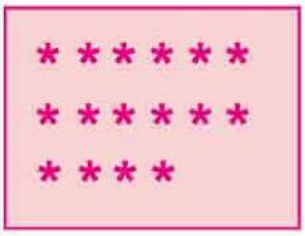
Chapter 2

Relation among Numbers

2.1 Rimjhim and Mukul are practising factors learned in previous classes. According to Rimjhim, the factors of 16 are 2, 4, 6 and 8.

Mukul – Rimjhim how can you say that 6 is a factor of 16? Can you divide 16 into groups of 6-6?

Rimjhim – Let me do it.



Oh! Two groups of 6-6 were formed but 2 remained the third time.

Mukul – This means that 6 is not a factor of 16, because 16 cannot be divided completely into groups of 6-6.

Rimjhim – Dividing equally means division, so can we say that all numbers which perfectly divide 16 are factors of 16?

2.2 Factors and Multiples

Rimjhim wants to find out those numbers which perfectly divide 8. She divides 8 by 8 and smaller numbers like this:

1) 8 (8	2) 8 (4	3) 8 (2	4) 8 (2
- 8	- 8	- 6	- 8
0	0	2	0
Quotient is 8 Remainder is 0	Quotient is 4 Remainder is 0	Quotient is 2 Remainder is 2	Quotient is 2 Remainder is 0

5) 8 (1	6) 8 (1	7) 8 (1	8) 8 (1
- 5	- 6	- 7	- 8
3	2	1	0
Quotient is 1 Remainder is 3	Quotient is 1 Remainder is 2	Quotient is 1 Remainder is 1	Quotient is 1 Remainder is 0

Rimjhim – 1, 2, 4 and 8 are numbers which perfectly divides 8. Hence 1, 2, 4 and 8 are factors of the number 8. So 8 can also be written as 1×8, 2×4. Factors are also known as "Upvartak".

Mukul – Rimjhim, we can also say that 8 is one multiple of 1, 2, 4, 8. (Hence '8' appears in the tables of 1, 2, 4, 8.)

Do and Learn

In the following table, write the factors against the numbers given:

Number	Factors
12	1, 2, 3, 4, 6, 12
24	.....
27	.....
17	.....
15	.....
7	.....

From the table above, can you say that '1' is the factor of every number?

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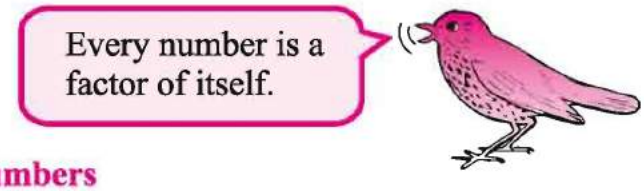

2.3 Prime and Non-prime numbers

Look at the factors of the numbers given below:

Number	Factors	Number of Factors
1	1	1
2	1,2	2
3	1,3	2
4	1,2,4	3
5	1,5	2
6	1,2,3,6	4
7	1,7	2
8	1,2,4,8	4

Table 2.1

In the table, 1 is the only number whose number of factors is 1, therefore 1 is neither Prime nor Non-prime.

Looking at the table, find out which numbers have only two factors?...

Those numbers which have only two factors (1 and the number itself) are known as Prime numbers, such as 2,3,5,7 etc.

Numbers with more than 2 factors are known as Non-prime or Composite numbers, such as 4, 6, 8, 9, 10 etc.

Number Game – Let us play a game where we can tell if a number is prime or not without factorization. First write the numbers 1-100 as shown below:

Step 1: Make a box ☐ on the number 1 as it is neither prime nor non-prime.

Step 2: Encircle number 2, and cross all its multiples such as 4, 6, 8 etc (except 2).

Step 3: The next number not crossed is 3. Encircle 3 and cross all its remaining multiples.

Step 4: Continue this process until all numbers have either been encircled or crossed. All encircled numbers are Prime numbers. After this game, how many Prime numbers did you get between 1–100? Write these numbers in sequence and match them with your friends.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2.4 Odd-Even Numbers

Kanak and Pritam were playing marbles.

Kanak – Look Pritam, let me teach you a game. Take any number of marbles in your hand and close your fist. Now, I have to tell if the marbles in your hand are in pairs or not. This game is also called Eki or Beki. Eki means that when you make pairs of marbles in your hand and if one marble is left without a pair then it is Eki, if all marbles are in pairs then it is called Beki. Kanak and Pritam played this game and wrote it in a table.

