

## Linear Equation and Inequalities in two variables

### 4.01. Introduction

To solve any problem it is denoted in mathematical form. We know that problem is based on one or more factors. In previous class we have solved such problems by denoting in equation form. According to problem these equations are based on one variable or two or more variables. If any equation represents a straight line then it is called linear equation. In general form  $ax + b = 0, a \neq 0$  where  $a, b$  are real numbers, represents a linear equation in one variable. The value of  $x$  which satisfies this equation is called solution of the equation. Graph of linear equation in one variable is a line parallel to any axis. In class IX we have draw graph of linear equations of two variables  $ax + by + c = 0; a, b \neq 0$ . The values of  $x$  and  $y$  for which equation satisfies, are called its solutions. Graph of linear equation in two variables is also a straight line. Each point  $(x, y)$  on this straight line express solution of this equation. Here, we consider two linear equations in two variables  $x, y$

$$a_1x + b_1y + c_1 = 0 \quad \dots (1)$$

$$a_2x + b_2y + c_2 = 0, \quad \dots (2)$$

Where,  $a_1, b_1, c_1, a_2, b_2, c_2$  all are real numbers and  $a_1, b_1$  and  $a_2, b_2$  are not zero. It means for equations (1) and (2)

$$a_1^2 + b_1^2 \neq 0 \quad \text{and} \quad a_2^2 + b_2^2 \neq 0$$

These type of equations are called pair of linear equations in two variables. This pair of equations is called simultaneous linear equations in two variables. Here we will discuss in details of the solution of these pair of equation and its consistency and inconsistency.

In this chapter we will also discuss about inequalities. If in linear equations, we replace the  $=$  sign by  $<$  or  $>$  or  $\leq$  or  $\geq$  (sign of inequations) then these equations are called inequations. Here, we will solve the inequations in two variables by graph method. This solution in the form of area. Solution of set of simultaneous linear inequations in two variables can be obtained in the form of common area.

### 4.02. Simultaneous Linear Equation of Two Variables

Pair of linear equations of two variables is called simultaneous linear equation system. For example  $5x + 2y = 17; 2x - 5y = 1$  or  $x + 2y = 3; 2x - y = 5$  etc.

Solving of pair of linear equation means to find values of two variables which satisfies both the equations. Solving of pair of linear equation can be understand by the following examples

**Example 1.** Simultaneous linear equations are

$$3x + 2y - 5 = 0; \quad 4x + 7y - 11 = 0$$

To find the nature of solution for  $x = 1$  and  $y = 1$

By Putting  $x = 1$  and  $y = 1$  in both equations, we get

$$3(1) + 2(1) - 5 = 0; \quad 4(1) + 7(1) - 11 = 0$$

then at  $x = 1$  and  $y = 1$  is required solution of pair of linear equations.

**Example 2.**  $2x + 7y = 11$ ;  $x - 3y = 5$  are simultaneous linear pair of equations. to find the nature of solutions for  $x = 2$  and  $y = 1$ .

Putting  $x = 2$  and  $y = 1$  in above two equations, we get

$$2(2) + 7(1) = 11$$

i.e, first equation satisfies

For second equation put the value of  $x$  and  $y$ .

$$2 - 3(1) = -1 \neq 5$$

So this equation does not satisfy at  $x = 2, y = 1$

i.e,  $x = 2, y = 1$  is not a solution of given pair of linear equations.

**Example 3.** To find the nature of solution of linear pair  $x + 2y - 5 = 0$ ;  $2x + 4y - 10 = 0$  for  $x = 1, y = 2$  and  $x = 3, y = 1$ .

**Case I :** at  $x = 1, y = 2$  simultaneous equations are

$$1 + 2(2) - 5 = 0$$

and

$$2(1) + 4(2) - 10 = 0$$

equations are satisfied.

**Case II :** at  $x = 3, y = 1$  simultaneous equations are

$$3 + 2(1) - 5 = 0$$

and

$$2(3) + 4(1) - 10 = 0$$

So, in this case also, equations are satisfied

i.e,  $x = 1$  and  $y = 2$  and  $x = 3$  and  $y = 1$  both are solutions of given pair of linear equations.

Similarly many other solutions for these equations are possible.

Here, from above examples it is clear that simultaneous linear equation has unique solutions or many solution or no solution. If any simultaneous linear pair of equations can be solve, either unique or more than one solution, then this type of linear equations are called consistent, and if it has no solution then these pair are called inconsistent. When we draw a linear equation of two variables on graph, a straight line is obtained in a plane, their position can be as follows :

- (i) Two lines intersect at a point
- (ii) Two lines do not intersect or they are parallel.
- (iii) Two lines coincide each other.

The linear equation of the lines are consistent or inconsistent. These cases can be understood by graphical representation.

### 4.03. Graphical Representation of Linear Equation and their Solution

Here, for the following examples we will find the nature of solution by graphical representation of linear pair of equations.

for example, represent the following linear pairs of equations graphically

(i)  $2x + 3y = 13$ ;  $5x - 2y = 4$

(ii)  $2x + 4y = 10$ ;  $3x + 6y = 12$

(iii)  $4x + 6y = 18$ ;  $2x + 3y = 9$

**Example 4.** Equations

$$2x + 3y = 13 \quad \dots (1)$$

$$5x - 2y = 4 \quad \dots (2)$$

Let us represent these equations graphically. For this, we need at least two solutions for each equation. First we try to get a point at  $x = 0$  or  $y = 0$ . So, these equations reduces to linear equations, so second variable can be find easily.

