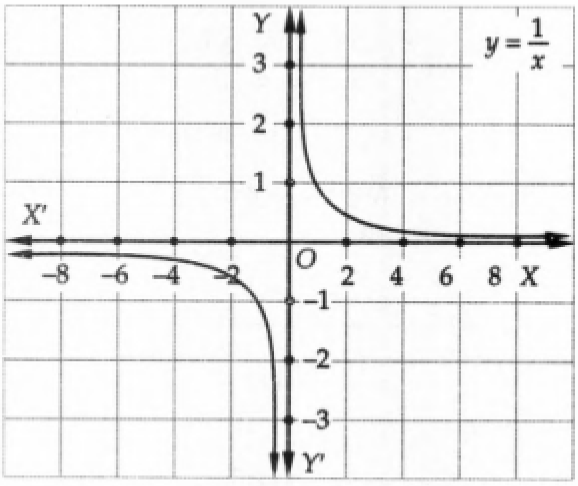
Clearly, $x = \frac{1}{y} \in R_0$ (domain) for all y $\in R_0$ (co-domain).

Thus, for each y $\in R_0$ (co-domain) there exits $x = \frac{1}{y} \in R_0$ (domain) such that $f(x) = \frac{1}{x} = y$

So, f : $R_0 \rightarrow R_0$ is onto.

Hence, f: $R_0 \rightarrow R_0$ is one-one onto.

This is also evident from the graph of f(x) as shown in fig.




Let us now consider f : N $\rightarrow R_0$ given by $f(x) = \frac{1}{x}$

For any x, y $\in$ N, we find that

f(x) = f(y) $\Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$

So, f: N $\rightarrow R_0$ is one-one.

We find that $\frac{2}{3}, \frac{3}{5}$ etc. in co-domain $R_0$ do not have their pre-image in domain N. So, f: N $\rightarrow R_0$ is not onto.

Thus, f: N $\rightarrow R_0$ is one-one but not onto.

34. $x - y + z = 4$

$x - 2y - 2z = 9$

$2x + y + 3z = 1$

Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$

Then, given system of equations can be rewritten as,

AX = C

Now, $AB = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

AB = 8I

$A^{-1} = \frac{1}{8}B \begin{bmatrix} \because A^{-1}AB = 8A^{-1}I \\ B = 8A^{-1} \end{bmatrix}$

$\Rightarrow A^{-1} = \frac{1}{8} \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{-7}{8} & \frac{1}{8} & \frac{3}{8} \\ \frac{5}{8} & \frac{-3}{8} & \frac{-1}{8} \end{bmatrix}$

Now, AX = C,

$\Rightarrow X = A^{-1}C$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{-7}{8} & \frac{1}{8} & \frac{3}{8} \\ \frac{5}{8} & \frac{-3}{8} & \frac{-1}{8} \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} \frac{-4}{2} + \frac{9}{2} + \frac{1}{2} \\ \frac{-28}{8} + \frac{9}{8} + \frac{3}{8} \\ \frac{20}{8} + \frac{-27}{8} + \frac{-1}{8} \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$

$\Rightarrow x = 3, y = -2, z = -1$

35. Given

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$

Here, we have

$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$\vec{b_2} = -2\hat{i} + 3\hat{j} + 8\hat{k}$

Thus,

$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ -2 & 3 & 8 \end{vmatrix}$

$= \hat{i}(24 - 18) - \hat{j}(16+12) + \hat{k}(6 - 6)$

$\vec{b_1} \times \vec{b_2} = 6\hat{i} - 28\hat{j} + 0\hat{k}$

$\therefore |\vec{b_1} \times \vec{b_2}| = \sqrt{6^2 + (-28)^2 + 0^2}$

$= \sqrt{36 + 784 + 9}$

$= \sqrt{820}$

$\vec{a_2} - \vec{a_1} = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (-5 + 4)\hat{k}$