Play this game with your friends and decide which numbers should be called Eki and which ones Beki?

Were you able to frame any rule? Numbers with 2, 4, 6, 8, 0 in units place are known as Even numbers. If 1, 3, 5, 7, 9, are in units place then the numbers are known as Odd numbers.

Score Card		
Kanak	Pritam	
15 marbles	Beki	Wrong
19 marbles	Eki	Right
24 marbles	Beki	Right
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Table 2.2

All those numbers which are perfectly divisible by 2 or are multiples of 2 are known as Even numbers.

Do and Learn

Write Even and Odd numbers separately.

(i) 357 (ii) 436 (iii) 77 (iv) 1900 (v) 5001

Even numbers.....Odd numbers.....

Exercise 2.1

1. Write all the factors of the following numbers:
   (i) 48 (ii) 36 (iii) 28 (iv) 100 (v) 125
2. Write the first five multiples of the following numbers:
   (i) 7 (ii) 12 (iii) 17 (iv) 15 (v) 18
3. Write all prime numbers between 10 and 30.
4. Write the smallest prime number.
5. Which of the following numbers have 6 as a factor?
   6,10, 12, 15, 18, 25, 30, 38, 46
6. Write 3 numbers which are multiples of 4 and 6
7. State whether True or False
   (i) 108 is a multiple of 9
   (ii) 7 is a factor of 27
   (iii) The sum of two prime numbers is an even number
   (iv) Every prime number is odd
   (v) 1 is a factor of every number
   (vi) Multiple of each number is less than the number itself
   (vii) Factor of each number is less than the number itself

2.5 Rules of Divisibility

2.5.1 On the basis of the units digit

(I) Divisibility by 2

We have learned about odd and even numbers. Now can you tell if every even number is divisible by 2? Take some odd and even numbers like 24, 15, 48, 26, 13, 11 and find their factors.

Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.

Factors of 15 are 1, 3, 5, 15.

Likewise find out factors of 26, 48, 13, 11.

Write the unit digit of those numbers whose one factor is 2.

Numbers		Numbers	
Even	Divisible by 2	Odd	Divisible by 2
22 28 50 36 — —	Yes	11 51 57 23 — —	No

Table 2.3

Hence now we can say, all numbers which have unit digits as 0, 2, 4, 6, 8, are divisible by 2, and 2 is a factor of these numbers.

(i) Divisibility by 10

Numbers	Divisible by 10 Yes/No
20 22 120 50 17 19 --- ---	

Table 2.4

Write some more numbers in the table. Do you find any pattern in the numbers divisible by 10 at its units place?

All those numbers which have 0 at units place, or which have 10 as one of their factors, are divisible by 10.

(i) Divisibility by 5

Numbers	Factors
45	1,3,5,9,45
40	1,2,4,5,8,10,20,40
32	.....
18	.....
25	.....

Now look at the unit digit of every number, which has 5 as one of the factors. **Hence we can say that all numbers having their unit digits 0 or 5 are divisible by 5.**

Do and Learn

1. Do all the numbers which have 0 and 5 at their units place, have 5 as one of their factors?
2. Are all these numbers divisible by 5?
3. Does any number not having 0 or 5 at its units place, have 5 as a factor?

2.5.2 On the basis of addition of digits

(i) Divisibility by 3

Teacher will arrange a game in the class.

1. Think about a number.
2. Add the digits of that number.
3. Divide the sum by 3.
4. Was it perfectly divided?
5. Divide the number by 3 directly.
6. Could it be divided perfectly.

Teacher will write the result on blackboard.

Numbers	Sum of Digit	Divisible by 3
39	3 + 9 = 12 ; 1 + 2 = 3	Yes
109	1 + 0 + 9 = 10 ; 1 + 0 = 1	No
507		
1008		
.....		

Table 2.5

Complete the table above –

Reena tried this rule on number 321 for divisibility by 3.

Sum of digits of 321 = 3+2+1=6

6 is divisible by 3.

$$\begin{array}{r} 3\overline{)321}\,(\,107 \\ -3 \\ \hline 021 \\ -21 \\ \hline 00 \end{array}$$

Hence we can say that any number with the sum of digits divisible by 3 is also divisible by 3.

