in the prime factorisation of 45, The prime factor 5 appears one time in the prime factorisations of 40 and 45, we take it only once.

Therefore, required LCM = $(2 \times 2 \times 2 \times 2)\times(3 \times 3) \times 5 = 720$

LCM can also be found in the following way :

Example 11 : Find the LCM of 20, 25 and 30.

Solution : We write the numbers as follows in a row :

2	20	25	30	**(A)**
2	10	25	15	**(B)**
3	5	25	15	**(C)**
5	5	25	5	**(D)**
5	1	5	1	**(E)**
	1	1	1	

So, LCM = $2 \times 2 \times 3 \times 5 \times 5$.

(A) Divide by the least prime number which divides atleast one of the given numbers. Here, it is 2. The numbers like 25 are not divisible by 2 so they are written as such in the next row.

(B) Again divide by 2. Continue this till we have no multiples of 2.

(C) Divide by next prime number which is 3.

(D) Divide by next prime number which is 5.

(E) Again divide by 5.

3.10 Some Problems on HCF and LCM

We come across a number of situations in which we make use of the concepts of HCF and LCM. We explain these situations through a few examples.

Example 12 : Two tankers contain 850 litres and 680 litres of kerosene oil respectively. Find the maximum capacity of a container which can measure the kerosene oil of both the tankers when used an exact number of times.

Solution : The required container has to measure both the tankers in a way that the count is an exact number of times. So its capacity must be an exact divisor of the capacities of both the tankers. Moreover, this capacity should be **maximum**. Thus, the maximum capacity of such a container will be the HCF of 850 and 680.

It is found as follows :

2	850
5	425
5	85
17	17
	1

2	680
2	340
2	170
5	85
17	17
	1

Hence,

$850 = 2 \times 5 \times 5 \times 17 \quad = \boxed{2} \times \boxed{5} \times \boxed{17} \times 5$ and

$680 = 2 \times 2 \times 2 \times 5 \times 17 \quad = \boxed{2} \times \boxed{5} \times \boxed{17} \times 2 \times 2$

The common factors of 850 and 680 are 2, 5 and 17.

Thus, the HCF of 850 and 680 is $2 \times 5 \times 17 = 170$.

Therefore, maximum capacity of the required container is 170 litres.

It will fill the first container in 5 and the second in 4 refills.

Example 13 : In a morning walk, three persons step off together. Their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps?

Solution : The distance covered by each one of them is required to be the same as well as minimum. The required minimum distance each should walk would be the lowest common multiple of the measures of their steps. Can you describe why? Thus, we find the LCM of 80, 85 and 90. The LCM of 80, 85 and 90 is 12240.

The required minimum distance is 12240 cm.

Example 14 : Find the least number which when divided by 12, 16, 24 and 36 leaves a remainder 7 in each case.

Solution : We first find the LCM of 12, 16, 24 and 36 as follows :

2	12	16	24	36
2	6	8	12	18
2	3	4	6	9
2	3	2	3	9
3	3	1	3	9
3	1	1	1	3
	1	1	1	1

Thus, LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$

144 is the least number which when divided by the given numbers will leave remainder 0 in each case. But we need the least number that leaves remainder 7 in each case.

Therefore, the required number is 7 more than 144. The required least number = 144 + 7 = 151.

EXERCISE 3.7

1. Renu purchases two bags of fertiliser of weights 75 kg and 69 kg. Find the maximum value of weight which can measure the weight of the fertiliser exact number of times.
2. Three boys step off together from the same spot. Their steps measure 63 cm, 70 cm and 77 cm respectively. What is the minimum distance each should cover so that all can cover the distance in complete steps?
3. The length, breadth and height of a room are 825 cm, 675 cm and 450 cm respectively. Find the longest tape which can measure the three dimensions of the room exactly.
4. Determine the smallest 3-digit number which is exactly divisible by 6, 8 and 12.
5. Determine the greatest 3-digit number exactly divisible by 8, 10 and 12.
6. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change simultaneously again?
7. Three tankers contain 403 litres, 434 litres and 465 litres of diesel respectively. Find the maximum capacity of a container that can measure the diesel of the three containers exact number of times.
8. Find the least number which when divided by 6, 15 and 18 leave remainder 5 in each case.
9. Find the smallest 4-digit number which is divisible by 18, 24 and 32.
10. Find the LCM of the following numbers :

    (a) 9 and 4 (b) 12 and 5 (c) 6 and 5 (d) 15 and 4

    Observe a common property in the obtained LCMs. Is LCM the product of two numbers in each case?
11. Find the LCM of the following numbers in which one number is the factor of the other.

    (a) 5, 20 (b) 6, 18 (c) 12, 48 (d) 9, 45

    What do you observe in the results obtained?

What have we discussed?

1. We have discussed multiples, divisors, factors and have seen how to identify factors and multiples.
2. We have discussed and discovered the following :
   (a) A factor of a number is an exact divisor of that number.
   (b) Every number is a factor of itself. 1 is a factor of every number.
   (c) Every factor of a number is less than or equal to the given number.
   (d) Every number is a multiple of each of its factors.
   (e) Every multiple of a given number is greater than or equal to that number.
   (f) Every number is a multiple of itself.
3. We have learnt that –
   (a) The number other than 1, with only factors namely 1 and the number itself, is a prime number. Numbers that have more than two factors are called composite numbers. Number 1 is neither prime nor composite.
   (b) The number 2 is the smallest prime number and is even. Every prime number other than 2 is odd.
   (c) Two numbers with only 1 as a common factor are called co-prime numbers.
   (d) If a number is divisible by another number then it is divisible by each of the factors of that number.
   (e) A number divisible by two co-prime numbers is divisible by their product also.
4. We have discussed how we can find just by looking at a number, whether it is divisible by small numbers 2,3,4,5,8,9 and 11. We have explored the relationship between digits of the numbers and their divisibility by different numbers.
   (a) Divisibility by 2,5 and 10 can be seen by just the last digit.
   (b) Divisibility by 3 and 9 is checked by finding the sum of all digits.
   (c) Divisibility by 4 and 8 is checked by the last 2 and 3 digits respectively.
   (d) Divisibility of 11 is checked by comparing the sum of digits at odd and even places.
5. We have discovered that if two numbers are divisible by a number then their sum and difference are also divisible by that number.
6. We have learnt that –
   (a) The Highest Common Factor (HCF) of two or more given numbers is the highest of their common factors.
   (b) The Lowest Common Multiple (LCM) of two or more given numbers is the lowest of their common multiples.

Basic Geometrical Ideas

Chapter 4

4.1 Introduction

Geometry has a long and rich history. The term 'Geometry' is the English equivalent of the Greek word '*Geometron*'. '*Geo*' means Earth and '*metron*' means Measurement. According to historians, the geometrical ideas shaped up in ancient times, probably due to the need in art, architecture and measurement. These include occasions when the boundaries of cultivated lands had to be marked without giving room for complaints. Construction of magnificent palaces, temples, lakes, dams and cities, art and architecture propped up these ideas. Even today geometrical ideas are reflected in all forms of art, measurements, architecture, engineering, cloth designing etc. You observe and use different objects like boxes, tables, books, the tiffin box you carry to your school for lunch, the ball with which you play and so on. All such objects have different shapes. The ruler which you use, the pencil with which you write are straight. The pictures of a bangle, the one rupee coin or a ball appear round.

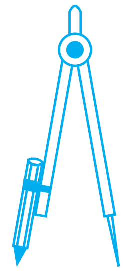
Here, you will learn some interesting facts that will help you know more about the shapes around you.

4.2 Points

By a sharp tip of the pencil, mark a dot on the paper. Sharper the tip, thinner will be the dot. This almost invisible tiny dot will give you an idea of a point.

A point determines a location.

These are some models for a point :



The tip of a compass

The sharpened end of a pencil

The pointed end of a needle.

If you mark three points on a paper, you would be required to distinguish them. For this they are denoted by a single capital letter like A,B,C.

•B

•A

•C

These points will be read as point A, point B and point C.

Of course, the dots have to be invisibly thin.

Try These

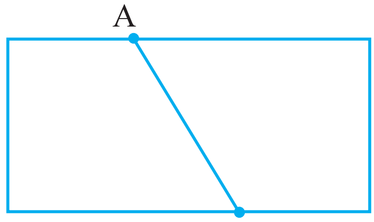
1. With a sharp tip of the pencil, mark four points on a paper and name them by the letters A,C,P,H. Try to name these points in different ways. One such way could be this

   A• •C

   P• •H

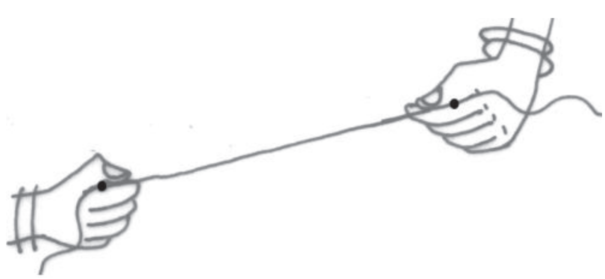
2. A star in the sky also gives us an idea of a point. Identify at least five such situations in your daily life.

4.3 A Line Segment

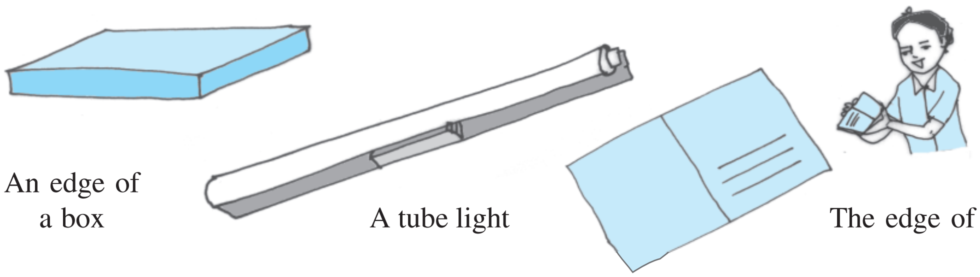


Fold a piece of paper and unfold it. Do you see a fold? This gives the idea of a line segment. It has two end points A and B.

Take a thin thread. Hold its two ends and stretch it without a slack. It represents a line segment. The ends held by hands are the end points of the line segment.



The following are some models for a line segment :



Try to find more examples for line segments from your surroundings.

Mark any two points A and B on a sheet of paper. Try to connect A to B by all possible routes. (Fig 4.1)

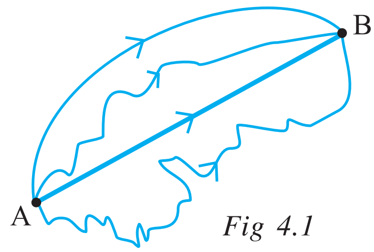


Fig 4.1

What is the shortest route from A to B?

This shortest join of point A to B (including A and B) shown here is a line segment. It is denoted by $\overline{AB}$ or $\overline{BA}$. The points A and B are called the end points of the segment.

Try These

1. Name the line segments in the figure 4.2. Is A, the end point of each line segment?

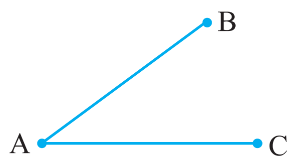


Fig 4.2

4.4 A Line

Imagine that the line segment from A to B (i.e. $\overline{AB}$) is extended beyond A in one direction and beyond B in the other direction without any end (see figure). You now get a model for a line.

Do you think you can draw a complete picture of a line? No. (Why?)

A line through two points A and B is written as $\overleftrightarrow{AB}$. It extends indefinitely in both directions. So it contains a countless number of points. (Think about this).

Two points are enough to fix a line. We say 'two points determine a line'.

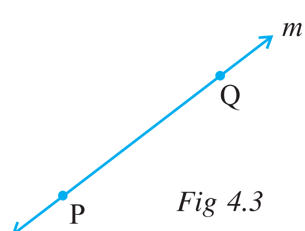


Fig 4.3

The adjacent diagram (Fig 4.3) is that of a line PQ written as $\overleftrightarrow{PQ}$. Sometimes a line is denoted by a letter like *l*, *m*.

4.5 Intersecting Lines

Look at the diagram (Fig 4.4). Two lines l_1 and l_2 are shown. Both the lines pass through point P. We say l_1 and l_2 intersect at P. If two lines have one common point, they are called *intersecting lines*.

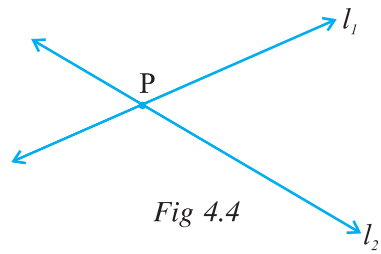
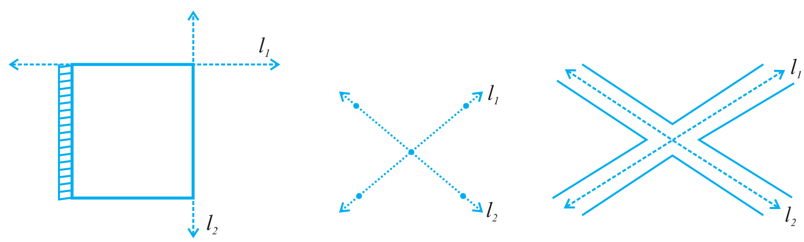


Fig 4.4

The following are some models of a pair of intersecting lines (Fig 4.5) :

Try to find out some more models for a pair of intersecting lines.



Two adjacement edges of your notebook | The letter X of the English alphabet | Crossing-roads

Fig 4.5

Do This

Take a sheet of paper. Make two folds (and crease them) to represent a pair of intersecting lines and discuss :

(a) Can two lines intersect in more than one point?

(b) Can more than two lines intersect in one point?

4.6 Parallel Lines

Let us look at this table (Fig 4.6). The top ABCD is flat. Are you able to see some points and line segments?

Are there intersecting line segments?

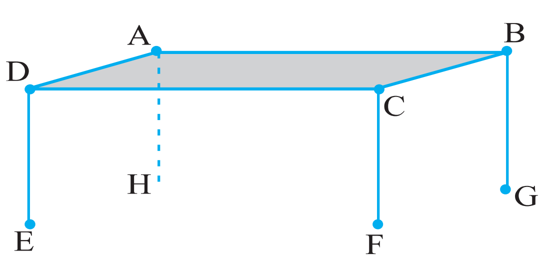


Fig 4.6

Yes, $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ intersect at the point B.

Which line segments intersect at A? at C? at D?

Do the lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{CD}$ intersect?

Do the lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ intersect?

You find that on the table's surface there are line segment which will not meet, however far they are extended. $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ form one such pair. Can you identify one more such pair of lines (which do not meet) on the top of the table?

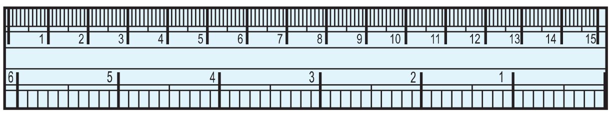
Think, discuss and write

Where else do you see parallel lines? Try to find ten examples.

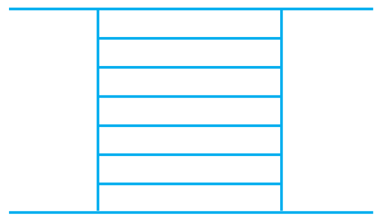
If two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, we write $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

If two lines l_1 and l_2 are parallel, we write $l_1 \parallel l_2$.

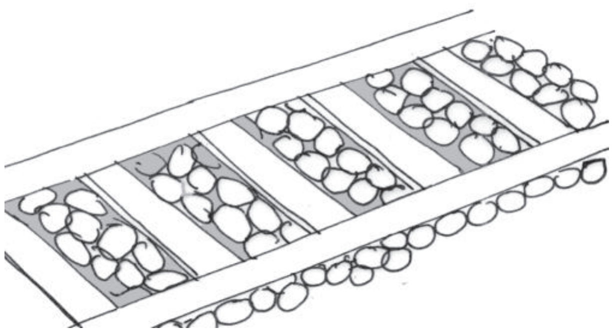
Can you identify parrallel lines in the following figures?



The opposite edges of ruler (scale)



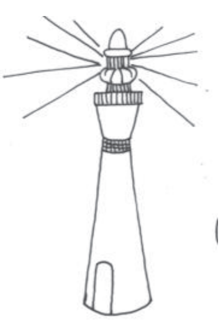
The cross-bars of this window



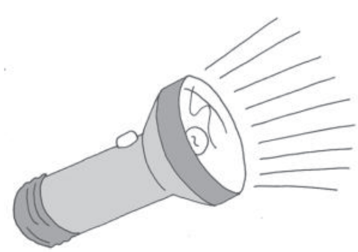
Rail lines

Lines like these which do not meet are said to be parallel; and are called **parallel lines**.

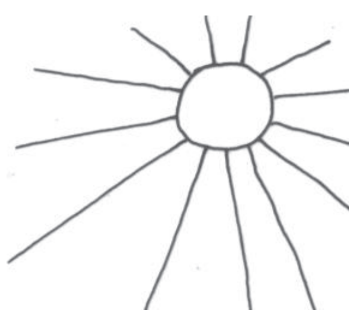
4.7 Ray



Beam of light from a light house



Ray of light from a torch



Sun rays

The following are some models for a ray :

A *ray* is a portion of a line. It starts at one point (called starting point) and goes endlessly in a direction.

Look at the diagram (Fig 4.7) of ray shown here. Two points are shown on the ray. They are (a) A, the starting point (b) P, a point on the path of the ray.

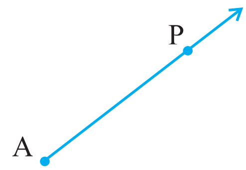


Fig 4.7

We denote it by $\overrightarrow{AP}$.

Think, discuss and write

If $\overrightarrow{PQ}$ is a ray,

(a) What is its starting point?

(b) Where does the point Q lie on the ray?

(c) Can we say that Q is the starting point of this ray?

Try These

1. Name the rays given in this picture (Fig 4.8).
2. Is T a starting point of each of these rays?

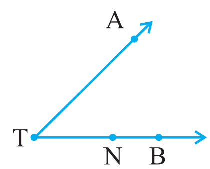


Fig 4.8

Here is a ray $\overrightarrow{OA}$ (Fig 4.9). It starts at O and passes through the point A. It also passes through the point B.

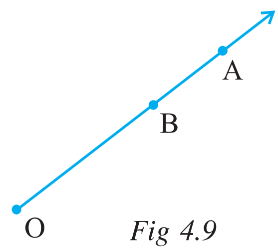


Fig 4.9

Can you also name it as $\overrightarrow{OB}$? Why?

$\overrightarrow{OA}$ and $\overrightarrow{OB}$ are same here.