When we put  $x = 0$  or  $y = 0$  in equation (1), we do not get value of another variable in integer. So, we put another values,

Putting  $x = 2$  in equation (1).

$$2 \times 2 + 3y = 13 \quad \text{or} \quad 3y = 13 - 4 = 9 \quad \text{or} \quad y = 3$$

and at  $x = 5$

$$2 \times 5 + 3y = 13 \quad \text{or} \quad 3y = 13 - 10 = 3 \quad \text{or} \quad y = 1$$

Thus, point are obtained according to the following table

$x$	2	5
$y$	3	1

Similarly in equation (2),  $x = 0$

$$5 \times 0 - 2y = 4 \quad y = -2$$

and at  $x = 2$

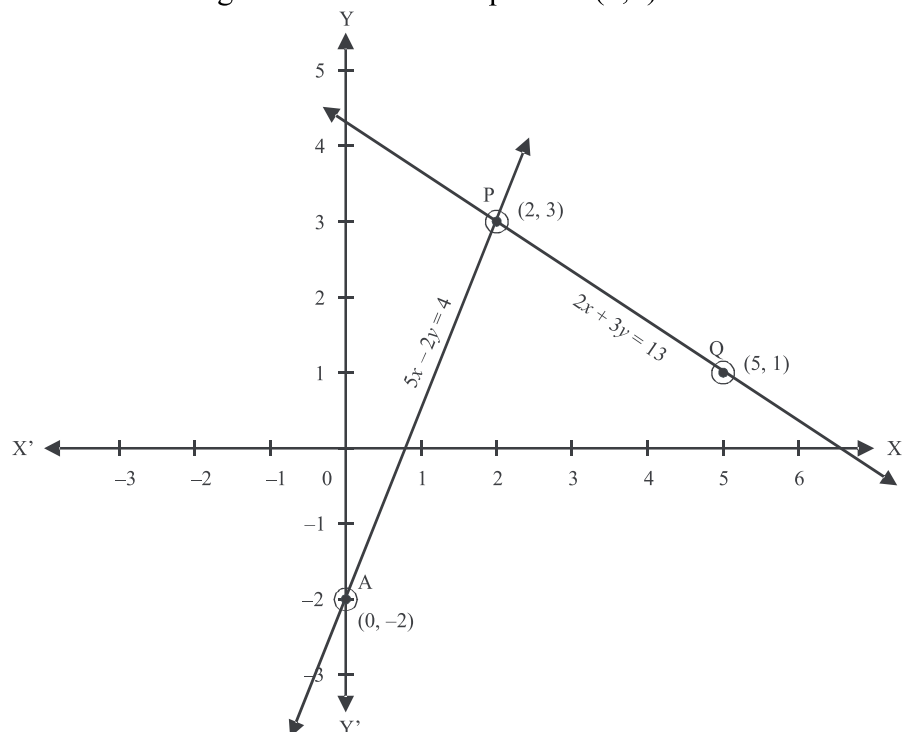
$$5 \times 2 - 2y = 4 \quad -2y = -6 \quad y = 3$$

Thus, for equation (2), points are obtained as follows :

$x$	0	2
$y$	-2	3

Now plot these points on graph paper and obtained following straight lines *i.e.*, draw  $XOX'$  and  $YOY'$  axis on graph paper join plotted points and obtained straight line.

In figure, we see that straight lines intersect at point  $P(2,3)$



**Fig.4.1**

**Example 5.** Pair of linear equations are

$$2x + 4y = 10 \quad \dots (1)$$

$$3x + 6y = 12 \quad \dots (2)$$

For equivalent graphical representation, we get points as follows :

Put  $y = 0$  in equation (1)

$$2x + 4 \times 0 = 10$$

or

$$x = 5$$

and put  $x = 1$  in equation (1)

$$2 \times 1 + 4y = 10$$

or

$$4y = 10 - 2 = 8$$

or

$$y = 2$$

For equation (1), points are obtained as follows

$x$	5	1
$y$	0	2

Similarly in equation (2), put  $x = 0$

$$3 \times 0 + 6y = 12$$

or

$$6y = 12$$

or

$$y = 2$$

and put

$$y = 0$$

$$3x + 6 \times 0 = 12$$

or

$$3x = 12$$

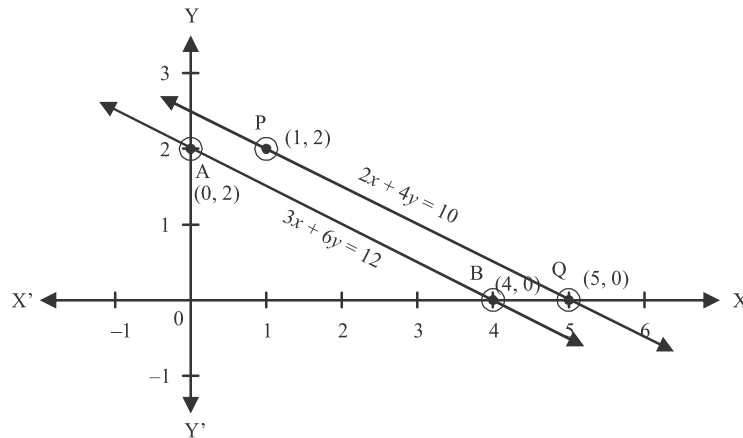
or

$$x = 4$$

Thus for equation (2) obtained as follows points :

$x$	5	1
$y$	0	2

Now plot all the points on graph paper and join them, we obtained following graph. In above figure, we see that two straight lines are parallel to each other.



**Fig. 4.2**

**Example 6.** Pair of linear equations are

$$4x + 6y = 18 \quad \dots (1)$$

$$2x + 3y = 9 \quad \dots (2)$$

From equation (1) and (2) points are obtained and plot the lines on graph.

