Mechanical Properties of Solids

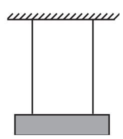
OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions (MCQs)

1. Four wires of the same material are stretched by the same load. The dimensions are given below. Which of them will elongate the most ?

(a) Length 100 cm, diameter 1 cm
(b) Length 200 cm, diameter 2 cm
(c) Length 300 cm, diameter 3 cm
(d) Length 400 cm, diameter 0.5 cm

2. Two wires of equal length and cross-section area are suspended as shown in figure. Their Young's modulus are Y_1 and Y_2 respectively. The equivalent Young's modulus will be



(a) $Y_1 + Y_2$ (b) $\frac{Y_1 + Y_2}{2}$
(c) $\frac{Y_1 Y_2}{Y_1 + Y_2}$ (d) $\sqrt{Y_1 Y_2}$

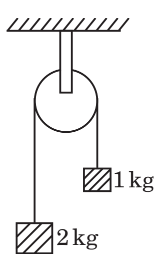
3. Two wires A and B have the same length and area of cross-section. But Young's modulus of A is two times the Young's modulus of B. Then the ratio of force constant of A to that of B is

(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) $\sqrt{2}$

4. One end of a horizontal thick copper wire of length $2L$ and radius $2R$ is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is

(a) 0.25 (b) 0.50
(c) 2.00 (d) 4.00

5. Two blocks of masses 1 kg and 2 kg are connected by a metal wire going over a smooth pulley as shown in figure. The breaking stress of the metal is $(40/3\pi) \times 10^6$ N/m^2. If g = 10 m s^{-2}, then the minimum radius of the wire used if it is not to break is



(a) 0.5 mm (b) 1 mm
(c) 1.5 mm (d) 2 mm

6. A wire of cross-section 4 mm^2 is stretched by 0.1 mm by a certain weight. How far (length) will the wire of same material and length but of area 8 mm^2 stretch under the action of same force?

(a) 0.05 mm (b) 0.10 mm
(c) 0.15 mm (d) 0.20 mm

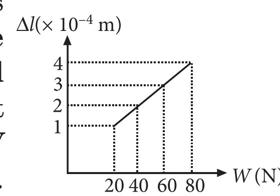
7. If Y and B represent Young's modulus and bulk modulus for a material, then in practice

(a) $Y < 3B$ (b) $Y = 3B$
(c) $Y > 3B$ (d) $B = 3Y$

8. The compressibility of water is 6×10^{-10} N^{-1}m^2. If one litre is subjected to a pressure of 4×10^7 N m^{-2}, the decrease in its volume is

(a) 10 cc (b) 24 cc
(c) 15 cc (d) 12 cc

9. The given graph shows the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is 10^{-6} m^2, then the Young's modulus of the material of the wire is



(a) 2×10^{11} N/m^2 (b) 2×10^{-11} N/m^2
(c) 3×10^{-12} N/m^2 (d) 2×10^{-13} N/m^2

10. A solid sphere of radius R, made of a material of bulk modulus K, is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid the fractional change in the radius of the sphere dR/R, is

(a) $Mg/2AK$ (b) $Mg/3AK$
(c) Mg/AK (d) $2Mg/3AK$

11. A steel wire of length 4.7 m and cross-sectional area 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of 4.0×10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel to that of copper?

(a) 1.8 : 1 (b) 2.8 : 1
(c) 3.8 : 1 (d) 4.8 : 1

12. A and B are two wires. The radius of A is twice that of B. If they are stretched by the same load, then the stress on B is

(a) equal to that of A (b) two times that of A
(c) four times that of A (d) half that of A.

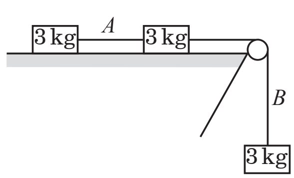
13. The Young's modulus of brass and steel are respectively 1.0×10^{11} N m^{-2} and 2.0×10^{11} N m^{-2}. A brass wire and a steel wire of the same length are extended by 1 mm each under the same force. If radii of brass and steel wires are R_B and R_S respectively, then

(a) $R_S = \sqrt{2} R_B$ (b) $R_S = \frac{R_B}{\sqrt{2}}$
(c) $R_S = 4R_B$ (d) $R_S = \frac{R_B}{2}$

14. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area 3A. If length of wire 1 is increased by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount?

(a) $4F$ (b) $6F$
(c) $9F$ (d) F

15. Three equal masses of 3 kg are connected by massless string of cross sectional area 0.005 cm^2 and Young's modulus 2×10^{11} N/m^2. In the absence of friction, the longitudinal strain in the wire



(a) A is 10^{-4} (b) B is 2×10^{-4}
(c) both (a) and (b) (d) none of these

16. A polyster fibre rope of diameter 3 cm has a breaking strength of 150 kN. If it is required to have 600 kN breaking strength. What should be the diameter of similar rope?

(a) 12 cm (b) 6 cm
(c) 3 cm (d) 1.5 cm

17. Two wires A and B of the same material have their lengths in the ratio of 1 : 2 and their diameters in the ratio of 2 : 1. If they are stretched with the same force, the ratio of the increase in the length of A to that of B will be

(a) 1 : 2 (b) 4 : 1
(c) 1 : 8 (d) 1 : 4

18. For a perfectly rigid body

(a) Young's modulus is infinite and bulk modulus is zero.
(b) Young's modulus is zero and bulk modulus is infinite.
(c) Young's modulus is infinite and bulk modulus is also infinite.
(d) Young's modulus is zero and bulk modulus is also zero.

19. A steel rod of length 1 m and radius 10 mm is stretched by a force 100 kN along its length. The stress produced in the rod is
($Y_{Steel} = 2 \times 10^{11}$ N m^{-2})

(a) 3.18×10^6 N m^{-2} (b) 3.18×10^7 N m^{-2}
(c) 3.18×10^8 N m^{-2} (d) 3.18×10^9 N m^{-2}

20. The pressure of a medium is changed from 1.01×10^5 Pa to 1.165×10^5 Pa and change in volume is 10% keeping temperature constant. Find the bulk modulus of the medium.