$\therefore \vec{a_2} - \vec{a_1} = 2\hat{i} + \hat{j} - \hat{k}$

Now, we have

$(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = (6\hat{i} - 28\hat{j} + 0\hat{k}) \cdot (2\hat{i} + \hat{j} - \hat{k})$

$= (6 \times 2) + ((-28) \times 1) + (0 \times (-1))$

= 12 - 28 + 0

= -16

Thus,the shortest distance between the given lines is

$\text{d} = \left|\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$

$\Rightarrow \text{d} = \left|\frac{-16}{\sqrt{820}}\right|$

$\therefore d = \frac{16}{\sqrt{820}}$ units

OR

Suppose the point (1, 0, 0) be P and the point through which the line passes be Q(1,-1,-10). The line is parallel to the vector $\vec{b} = 2\hat{i} - 3\hat{j} + 8\hat{k}$

Now,

$\overrightarrow{PQ} = 0\hat{i} - \hat{j} - 10\hat{k}$

$\therefore \vec{b} \times \overrightarrow{PQ}$ = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 8 \\ 0 & -1 & -10 \end{vmatrix}$

= $38\hat{i} + 20\hat{j} - 2\hat{k}$

$\Rightarrow |\vec{b} \times \overrightarrow{PQ}| = \sqrt{38^2 + 20^2 + 2^2}$

= $\sqrt{1444 + 400 + 4}$

= $\sqrt{1848}$

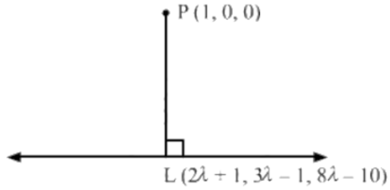
d = $\frac{|\vec{b} \times \overrightarrow{PQ}|}{|\vec{b}|}$

= $\frac{\sqrt{1848}}{\sqrt{77}}$

= $\sqrt{24}$

= $2\sqrt{6}$

Suppose L be the foot of the perpendicular drawn from the point P(1,0,0) to the given line-




The coordinates of a general point on the line

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ are given by

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda$

$\Rightarrow x = 2\lambda + 1$

$y = -3\lambda - 1$

$z = 8\lambda - 10$

Suppose the coordinates of L be

$(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$

Since,The direction ratios of PL are proportional to,

$2\lambda + 1 - 1, -3\lambda - 1 - 0, 8\lambda - 10 - 0$, i.e., $2\lambda, -3\lambda - 1, 8\lambda - 10$

Since,The direction ratios of the given line are proportional to 2, -3, 8, but PL is perpendicular to the given line.

$\therefore 2(2\lambda) - 3(-3\lambda - 1) + 8(8\lambda - 10)$ = 0

$\Rightarrow \lambda$ = 1 Substituting $\lambda$ = 1 in $(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$ we get the coordinates of L as (3, -4, -2). Equation of the line PL is given by

$\frac{x-1}{3-1} = \frac{y-0}{-4-0} = \frac{z-0}{-2-0}$

= $\frac{x-1}{1} = \frac{y}{-2} = \frac{z}{-1}$

$\Rightarrow \vec{r} = \hat{i} + \lambda(\hat{i} - 2\hat{j} - \hat{k})$

**Section E**

36. i. According to given information, we construct the following table.

Given, total applications = 2000

| | **Folk Genre** | **Classical Genre** |
|---|---|---|
| | 960 (given) | 2000 - 960 = 1040 |
| | | |

| | | |
|---|---|---|
| Below 18 | 192 (given) | 1040 - 104 = 936 |
| 18 or Above 18 | 960 - 192 = 768 | 104 (given) |

