MISCELLANEOUS EXERCISE

Q.No 1. The relation f is defined by $f(x) = \begin{cases} x^2 ; & 0 \leq x \leq 3 \\ 3x ; & 3 \leq x \leq 10 \end{cases}$

and g is defined by $g(x) = \begin{cases} x^2 ; & 0 \leq x \leq 2 \\ 3x ; & 2 \leq x \leq 10 \end{cases}$

Show that f is a function but g is not a function.

Sol (i) The given relation is $f(x) = \begin{cases} x^2 ; & 0 \leq x \leq 3 \\ 3x ; & 3 \leq x \leq 10 \end{cases}$

The relationship can be written as.

$f(x) = x^2$ in $0 \leq x < 3$ and is well defined.

and $f(x) = 3x$ in $3 < x \leq 10$ and is well defined

Now At $x = 3$ $f(x) = x^2$ and $f(x) = 3x$.

From $f(x) = x^2$; $f(3) = (3)^2 = 9$

and from $f(x) = 3x$; $f(3) = 3 \times 3 = 9$

$\therefore$ f is well at $x = 3$ also.

Hence f is a function.

(ii) $g(x) = \begin{cases} x^2 ; & 0 \leq x \leq 2 \\ 3x ; & 2 \leq x \leq 10 \end{cases}$

Now $g(x) = x^2$ is well defined in $0 \leq x < 2$

and $g(x) = 3x$ is well defined in $2 < x \leq 10$.

But At $x = 2$; $g(x) = x^2 \Rightarrow g(2) = 2^2 = 4$

Also $g(x) = 3x \Rightarrow g(2) = 3 \times 2 = 6$

$\therefore$ At $x = 2$ $g(x)$ has two values.

$\therefore$ Relation g is not a relation.

Q No 2: If $f(x) = x^2$, find $\dfrac{f(1.1) - f(1)}{(1.1) - 1}$

Sol:

Here $f(x) = x^2$.

$$\therefore \frac{f(1.1) - f(1)}{(1.1) - 1} = \frac{(1.1)^2 - (1)^2}{0.1} = \frac{1.21 - 1}{0.1} = \frac{21}{100} \times \frac{10}{1} = \frac{21}{10} = 2.1$$

Q No 3: Find the domain of the function $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - 8x + 12}$

Sol:

$$f(x) = \frac{x^2 + 12x + 1}{x^2 - 8x + 12}$$

For f to be defined $x^2 - 8x + 12 \neq 0$

Now $x^2 - 8x + 12 = 0$ ie $(x-2)(x-6) = 0$

ie $x^2 - 8x + 12 = 0$ when $x = 2, 6$.

$\therefore$ For f to be defined $x \neq 2, 6$

$\because$ f is defined for all real nos. except $2, 6$.

$\therefore D_f = R - \{2, 6\}$

Q No. 4: Find the domain and Range of the real function f defined by $f(x) = \sqrt{(x-1)}$

Sol:

Here $f(x) = \sqrt{x-1}$

For f to be defined. $x - 1 \geq 0$ or $x \geq 1$

$\therefore D_f = [1, \infty)$

Put $f(x) = y$.

$\therefore y = \sqrt{x-1}$

For $x \geq 1$, $y \geq 0$ $\therefore R_f = [0, \infty)$

Q No 5 Find the domain and Range of Real function f defined by $f(x) = |x - 1|$

Sol.

$f(x) = |x - 1|$

$\because$ f is defined for all real values of x.

$\therefore D_f = R$.

Now $|x - 1| \geq 0 \ \forall \ x \in R$

$\Rightarrow f(x) \geq 0 \ \forall \ x \in R \Rightarrow R_f = [0, \infty)$

Q No 6: Let $f(x) = \left\{\left(x, \frac{x^2}{1+x^2}\right); x \in R\right\}$ be a function from R to R. Determine the range of f.

Sol.:

Sol.: $f(x) = \left\{\left(x, \frac{x^2}{1+x^2}\right); x \in R\right\}$

Now $f(x) = \frac{x^2}{1+x^2}$

We know that $\forall x \in R \quad x^2 \geqslant 0$

$\Rightarrow \quad 1+x^2 \geqslant 0+1 \quad$ ie $\quad 1+x^2 \geqslant 1$ and hence $1+x^2 > 0$

$\Rightarrow \quad \frac{x^2}{1+x^2} \geqslant 0 \quad$ ie $\quad 0 \leqslant \frac{x^2}{1+x^2}$

Also $\quad \forall x \in R \quad x^2 < 1+x^2$

$\Rightarrow \quad \frac{x^2}{1+x^2} < \frac{1+x^2}{1+x^2} \quad [\because 1+x^2 > 0]$

$\Rightarrow \quad \frac{x^2}{1+x^2} < 1$

$\therefore \quad 0 \leqslant \frac{x^2}{1+x^2} < 1$

ie $\quad 0 \leqslant f(x) < 1 \quad$ ie $\quad f(x) \in [0, 1)$

$\therefore \quad R_f = [0, 1)$

Q No 7. Let $f, g : R \to R$ be defined respectively by $f(x) = (x+1)$, $g(x) = 2x-3$. Find $f+g$, $f-g$ and $\frac{f}{g}$.

