1. Mathematical logic

EXERSICE 1.1

State which of the following sentences are statements. Justify your answer if it is a statement. Write down its truth value.

i) A triangle has 'n' sides

Ans. it is a statement which is false, hence its truth value is 'F'.

ii) The sum of interior angles of a triangle is 180°

Ans. It is a statement which is true, hence its truth value is 'T'.

iii) You are amazing!

Ans. It is an exclamatory sentence, hence it is not a statement

iv) Please grant me a loan.

Ans. It is an imperative sentence, hence it is not a statement

v) $\sqrt{-4}$ is an irrational number.

Ans. It is a statement which is false, hence its truth value is 'F'.

**vi) $x^2 - 6x + 8 = 0$
implies $x = -4$
or $x = -2$.**

Ans. It is a statement which is false, hence its truth value is 'F'

vii) He is an actor.

Ans. It is an open sentence, hence it is not a statement.

viii) Did you eat lunch yet?

Ans. It is an interrogative sentence, hence it is not a statement

ix) Have a cup of cappuccino.

Ans. It is an imperative sentence, hence it is not a statement.

x) $(x+y)^2 2 = x^2 + 2xy +$
$\rightarrow y^2$ **for all** $x, y \in R$.

Ans. It is a mathematical identity which is true, hence its truth value is 'T'

xi) Every real number is a complex number.

Ans. It is a statement which is true, hence its truth value is 'T'

xii) 1 is a prime number.

Ans. It is a statement which is false, hence its truth value is 'F

xiii) With the sunset the day ends.

Ans. It is a statement which is true, hence its truth valy is T'.

xiv) $1\,! = 0$

Ans. It is a statement which is false, hence its truth value is 'F'

xv) $3 + 5 > 11$

Ans. It is a statement which is false, hence its truth value is 'F'.

xvi) The number Π is an irrational number.

Ans. It is a statement which is true, hence its truth value is 'T'.

xvii) $x^2 - y^2 = (x+y)$
$\rightarrow (x-y)$ **for all** $x, y \in R$.

Ans. It is a mathematical identity which is true, hence its truth value is 'T'.

xviii) The number 2 is the only even prime number.

Ans. It is a statement which is true, hence its truth value is 'T'.

xix) Two co-planar lines are either parallel or intersecting.

Ans. It is a statement which is true, hence its truth value is 'T'.

xx) The number of arrangements of 7 girls in a row for a photograph is 7!.

Ans. It is a statement which is true, hence its truth value is 'T'

xxi) Give me a compass box.

Ans. It is an imperative sentence, hence it is not a statement

xxii) Bring the motor car here.

Ans. It is an imperative sentence, hence it is not a statement

xxiii) It may rain today.

Ans. It is an open sentence, hence it is not a statement

xxiv) If $a + b < 7$, where $a \geq 0$ and $b \geq 0$ then $a < 7$ and $b < 7$.

Ans. It is a statement which is true, hence its truth value is 'T'.

xxv) Can you speak in English?

Ans. It is an interrogative sentence, hence it is not statement.

EXERCISE 1.2

Ex. 1: Express the following statements in symbolic form.

i) e is a vowel or $2 + 3 = 5$

Ans. Let *p:* e is a vowel.
q: $2 + 5 = 5$.
Then the symbolic form of the given statement is
$p \vee q$.

ii) Mango is a fruit but potato is a vegetable.

Ans. Let *p*: Mango is a fruit.
q: Potato is a vegetable.

Then the symbolic form of the given statement is
$p \wedge q$.

iii) Milk is white or grass is green.

Ans. Let *p*: Milk is white.
q: Grass is green.
Then the symbolic form of given statement is
$p \vee q$.

iv) I like playing but not singing.

Ans. Let *p*: I like playing.
q: I am not singing.
Then the symbolic form of given statement is
$p \wedge q$.

v) Even though it is cloudy, it is still raining.

Ans. The given statement is equivalent to:
It is cloudy and it is still raining.
Let: *p*: It is cloudy.
q: It is still raining.
Then the symbolic form of given statements is
$p \wedge q$.

Ex. 2: Write the truth values of following statements.

i) Earth is a planet and Moon is a star.

Ans. Let: *p*: Earth is a planet.
q: Moon is a star.
Then the symbolic form of given statements is
$p \wedge q$.

The truth values of p and q are T and F respectively.
$\therefore$ The truth value of $p \wedge q$
$is\ F. \ldots\ldots [T \wedge F = F]$

ii) 16 is an even number and 8 is a perfect square.

