CHAPTER

2

INVERSE TRIGONOMETRIC FUNCTIONS

Syllabus

➢ *Definition, range, domain, principal value branch.*

STAND ALONE MCQs

(1 Mark each)

Q. 1. The value of $\sin^{-1}\left(\cos\dfrac{3\pi}{5}\right)$ is

(A) $\dfrac{\pi}{10}$

(B) $\dfrac{3\pi}{5}$

(C) $-\dfrac{\pi}{10}$

(D) $\dfrac{-3\pi}{5}$

[CBSE OD Set-I 2020]

Ans. Option (C) is correct.

Explanation:

$$= \sin^{-1}\left[\cos\left(\dfrac{3\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[\cos\left(\dfrac{\pi}{2}+\dfrac{\pi}{10}\right)\right]$$

$$= \sin^{-1}\left(-\sin\dfrac{\pi}{10}\right) \quad \left[\because \cos\left(\dfrac{\pi}{2}+x\right)=-\sin x\right]$$

$$= -\sin^{-1}\left(\sin\dfrac{\pi}{10}\right) \quad [\because \sin^{-1}(-x)=-\sin^{-1}x]$$

$$= -\dfrac{\pi}{10} \quad \left[\because \sin^{-1}(\sin x)=x, x \in \left(\dfrac{-\pi}{2},\dfrac{\pi}{2}\right)\right]$$

Q. 2. The value of $\tan\left[\dfrac{1}{2}\cos^{-1}\left(\dfrac{\sqrt{5}}{3}\right)\right]$ is

(A) $\dfrac{3+\sqrt{5}}{2}$

(B) $\dfrac{3-\sqrt{5}}{2}$

(C) $\dfrac{-3+\sqrt{5}}{2}$

(D) $\dfrac{-3-\sqrt{5}}{2}$

Ans. Option (B) is correct.

Explanation:

$$x = \tan\left[\dfrac{1}{2}\cos^{-1}\left(\dfrac{\sqrt{5}}{3}\right)\right]$$

Let $\cos^{-1}\dfrac{\sqrt{5}}{3} = \theta$

$$\cos\theta = \dfrac{\sqrt{5}}{3}$$

$$\Rightarrow \quad x = \tan\dfrac{1}{2}\theta$$

$$\Rightarrow \quad x = \dfrac{\sin\dfrac{\theta}{2}}{\cos\dfrac{\theta}{2}}$$

$$\because \quad \sin\dfrac{\theta}{2} = \dfrac{\sqrt{1-\dfrac{\sqrt{5}}{3}}}{\sqrt{2}}$$

$$\Rightarrow \quad \cos\dfrac{\theta}{2} = \dfrac{\sqrt{1+\dfrac{\sqrt{5}}{3}}}{\sqrt{2}}$$

$$x = \dfrac{\sqrt{1-\dfrac{\sqrt{5}}{3}}}{\sqrt{1+\dfrac{\sqrt{5}}{3}}}$$

$$= \frac{\sqrt{3-\sqrt{5}}}{\sqrt{3+\sqrt{5}}}$$

$$= \frac{\sqrt{3-\sqrt{5}}}{\sqrt{3+\sqrt{5}}} \times \frac{\sqrt{3-\sqrt{5}}}{\sqrt{3-\sqrt{5}}}$$

$$= \frac{3-\sqrt{5}}{\sqrt{(3)^2-(\sqrt{5})^2}}$$

$$= \frac{3-\sqrt{5}}{\sqrt{9-5}}$$

$$= \frac{3-\sqrt{5}}{2}$$

Q. 3. Which of the following is the principal value branch of $\cos^{-1}x$?

(A) $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ (B) $\left[0, \dfrac{\pi}{2}\right]$

(C) $[0, \pi]$ (D) $(0, \pi) - \left\{\dfrac{\pi}{2}\right\}$

Ans. Option (C) is correct.