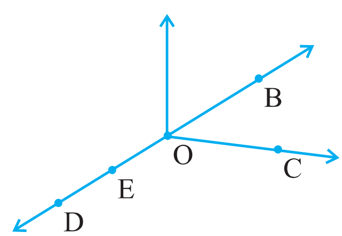
Can we write $\overrightarrow{OA}$ as $\overrightarrow{AO}$? Why or why not?

Draw five rays and write appropriate names for them.

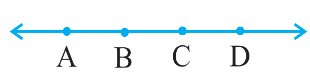
What do the arrows on each of these rays show?

EXERCISE 4.1

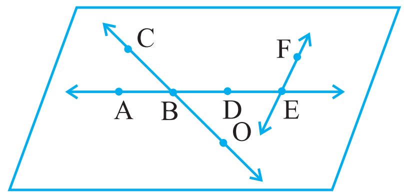
1. Use the figure to name :
   (a) Five points
   (b) A line
   (c) Four rays
   (d) Five line segments



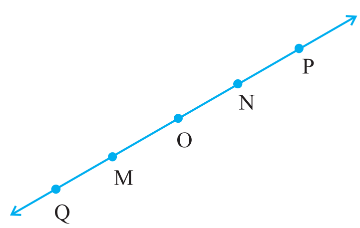
2. Name the line given in all possible (twelve) ways, choosing only two letters at a time from the four given.



3. Use the figure to name :
   (a) Line containing point E.
   (b) Line passing through A.
   (c) Line on which O lies
   (d) Two pairs of intersecting lines.



4. How many lines can pass through (a) one given point? (b) two given points?
5. Draw a rough figure and label suitably in each of the following cases:
   (a) Point P lies on $\overline{AB}$.
   (b) $\overleftrightarrow{XY}$ and $\overleftrightarrow{PQ}$ intersect at M.
   (c) Line l contains E and F but not D.
   (d) $\overleftrightarrow{OP}$ and $\overleftrightarrow{OQ}$ meet at O.
6. Consider the following figure of line $\overleftrightarrow{MN}$. Say whether following statements are true or false in context of the given figure.
   (a) Q, M, O, N, P are points on the line $\overleftrightarrow{MN}$.
   (b) M, O, N are points on a line segment $\overline{MN}$.
   (c) M and N are end points of line segment $\overline{MN}$.
   (d) O and N are end points of line segment $\overline{OP}$.
   (e) M is one of the end points of line segment $\overline{QO}$.
   (f) M is point on ray $\overrightarrow{OP}$.
   (g) Ray $\overrightarrow{OP}$ is different from ray $\overrightarrow{QP}$.
   (h) Ray $\overrightarrow{OP}$ is same as ray $\overrightarrow{OM}$.
   (i) Ray $\overrightarrow{OM}$ is not opposite to ray $\overrightarrow{OP}$.
   (j) O is not an initial point of $\overrightarrow{OP}$
   (k) N is the initial point of $\overrightarrow{NP}$ and $\overrightarrow{NM}$.



4.8 Curves

Have you ever taken a piece of paper and just doodled? The pictures that are results of your doodling are called *curves*.

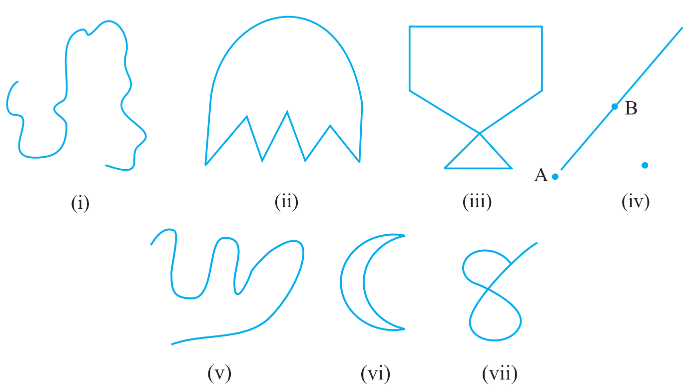


Fig 4.10

You can draw some of these drawings without lifting the pencil from the paper and without the use of a ruler. These are all curves (Fig 4.10).

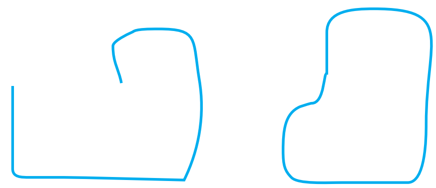
'Curve' in everyday usage means "not straight". In Mathematics, a curve can be straight like the one shown in fig 4.10 (iv).

Observe that the curves (iii) and (vii) in Fig 4.10 cross themselves, whereas the curves (i), (ii), (v) and (vi) in Fig 4.10 do not. If a curve does not cross itself, then it is called a **simple curve**.

Draw five more simple curves and five curves that are not simple.

Consider these now (Fig 4.11).

What is the difference between these two? The first i.e. Fig 4.11 (i) is an **open curve** and the second i.e. Fig 4.11(ii) is a **closed curve**. Can you identify some closed and open curves from the figures Fig 4.10 (i), (ii), (v), (vi)? Draw five curves each that are open and closed.



(i) (ii)

Fig 4.11

Position in a figure

A court line in a tennis court divides it into three parts : inside the line, on the line and outside the line. You cannot enter inside without crossing the line.

A compound wall separates your house from the road. You talk about 'inside' the compound, 'on' the boundary of the compound and 'outside' the compound.

In a closed curve, thus, there are three parts.

(i) interior ('inside') of the curve

(ii) boundary ('on') of the curve and

(iii) exterior ('outside') of the curve.

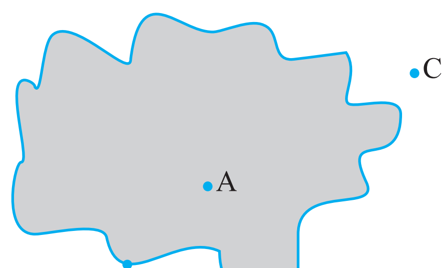


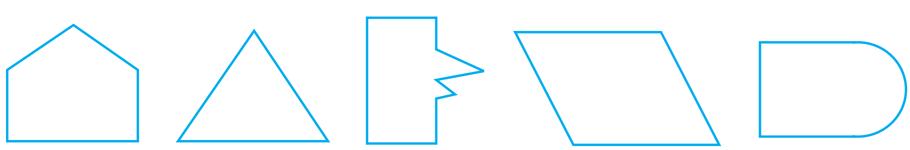
Fig 4.12

In the figure 4.12, A is in the interior, C is in the exterior and B is on the curve.

The interior of a curve together with its boundary is called its "**region**".

4.9 Polygons

Look at these figures 4.13 (i), (ii), (iii), (iv) and (v).



(i) (ii) (iii) (iv) (v)

Fig 4.13

What can you say? Are they closed? How does each one of them differ from the other? (i), (ii), (iii) and (iv) are special because they are made up entirely of line segments. They are called **polygons.**

So, a figure is a polygon if it is a simple closed figure made up entirely of line segments. Draw ten differently shaped polygons.

Do This

Try to form a polygon with

1. Five matchsticks.
2. Four matchsticks.
3. Three matchsticks.
4. Two matchsticks.

In which case was it not possible? Why?

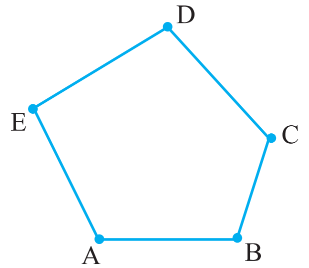


Fig 4.14

Sides, vertices and diagonals

Examine the figure given here (Fig 4.14).

Give justification to call it a polygon.

The line segments forming a polygon are called its *sides*.

What are the sides of polygon ABCDE? (Note how the corners are named in order.)

Sides are $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}$ and $\overline{EA}$.

The meeting point of a pair of sides is called its *vertex*.

Sides $\overline{AE}$ and $\overline{ED}$ meet at E, so E is a vertex of the polygon ABCDE. Points B and C are its other vertices. Can you name the sides that meet at these points?

Can you name the other vertices of the above polygon ABCDE?

Any two sides with a common end point are called the *adjacent sides* of the polygon.

Are the sides $\overline{AB}$ and $\overline{BC}$ adjacent? How about $\overline{AE}$ and $\overline{DC}$?

The end points of the same side of a polygon are called the *adjacent vertices*. Vertices E and D are adjacent, whereas vertices A and D are not adjacent vertices. Do you see why?

Consider the pairs of vertices which are not adjacent. The joins of these vertices are called the diagonals of the polygon.

In the figure 4.15, $\overline{AC}, \overline{AD}, \overline{BD}, \overline{BE}$ and $\overline{CE}$ are diagonals.

Is $\overline{BC}$ a diagonal, Why or why not?

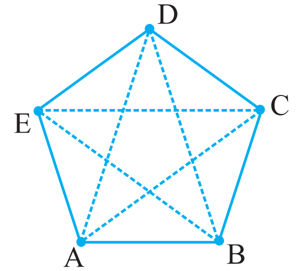


Fig 4.15

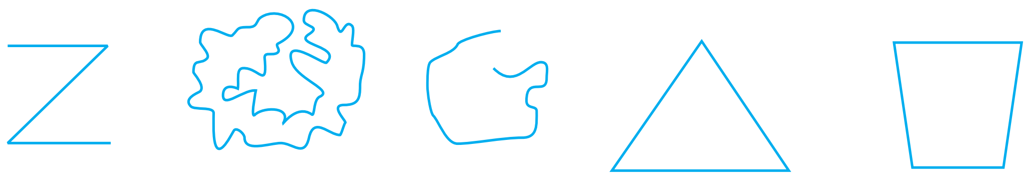
If you try to join adjacent vertices, will the result be a diagonal?

Name all the sides, adjacent sides, adjacent vertices of the figure ABCDE (Fig 4.15).

Draw a polygon ABCDEFGH and name all the sides, adjacent sides and vertices as well as the diagonals of the polygon.

EXERCISE 4.2

1. Classify the following curves as (i) Open or (ii) Closed.



(a) (b) (c) (d) (e)

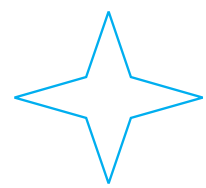
2. Draw rough diagrams to illustrate the following :

   (a) Open curve (b) Closed curve.

3. Draw any polygon and shade its interior.

4. Consider the given figure and answer the questions :

   (a) Is it a curve? (b) Is it closed?



5. Illustrate, if possible, each one of the following with a rough diagram:

   (a) A closed curve that is not a polygon.

   (b) An open curve made up entirely of line segments.

   (c) A polygon with two sides.

4.10 Angles

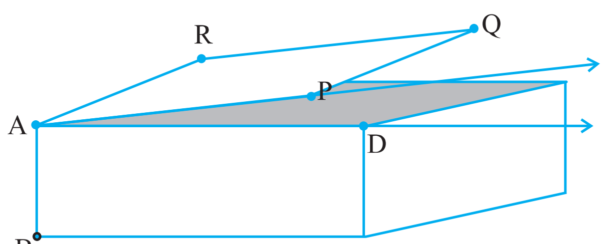


Fig 4.16

Angles are made when corners are formed.

Here is a picture (Fig 4.16) where the top of a box is like a hinged lid. The edges AD of the box and AP of the door can be imagined as two rays $\overrightarrow{AD}$ and $\overrightarrow{AP}$. These two rays have a common end point A. The two rays here together are said to form an angle.

An angle is made up of two rays starting from a common end point.

The two rays forming the angle are called the *arms* or *sides* of the angle.

The common end point is the *vertex* of the angle.

This is an angle formed by rays $\overrightarrow{OP}$ and $\overrightarrow{OQ}$

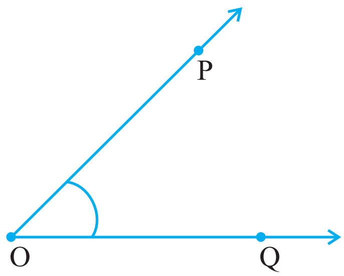


Fig 4.17

(Fig 4.17). To show this we use a small curve at the vertex. (see Fig 4.17). O is the vertex. What are the sides? Are they not $\overrightarrow{OP}$ and $\overrightarrow{OQ}$? How can we name this angle? We can simply say that it is an angle at O. To be more specific we identify some two points, one on each side and the vertex to name the angle. Angle POQ is thus a better way of naming the angle. We denote this by $\angle POQ$.

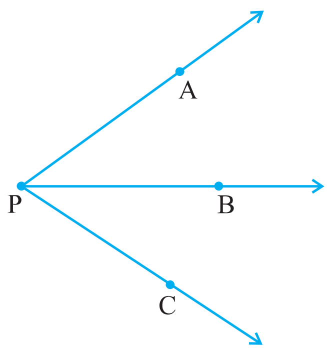


Fig 4.18

Think, discuss and write

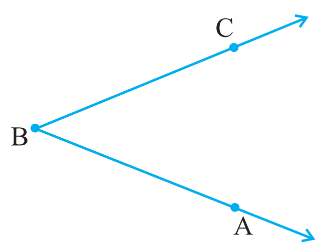
Look at the diagram (Fig 4.18). What is the name of the angle? Shall we say $\angle P$? But then which one do we mean? By $\angle P$ what do we mean? Is naming an angle by vertex helpful here? Why not?

By $\angle P$ we may mean $\angle APB$ or $\angle CPB$ or even $\angle APC$! We need more information.

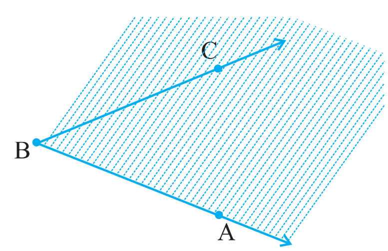
Note that in specifying the angle, the vertex is always written as the middle letter.

Take any angle, say $\angle ABC$.

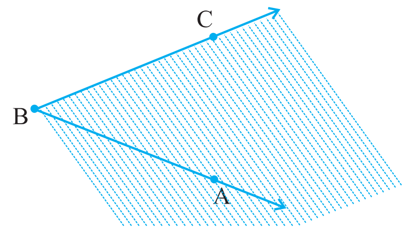
Do This



Shade that portion of the paper bordering $\overrightarrow{BA}$ and where $\overrightarrow{BC}$ lies.



Now shade in a different colour the portion of the paper bordering $\overrightarrow{BC}$ and where $\overrightarrow{BA}$ lies.



The portion common to both shadings is called the interior of $\angle ABC$ (Fig 4.19). (Note that **the interior** is not a restricted area; it extends indefinitely since the two sides extend indefinitely).

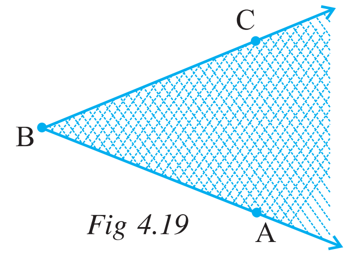


Fig 4.19

In this diagram (Fig 4.20), X is in the interior of the angle, Z is not in the interior but in the exterior of the angle; and S is on the $\angle PQR$. Thus, the angle also has three parts associated with it.

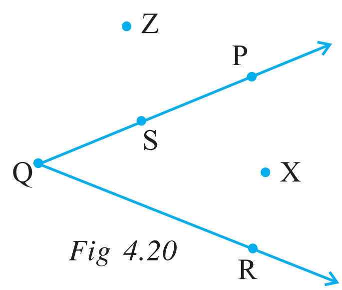
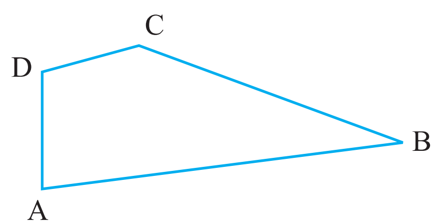


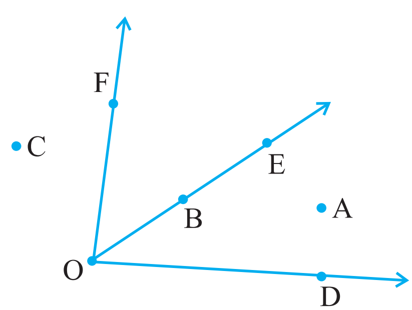
Fig 4.20

EXERCISE 4.3

1. Name the angles in the given figure.



2. In the given diagram, name the point(s)
   (a) In the interior of $\angle DOE$
   (b) In the exterior of $\angle EOF$
   (c) On $\angle EOF$



3. Draw rough diagrams of two angles such that they have
   (a) One point in common.
   (b) Two points in common.
   (c) Three points in common.
   (d) Four points in common.
   (e) One ray in common.

4.11 Triangles

A triangle is a three-sided polygon. In fact, it is the polygon with the least number of sides.

Look at the triangle in the diagram (Fig 4.21). We write ΔABC instead of writing "Triangle ABC".

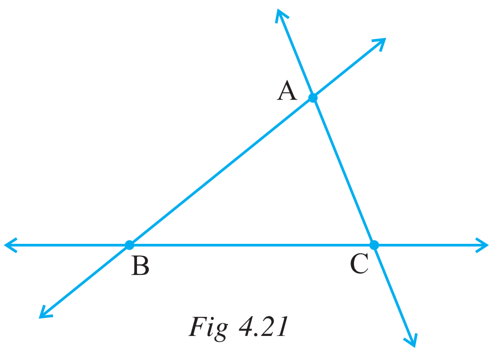


Fig 4.21

In ΔABC, how many sides and how many angles are there?

The three sides of the triangle are $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$. The three angles are $\angle BAC$, $\angle BCA$ and $\angle ABC$. The points A, B and C are called the vertices of the triangle.

Being a polygon, a triangle has an exterior and an interior. In the figure 4.22, P is in the interior of the triangle, R is in the exterior and Q on the triangle.

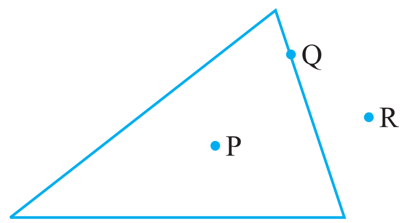


Fig 4.22

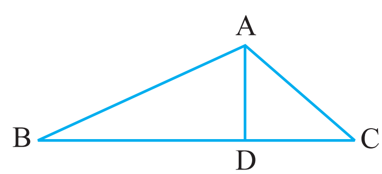
EXERCISE 4.4

1. Draw a rough sketch of a triangle ABC. Mark a point P in its interior and a point Q in its exterior. Is the point A in its exterior or in its interior?
2. (a) Identify three triangles in the figure.

   (b) Write the names of seven angles.

   (c) Write the names of six line segments.

   (d) Which two triangles have $\angle B$ as common?



4.12 Quadrilaterals

A four sided polygon is a *quadrilateral*. It has 4 sides and 4 angles. As in the case of a triangle, you can visualise its interior too.

Note the cyclic manner in which the vertices are named.