Ans. Let: *p*: 16 is a even number.
q: 8 is a perfect square.

Then the symbolic form of given statements is
$p \wedge q.$

The truth values of p and q are T and F respectively.
$\therefore$ The truth value of $p \wedge q$
$is\ F. \ldots\ldots [T \wedge F = F]$

iii) A quadratic equation has two distinct roots or 6 has three prime factors.

Ans. Let: *p*: A quadratic equation has two distinct roots.
q: 6 has three prime factors.
Then the symbolic form of given statements is
$p \vee q.$
The truth values of p and q are T and F respectively.
$\therefore$ The truth value of $p \vee q$
$is\ F. \ldots\ldots [T \wedge F = F]$

iv) The Himalayas are the highest mountains but they are part of India in the North East.
Ans. Let: *p*: Himalayas are the highest mountains.
q: They are the part of india in north-east.
Then the symbolic form of given statements is
$p \wedge q.$
The truth values of p and q are T.
$\therefore$ The truth value of $p \wedge q$
$is\ F. \ldots\ldots [T \wedge T = T]$

EXERCISE 1.3

1. Write the negation of each of the following statements.

i) All men are animals.

Ans. Some men are not animal

ii) − 3 is a natural number.

Ans. -3 is a neutral number

iii) It is false that Nagpur is capital of Maharashtra

Ans. Nagpur is the capital of maharashtra

iv) $2 + 3 \neq 5$

Ans. $2 + 3 = 5$

2. Write the truth value of the negation of each of the following statements.

i)$\sqrt{5}$ is an irrational number

Ans. Let p: $\sqrt{5}$
Is irrational number.
The truth value of p is T.
Therefore, the truth value of $\sim$ p if F.

ii) London is in England

Ans. Let p: London is in England.
The truth value of p is T.
Therefore, the truth value of $\sim$ p is F0.

iii) For every $x \in N, x + 3 < 8$.

Ans. Let p: For every $x \in$ N, $x + 3 < 8$.
The truth value of p is F.
Therefore, the truth value of $\sim$ p is T.

EXERCISEL 1.4

Ex. 1: Write the following statements in symbolic form.

i) If triangle is equilateral then it is equiangular.

Ans. Let p: Triangle is equilateral.
q: It is equiangular.
Then the symbolic form of the given statement is
p àq.

ii) It is not true that "i" is a real number.

Ans. Let p: 'i' is a real number.
Then the symbolic form of the given statement is
$\sim$p.

iii) Even though it is not cloudy, it is still raining.

Ans. Let p: It is cloudy.
q: It is still raining
Then the symbolic form of the given statement is
$\sim p \wedge q$.

iv) Milk is white if and only if the sky is not blue.

Ans. Let p: Milk is white
q: Sky is blue.
Then the symbolic form of given statement is
$p \leftrightarrow (\sim q)$.

v) Stock prices are high if and only if stocks are rising.

Ans. Let p: Stock prices are high.
q: stocks are rising:
Then the symbolic form of the given statement is
$p \leftrightarrow q$.

vi) If Kutub-Minar is in Delhi then Taj-Mahal is in Agra.

Ans. Let p: Kutub-minar is in Delhi,
q: Taj Mahal is in Agra.
Then the symbolic form of the given statement is
pàq.

Ex. 2: Find truth value of each of the following statements.

i) It is not true that 3 − 7i is a real number.

Ans. Let p: 3-7i is a real number.
Then the symbolic form of the given statement is $\sim p$. The truth value of p is F.
the truth value of $\sim p$ is T. ……[$\sim F \equiv T$]

ii) If a joint venture is a temporary partnership, then discount on purchase is credited to the supplier.

Ans. Let p: Joint venture is a temporary partnership.
q: Discount on purchases is credited to supplier
Then the symbolic form of the given statement is

pàq.
The truth values of p and q are T and F respectively.
∴The truth value of p àq is F. …….[TàF ≡ F]

iii) Every accountant is free to apply his own accounting rules if and only if machinery is an asset.
Ans. Let p: Every accountant is free to apply his own accounting rules.
q: Machinery is an asset.
Then the symbolic form of the given statement is
$p \leftrightarrow q$.
The truth values of p and q are F and T respectively.
∴The truth value of $p \leftrightarrow q$ is F. …….[$F \leftrightarrow T \equiv F$]

iv) Neither 27 is a prime number nor divisible by 4.

