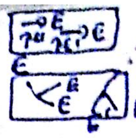


Homogenous - same elastic properties at any pt. in a given direcⁿ

Isotropic - " " " in any direcⁿ at " " pt.

True stress vs Engg. stress: $\sigma_T = \sigma_{Engg}(1+\epsilon_{Engg})$

Impact load: $\sigma_{impact} = \sigma_{static} \times I.F.$ $\quad I.F. = 1+\sqrt{1+\frac{2h}{\delta_{st}}}$

- why use longer structure when I.L.
- why use spring

Type of stress by moments: B.m. → Normal; T.m. → Shear

Types of loads & their effects: 1- Axial load: Axial stress 2- EAL → Axial stress, const. B.m.

3- TSL → const. S.F., varying B.m. 4- ETSL → const. T.m., const. S.F., varying B.m.

Stress: magnitude of internal resisting force developed at a pt. against deformation caused due to ext. applied loads.

* Stress in 1 direcⁿ causes strain in 3 directions.

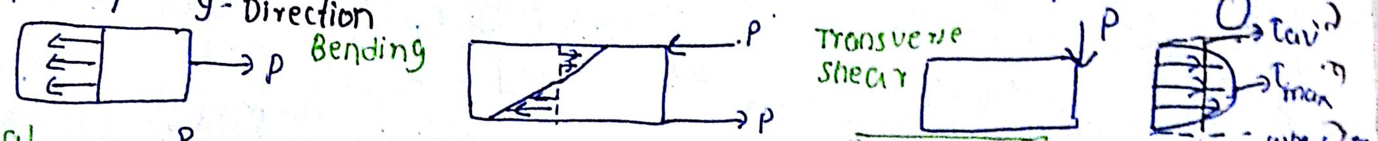
* Stress developed in that direcⁿ in which change of dimension is restricted either completely or partially. (This restricⁿ e.g. internal elasticity)

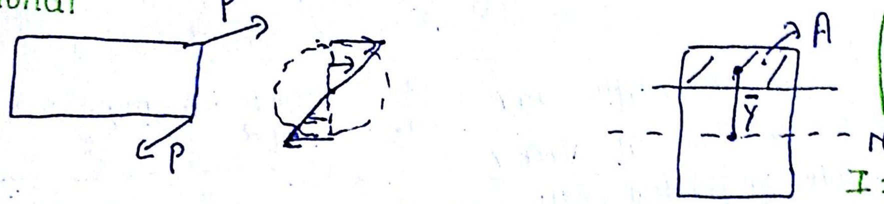
Normal stress → Axial $\pm P/A$; Bending $\pm M/Z$

Shear stress → Transverse $\tau_{av} = \frac{P}{A}$ $\quad \tau = K\cdot\tau_{av}$ $\quad K$ → 3/2 (rectangle), 4/3 (circle); Torsional $\tau_s = \frac{T}{2P}$

σ_{xy} or τ_{xy} x - Face, y - Direction

Axial — Bending — Transverse Shear — Torsional (S)





$\tau = \frac{F\cdot A\cdot\bar{y}}{I b}$

A = Area above τ found

$\bar{y}$ = distance of centroid of hatched portion from NA

I = MoI about NA of whole body.

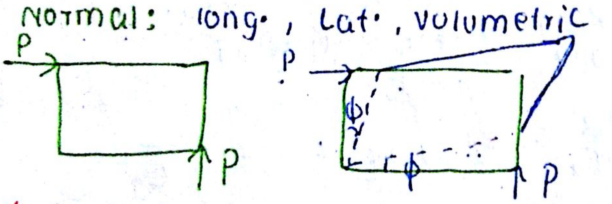
b = width of x-secn where calculated

Strain: if relative position of particles change.

Normal: long., Lat., volumetric

Shear: ↳ change in initial Rt. ∠ b/w 2 line elements which were ∥ to x- and y- axis

*Every long ε is associated with 2 Lat. ε.



$\gamma = 2\phi$ - can find by $\frac{\tau}{G}$

* Can't find $\epsilon_x, \epsilon_y, \epsilon_z$ theoretically ∴ use elastic constants

$E = 2G(1+\mu)$

$E = 3K(1-2\mu)$

$E = \frac{\text{Normal stress}}{\text{long. strain}}$ $\quad G = \frac{\text{Shear stress}}{\text{Shear strain}}$ $\quad K = \frac{\text{Normal stress}}{\text{volumetric strain}} = \frac{-P\cdot v}{dv}$ $\quad E = \frac{9KG}{3K+G}$

$e_v = \frac{-dP}{K}$ (for hydrostatic condⁿ) $\quad e_v = e_x + e_y + e_z$ (for all others)

For Triaxial loading: $\epsilon_x = \frac{1}{E}(\sigma_x - \mu\sigma_y - \mu\sigma_z)$ same for ϵ_y & ϵ_z

Strain Energy: Energy absorbed when a material is strained. modulus of R.: P.R./vol

Resilience: Till the elastic limit proof R.: max. in elastic limit. Max. S.E. absorbing capacity within E.L. per unit vol.

$P.R. = \frac{1}{2}P_{EL}\cdot\delta_{EL}$ $\quad M.R. = \frac{1}{2}\sigma_{EL}\cdot\epsilon_{EL} = \frac{(\sigma_{EL})^2}{2E}$

Toughness & modulus of T: upto prior to fracture.

STRENGTHS

Ductile: $(YS)_t > (YS)_T > (YS)_{Shear}$

Brittle: $S_{uc} > S_{us} > S_{ut}$

EF = Strain hardening

F = Ultimate pt.

FG = Necking

G = fracture pt.

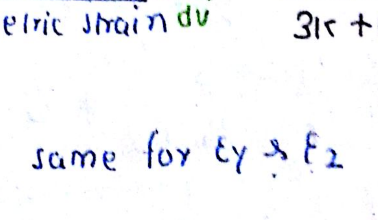
A = Prop. limit i.e. H.L.W

B = Elastic Region

C = UYP D = LYP (2 only in LCS)

Y. strength = LYP.

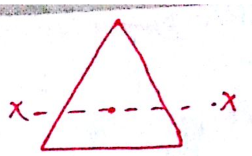
DE = Plastic deformation w/o any ↑ in load by change in int. structure.