Put  $x = 0$  in equation (1)

$$4 \times 0 + 6y = 18$$

or  $6y = 18$

or  $y = 3$

put  $y = 1$

$$4x + 6 \times 1 = 18$$

or  $4x = 18 - 6 = 12$

or  $x = 3$

Thus, following points are obtained as follows :

$x$	0	3
$y$	3	1

For equation (2), put  $x = 0$

$$2 \times 0 + 3y = 9$$

or  $3y = 9$

or  $y = 3$

and put  $y = 1$

or  $2x + 3 \times 1 = 9$

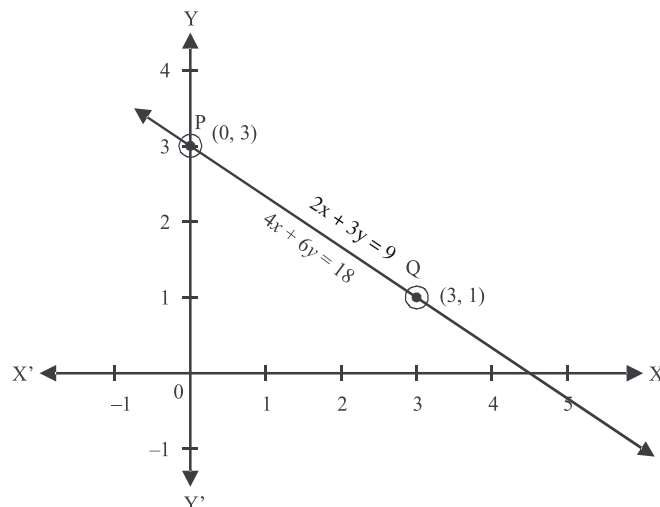
or  $2x = 9 - 3 = 6$

or  $x = 3$

Similarly for equation (2) following points are obtained.

$x$	0	3
$y$	3	1

Plot all the points on graph paper and join them.



**Fig. 4.3**

In the above graph fig 4.3 two straight lines coincide each other. It is clear that two equation represents equal lines.

*i.e.* , equations are equivalent.

In the above examples 4,5,6 equations can be written in general form as :

$$a_1x + b_1y + c_1 = 0 \quad \dots (1)$$

and  $a_2x + b_2y + c_2 = 0 \quad \dots (2)$

Here, we prepare comparative table for coefficients of  $x$ ,  $y$  and contents.

**Comparitive Table**

Example No.	Pair of linear Equations	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the the Ratio of Coefficients	Nature of lines	Algebraic Interpretation
(i)	$2x + 3y = 13$ $5x - 2y = 4$	$\frac{2}{5}$	$\frac{3}{-2}$	$\frac{13}{4}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Unique Solution
(ii)	$2x + 4y = 10$ $3x + 6y = 12$	$\frac{2}{3}$	$\frac{4}{6}$	$\frac{10}{12}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No Solution
(iii)	$4x + 6y = 18$ $2x + 3y = 9$	$\frac{4}{2}$	$\frac{6}{3}$	$\frac{18}{9}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinity many Solution

From above table, it is clear that linear pair

$$a_1x + b_1y + c = 0$$

$$a_2x + b_2y + c = 0$$

(i) Intersecting, then relation coefficients is

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Such linear pair has unique solution and intersection point  $(x, y)$  will be required solution.

(ii) Parallel, then relation between coefficients is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Such linear pair has no solution.

(iii) Coincident, then relation between coefficients is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Such linear pair has infinitely many solutions *i.e.*, for each point value  $x, y$  will its solutions.

Conversely, if relation in coefficients are given then we can know nature of linear pair of equations represented. Linear equations can be solve graphically by following steps.

### Graphical Method :

**Step 1 :** Write given linear equations in standard form

$$i.e., \quad a_1x + b_1y + c_1 = 0 \quad \dots (1)$$

$$a_2x + b_2y + c_2 = 0 \quad \dots (2)$$

**Step 2 :** Prepare table of corresponding points of two equations.

Draw XOY' and YOY' on graph paper and then plot these points. Let lines  $L_1$  and  $L_2$  corresponds to equations (1) and (2).

**Step 3 :** If lines  $L_1$  and  $L_2$  intersect at point  $(\alpha, \beta)$  then  $x = \alpha$  and  $y = \beta$  will be solution of given linear pair of equations.

**Step 4 :** If lines  $L_1$  and  $L_2$  are parallel then there will be no solutions *i.e.*, linear pair equations will be inconsistent.

**Step 5 :** If lines  $L_1$  and  $L_2$  are consistent then there will be infinite solutions *i.e.*, two lines may be expressed as same line and each point  $(\alpha, \beta)$  of this line will be obtained in the form of many solutions  $(x = \alpha, y = \beta)$  of linear pair of equations.

Here, we can understand more from the following examples :

**Example 7.** Solve the following pairs of linear equations, graphically

$$(i) \quad 3x + 2y - 11 = 0$$

$$2x - 3y + 10 = 0$$

$$(ii) \quad 2x + 3y = 8$$

$$x - 2y = -3$$

$$(iii) \quad 2x + y - 6 = 0$$

$$4x - 2y - 4 = 0$$

**Sol :** (i) Given equation can be written as

$$3x + 2y = 11 \quad \dots (i)$$

$$2x - 3y = -10 \quad \dots (ii)$$

Obtain points table from equation (1)

$$\text{at} \quad y = 1$$

$$3x + 2 \times 1 = 11$$

$$\text{or} \quad 3x = 11 - 2 = 9$$

$$\text{or} \quad x = 3$$

Similarly at  $x = 1$  in equation (1)