Ans. Let p: 27 is a prime number.
q: 27 is divisible by 4.
Then the symbolic form of the given statement is
$\sim p \wedge \sim q$.
The truth value of both p and q are F.
∴ the truth value of $\sim p \wedge \sim q$ is T. ……[$\sim F \wedge \sim q$ is T.]

v) 3 is a prime number and an odd number.

Ans. Let p: 3 is a prime number.
q: 3 is an odd number.
Then the symbolic form of the given statement is
$p \wedge q$.
The truth values of both p and q are T.
∴ The truth value of $p \wedge q$ is T. ……[$T \wedge T \equiv T$]

3. If p and q are true and r and s false, find the truth value of each of the following statements:

(i) $p \wedge (q \wedge r)$

Ans. Solution: Truth values of p and q are T and truth values of r and s are F.
$p \wedge (q \wedge r) \equiv T \wedge (T \wedge F)$
$\equiv T \wedge F \equiv F$
Hence, the truth value of the given statement is **False.**

(ii) (pàq)∨(r∧s)

Ans. (pàq)∨(r∧s) ≡ (TàT)∨(F∧F)
≡T∨F ≡T
Hence, the truth value of the given statement is **true.**

(iii) ~[(~p∨s)∧(~q∧r)]

Ans. ~[(~p∨s)∧(~q∧r)]≡~[(~T∨F)∧(~T∧F)]
≡~[(F∨F)∧(F∧F)]
≡~(F∧F)
≡~F≡T
Hence, the truth value of the given statement is **true.**

(iv) (pàq)↔~(p∨q)

Ans. (pàq)↔~(p∨q)≡(TàT)↔~(T∨T)
≡T ↔~(T)
≡ T ↔ F ≡ F
Hence, the truth value of the given statement is **false.**

(v) [(p∨s)àr]∨~[~(pàq)∨s]

Ans. [(p∨s)àr]∨~[~(pàq)∨s]
≡ [(T∨F)àF]∨~[~(TàT)∨F]
≡ (TàF)∨~(~T∨F)
≡ F∨~(F∨F)
≡ F∨~F ≡ F∨T ≡ T
Hence, the truth value of given statement is **true.**

(vi) ~[p∨(r∧s)]∧~[(r∧~s)∧q]

Ans. ~[T∨(F∧F)]∧~[(F∧~F)∧T]
≡~[T∨(F∧F)]∧~[(F∧~F)∧T]
≡~[T∨F]∧~[(F∧T)∧T]
≡~T∧~(F∧T)
≡F∧~F ≡ F∧T ≡F
Hence, the truth value of the given statement is **false.**

Ex. 4: Assuming that the following statements are true,
p: Sunday is holiday,
q: Ram does not study on holiday, find the truth values of the following statements.

i) Sunday is not holiday or Ram studies on holiday.

Ans. The symbolic form of the statement is ~p∨ ~q.

p	q	~p	~q	~p∨à~q
T	T	F	F	F

∴ truth value of given statement is F.

ii) If Sunday is not holiday then Ram studies on holiday.

Ans. The symbolic form of the given statement is ~pà ~q.

p	q	~p	~q	~p à~q
T	T	F	F	T

∴truth value of given statement is T.

iii) Sunday is a holiday and Ram studies on holiday.

Ans. The symbolic form of the given statement is p∧~q.

p	Q	~q	p∧~q
T	T	F	F

∴truth value of given statement is F.

Ex. 5: If p: He swims
q: Water is warm Give the verbal statements for the following symbolic statements.

i) p ↔ ~ q

Ans. He swims if and only if water is not warm.

ii) ~ (p ∨ q)

Ans. It is not true that he swims or water is warm.

iii) q → p

Ans. If water is warm, then he swims.

iv) q ∧ ~ p

Ans. Water is warm and he does not swim.