$E = 2G(1+\mu) = 3K(1-2\mu)$.

Cir Tube $J = \text{Area} \times R^2$

$J = \pi \cdot D \cdot t \cdot \frac{D^2}{4}$



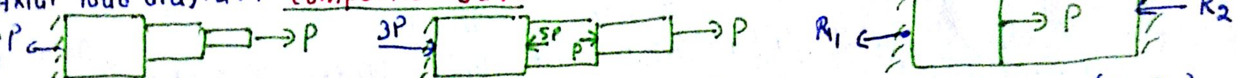
$I_{xx} = \frac{b \cdot h^3}{36}$

$I_{base} = \frac{bh^3}{12}$ बाहरी = base

→ write Reacn
→ use Axial load diagram **Compound Bars**

* write both Reacn opp. to net load

SERIES:



$P_1 = P_2 = P_3 = P$

$\delta = \delta_1 + \delta_2 + \delta_3$

$P_1 = -3P \quad P_2 = 2P \quad P_3 = P$

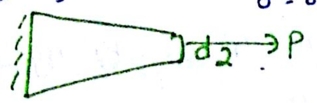
$\delta = \delta_1 + \delta_2 + \delta_3$

$P_1 = R_1 \quad P_2 = -(P - R_1)$

$R_1 + R_2 = P \quad \delta_1 = \delta_2$

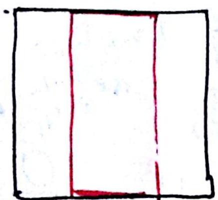
$\boxed{\delta_1 + \delta_2 = 0}$

Tapered bar



$\boxed{\delta = \frac{4PL}{\pi \cdot d_1 \cdot d_2 \cdot E}}$

PARALLEL: I: Equal length



$\boxed{\delta_1 = \delta_2}$

$\epsilon_1 = \epsilon_2$

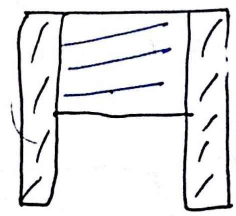
$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$

$\frac{P_1}{P_2} = \frac{A_1 \cdot E_1}{A_2 \cdot E_2}$

$\boxed{P = P_1 + P_2}$

$P = \sigma_1 A_1 + \sigma_2 A_2$

II unequal length



$\boxed{\delta_1 = \delta_2}$ $\epsilon_1 \cdot l_1 = \epsilon_2 l_2$

$\frac{\sigma_1}{E_1} \cdot l_1 = \frac{\sigma_2 \cdot l_2}{E_2}$

$\frac{P_1}{P_2} = \frac{A_1 \cdot E_1 \cdot l_2}{A_2 \cdot E_2 \cdot l_1}$

$\boxed{P = P_1 + P_2 =}$ $\sigma_1 A_1 + \sigma_2 A_2$

Elongation under self wt: → Prismatic bar $= \frac{PL}{2AE}$ $P = \text{self wt.} = mg = \gamma \cdot A \cdot L$, $\delta \propto L^2$

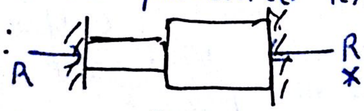
→ Conical bar $= \frac{\gamma \cdot L^2}{6E}$

Thermal Stresses: when complete or partial Restricn on change of length due to Temp. chang. when no Restriction → Strain is Present but not stress

Completely prevented: $\Delta l = l \cdot \alpha \cdot \Delta T$ $\quad \epsilon = \Delta l / l = \alpha \cdot \Delta T$ $\quad \sigma = E \cdot \epsilon = E \cdot \alpha \cdot \Delta T$

Partially prevented: prevented length $= \lambda$ $\quad \epsilon = \Delta l / \lambda$ $\quad \sigma = E \cdot \epsilon$

SERIES BARS:

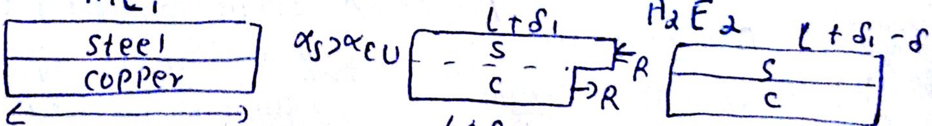


$\boxed{\text{Reacn: same}}$

* $\frac{R \cdot L_1}{A_1 \cdot E_1} + \frac{R \cdot L_2}{A_2 \cdot E_2} = \alpha_1 \cdot \Delta T \cdot L_1 + \alpha_2 \cdot \Delta T \cdot L_2$

* Juncn movt. $= \frac{RL_1}{A_1 E_1} - \alpha_1 \Delta T \cdot L_1 = \alpha_2 \cdot \Delta T \cdot L_2 - \frac{R \cdot L_2}{A_2 E_2}$

Parallel Bars:



$\alpha_s > \alpha_{cu}$

$\delta_1 - \frac{R}{A_1} \cdot \frac{L_1}{E_1} = \frac{R}{A_2} \cdot \frac{L_2}{E_2} + \delta_2$ [if $\delta_1 > \delta_2$]

$\boxed{\text{Reacn are same}}$

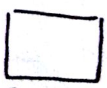
Bending: $\boxed{\frac{M_R}{I_{NA}} = \frac{\sigma_b}{y} = \frac{E}{R}}$ Euler's Bernaulli Bending eqn. $M_R = Z \sigma_b$ ∵ if ↑ Z ∴ ↑ Moment of Resistance to bending

$\sigma_b = \frac{M_R}{Z}$

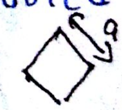
In square side a → Side H-V $I = a^4/12$ $Z = \frac{a^3}{6}$ ∴ if ↑ Z ∴ Then ↓ σ Induced.