$$3 \times 1 + 2y = 11$$

or  $2y = 11 - 3 = 8$

or  $y = 4$

Thus point table of equation (1) is as follows

$x$	3	1
$y$	1	4

Now prepare point table of equation (2) is as follows

at  $y = 0$ ,

$$2x - 3 \times 0 = -10$$

or  $x = -5$

Similarly at  $y = 2$ ,  $2x - 3 \times 2 = -10$

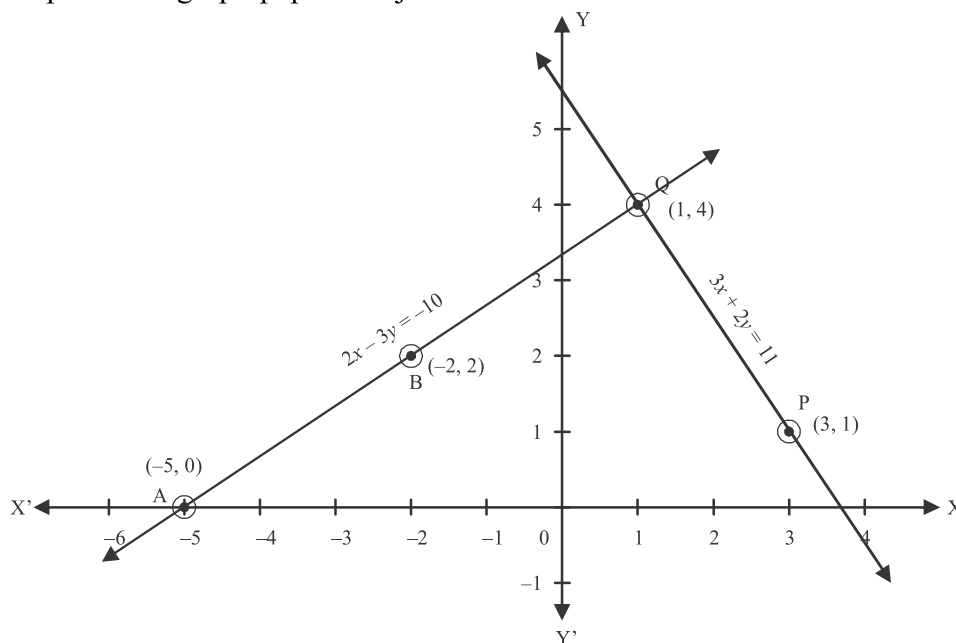
or  $2x = -4$

or  $x = -2$

So, points tables of equation (2) is as follows

$x$	-5	-2
$y$	0	2

Plot all the points on graph paper and join them



**Fig. 4.4**

From above representation, it is clear that two lines intersect each other at  $(1, 4)$  so,  $x = 1$  and  $y = 4$  is required solution of given linear pair *i.e.*,  $x = 1, y = 4$  satisfy both the equations. So, solution is verified.

**Sol :** (ii) Given pair of equation is

$$2x + 3y = 8 \quad \dots (1)$$

$$x - 2y = -3 \quad \dots (2)$$



Obtain the point table for equation (1)

at  $x = 1$ ,  $2 \times 1 + 3y = 8$

or  $3y = 8 - 2 = 6$

or  $y = 2$

Similarly  $y = 0$ ,  $2x + 3 \times 0 = 8$

or  $x = 4$

So, following table is obtained for equation (1)

$x$	1	4
$y$	2	0

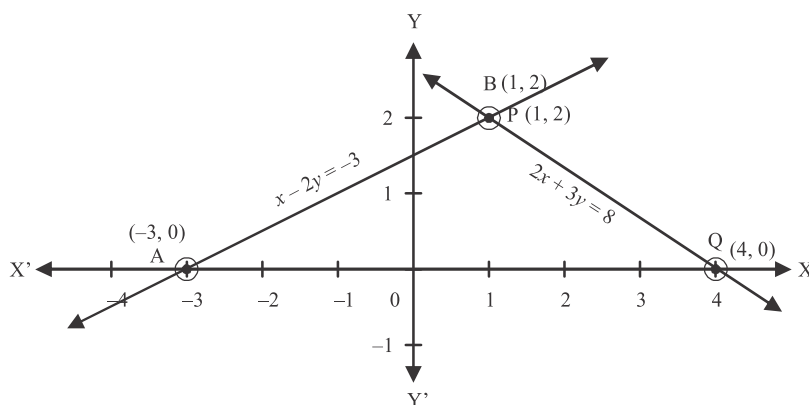
Now, we obtained point table for equation (2).

In equation (2) at  $y = 0$ ,  $x - 2 \times 0 = -3$   
 $x = -3$

Similarly at  $x = 1$ ,  $1 - 2y = -3$   
 $-2y = -4$   
 $y = 2$

Similarly following table is obtained for equation (2)

$x$	-3	1
$y$	0	2



**Fig. 4.5**

With the help of above tables plot the points on graph paper and join them.

From above graphical representation it is clear that two lines intersect each other at point (1,2). So  $x = 1$ ,  $y = 2$  is required solution of given linear pair.  $x = 1$  and  $y = 2$  satisfy both the equations.

**Sol :** (iii) Writing given equations as

$$2x + y = 6 \quad \dots (1)$$

$$4x - 2y = 4 \quad \dots (2)$$

Obtain point table for equation (1)

at  $x = 0$ ,  $2 \times 0 + y = 6$

or  $y = 6$

and at  $x = 1$ ,  $2 \times 1 + y = 6$

or

$$y = 6 - 2 = 4$$

So, following table is obtained for equation (1)

$x$	0	1
$y$	6	4

Now obtain point table for equation (2).