Explanation: As we know that the principal value of $\cos^{-1}x$ is $[0, \pi]$.
$$y = \cos^{-1}x$$

Q. 4. Which of the following is the principal value branch of $\mathrm{cosec}^{-1}x$?

(A) $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ (B) $[0, \pi] - \left\{\dfrac{\pi}{2}\right\}$

(C) $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ (D) $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right] - \{0\}$

Ans. Option (D) is correct.

Explanation : As we know that the principal value of $\mathrm{cosec}^{-1}x$ is $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right] - \{0\}$.
$$y = \mathrm{cosec}^{-1}x$$

Q. 5. The value of $\sin^{-1}\left[\cos\left(\dfrac{33\pi}{5}\right)\right]$ is

(A) $\dfrac{3\pi}{5}$ (B) $\dfrac{-7\pi}{5}$

(C) $\dfrac{\pi}{10}$ (D) $\dfrac{-\pi}{10}$

Ans. Option (D) is correct.

Explanation: Let,

$$\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right] = \sin^{-1}\left[\cos\left(6\pi + \frac{3\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$$

$$[\because \cos(2n\pi + \theta) = \cos\theta]$$

$$= \sin^{-1}\left[\cos\left(\frac{\pi}{2} + \frac{\pi}{10}\right)\right]$$

$$= \sin^{-1}\left(-\sin\frac{\pi}{10}\right)$$

$$\left[\because \cos\left(\frac{\pi}{2} + x\right) = -\sin x\right]$$

$$= -\sin^{-1}\left(\sin\frac{\pi}{10}\right)$$

$$[\because \sin^{-1}(-x) = -\sin^{-1}x]$$

$$= -\frac{\pi}{10}$$

$$\left[\because \sin^{-1}(\sin x) = x, x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right]$$

Q. 6. The domain of function $\cos^{-1}(2x - 1)$ is

(A) $[0, 1]$ (B) $[-1, 1]$

(C) $(-1, 1)$ (D) $[0, \pi]$

Ans. Option (A) is correct.

Explanation:
We have $\cos^{-1}(2x - 1)$
$$\Rightarrow -1 \le 2x - 1 \le 1 \qquad [\because x \in [-1, 1]]$$
$$\Rightarrow \quad 0 \le 2x \le 2$$
$$\Rightarrow \quad 0 \le x \le 1$$
$$\Rightarrow \quad \in [0, 1]$$

Q. 7. The value of $\cos^{-1}\left(\cos\dfrac{3\pi}{2}\right)$ is

(A) $\dfrac{\pi}{2}$ (B) $\dfrac{3\pi}{2}$

(C) $\dfrac{5\pi}{2}$ (D) $\dfrac{7\pi}{2}$

Ans. Option (A) is correct.

Explanation: We have,

$$\cos^{-1}\left(\cos\frac{3\pi}{2}\right) = \cos^{-1}\left[\cos\left(2\pi - \frac{\pi}{2}\right)\right]$$

$$\left[\because \cos\left(2\pi - \frac{\pi}{2}\right) = \cos\frac{\pi}{2}\right]$$

$$= \cos^{-1}\cos\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$$

$$\left[\because \cos^{-1}(\cos x) = x, x \in [0, \pi]\right]$$

Q. 8. The value of expression $2\sec^{-1}2 + \sin^{-1}\left(\dfrac{1}{2}\right)$ is

(A) $\dfrac{\pi}{6}$ (B) $\dfrac{5\pi}{6}$

(C) $\dfrac{7\pi}{6}$ (D) 1

Ans. Option (B) is correct.

Explanation: We have,

$$2\sec^{-1}2+\sin^{-1}\left(\frac{1}{2}\right)$$

$$=2\sec^{-1}\sec\frac{\pi}{3}+\sin^{-1}\sin\frac{\pi}{6}$$

$$=2\times\frac{\pi}{3}+\frac{\pi}{6}$$

$$\left[\because\sec^{-1}(\sec x)=x\text{ and }\sin^{-1}(\sin x)=x\right]$$

$$=\frac{4\pi+\pi}{6}$$

$$=\frac{5\pi}{6}$$

Q. 9. What is the value of $\sec^2(\tan^{-1}2)$

    (A) 1   **(B)** 4

    (C) 5   **(D)** 3

Ans. Option (C) is correct.