This quadrilateral ABCD (Fig 4.23) has four sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$. It has four angles $\angle A$, $\angle B$, $\angle C$ and $\angle D$.

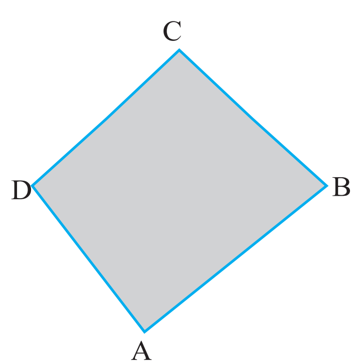
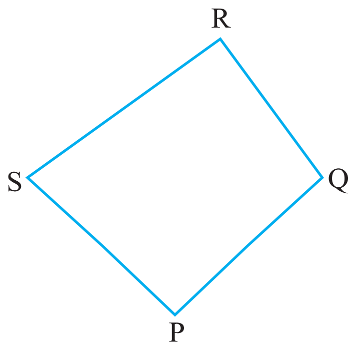
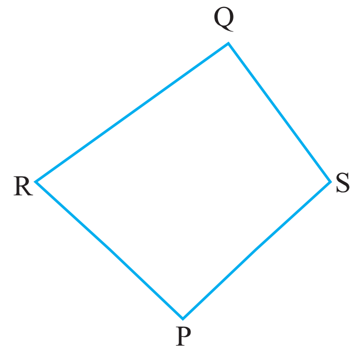


Fig 4.23

In any quadrilateral ABCD, $\overline{AB}$ and $\overline{BC}$ are



This is quadrilateral PQRS.

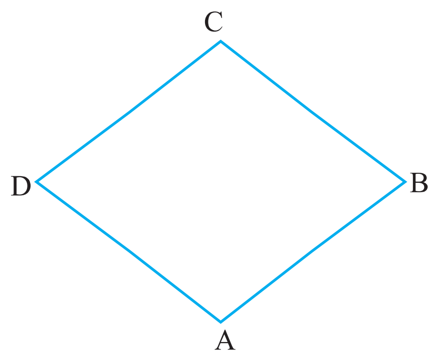


Is this quadrilateral PQRS?

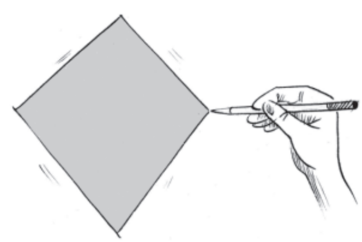
adjacent sides. Can you write other pairs of adjacent sides?

$\overline{AB}$ and $\overline{DC}$ are *opposite sides*; Name the other pair of opposite sides.

$\angle A$ and $\angle C$ are said to be *opposite angles*; similarly, $\angle D$ and $\angle B$ are opposite angles. Naturally $\angle A$ and $\angle B$ are *adjacent angles*. You can now list other pairs of adjacent angles.



EXERCISE 4.5



1. Draw a rough sketch of a quadrilateral PQRS. Draw its diagonals. Name them. Is the meeting point of the diagonals in the interior or exterior of the quadrilateral?
2. Draw a rough sketch of a quadrilateral KLMN. State,
   (a) two pairs of opposite sides,
   (b) two pairs of opposite angles,
   (c) two pairs of adjacent sides,
   (d) two pairs of adjacent angles.
3. **Investigate :**
   Use strips and fasteners to make a triangle and a quadrilateral.
   Try to push inward at any one vertex of the triangle. Do the same to the quadrilateral.
   Is the triangle distorted? Is the quadrilateral distorted? Is the triangle rigid?
   Why is it that structures like electric towers make use of triangular shapes and not quadrilaterals?

4.13 Circles

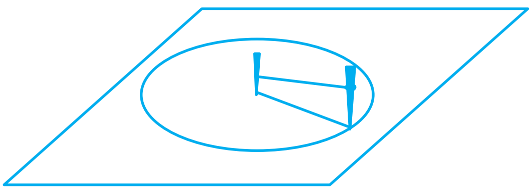
In our environment, you find many things that are round, a wheel, a bangle, a coin etc. We use the round shape in many ways. It is easier to roll a heavy steel tube than to drag it.

A circle is a simple closed curve which is not a polygon. It has some very special properties.

- Place a bangle or any round shape; trace around to get a circular shape.
- If you want to make a circular garden, how will you proceed?

Do This

Take two sticks and a piece of rope. Drive one stick into the ground. This is the centre of the proposed circle. Form two loops, one at each end of the rope. Place one loop around the stick at the centre. Put the other around the other stick. Keep the sticks vertical to the ground. Keep the rope taut all the time and trace the path. You get a circle.



Naturally every point on the circle is at equal distance from the centre.

Parts of a circle

Here is a circle with *centre* C (Fig 4.24)

A, P, B, M are points on the circle. You will see that CA = CP = CB = CM.

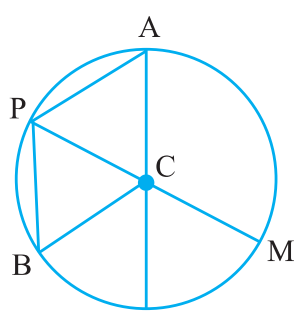


Fig 4.24

Each of the segments $\overline{CA}$, $\overline{CP}$, $\overline{CB}$, $\overline{CM}$ is *radius* of the circle. The radius is a line segment that connects the centre to a point on the circle. $\overline{CP}$ and $\overline{CM}$ are radii (plural of 'radius') such that C, P, M are in a line. $\overline{PM}$ is known as *diameter* of the circle.

Is a diameter double the size of a radius? Yes.

$\overline{PB}$ is a *chord* connecting two points on a circle. Is $\overline{PM}$ also a chord?

An arc is a portion of circle.

If P and Q are two points you get the arc PQ. We write it as $\overset{\frown}{PQ}$ (Fig 4.25).

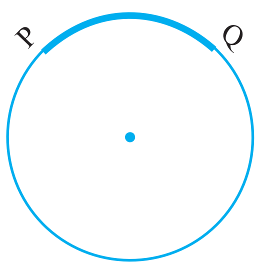


Fig 4.25

As in the case of any simple closed curve you can think of the *interior* and *exterior* of a circle. A region in the interior of a circle enclosed by an arc on one side and a pair of radii on the other two sides is called a *sector* (Fig 4.26).

A region in the interior of a circle enclosed by a chord and an arc is called a *segment* of the circle.

Take any circular object. Use a thread and wrap it around the object once. The length of the thread is the distance covered to travel around the object once. What does this length denote?

The distance around a circle is its *circumference*.

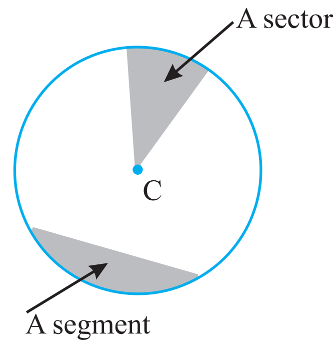


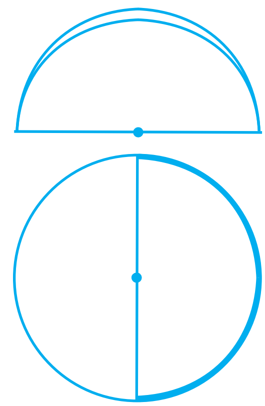
Fig 4.26

- Take a circular sheet. Fold it into two halves. Crease the fold and open up. Do you find that

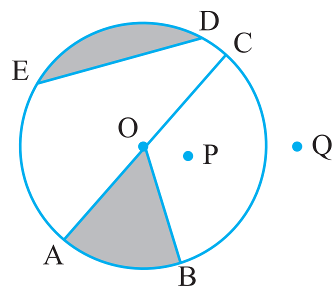
Do This

The circular region is halved by the diameter?

A diameter of a circle divides it into two equal parts; each part is a *semi-circle*. A semi-circle is half of a circle, with the end points of diameter as part of the boundary.



EXERCISE 4.6



1. From the figure, identify :
   (a) the centre of circle (b) three radii
   (c) a diameter (d) a chord
   (e) two points in the interior (f) a point in the exterior
   (g) a sector (h) a segment
2. (a) Is every diameter of a circle also a chord?
   (b) Is every chord of a circle also a diameter?
3. Draw any circle and mark
   (a) its centre (b) a radius
   (c) a diameter (d) a sector
   (e) a segment (f) a point in its interior
   (g) a point in its exterior (h) an arc
4. Say true or false :
   (a) Two diameters of a circle will necessarily intersect.
   (b) The centre of a circle is always in its interior.

What have we discussed?

1. A point determines a location. It is usually denoted by a capital letter.
2. A line segment corresponds to the shortest distance between two points. The line segment joining points A and B is denoted by $\overline{AB}$.
   $\overline{AB}$ and $\overline{BA}$ denote the same line segment.

3. A line is obtained when a line segment like $\overline{AB}$ is extended on both sides indefinitely; it is denoted by $\overleftrightarrow{AB}$ or sometimes by a single small letter like *l*.
4. Two distinct lines meeting at a point are called *intersecting lines*.
5. Two lines in a plane are said to be parallel if they do not meet.
6. A ray is a portion of line starting at a point and going in one direction endlessly.
7. Any drawing (straight or non-straight) done without lifting the pencil may be called a curve. In this sense, a line is also a curve.
8. A simple curve is one that does not cross itself.
9. A curve is said to be closed if its ends are joined; otherwise it is said to be open.
10. A polygon is a simple closed curve made up of line segments. Here,
    (i) The line segments are the sides of the polygon.
    (ii) Any two sides with a common end point are adjacent sides.
    (iii) The meeting point of a pair of sides is called a *vertex*.
    (iv) The end points of the same side are adjacent vertices.
    (v) The join of any two non-adjacent vertices is a diagonal.
11. An angle is made up of two rays starting from a common end point.

    Two rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ make $\angle AOB$ (or also called $\angle BOA$).

    An angle leads to three divisions of a region:

    On the angle, the interior of the angle and the exterior of the angle.
12. A triangle is a three-sided polygon.
13. A quadrilateral is a four-sided polygon. (It should be named cyclically).

    In any quadrilateral ABCD, $\overline{AB}$ & $\overline{DC}$ and $\overline{AD}$ & $\overline{BC}$ are pairs of opposite sides. $\angle A$ & $\angle C$ and $\angle B$ & $\angle D$ are pairs of opposite angles. $\angle A$ is adjacent to $\angle B$ & $\angle D$; similar relations exist for other three angles.
14. A circle is the path of a point moving at the same distance from a fixed point.

    The fixed point is the centre, the fixed distance is the radius and the distance around the circle is the *circumference*.

    A *chord* of a circle is a line segment joining any two points on the circle.

    A *diameter* is a chord passing through the centre of the circle.

    A sector is the *region* in the interior of a circle enclosed by an arc on one side and a pair of radii on the other two sides.

    A *segment* of a circle is a region in the interior of the circle enclosed by an arc and a chord.

    The diameter of a circle divides it into *two semi-circles*.

Understanding Elementary Shapes

Chapter 5

5.1 Introduction

All the shapes we see around us are formed using curves or lines. We can see corners, edges, planes, open curves and closed curves in our surroundings. We organise them into line segments, angles, triangles, polygons and circles. We find that they have different sizes and measures. Let us now try to develop tools to compare their sizes.

5.2 Measuring Line Segments

We have drawn and seen so many line segments. A triangle is made of three, a quadrilateral of four line segments.

A line segment is a fixed portion of a line. This makes it possible to measure a line segment. This measure of each line segment is a unique number called its "length". We use this idea to compare line segments.

To compare any two line segments, we find a relation between their lengths. This can be done in several ways.

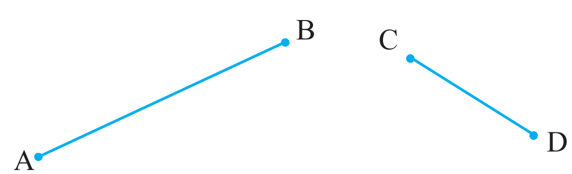
(i) Comparison by observation:

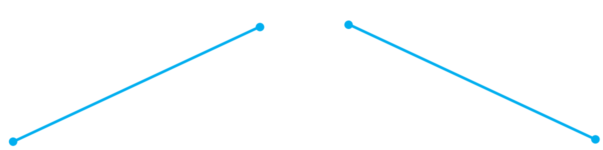
By just looking at them can you tell which one is longer?

You can see that $\overline{AB}$ is longer.

But you cannot always be sure about your usual judgment.

For example, look at the adjoining segments :

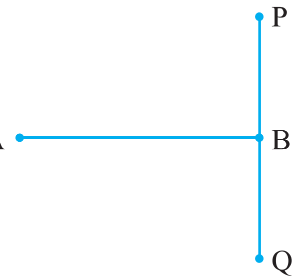




The difference in lengths between these two may not be obvious. This makes other ways of comparing necessary.

In this adjacent figure, $\overline{AB}$ and $\overline{PQ}$ have the same lengths. This is not quite obvious.

So, we need better methods of comparing line segments.



(ii) Comparison by Tracing

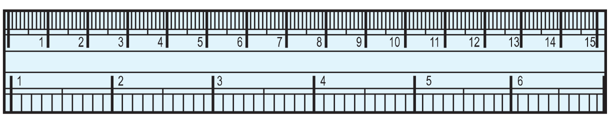
To compare $\overline{AB}$ and $\overline{CD}$, we use a tracing paper, trace $\overline{CD}$ and place the traced segment on $\overline{AB}$.

Can you decide now which one among $\overline{AB}$ and $\overline{CD}$ is longer?

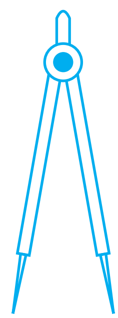
The method depends upon the accuracy in tracing the line segment. Moreover, if you want to compare with another length, you have to trace another line segment. This is difficult and you cannot trace the lengths everytime you want to compare them.

(iii) Comparison using Ruler and a Divider

Have you seen or can you recognise all the instruments in your instrument box? Among other things, you have a ruler and a divider.



Ruler

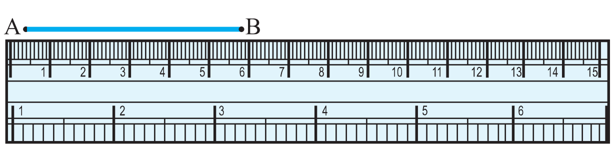


Divider

Note how the ruler is marked along one of its edges. It is divided into 15 parts. Each of these 15 parts is of length 1cm.

Each centimetre is divided into 10subparts. Each subpart of the division of a cm is 1mm.

> 1 mm is 0.1 cm.
> 2 mm is 0.2 cm and so on .
> 2.3 cm will mean 2 cm and 3 mm.



How many millimetres make one centimetre? Since 1cm = 10 mm, how will we write 2 cm? 3mm? What do we mean by 7.7 cm?

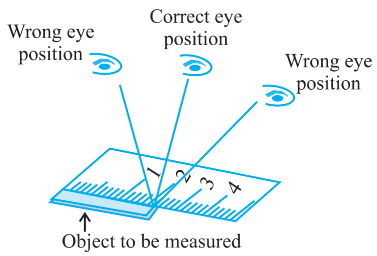
Place the zero mark of the ruler at A. Read the mark against B. This gives the length of $\overline{AB}$. Suppose the length is 5.8 cm, we may write,

Length AB = 5.8 cm or more simply as AB = 5.8 cm.

There is room for errors even in this procedure. The thickness of the ruler may cause difficulties in reading off the marks on it.

Think, discuss and write

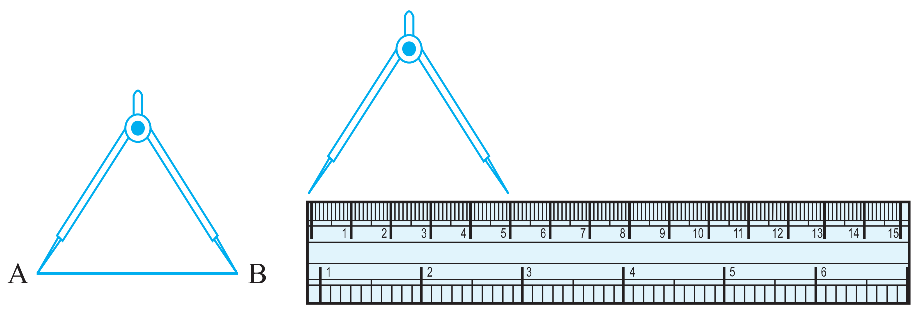
1. What other errors and difficulties might we face?
2. What kind of errors can occur if viewing the mark on the ruler is not proper? How can one avoid it?



Positioning error

To get correct measure, the eye should be correctly positioned, just vertically above the mark. Otherwise errors can happen due to angular viewing.

Can we avoid this problem? Is there a better way?
Let us use the divider to measure length.



Open the divider. Place the end point of one of its arms at A and the end point of the second arm at B. Taking care that opening of the divider is not disturbed, lift the divider and place it on the ruler. Ensure that one end point is at the zero mark of the ruler. Now read the mark against the other end point.

Try These

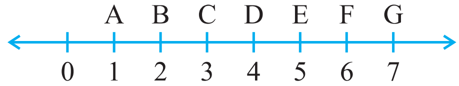
1. Take any post card. Use the above technique to measure its two adjacent sides.
2. Select any three objects having a flat top. Measure all sides of the top using a divider and a ruler.

EXERCISE 5.1

1. What is the disadvantage in comparing line segments by mere observation?
2. Why is it better to use a divider than a ruler, while measuring the length of a line segment?
3. Draw any line segment, say $\overline{AB}$. Take any point C lying in between A and B. Measure the lengths of AB, BC and AC. Is AB = AC + CB?

   [Note : If A,B,C are any three points on a line such that AC + CB = AB, then we can be sure that C lies between A and B.]

4. If A,B,C are three points on a line such that AB = 5 cm, BC = 3 cm and AC = 8 cm, which one of them lies between the other two?
5. Verify, whether D is the mid point of $\overline{AG}$.



6. If B is the mid point of $\overline{AC}$ and C is the mid point of $\overline{BD}$, where A,B,C,D lie on a straight line, say why AB = CD?
7. Draw five triangles and measure their sides. Check in each case, if the sum of the lengths of any two sides is always less than the third side.

5.3 Angles – 'Right' and 'Straight'

You have heard of directions in Geography. We know that China is to the north of India, Sri Lanka is to the south. We also know that Sun rises in the east and sets in the west. There are four main directions. They are North (N), South (S), East (E) and West (W).

Do you know which direction is opposite to north?

Which direction is opposite to west?

Just recollect what you know already. We now use this knowledge to learn a few properties about angles.

Stand facing north.