(a) 1×10^3 Pa (b) 3×10^5 Pa
(c) 2×10^4 Pa (d) 1.55×10^5 Pa

21. The area of a cross-section of steel wire is 0.1 cm^2 and Young's modulus of steel is 2×10^{11} N m^{-2}. The force required to stretch by 0.1% of its length is

(a) 2000 N (b) 1000 N
(c) 1500 N (d) 1700 N

22. A light rod of length 200 cm is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its ends. One of the wires is made of steel and is of cross-section 0.1 cm^2 and the other of brass of cross-section 0.2 cm^2. Along the rod at which distance a weight may be hung to produce equal stresses in both the wires?
($Y_{Steel} = 2 \times 10^{11}$ N m^{-2}, $Y_{Brass} = 1 \times 10^{11}$ N m^{-2})

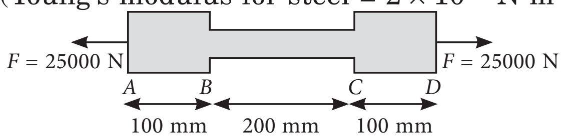
(a) $\frac{4}{3}$ m from steel wire

(b) $\frac{4}{3}$ m from brass wire

(c) 1 m from steel wire

(d) $\frac{1}{4}$ m from brass wire

23. A steel bar *ABCD* 40 cm long is made up of three parts *AB*, *BC* and *CD*, as shown in figure. The rod is subjected to a pull of 25 kN. Diamerter of parts *AB* and *CD* is 50 mm while diametre of port *BC* is 25 mm. The total extension of the rod is (Young's modulus for steel = 2×10^{11} N m^{-2})



(a) 0.0637 mm (b) 0.0647 mm
(c) 0.0657 mm (d) 0.0667 mm

24. The bulk modulus of water if its volume changes from 100 litre to 99.5 litre under a pressure of 100 atm is
(Take 1 atm = 10^5 N m^{-2})
(a) 2×10^7 N m^{-2} (b) 2×10^8 N m^{-2}
(c) 2×10^9 N m^{-2} (d) 2×10^{10} N m^{-2}

25. A steel cable with a radius 2 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 N m^{-2}, the maximum load the cable can support is
(a) $4\pi \times 10^5$ N (b) $4\pi \times 10^4$ N
(c) $2\pi \times 10^5$ N (d) $2\pi \times 10^4$ N

26. Assuming that shear stress at the base of a mountain is equal to the force per unit area due to its weight. Calculate the maximum possible height of a mountain on the earth if breaking stress of a typical rock is 3×10^8 N m^{-2} and its density is 3×10^3 kg m^{-3}.
(Take g = 10 m s^{-2})
(a) 4 km (b) 8 km
(c) 10 km (d) 16 km

27. A steel wire of length 4.5 m and cross-sectional area 3×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of 4×10^{-5} m^2 under a given load. The ratio of the Young's modulus of steel to that of copper is
(a) 1.3 (b) 1.5
(c) 1.7 (d) 1.9

28. The length of an iron wire is L and area of cross-section is A. The increase in length is l on applying the force F on its two ends. Which of the following statements is correct ?
(a) Increase in length is inversely proportional to its length L.
(b) Increase in length is proportional to area of cross-section A.
(c) Increase in length is inversely proportional to cross-section A.
(d) Increase in length is proportional to Young's modulus.

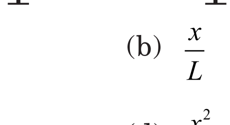
29. A wire of length L and radius r fixed at one end and a force F applied to the other end produces an extension l. The extension produced in another wire of the same material of length 2 L and radius $2r$ by a force $2F$, is
(a) l (b) $2l$
(c) $4l$ (d) $\frac{l}{2}$

30. A glass slab is subjected to a pressure of 10 atm. The fractional change in its volume is (Bulk modulus of glass = 37×10^9 N m^{-2}, 1 atm = 1×10^5 N m^{-2})
(a) 2.7×10^{-2} (b) 2.7×10^{-3}
(c) 2.7×10^{-4} (d) 2.7×10^{-5}

31. A rigid bar of mass M is supported symmetrically by three wires each of length L. Those at each end are of copper and the middle one is of iron. The ratio of their diameters, if each is to have the same tension, is equal to
(a) $\frac{Y_{copper}}{Y_{iron}}$ (b) $\sqrt{\frac{Y_{iron}}{Y_{copper}}}$
(c) $\frac{Y_{iron}^2}{Y_{copper}^2}$ (d) $\frac{Y_{iron}}{Y_{copper}}$

32. The length of an elastic spring is a metres when a force of 4 N is applied, and b metres when the 5 N force is applied. Then the length of the spring when the 9 N force is applied is
(a) $a + b$ (b) $9b - 9a$
(c) $5b - 4a$ (d) $4a - 5b$

33. A mild steel wire of length $2L$ and cross-sectional area A is stretched, well within elastic limit, horizontally between two pillars as shown in the figure. A mass m is suspended from the mid-point of the wire. Strain in the wire is



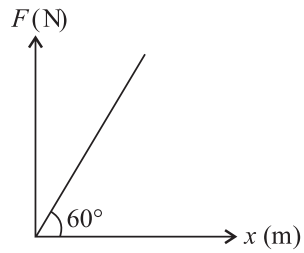
(a) $\frac{x^2}{2L^2}$ (b) $\frac{x}{L}$
(c) $\frac{x^2}{L}$ (d) $\frac{x^2}{2L}$

34. The volume change of a solid copper cube 10 cm on an edge, when subjected to a pressure of 7 MPa is (Bulk modulus of copper = 140 GPa)

(a) 5×10^{-2} cm^3 (b) 10×10^{-2} cm^3
(c) 15×10^{-2} cm^3 (d) 20×10^{-2} cm^3

35. How much pressure should be applied on a litre of water if it is to be compressed by 0.1%? (Bulk modulus of water = 2100 MPa)
(a) 2100 kPa (b) 210 kPa
(c) 2100 MPa (d) 210 MPa

36. A graph between the restoring force (F) of a wire and the extension (x) produced in it, is shown in the figure. Then $\frac{YA}{L}$ (in SI unit) is



(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$
(c) 3 (d) $\frac{1}{3}$

37. The following four wires of length L and radius r are made of the same material. Which of these will have the largest extension, when the same tension is applied?
(a) L = 100 cm, r = 0.2 mm
(b) L = 200 cm, r = 0.4 mm
(c) L = 300 cm, r = 0.6 mm
(d) L = 400 cm, r = 0.8 mm

38. The radii and Young's moduli of two uniform wires A and B are in the ratio 2 : 1 and 1 : 2 respectively. Both wires are subjected to the same longitudinal force. If the increase in length of the wire A is one percent, the percentage increase in length of the wire B is
(a) 1.0 (b) 1.5
(c) 2.0 (d) 3.0

39. The following four wires of length L and radius r are made of the same material. Which of these will have the largest extension, when the same tension is applied?
(a) L = 100 cm, r = 0.2 mm
(b) L = 200 cm, r = 0.4 mm
(c) L = 300 cm, r = 0.6 mm
(d) L = 400 cm, r = 0.8 mm

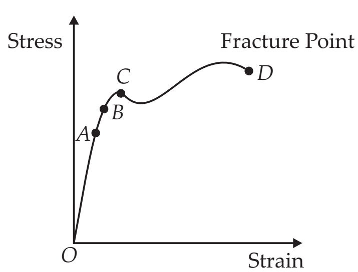
40. A force of 6×10^6 N m^{-2} required for breaking a material. The density ρ of the material is 3×10^3 kg m^{-3}. If the wire is to break under its own weight, then the length of the wire made of that material should be (Given, g = 10 m s^{-2})
(a) 20 m (b) 200 m
(c) 100 m (d) 2000 m

Case Based MCQs

Case I : Read the passage given below and answer the following questions from 41 to 45.