Explanation :

$$\sec^2(\tan^{-1}2)=\sec^2\left(\sec^{-1}\sqrt{1+2^2}\right)$$

$$=\sec^2(\sec^{-1}\sqrt{5})$$

$$=(\sqrt{5})^2$$

$$=5$$

Q. 10. The principal value of

$$\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)+4\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\text{ is}$$

    (A) $\dfrac{\pi}{3}$ **(B)** $\dfrac{\pi}{6}$

    (C) $\dfrac{4\pi}{3}$ **(D)** $\dfrac{3\pi}{4}$

Ans. Option (C) is correct.

Explanation :

$$\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)+4\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$=\cos^{-1}\left(\cos\frac{\pi}{3}\right)+2\sin^{-1}\left(\sin\frac{\pi}{6}\right)+4\tan^{-1}\left(\tan\frac{\pi}{6}\right)$$

$$=\frac{\pi}{3}+2\times\frac{\pi}{6}+4\times\frac{\pi}{6}$$

$$=\frac{2\pi+2\pi+4\pi}{6}$$

$$=\frac{8\pi}{6}$$

$$=\frac{4\pi}{3}$$

Q. 11. The principal value of $\cot^{-1}(-\sqrt{3})$ is

    (A) $\dfrac{5\pi}{6}$ **(B)** $\dfrac{\pi}{2}$

    (C) $\dfrac{\pi}{4}$ **(D)** $\dfrac{\pi}{3}$

Ans. Option (A) is correct.

Explanation :

Let $\qquad\cot^{-1}(-\sqrt{3})=\theta$

$\Rightarrow\qquad\qquad\qquad\cot\theta=-\sqrt{3}$

$\Rightarrow\qquad\qquad\qquad\cot\theta=-\cot\dfrac{\pi}{6}$

$\qquad\qquad\qquad\qquad\qquad=\cot\left(\pi-\dfrac{\pi}{6}\right)$

$\Rightarrow\qquad\qquad\qquad\cot\theta=\cot\dfrac{5\pi}{6}$

$\Rightarrow\qquad\qquad\qquad\qquad\theta=\dfrac{5\pi}{6}\in(0,\pi)$

$\therefore$ Principal value of $\cot^{-1}(-\sqrt{3})$ is $\dfrac{5\pi}{6}$

Q. 12. Domain of $\sin^{-1}x$ is:

    (A) $[-1,\infty)$ **(B)** $[-1,1]$

    (C) $(-1,1)$ **(D)** None of these.

Ans. Option (B) is correct.

Explanation : Domain of $\sin^{-1}x$ is $[-1,1]$

Q. 13. Range of $\cos^{-1}x$ is:

    (A) $\left[0,\dfrac{\pi}{2}\right]$ **(B)** $\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$

    (C) $\left[\dfrac{\pi}{2},\dfrac{3\pi}{2}\right]$ **(D)** $[0,\pi]$

Ans. Option (D) is correct.

Explanation : The branch with range $[0,\pi)$ is called the **principal value branch** of the function $\cos^{-1}x$.

Q. 14. Domain of $\sec^{-1}x$ is:

    (A) $R-(-1,1)$ **(B)** R

    (C) $[-1,1]$ **(D)** $R-(0,1)$

Ans. Option (A) is correct.

Q. 15. The value of $\tan^{-1}(\sqrt{3})-\sec^{-1}(-2)$ is:

    (A) π **(B)** $-\dfrac{\pi}{3}$

    (C) $\dfrac{\pi}{3}$ **(D)** $\dfrac{2\pi}{3}$

Ans. Option (B) is correct.