Do This

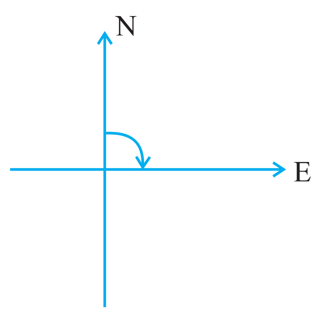
Turn clockwise to east.

We say, you have turned through a **right angle.**

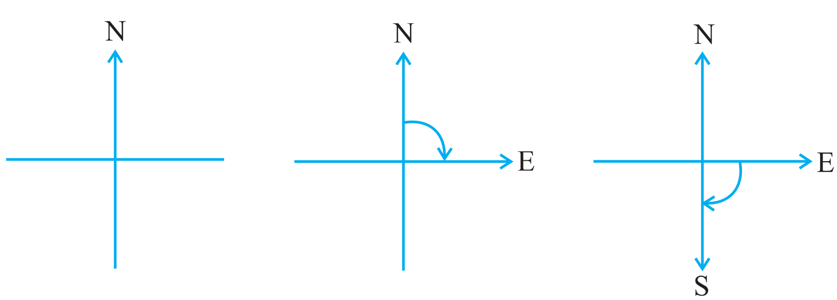
Follow this by a 'right-angle-turn', clockwise.

You now face south.

If you turn by a right angle in the anti-clockwise direction, which direction will you face? It is east again! (Why?)



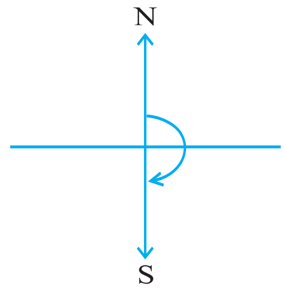
Study the following positions :



You stand facing north

By a 'right-angle-turn' clockwise, you now face east

By another 'right-angle-turn' you finally face south.



From facing north to facing south, you have turned by two right angles. Is not this the same as a single turn by two right angles?

The turn from north to east is by a right angle.

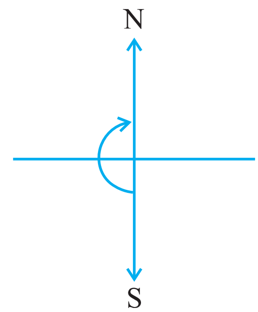
The turn from north to south is by two right angles; it is called a **straight angle**. (NS is a straight line!)

Stand facing south.

Turn by a straight angle.

Which direction do you face now?

You face north!

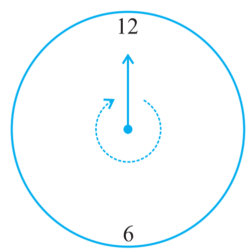


To turn from north to south, you took a straight angle turn, again to turn from south to north, you took another straight angle turn in the same direction. Thus, turning by two straight angles you reach your original position.

Think, discuss and write

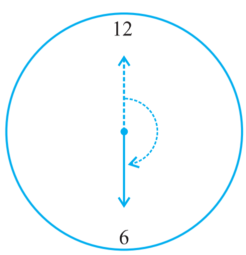
By how many right angles should you turn in the same direction to reach your original position?

Turning by two straight angles (or four right angles) in the same direction makes a full turn. This one complete turn is called one revolution. The angle for one revolution is a **complete angle**.



We can see such revolutions on clock-faces. When the hand of a clock moves from one position to another, it turns through an **angle.**

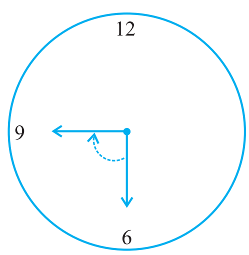
Suppose the hand of a clock starts at 12 and goes round until it reaches at 12 again. Has it not made one revolution? So, how many right angles has it moved? Consider these examples :



From 12 to 6

$\frac{1}{2}$ of a revolution.

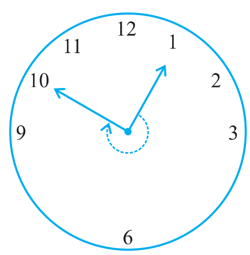
or 2 right angles.



From 6 to 9

$\frac{1}{4}$ of a revolution

or 1 right angle.



From 1 to 10

$\frac{3}{4}$ of a revolution

or 3 right angles.

Try These

1. What is the angle name for half a revolution?
2. What is the angle name for one-fourth revolution?
3. Draw five other situations of one-fourth, half and three-fourth revolution on a clock.

Note that there is no special name for three-fourth of a revolution.

EXERCISE 5.2

1. What fraction of a clockwise revolution does the hour hand of a clock turn through, when it goes from

   (a) 3 to 9 (b) 4 to 7 (c) 7 to 10

   (d) 12 to 9 (e) 1 to 10 (f) 6 to 3

2. Where will the hand of a clock stop if it

   (a) starts at 12 and makes $\frac{1}{2}$ of a revolution, clockwise?

   (b) starts at 2 and makes $\frac{1}{2}$ of a revolution, clockwise?

   (c) starts at 5 and makes $\frac{1}{4}$ of a revolution, clockwise?

   (d) starts at 5 and makes $\frac{3}{4}$ of a revolution, clockwise?

3. Which direction will you face if you start facing

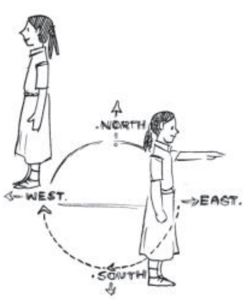
   (a) east and make $\frac{1}{2}$ of a revolution clockwise?

   (b) east and make $1\frac{1}{2}$ of a revolution clockwise?

   (c) west and make $\frac{3}{4}$ of a revolution anti-clockwise?

   (d) south and make one full revolution?

   (Should we specify clockwise or anti-clockwise for this last question? Why not?)



4. What part of a revolution have you turned through if you stand facing

   (a) east and turn clockwise to face north?

   (b) south and turn clockwise to face east?

   (c) west and turn clockwise to face east?

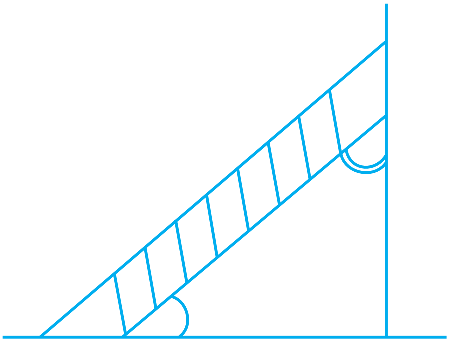
5. Find the number of right angles turned through by the hour hand of a clock when it goes from

   (a) 3 to 6 (b) 2 to 8 (c) 5 to 11

   (d) 10 to 1 (e) 12 to 9 (f) 12 to 6

6. How many right angles do you make if you start facing
   (a) south and turn clockwise to west?
   (b) north and turn anti-clockwise to east?
   (c) west and turn to west?
   (d) south and turn to north?
7. Where will the hour hand of a clock stop if it starts
   (a) from 6 and turns through 1 right angle?
   (b) from 8 and turns through 2 right angles?
   (c) from 10 and turns through 3 right angles?
   (d) from 7 and turns through 2 straight angles?

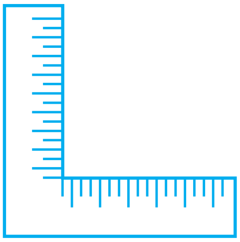
5.4 Angles – 'Acute', 'Obtuse' and 'Reflex'



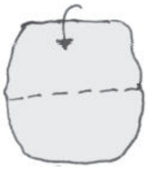
We saw what we mean by a right angle and a straight angle. However, not all the angles we come across are one of these two kinds. The angle made by a ladder with the wall (or with the floor) is neither a right angle nor a straight angle.

Think, discuss and write

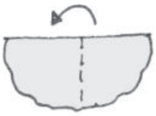
Are there angles smaller than a right angle?
Are there angles greater than a right angle?
Have you seen a carpenter's square? It looks like the letter "L" of English alphabet. He uses it to check right angles. Let us also make a similar 'tester' for a right angle.



Do This



Step 1
Take a piece of paper



Step 2
Fold it somewhere in the middle

Step 3
Fold again the straight edge. Your tester is ready

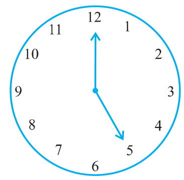
Observe your improvised 'right-angle-tester'. [Shall we call it RA tester?] Does one edge end up straight on the other?

Suppose any shape with corners is given. You can use your RA tester to test the angle at the corners.

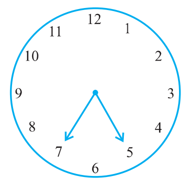
Do the edges match with the angles of a paper? If yes, it indicates a right angle.

Try These

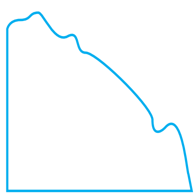
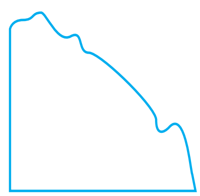
1. The hour hand of a clock moves from 12 to 5. Is the revolution of the hour hand more than 1 right angle?



2. What does the angle made by the hour hand of the clock look like when it moves from 5 to 7. Is the angle moved more than 1 right angle?

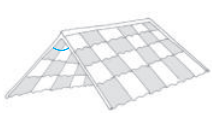


3. Draw the following and check the angle with your RA tester.
   (a) going from 12 to 2 (b) from 6 to 7
   (c) from 4 to 8 (d) from 2 to 5
4. Take five different shapes with corners. Name the corners. Examine them with your tester and tabulate your results for each case :

Corner	**Smaller than** 	**Larger than** 
A	....	....
B	....	....
C	....	....
⋮		

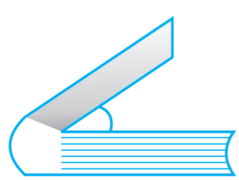
Other names

- An angle smaller than a right angle is called an **acute angle**. These are acute angles.



Roof top

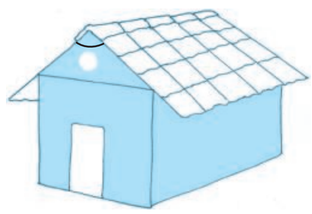
Sea-saw



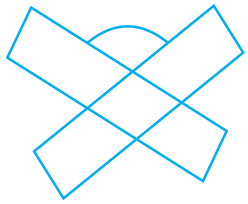
Opening book

Do you see that each one of them is less than one-fourth of a revolution? Examine them with your RA tester.

- If an angle is larger than a right angle, but less than a straight angle, it is called an **obtuse angle**. These are obtuse angles.



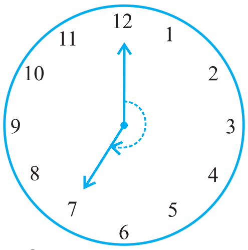
House



Book reading desk

Do you see that each one of them is greater than one-fourth of a revolution but less than half a revolution? Your RA tester may help to examine.

Identify the obtuse angles in the previous examples too.



- A reflex angle is larger than a straight angle.
  It looks like this. (See the angle mark)
  Were there any reflex angles in the shapes you made earlier?
  How would you check for them?

Try These

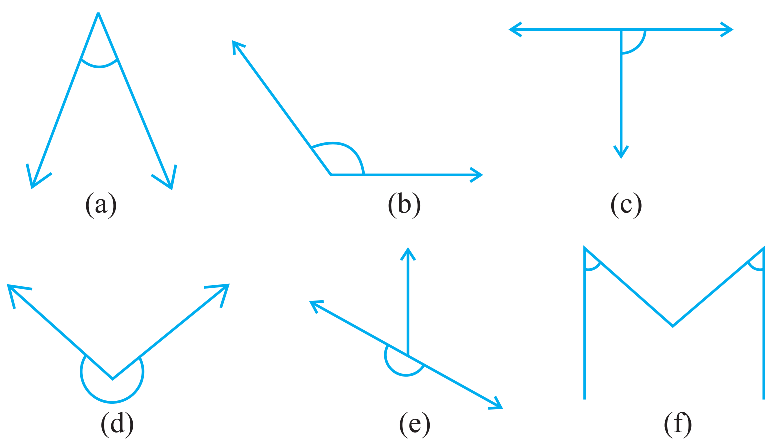
1. Look around you and identify edges meeting at corners to produce angles. List ten such situations.
2. List ten situations where the angles made are acute.
3. List ten situations where the angles made are right angles.
4. Find five situations where obtuse angles are made.
5. List five other situations where reflex angles may be seen.

EXERCISE 5.3

1. Match the following :

(i)	Straight angle	(a)	Less than one-fourth of a revolution
(ii)	Right angle	(b)	More than half a revolution
(iii)	Acute angle	(c)	Half of a revolution
(iv)	Obtuse angle	(d)	One-fourth of a revolution
(v)	Reflex angle	(e)	Between $\frac{1}{4}$ and $\frac{1}{2}$ of a revolution
		(f)	One complete revolution

2. Classify each one of the following angles as right, straight, acute, obtuse or reflex :



5.5 Measuring Angles

The improvised 'Right-angle tester' we made is helpful to compare angles with a right angle. We were able to classify the angles as acute, obtuse or reflex.

But this does not give a precise comparison. It cannot find which one among the two obtuse angles is greater. So in order to be more precise in comparison, we need to 'measure' the angles. We can do it with a 'protractor'.

The measure of angle

We call our measure, 'degree measure'. One complete revolution is divided into 360 equal parts. Each part is a **degree**. We write 360° to say 'three hundred sixty degrees'.

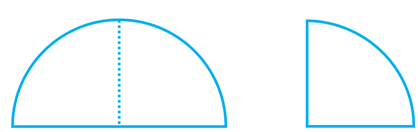
Think, discuss and write

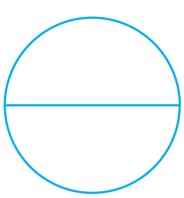
How many degrees are there in half a revolution? In one right angle? In one straight angle?

How many right angles make 180°?, 360°?

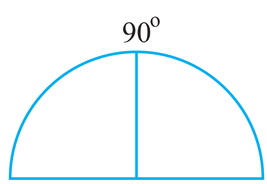
Do This

1. Cut out a circular shape using a bangle or take a circular sheet of about the same size.
2. Fold it twice to get a shape as shown. This is called a quadrant.



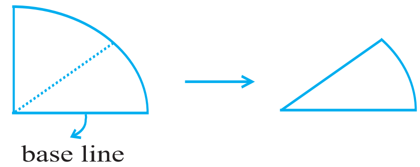


3. Open it out. You will find a semi-circle with a fold in the middle. Mark 90° on the fold.

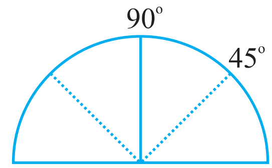


4. Fold the semicircle to reach

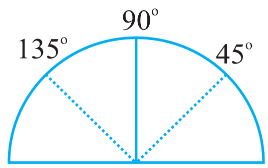
the quadrant. Now fold the quadrant once more as shown. The angle is half of 90° i.e. 45°.



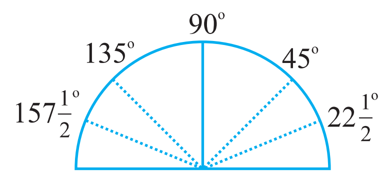
5. Open it out now. Two folds appear on each side. What is the angle upto the first new line? Write 45° on the first fold to the left of the base line.



6. The fold on the other side would be 90° + 45° = 135°



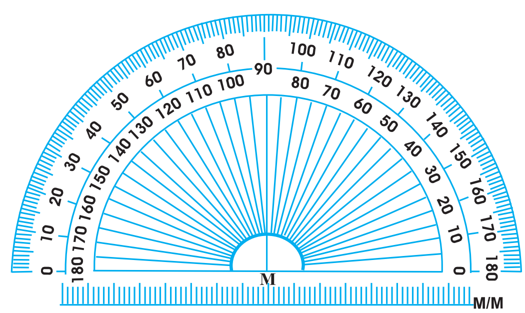
7. Fold the paper again upto 45° (half of the quadrant). Now make half of this. The first fold to the left of the base line now is half of 45° i.e. $22\frac{1}{2}^{\circ}$. The angle on the left of 135° would be $157\frac{1}{2}^{\circ}$.



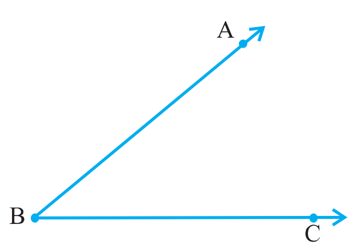
You have got a ready device to measure angles. This is an approximate protractor.

The Protractor

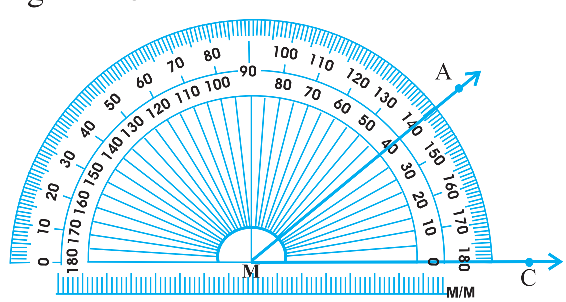
You can find a readymade protractor in your 'instrument box'. The curved edge is divided into 180 equal parts. Each part is equal to a 'degree'. The markings start from 0° on the right side and ends with 180° on the left side, and vice-versa.



Suppose you want to measure an angle ABC.



Given ∠ABC



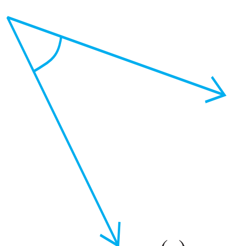
Measuring ∠ABC

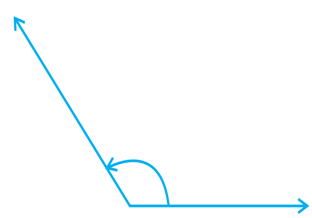
1. Place the protractor so that the mid point (M in the figure) of its straight edge lies on the vertex B of the angle.
2. Adjust the protractor so that $\overline{BC}$ is along the straight-edge of the protractor.
3. There are two 'scales' on the protractor : read that scale which has the 0° mark coinciding with the straight-edge (i.e. with ray $\overrightarrow{BC}$).
4. The mark shown by $\overline{BA}$ on the curved edge gives the degree measure of the angle.

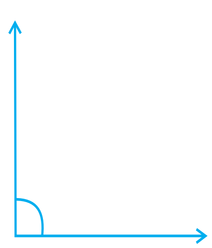
   We write m $\angle ABC = 40°$, or simply $\angle ABC = 40°$.