EXERSICE 1.5

Ex. 1: Use quantifiers to convert each of the following open sentences defined on N, into a true statement.

i) $x^2 + 3x - 10 = 0$

Ans. ∃ x ∈ N, Such that $x^2 +$
$3x - 10 = 0$ is a true statement

(x = 2 ∈ N satisfy x^2
→ $+3x - 10 = 0$)

ii) $3x - 4 < 9$

Ans. ∃ x ∈ N, such that $3x - 4 < 9$ is a true statement.
(x = 1, 2, 3, 4 ∈ N satisfy $3x - 4 < 9$)

iii) $n^2 \geq 1$

Ans. ∀ n ∈ N, $n^2 \geq 1$ *is a true statement.*
(All n ∈ N satisfy $n^2 \geq 1$)

iv) $2n - 1 = 5$

Ans. ∃ x ∈ N, such that $2n - 1 = 5$ is a true statement.
(n = 3 ∈ N satisfy $2n - 1 = 5$)

v) $Y + 4 > 6$

Ans. ∃ y ∈ N, such that $y + 4 > 6$ is a true statement.
(y = 3, 4, 5, ….. ∈ N satisfy $y + 4 > 6$)

vi) $3y - 2 \leq 9$

Ans. $\exists$ y $\in$ N, such that $3y - 2 \leq 9$ is a true statement.
(y = 1, 2, 3 $\in$ N satisfy $3y - 2 \leq 9$).

Ex. 2: If B = {2, 3, 5, 6, 7} determine the truth value of each of the following.

i) " $\forall$ x $\in$ B such that x is prime number.

Ans. x = 6 $\in$ B not satisfy whit x is a prime number.
So, the given statement is **false,** hence is truth value is F.

ii) $\exists$ n $\in$ B, such that n + 6 > 12.

Ans. Clearly n = 7 $\in$ B satisfies n + 6 > 12.
So, the given statement is **true,** hence its truth value is T.

iii) $\exists$ n $\in$ B, such that 2n + 2 < 4.

Ans. No element n $\in$ B satisfy 2n + 2 < 4.
So, the given statement is **false,** hence its truth value is F.

iv) "$\forall$ y $\in$ B such that
$\rightarrow$ y^2 is negative.

Ans. No element y $\in$ B
$\rightarrow$ satisfy y^2 *is nagative*

So, the given statement is **false,** hence its truth value is F.

v) " $\forall$y $\in$ B such that (y – 5) $\in$ N

Ans. y = 2 $\in$ B, y = 3 $\in$ B and y = 5 $\in$ B do not satisfy (y = 5) $\in$ N.
So, the given statement is **false,** hence its true value is F.

EXERSICE 1.6

1. Prepare truth tables for the following statement patterns.

i) $p \rightarrow (\sim p \vee q)$

Ans. Solution: Here are two statements and three connectives.
∴ There are 2 x 2 =4 rows and 2 + 3 = 5 columns in the truth table.

p	q	~p	~pVq	pà(~pVq)
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

ii) (~ p ∨ q) ∧ (~ p ∨ ~ q)

Ans.

p	q	~p	~q	~pVq	~pV~q	(~pVq)V(~pVq~)
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

iii) (p ∧ r) → (p ∨ ~ q)
Ans. Here are three statements and 4 connectives.
∴ there are 2 x 2 x 2 = 8 rows and 3 + 4 = 7 columns in the truth table.

p	q	r	~q	p∧r	pV~q	(p∧r)à(pV~q)
T	T	T	F	T	T	T
T	T	F	F	F	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	F	F	F	T
F	T	F	F	F	F	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

iv) (p ∧ q) ∨ ~ r

Ans.

p	q	r	~r	p∧q	(p∧q)V~r
T	T	T	F	T	T
T	T	F	T	T	T

T	F	T	F	F	F
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	F	T
F	F	T	F	F	F
F	F	F	T	F	T

2. Examine whether each of the following statement patterns is a tautology, a contradiction or a contingency

i) q ∨ [~ (p ∧ q)]

Ans.

p	q	p⊥q	~(p⊥q)	q⊦[~(p⊥q)]
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

All the entries in the last column of the above truth table are T.
∴ q∨[~ (p∧q)] is a **tautology.**

ii) (~ q ∧p) ∧(p ∧ ~ p)

Ans.

p	q	p	q	~q∧p	p∧~p	(~q∧p)∧(p∧~p)
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	F	F
F	F	T	T	F	F	F

All the entries in the last column of the above truth table are F.
∴ (~q∧p)∧(p∧~p) is a **contradiction.**

iii) (p ∧ ~ q) → (~ p ∧ ~ q)

Ans.

p	q	~p	~q	p∧~q	~p∧~q	(p∧~q)à(~p∧~q)
T	T	F	F	F	F	T
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	F	T	T	F	T	T

The entries in the last column are neither all T nor all F.
∴ (p∧~q)à(~p∧~q) is a **contingency.**

iv) ~ p → (p → ~ q)

Ans.