(ii) Divisibility by 9

Numbers	Sum of Digit	Divisible by 9
1827	1 + 8 + 2 + 7 = 18	Yes
1227		
3395		
145		
.....		

Table 2.6

Complete the table above. On the basis of this can you find any pattern for the divisibility by 9?

If sum of digits of any number is divisible by 9 then that number is also divisible by9.

Do and Learn

Sum of digits of 3672: 3+6+7+2=18

Is it divisible by 9?

Find out 3672 ÷ 9

(iii) Divisibility by 6

Check the divisibility by 2 and 3 on the number 216.

Numbers	Divisible by 2	Divisible by 3	Divisible by 6
216	Yes	Yes	Yes
58	Yes	No	No
108			
103			
.....			

Table 2.7

Write some more numbers in the table above and complete the table.
Do you see any pattern of divisiblity by 6?
If any number is divisible by 2 and 3 separately, then it is also divisible by 6.

Do and Learn

Find out the divisibility by 6 for the numbers 336, 123, 1002, 4236.

(iv) Divisibility by 4

If a number has its last two digits divisible by 4 or if its tens and units digits are 0, then it is divisible by 4. Take some numbers and check this pattern.

Meena took one number 9212. Its last two digits are 12 which is divisible by 4. Now you try to divide it.

(v) Divisibility by 8

If the number framed by last three digits i.e. units, tens and hundred is divisible by 8 or if any number has 0 as its units, tens and hundreds digits, then the number is divisible by 8. Test this pattern in the table below:

Numbers	Number formed by hundreds, tens and units	Divisible by 8 Yes/No
1. 30480	480 ÷ 8 = 60	Yes
2. 42108	108 ÷ 8 = ...	.....
3. 1324	324 ÷ 8 = ...	.....
4.	..... = ...	.....
5	..... = ...	.....

Table 2.8

(vi) Divisibility by 11

Is the number 72325 divisible by 11? The odd place digits in the number 72325 are 7, 3, 5.

Sum of these digits equals 7+3+5=15.

Likewise sum of the even placed digits are 2+2=4. Now (sum of odd placed digits-sum of even placed digits) = 15-4 = 11, which is divisible by 11.

Hence the number 72325 is also divisible by 11.

Likewise you also fill in the table and find out which other numbers are divisible by 11?

```
11) 72325 ( 6575
   -66
    63
   -55
    82
   -77
     55
    -55
     0
```

S.N.	Numbers	Sum of even -placed digits	Sum of odd-placed digits	Difference divisible by 11 or not
1	3333			
2	15708			
3	12345			
4	130303			

Table 2.9

Can you define a rule for divisibility by 11 on the basis of the above table?

All those numbers which have the difference between sum of its odd place digits and even place digits as zero (0) or multiple of 11 is divisible by 11.

2.6 Common multiple and prime factor

Common multiple

What are the multiple of 3 and 4?

Multiple of 3= 3, 6,9,12,15,18,21,24…..(write some more multiples)

Multiple of 4= 4,8,12,16,20,24,28,36……..(write some more multiples)

Now select the common multiple of 3 and 4.

12,24,36…are those numbers which are multiple of both 3 and 4. These are called common multiple of 3 and 4.

Prime factor

We learnt finding out factors of numbers. Observe the factors of number 18.

$18 = 2 \times 9$ and $18 = 3 \times 6$

$= 2 \times 3 \times 3$ $= 3 \times 2 \times 3$

Now we can see that the factors of 18 done by above methods are prime numbers. These types of factors of any number are called prime factors. We can obtain factors of any number like this as well:

2	24
2	12
2	6
3	3
	1

Exercise 2.2

1. Find out the prime factors of following numbers
   (i) 28 (ii) 54 (iii) 96 (iv) 148 (v) 156
2. Write the prime factors of the smallest number of four digits.
3. Find out common factors of the following:
   (i) 24,36 (ii) 35,40 (iii)12,18,30 (iv)14,25,35
4. Find out first 3 common multiple of the following
   (i) 4 and 5 (ii) 8 and 12 (iii) 2, 4, 10 (iv) 3,9,15
5. Write all the numbers smaller than 50 , which are common multiple of 2 and 3

2.7 Highest Common factor(HCF)

2.7.1 By method of prime factorisation

We have learned about factors. Let's learn about properties of factors.