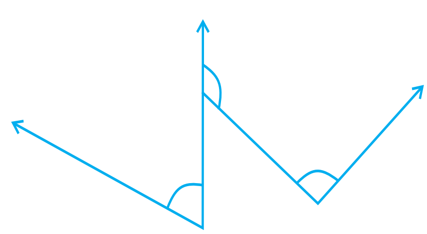
EXERCISE 5.4

1. What is the measure of (i) a right angle? (ii) a straight angle?
2. Say True or False :
   (a) The measure of an acute angle < 90°.
   (b) The measure of an obtuse angle < 90°.
   (c) The measure of a reflex angle > 180°.
   (d) The measure of one complete revolution = 360°.
   (e) If $m\angle A = 53°$ and $m\angle B = 35°$, then $m\angle A > m\angle B$.
3. Write down the measures of
   (a) some acute angles. (b) some obtuse angles.
   (give at least two examples of each).
4. Measure the angles given below using the Protractor and write down the measure.


(a)


(b)

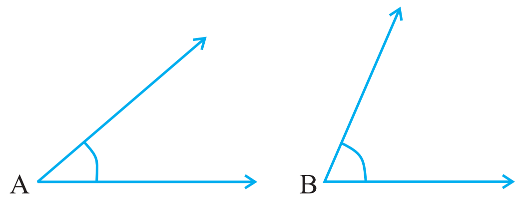

(c)


(d)

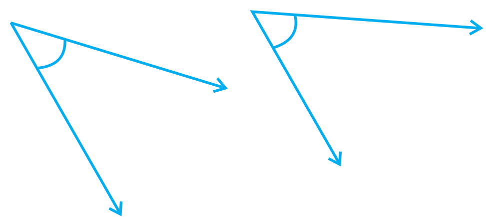
5. Which angle has a large measure?
   First estimate and then measure.

   Measure of Angle A =

   Measure of Angle B =

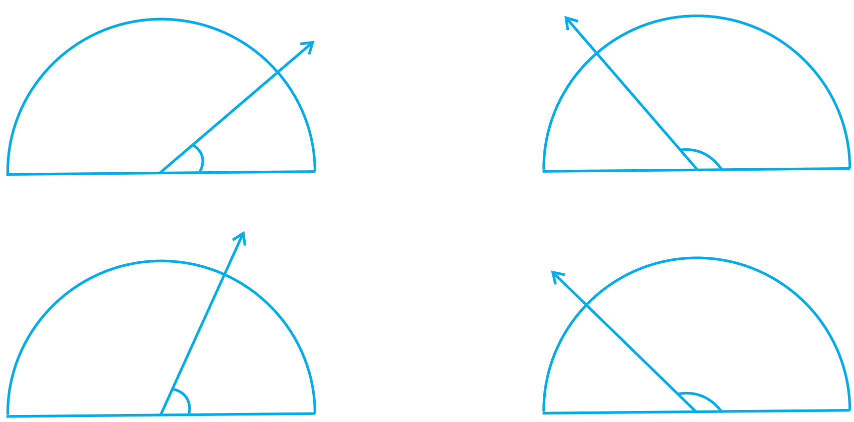


6. From these two angles which has larger measure? Estimate and then confirm by measuring them.

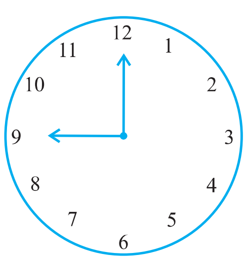


7. Fill in the blanks with acute, obtuse, right or straight :
   (a) An angle whose measure is less than that of a right angle is______.
   (b) An angle whose measure is greater than that of a right angle is ______.
   (c) An angle whose measure is the sum of the measures of two right angles is _____.
   (d) When the sum of the measures of two angles is that of a right angle, then each one of them is ______.
   (e) When the sum of the measures of two angles is that of a straight angle and if one of them is acute then the other should be _______.

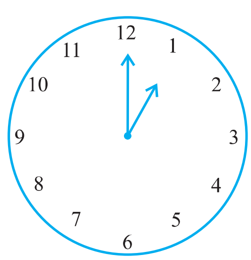
8. Find the measure of the angle shown in each figure. (First estimate with your eyes and then find the actual measure with a protractor).



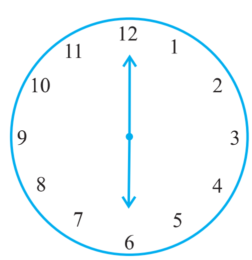
9. Find the angle measure between the hands of the clock in each figure :



9.00 a.m.



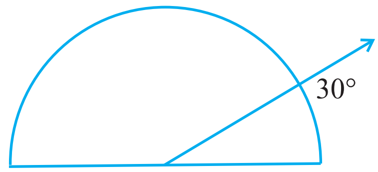
1.00 p.m.



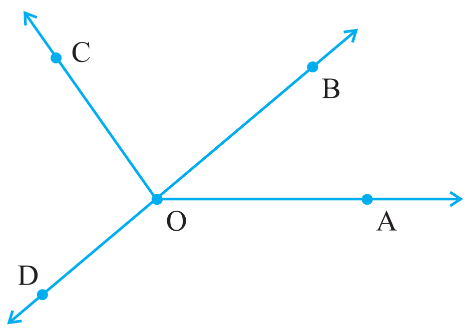
6.00 p.m.

10. **Investigate**

In the given figure, the angle measures 30°. Look at the same figure through a magnifying glass. Does the angle becomes larger? Does the size of the angle change?



11. Measure and classify each angle :



Angle	Measure	Type
∠AOB		
∠AOC		
∠BOC		
∠DOC		
∠DOA		
∠DOB		

5.6 Perpendicular Lines

When two lines intersect and the angle between them is a right angle, then the lines are said to be **perpendicular**. If a line AB is perpendicular to CD, we write $AB \perp CD$.

Think, discuss and write

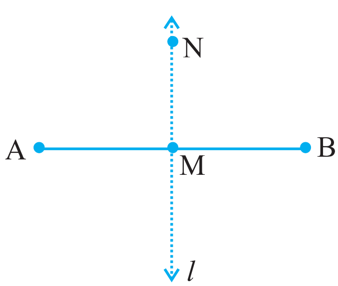
If $AB \perp CD$, then should we say that $CD \perp AB$ also?

Perpendiculars around us!

You can give plenty of examples from things around you for perpendicular lines (or line segments). The English alphabet T is one. Is there any other alphabet which illustrates perpendicularity?

Consider the edges of a post card. Are the edges perpendicular?

Let $\overline{AB}$ be a line segment. Mark its mid point as M. Let MN be a line perpendicular to $\overline{AB}$ through M.



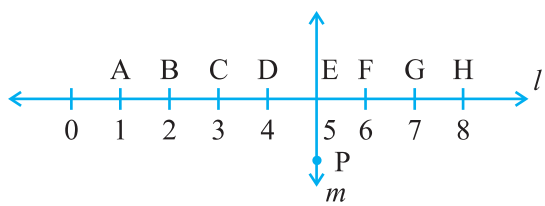
Does MN divide $\overline{AB}$ into two equal parts?

MN bisects $\overline{AB}$ (that is, divides $\overline{AB}$ into two equal parts) and is also perpendicular to $\overline{AB}$.

So we say MN is the **perpendicular bisector** of $\overline{AB}$.
You will learn to construct it later.

EXERCISE 5.5

1. Which of the following are models for perpendicular lines :
   (a) The adjacent edges of a table top.
   (b) The lines of a railway track.
   (c) The line segments forming the letter 'L'.
   (d) The letter V.
2. Let $\overline{PQ}$ be the perpendicular to the line segment XY. Let $\overline{PQ}$ and $\overline{XY}$ intersect in the point A. What is the measure of $\angle PAY$?
3. There are two set-squares in your box. What are the measures of the angles that are formed at their corners? Do they have any angle measure that is common?
4. Study the diagram. The line l is perpendicular to line m
   (a) Is CE = EG?



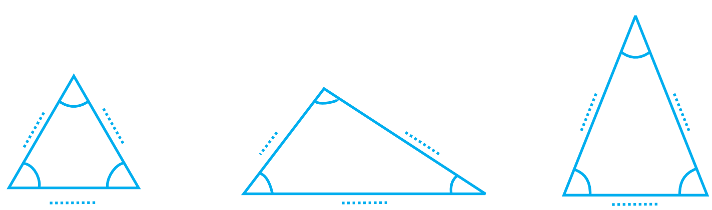
   (b) Does PE bisect CG?
   (c) Identify any two line segments for which PE is the perpendicular bisector.
   (d) Are these true?
      (i) AC > FG
      (ii) CD = GH
      (iii) BC < EH.

5.7 Classification of Triangles

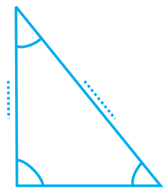
Do you remember a polygon with the least number of sides? That is a triangle. Let us see the different types of triangle we can get.

Do This

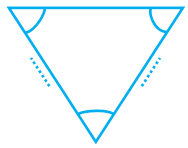
Using a protractor and a ruler find the measures of the sides and angles of the given triangles. Fill the measures in the given table.



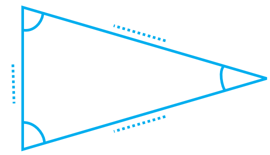
(a) (b) (c)



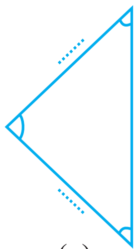
(d)



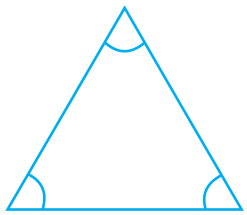
(e)



(f)



(g)



(h)

The measure of the angles of the triangle	What can you say about the angles?	Measures of the sides
(a) ...60^0..., ...60^0.., ...60^0.....,	All angles are equal	
(b) ...., ..., ...,	.... angles ....,	
(c) ...., ..., ...,	.... angles ....,	
(d) ...., ..., ...,	.... angles ....,	
(e) ...., ..., ...,	.... angles ....,	
(f) ...., ..., ...,	.... angles ....,	
(g) ...., ..., ...,	.... angles ....,	
(h) ...., ..., ...,	.... angles ....,	

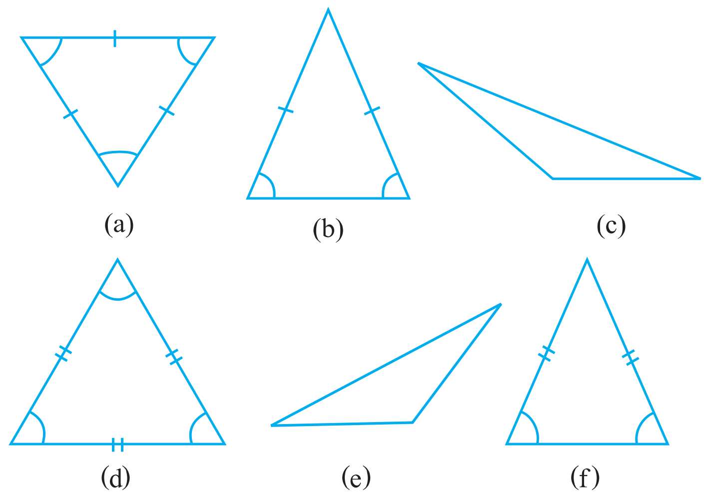
Observe the angles and the triangles as well as the measures of the sides carefully. Is there anything special about them?

What do you find?

- Triangles in which all the angles are equal.
  If all the angles in a triangle are equal, then its sides are also ....
- Triangles in which all the three sides are equal.
  If all the sides in a triangle are equal, then its angles are............. .
- Triangle which have two equal angles and two equal sides.
  If two sides of a triangle are equal, it has .... equal angles.
  and if two angles of a triangle are equal, it has .... equal sides.
- Triangles in which no two sides are equal.
  If none of the angles of a triangle are equal then none of the sides are equal.
  If the three sides of a triangle are unequal then, the three angles are also............. .

Take some more triangles and verify these. For this we will again have to measure all the sides and angles of the triangles.

The triangles have been divided into categories and given special names. Let us see what they are.



Naming triangles based on sides

A triangle having all three unequal sides is called a **Scalene Triangle** [(c), (e)].

A triangle having two equal sides is called an **Isosceles Triangle** [(b), (f)].

A triangle having three equal sides is called an **Equilateral Triangle** [(a), (d)].

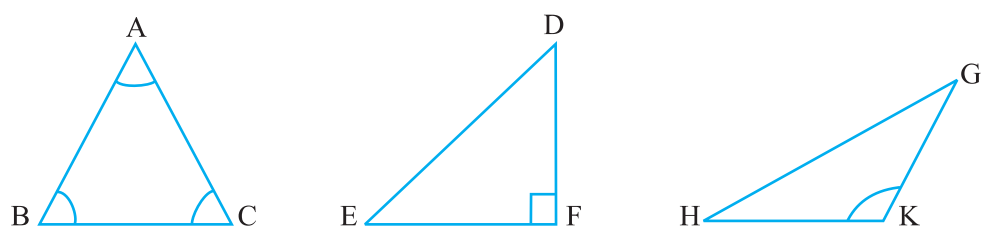
Classify all the triangles whose sides you measured earlier, using these definitions.

Naming triangles based on angles

If each angle is less than 90°, then the triangle is called an **acute angled triangle**.

If any one angle is a right angle then the triangle is called a **right angled triangle**.

If any one angle is greater than 90°, then the triangle is called an **obtuse angled triangle**.



Acute Angled Triangle

Right Angled Triangle

Obtuse Angled Triangle

Name the triangles whose angles were measured earlier according to these three categories. How many were right angled triangles?

Do This

Try to draw rough sketches of

(a) a scalene acute angled triangle.

(b) an obtuse angled isosceles triangle.

(c) a right angled isosceles triangle.

(d) a scalene right angled triangle.

Do you think it is possible to sketch

(a) an obtuse angled equilateral triangle ?

(b) a right angled equilateral triangle ?

(c) a triangle with two right angles?

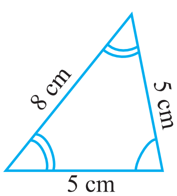
Think, discuss and write your conclusions.

EXERCISE 5.6

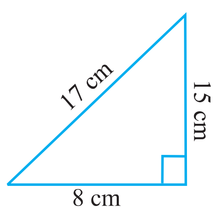
1. Name the types of following triangles :
   (a) Triangle with lengths of sides 7 cm, 8 cm and 9 cm.
   (b) ΔABC with AB = 8.7 cm, AC = 7 cm and BC = 6 cm.
   (c) ΔPQR such that PQ = QR = PR = 5 cm.
   (d) ΔDEF with $m\angle D = 90°$
   (e) ΔXYZ with $m\angle Y = 90°$ and XY = YZ.
   (f) ΔLMN with $m\angle L = 30°$, $m\angle M = 70°$ and $m\angle N = 80°$.
2. Match the following :

Measures of Triangle	Type of Triangle
(i) 3 sides of equal length	(a) Scalene
(ii) 2 sides of equal length	(b) Isosceles right angled
(iii) All sides are of different length	(c) Obtuse angled
(iv) 3 acute angles	(d) Right angled
(v) 1 right angle	(e) Equilateral
(vi) 1 obtuse angle	(f) Acute angled
(vii) 1 right angle with two sides of equal length	(g) Isosceles

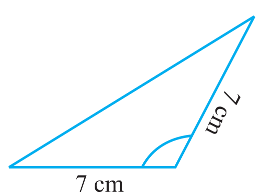
3. Name each of the following triangles in two different ways: (you may judge the nature of the angle by observation)



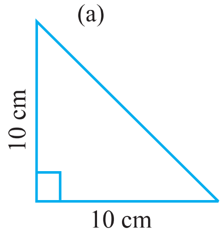
(a)



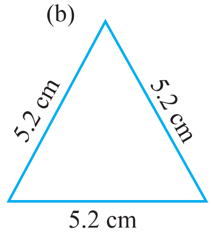
(b)



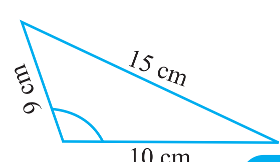
(c)



(d)



(e)



(f)

4. Try to construct triangles using match sticks. Some are shown here.

   Can you make a triangle with

   (a) 3 matchsticks?

   (b) 4 matchsticks?

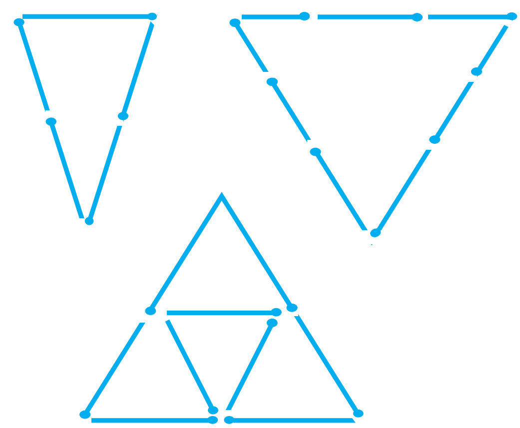
   (c) 5 matchsticks?

   (d) 6 matchsticks?

   (Remember you have to use all the available matchsticks in each case)

   Name the type of triangle in each case.

   If you cannot make a triangle, think of reasons for it.



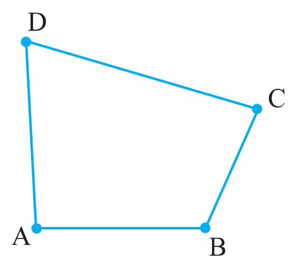
5.8 Quadrilaterals

A quadrilateral, if you remember, is a polygon which has four sides.

Do This

1. Place a pair of unequal sticks such that they have their end points joined at one end. Now place another such pair meeting the free ends of the first pair.
   What is the figure enclosed?
   It is a quadrilateral, like the one you see here.

   The sides of the quadrilateral are $\overline{AB}$, $\overline{BC}$, ____, ____.
   There are 4 angles for this quadrilateral.
   They are given by $\angle BAD$, $\angle ADC$, $\angle DCB$ and ______.
   BD is one diagonal. What is the other?
   Measure the length of the sides and the diagonals.
   Measure all the angles also.



2. Using four unequal sticks, as you did in the above activity, see if you can form a quadrilateral such that

   (a) all the four angles are acute.

   (b) one of the angles is obtuse.

   (c) one of the angles is right angled.

   (d) two of the angles are obtuse.

   (e) two of the angles are right angled.

   (f) the diagonals are perpendicular to one another.

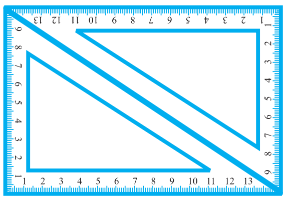
Do This

You have two set-squares in your instrument box. One is 30° – 60° – 90° set-square, the other is 45°– 45°– 90° set square.

You and your friend can jointly do this.

(a) Both of you will have a pair of 30°– 60°– 90° set-squares. Place them as shown in the figure.

Can you name the quadrilateral described?

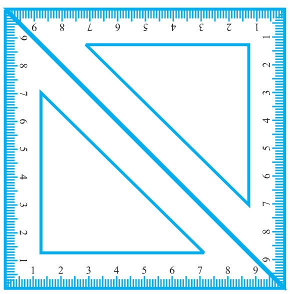


What is the measure of each of its angles?