Stress-Strain Curve

The graph shown below shows qualitatively the relation between the stress and the strain as the deformation gradually increases. Within Hooke's limit for a certain region stress and strain relation is linear. Beyond that up to a certain value of strain the body is still elastic and if deforming forces are removed the body recovers its original shape.



41. If deforming forces are removed up to which point the curve will be retraced?
(a) upto OA only
(b) upto OB
(c) upto C
(d) Never retraced its path

42. In the above question, during loading and unloading the force exerted by the material are conservative up to
(a) OA only (b) OB only
(c) OC only (d) OD only

43. During unloading beyond B, say C, the length at zero stress in now equal to
(a) less than original length
(b) greater than original length
(c) original length
(d) can't be predicted

44. The breaking stress for a wire of unit cross-section is called

(a) yield point (b) elastic fatigue

(c) tensile strength (d) Young's modulus

45. Substances which can be stretched to cause large strains are called

(a) isomers (b) plastomers

(c) elastomers (d) polymers

Case II : Read the passage given below and answer the following questions from 46 to 50.

Hooke's law

According to Hooke's law, within the elastic limit, the stress applied to a body is directly proportional to the corresponding strain.

Stress $\propto$ Strain

or Stress $= E \times$ Strain or $\dfrac{\text{Stress}}{\text{Strain}} = E$

Where E is the constant of proportionality and is known as coefficient of elasticity or modulus of elasticity.

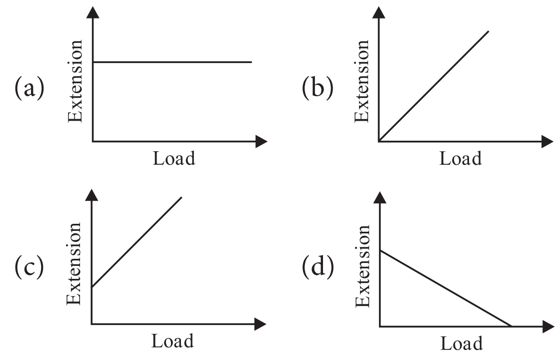
Hooke's law is an empirical law and is found to be valid for most materials. However, there are some materials which do not exhibit this linear relationship.

46. According to Hooke's law of elasticity, if stress is increased, the ratio of stress to strain

(a) decreases (b) increases

(c) becomes zero (d) remains constant

47. Within elastic limit, which of the following graphs correctly represents the variation of extension in the length of a wire with the external load?



48. According to Hooke's law, if stress is reduced to one-third, the ratio of stress to strain

(a) is increased to three time

(b) is decreased

(c) is zero

(d) remains constant.

49. Hooke's law defines

(a) stress

(b) strain

(c) modulus of elasticity

(d) elastic limit.

50. Whenever a material is loaded with elasitic limits, stress is .... strain.

(a) equal to

(b) directly proportional to

(c) inxessely propotional to

(d) None of the above given

Assertion & Reasoning Based MCQs

For question numbers 51-60, two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is NOT the correct explanation of A

(c) A is true but R is false

(d) A is false and R is also false

51. Assertion (A) : Spring balances show incorrect readings after they had been used for a long time interval.

Reason (R) : On using for long time, spring balance loses its elastic strength.

52. Assertion (A) : Breaking stress is fixed for a material, but breaking force will vary, depending on area of cross section of the wire.

Reason (R) : While using a material, the working stress is always kept much lower than that of breaking stress so that safety factor (= breaking stress/working stress) may have a large value.

53. Assertion (A) : Glassy solids have sharp melting point.

Reason (R) : The bonds between the atoms of glassy solids get broken at the same temperature.

54. Assertion (A) : Bulk modulus of elasticity (K) represents incompressibility of the material.

Reason (R) : Bulk modulus of elasticity is proportional to change in pressure.

55. Assertion (A) : Strain is a unitless quantity.
Reason (R) : Strain is equivalent to force.

56. Assertion (A) : The crane which is used to lift and move the heavy load is provided with thick and strong metallic ropes to which the load to be lifted is attached.
Reason (R) : The thickness of the metallic rope used in the crane is decided from the knowledge of elastic limit of the material of the rope and the factor of safety.

57. Assertion (A) : Young's modulus for a perfectly plastic body is zero.
Reason (R) : For a perfectly plastic body, restoring force is zero.

58. Assertion (A) : Identical springs of steel and copper are equally stretched. More work will be done on the steel spring.
Reason (R) : Steel is more elastic than copper.

59. Assertion (A) : Two identical solid balls, one of ivory and the other of wet-clay are dropped from the same height on the floor. Both the balls will rise to same height after bouncing.
Reason (R) : Ivory and wet-clay have same elasticity.

60. Assertion (A) : Steel is more elastic than rubber.
Reason (R) : Under given deforming force, steel is deformed less than rubber.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

1. Write dimensional formula of Young's modulus.

2. What is the value of bulk modulus for an incompressible liquid?

3. What is the value of compressibility for an incompressible liquid ?

4. Give the relation between bulk modulus and compressibility.

5. What is elastic fatigue?

6. What do you mean by 'permanent set' in a body?

7. Stress and pressure are both force per unit area. Then in what respect does stress differ from pressure?

8. The ratio stress/strain remains constant for small deformation. What will be the effect on this ratio when the deformation made is very large?

9. Arrange the value of bulk modulus of elasticity of solids, liquids and gases according to their magnitude. Give reason.

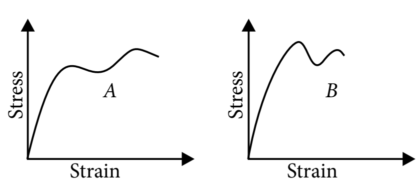
10. What does the slope of stress versus strain graph give?

Short Answer Type Questions (SA-I)

11. What is a perfectly plastic body? Give an example.

12. No material is perfectly elastic. Why?

13. The stress-strain graphs for materials A and B are shown in figure.



The graphs are drawn to the same scale.

(a) Which is more brittle?
(b) Which of the two is the stronger material?

14. Define modulus of elasticity. Name its three components.

15. A cable is replaced by another cable of the same length and material but twice the diameter. How will this affect the elongation under a given load?

16. What is the percentage increase in the length of a wire of diameter 2.5 mm stretched by a force of 100 kgf? Young's modulus of elasticity of the wire is 12.5×10^{11} dyne cm^{-2}.

17. Why does modulus of elasticity of most of the materials decrease with the increase of temperature?

18. (a) Is it possible to double the length of a metallic wire by applying a force over it?

(b) Is elastic limit a property of the material of the wire?

19. A wire stretches by a certain amount under a load. If the load and radius are both increased to four times, find the stretch caused in the wire.