p	q	~p	~q	pà~q	~pà(pà~q)
T	T	F	F	F	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

All the entries in the last column of the truth table are T.
∴ ~pà (pà~q) is a **tautology.**

3. Prove that each of the following statement pattern is a tautology.

i) (p ∧ q) → q

Ans.

p	q	p∧q	(p∧q)àq
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

All the entries in the last column of the above truth table are T.
∴ (p∧q)àq is a **tautology.**

ii) (p → q) ↔ (~ q → ~ p)

Ans.

p	q	~p	~q	pàq	~qà~p	(pàq) ↔ (~qà~p)
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

All the entries in the last column of the above truth table are T.
∴ (pàq)↔(~qà~p) is a **tautology.**

iii) (~ p ∧ ~ q) → (p → q)
Ans.

p	q	~p	~q	~p∧~q	qàp	(~p∧~q) à (q∧p)
T	T	F	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

All the entries in the last column of the above truth table are T.
∴ (~P∧~q à (p → q) is a **tautology.**

iv) (~ p ∨ ~ q) ↔ ~ (p ∧ q)
Ans.

p	q	~p	~q	~p∨~q	p∧q	~(p∧q)	(~p∨~q) ↔ ~(p∧q)
T	T	F	F	F	T	F	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	T	F	T	T

All the entries in the last column of the above truth table are T.
∴ (~p∨~q) ↔ ~ (p∧g) is a **tautology.**

4. Prove that each of the following statement pattern is a contradiction.
i) (p ∨ q) ∧ (~ p ∧ ~ q)
Ans.

p	q	~p	~q	p∧q	~p∧~q	(p∨q) ∧ (~p∧~q)
T	T	F	F	T	F	F

T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

All the entries in the last column of the above truth table are F.
∴ (p∨q)∧(~p∧~q) is a **contradiction.**

ii) (p ∧ q) ∧ ~ p

Ans.

p	q	~p	p∧q	(p∧q)∧~p
T	T	F	T	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

All the entries in the last column of the above truth table are T.
∴ (p∧q) ∧ ~p is a **contradiction.**

iii) (p ∧ q) ∧ (~ p ∨ ~ q)

Ans.

p	q	~p	~q	p∧q	~p∨~q	(p∧q)∧(~p∨~q)
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

All the entries in the last column of the above truth table are F.
∴ (p∧q)∧(~p∧~q) is a **contradiction.**

iv) (p → q) ∧ (p ∧ ~ q)
Ans.

p	q	~q	pàq	p∧~q	(pàq)∧(p∧~q)
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	T	F	F

F	F	T	T	F	F

All the entries in the last column of the above truth table are F.
∴ (p∧àq)à(p∧~q) is a **contradiction.**

5. Show that each of the following statement pattern is a contingency.

i) (p ∧ ~ q) → (~ p ∧ ~ q)

Ans.

p	q	~p	~q	p∧~q	~p∧~q	(p∧~q)à(~p∧~q)
T	T	F	F	F	F	T
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	F	T	T	F	T	T

The entries in the last column of the above truth table are neither all T nor all F.
∴ (p∧ ~q) à (~p∧~q) is a **contingency.**

ii) (p → q) ↔ (~ p ∨ q)

Ans.

p	q	~p	pàq	~p∧q	(pàq)↔(~p∧q)
T	T	F	T	F	F
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	F	F

The entries in the last column of the above truth table are neither all T nor all F.
∴ (pàq) ↔ (~p∧q) is a **contingency.**

iii) p ∧ [(p → ~ q) → q]

Ans.

p	q	~q	pà~q	(pà~q)àq	P∧[(pà~q)àq]
T	T	F	F	T	T

T	F	T	T	F	F
F	T	F	T	T	F
F	F	T	T	F	F

The entries in the last column of the above truth table are neither all T nor all F.
∴ p∧ [pà ~q)àq] is a **contingency.**

iv) (p → q) ∧ (p → r)

Ans.

p	q		pàq	pàr	(pàq)∧(pàr)
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	F	T	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

The entries in the last column of the above truth table are neither all T nor all F.
∴ (pà q)∧(pàr) is a **contingency.**

6. Using the truth table, verify

i) p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r)

Ans.

1	2	3	4	5	6	7	8
p	q	R	q∧r	p∨(q∧r)	p∨q	p∨r	(p∧q)∧(p∨r)
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

The entries in columns 5 and 8 are identical.
∴ p∨(q∧r) ≡ (p∨q)∧(p∨r).

ii) p → (p → q) ≡ ~ q → (p → q)
Ans.

