Verify the Algebraic Identity $a^3-b^3 = (a-b)(a^2+ab+b^2)$

OBJECTIVE

To verify the algebraic identity $a^3-b^3 = (a-b)(a^2+ab+b^2)$.

Materials Required

1. Acrylic sheet
2. Geometry box
3. Scissors
4. Adhesive/Adhesive tape
5. Cutter

Prerequisite Knowledge

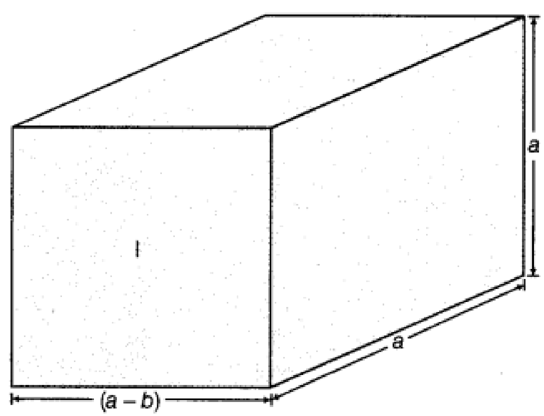
1. Concept cuboid and its volume.
2. Concept of cube and its volume.

Theory

1. For concept of cuboid and its volume refer to Activity 7.
2. For concept of cube and its volume refer to Activity 7.

Procedure

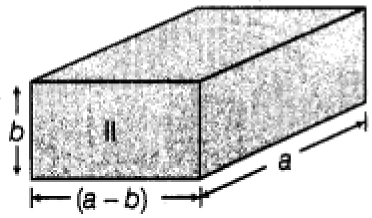
1. Using acrylic sheet, make a cuboid of dimensions $(a – b) \times a \times a$, where $b < a$. (see Fig. 10.1)



Volume = $(a - b)a^2$ cu units

Fig. 10.1

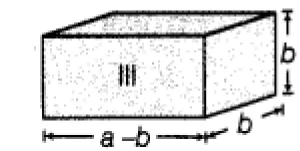
2. Using acrylic sheet, make another cuboid of dimensions (a-b) x a x b, where b < a. (see Fig. 10.2).



Volume = $(a-b)ab$ cu units

Fig. 10.2

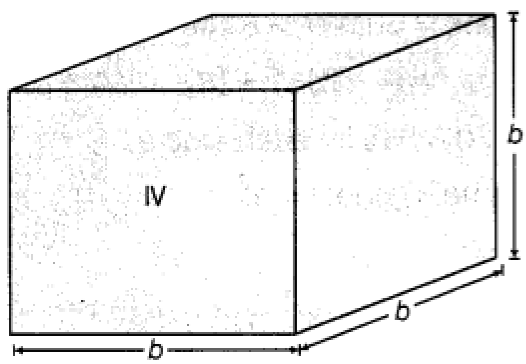
3. Now, make one more cuboid of dimensions (a-b) x b x b. (see Fig. 10.3)



Volume = $(a-b)b^2$ cu units

Fig. 10.3

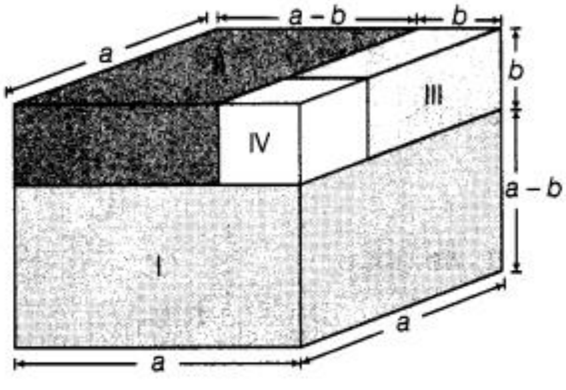
4. Now, make a cube of dimensions b x b x b. (see Fig. 10.4)