20. State Hooke's law. Calculate the fractional compression, $\Delta V/V$, of water at the bottom of the ocean having depth 3000 m. The bulk modulus of water is 2.2×10^9 N m^{-2}. (Take g = 10 m s^{-2})

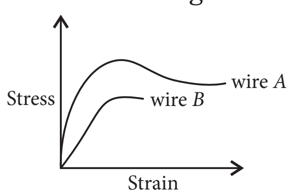
Short Answer Type Questions (SA-II)

21. Define the term strain. Why it has no units and dimensions? What are different types of strain?

22. Define the term stress. Give its units and dimensions. Describe the different types of stress.

23. On the basis of stress-strain curves, distinguish between ductile and brittle materials.

24. Stress strain curve for two wires of material A and B are as shown in figure.



(a) Which material in more ductile?

(b) Which of the two is stronger material?

(c) Which material is more brittle?

25. A structural steel rod has a radius of 10 mm and a length of 1 m. A 100 kN force F stretches it along its length. Calculate (a) the stress, (b) elongation, and (c) strain on the rod. Given that the Young's modulus of the structural steel is 2.0×10^{11} N m^{-2}.

26. Compute the bulk modulus of water from the following data : Initial volume = 100.0 litre, pressure increase = 100.0 atm (1 atm = 1.013×10^5 Pa), final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

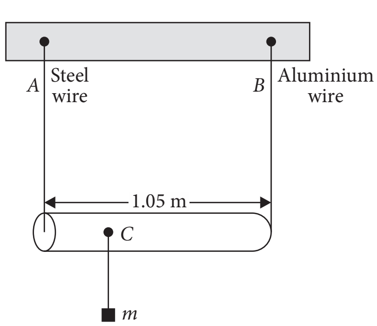
27. A cable is replaced by another cable of the same length and material but of half the diameter.

(a) How does this affect its elongation under a given load?

(b) How many times will be the maximum load it can now support without exceeding the elastic limit?

Long Answer Type Questions (LA)

28. A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in figure. The cross-sectional areas of wires A and B are 1.0 mm^2 and 2.0 mm^2 respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires?

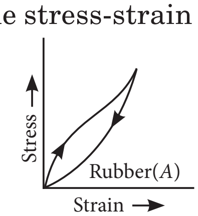


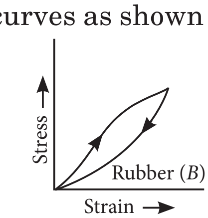
29. (a) A wire 50 cm long and 1 mm^2 in cross section has the Young's modulus, $Y = 2 \times 10^{10}$ N m^{-2}. How much work is done in stretching the wire through 1 mm?

(b) What is the length of a wire that breaks under its own weight when suspended vertically? Breaking stress = 5×10^7 N m^{-2}

Density of the material of the wire = 3×10^3 kg/m^3

30. Two different types of rubber are found to have the stress-strain curves as shown in figure





(a) In which significant ways do these curves shown in figure differ from the stress-strain curve of a metal wire ?

(b) A heavy machine is to be installed in a factory. To absorb vibrations of the machine, a block of rubber is placed between the machinery and the floor. Which of the two rubber A and B would you prefer to use for this purpose? Why?

(c) Which of the two rubber materials would you choose for a car tyre?

ANSWERS

OBJECTIVE TYPE QUESTIONS

1. (d) : The elongation produced in a wire is $\Delta L = \dfrac{4FL}{\pi D^2 Y}$

As F and Y are same for all four wires, $\therefore\ \Delta L \propto \dfrac{L}{D^2}$

For wire 1, $\dfrac{L}{D^2} = \dfrac{100}{1^2} = 100$

For wire 2, $\dfrac{L}{D^2} = \dfrac{200}{2^2} = 50$

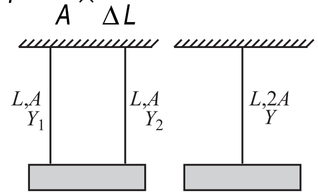
For wire 3, $\dfrac{L}{D^2} = \dfrac{300}{3^2} = \dfrac{100}{3}$

For wire 4, $\dfrac{L}{D^2} = \dfrac{400}{(0.5)^2} = 1600$

Thus the wire 4 will elongate the most.

2. (b) : Young's modulus,

$$Y = \frac{F}{A} \times \frac{L}{\Delta L}$$



Force constant of a wire is

$$k = \frac{F}{\Delta L} = \frac{YA}{L}$$

Refer figure, the equivalent force constant is $k_{eq} = k_1 + k_2$

or $\dfrac{Y(2A)}{L} = \dfrac{Y_1 A}{L} + \dfrac{Y_2 A}{L}$ or $Y = \dfrac{Y_1 + Y_2}{2}$

3. (b) : Force constant, $k = \dfrac{YA}{L}$

As A and L are same for both the wires,

$$\therefore \quad \frac{k_A}{k_B} = \frac{Y_A}{Y_B} = 2$$

4. (c) : 

Both the rods are made up of same kind of material, i.e., their Young's modulus is same.

$$k_1 = \frac{4\pi R^2 Y}{2L};\ k_2 = \frac{\pi R^2 Y}{L}$$

Force constants of two rods are,

$$\therefore \quad F = k_1 x = k_2 y \Rightarrow \frac{y}{x} = \frac{k_1}{k_2} = 2$$

5. (b) : $T = \dfrac{2m_1 m_2}{m_1 + m_2} g = \dfrac{2 \times 1 \times 2}{1+2} \times 10\ \text{N} = \dfrac{40}{3}\ \text{N}$

If r is the minimum radius, then

Breaking stress $= \dfrac{\frac{40}{3}}{\pi r^2}$ or $\dfrac{40}{3\pi} \times 10^6 = \dfrac{40}{3\pi r^2}$

or $r^2 = \dfrac{1}{10^6}$ or $r = \dfrac{1}{10^3}$ m $= 1$ mm

6. (a) : Here, $A_1 = 4\ \text{mm}^2 = 4 \times 10^{-6}\ \text{m}^2$,

$\Delta L_1 = 0.1 \times 10^{-3}$ m

$A_2 = 8\ \text{mm}^2 = 8 \times 10^{-6}\ \text{m}^2$; $Y_2 = Y_1$; $L_2 = L_1$; $F_2 = F_1$; $\Delta L_2 = ?$

$$\Delta L_1 = \frac{F_1 L_1}{A_1 Y_1}, \quad \Delta L_2 = \frac{F_2 L_2}{A_2 Y_2}$$

$$\therefore \quad \frac{\Delta L_2}{\Delta L_1} = \frac{A_1}{A_2} = \frac{4 \times 10^{-6}}{8 \times 10^{-6}} = \frac{1}{2}$$

or $\Delta L_2 = \dfrac{\Delta L_1}{2} = 0.05 \times 10^{-3}\ \text{m} = 0.05\ \text{mm}$

7. (a) : $\dfrac{Y}{3B} = 1 - 2\sigma$

But σ, in practice lies between zero and $\dfrac{1}{2}$, so $0 < 1 - 2\sigma < 1$

Hence, $\dfrac{Y}{3B} < 1$ or $Y < 3B$

8. (b) : Bulk modulus, $B = -\dfrac{P}{(\Delta V / V)}$

–ve sign shows that with an increase in pressure, a decrease in volume occurs.