Most possible factors of 30,36 and 42 are

30 =	1	2	3	5	6	10	15	30	
36 =	1	2	3	4	6	9	12	18	36
42 =	1	2	3	6	7	14	21	42	

Hence we see 1, 2, 3 and 6 are common factors of 30, 36 and 42. 6 is the largest number and all 30, 36, 42 are divisible by 6. These kinds of numbers are called HCF. Let's understand its use in daily life.

Example 1 Asha, Nisha and Shyam have roll of ribbons 14 meter, 35 meter and 21 meter respectively. All of three want to cut the ribbon in equal largest pieces so that no ribbon must be left. Then how much long pieces of ribbon will they cut equal in length?

Solution Asha, Nisha and Shyam can respectively cut ribbons with the following measurement:

14	=	1	2	7	14
35	=	1	5	7	35
21	=	1	3	7	21

7 is the greatest common factor of 14,35 and 21.Therefore 7 meter is the greatest measurement for cutting the ribbons of length 14,35 and 21 and equal as well for cutting the equal ribbons in the above example. It is also HCF.

Example2 Find out the HCF of 24,36 and 60 by the method of prime factor.

Solution

2	24
2	12
2	6
3	3
	1

2	36
2	18
3	9
3	3
	1

2	60
2	30
3	15
5	5
	1

24	=	2	x	2	x	2	x	3
36	=	2	x	2	x	3	x	3
60	=	2	x	2	x	3	x	5

Common Factors of 24, 36 and 60 = $2 \times 2 \times 3$

Hence HCF of 24, 36, 60 = $2 \times 2 \times 3 = 12$

Do and Learn

Raju's Cow Gives 15 litres and buffalo give 20 litres milk. Find out The maximum measurement for measuring pot for both type of milk exactly.

2.7.2 Vedic Method

In Vedic method the formula (Addition – Subtraction) is used to find out H.C.F, let's Practice It

Example 3 Find out H.C.F of 24 and 36

Solution First difference of numbers = 36-24=12

Therefore possible H.C.F=12

Second Difference 24-12=12, First difference=Second difference. Hence H.C.F of 24 and 36=12

Example 4 Find out H.C.F of 145 And 232

Solution First difference 232-145=87 therefore possible H.C.F is 87

Second difference 145-87=58 therefore possible H.C.F is 58

Third difference 87-58=29 therefore possible H.C.F is 29

Fourth difference 58-29=29 therefore H.C.F is 29

H.C.F of 145 and 232= 29

Example 5 Find out H.C.F of 18, 54, 81

Solution Addition Of two numbers 18+81=99

First difference 18+81-54=45 therefore possible H.C.F is 45

Second difference 54-45=9 therefore possible H.C.F is 9

Possible H.C.F 9 is a factor of 45.

Hence H.C.F 18, 54, 81=9

Do and Learn

Find out H.C.F by Vedic method

(i) 8,12 (ii) 38, 57 (iii) 117, 195 (iv) 99, 165, 231

Exercise 2.3

1. Find out H.C.F of the following numbers
   (I) 36,84 (ii)28,42 (iii)13,26,52 (iv)15,35,40 (v)23,31,93
2. Find out H.C.F of the following
   (i) Two successive numbers (ii) two successive even numbers
   (iii) two successive odd numbers

3. Width and length of the floor is 25meters and 30meters respectively. Find out the length of the longest rope which can be use to measure length and width of the room exactly.
4. Three oil tankers are of capacity 96liters, 100liters and 144liters. Find out maximum measurements to measure the oil of all three tankers exactly.
5. Find out the length of the longest rope to measure distances of 36meters, 54meters, 90meters?

2.8 Lowest common multiple (L.C.M)

Teacher asks students a puzzle in the class.

If i make a pile of "4-4 or 5-5 berries

How many minimum berries would be distributed equally both the time"

Leela- It means that each pile must have equal berries and both of the piles must be distributed so that nothing left, nothing less

Teacher- Yes, now tell us that how many minimum berries are there in each pile?

Kamal – If berries are in the numbers 4, 8 , 12 ,16 ,20 ,24 ,28 ,32 ,36 ,40 etc then these berries can be divided into the groups of 4-4

Leela- If berries are in the number 5, 10, 15, 20, 25, 30, 35, 45 then we can distribute it in the groups of 5-5

The smallest number which is divisible by two or more than two numbers wholly, is called lowest common multiple of those numbers.