1	2	3	4	5	6
P	q	~q	pàq	pà(qàp)	~qà(pàq)
T	T	F	T	T	T
T	F	T	F	F	F
F	T	F	T	T	T
F	F	T	T	T	T

The entries in columns 5 and 6 are identical.
∴ pà(pàq) ≡ ~qà(pàq).

iii) ~ (p → ~ q) ≡ p ∧ ~ (~ q) ≡ p ∧ q

Ans.

1	2	3	4	5	6	7	8	
p	q	~q	pà~q	~(pà~q)	~(~q)	p∧~(~q)	p∧q	
T	T	F	F	T	T	T	T	
T	F	T	T	F	F	F	F	
F	T	F	T	F	T	F	F	
F	F	T	T	F	F	F	F	

The entries in columns 5, 7 and 8 are identical.
∴ ~(pà~q) ≡ p∧~(~q) ≡ p∧q.

iv) ~ (p ∨ q) ∨ (~ p ∧ q) ≡ ~ p

Ans.

1	2	3	4	5	6
p	q	p↔q	pàq	qàp	(pàq)∧(qàp)
T	T	T	T	T	T
T	F	F	F	T	F

F	T	F	T	F	F
F	F	T	T	T	T

The entries in columns 3 and 7 are identical.
∴ ~(p∧q)∨(~p∧q) = ~p.

7. Prove that the following pairs of statement patterns are equivalent.
i) p ∨ (q ∧ r) and (p ∨ q) ∧ (p ∨ r)
Ans.

1	2	3	4	5	6	7	8
p	q	R	q∧r	p∨(q∧r)	p∨q	p∨r	(p∧q)∧(p∨r)
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

The entries in columns 5 and 8 are identical.
∴ p∨(q∧r) ≡ (p∨q)∧(p∨r).

ii) p ↔ q and (p → q) ∧ (q → p)

Ans.

1	2	3	4	5	6
p	q	p↔q	pàq	qàp	(pàq)∧(qàp)
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

The entries in column 3 and 6 are identical.
∴ p↔q ≡ (pàq)∧(qàp).

iii) p → q and ~ q → ~ p and ~ p ∨ q

Ans.

1	2	3	4	5	6	7
p	q	~p	~q	pàq	~qà~p	~p∨q
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

The entries of the column 5, 6 and 7 are identical.
∴pàq ≡ ~qà ~p ≡ ~ p∨q.

iv) ~ (p ∧ q) and ~ p ∨ ~ q.

Ans.

1	2	3	4	5	6	7
p	q	~p	~q	p∧q	~(p∧q)	~p∨~q
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

The entries in column 6 and 7 are identical.
∴ ~(p∧q) ≡ ~p∨~q.

EXERSICE 1.7

1. Write the dual of each of the following:

i) (p ∨ q) ∨ r

Ans. (p∧q) ∧r

ii) ~ (p ∨ q) ∧ [p ∨ ~ (q ∧ ~ r)]
Ans. ~(p∧q)∨[p∧~(q∨~r)]

iii) $p \vee (q \vee r) \equiv (p \vee q) \vee r$

Ans. $p\wedge(q\wedge r)\equiv(p\wedge q)\wedge r$.

iv) $\sim (p \wedge q) \equiv \sim p \vee \sim q$

Ans. $\sim(p\vee q)\equiv \sim p\wedge\sim q$.

2. Write the dual statement of each of the following compound statements.

i) 13 is prime number and India is a democratic country.

Ans. 13 is prime number or India is a democratic country.

ii) Karina is very good or every body likes her.

Ans. Karina is very good and and everybody likes her.

iii) Radha and Sushmita can not read Urdu.

Ans. Radha or sushmita can not read urdu.

iv) A number is real number and the square of the number is non negative.

Ans. A number is real number or the square of the number is non-negative.

EXERSICE 1.8

1. Write negation of each of the following statements.

i) All the stars are shining if it is night.

Ans. The given statement can be written as:
If it is night, then all the stars are shining.
Let p: It is night.
q: All the stars are shining:
Then the symbolic form of the given statement is
pàq.
Since, ~(pàq) ≡ P∧~q, the negation of given statement is:
"It is night and all the stars are not shining.

ii) $\forall n \in N, n + 1 > 0$.