Compressibility, $K = \frac{1}{B} = -\frac{\Delta V}{PV}$

Decrease in volume, $\Delta V = PVK$

$= 4 \times 10^7 \times 1 \times 6 \times 10^{-10}$

$= 24 \times 10^{-3}$ litre

$= 24 \times 10^{-3} \times 10^3\ cm^3 = 24$ cc

9. (a) : $Y = \frac{F/A}{\Delta l / l} = \frac{F}{A}\frac{l}{\Delta l}$

$Y = \frac{20 \times 1}{10^{-6} \times 10^{-4}} = 2 \times 10^{11}\ \mathrm{N\ m^{-2}}$

10. (b) : Bulk modulus,

$$K = \frac{\text{Stress}}{\text{Volume strain}} = \frac{F/A}{\Delta V/V} = \frac{Mg/A}{\Delta V/V} \text{ or } \frac{\Delta V}{V} = \frac{Mg}{AK} \quad \text{...(i)}$$

Volume of sphere, $V = \frac{4}{3}\pi R^3$

$$\therefore \quad \Delta V = 4\pi R^2\, \Delta R \quad \therefore \quad \frac{\Delta V}{V} = \frac{4\pi R^2\ \Delta R}{\frac{4}{3}\pi R^3} = 3\frac{\Delta R}{R}$$

$$\text{or} \quad \frac{Mg}{AK} = 3\frac{\Delta R}{R} \text{ or } \frac{\Delta R}{R} = \frac{Mg}{3AK} \quad \therefore \quad \frac{\delta R}{R} = \frac{Mg}{3AK}.$$

11. (a) : Given, for steel wire,

$A_1 = 3.0 \times 10^{-5}\ m^2$, $L_1 = 4.7$ m, $\Delta L_1 = \Delta L$, $F_1 = F$

For copper wire,

$A_2 = 4.0 \times 10^{-5}\ m^2$, $L_2 = 3.5$ m, $\Delta L_2 = \Delta L$, $F_2 = F$

Let Y_1, Y_2 be the Young's modulus of steel wire and copper wire respectively.

$$\therefore \quad Y_1 = \frac{F_1 L_1}{A_1 \Delta L_1} = \frac{F \times 4.7}{3.0 \times 10^{-5} \times \Delta L}$$

$$\text{and } Y_2 = \frac{F_2 L_2}{A_2 \Delta L_2} = \frac{F \times 3.5}{4 \times 10^{-5} \times \Delta L}$$

$$\therefore \quad \frac{Y_1}{Y_2} = \frac{4.7 \times 4 \times 10^{-5}}{3.0 \times 10^{-5} \times 3.5} = 1.8$$

Hence $Y_1 : Y_2 = 1.8 : 1$.

12. (c) : Stress $= \frac{F}{A}$

Let r be the radius of B.

For B, Stress $= \frac{F}{\pi r^2}$. For A, $\frac{F}{\pi(2r)^2} = \frac{F}{\pi 4r^2}$

$$\therefore \quad \frac{F}{\pi r^2} = 4\left[\frac{F}{\pi r^2}\right]; \text{ Stress on } B = 4 \text{ (Stress on } A)$$

13. (b) : Increase in length, $dL = \frac{FL}{YA} = \frac{FL}{Y\pi R^2}$

As F, L and ΔL are same hence,

YR^2 = constant

$$\therefore \quad 2.0 \times 10^{11} R_S^2 = 1.0 \times 10^{11} R_B^2 \Rightarrow R_S = \frac{R_B}{\sqrt{2}}$$

14. (c) : As $Y = \frac{FL}{A\Delta L}$

$$\therefore \quad F = \frac{YA\Delta L}{L} = \frac{YA^2\Delta L}{AL} = \frac{YA^2\Delta L}{V} = \frac{YA^2\Delta x}{V}$$

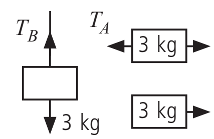
where $AL = V$ = Volume of wire.

Young's modulus is the same as both the wires are made of same material. It is given that both the wires have same volume and same extension in length.

$$\therefore \quad \frac{F'}{F} = \frac{A'^2}{A^2} = \frac{(3A)^2}{A^2} = 9 \quad \text{or} \quad F' = 9F$$

15. (c) : $3g - T_B = 3a$

$T_A = 3a$



$T_B - T_A = 3a$ are the equations defining the motion

Solving we get, $a = \frac{3g}{9} = \frac{g}{3}$, $T_A = g$ and $T_B = 2g$

So, strain in $A = \frac{T_A}{aY} = \frac{g}{0.005 \times 10^{-4} \times Y} = 10^{-4}$

strain in $B = \frac{T_B}{aY} = \frac{2g}{0.005 \times Y \times 10^{-4}} = 2 \times 10^{-4}$

So, choice (c) is the correct answer.

16. (b) : $Y = \frac{FL}{A\Delta L}$

The breaking strength $F \propto A$

$$\therefore \quad \frac{F_2}{F_1} = \frac{A_2}{A_1} = \frac{\pi D_2^2/4}{\pi D_1^2/4} = \frac{D_2^2}{D_1^2}$$

$$\text{or} \quad D_2 = D_1\left(\frac{F_2}{F_1}\right)^{1/2} = 3\left(\frac{600}{150}\right)^{1/2} = 6 \text{ cm}$$

17. (c) : If a wire of length L and radius r is stretched by ΔL by applying a force F, then $\Delta L = \frac{FL}{\pi r^2 Y}$

$$\therefore \ \Delta L_A = \frac{FL_A}{\pi r_A^2 Y} \text{ and } \Delta L_B = \frac{FL_B}{\pi r_B^2 Y} \text{ or } \frac{\Delta L_A}{\Delta L_B} = \frac{L_A}{L_B}\left(\frac{r_B}{r_A}\right)^2$$

Given, $\frac{L_A}{L_B} = \frac{1}{2}$ and $\frac{r_A}{r_B} = \frac{2}{1}$

$$\therefore \quad \frac{\Delta L_A}{\Delta L_B} = \frac{1}{2} \times \frac{1}{(2)^2} = \frac{1}{8}$$

18. (c) : For a perfectly rigid body, both Young's modulus and bulk modulus is infinite.