Do and Learn

Two bells start ringing together. First bell rings after every three minutes and second bell rings after every five minutes then after how much time both bells will ring together?

2.8.1 Methods of finding out lowest common multiple

1. Prime Factor Method

L.C.M. of 48 and 30 can be found out by prime factor method

Step 1 Find out prime factors of each number

$48 = 2\times2\times2\times2\times3$

$30 = 2\times3\times5$

2	48
2	24
2	12
2	6
3	3
	1

2	30
3	15
5	5
	1

Step 2 In these prime factors the prime factor 2 appears the maximum of four times (that is in 48) and 3 and 5 appear maximum 1-1 times each. Therefore the desired L.C.M $= 2\times2\times2\times2\times3\times5 = 240$

2. Method of division

Let's find out the L.C.M of 18,24,30 by division method.

Step 1 Write the numbers in series as shown below:

2	18, 24, 30
2	9, 12, 15
2	9, 6, 15
3	9, 3, 15
3	3, 1, 5
5	1, 1, 5
	1, 1, 1

Step 2 Divide by the smallest possible numbers. Numbers which cannot be divided by that number are written as it is in the next line.

Step 3 This is continued till the numbers are divided. Then the division is repeated with the next prime number till all the numbers have been divided completely.

Step 4 The multiplication of divisors from each row is the L.C.M. Therefore the L.C.M of 18, 24, 30 is $2\times2\times2\times3\times3\times5 = 360$

3. Vedic method

Let's find the L.C.M of 12 and 16 by Vedic method.

Step 1 12 and 16 can be written in different form as $\frac{12}{16}$.(According to formula)

Step 2 Do prime factorization of 12 and 16. $\frac{12}{16} = \frac{2\times2\times3}{2\times2\times2\times2}$

Step 3 Now remove the common numbers from numerator and denominator. $\frac{12}{16} = \frac{3}{4}$

Step 4 By the method of cross multiplication $12\times4 = 16\times3 = 48$. Hence L.C.M of 12 and 16 is 48.

Do and Learn

1. Find out the L.C.M of 48,64 and 80 by division method.
2. Find out the L.C.M of 24 and 30 by Vedic method.

Exercise 2.4

1. Find out L.C.M of the following.
   (i) 10,15
   (ii) 14,28
   (iii) 12,18 and 27
   (iv) 48, 56 and 72
2. Minimum how many mangoes can be divided into the group of 5 and 6 compeletly?
3. Sneha and Vansh go to market every 3'rd and 5'th day. They went to market today. After how many days will they go to market together?
4. Harish, kareem and rakesh take a round of the ground respectivly in 6,8 and 12 minutes. If all of three start at 6'O clock then after how long time they will be together?
5. Find out the smallest number which is divisible by 16,20 and 24 completely.
6. A blue bulb keeps flashing on the interval of every 60 second and a red bulb keeps flashing on the interval of every 90 seconds. If both the bulbs are switched on at 5'o clock together then after how much time both will flash together?

We learnt

1. (i) One factor of a number is a whole divisor of that number.
   (ii) Each number is a factor of itself.1 is a factor of every number.
   (iii) Every factor of a given number is smaller or equivalent to the number.
   (iv) Each number is a multiple of its each factor.
   (v) Every multiple of a given number is greater than the number or equivalent to the number.
   (vi) Every number is a multiple of itself.

2. (i) The number which has only two factors (The number itself and 1) is called a prime number.

(ii) 2 is the smallest prime number which is also an even number. All the prime numbers are odd numbers.

3. We can check the divisibility of any number by 2, 3, 4, 5, 8, 9, 10 and 11.

(i) Divisibility by 2, 5 and 10 can be checked by observing the digit at units place of the given number.

(ii) Divisibility by 3 and 9 can be checked by adding the digits of the given number.

(iii) Divisibilty by 4 can be checked by the digits at tens and units place and divisibility by 8 can be checked by the digits at the unit, tens and hundreds place.

(iv) Divisibilty by 11 can be checked by the sum of the digits at the odd and even places.

4. If two numbers can be divided by a number, then the sum and difference of those numbers can also be divided by that same number.

5. The HCF of two or more numbers, is biggest among all its common factors.

6. The L.C.M. of two or more number is the smallest common among its all multiples through vedic method we can also find the LCM and HCF of numbers.