In equation (2) at  $x = 0$

$$4 \times 0 - 2y = 4$$

$$y = -2$$

and  $y = 0$

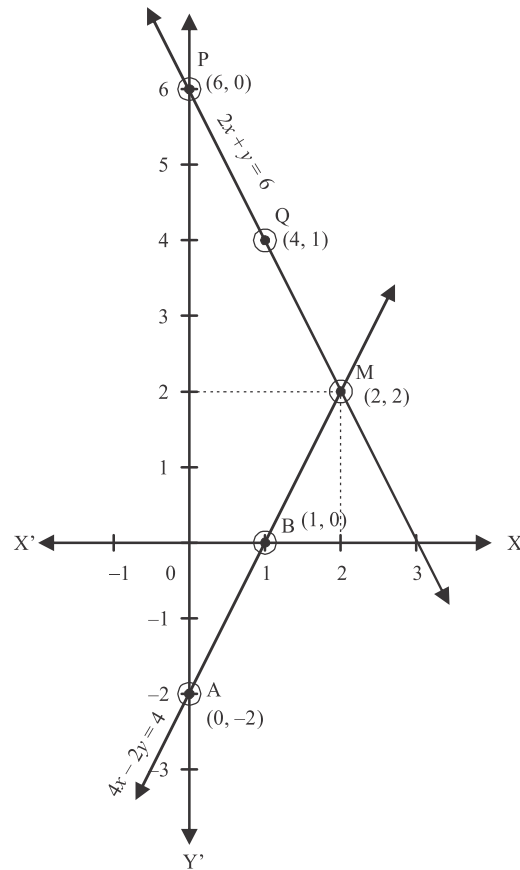
$$4x - 2 \times 0 = 4$$

$$x = 1$$

Following table is obtained for equation (2)

$x$	0	1
$y$	-2	0

With the help of above tables linear pair is graphically represented.



**Fig.4.6**

From the graph it is clear that two lines intersect each other at point (2, 2). So,  $x = 2, y = 2$  is required solution of given linear pair and  $x = 2, y = 2$  satisfy given equations.

### Exercise 4.1

- On comparing the ratios  $\frac{a_1}{a_2}, \frac{b_1}{b_2}$  and  $\frac{c_1}{c_2}$ , find out whether pairs of linear equations are consistent or inconsistent.
  - $2x - 3y = 8$ ;  $4x - 6y = 9$
  - $3x - y = 2$ ;  $6x - 2y = 4$
  - $2x - 2y = 2$ ;  $4x - 4y = 5$
  - $\frac{4}{3}x + 2y = 8$ ;  $2x + 3y = 12$
- Solve the following pairs of linear equations, graphically and find the nature of that solution.
  - $x + y = 3$ ;  $3x - 2y = 4$
  - $2x - y = 4$ ;  $x + y = -1$
  - $x + y = 5$ ;  $2x + 2y = 10$
  - $3x + y = 2$ ;  $2x - 3y = 5$
- Solve the following linear pairs by graphical method and find co-ordinates of point where lines representing by them cuts  $y$ - axis.
  - $2x - 5y + 4 = 0$ ;  $2x + y - 8 = 0$
  - $3x + 2y = 12$ ;  $5x - 2y = 4$
- Solve the following linear pair by graphical method and find coordinated of vertices of triangle, formed by  $y$ - axis and lines formed by linear pair.
$$4x - 5y = 20; \quad 3x + 5y = 15$$

### 4.04. Linear Inequalities in Two Variables

A mathematical statement in which variable and sign  $>$ ,  $<$ ,  $\geq$  or  $\leq$  are presents, called inequality. Inequality may be of one variable or more than one variable. Let  $a$  is a non-zero real number then for variables  $x$ , inequalities  $ax + b < 0$ ,  $ax + b \leq 0$ ,  $ax + b > 0$  and  $ax + b \geq 0$  and called linear inequalities of one variable.

If number of variables is two then it is called as inequalities of two variables. For example, in general  $2x + 3y \leq 6$  and  $x + y < 4$ . Inequalities of two variables can be defined as : Let  $a, b$  are two non-zero real; numbers for variables  $x$  and  $y$  inequalities  $ax + by < c$ ,  $ax + by \leq c$ ,  $ax + by > c$  or  $ax + by \geq c$  are called linear inequalities of two variables.

In this appendix we will study about the solution of linear inequalities of two variables. Many solutions are possible for these inequalities. Set of all possible solutions is called set of a inequality.

### 4.05. Solution of Linear Inequalities of Two Variables by Graphical Method

Here, we will solve linear inequalities of two variables by graphical method. In co-ordinate geometry we have studied that straight line  $ax + by = c$ , is represented by joining the points which satisfy the equation in the plane  $x, y$  on graph paper that is relative to  $x$  - axis and  $y$  - axis.

Straight line  $ax + by = c$ , divides  $x, y$  - plane in two parts. *i.e.* these divided areas can be expressed by  $ax + by \leq c$  and  $ax + by \geq c$ .

These are expressed in the following sets in the forms of closed and open semi heavenly area.

In expression of sets :

Set  $\{(x, y) : ax + by = c\}$  Straight line

Set  $\{(x, y) : ax + by \leq c\}$  and  $\{(x, y) : ax + by \geq c\}$  closed semi heavenly area and Set  $\{(x, y) : ax + by < c\}$  and  $\{(x, y) : ax + by > c\}$  expose open semi heavenly area. All these semi heavenly areas the solution of

inequalities are called solution set.

Hence linear inequalities can be solved by graphical method in following steps.