Volume = b^3 cu units

Fig. 10.4

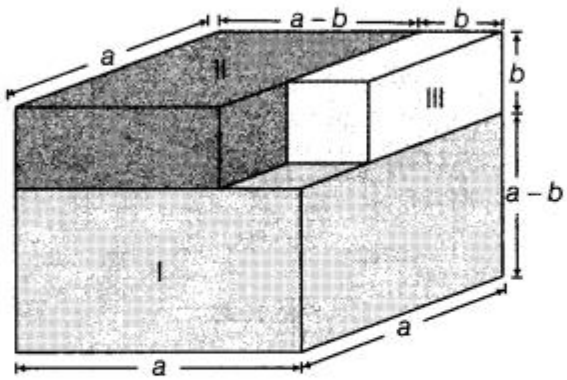
5. Arrange the cube and cuboids obtained in Fig. 10.1 to 10.4 to form a solid as shown in Fig. 10.5, which is a cube of side a units.



Volume = a^3 cu units

Fig. 10.5

6. Now, remove a cube of side b units from the solid obtained in Fig. 10.5, thus we obtain solid as shown in Fig. 10.6.



Volume = $a^3 - b^3$ cu units

Fig. 10.6

Demonstration

For Fig. 10.1, volume of cuboid = (a-b) x a x a = $(a-b)a^2$
For Fig. 10.2, volume of cuboid = (a-b) x a x b = (a-b)ab
For Fig. 10.3, volume of cuboid = (a – b) x b x b = $(a - b)b^2$
For Fig. 10.4, volume of cube =b^3
For Fig. 10.5, volume of cube = Sum of volume of all cubes and cuboids
= $(a - b)a^2 + (a - b)ab + (a - b)b^2 + b^3$ …..(i)
The cube obtained in Fig. 10.5 has its each side a.
Its volume = $(\text{side})^3 = a^3$ …..(ii)
From Eqs. (i) and (ii), we get
$a^3 = (a - b)a^2 + (a - b)ab + (a - b)b^2 + b^3$ …..(iii)
For Fig. 10.6, volume of solid obtained = $a^3 - b^3$
= $(a - b)a^2 + (a - b)ab + (a - b)b^2 + b^3 - b^3$ [from Eq.(iii)]
= $(a - b)a^2 + (a - b)ab + (a - b)b^2 = (a-b)(a^2 + ab + b^2)$

Therefore, $a^3-b^3 = (a-b)(a^2+ab+b^2)$
Here, volume is in cubic units.

Observation

On actual measurement, we get
a = ……. , b = ……. ,
So, a^2 =…….. , b^2 = ……. ,
(a- b) = ……. , ab = ……. ,
a^3 =…….. , b^3 = ……. ,
Hence, $a^3-b^3 = (a-b)(a^2+ab+b^2)$.

Result

The algebraic identity $a^3-b^3 = (a-b)(a^2+ab+b^2)$ has been verified.

Application

The identity can be used in simplification and factorisation of algebraic expressions.

Viva Voce

Question 1:
What is the expanded form of $a^3 - b^3$?
Answer:
Expanded form of $a^3-b^3 = (a-b)(a^2+ab+b^2)$.

Question 2:
If $(a^2 + ab + b^2) = 0$, then what will be the value of $a^3 - b^3$?
Answer:
$a^3-b^3 = (a-b)(a^2+ab+b^2)$
$a^3-b^3=0$

Question 3:
If x = y, then what will be the value of $x^3 - y^3$?
Answer:
Now, $x^3 - y^3 = x^3 - x^3 = 0$

Question 4:
What is degree of the expression $y^3 - x^3$?
Answer:
The degree of given expression is 3.

Question 5:

If we replace a by -a and b by -b, then what is the expansion of $a^3 - b^3$?

Answer:

$a^3 + b^3 = (-a^3)-(-b^3)$

$=-a^3+b^3$

$b^3 -a^3 =(b-a)(b^2 +a^2 +ab)$

Suggested Activity

Verify the algebraic identity $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ by using x =7, y = 5.