This quadrilateral is a **rectangle.**

One more obvious property of the rectangle you can see is that opposite sides are of equal length.

What other properties can you find?



(b) If you use a pair of 45°– 45°–90° set-squares, you get another quadrilateral this time.

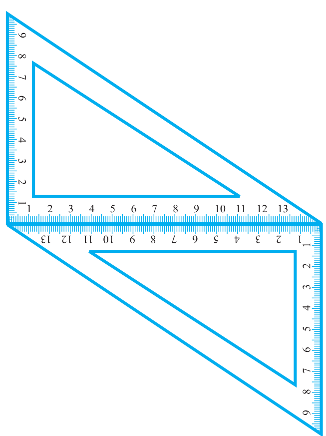
It is a **square**.

Are you able to see that all the sides are of equal length? What can you say about the angles and the diagonals? Try to find a few more properties of the square.

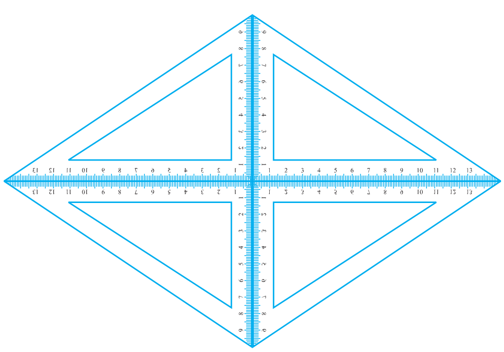
(c) If you place the pair of 30° – 60° – 90° set-squares in a different position, you get a **parallelogram**. Do you notice that the opposite sides are parallel?

Are the opposite sides equal?

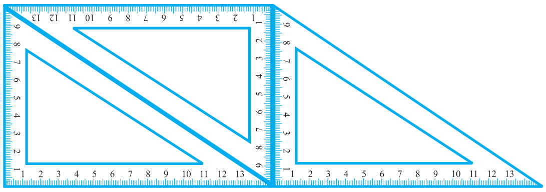
Are the diagonals equal?



(d) If you use four 30° – 60° – 90° set-squares you get a **rhombus**.



(e) If you use several set-squares you can build a shape like the one given here.



Here is a quadrilateral in which two sides are parallel.

It is a **trapezium.**

Here is an outline-summary of your possible findings. Complete it.

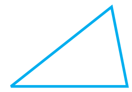
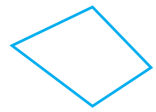
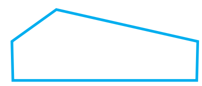
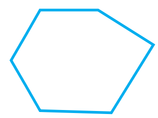
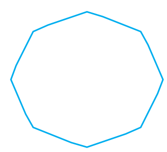
Quadrilateral	Opposite sides		All sides	Opposite Angles	Diagonals	
	Parallel	Equal	Equal	Equal	Equal	Perpen-dicular
Parallelogram	Yes	Yes	No	Yes	No	No
Rectangle			No			
Square						Yes
Rhombus				Yes		
Trapezium		No				

EXERCISE 5.7

1. Say True or False :
   (a) Each angle of a rectangle is a right angle.
   (b) The opposite sides of a rectangle are equal in length.
   (c) The diagonals of a square are perpendicular to one another.
   (d) All the sides of a rhombus are of equal length.
   (e) All the sides of a parallelogram are of equal length.
   (f) The opposite sides of a trapezium are parallel.
2. Give reasons for the following :
   (a) A square can be thought of as a special rectangle.
   (b) A rectangle can be thought of as a special parallelogram.
   (c) A square can be thought of as a special rhombus.
   (d) Squares, rectangles, parallelograms are all quadrilaterals.
   (e) Square is also a parallelogram.
3. A figure is said to be regular if its sides are equal in length and angles are equal in measure. Can you identify the regular quadrilateral?

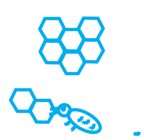
5.9 Polygons

So far you studied polygons of 3 or 4 sides (known as triangles and quardrilaterals respectively). We now try to extend the idea of polygon to figures with more number of sides. We may classify polygons according to the number of their sides.

Number of sides	Name	Illustration
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
8	Octagon	

You can find many of these shapes in everyday life. Windows, doors, walls, almirahs, blackboards, notebooks are all usually rectanglular in shape. Floor tiles are rectangles. The sturdy nature of a triangle makes it the most useful shape in engineering constructions.

The triangle finds application in constructions.

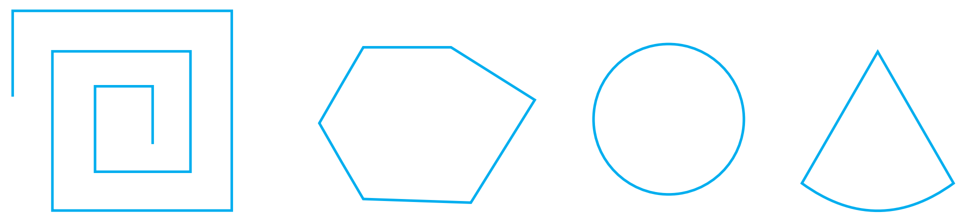


A bee knows the usefulness of a hexagonal shape in building its house .

Look around and see where you can find all these shapes.

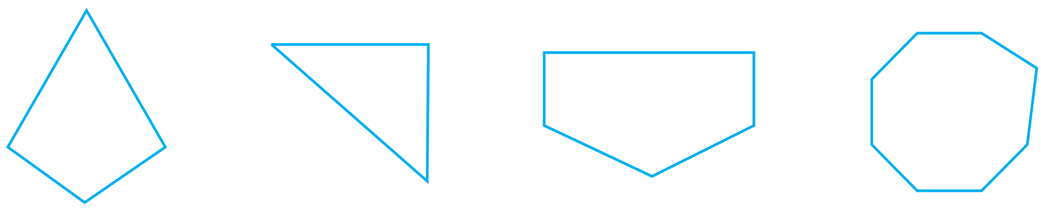
EXERCISE 5.8

1. Examine whether the following are polygons. If any one among them is not, say why?



(a) (b) (c) (d)

2. Name each polygon.



(a) (b) (c) (d)

Make two more examples of each of these.

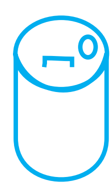
3. Draw a rough sketch of a regular hexagon. Connecting any three of its vertices, draw a triangle. Identify the type of the triangle you have drawn.
4. Draw a rough sketch of a regular octagon. (Use squared paper if you wish). Draw a rectangle by joining exactly four of the vertices of the octagon.
5. A diagonal is a line segment that joins any two vertices of the polygon and is not a side of the polygon. Draw a rough sketch of a pentagon and draw its diagonals.

5.10 Three Dimensional Shapes

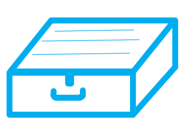
Here are a few shapes you see in your day-to-day life. Each shape is a solid. It is not a 'flat' shape.

The ball is a sphere.

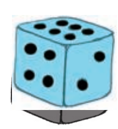
The ice-cream is in the form of a cone.



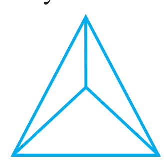
This can is a cylinder.



The box is a cuboid.



The playing die is a cube.



This is the shape of a pyramid.

Name any five things which resemble a sphere.

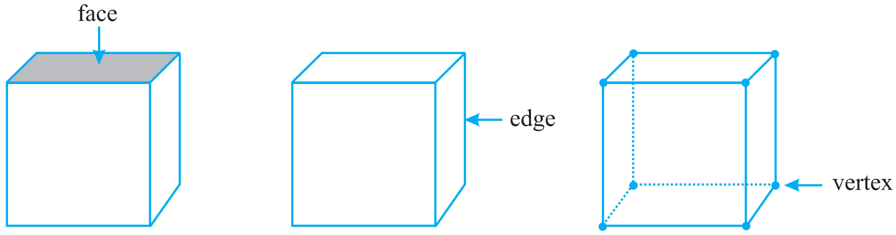
Name any five things which resemble a cone.

Faces, edges and vertices

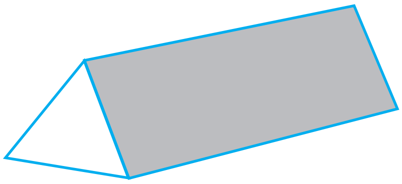
In case of many three dimensional shapes we can distinctly identify their faces, edges and vertices. What do we mean by these terms: Face, Edge and Vertex? (Note 'Vertices' is the plural form of 'vertex').

Consider a cube, for example.

Each side of the cube is a flat surface called a flat **face** (or simply a **face**)**.** Two faces meet at a *line segment* called an **edge.** Three edges meet at a point called a **vertex.**



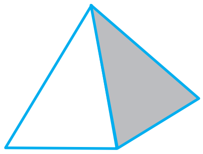
Here is a diagram of a **prism.**



Have you seen it in the laboratory? One of its faces is a triangle. So it is called a triangular prism.

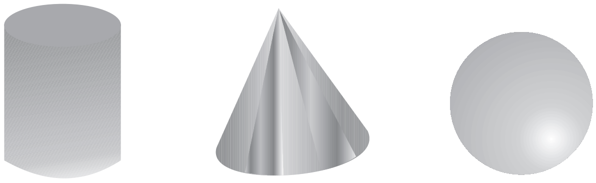
The triangular face is also known as its base. A prism has two identical bases; the other faces are rectangles.

If the prism has a rectangular base, it is a rectangular prism. Can you recall another name for a rectangular prism?



A pyramid is a shape with a single base; the other faces are triangles.

Here is a square pyramid. Its base is a square. Can you imagine a triangular pyramid? Attempt a rough sketch of it.



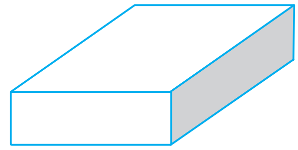
The cylinder, the cone and the sphere have no straight edges. What is the base of a cone? Is it a circle? The cylinder has two bases. What shapes are they? Of course, a sphere has no flat faces! Think about it.

Do This

1. A cuboid looks like a rectangular box.

   It has 6 faces. Each face has 4 edges.

   Each face has 4 corners (called vertices).

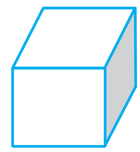


2. A cube is a cuboid whose edges are all of equal length.

   It has _______ faces.

   Each face has _______ edges.

   Each face has _______ vertices.

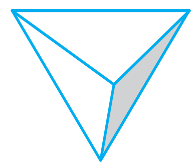


3. A triangular pyramid has a triangle as its base. It is also known as a tetrahedron.

   Faces : ________

   Edges : ________

   Corners : ________

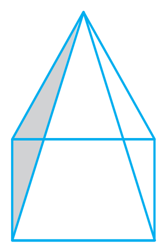


4. A square pyramid has a square as its base.

   Faces : ________

   Edges : ________

   Corners : ________

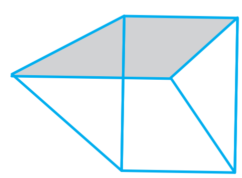


5. A triangular prism looks like the shape of a Kaleidoscope. It has triangles as its bases.

   Faces : ________

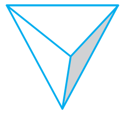
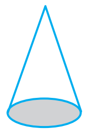
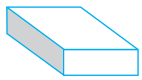
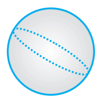
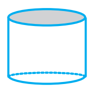
   Edges : ________

   Corners : ________



EXERCISE 5.9

1. Match the following :

(a) Cone	(i)	
(b) Sphere	(ii)	
(c) Cylinder	(iii)	
(d) Cuboid	(iv)	
(e) Pyramid	(v)	

Give two new examples of each shape.

2. What shape is
   (a) Your instrument box? (b) A brick?
   (c) A match box? (d) A road-roller?
   (e) A sweet laddu?

What have we discussed?

1. The distance between the end points of a line segment is its *length*.
2. A graduated *ruler* and the *divider* are useful to compare lengths of line segments.
3. When a hand of a clock moves from one position to another position we have an example for an *angle*.
   One full turn of the hand is 1 *revolution*.
   A *right angle* is ¼ revolution and a *straight angle* is ½ a revolution .
   We use a *protractor* to measure the size of an angle in degrees.
   The measure of a right angle is 90° and hence that of a straight angle is 180°.
   An angle is *acute* if its measure is smaller than that of a right angle and is *obtuse* if its measure is greater than that of a right angle and less than a straight angle.
   A *reflex angle* is larger than a straight angle.

4. Two intersecting lines are *perpendicular* if the angle between them is 90°.
5. The *perpendicular bisector* of a line segment is a perpendicular to the line segment that divides it into two equal parts.
6. Triangles can be classified as follows based on their angles:

Nature of angles in the triangle	*Name*
Each angle is acute	Acute angled triangle
One angle is a right angle	Right angled triangle
One angle is obtuse	Obtuse angled triangle

7. Triangles can be classified as follows based on the lengths of their sides:

Nature of sides in the triangle	*Name*
All the three sides are of unequal length	Scalene triangle
Any two of the sides are of equal length	Isosceles triangle
All the three sides are of equal length	Equilateral triangle

8. Polygons are named based on their sides.

Number of sides	*Name of the Polygon*
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
8	Octagon

9. Quadrilaterals are further classified with reference to their properties.

Properties	*Name of the Quadrilateral*
One pair of parallel sides	Trapezium
Two pairs of parallel sides	Parallelogram
Parallelogram with 4 right angles	Rectangle
Parallelogram with 4 sides of equal length	Rhombus
A rhombus with 4 right angles	Square

10. We see around us many *three dimensional shapes*. Cubes, cuboids, spheres, cylinders, cones, prisms and pyramids are some of them.

Integers

Chapter 6

6.1 Introduction

Sunita's mother has 8 bananas. Sunita has to go for a picnic with her friends. She wants to carry 10 bananas with her. Can her mother give 10 bananas to her? She does not have enough, so she borrows 2 bananas from her neighbour to be returned later. After giving 10 bananas to Sunita, how many bananas are left with her mother? Can we say that she has zero bananas? She has no bananas with her, but has to return two to her neighbour. So when she gets some more bananas, say 6, she will return 2 and be left with 4 only.

Ronald goes to the market to purchase a pen. He has only Rs 12 with him but the pen costs Rs 15. The shopkeeper writes Rs 3 as due amount from him. He writes Rs 3 in his diary to remember Ronald's debit. But how would he remember whether Rs 3 has to be given or has to be taken from Ronald? Can he express this debit by some colour or sign?

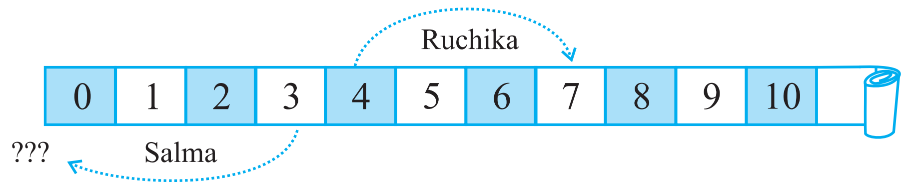
Ruchika and Salma are playing a game using a number strip which is marked from 0 to 25 at equal intervals.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

To begin with, both of them placed a coloured token at the zero mark. Two coloured dice are placed in a bag and are taken out by them one by one. If the die is red in colour, the token is moved forward as per the number shown on throwing this die. If it is blue, the token is moved backward as per the number shown when this die is thrown. The dice are put back into the bag after each move so that both of them have equal chance of getting either die. The one who reaches the 25th mark first is the winner. They play the game. Ruchika gets the red die and gets four on the die after throwing it. She, thus, moves the token to mark four on the strip. Salma also happens to take out the red die and wins 3 points and, thus, moves her token to number 3.

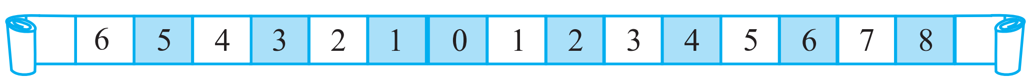
In the second attempt, Ruchika secures three points with the red die and Salma gets 4 points but with the blue die. Where do you think both of them should place their token after the second attempt?

Ruchika moves forward and reaches 4 + 3 i.e. the 7th mark.



Whereas Salma placed her token at zero position. But Ruchika objected saying she should be behind zero. Salma agreed. But there is nothing behind zero. What can they do?

Salma and Ruchika then extended the strip on the other side. They used a blue strip on the other side.



Now, Salma suggested that she is one mark behind zero, so it can be marked as blue one. If the token is at blue one, then the position behind blue one is blue two. Similarly, blue three is behind blue two. In this way they decided to move backward. Another day while playing they could not find blue paper, so Ruchika said, let us use a sign on the other side as we are moving in opposite direction. So you see we need to use a sign going for numbers less than zero. The sign that is used is the placement of a minus sign attached to the number. This indicates that numbers with a negative sign are less than zero. These are called ***negative numbers*.**

Do This

(Who is where?)

Suppose David and Mohan have started walking from zero position in opposite directions. Let the steps to the right of zero be represented by '+' sign and to the left of zero represented by '–' sign. If Mohan moves 5 steps to the right of zero it can be represented as +5 and if David moves 5 steps to the left of zero it can be represented as – 5. Now represent the following positions with + or – sign :

(a) 8 steps to the left of zero.

(b) 7 steps to the right of zero.

(c) 11 steps to the right of zero.

(d) 6 steps to the left of zero.

Do This

(Who follows me?)

We have seen from the previous examples that a movement to the right is made if the number by which we have to move is positive. If a movement of only 1 is made we get the successor of the number.

Write the succeeding number of the following :

Number	Successor
10	
8	
– 5	
– 3	
0	

A movement to the left is made if the number by which the token has to move is negative.

If a movement of only 1 is made to the left, we get the predecessor of a number.

	–8	–7	–6	–5	–4	–3	–2	–1	0	1	2	3	4	5	6	7	8	

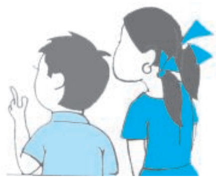
Now write the preceding number of the following :

Number	Predecessor
10	
8	
5	
3	
0	

6.1.1 Tag me with a sign

We have seen that some numbers carry a minus sign. For example, if we want to show Ronald's due amount to the shopkeeper we would write it as – 3.

Following is an account of a shopkeeper which shows profit and loss from the sale of certain items. Since profit and loss are opposite situations and if profit is represented by '+' sign, loss can be represented by '–' sign.



Some of the situations where we may use these signs are :

Name of items	Profit	Loss	Representation with proper sign
Mustard oil	₹ 150		....
Rice		₹ 250	....
Black pepper	₹ 225		....
Wheat	₹ 200		....
Groundnut oil		₹ 330	....

The height of a place above sea level is denoted by a positive number. Height becomes lesser and lesser as we go lower and lower. Thus, below the surface of the sea level we can denote the height by a negative number.