Ans. (ii) The negation of the given statement is:
'∃ n ∈ N, such that n +1 ≤ 0.'

iii) ∃ n ∈ N, $(n^2 + 2)$ is odd number

Ans. The negation of the given statement is:
'∀ n ∈ N, $n^2 + 2$ is not an odd number.' +

iv) Some continuous functions are differentiable.

Ans. The negation of given statement is: 'All continuous functions are not differentiable.'

2. Using the rules of negation, write the negations of the following:
i) (p → r) ∧ q

Ans. The negation of (pàr)∧ q is:
~ [pàr)∧q] ≡ ~ (pàr) ∨ (~q)
……[Negation of conjunction]
≡ (p∧ ~r)∨(~q)
……[Negation of implication]

ii) ~ (p ∨ q) → r

Ans. The negation of ~(p∨q)àr is:
~[~(p∨q) à r]≡ ~(p∨q)∧(~r)
……[Negation of implication]
≡(~p∧ ~q)∧(~r)
……[Negation of disjunction]

iii) (~ p ∧ q) ∧ (~ q ∨ ~ r)

Ans. The negation of (~p ∧ q)∧(~q∨~r) is:
~[(~p∧q)∧(~q∨~r)]≡ ~p∧q)∨~(~q∨~r)
…….[Negation of conjunction]
≡[~(~p) ∨~ q]∨[~(~q)∧~(~r)]
……..[Negation of conjunction and disjunction]
≡(p∨~q)∨(q∧r) …[Negation of negation]]

3. Write the converse, inverse and contrapositive of the following statements.

i) If it snows, then they do not drive the car.

Ans. Let p: It snows.
q: They do not drive the car.
Then the symbolic form of the given statement is p à q.
Converse: qàp is the converse of pàq.
i.e. If they do not drive the car, then it snows.
Inverse: ~pà~q is the inverse of pàq.
i.e. If it does not snow, then they drive the car.
Contrapositive: ~qà ~p is the contrapositive of pàq. i.e. If they drive the car, then it does not snow.

ii) If he studies, then he will go to college.

Ans. Let p: He studies.
q: He will go to college.
Then two symbolic form of the given statement is pàq.
Converse: qàp is the converse of pàq.
i.e. If he will go to college, then he studies.
Inverse: ~pà~q is the inverse of pàq. i.e. If he does not study, then he will not go to college, **Contrapositive:** ~qà~p is the contrapositive of pàq. i.e. If he will not go to college, then he does not study.

4. With proper justification, state the negation of each of the following.
i) (p → q) ∨ (p → r)

Ans. The negation of (pàq)∨(pàr) is:
~[(pàq)∨(pàr)]≡ ~(pàq)∧~(pàr)
.....[Negation of disjunction]
≡ (p∧~q)∧(p∧~r)
......[Negation of implication)

ii) (p ↔ q) ∨ (~ q → ~ r)

Ans. The negation of (p ↔ q)∨(~9à ~r) is:
~[(p ↔ q)∨(~qà~r)] ≡à(p↔q)∧~(~qà~r)
......[Negation of disjunction]
≡[(p∧~q)∨(q∧~p)]∧[~q∧~(~r)]
.....[Negation of biconditional and implication)
≡[(p∧~9)∨(q∧ ~p)]∧(~q∧r)
.....[Negation of negation]

iii) (p → q) ∧ r

Ans. The negation of (pàq)∧r is:
≡(pàg)∧r] ≡ ~ (pàq)∨(~r)
... [Negation of conjunction]
≡ (p∧~q)∨(~r)
...[Negation of implication]

EXERSICE 1.9

1. Without using truth table, show that

i) p ↔ q ≡ (p ∧ q) ∨ (~ p ∧ ~ q)

Ans. LHS = p↔q
≡ (pàq)∧(qàp)
≡ (~p∨q)∧(~q∨p) ... (Conditional law)
≡ [~p∧(~q∨p)]∨[q∧(~q∨p]
... (Distributive law)
≡ [(~p∧~q)∨(~p∧p)]∨[(q∧~q)∨(q∧p)]
... (Distributive law)
≡ [(~p∧~q)∨c]∨[c∨(q∧p)]
... (Complement law)
≡ (~p∧~q)∨(q∧p) ... (Identity law)
≡ (~p ∧ ~q)∨(p ∧ q) ... (Commutative law)
≡ (p ∧ q) ∨ (~p∧~q) ... (Commutative law)
≡ RHS.