Sol

$f(x) = x+1 \quad ; \quad g(x) = 2x-3$

$\therefore \quad (f+g)(x) = f(x)+g(x) = (x+1)+(2x-3) = 3x-2$

$(f-g)x = f(x)-g(x) = (x+1)-(2x-3) = -x+4 = 4-x$

$\frac{f}{g}(x) = \frac{f(x)}{g(x)}; \; g(x) \neq 0 = \frac{x+1}{2x-3}; \quad 2x-3 \neq 0$

ie $\quad \frac{f}{g}(x) = \frac{x+1}{2x-3}; \quad x \neq 3/2$

Q No 8 Let $f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$ be a linear function from Z into Z. Find $f(x)$.

Sol.

$$f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$$

Let $f(x) = ax+b$ be a linear function.

$\therefore$ $f(1) = 1 \Rightarrow a+b = 1$ ...(1)

$f(2) = 3 \Rightarrow 2a+b = 3$ ...(2)

Solving (1) and (2) for a and b, we get

$a = 2$ and $b = -1$

$\therefore$ $f(x) = 2x - 1$

Now $f(0) = 0\times 2 - 1 = -1$

and $f(-3) = -1\times 2 - 1 = -3$

$\therefore$ $(0,-1)$ and $(-1,-3)$ satisfy $f(x) = 2x - 1$.

$\therefore$ Required function is $f(x) = 2x - 1$

Q No. 9: Let R be a relation from N into N defined by.
$R = \{(a,b);\ a, b \in N \text{ and } a = b^2\}$. Are the following true?

(i) $(a,a) \in R \ \forall\ a \in N$

(ii) $(a,b) \in R \Rightarrow (b,a) \in R$

(iii) $(a,b) \in R,\ (b,c) \in R \Rightarrow (a,c) \in R$. Justify your ans.

Sol.

(i) False

$\because$ $a = a^2$ only when $a = 1$ $\therefore$

$\therefore$ $(a,a) \notin \ \forall\ a \in N$.

(ii) False

$\because$ $(4,2) \in R$ as $4 = 2^2$.

But $(2,4) \notin R$ as $2 \neq 4^2$.

(iv) False

$\because$ $(16,4) \in R$, $(4,2) \in R$ But $(16,2) \notin R$.

Q No 10 Let $A = \{1,2,3,4\}$; $B = \{1,5,9,11,15,16\}$ and

$f = \{(1,5)\ (2,9)\ (3,1)\ (4,5)\ (2,11)\}$. Are the following true?

(i) f is a relation from A into B (ii) f is a function from A to B

Justify your answer in each case.

Sol: $A = \{1,2,3,4\}$; $B = \{1,5,9,11,15,16\}$ and

$f = \{(1,5), (2,9)\ (3,1)\ (4,5), (2,11)\}$

(i) True as f is a subset of $A \times B$

(ii) False as $(2,9)$ and $(2,11)$ have same first component

ie 2 is associated with two elements 9 and 11

$\therefore$ f is not a function.

Q No 11 Let f be a subset of $Z \times Z$ defined by

$f = \{(ab, a+b) : a, b \in Z\}$ is f a function from Z into Z? Justify.

Sol: $f = \{(ab, a+b); a, b \in Z\}$

Now $2 \times 6 = 12$ and $2+6 = 8$ $\Rightarrow (12,8) \in R$

$4 \times 3 = 12$ and $4+3 = 7$ $\Rightarrow (12,7) \in R$

$\Rightarrow$ 12 is associated with two elements 8 and 7 of Z.

$\therefore$ f is not a function.

Q No. 12. Let $A = \{9,10,11,12,13\}$ and $f : A \to N$ be defined by

$f(n)$ = The highest prime factor of n. Find Range of f.

Sol:

$9 = 3 \times 3$ $\therefore f(9) = 3$

$10 = 2 \times 5$ $\therefore f(10) = 3$

$11 = 11$ $\therefore f(11) = 11$

$12 = 2 \times 2 \times 3$ $\therefore f(12) = 3$

$13 = 13$ $\therefore f(13) = 13$

$\therefore f = \{(9,3), (10,5)\ (11,11)\ (12,3)\ (13,13)\}$

$\therefore$ Range of $f = \{3, 5, 11, 13\}$

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