Explanation : $\tan^{-1}(\sqrt{3})-\sec^{-1}(-2)$

$$=\frac{\pi}{3}-\frac{2\pi}{3}$$

$$=-\frac{\pi}{3}$$

Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is True

Q. 1. Assertion (A): $\sin^{-1}\left(\sin\dfrac{2\pi}{3}\right) = \dfrac{2\pi}{3}$

Reason (R): $\sin^{-1}(\sin\theta) = \theta$, if $\theta \in \left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$

Ans. Option (D) is correct.

> *Explanation:*
>
> The principal value branch of $\sin^{-1}x$ is $\left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$
>
> Let $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$
>
> $\sin^{-1}(\sin\theta) = \sin^{-1}x = \theta$
>
> $\sin^{-1}(\sin\theta) = \theta$, if $\theta \in \left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$.
>
> Hence R is true.
>
> $\sin^{-1}\left(\sin\dfrac{2\pi}{3}\right) \neq \dfrac{2\pi}{3}$, since $\dfrac{2\pi}{3} \notin \left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$
>
> Hence A is false.

Q. 2. Assertion (A): Range of $\tan^{-1}x$ is $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$

Reason (R): Domain of $\tan^{-1}x$ is R.

Ans. Option (B) is correct.

> *Explanation:* Domain of $\tan x$ is the set $\{x : x \in R$ and $x \neq (2n+1)\dfrac{\pi}{2}$, $n \in Z\}$ and Range is R.
>
> $\Rightarrow \tan x$ is not defined for odd multiples of $\dfrac{\pi}{2}$.
>
> If we restrict the domain of tangent function to $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$, then it is one-one and onto with its range as R. Actually tan x restricted to any of the intervals $\left(\dfrac{-3\pi}{2}, \dfrac{-\pi}{2}\right), \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right), \left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)$ etc., is bijective and its range is R.
>
> Thus $\tan^{-1}x$ can be defined as a function whose domain is R and range could be any of the intervals $\left(\dfrac{-3\pi}{2}, \dfrac{-\pi}{2}\right), \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right), \left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)$ and soon.

∴ Both A and R are true but R is not correct explanation of A.

Q. 3. Assertion (A): Principal value of $\sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$ is $\dfrac{\pi}{4}$

Reason (R): Principal value of $\cot^{-1}\left(\dfrac{-1}{\sqrt{3}}\right)$ is $\dfrac{\pi}{3}$

Ans. Option (C) is correct.

> *Explanation:*
>
> $$\sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = \sin^{-1}\left(\sin\dfrac{\pi}{4}\right)$$
>
> $$= \dfrac{\pi}{4}$$
>
> $$\cot^{-1}\left(\dfrac{-1}{\sqrt{3}}\right) = y$$
>
> $$\cot y = \dfrac{-1}{\sqrt{3}}$$
>
> $$= -\cot\left(\dfrac{\pi}{3}\right)$$
>
> $$= \cot\left(\pi - \dfrac{\pi}{3}\right)$$
>
> $$= \cot\left(\dfrac{2\pi}{3}\right)$$
>
> $$\Rightarrow \cot^{-1}\left(\dfrac{-1}{\sqrt{3}}\right) = \dfrac{2\pi}{3}$$
>
> Hence Assertion is correct and Reason is incorrect.

Q. 4. Assertion (A): Range of $\cot^{-1}x$ is $(0, \pi)$

Reason (R): Domain of $\tan^{-1}x$ is R.

Ans. Option (B) is correct.

Q. 5. Assertion (A): Principal value of $\cos^{-1}(1)$ is π

Reason (R): Value of $\cos 0°$ is 1

Ans. Option (D) is correct.