Let $E_1$ = Event that application for folk genre

$E_2$ = Event that application for classical genre

A = Event that application for below 18

B = Event that application for 18 or above 18

$\therefore P(E_2) = \frac{1040}{2000}$

and $P(B \cap E_2) = \frac{104}{2000}$

Required Probability = $\frac{P(B \cap E_2)}{P(E_2)}$

$= \frac{\frac{104}{2000}}{\frac{1040}{200}} = \frac{1}{10}$

ii. Required probability = $P\left(\frac{\text{folk}}{\text{below 18}}\right)$

$= P\left(\frac{E_1}{A}\right)$

$= \frac{P(E_1 \cap A)}{P(A)}$

Now, $P(E_1 \cap A) = \frac{192}{2000}$

and $P(A) = \frac{192+936}{2000} = \frac{1128}{2000}$

$\therefore$ Required probability = $\frac{\frac{192}{2000}}{\frac{1128}{2000}} = \frac{192}{1128} = \frac{8}{47}$

iii. Here,

P(A) = 0.4, P(B) = 0.8 and P(B|A) = 0.6

$\because P(B|A) = \frac{P(B \cap A)}{P(A)}$

$\Rightarrow P(B \cap A) = P(B|A).P(A)$

$= 0.6 \times 0.4 = 0.24$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

= 0.4 + 0.8 - 0.24

= 1.2 - 0.24 = 0.96

**OR**

Since, A and B are independent events, A' and B' are also independent. Therefore,

$P(A' \cap B') = P(A') \cdot P(B')$

= (1 - P(A)(1 - P(B))

$= \left(1 - \frac{3}{5}\right)\left(1 - \frac{4}{9}\right)$

$= \frac{2}{5} \cdot \frac{5}{9}$

$= \frac{2}{9}$

37. i. Force applied by team A

$= \sqrt{4^2 + 0^2}$

= 4 N

Force applied by team B

$= \sqrt{(-2)^2 + 4^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20}$

$= 2\sqrt{5}$ N

Force applied by team C

$= \sqrt{(-3)^2 + (-3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Hence, the force applied by team B is maximum.

So, Team 'B' will win.

ii. Sum of force applied by team A, B and C

$= (4 + (-2) + (-3))\hat{i} + (0 + 4 + (-3))\hat{j}$

$= -\hat{i} + \hat{j}$

Magnitude of team combine force

$= \sqrt{(-1)^2 + (1)^2}$

$= \sqrt{2}N$

iii. Force applied by team B

$= \sqrt{(-2)^2 + 4^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20}$

$= 2\sqrt{5}$ N

OR

Force applied by team A

$= \sqrt{4^2 + 0^2}$

= 4 N

38. i. $P(x_1, y_1)$ is on the curve $y = x^2 + 7 \Rightarrow y_1 = x_1^2 + 7$

Distance from $p(x_1, x_1^2 + 7)$ and (3, 7)

$D = \sqrt{(x_1 - 3)^2 + (x_1^2 + 7 - 7)^2}$

$\Rightarrow \sqrt{(x_1 - 3)^2 + (x_1^2)^2}$

$\Rightarrow D = \sqrt{x_1^4 + x_1^2 - 6x_1 + 9}$

ii. $D = \sqrt{x_1^4 + x_1^2 - 6x_1 + 9}$

$D' = x_1^4 + x_1^2 - 6x_1 + 9$

$\frac{dD'}{dx} = 4x_1^3 + 2x_1 - 6 = 0$

$\frac{dD'}{dx} = 2x_1^3 + x_1 - 3 = 0$

$\Rightarrow (x_1 - 1)(2{x_1}^2 + 2x_1 + 3) = 0$

$x_1 = 1$ and $2x_1^2 + 2x_1 + 3 = 0$ gives no real roots

The critical point is (1, 8).

iii. $\frac{dD'}{dx} = 4x_1^3 + 2x_1 - 6$

$\frac{d^2D'}{dx^2} = 12x_1^2 + 2$

$\left.\frac{d^2D'}{dx^2}\right]_{x_1=1} = 12 + 2 = 14 > 0$

Hence distance is minimum at (1, 8).

OR

$D = \sqrt{x_1^4 + x_1^2 - 6x_1 + 9}$

$D = \sqrt{1 + 1 - 6 + 9} = \sqrt{5}$ units