→ diag H-V $I = a^4/12$ $Z = \frac{a^3}{6\sqrt{2}}$

* Better secn if more material is at extreme.



is better than

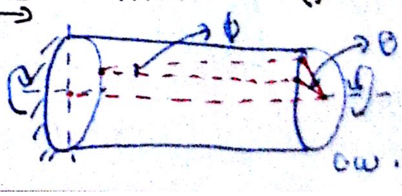


TORSION: θ = ∠. Twist = ∠. by which Radial line Twisted

$\theta \propto l$ $\quad \phi$ = Shear ∠ = " " surface "

$\phi \propto r$

ϕ = S.strain $= \frac{\theta \cdot r}{l} = \frac{\tau}{G}$ $\quad \boxed{\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}}$

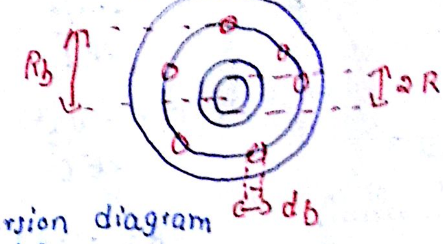


Hollow vs Solid shafts: ① For a given dia (Radial space is constrained) Solid shaft give ↑ power Transmission
② For same wt. ($D_{OH} > D_S$) Then hollow provides more power T.

Torsional stiffness: $S\theta = \frac{T}{\theta} = \frac{GJ}{l}$

Shaft coupling: $T = n_b \cdot \frac{\pi}{4} d_b^2 \cdot \tau_b \cdot R_b$; R_b = Radious of bolt circle (T) ; d_b = dia of bolt ; n_b = no. of bolts ; τ_b = shear stress in bolt

Key: Shearing $T = FR$; F = Res. force in each bolt
$T = \tau_K \cdot l b \cdot R$
crushing
$T = FR = \sigma_{ck} \cdot l \frac{t}{2} \cdot R$

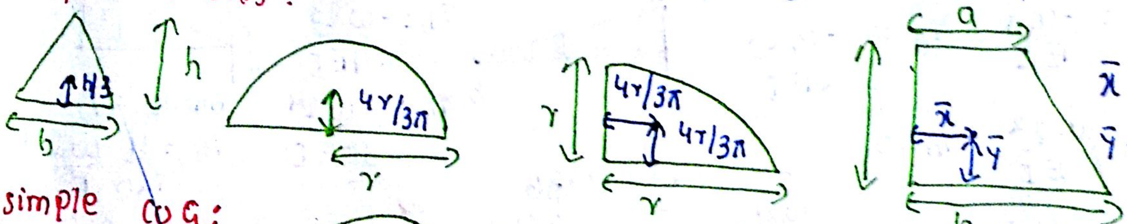


Torsion in series shafts: - similar to previous - Draw Torsion diagram
* find T for each part
1 end fixed $\theta = \theta_1 + \theta_2 + \theta_3$
2 " " $\theta_1 + \theta_2 + \theta_3 = 0$

Parallel: $T = T_1 + T_2$ $\theta_1 = \theta_2$

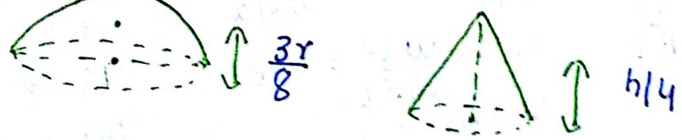
Centre of gravity: pt. thru which whole wt. of body may be assumed to act ; may not lies inside body ⟷ Depends on (i) shape
Centroid: " " " whole Area " figure " " " ; Analogous when bodies hv only Area

Simple centroids:



$\bar{x} = a^2+b^2+ab/3(a+b)$
$\bar{y} = (2a+b)h/3(a+b)$
$A = \frac{(a+b)h}{2}$

simple CoG:

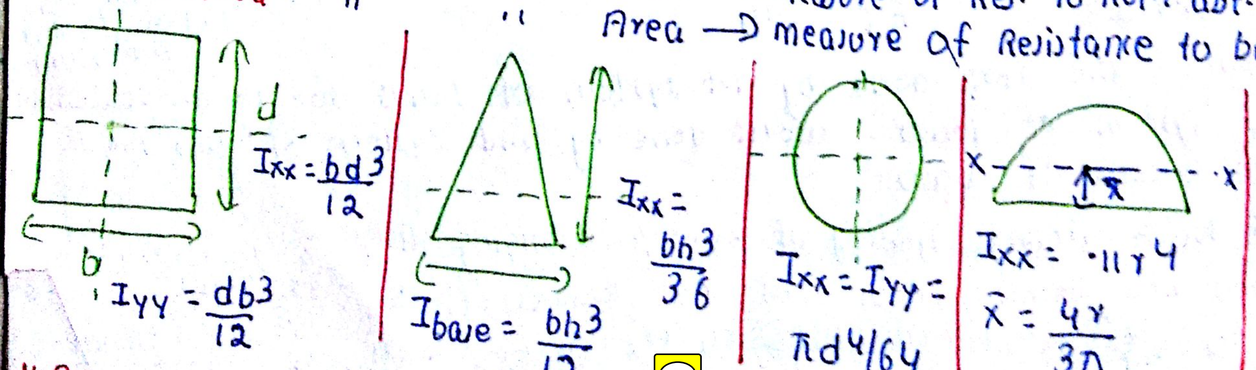


Centroid for composite Areas: $\bar{x} = \frac{\Sigma ax}{\Sigma a}$ $\bar{y} = \frac{\Sigma ay}{\Sigma a}$ # if a hole then treat it as a -ve area & proceed

CoG: $\bar{x} = \frac{\Sigma w \cdot x}{\Sigma w} = \frac{\Sigma v \cdot x}{\Sigma v} = \frac{\Sigma l \cdot x}{\Sigma l}$ $\bar{y} = \frac{\Sigma wy}{\Sigma w} = \frac{\Sigma v \cdot y}{\Sigma v} = \frac{\Sigma l \cdot y}{\Sigma l}$
(if same material) (if same material + same x secn Area)

moment of any qty. about any pt. = product of that qty. with ⊥r distance upto that pt. 2nd moment = if ⊥r distance multiplied 1 more time.
* Always has to be defined wrt a pt. or a line.