> *Explanation:* In case of Assertion:
>
> $$\cos^{-1}(1) = y$$
> $$\cos y = 1$$
> $$\cos y = \cos 0° \qquad [\because \cos 0° = 1]$$
> $$\therefore \qquad y = 0$$
>
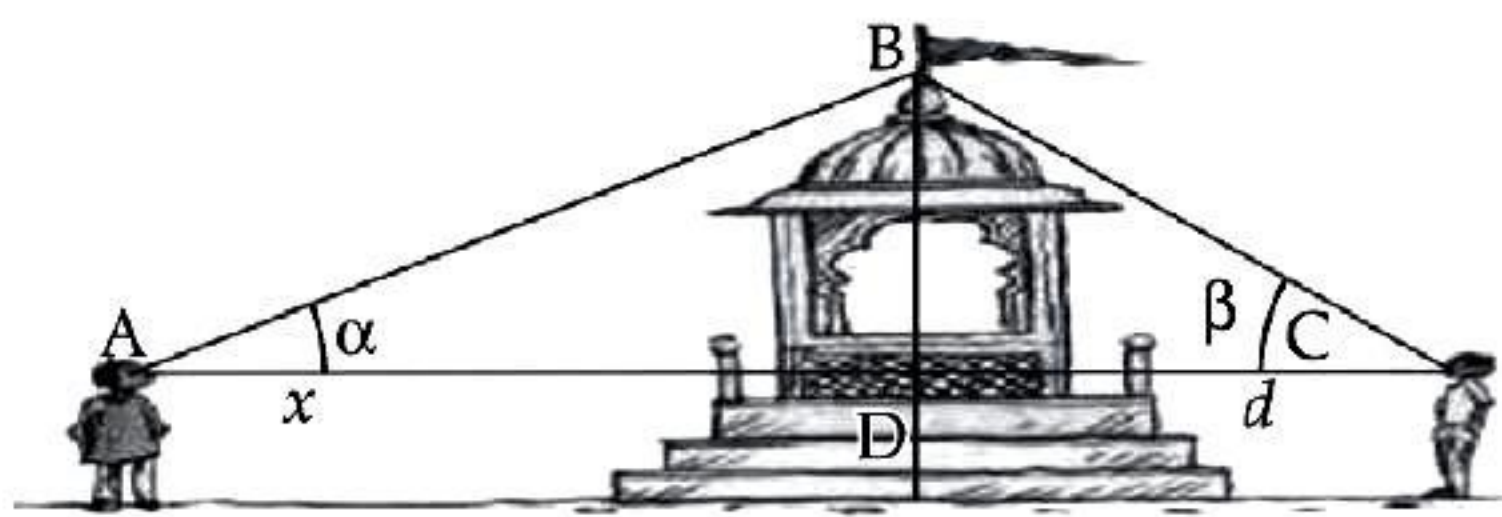> $\Rightarrow$ Principal value of $\cos^{-1}(1)$ is 0
> Hence Assertion is in correct.
> Reason is **correct.**

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

Two men on either side of a temple of 30 metres high observe its top at the angles of elevation α and β respectively. (as shown in the figure above). The distance between the two men is $40\sqrt{3}$ metres and the distance between the first person A and the temple is $30\sqrt{3}$ meters. **[CBSE QB-2021]**

Q. 1. $\angle CAB = \alpha =$

(A) $\sin^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$ (B) $\sin^{-1}\left(\dfrac{1}{2}\right)$

(C) $\sin^{-1}(2)$ (D) $\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

Ans. Option (B) is correct.

Explanation: In Δ BDA

$$\sin\alpha = \frac{BD}{AB}$$

$$AB^2 = AD^2 + BD^2$$
$$= (30\sqrt{3})^2 + (30)^2$$
$$= (60)^2$$
$$AB = 60\,\text{m}$$

Now, $\quad \sin\alpha = \dfrac{30}{60}$

$$\sin\alpha = \frac{1}{2}$$

i.e. $\quad \angle CAB = \alpha = \sin^{-1}\left(\dfrac{1}{2}\right)$

Q. 2. $\angle CAB = \alpha =$

(A) $\cos^{-1}\left(\dfrac{1}{5}\right)$ (B) $\cos^{-1}\left(\dfrac{2}{5}\right)$

(C) $\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ (D) $\cos^{-1}\left(\dfrac{4}{5}\right)$

Ans. Option (C) is correct.