Try These

Write the following numbers with appropriate signs :

(a) 100 m below sea level.
(b) 25°C above 0°C temperature.
(c) 15°C below 0°C temperature.
(d) any five numbers less than 0.

If earnings are represented by '+' sign, then the spendings may be shown by a '–' sign. Similarly, temperature above 0°C is denoted a '+' sign and temperature below 0°C is denoted by '–' sign.

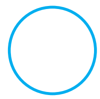
For example, the temperature of a place 10° below 0°C is written as –10°C.

6.2 Integers

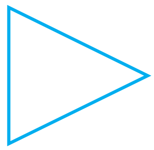
The first numbers to be discovered were natural numbers i.e. 1, 2, 3, 4,... If we include zero to the collection of natural numbers, we get a new collection of numbers known as whole numbers i.e. 0, 1, 2, 3, 4,... You have studied these numbers in the earlier chapter. Now we find that there are negative numbers too. If we put the whole numbers and the negative numbers together, the new collection of numbers will look like 0, 1, 2, 3, 4, 5,..., –1, – 2, – 3, –4, –5, ... and this collection of numbers is known as Integers. In this collection, 1, 2, 3, ... are said to be positive integers and – 1, – 2, – 3,.... are said to be negative integers.

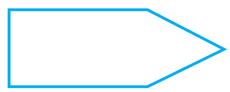
Let us understand this by the following figures. Let us suppose that the figures represent the collection of numbers written against them.


Natural numbers

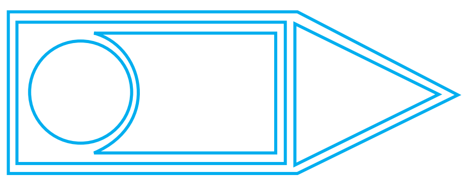

Zero


Whole numbers

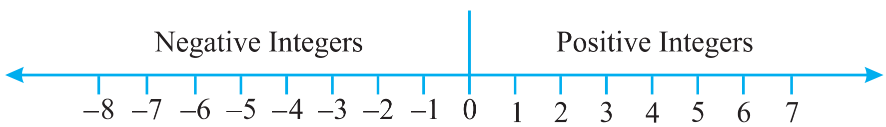

Negative numbers


Integers

Then the collection of integers can be understood by the following diagram in which all the earlier collections are included :

Integers


6.2.1 Representation of integers on a number line



Draw a line and mark some points at equal distance on it as shown in the figure. Mark a point as zero on it. Points to the right of zero are positive integers and are marked + 1, + 2, + 3, etc. or simply 1, 2, 3 etc. Points to the left of zero are negative integers and are marked – 1, – 2, – 3 etc.
In order to mark – 6 on this line, we move 6 points to the left of zero. (Fig 6.1)

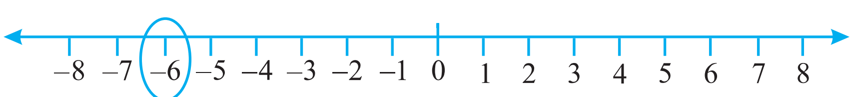


Fig 6.1

In order to mark + 2 on the number line, we move 2 points to the right of zero. (Fig 6.2)

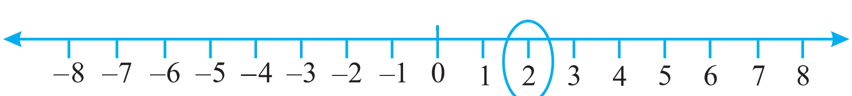


Fig 6.2

6.2.2 Ordering of integers

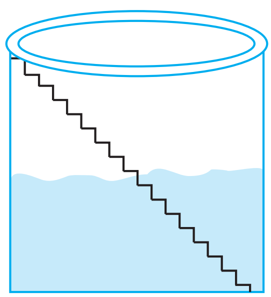
Raman and Imran live in a village where there is a step well. There are in all 25 steps down to the bottom of the well.

Try These

Mark –3, 7, –4, –8, –1 and – 3 on the number line.

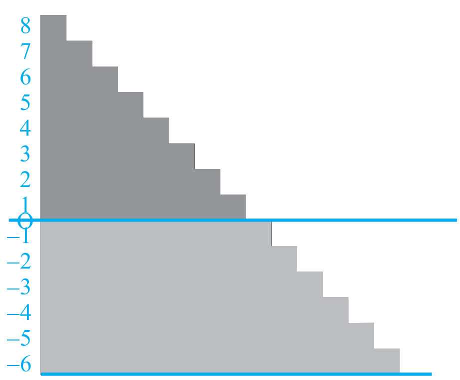
One day Raman and Imran went to the well and counted 8 steps down to water level. They decided to see how much water would come in the well during rains. They marked zero at the existing level of water and marked 1,2,3,4,... above that level for each step. After the rains they noted that the water level rose up to the sixth step. After a few months, they noticed that the water level had fallen three steps below the zero mark. Now, they started thinking about marking the steps to note the fall of water level. Can you help them?

Suddenly, Raman remembered that at one big dam he saw numbers marked even below zero. Imran pointed out that there should be some way to distinguish between numbers which are above zero and below zero. Then Raman recalled that the numbers which were below zero had minus sign in front of them. So they marked one step below zero as – 1 and two steps below zero as – 2 and so on.



So the water level is now at – 3 (3 steps below zero). After that due to further use, the water level went down by 1 step and it was at – 4. You can see that – 4 < – 3.

Keeping in mind the above example, fill in the boxes using > and < signs.



0	☐	– 1	– 100	☐	–101
– 50	☐	– 70	50	☐	–51
– 53	☐	– 5	– 7	☐	1

Let us once again observe the integers which are represented on the number line.

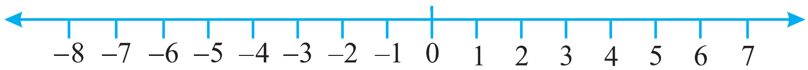


Fig 6.3

We know that 7 > 4 and from the number line shown above, we observe that 7 is to the right of 4 (Fig 6.3).

Similarly, 4 > 0 and 4 is to the right of 0. Now, since 0 is to the right of –3 so, 0 > – 3. Again, – 3 is to the right of – 8 so, – 3 > – 8.

Thus, we see that on a number line the number increases as we move to the right and decreases as we move to the left.

Therefore, – 3 < – 2, – 2 < – 1, – 1 < 0, 0 < 1, 1 < 2, 2 < 3 so on.

Hence, the collection of integers can be written as..., –5, –4, – 3, – 2, – 1, 0, 1, 2, 3, 4, 5...

Try These

Compare the following pairs of numbers using > or <.

0	☐	– 8;	– 1	☐	– 15
5	☐	– 5;	11	☐	15
0	☐	6;	– 20	☐	2

From the above exercise, Rohini arrived at the following conclusions :

(a) Every positive integer is larger than every negative integer.

(b) Zero is less than every positive integer.

(c) Zero is larger than every negative integer.

(d) Zero is neither a negative integer nor a positive integer.

(e) Farther a number from zero on the right, larger is its value.

(f) Farther a number from zero on the left, smaller is its value.

Do you agree with her? Give examples.

Example 1 : By looking at the number line, answer the following questions : Which integers lie between – 8 and – 2? Which is the largest integer and the smallest integer among them?

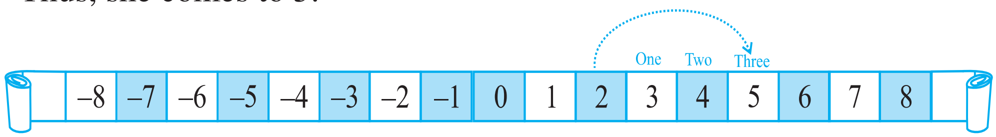
Solution : Integers between – 8 and – 2 are – 7, – 6, – 5, – 4, – 3. The integer – 3 is the largest and – 7 is the smallest.

If, I am not at zero what happens when I move?

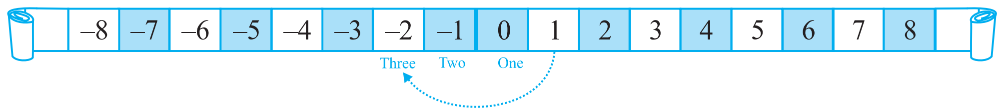
Let us consider the earlier game being played by Salma and Ruchika.

Suppose Ruchika's token is at 2. At the next turn she gets a red die which after throwing gives a number 3. It means she will move 3 places to the right of 2.

Thus, she comes to 5.



If on the other hand, Salma was at 1, and drawn a blue die which gave her number 3, then it means she will move to the left by 3 places and stand at – 2.



By looking at the number line, answer the following question :

Example 2 : (a) One button is kept at – 3. In which direction and how many steps should we move to reach at – 9?

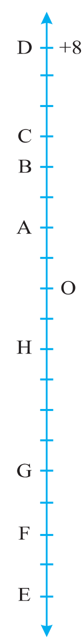
(b) Which number will we reach if we move 4 steps to the right of – 6.

Solution : (a) We have to move six steps to the left of – 3.

(b) We reach – 2 when we move 4 steps to the right of – 6.

EXERCISE 6.1

1. Write opposites of the following :
   (a) Increase in weight (b) 30 km north (c) 326 BC
   (d) Loss of Rs 700 (e) 100 m above sea level
2. Represent the following numbers as integers with appropriate signs.
   (a) An aeroplane is flying at a height two thousand metre above the ground.
   (b) A submarine is moving at a depth, eight hundred metre below the sea level.
   (c) A deposit of rupees two hundred.
   (d) Withdrawal of rupees seven hundred.
3. Represent the following numbers on a number line :
   (a) + 5 (b) – 10 (c) + 8
   (d) – 1 (e) – 6
4. Adjacent figure is a vertical number line, representing integers. Observe it and locate the following points :



   (a) If point D is + 8, then which point is – 8?

(b) Is point G a negative integer or a positive integer?

(c) Write integers for points B and E.

(d) Which point marked on this number line has the least value?

(e) Arrange all the points in decreasing order of value.

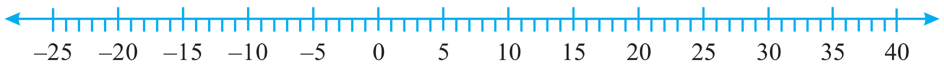
5. Following is the list of temperatures of five places in India on a particular day of the year.

Place	**Temperature**	
Siachin	10°C below 0°C	....
Shimla	2°C below 0°C	....
Ahmedabad	30°C above 0°C	....
Delhi	20°C above 0°C	....
Srinagar	5°C below 0°C	....

(a) Write the temperatures of these places in the form of integers in the blank column.

(b) Following is the number line representing the temperature in degree Celsius.

Plot the name of the city against its temperature.



(c) Which is the coolest place?

(d) Write the names of the places where temperatures are above 10°C.

6. In each of the following pairs, which number is to the right of the other on the number line?

(a) 2, 9 (b) – 3, – 8 (c) 0, – 1

(d) – 11, 10 (e) – 6, 6 (f) 1, – 100

7. Write all the integers between the given pairs (write them in the increasing order.)

(a) 0 and – 7 (b) – 4 and 4

(c) – 8 and – 15 (d) – 30 and – 23

8. (a) Write four negative integers greater than – 20.

(b) Write four negative integers less than – 10.

9. For the following statements, write True (T) or False (F). If the statement is false, correct the statement.

(a) – 8 is to the right of – 10 on a number line.

(b) – 100 is to the right of – 50 on a number line.

(c) Smallest negative integer is – 1.

(d) – 26 is greater than – 25.

10. Draw a number line and answer the following :
    (a) Which number will we reach if we move 4 numbers to the right of – 2.
    (b) Which number will we reach if we move 5 numbers to the left of 1.
    (c) If we are at – 8 on the number line, in which direction should we move to reach – 13?
    (d) If we are at – 6 on the number line, in which direction should we move to reach – 1?

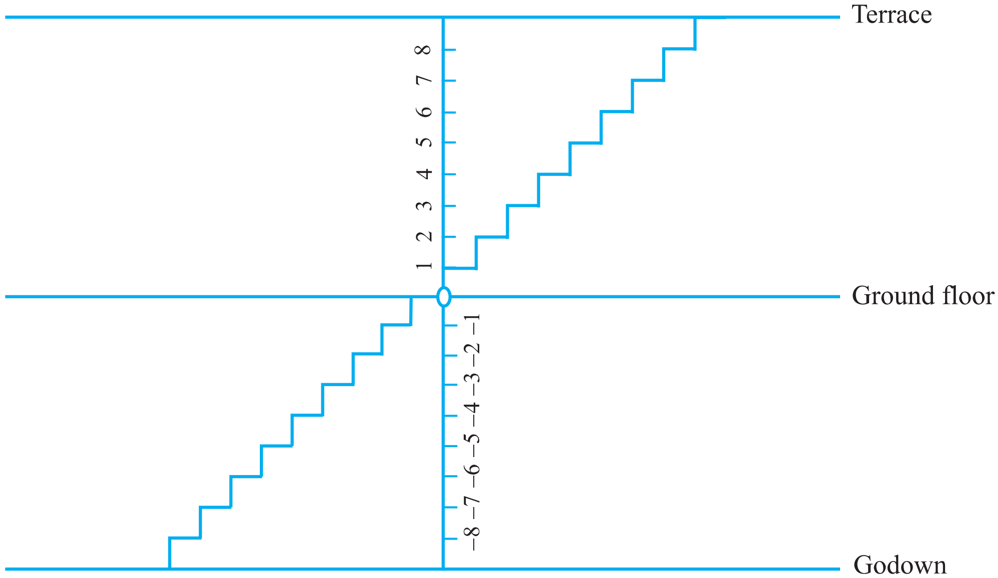
6.3 Additon of Integers

Do This

(Going up and down)

In Mohan's house, there are stairs for going up to the terrace and for going down to the godown.

Let us consider the number of stairs going up to the terrace as positive integer, the number of stairs going down to the godown as negative integer, and the number representing ground level as zero.



Do the following and write down the answer as integer :

(a) Go 6 steps up from the ground floor.
(b) Go 4 steps down from the ground floor.
(c) Go 5 steps up from the ground floor and then go 3 steps up further from there.
(d) Go 6 steps down from the ground floor and then go down further 2 steps from there.

(e) Go down 5 steps from the ground floor and then move up 12 steps from there.

(f) Go 8 steps down from the ground floor and then go up 5 steps from there.

(g) Go 7 steps up from the ground floor and then 10 steps down from there.

Ameena wrote them as follows :

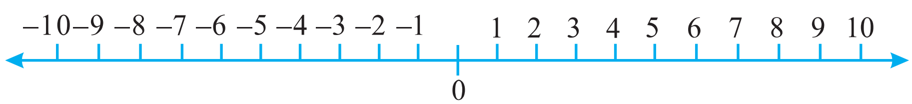
(a) + 6 (b) – 4 (c) (+5) + (+ 3) = + 8 (d) (– 6) + (–2) = – 4

(e) (– 5) + (+12) = + 7 (f) (– 8) + (+5) = – 3 (g) (+7) + (–10) = 17

She has made some mistakes. Can you check her answers and correct those that are wrong?

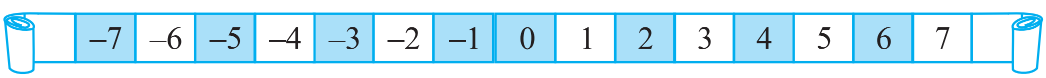
Try These

Draw a figure on the ground in the form of a horizontal number line as shown below. Frame questions as given in the said example and ask your friends.



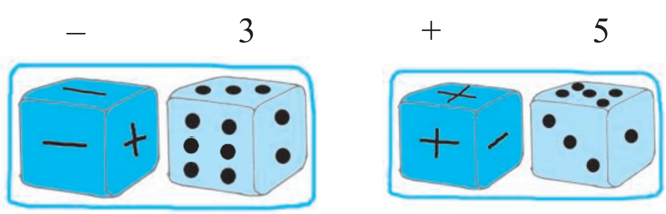
A Game

Take a number strip marked with integers from + 25 to – 25.

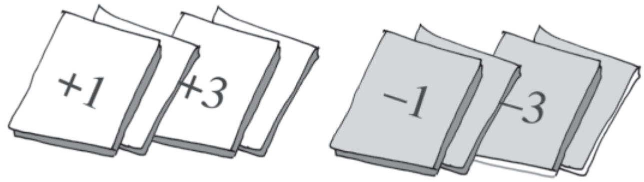


Take two dice, one marked 1 to 6 and the other marked with three '+' signs and three '–' signs.

Players will keep different coloured buttons (or plastic counters) at the zero position on the number strip. In each throw, the player has to see what she has obtained on the two dice. If the first die shows 3 and the second die shows – sign, she has –3. If the first die shows 5 and the second die shows '+' sign, then, she has +5.



Whenever a player gets the + sign, she has to move in the forward direction (towards + 25) and if she gets '–' sign then she has to move in the backward direction (towards – 25).



Each player will throw both dice simultaneously. A player whose counter touches –25 is out of the game and the one whose counter touches + 25 first, wins the game.

You can play the same game with 12 cards marked with + 1, + 2, + 3, + 4, + 5 and + 6 and –1, – 2, ...– 6. Shuffle the cards after every attempt.

Kamla, Reshma and Meenu are playing this game.

Kamla got + 3, + 2, + 6 in three successive attempts. She kept her counter at the mark +11.

Reshma got – 5, + 3, + 1. She kept her counter at – 1. Meenu got + 4, – 3, –2 in three successive attempts; at what position will her counter be? At –1 or at + 1?

Do This

Take two different coloured buttons like white and black. Let us denote one white button by (+ 1) and one black button by (– 1). A pair of one white button (+ 1) and one black button (– 1) will denote zero i.e. [1 + (– 1) = 0]

In the following table, integers are shown with the help of coloured buttons.

Coloured Button	Integers
○ ○ ○ ○ ○	5
● ● ●	– 3
○ ●	0

Let us perform additions with the help of the coloured buttons.
Observe the following table and complete it.

○ ○ ○ + ○ ○ = ○ ○ ○ ○ ○	(+ 3) + (+ 2) = +5
● ● + ● = ● ● ●	(– 2) + (– 1) = –3
○ ○ ○ ○ + ○ = ○ ○ ○ ○ ○	....
● ● ● ● + ● ● = ...	....

Try These

Find the answers of the following additions:

(a) (– 11) + (– 12)

(b) (+ 10) + (+ 4)

(c) (– 32) + (– 25)

(d) (+ 23) + (+ 40)

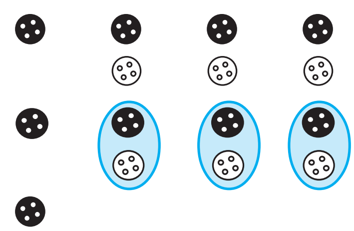
You add when you have two positive integers like (+3) + (+2) = +5 [= 3 + 2]. You also add when you have two negative integers, but the answer will take a minus (–) sign like (–2) + (–1) = – (2+1) = –3.