MoI < mass - 2nd moment of mass ⟶ measure of Res. to Rotn abt. the pt. about which force
Area - " " Area ⟶ measure of Resistance to bending " "



$I_{xx} = \frac{bd^3}{12}$, $I_{yy} = \frac{db^3}{12}$

$I_{xx} = \frac{bh^3}{36}$, $I_{base} = \frac{bh^3}{12}$

$I_{xx} = I_{yy} = \pi d^4/64$

$I_{xx} = \cdot 11 r^4$, $\bar{x} = \frac{4r}{3\pi}$

Quarter circle $I_{xx} = I_{yy} = \cdot 055 r^4$
Ellipse $I_{xx} = \pi ab^3/4$ $I_{yy} = \pi ba^3/4$
Thin Ring $I_{xx} = I_{yy} = \pi D^3 t/8$ $I_P = \pi D^3 t/8$

II Axes: MoI of a lamina about any Axis in plane of lamina = MoI of lamina about a ∥ centroidal axis in plane of lamina + Area of lamina X square of distance b/w axes

$\perp^r$ Axis: I_{ox}, I_{oy} = MoI of lamina about 2 mutually $\perp^r$ in plane axis
I_{oz} = MoI of lamina abount an axis $\perp^r$ to its plane & passing thru pt. of intersecn of ox & oy

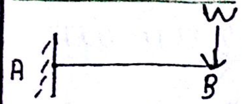
$$\boxed{I_{oz} = I_{ox} + I_{oy}}$$

DEFLECTION: ① Double I method: $M = EI \cdot \frac{d^2y}{dx^2}$ | $\int M dx + c_1 = EI\theta$ | $\iint M dx + c_1 x + c_2 = EIy$

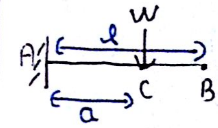
$2 \to 3$
$6 \to 8$
$24 \to 30$

$\theta = -ve$ (c.w.) = Radian
$y = -ve$ (↓)

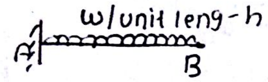
* In all these derivations take x from LHS
* don't consider Reacn.



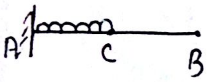
$\theta_b = \frac{-Wl^2}{2EI}$ $\quad y_b = \frac{-Wl^3}{3EI}$



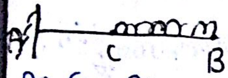
$\theta_c = \theta_b = \frac{-Wa^2}{2EI}$ $\quad y_c = \frac{-Wa^3}{3EI}$ $\quad y_B = y_c + \left(\frac{Wa^2}{2EI}\right)\cdot(l-a)$ (down)



$\theta_b = \frac{-wl^3}{6EI}$ $\quad y_B = \frac{-wl^4}{8EI}$



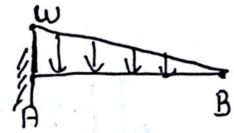
$\theta_c = \theta_b = \frac{-w\cdot a^3}{6EI}$ $\quad y_c = \frac{-wa^4}{8EI}$ $\quad y_B = y_c + \left(\frac{wa^3}{6EI}\right)\cdot(l-a)$



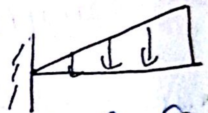
$\theta_b = \frac{-w}{6EI}(l^3 - a^3)$ $\quad y_B = \frac{-w}{24EI}(3l^4 - 4la^3 + a^4)$

Do ③ - ④

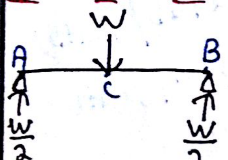
$\theta_b = \frac{-Ml}{EI}$ $\quad y_B = \frac{-Ml^2}{2EI}$



$\theta_b = \frac{-wl^3}{24EI}$ $\quad y_b = \frac{-wl^4}{30EI}$

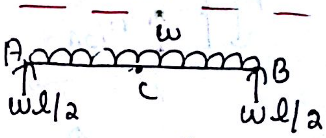


Ans = ③ - ⑦.
$y_B = \frac{11}{120}\frac{wl^4}{EI}$



$\theta_A = \frac{-W\cdot l^2}{16EI}$ $\quad y_A = 0$
$\theta_c = 0$ $\quad y_c = \frac{-W\cdot l^3}{48EI}$
$\theta_B = \frac{Wl^2}{16EI}$ $\quad y_B = 0$

if not middle $y = \frac{W\cdot a^2 b^2}{3EIl}$



$\theta_A = \frac{-wl^3}{24EI} = -\theta_B$
$y_A = y_B = 0$
$\theta_c = 0$ $\quad y_c = \frac{-5}{384}\cdot\frac{wl^4}{EI}$

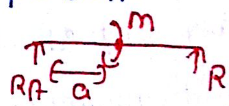
* Take x from left uptill $l/2$.

$16 \to 48$
$24 \to \frac{5}{384}$

② Macaulay's method: -use for multiple loads
- position for max. deflecn asked use
- For SSBs - Find Total deflecn & slope eqn.
- For Respective δ, θ use eqn. only upto that pt.

dummy upward

③ M-A method:



$EIy'' = R_a x \,|\, + M(x-a)^0$

$y = \frac{R_a \cdot x^3}{6} + c_1 x + c_2 \,|\, + \frac{M(x-a)^2}{2}$

~~* while writing eqn. for M consider acc. to cw(+) ACW (-) & not acc. to hogging or sagging.~~

Fixed Beam: $\delta = \frac{Wa^3b^3}{3EIl^3}$ $\quad \delta_s = \frac{w\cdot l^4}{EI}\cdot\frac{1}{384}$.