19. (c) : Here, $r = 10$ mm $= 10 \times 10^{-3}$ m $= 10^{-2}$ m

$L = 1$ m, $F = 100$ kN $= 100 \times 10^3$ N $= 10^5$ N

Stress produced in the rod is

$$\text{Stress} = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{100 \times 10^3 \text{N}}{3.14 \times (10^{-2}\text{m})^2} = 3.18 \times 10^8 \text{ N m}^{-2}$$

20. (d) : Bulk modulus of elasticity, $B = \dfrac{\Delta P}{\dfrac{-\Delta V}{V}}$

Since pressure increases, volume decreases. Therefore,

$$\frac{\Delta V}{V} = -\frac{10}{100} \Rightarrow B = -\frac{(1.165 - 1.01) \times 10^5}{-\dfrac{10}{100}} = 1.55 \times 10^5 \text{ Pa}$$

21. (a) : Here,

$A = 0.1$ cm^2 $= 0.1 \times 10^{-4}$ m^2

$Y = 2 \times 10^{11}$ N m^{-2}

$$\frac{\Delta L}{L} = 0.1\% = \frac{0.1}{100} = 0.1 \times 10^{-2}$$

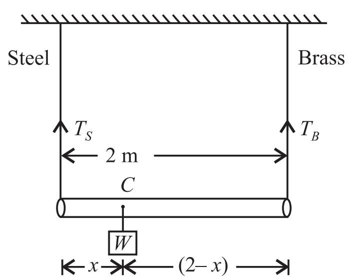
As $\quad Y = \dfrac{F/A}{\Delta L/L}$

$\therefore \quad F = Y\dfrac{\Delta L}{L}A$

$= 2 \times 10^{11}$ N m^{-2} $\times 0.1 \times 10^{-2} \times 0.1 \times 10^{-4}$ m^2

$= 2 \times 10^3$ N $= 2000$ N

22. (a) : The situation is as shown in the figure.



Let a weight W be suspended at a distance x from steel wire. Let T_S and T_B be tensions in the steel and brass wires respectively.

$\therefore \quad$ Stress in steel wire $= \dfrac{T_S}{A_S}$

Stress in brass wire $= \dfrac{T_B}{A_B}$

For equal stress in both the wires

$$\frac{T_S}{A_S} = \frac{T_B}{A_B}$$

$$\frac{T_S}{T_B} = \frac{A_S}{A_B} = \frac{0.1 \text{ cm}^2}{0.2 \text{ cm}^2} = \frac{1}{2} \qquad \text{... (i)}$$

For the rotational equilibrium of the rod,

$T_S x = T_B(2 - x)$

$$\frac{2 - x}{x} = \frac{T_S}{T_B} = \frac{1}{2} \qquad \text{[Using (i)]}$$

$4 - 2x = x \quad$ or $\quad 3x = 4 \quad$ or $\quad x = \dfrac{4}{3}$ m

23. (a) : The axial force 25 kN is transmitted to each of the three parts of the bar.

Stress in part AB is $\dfrac{25000 \text{ N}}{\dfrac{\pi}{4}(50)^2 \text{mm}^2} = \dfrac{40}{\pi} = 12.73 \text{ N/mm}^2$

Stress in part $BC = \dfrac{25000}{\dfrac{\pi}{4}(25)^2} = 50.93 \text{ N/mm}^2$

Stress in part $CD = 12.73$ N/mm^2

Therefore, total extension of the rod = extension in the parts $AB + BC + CA$

$$= \left(\frac{12.73 \text{ N/mm}^2}{2 \times 10^5 \text{ N/mm}^2} \times 100 \text{ mm}\right) \times 2 + \frac{50.93 \text{ N/mm}^2}{2 \times 10^5 \text{ N/mm}^2} \times 200 \text{ mm}$$

$$= \frac{12732}{2 \times 10^5} \text{ mm} = 0.0637 \text{ mm}$$

24. (c) : According to definition of bulk modulus,

$$B = -\frac{P}{\Delta V / V}$$

Here, $P = 100$ atm $= 100 \times 10^5$ N m^{-2} $= 10^7$ N m^{-2}

$\Delta V = (99.5 - 100)$ litre $= -0.5$ litre

$V = 100$ litre

$$\therefore \quad B = -\frac{10^7 \text{ N m}^{-2}}{(-0.5 \text{ litre}/100 \text{ litre})} = 2 \times 10^9 \text{ N m}^{-2}$$

25. (b) : Here, $r = 2$ cm $= 2 \times 10^{-2}$ m

Maximum load = Maximum stress × Area of cross-section

$= 10^8$ N m^{-2} $\times \pi \times (2 \times 10^{-2}$ m$)^2 = 4\pi \times 10^4$ N

26. (c) : For a mountain of height h and base area A, weight $W = Ah\rho g$

So, pressure at the base due to its own weight is

$$P = \frac{W}{A} = \frac{Ah\rho g}{A} = h\rho g$$

The mountain will exist, if

$h\rho g \leq$ Breaking stress

$$h \leq \frac{\text{Breaking stress}}{\rho g}$$

$$h \leq \frac{30 \times 10^7}{3 \times 10^3 \times 10} = 10^4 \ m = 10 \ km$$

$\therefore \quad h_{max} = 10$ km

27. (c) : For copper wire, $L_C = 3.5$ m,

$A_C = 4 \times 10^{-5}$ m^2

For steel wire, $L_S = 4.5$ m,

$A_S = 3 \times 10^{-5}$ m^2

$$\text{As Young's modulus}, Y = \frac{(F/A)}{(\Delta L/L)}$$

As applied force F and extension ΔL are same for steel and copper wires

$$\therefore \quad \frac{F}{\Delta L} = \frac{Y_S A_S}{L_S} = \frac{Y_C A_C}{L_C}$$

where the subscripts C and S refers to copper and steel respectively.

$$\therefore \quad \frac{Y_S}{Y_C} = \frac{L_S}{L_C} \times \frac{A_C}{A_S} = \frac{(4.5\text{ m}) \times (4 \times 10^{-5}\text{ m}^2)}{(3.5\text{ m}) \times (3 \times 10^{-5}\text{ m}^2)} = 1.7$$

28. (c) : $dl = \dfrac{fl}{YA} \Rightarrow dl \propto \dfrac{1}{A}$

29. (a) : When strain is small, the ratio of the longitudinal stress to the corresponding longitudinal strain is called the Young's modulus (Y) of the material of the body.

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{l/L}$$

where F is force, A the area, l the change in length and L the original length.

$$\therefore \quad Y = \frac{FL}{\pi r^2 l} \Rightarrow l = \frac{FL}{\pi r^2 Y}$$

r being radius of the wire.