**Step 1 :** Write given inequality in equation form it will represent a line.

**Step 2 :** By putting  $x = 0$  and  $y = 0$  in the equation of straight line ; we get, meeting point at  $y$  and  $x$  axis respectively

**Step 3 :** Join both the obtained points which represents a straight line.

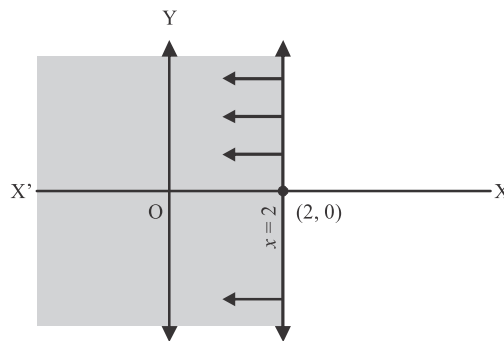
**Step 4 :** Take a point (may be origin) and put its co-ordinates in inequations. If these coordinates satisfy the inequality, then shade that area from line to point. This shaded part is the required solution of inequality. If origin does not satisfy the inequality then shaded portion will be in opposite side from line and this area will be the required solutions of inequality.

Solution of any inequality can be understood by the following examples.

**Example 8.** Solve the following inequalities graphically.

(i)  $x \leq 2$                       (ii)  $2x - y \geq 1$                       (iii)  $|y - x| \leq 3$

**Solution :** (i) Replace inequality  $x \leq 2$  into equation we get  $x = 2$ . It is clear that this line is parallel to  $y$ -axis and will pass through point  $(2, 0)$  of  $x$ -axis. Following graph is obtained accordingly.



**Fig. 4.07**

Now inequality  $x \leq 2$ , satisfies by origin  $(0,0)$ . So, shaded portion from line  $x = 2$  to indefinite extent will be the required solution set.

(ii) Replace inequality  $2x - y \geq 1$  into equation from we get  $2x - y = 1$

By putting  $x = 0$ , we get  $y = -1$ . So point  $(0, -1)$  cuts  $y$ -axis similarly by putting  $y=0$ ,

$x = \frac{1}{2}$  so point  $\left(\frac{1}{2}, 0\right)$  cuts  $x$ -axis we get the graph fig 4.08.

Now inequality  $2x - y \geq 1$  does not satisfy with origin  $(0, 0)$  i.e.,  $2 \times 0 - 0 \geq 1$  is not true.

So, shaded portion opposite to origin to line  $2x - y = 1$  will be the required solution set.

(iii) Here given inequality is  $|y - x| \leq 3$ . It can be written as  $-3 \leq y - x \leq 3$

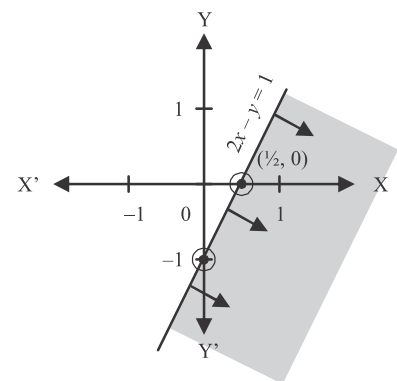
Again it can be written as following two in equalities

$$-3 \leq y - x; \quad y - x \leq 3$$

and

$$x - y - 3 \leq 0$$

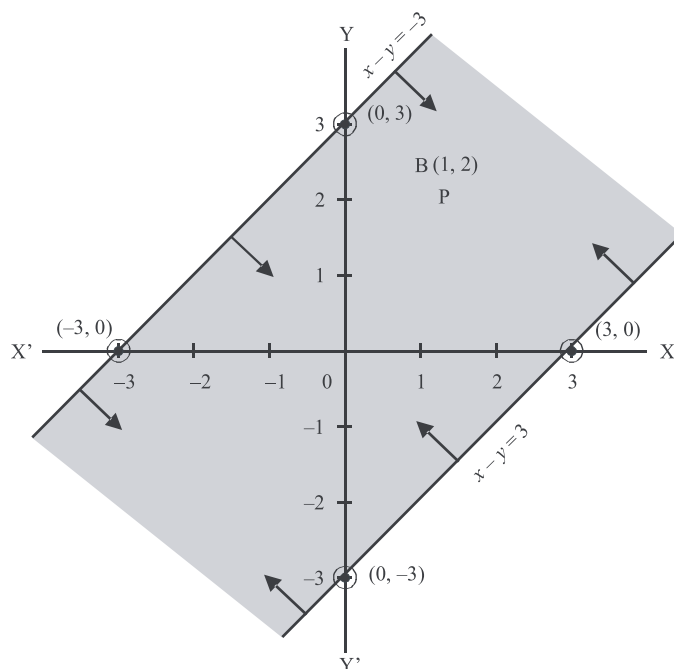
... (i)



**Fig. 4.08**

and  $x - y + 3 \geq 0$  . . . (ii)

By plotting inequality (1) in equation form  $x - y - 3 = 0$  is obtained, we get points (3, 0) and Y-axis respectively. Similarly by replacing inequality (2) in equation form we get  $x - y + 3 = 0$ , we get points (0, -3) and (0, 3) at x and y axis respectively. Now graph of these two lines are obtained in fig 4.09 :



**Fig. 4.09**

Now inequality  $x - y - 3 \leq 0$  satisfies by origin (0, 0) i.e.,  $0 - 0 - 3 \leq 0$  is true. So its shaded part will be the side of origin from line. Second inequality  $x - y + 3 \geq 0$  also satisfy by origin (0, 0) i.e.,  $0 - 0 + 3 \geq 0$  is true so its shaded area will be the side of origin from line. Thus shaded area between two lines will be required solution set.