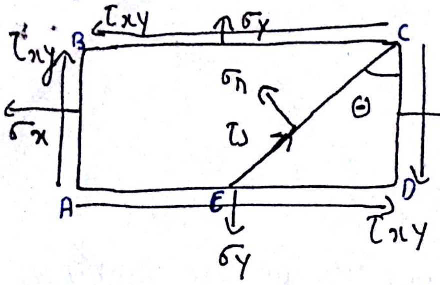
Maxwell's Theorem: The work done by 1st system of loads due to displacement caused by 2nd system of loads = work done by 2nd system of loads due to d. caused by 1st system of loads.

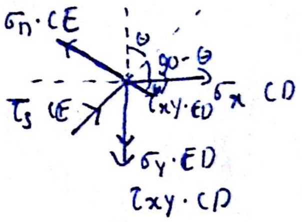
Auto Frettage: To ↑se P. carrying capacity of P.V. w/o changing dimn & material.

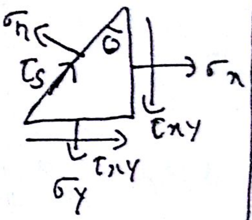
Thin cylindrical shell with hemispherical ends: Cyl. Thickness $= t_1$ $\quad e_{c_1} = \frac{Pd}{4tE}(2-\mu)$
H.S. thickness $= t_2$ $\quad e_{c_2} = \frac{Pd}{4tE}(1-\mu)$ $\quad$ For no distortion $e_{c_1} = e_{c_2}$ $\quad \boxed{t_1/t_2 = 2-\mu/1-\mu}$ $\quad \boxed{\text{H.S. is thinner}}$
- For same max. stress $Pd/2t_1 = Pd/4t_2$ $\quad \boxed{t_2 = .5t_1}$

Why η_l & η_c: Simple formula assumes that structure is seamless solid w/o any Jts. but in actual made by C. & L. Joints.

① **Combined Stresses** - when both normal, shear then to find max. normal and max. shear to be used in designing.





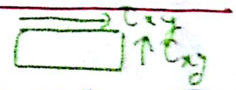


IMP.
θ = c.w. from x-face is +ve
σ_n = if Elongation on plane then +ve
τ = imaginary opp. τ on left side. if gives c.w. Then +ve

$$\sigma_n = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cdot\cos 2\theta + \tau_{xy}\cdot\sin 2\theta$$

* on complimentary plane i.e. $\theta = 90+\theta$

$$\tau_s = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cdot\cos 2\theta$$

①.  Then use $\tau_{xy} = -ve$

- $\sigma_n + \sigma_n' = \sigma_x + \sigma_y = \sigma_1 + \sigma_2$ ↳ use this to find σ_n if another σ_n found.
- $\tau_s' = -\tau_s$

principal planes: - where τ_s 0 - where only σ_n (either max. or min.)
- These planes 90° apart.
- $\tan 2\theta = \dfrac{2\cdot\tau_{xy}}{\sigma_x-\sigma_y}$ (gives θ_1, θ_2) - find $(\sigma_n)_{\theta_1}, (\sigma_n)_{\theta_2}$ more mag. will be σ_1.
- Directly magnitude $\sigma_{1,2} = \dfrac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$
  ↳ does not give location.

max. shear stress planes: will also have normal stress. $d\tau_s/d\theta = 0$

$\tan 2\theta = \dfrac{\sigma_y-\sigma_x}{2\cdot\tau_{xy}}$ (θ_3, θ_4)

* In τ_{max} Problem $\tau_{max} = \dfrac{\sigma_1-\sigma_2}{2}$
* In design " $\tau_{max}\left\{\left|\dfrac{\sigma_1}{2}\right|, \left|\dfrac{\sigma_1-\sigma_2}{2}\right|\right\}$ max. of these two

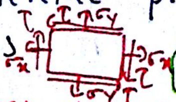
i) CASE- if only shear (Pure Torsion): $\sigma_1 = \tau$ (45°) $\sigma_2 = -\tau$ (135°) $\tau_{max} = \tau$ (0, 90°)
Ductile shaft (ms) } failure ∵ τ_{max} at 0°
Brittle " (CI) } failure ∵ σ_{max} at 45°

- when a pt. is subjected to only normal stresses $(\sigma_x, \sigma_y, \sigma_z)$ Then these become the P. stresses & x, y, z faces become P. Planes

ii) CASE- pure Tension: $\sigma_1 = \sigma$ (0°) $\sigma_2 = 0$ $\tau_{max} = \sigma$ (45°)
iii) ↳ Reverse in failure

② **MOHRS CIRCLE**: ① τ ↑ → σ ② mark 2 pts. < A stress condn of x-face (σ, τ); B " " y-" $(\sigma, -\tau)$
③ Join them and draw circle with AB as diameter
④ Principal planes - where circle cuts x-axis < higher mag. σ_1; lower " σ_2
⑤ τ_{max} planes - where max. y mag. [Radii ∥ to y-axis]
⑥ pure shear " - where circle cuts y-axis
⑦ Any pt. on circle with (σ_n, τ_s) ∠ of obliquity $\tan\phi = \tau_s/\sigma_n$
⑧ Every plane is Represented by a Radius and ∠s. = $\frac{1}{2}$ ∠. by this radius wrt reference planes.

To find θ: Always find ∠ wrt rad. ius of x-face i.e. line Joining centre of circle with the pt. representing condn of x-face. Find ∠ in c.w. for θ. Then divide by $2\theta(\theta)$.

* 7 cases  A = σ_x, τ; $\boxed{B = \sigma_x, -\tau}$ → $\epsilon_{1,2} = \frac{1}{2}\left\{(\epsilon_x+\epsilon_y) \pm \sqrt{(\epsilon_x-\epsilon_y)^2 + 4\left(\frac{\gamma_{xy}}{2}\right)^2}\right\}$

③ **principal strains** (ϵ_1, ϵ_2) and max. shear strain (γ_{max}):

* $\epsilon_n = \left(\dfrac{\epsilon_x+\epsilon_y}{2}\right) + \left(\dfrac{\epsilon_x-\epsilon_y}{2}\right)\cos 2\theta + \dfrac{\gamma_{xy}}{2}\cdot\sin 2\theta$ * $\dfrac{\gamma_s}{2} = -\left(\dfrac{\epsilon_x-\epsilon_y}{2}\right)\sin 2\theta + \dfrac{\gamma_{xy}}{2}\cdot\cos 2\theta$

* $\epsilon_n + \epsilon_n' = \epsilon_x + \epsilon_y = \epsilon_1 + \epsilon_2$ * $\gamma_s + \gamma_s' = 0$ * Principal Planes $\tan 2\theta = \dfrac{\gamma_{xy}}{\epsilon_x-\epsilon_y}$.