Explanation: In Δ BDA

$$\cos\alpha = \frac{AD}{AB}$$

$$\cos\alpha = \frac{30\sqrt{3}}{60}$$

$$\alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$\therefore \quad \angle CAB = \alpha = \cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

Q. 3. $\angle BCA = \beta =$

(A) $\tan^{-1}\left(\dfrac{1}{2}\right)$ (B) $\tan^{-1}(2)$

(C) $\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ (D) $\tan^{-1}\left(\sqrt{3}\right)$

Ans. Option (D) is correct.

Explanation:
$$DC = AC - AD$$
$$= 40\sqrt{3} - 30\sqrt{3}$$
$$= 10\sqrt{3}\,\text{m}$$

In ΔBDC

$$\tan\beta = \frac{BD}{DC} = \frac{30}{10\sqrt{3}} = \sqrt{3}$$

$$\angle BCA = \beta = \tan^{-1}(\sqrt{3})$$

Q. 4. $\angle ABC =$

(A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{6}$

(C) $\dfrac{\pi}{2}$ (D) $\dfrac{\pi}{3}$

Ans. Option (C) is correct.

Explanation: Since,

$$\sin\alpha = \frac{1}{2}$$

i.e., $\quad \sin\alpha = \sin 30°$

$$\left[\because \sin 30° = \frac{1}{2}\right]$$

$\therefore \quad \alpha = 30°$

we, have $\quad \tan\beta = \sqrt{3}$

$\tan\beta = \tan 60°$

$\therefore \quad \beta = 60°$

Now, In ΔABC

$$\angle ABC + \angle BCA + \angle CAB = 180°$$
$$\angle ABC + 60° + 30° = 180°$$
$$\angle ABC = 90°$$

$\therefore \quad \angle ABC = \dfrac{\pi}{2}$
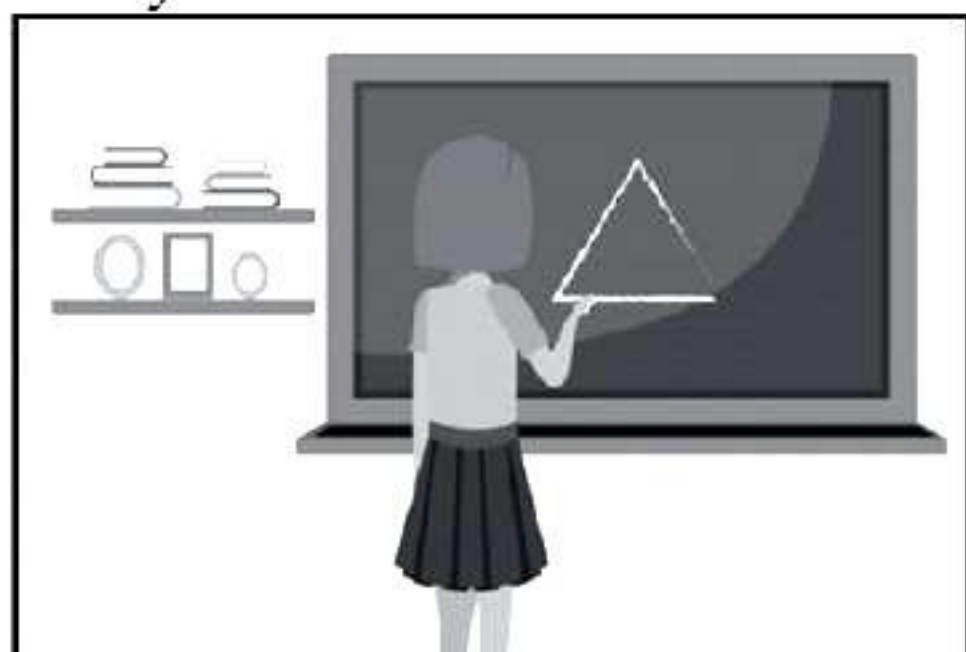
Q. 5. Domain and Range of $\cos^{-1}x =$

    (A) $(-1, 1)$, $(0, \pi)$ (B) $[-1, 1]$, $(0, \pi)$

    (C) $[-1, 1]$, $[0, \pi]$ (D) $(-1, 1)$, $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

Ans. Option (C) is correct.