### Exercise 4.2

- Show the solution set of the following inequalities, graphically.
  - $x \geq 2$
  - $y \leq -3$
  - $x - 2y < 0$
  - $2x + 3y \leq 6$
- Solve the following inequalities by graphical method :
  - $|x| \leq 3$
  - $3x - 2y \leq x + y - 8$
  - $|x - y| \geq 1$

### Miscellaneous Exercise 4

- For which value of  $k$ , linear pair  $x + y - 4 = 0$ ;  $2x + ky - 3 = 0$  have no solution :
  - 0
  - 2
  - 6
  - 8
- For which value of  $k$ , linear pair  $3x - 2y = 0$  and  $kx + 5y = 0$  have infinite solutions :
  - $\frac{1}{2}$
  - 3
  - $\frac{-5}{3}$
  - $\frac{-15}{2}$

3. Linear pair  $kx - y = 2$ ;  $6x - 2y = 3$  have unique solution. If  
 (a)  $k = 2$  (b)  $k = 3$  (c)  $k \neq 3$  (d)  $k \neq 0$
4. Inequalities  $x \geq 0, y \geq 0$  express corresponding equation to :  
 (a)  $x$ -axis (b)  $y$ -axis (c)  $x$  and  $y$ -axis (d) line
5. For line corresponding to inequality  $y - 3 \leq 0$ , following statement is true :  
 (a) parallel to  $x$ -axis (b) parallel to  $y$ -axis  
 (c) divides  $x$ -axis (d) passes through origin
6. Write the number of solution of following linear pairs  
 $x + 2y - 8 = 0$ ;  $2x + 4y = 16$
7. If pair of equations  $2x + 3y = 7$ ;  $(a + b)x + (2a - b)y = 21$  have infinite many solutions then find  $a$  and  $b$ .
8. Shade the solution set of inequality  $|x| \leq 3$
9. Shade the solution set of inequality  $2x + 3y \geq 3$
10. Solve the linear pair of equations graphically and with the help of this find value of 'a' where as  
 $4x + 3y = a$ ,  $x + 3y = 6$ ;  $2x - 3y = 12$ .
11. Solve the linear pair of equations graphically and find the co-ordinates of that points where lines represent them, cut at  $y$ -axis  $3x + 2y = 12$ ;  $5x - 2y = 4$ .

### Important Points

1. If  $a, b, c$  are real numbers then linear equation with two variables  $x$  and  $y$  can be expressed in general form as  $ax + by + c = 0$ , where  $a, b \neq 0$ .
2. Pair of linear equation with two variables, in general form can be expresses as  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$   
 Values of  $x, y$  which satisfy the two equations is solution of simultaneous equation.
3. Pair of linear equation of two variable are called consistent if this pair has at least one solution. If any pair has no solution then it is called inconsistent pair.
4. Consider relation between the coefficient of linear pair

$$a_1x + b_1y + c = 0 \text{ and } a_2x + b_2y + c = 0$$

We can check the nature and existence of solution :

- (i) Intersecting when,  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$  (Unique, consistent)
- (ii) Parallel when,  $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$  (No solution, inconsistent)
- (iii) Coincident when,  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$  (Infinite many solution consistent)

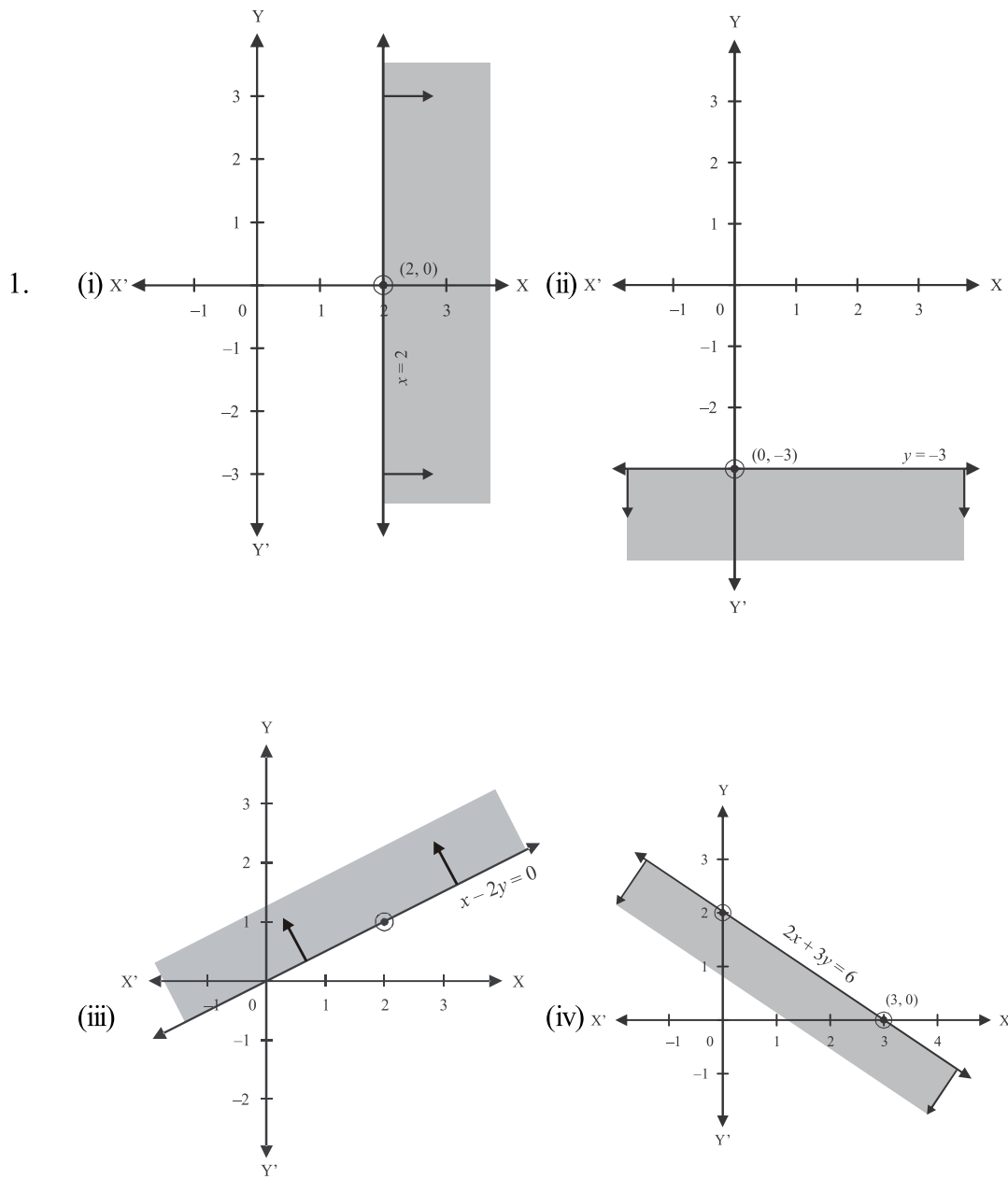
5. A linear pair with two variables can be solved graphically in following steps :
  - (i) From the equations of lines we obtain table of corresponding points and then show graphically.
  - (ii) If two lines intersect at point  $(\alpha, \beta)$  then  $x = \alpha, y = \beta$  will be required solution of given pair.
  - (iii) If lines are consistent then they have infinite many solution and two lines can be represent as same line so each point  $(\alpha, \beta)$  will be obtained in the form of solution  $x = \alpha, y = \beta$ .
  - (iv) If lines are parallel then there will be no solution.
6. If  $a, b$  are two non-zero real numbers, then for variables  $x$  and  $y$ , inequalities  $ax + by < c$ ,  $ax + by \leq c$ ,  $ax + by > c$  or  $ax + by \geq c$  are called linear inequalities of two variables.
7. Linear inequalities of two variable can be solved graphically in following steps :
  - (i) Write given inequalities in the equation form.
  - (ii) Putting  $x = 0$  and  $y = 0$  in above equations, obtained meeting points at  $x$ -axis and  $y$ -axis join these points.
  - (iii) Now check whether the inequality satisfy by origin  $(0, 0)$ . If satisfy, then solution set will be shaded part of corresponding side to origin. If origin not satisfies the inequality then solution set will be shaded part of opposite side of origin from line.
  - (iv) So common shaded area of all linear inequalities will be required solution of system.
8. Required solution will be common region which will satisfy all the inequalities. This solution set may be null set, bounded or unbounded area.