Strain Rosettes: 1- Rectangular $\varepsilon_{0^\circ} = \varepsilon_x$ ε_{45° $\varepsilon_{90^\circ} = \varepsilon_y$.
2- Delta Δ (0, 60, 120) 3- Star λ (0, 120, 240)
mohrs circle for strain $\frac{\gamma}{2}$ L ε

④ Principal stress ($\sigma_1\ \sigma_2\ \sigma_3$) and P. strains ($\varepsilon_1\ \varepsilon_2\ \varepsilon_3$):

$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \mu(\sigma_2+\sigma_3)\right]$ $\quad \varepsilon_1 \neq \frac{\sigma_1}{E}$ $\quad \sigma_1 = \frac{E}{1-\mu^2}(\varepsilon_1 + \mu\varepsilon_2)$

⑤ Theories of failure: used b/c failure stress not known for combined stresses.

1- MPST, MNST, Rankine: $\sigma_1 \le \frac{S_{yt}}{N}$ - use it for brittle - Not for ductile b/c shear not considered

2- MSST, Guest & Tresca: max. of $\left(\left|\frac{\sigma_1-\sigma_2}{2}\right|, \left|\frac{\sigma_1}{2}\right|\right) \le \frac{S_{ys}}{N}$ or $\frac{S_{yt}}{2N}$ - good for Ductile

3- M. pri. strain T, St. venant: $\sigma_1 - \mu\sigma_2 \le \frac{S_{yt}}{N}$

4- Total Strain Energy T, HAIGH: Total S.E./vol $\le$ (S.E./vol) yp

$\underbrace{\frac{1}{2}\sigma_1\varepsilon_1 + \frac{1}{2}\sigma_2\varepsilon_2 + \frac{1}{2}\sigma_3\varepsilon_3}_{\text{Total S.E./vol}}$ $\quad \sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 \le \left(\frac{S_{yt}}{N}\right)^2$

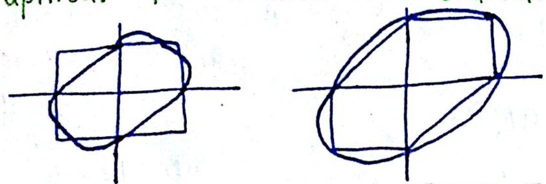
5- MDET, von-mises: MAX. Shear Strain E. Theory: $\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \le \left(\frac{S_{yt}}{N}\right)^2$

Total S.E./vol = Volumetric S.E./vol + D.E./vol
Vol. S.E./vol $= \frac{(\sigma_1+\sigma_2+\sigma_3)^2}{3}\frac{(1-2\mu)}{2E}$

* Reln b/w S_{ys}, S_{yt} for pure shear: [use $\tau = S_{ys}$]

$\frac{S_{ys}}{S_{yt}} = 1, .5, .77, .62, .577$

* graphical for ductile in Biaxial:



Safe within the figure.

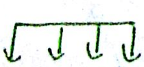
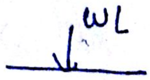
⑥ support in beams:
→ FIXED 3 motion Restrict ∴ 3 Reacn R_V, R_H, M
→ Hinge 2 " " ∴ 2 " R_V, R_H
→ Roller 1 " " ∴ 1 " R_V

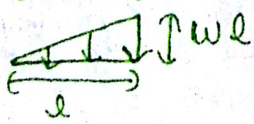
Fixed: At End Pt. moment exists.
* SSB supports hv 0 M ∴ balance about it

* For Equillibrium
$\Sigma H = \Sigma V = \Sigma M = 0$.

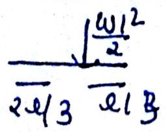
* Reacn always ⊥r to plane supporting the Roller.

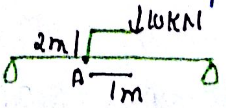
To find Reacn: 1- cantilever ∴ introduce opp. net moment at fixed End as Reacn
2- SSB: ΣM about hinge or Roller support = 0

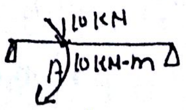
*  w N/m Put wl at centroid as pt. load 

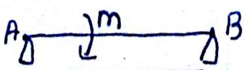
* 

Total wt. = $\frac{wl \cdot l}{2}$ put as pt. load at centroid

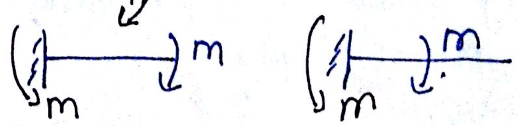


* 



* if only moment acting 

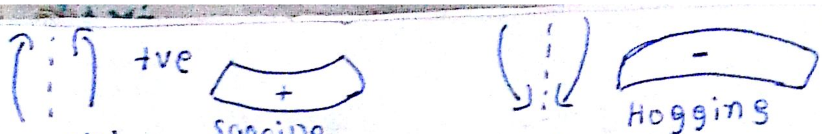
$R_A = \frac{-m}{l}$ $\quad R_B = \frac{m}{L}$



⑦ Shear force - At a given secn it is the algebraic sum of all vertical forces acting either on LHS or RHS of secn.