For another wire, extension,

$$l' = \frac{(2F)(2L)}{\pi(2r)^2 Y} = \frac{FL}{\pi r^2 Y} = l$$

30. (d) : Bulk modulus, $B = \dfrac{P}{\Delta V/V}$

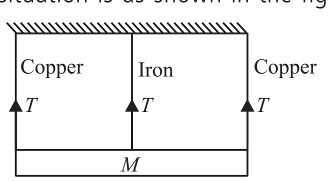
$\therefore$ Fractional change in volume, $\dfrac{\Delta V}{V} = \dfrac{P}{B}$

Here, $P = 10$ atm $= 10 \times 1 \times 10^5$ N m^{-2}

$B = 37 \times 10^9$ N m^{-2}

$$\therefore \quad \frac{\Delta V}{V} = \frac{1 \times 10^6\text{ N m}^{-2}}{37 \times 10^9\text{ N m}^{-2}} = 0.027 \times 10^{-3} = 2.7 \times 10^{-5}$$

31. (b) : The situation is as shown in the figure.



Let T be tension in each wire.

As the bar is supported symmetrically by the three wires, therefore extension in each wire is same.

$$\text{As } Y = \frac{F/A}{\Delta L/L}$$

If D is the diameter of the wire, then

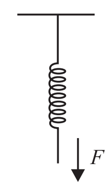
$$Y = \frac{F/\pi(D/2)^2}{\Delta L/L} = \frac{4FL}{\pi D^2 L}$$

As per the conditions of the problem, F (tension), length L, and extension ΔL is same for each wire.

$$\therefore \quad Y \propto \frac{1}{D^2} \text{ or } D \propto \sqrt{\frac{1}{Y}}$$

$$\therefore \quad \frac{D_{\text{copper}}}{D_{\text{iron}}} = \sqrt{\frac{Y_{\text{iron}}}{Y_{\text{copper}}}}$$

32. (c) : From Hooke's law, restoring force F is $F = kl$ where k is spring constant. When L is original length of spring, and k the spring constant, then



$$L + \left(\frac{5}{k}\right) = b$$

$$\text{Also, } L + \left(\frac{4}{k}\right) = a$$

$$\therefore \quad \frac{5}{k} - \frac{4}{k} = b - a \Rightarrow k = \frac{1}{b-a}$$

$$\therefore \quad L = b - \frac{5}{k}$$

$\Rightarrow \quad L = b - 5(b - a) = 5a - 4b$

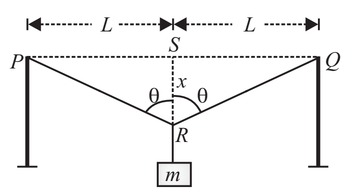
When tension is 9 N.

Length of spring $= L + 9/k$

Length of spring $= (5a - 4b) + 9(b - a)$

Length of spring $= 5b - 4a$

33. (a) :



Refer figure,

Increase in length, $\Delta L = (PR + RQ) - PQ$

$= 2PR - PQ$

$$\Delta L = 2(L^2 + x^2)^{1/2} - 2L = 2L\left(1 + \frac{x^2}{L^2}\right)^{1/2} - 2L$$

$$= 2L\left[1 + \frac{1}{2}\frac{x^2}{L^2}\right] - 2L \text{ (By binomial theorem)}$$

$$= \frac{x^2}{L}$$

$\therefore \quad \text{Strain} = \frac{\Delta L}{2L} = \frac{x^2}{2L^2}$

34. (a) : Here, L = 10 cm = 10 × 10^{-2} m

P = 7 MPa = 7 × 10^6 Pa

B = 140 GPa = 140 × 10^9 Pa

As $B = \frac{P}{\Delta V / V}$

$$\therefore \quad \Delta V = \frac{PV}{B} = \frac{PL^3}{B} = \frac{(7\times10^6 \text{ Pa})(10\times10^{-2} \text{ m})^3}{140\times10^9 \text{ Pa}}$$

= 5 × 10^{-8} m^3 = 5 × 10^{-2} cm^3

35. (a) : Here, V = 1 litre = 10^{-3} m^{-3},

B = 2100 MPa = 2100 × 10^6 Pa

Volume strain = $\frac{\Delta V}{V} = \frac{0.1}{100} = 1\times10^{-3}$

Bulk modulus, $B = \frac{P}{\Delta V / V}$

$$P = B\frac{\Delta V}{V} = 2100\times10^6 \times 1\times10^{-3}$$

= 21 × 10^5 Pa = 2100 × 10^3 Pa = 2100 kPa

36. (a) : $Y = \frac{F/A}{x/L} = \frac{FL}{Ax}$

$\frac{F}{x} = \frac{YA}{L}, \quad \therefore \quad \text{Slope} = \frac{YA}{L} = \tan 60° = \sqrt{3}$

37. (a) : Young's modulus, $Y = \frac{F}{A}\frac{L}{\Delta L} = \frac{T}{\pi r^2}\frac{L}{\Delta L}$

$\Delta L = \frac{T}{\pi r^2}\frac{L}{Y}$

where the symbols have their usual meanings.

As the four wires are made of the same material, therefore Young's modulus is the same for four wires.

As the T and Y are the same for the four wires.

$\therefore \quad \Delta L \propto \frac{L}{r^2}$

$\frac{L}{r^2}$ is maximum for wire of length L = 100 cm and radius r = 0.2 mm.

38 (c) : Young's modulus, $Y = \frac{FL}{\pi r^2 \Delta L}$

or $\quad \frac{\Delta L}{L} = \frac{F}{\pi r^2 Y}$

For the same force,

$$\therefore \quad \frac{\Delta L_B / L_B}{\Delta L_A / L_A} = \left(\frac{r_A}{r_B}\right)^2\left(\frac{Y_A}{Y_B}\right) = \left(\frac{2}{1}\right)^2\left(\frac{1}{2}\right) = 2$$

$$\frac{\Delta L_B}{L_B} = 2\left(\frac{\Delta L_A}{L_A}\right)$$

$$\frac{\Delta L_B}{L_B}\times100 = 2\left(\frac{\Delta L_A}{L_A}\times100\right) = 2\%$$

39. (a) : Extension of wire, $e = \frac{FL}{AY}$

$$e = \frac{FL}{\pi r^2 Y}$$

F and Y are same for all wires,

$\therefore \quad e \propto \frac{L}{r^2}$

$e_a = \frac{100}{0.2\times0.2} = 2500$ and $e_b = \frac{200}{0.4\times0.4} = 1250$

$e_c = \frac{300}{0.6\times0.6} = 833.4$ and $e_d = \frac{400}{0.8\times0.8} = 625$

We can see that, first wire have the largest extension.

40. (b) : When a wire is pulled, it stretches (undergoes strain) upto a certain limit, the amount it stretches is proportional to the load divided by the cross-sectional area of the wire.

$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{mg}{A} = \frac{V\rho g}{A} = \frac{LA\rho g}{A}$

$\therefore \quad$ Stress = $L\rho g$

Given, stress = 6 × 10^6 N m^{-2}, ρ = 3 × 10^3 kg m^{-3}

g = 10 m s^{-2}

$L = \frac{\text{Stress}}{\rho g} = \frac{6\times10^6}{3\times10^3\times10} = 2\times10^2 = 200 \text{ m}$

41. (b)

42. (b) : Point B is the elastic limit

43. (b) : Beyond B even if deforming forces are removed still some deformation is left.