### Answer Sheet

#### Exercise 4.1

1. (i) Inconsistent, (ii) Consistent, (iii) Inconsistent, (iv) Consistent
2. (i) Unique solution  $x = 2, y = 1$  (ii) Unique solution  $x = 1, y = -2$   
(iii) Infinite many solutions (iv) Unique solution  $x = 1, y = -1$
3. (i)  $x = 3, y = 2$ ;  $(0, 4/5), (0, 8)$  (ii)  $x = 2, y = 3$ ;  $(0, 6), (0, -2)$
4.  $x = 5, y = 0$ ;  $(5, 0), (0, 3), (0, -4)$

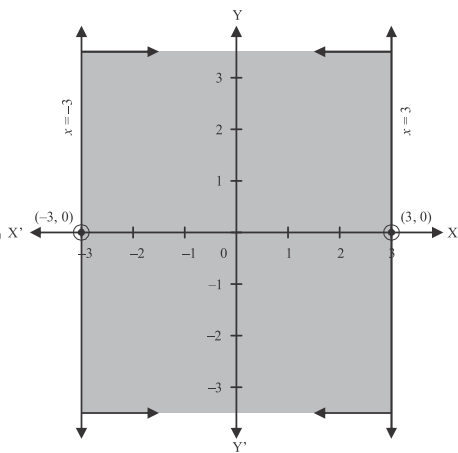
## Exercise 4.2



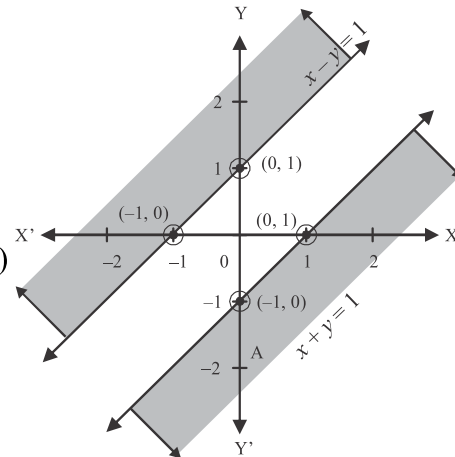


2.

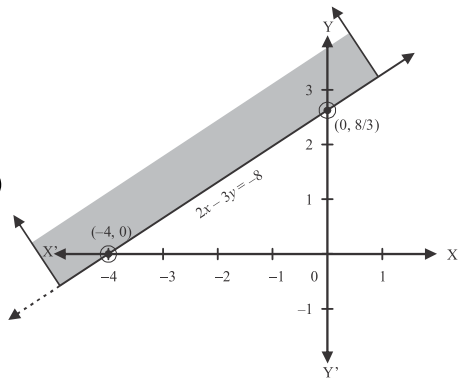
(i)



(ii)



(iii)



### Miscellaneous Exercise 4

1. (b)

2. (d)

3. (c)

4. (c)

5. (a)

6. Infinite many solutions

7.  $a = 5, b = 1$

10.  $x = 6, y = 0$ , so  $a = 24$

11.  $x = 2, y = 3$ ;  $(0, 6)$  and  $(0, -2)$