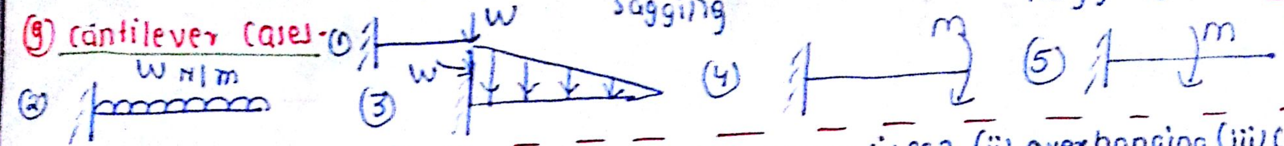
if from RHS(↓), LHS(↑) +ve ↑ ↓ +ve (if causes c.w. moment)

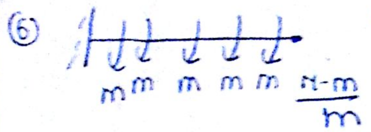
⑧ **Bending moment:**



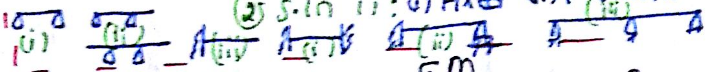
LHS
C.w. – +ve
A.c.w. – −ve
RHS
C.w. – −ve
A.c.w. – +ve

⑨ **Cantilever cases** –

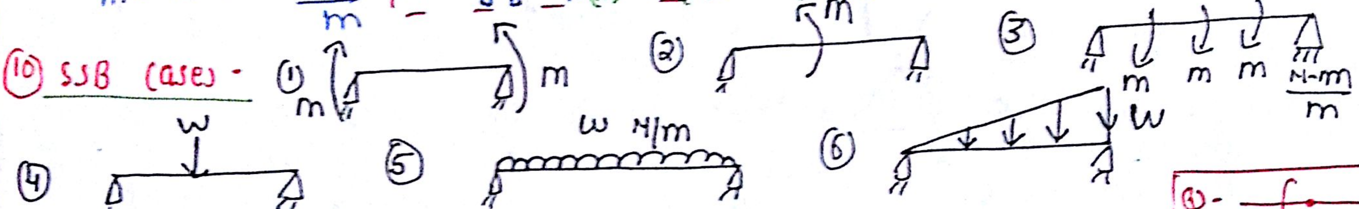




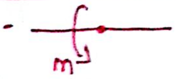
Types of Beams: ① S. Determinate: (i) SSB (ii) overhanging (iii) cantilever
② S.In " : (i) Fixed (ii) P. cantilever (iii) continuous



⑩ **SSB cases** –



* Always at fixed supports – SF, BM ≠ 0
" " S S supports – SF ≠ 0 but BM = 0
For distributed loads – order of SF = 1 more than load
" " BM = 1 " " SF



on drawing BMD from RHS consider it as → m ∴ +ve

when SF changes its sign i.e. SF = 0, BM becomes maximum
" BM " " i.e. BM = 0 Pt. of contraflexure
Area of loading diag. b/w 2 pts. = Difference of SF b/w those pts.
" " SF " " = " " BM " "

⑪ **Deflection:** $\boxed{M = EI\frac{d^2y}{dx^2}}$ – based only on bending eqn. and effect of shear force neglected.

$x \to$ +ve $y \uparrow$ +ve   M (+ve) (−ve)   Slope −ve if C.w. Rotn

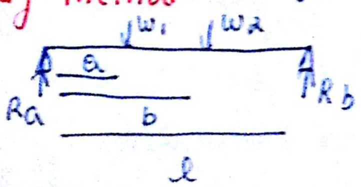
(i) Double Integration method – suitable for single load
(ii) Moment Area " – " " " , simple BMD
(iii) Macaulays " – " " >1 "

Boundary condn: SS end (y = 0, M = 0) fixed end (y = 0, θ = 0) free end (M = 0; y, θ = max.)

I DI method – $\boxed{EI \cdot y = \iint M \cdot dx + C_1 x + C_2}$ – use boundary conditions

II Macaulay method – Though multiple loads but only 1 eqn. is constructed

E.g.



$M_{x-x} = R_a \cdot x \;|\; - w_1(x-a) \;|\; - w_2(x-b)$

$EIy = \frac{R_a x^3}{6} + C_1 x + C_2 \;|\; - \frac{w_1(x-a)^3}{6} \;|\; - \frac{w_2(x-b)^3}{6}$

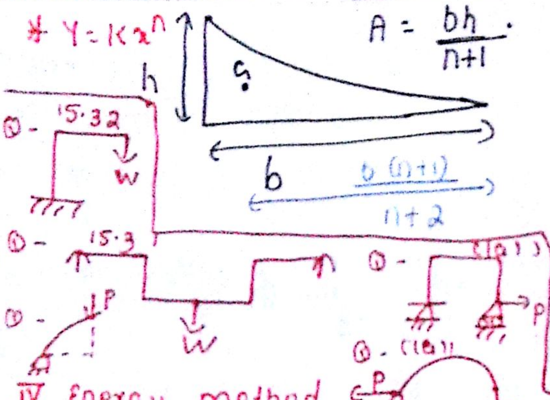
* while applying boundary condn use the expression which is in range of x.

III M-Area method – $\theta_b - \theta_c = \frac{1}{EI}$ [Area of BMD b/w b and c]

- $y_b - y_c = \frac{1}{EI}$ (Moment of area of BMD b/w b and c) $= \frac{A\bar{x}}{EI}$

where $\bar{x}$ = distance of centroid of area from b.

* $Y = kx^n$



$A = \frac{bh}{n+1}$

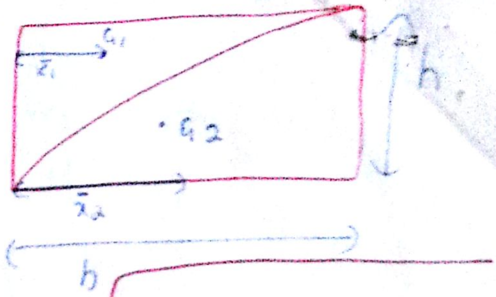
* $Y = kx^n$

$A_1 = \frac{bh}{(n+1)}$

$A_2 = \frac{bh \cdot n}{n+1}$

$\bar{x}_1 = \frac{b}{n+2}$

$\bar{x}_2 = \left(\frac{b}{n+2}\right)\left(\frac{n+3}{n}\right)$



IV Energy method

use for (i) Y, θ
(ii) if shearing, bending
(iii) & Total S.E. known

$$U = \int_a^b \frac{(M_{x-x})^2 \cdot dx}{2EI} \qquad Y = \frac{\partial U}{\partial W} \qquad \theta = \frac{\partial U}{\partial m}$$

- where W, m are load and moment at pt. where Y, θ found.
- if not then add dummy and Remove from final expression.
- Treat all loads, moments as variables & partially differentiate.

