Factorization

Introduction

Factorization of an algebraic expression is the process of writing the algebraic expression as a product of two or more linear factors. Each multiple of the algebraic expression is called factors of the algebraic expression. Thus the process of splitting the given algebraic expression into the product of two or more linear factors is called factorization.

According to the factor theorem if $f(x)$ is polynomial which is completely divisible by another polynomials $g(x)=x-a$, then $x-a$ is called the factor of the polynomial $f(x)$ and $f(a)=0$ for all value of a

Methods of Factorization

Different algebraic expressions can be factorized by different methods. Monomials can be easily written into their linear factors. Binomials can be factorized by using identities given in previous chapter. The quadratic equation can be factorized by splitting the middle term and cubic equation can be factorized by first dividing it by linear factors and then reducing it to the quadratic form and then applying the middle term splitting.

The other methods of factorization are by grouping the terms having the common coefficients or having some common variables.

Degree of the Polynomials

The degree of the polynomials is the highest of power of the variable in the given polynomials. If the degree of the polynomial is zero then it is called constant polynomial.

If the degree of the polynomial is one then it is called linear polynomial and if the degree of the polynomial is two then it is called quadratic polynomial.

For cubic polynomial the degree is three and if the degree is four then it is called biquadrate polynomial.

Illustrative EXAMPLE

The polynomial $ax+b=0$ is a linear polynomial.

The polynomial $ax^2+bx+c=0$ is a quadratic polynomial.

The polynomial $dx^3+ax^2+bx+c=0$ is a cubic polynomial.

The polynomial $ex^4+dx^3+ax^2+bx+c=0$ is a biquadrate polynomial.

Illustrative EXAMPLE

Factories: $y^2+3y+y+3$

Solution:

$=y^2+3y+y+3=y(y+3)+1(y+3)=(y+3)(y+1)$

Illustrative EXAMPLE

Factories: $x^2+\frac{1}{x^2}+2-2x-\frac{2}{x}$

Solution:

$=x^2+\frac{1}{x^2}+2-2x-\frac{2}{x}=\left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)=$

$\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}-2\right)$

You Must KNOW

- A prime number has no factors other than 1 and itself.
- There are infinite number of prime numbers.
- The only prime number that is even is 2.
- 1 is not a prime number.
- Every composite number can be expressed as product of primes.

SUMMARY

- When we factories an expression, we write it as a product of factors.
- We can factories the expression by splitting the middle term if it is a quadratic equation.
- Division can be carried out through reducing the given expression or by merely performing the division.
- For division dividend $= Divisor \times quotient + remainder$.
- Division can also be carried out by taking out the common factors and cancelling out the common terms in both numerators and denominators.

Commonly Asked QUESTIONS

Factories $4x^2+20x+3xy+15y$ and choose the correct option.

(a) $(x+5)(4x+3y)$ (b) $(x-5)(5x+3y)$

(c) $(x+5)(5x-3y)$ (d) $(x+5)(x+3y)$

(e) None of these

Answer: (a)

Explanation

$=(4x^2+20x)+(3yx+15y)$

$=4x(x+5)+2y(x+5)$

$=(x+5)(4x+3y)$

Factories x^3-27 and choose the correct option.

(a) $(x-3)(x^2+3x+9)$

(b) $(x+3)(x^2+3x+9)$

(c) $(x-3)(x^2-3x+9)$

(d) $(x-3)(x^2+3x-9)$

(e) None of these

Answer: (a)

Explanation

$x^3-27=(x-3)(x^2+3x+9)$

Factories the given polynomial

$\frac{9}{16}x^2+\frac{4}{9}y^2+4z^2-xy-\frac{8}{3}yz+3zx$.

(a) $\left(\frac{3}{4}x-\frac{2}{3}y+2z\right)^2$ (b) $\left(\frac{3}{4}x+\frac{2}{3}y+2z\right)^2$

(c) $\left(\frac{3}{4}x+\frac{2}{3}y-2z\right)^2$ (d) $\left(\frac{3}{4}x-\frac{2}{3}y-2z\right)^2$

(e) None of these

Answer: (a)

Factories the given polynomial $36u^2+\frac{1}{25}v^2+25w^2-\frac{12}{5}uv-\frac{8}{5}vw+48wu$.