  II. Read the following text and answer the following questions on the basis of the same:

    In the school project Sheetal was asked to construct a triangle and name it as ABC. Two angles A and B were given to be equal to $\tan^{-1}\dfrac{1}{2}$ and $\tan^{-1}\dfrac{1}{3}$ respectively.
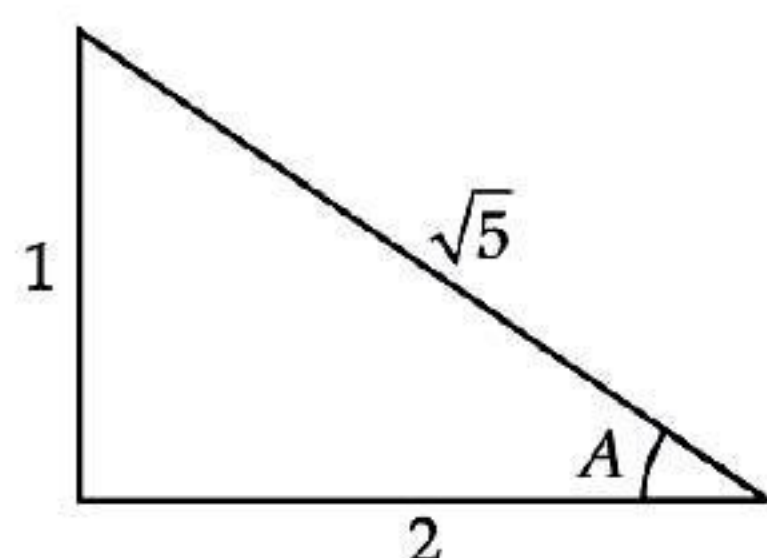

Q. 1. The value of $\sin A$ is _____.

    (A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$

    (C) $\dfrac{1}{\sqrt{5}}$ (D) $\dfrac{2}{\sqrt{5}}$

Ans. Option (C) is correct.

    Explanation:

$$A = \tan^{-1}\dfrac{1}{2}$$
$$\Rightarrow \quad \tan A = \dfrac{1}{2}$$
$$\therefore \quad \sin A = \dfrac{1}{\sqrt{5}}$$

Q. 2. $\cos(A + B + C) =$ _____.

    (A) 1 (B) 0

    (C) -1 (D) $\dfrac{1}{2}$

Ans. Option (C) is correct.

    Explanation: Since ABC is a triangle,
$$\therefore \quad A + B + C = 180°$$
$$\cos(A + B + C) = \cos 180°$$
$$= -1$$

Q. 3. If $B - \cos^{-1}x$, then $x -$ _____.

    (A) $\dfrac{1}{\sqrt{5}}$ (B) $\dfrac{3}{\sqrt{10}}$

    (C) $\dfrac{1}{\sqrt{10}}$ (D) $\dfrac{2}{\sqrt{5}}$

Ans. Option (B) is correct.

    Explanation:

    Given
$$B = \tan^{-1}\dfrac{1}{3}$$
$$\Rightarrow \quad \tan B = \dfrac{1}{3}$$
$$\therefore \quad \cos B = \dfrac{3}{\sqrt{10}}$$
$$B = \cos^{-1}\dfrac{3}{\sqrt{10}}$$
$$\Rightarrow \quad x = \dfrac{3}{\sqrt{10}}$$

Q. 4. If $A = \sin^{-1}x$; then the value of x is:

    (A) $\dfrac{1}{\sqrt{5}}$ (B) $\dfrac{2}{\sqrt{5}}$

    (C) $\dfrac{1}{\sqrt{10}}$ (D) $\dfrac{3}{\sqrt{10}}$

Ans. Option (A) is correct.