(12) pressure vessels:

σ_l in thickness → Tensile
σ_r " " in radial direc^n (C)
σ_h " " along circumference (T)

D/t ≥ 20 Thick   D/t < 20 Thin

Thin vessels.

σ_l → C. failure
σ_h → L. failure

$\sigma_h = \frac{Pd}{2t}$ (Internal dia.) $\sigma_l = \frac{Pd}{4t}$

∵ σ_r very small ∴ Biaxial Problem
$= -P =$ comp.

Thin - Gas cylinder, P. Cooker

$\sigma_1 = \sigma_h \quad \sigma_2 = \sigma_l \quad \tau_{max} = \frac{\sigma_h - \sigma_l}{2} = \frac{Pd}{8t}$

* if efficiencies: $\sigma_l = \frac{Pd}{4t \cdot \eta_c}$ $\sigma_c = \frac{Pd}{2t \cdot \eta_l}$

* if cyl P.V. closed at 1 end & open at other → $\sigma_l = 0$

$\epsilon_1 = \epsilon_h = \frac{\sigma_1 - \mu\sigma_2}{E} = \frac{\delta D}{D}$ ‖ $\epsilon_2 = \epsilon_l = \frac{\sigma_2 - \mu\sigma_1}{E} = \frac{\delta l}{l}$ ‖ $\epsilon_v = 2\cdot\epsilon_h + \epsilon_l = \frac{dv}{V} = \frac{PD}{4tE}(5-4\mu)$

* MPST & MSST same.

Thick - Hydraulic cylinder, Gun barrel

* supply of extra fluid : (i) $\epsilon_v = \frac{dv_1}{V}$ (size of vessel ↑ ses)
(ii) $\frac{dv_2}{V} = \frac{P}{K}$ (volume of fluid ↓ ses)

* incompressible - $\Delta v = dv_1$
* compressible - $\Delta v = dv_1 + dv_2$

both dv_1 + dv_2 has to be supplied

→ Adv. → Sur. less stress

* Thin spherical: $\sigma_h = \sigma_l = \frac{PD}{4t}$ $\sigma_1 = \sigma_2 = \frac{PD}{4t}$ $\tau_{max} = \frac{\sigma_1}{2} = \frac{\sigma_2}{2} = \frac{PD}{8t}$

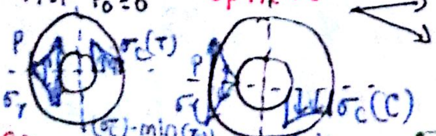
$\epsilon_h = \frac{\sigma_1 - \mu\sigma_1}{E} = \frac{PD}{4t}(1-\mu) = \frac{\delta D}{D}$ $e_v = 3 \cdot e_h = \frac{3}{4}\cdot\frac{PD}{tE}(1-\mu)$

Thin: Stress at a pt. on Shell

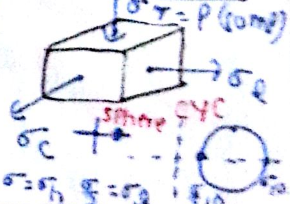
Thick → cylinder, sphere
$P_i = P$, $P_o = 0$

Cylinder: $P_r = b/r^2 - a$, $\sigma_c = b/r^2 + a$, $\sigma_r = -P$

Sphere: $P_r = 2b/r^3 - a$, $\sigma_c = 2b/r^3 + a$, $\sigma_r = -P$



Practice Q. cyl with closed ends
E.g. 10.12:
∵ $\sigma_r = -P$
$\sigma_c = \frac{b}{r^2} + a$ (T) $\sigma_r = \frac{b}{r^2} - a$ (comp)
$\sigma_l = \frac{r_1^2 \cdot P}{r_2^2 - r_1^2}$ (T) $\tau_{max} = \frac{\sigma_c - \sigma_r}{2} = \frac{b}{r^2}$



(13) coloums & struts:

* Slenderness Ratio $S = \frac{l_e}{K}$
$\sigma_e = \pi^2 \cdot E / S^2$

$P_{Euler} = \frac{\pi^2 \cdot E I_{min}}{l_e^2}$ For long coloumn

Both Ends hinged	→ l	1
1 fixed, 1 hinged	→ $l/\sqrt{2}$	.77
Both fixed	→ $l/2$	.5
1 fixed, 1 free	→ $2l$	2

$\frac{1}{P_{Ran}} = \frac{1}{P_E} + \frac{1}{P_c}$ $P_c = \sigma_c \cdot A$

→ using $I = AK^2$ → Rankine

$P_{Ran} = \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c}{\pi^2 \cdot E} \cdot \frac{l_e^2}{K^2}} = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l_e^2}{K^2}\right)}$ $a = \frac{\sigma_c}{\pi^2 \cdot E}$

Johnson's

$P = \sigma_c A \cdot \left[1 - a\left(\frac{l_e}{K}\right)^2\right]$

$\frac{P}{A} = \sigma_c - \sigma_c \cdot a \cdot \left(\frac{l_e}{K}\right)^2 = \sigma_c - b\cdot\left(\frac{l_e}{K}\right)^2$ where $b = \sigma_c \cdot a = \frac{\sigma_c^2}{\pi^2 \cdot E}$

Pressure vessels Problems : Eg. 10·17, 10·18

$$\sigma_c = \frac{Pd}{2t} \qquad \sigma_l = \frac{W}{\pi d t} = \frac{Pd}{4t}$$

or

$$P \cdot \frac{\pi}{4} d^2 + \sigma_l \cdot \pi d t = F$$