Now add one positive integer with one negative integer with the help of these buttons. Remove buttons in pairs i.e. a white button with a black button [since (+ 1) + (– 1) = 0]. Check the remaining buttons.

(a) (– 4) + (+ 3)

= (– 1) + (– 3) + (+ 3)

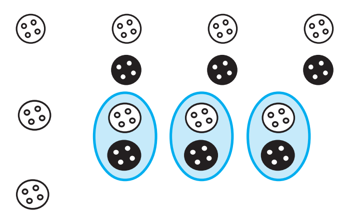
= (– 1) + 0 = – 1



(b) (+ 4) + (– 3)

= (+ 1) + (+ 3) + (– 3)

= (+ 1) + 0 = + 1



You can see that the answer of 4 – 3 is 1 and – 4 + 3 is – 1.

So, when you have one positive and one negative integer, you must subtract, but answer will take the sign of the bigger integer (Ignoring the signs of the numbers decide which is bigger). Some more examples will help :

Try These

Find the solution of the following:

(a) (– 7) + (+ 8)

(b) (– 9) + (+13)

(c) (+ 7) + (– 10)

(d) (+12) + (– 7)

(c) (+ 5) + (– 8) = (+ 5) + (– 5) + (– 3) = 0 + (– 3)
= (– 3)

(d) (+ 6) + (– 4) = (+ 2) + (+ 4) + (– 4) = (+ 2) + 0
= + 2

6.3.1 Addition of integers on a number line

It is not always easy to add integers using coloured buttons. Shall we use number line for additions?

(i) Let us add 3 and 5 on number line.

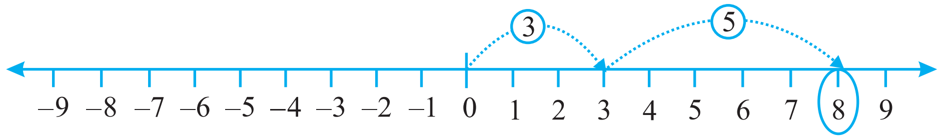


Fig 6.4

On the number line, we first move 3 steps to the right from 0 reaching 3, then we move 5 steps to the right of 3 and reach 8. Thus, we get 3 + 5 = 8 (Fig 6.4)

(ii) Let us add – 3 and – 5 on the number line.

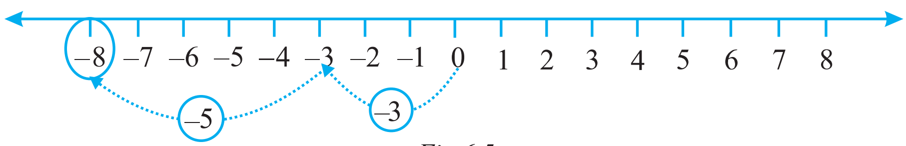


Fig 6.5

On the number line, we first move 3 steps to the left of 0 reaching – 3, then we move 5 steps to the left of – 3 and reach – 8. (Fig 6.5)

Thus, (– 3) + (– 5) = – 8.

We observe that when we add two positive integers, their sum is a positive integer. When we add two negative integers, their sum is a negative integer.

(iii) Suppose we wish to find the sum of (+ 5) and (– 3) on the number line. First we move to the right of 0 by 5 steps reaching 5. Then we move 3 steps to the left of 5 reaching 2. (Fig 6.6)

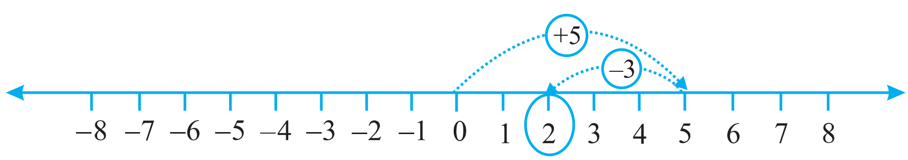


Fig 6.6

Thus, (+ 5) + (– 3) = 2

(iv) Similarly, let us find the sum of (– 5) and (+ 3) on the number line. First we move 5 steps to the left of 0 reaching – 5 and then from this point we move 3 steps to the right. We reach the point – 2.

Thus, (– 5) + (+3) = – 2. (Fig 6.7)

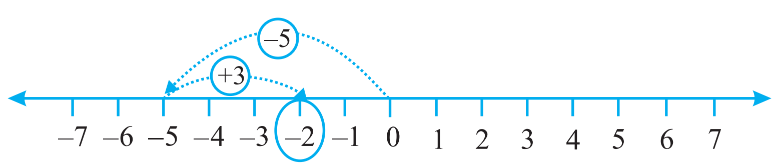


Fig 6.7

Try These

1. Find the solution of the following additions using a number line :
   (a) (– 2) + 6 (b) (– 6) + 2
   Make two such questions and solve them using the number line.
2. Find the solution of the following without using number line :
   (a) (+ 7) + (– 11)
   (b) (– 13) + (+ 10)
   (c) (– 7) + (+ 9)
   (d) (+ 10) + (– 5)
   Make five such questions and solve them.

When a positive integer is added to an integer, the resulting integer becomes greater than the given integer. When a negative integer is added to an integer, the resulting integer becomes less than the given integer.

Let us add 3 and – 3. We first move from 0 to + 3 and then from + 3, we move 3 points to the left. Where do we reach ultimately?

From the Figure 6.8, 3 + (– 3) = 0. Similarly, if we add 2 and – 2, we obtain the sum as zero. Numbers such as 3 and – 3, 2 and – 2, when added to each other give the sum zero. They are called **additive inverse** of each other.

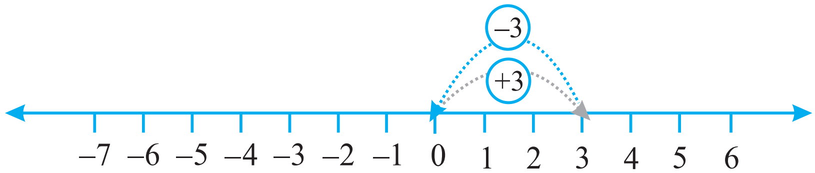


Fig 6.8

What is the additive inverse of 6? What is the additive inverse of – 7?

Example 3 : Using the number line, write the integer which is

(a) 4 more than –1

(b) 5 less than 3

Solution : (a) We want to know the integer which is 4 more than –1. So, we start from –1 and proceed 4 steps to the right of –1 to reach 3 as shown below:

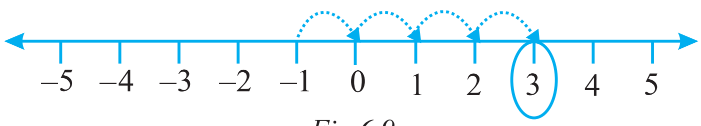


Fig 6.9

Therefore, 4 more than –1 is 3 (Fig 6.9).

(b) We want to know an integer which is 5 less than 3; so we start from 3 and move to the left by 5 steps and obtain –2 as shown below :

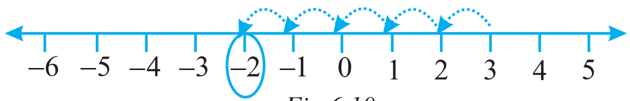


Fig 6.10

Therefore, 5 less than 3 is –2. (Fig 6.10)

Example 4 : Find the sum of (– 9) + (+ 4) + (– 6) + (+ 3)

Solution : We can rearrange the numbers so that the positive integers and the negative integers are grouped together. We have

(– 9) + (+ 4) + (– 6) + (+ 3) = (– 9) + (– 6) + (+ 4) + (+ 3) = (– 15) + (+ 7) = – 8

Example 5 : Find the value of (30) + (– 23) + (– 63) + (+ 55)

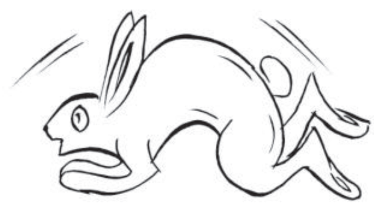
Solution : (30) + (+ 55) + (– 23) + (– 63) = 85 + (– 86) = – 1

Example 6 : Find the sum of (– 10), (92), (84) and (– 15)

Solution : (– 10) + (92) + (84) + (– 15) = (– 10) + (– 15) + 92 + 84
= (– 25) + 176 = 151

EXERCISE 6.2

1. Using the number line write the integer which is :
   (a) 3 more than 5
   (b) 5 more than –5
   (c) 6 less than 2
   (d) 3 less than –2



2. Use number line and add the following integers :
   (a) 9 + (– 6)
   (b) 5 + (– 11)
   (c) (– 1) + (– 7)
   (d) (– 5) + 10
   (e) (– 1) + (– 2) + (– 3)
   (f) (– 2) + 8 + (– 4)
3. Add without using number line :
   (a) 11 + (– 7)
   (b) (– 13) + (+ 18)
   (c) (– 10) + (+ 19)
   (d) (– 250) + (+ 150)
   (e) (– 380) + (– 270)
   (f) (– 217) + (– 100)

4. Find the sum of :
   (a) 137 and – 354 (b) – 52 and 52
   (c) – 312, 39 and 192 (d) – 50, – 200 and 300
5. Find the sum :
   (a) (– 7) + (– 9) + 4 + 16
   (b) (37) + (– 2) + (– 65) + (– 8)

6.4 Subtraction of Integers with the help of a Number Line

We have added positive integers on a number line. For example, consider 6+2. We start from 6 and go 2 steps to the right side. We reach at 8. So, 6 + 2 = 8. (Fig 6.11)

Fig 6.11

We also saw that to add 6 and (–2) on a number line we can start from 6 and then move 2 steps to the left of 6. We reach at 4. So, we have, 6 + (–2) = 4. (Fig 6.12)

Fig 6.12

Thus, we find that, to add a positive integer we move towards the right on a number line and for adding a negative integer we move towards left.

We have also seen that while using a number line for whole numbers, for subtracting 2 from 6, we would move towards left. (Fig 6.13)

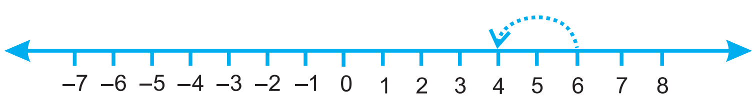


Fig 6.13

i.e. 6 – 2 = 4

What would we do for 6 – (–2)? Would we move towards the left on the number line or towards the right?

If we move to the left then we reach 4.

Then we have to say 6 – (–2) = 4. This is not true because we know 6 – 2 = 4 and 6 – 2 ≠ 6 – (–2).

So, we have to move towards the right. (Fig 6.14)

Fig 6.14

i.e. 6 – (–2) = 8

This also means that when we subtract a negative integer we get a greater integer. Consider it in another way. We know that additive inverse of (–2) is 2. Thus, it appears that adding the additive inverse of –2 to 6 is the same as subtracting (–2) from 6.

We write 6 – (–2) = 6 + 2.

Let us now find the value of –5 – (–4) using a number line. We can say that this is the same as –5 + (4), as the additive inverse of –4 is 4.

We move 4 steps to the right on the number line starting from –5. (Fig 6.15)

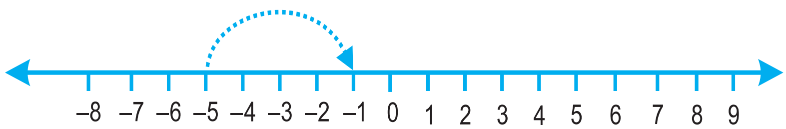


Fig 6.15

We reach at –1.

i.e. –5 + 4 = –1. Thus, –5 – (–4) = –1.

Example 7 : Find the value of – 8 – (–10) using number line

Solution : – 8 – (– 10) is equal to – 8 + 10 as additive inverse of –10 is 10. On the number line, from – 8 we will move 10 steps towards right. (Fig 6.16)

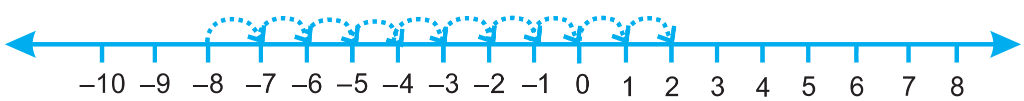


Fig 6.16

We reach at 2. Thus, –8 – (–10) = 2

Hence, to subtract an integer from another integer it is enough to add the additive inverse of the integer that is being subtracted, to the other integer.

Example 8 : Subtract (– 4) from (– 10)

Solution : (– 10) – (– 4) = (– 10) + (additive inverse of – 4)

= –10 + 4 = – 6

Example 9 : Subtract (+ 3) from (– 3)

Solution : $(-3) - (+3) = (-3) + (\text{additive inverse of } +3)$

$$= (-3) + (-3) = -6$$

EXERCISE 6.3

1. Find

   (a) $35 - (20)$ (b) $72 - (90)$

   (c) $(-15) - (-18)$ (d) $(-20) - (13)$

   (e) $23 - (-12)$ (f) $(-32) - (-40)$

2. Fill in the blanks with >, < or = sign.

   (a) $(-3) + (-6)$ ______ $(-3) - (-6)$

   (b) $(-21) - (-10)$ ______ $(-31) + (-11)$

   (c) $45 - (-11)$ ______ $57 + (-4)$

   (d) $(-25) - (-42)$ ______ $(-42) - (-25)$

3. Fill in the blanks.

   (a) $(-8) +$ ______ $= 0$

   (b) $13 +$ ______ $= 0$

   (c) $12 + (-12) =$ _____

   (d) $(-4) +$ _____ $= -12$

   (e) _____ $- 15 = -10$

4. Find

   (a) $(-7) - 8 - (-25)$

   (b) $(-13) + 32 - 8 - 1$

   (c) $(-7) + (-8) + (-90)$

   (d) $50 - (-40) - (-2)$

What have we discussed?

1. We have seen that there are times when we need to use numbers with a negative sign. This is when we want to go below zero on the number line. These are called *negative numbers*. Some examples of their use can be in temperature scale, water level in lake or river, level of oil in tank etc. They are also used to denote debit account or outstanding dues.

2. The collection of numbers..., $-4, -3, -2, -1, 0, 1, 2, 3, 4$, ... is called *integers*.

   So, $-1, -2, -3, -4$, ... called negative numbers are negative integers and $1, 2, 3, 4$, ... called positive numbers are the positive integers.

3. We have also seen how one more than given number gives a successor and one less than given number gives predecessor.

4. We observe that

   (a) When we have the same sign, add and put the same sign.

   (i) When two positive integers are added, we get a positive integer [e.g. $(+3) + (+2) = +5$].

   (ii) When two negative integers are added, we get a negative integer [e.g. $(-2) + (-1) = -3$].

   (b) When one positive and one negative integers are added we subtract them as whole numbers by considering the numbers without their sign and then put the sign of the bigger number with the subtraction obtained. The bigger integer is decided by ignoring the signs of the integers [e.g. $(+4) + (-3) = +1$ and $(-4) + (+3) = -1$].

   (c) The subtraction of an integer is the same as the addition of its additive inverse.

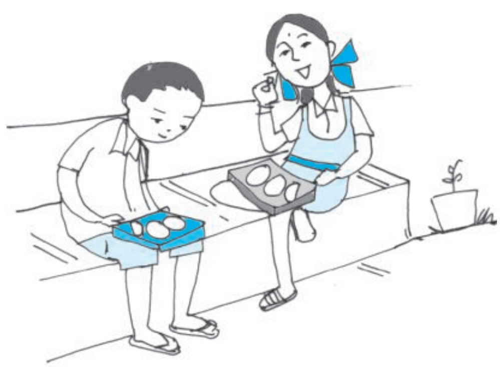
5. We have shown how addition and subtraction of integers can also be shown on a number line.

Fractions

Chapter 7

7.1 Introduction

Subhash had learnt about fractions in Classes IV and V, so whenever possible he would try to use fractions. One occasion was when he forgot his lunch at home. His friend Farida invited him to share her lunch. She had five pooris in her lunch box. So, Subhash and Farida took two pooris each. Then Farida made two equal halves of the fifth poori and gave one-half to Subhash and took the other half herself. Thus, both Subhash and Farida had 2 full pooris and one-half poori.



2 pooris + half-poori–Subhash
2 pooris + half-poori–Farida

Where do you come across situations with fractions in your life?

Subhash knew that one-half is written as $\frac{1}{2}$. While eating he further divided his half poori into two equal parts and asked Farida what fraction of the whole poori was that piece? (Fig 7.1)

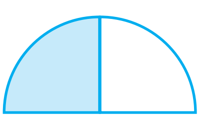


Fig 7.1

Without answering, Farida also divided her portion of the half puri into two equal parts and kept them beside Subhash's shares. She said that these four equal parts together make one

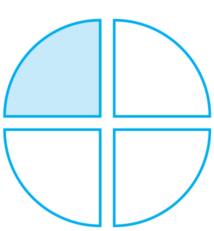


Fig 7.2

whole (Fig 7.2). So, each equal part is one-fourth of one whole poori and 4 parts together will be $\frac{4}{4}$ or 1 whole poori.

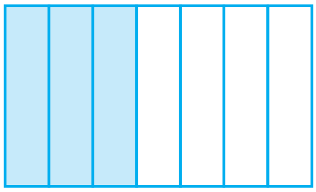


Fig 7.3

Fig 7.4

When they ate, they discussed what they had learnt earlier. Three parts out of 4 equal parts is $\frac{3}{4}$. Similarly, $\frac{3}{7}$ is obtained when we divide a whole into seven equal parts and take three parts (Fig 7.3). For $\frac{1}{8}$, we divide a whole into eight equal parts and take one part out of it (Fig 7.4).

Farida said that we have learnt that **a fraction is a number representing part of a whole. The whole may be a single object or a group of objects.** Subhash observed that **the parts have to be equal.**

7.2 A Fraction

Let us recapitulate the discussion.
A fraction means a part of a group or of a region.

$\frac{5}{12}$ is a fraction. We read it as "five-twelfths".

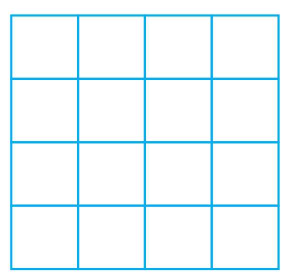
What does "12" stand for? It is the number of equal parts into which the whole has been divided.

What does "5" stand for? It is the number of equal parts which have been taken out.

Here 5 is called the numerator and 12 is called the denominator.

Name the numerator of $\frac{3}{7}$ and the denominator of $\frac{4}{15}$.

Play this Game



You can play this game with your friends.

Take many copies of the grid as shown here.

Consider any fraction, say $\frac{1}{2}$.

Each one of you should shade $\frac{1}{2}$ of the grid.