ii) p ∧ [(~ p ∨ q) ∨ ~ q] ≡ p

Ans. LHS = p∧[(~p∨q)(~q)]
≡ p∧ [~p∨(q∨~q)] ... (Associative law)
≡ p∧ p∧[~p∨t] ... (Complement law)
≡ p∧t ... (Identity law)
≡ p ... (Identity law)
= RHS.

iii) ~ [(p ∧ q) → ~ q] ≡ p ∧ q

Ans. LSH = ~[(p∧q)à(~q)]
≡ (p∧q)∧~(~q) ... (Negation of law)
≡ (p∧q)∧q ... (Negation of negation)
≡ p ∧ (q∧q) ... (Associative law)

≡ p ∧ q   … (Idempotent law)
= RHS

iv) ~ r → ~ (p ∧ q) ≡ [~ (q → r)] → ~ p

Ans. LSH = ~r à (p∧q)
≡ ~r à (~p∨~q) … (De Morgan's law)
≡ ~ (~r)∨(~p∨~q) … (Conditional law)
≡ r∨(~p∨~q) … (Involution law)
≡ r∨~q∨~p … (Commutative law)
≡ (~q∨r)∨(~p) … (Commutative law)
≡ (qàr)∨(~p)   .., (Conditional law)
≡ ~ (qàr)à(~p)   … (Conditional law)
= RHS.

v) (p ∨ q) → r ≡ (p → r) ∧ (q → r)

Ans. LSH = (p∨q)àr
≡ ~(p∨q)∨r   … (Conditional law)
≡ (~p∧~q)∨r … (De Morgan' s law)
≡ (~p∨r)∧(~q∨r) … (Distributive law)
≡ (pàr)∧(qàr) … (Conditional law)
= RHS.

2. Using the algebra of statement, prove that

i) [p ∧ (q ∨ r)] ∨ [~ r ∧ ~ q ∧ p] ≡ p

Ans. LHS = [p∧(q∨r)]∨[~r∧~q∧p]
≡ [p∧(q∨r)]∨[(~r∧~q)∧p]
… (Associative law)
≡ [p∧(q∨r)]∨[~(~q∧~r)∧p]
… (Commutative law)
≡ [p∧(q∨r)]∨[~(q∨r)∧p]
… (De Morgan's law)
≡ [p∧(q∨r)]∨[p∧~(q∨r)]
… (Commutative law)
≡ p∧[(q∨r)∨~(q∨r)]   … (Distributive law)
≡ p ∧ t   … (Complement law)
≡p   …. (Identify law)
= RHS.

ii) $(p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q) \equiv p \vee \sim q$

Ans. LHS = $(p\wedge q)\vee(p\wedge\sim q)\vee(\sim p\wedge\sim q)$
$\equiv (p\wedge q)\vee[(p\wedge\sim q)\vee(\sim p\wedge\sim q)]$ ... (Associative law)
$\equiv (p\wedge q)\vee[(\sim q\wedge p)\vee(\sim q\wedge\sim p)]$ ... (Commutative law)
$\equiv (p\wedge q)\vee[\sim q\wedge(p\vee\sim p)]$ ... (Distributive law)
$\equiv (p\wedge q)\vee(\sim q\wedge t)$ ... (Complement law)
$\equiv (p\wedge q)\vee(\sim q)$ ... (Identity law)
$\equiv (p\vee\sim q)\wedge(q\vee\sim q)$ ... (Distributive law)
$\equiv (p\vee\sim q)\wedge t$ ... (Complement law)
$\equiv p\vee\sim q$ ... (Identity law)
= RHS.

iii) $(p \vee q) \wedge (\sim p \vee \sim q) \equiv (p \vee \sim q) \wedge (\sim p \vee q)$

Ans. LHS = $(p\vee q)\wedge(\sim p\vee\sim q)$
$\equiv [p\wedge(\sim p\vee\sim q)]\vee[q\wedge(\sim p\vee\sim q)]$
... (Distributive law)
$\equiv [(p\wedge\sim p)\vee(\sim p\vee\sim q)]\vee[(q\wedge\sim p)\vee(q\wedge\sim q)]$
... (Distributive law)
$\equiv [c\vee(p\wedge\sim q)]\vee[(q\wedge\sim p)\vee c]$
... (Complement law)
$\equiv (p\wedge\sim q)\vee(q\wedge\sim p)$ ... (Identity law)
$\equiv (p\wedge\sim q)\vee(\sim p\wedge q)$ ... (Commutative law)
= RHS.