    Explanation:
$$A = \tan^{-1}\dfrac{1}{2}$$
$$\Rightarrow \quad \tan A = \dfrac{1}{2}$$
$$\therefore \quad \sin A = \dfrac{1}{\sqrt{5}}$$
$$A = \sin^{-1}\left(\dfrac{1}{\sqrt{5}}\right)$$
$$\Rightarrow \quad x = \dfrac{1}{\sqrt{5}}$$

Q. 5. The third angle, $\angle C =$ _____.

    (A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{2}$

    (C) $\dfrac{\pi}{3}$ (D) $\dfrac{3\pi}{4}$

Ans. Option (D) is correct.

    Explanation:
$$\angle C = \pi - (A + B)$$
$$= \pi - \dfrac{\pi}{4}$$
$$= \dfrac{3\pi}{4}$$

  III. Read the following text and answer the following questions on the basis of the same:

    The value of an inverse trigonometric functions which lies in the range of Principal branch is called the principal value of that inverse trigonometric functions.

Q. 1. Principal value of $\sin^{-1}\left(\dfrac{1}{2}\right)$ is

(A) $\dfrac{\pi}{6}$ 　　　　　(B) $\dfrac{\pi}{3}$

(C) $\dfrac{\pi}{4}$ 　　　　　(D) $\dfrac{\pi}{2}$

Ans. Option (A) is correct.

Explanation:

$$\sin^{-1}\left(\dfrac{1}{2}\right) = y$$

$$\sin y = \dfrac{1}{2}$$

Principal value branch of $\sin^{-1}$ is $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$

and 　　$\sin\left(\dfrac{\pi}{6}\right) = \dfrac{1}{2}$

$\Rightarrow$ Principal vlaue of $\sin^{-1}\left(\dfrac{1}{2}\right)$ is $\dfrac{\pi}{6}$

Q. 2. Principal value of $\tan^{-1}(1)$

(A) $\dfrac{\pi}{4}$ 　　　　　(B) $\dfrac{\pi}{2}$

(C) π 　　　　　(D) $\dfrac{\pi}{3}$

Ans. Option (A) is correct.

Explanation:

$$\tan^{-1}(1) = \tan^{-1}\left(\tan\dfrac{\pi}{4}\right)$$

$$= \dfrac{\pi}{4}$$

Q. 1. Principal value of $\cot^{-1}(\sqrt{3})$ is :

(A) $\dfrac{\pi}{3}$ 　　　　　(B) π

(C) $\dfrac{\pi}{6}$ 　　　　　(D) $\dfrac{\pi}{2}$

Ans. Option (C) is correct.

Explanation:

$$\cot^{-1}(\sqrt{3}) = \cot^{-1}\left(\cot\dfrac{\pi}{6}\right)$$

$$= \dfrac{\pi}{6}$$

Q. 4. Principal value of $\sin^{-1}(1) + \sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$ is

(A) 2π 　　　　　(B) π

(C) $\dfrac{3\pi}{4}$ 　　　　　(D) $\dfrac{\pi}{3}$

Ans. Option (C) is correct.

Explanation:

$$\sin^{-1}(1) + \sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = \dfrac{\pi}{2} + \dfrac{\pi}{4}$$

$$= \dfrac{3\pi}{4}$$

Q. 5. Principal value of $2\cos^{-1}(1) + 5\tan^{-1}(1)$ is:

(A) $\dfrac{3\pi}{4}$ 　　　　　(B) $\dfrac{\pi}{4}$

(C) $\dfrac{\pi}{2}$ 　　　　　(D) $\dfrac{5\pi}{4}$

Ans. Option (D) is correct.

Explanation:

$$2\cos^{-1}(1) + 5\tan^{-1}(1)$$

$$= 2 \times 0 + 5 \times \dfrac{\pi}{4}$$

$$= \dfrac{5\pi}{4}$$

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