(a) $\left(6u-\frac{v}{5}-5w\right)^2$ (b) $\left(6u+\frac{v}{5}-5w\right)^2$

(c) $\left(6u-\frac{v}{5}+5w\right)^2$ (d) $\left(6u+\frac{v}{5}+5w\right)^2$

(e) None of these

Answer: (c)

Simplify the given expression: $=\frac{0.86\times0.86\times0.86+0.14\times0.14\times0.14}{0.86\times0.86-0.86\times0.14\times0.14\times0.14}$

(a) 1 (b) 0

(c) 2 (d) -1

(e) None of these

Answer: (a)

Factories the expression given by $18x^3y^3-27x^2y^3+36x^3y^2$

(a) $9x^2y^2(2xy-3y+4x)$ (b) $9x^2y^2(2xy+3y+4x)$

(c) $9x^2y^2(2xy-3y+4x)$ (d) $9x^2y^2(2xy-3y-4x)$

(e) None of these

Answer: (a)

Self Evaluation TEST

Duration 10 Minutes

---

1. **Factories the given expression.**

(a) $(un+vm)(um-vn)$ (b) $(un+vm)(um+vn)$

(c) $(un-vm)(um-vn)$ (d) $(un-vm)(um+vn)$

(e) None of these

---

2. **Find the remainder when $30y^4+11y^3-42y^2-24y+3$ is divided by $3y^2+2y$?**

(a) 0 (b) $2y+3$

(c) 3 (d) $y-2$

(e) None of these

---

3. **Factories:** $(p+q)(2p+5)-(p+q)(p+3)$ **.**

(a) $(p-q)(p+2)$ (b) $(p-q)(p-2)$

(c) $(p-q)(p+2)$ (d) $(p+q)(p+2)$

(e) None of these

---

4. **The value of** $52xyz(xy+yz+xz+y^2)(z+x)\div 104xy(xy+xz+yz+y^2)$ **is —**

(a) $\frac{z(z+x)}{2}$ (b) $\frac{z(z-x)}{2}$

(c) $\frac{x(z-x)}{2}$ (d) $\frac{x(z+x)}{2}$

(e) None of these

---

5. **Simplify the given expression** $39y^2(50y^2-98)\div 26y^2(5y+7)$

(a) $3(5y+8)$

(b) $15y+21$

(c) $15y-21$

(d) $18y-21$

(e) None of these

---

6. **The value of** $(x-y)(x+y)(x^2+y^2)\ (x^2+y^2)^2\ -2x^2y^2)$ **is:**

(a) x^8-y^8 (b) x^8+y^8

(c) x^6-y^6 (d) x^6+y^6

(e) None of these

---

7. **The factor of** $6-y-2y^2$ **is:**

(a) $y-2$ (b) $y-3$

(c) $-2y+3$ (d) $y+3$

(e) None of these

---

8. **The quotient when** $6ab-b^2+12ac-2bc$ **is divided by** $(6a-b)$ **is:**

(a) $b+2c$ (b) $b-2c$

(c) $2b+c$ (d) $2b-c$

(e) None of these

---

9. **The greatest common factor of** $3m^2n^2+4n^2p^2+12m^2n^2p^2$ **is:**

(a) n^2 (b) m^2

(c) p^2 (d) mnp

(e) None of these

---

10. **The remainder when** $(m-2n)^2-4m+8n$ **is divided by** $(m-2n-4)$ **is:**

(a) 0 (b) m - 2n

(c) 1 (d) m + n

(e) None of these

Answers – Self Evaluation Test																					
1.	C	**2.**	C	**3.**	D	**4.**	A	**5.**	A	**6.**	A	**7.**	C	**8.**	A	**9.**	B	**10.**	A		