44. (c) : The breaking stress for a wire of unit cross-section is called tensile strength.

45. (c) : Substances which can be stretched to cause large strains are called elastomers.

46. (d) : According to Hooke's law, within the elastic limit, stress is directly proportional to the strain *i.e.*

Stress ∝ Strain or Stress = k strain

$\frac{\text{Stress}}{\text{Strain}} = k$

where k is the proportionality constant and is known as modulus of elasticity.

47. (b) : According to Hooke's law,

Within elastic limit,

Extension ∝ Load applied

Hence, option (b) represents the correct graph.

48. (d)

49. (c) : Hooke's law gives the relation between stress and strain, which is given by the modulus of elasticity.

50. (b)

51. (a) : When a spring balance has been used for a long time, the spring in the balance fatigued and there is loss of strength of the spring. In such a case, the extension in the spring is more for a given load and hence the balance gives wrong readings.

52. (b)

53. (d) : In a glassy solid, *i.e.*, amorphous solid, the various bonds between the atoms or ions or molecules of a solid are not equally strong. Different bonds are broken at different temperatures. Hence there is no sharp melting point for a glassy solid.

54. (a) : Bulk modulus of elasticity measures how good the body is to regain its original volume on being compressed. Therefore, it represents incompressibility of the material. $K = \frac{-PV}{\Delta V}$ where P is increase in pressure, ΔV is change in volume.

55. (c) : Strain is the ratio of change in dimensions of the body to the original dimensions. Because this is a ratio, therefore it is a dimensionless quantity.

56. (b)

57. (a) : Young's modulus of a material, $Y = \frac{\text{Stress}}{\text{Strain}}$

Here, stress $= \frac{\text{Restoring force}}{\text{Area}}$ $\therefore\ Y = 0$.

58. (a) : Work done $= \frac{1}{2} \times \text{Stress} \times \text{Strain} = \frac{1}{2} \times Y \times (\text{Strain})^2$.

59. (d) : Ivory is more elastic than wet-clay. Hence the ball of ivory will rise to a greater height. In fact the ball of wet-clay will not rise at all, it will be somewhat flattened permanently.

60. (a) : Elasticity is a measure of tendency of the body to regain its original configuration. As steel is deformed less than rubber therefore steel is more elastic than rubber.

SUBJECTIVE TYPE QUESTIONS

1. Dimensional formula of, $[Y] = [ML^{-1}T^{-1}]$.

2. Infinity.

3. Zero.

4. Compressibility is the reciprocal of bulk modulus.
$k = 1/B$

5. The property of an elastic body by virtue of which its behaviour becomes less elastic under the action of repeated alternating deforming force is called elastic fatigue.

6. Permanent set is the amount by which a material is stressed beyond its yield point and if the load is removed the material does not come back to its original shape or size.

7. Pressure is the external force per unit area, while stress is the internal restoring force which comes into play in a deformed body acting transversely, per unit area of the body.

8. When the deforming force is applied beyond elastic limit, the strain produced is more than that has been observed within elastic limit. Due to which the ratio stress/strain will decrease.

9. Under constant stress, magnitude of bulk modulus of elasticity of material is inversely proportional to volumetric strain.

$$B \propto \frac{1}{\Delta V / V}$$

Since, gases has higher compressibility than liquid and solids have the least, therefore $B_s > B_l > B_g$.

10. The slope of stress-strain graph gives modulus of elasticity.

11. If, on removal of deforming force, a body does not regain its original configuration even a little then it is said to be perfectly plastic. For example, putty.

12. All materials undergo a change in their original state, howsoever small it may be, after the removal of deforming force. Hence, there is no such material which is perfectly elastic.

13. (a) Material B is more brittle than A, because its plastic range of extension is very small.

(b) Material A is the stronger of the two materials. It is because, it can bear greater stress before the wire of this material breaks.

14. The ratio of stress and strain is called modulus of elasticity. Modulus of elasticity has three components.

(a) Young's modulus (b) Shear modulus
(c) Bulk modulus

15. Let Y be the Young's modulus of the material of the wire, L the length and D the diameter. Let the wire be loaded with a mass M. If Δl is the elongation, we can write,

$$\Delta l = \frac{Mg\,L}{\pi\left(\frac{D}{2}\right)^2 Y} \quad \text{or} \quad \Delta l = \frac{4\,Mg\,L}{\pi\,D^2 Y}$$

When the diameter is doubled for the same length (L) and mass (M), the elongation is given by,

$$\Delta l_1 = \frac{Mg\,L}{\pi\left(\frac{2D}{2}\right)^2 Y} = \frac{Mg\,L}{\pi\,D^2 Y}$$

$$\therefore \quad \frac{\Delta l_1}{\Delta l} = \frac{Mg\,L}{\pi D^2 Y} \times \frac{\pi\,D^2 Y}{4\,Mg\,L} = \frac{1}{4} \quad \text{or} \quad \Delta l_1 = \frac{1}{4}\Delta l$$

Therefore, the elongation is one-fourth the elongation with the diameter D of the wire.

16. $r =$ 1.25 mm = 0.125 cm,

F = 100 × 9.8 N = 980 N = 98 × 10^6 dyne

Y = 12.5 × 10^{11} dyne cm^{-2}

$$\therefore \quad Y = \frac{Fl}{A \times \Delta l} \quad \text{or} \quad \frac{\Delta l}{l} = \frac{F}{AY}$$

$$\text{or} \quad \frac{\Delta l}{l} \times 100 = \frac{F}{\pi r^2 Y} \times 100$$

$$= \frac{98 \times 10^6 \times 7 \times 100}{22 \times (0.125)^2 \times 12.5 \times 10^{11}}$$

$= 15.965 \times 10^{-2} = 0.16\%$

17. As the temperature increases, the interatomic forces of attraction become weaker. For given stress, a larger strain or deformation is produced at a higher temperature. Hence the modulus of elasticity (stress/strain) decreases with the increase of temperature.

18. (a) No; it is not possible because within elastic limit strain is only of the order of 10^{-3}. Wires actually break much before it is stretched to double the length.

(b) No; it also depends on the radius of the wire.

19. Young's modulus,

$$Y = \frac{Fl}{\pi r^2 \cdot \Delta l} \Rightarrow \Delta l = \frac{Fl}{\pi r^2 \cdot Y} \qquad \text{...(i)}$$

When load F and radius r are increased to four times,

$$Y = \frac{(4F)l}{\pi (4r)^2 \Delta l'} \Rightarrow \Delta l' = \frac{Fl}{4\pi r^2 \cdot Y} \qquad \text{...(ii)}$$

Using eq. (i) and (ii), we get $\Delta l' = \dfrac{\Delta l}{4}$

Clearly, if the load and radius are increased to four times, the final elongation will be 1/4th of the initial elongation.

20. Hooke's law : For small deformations the stress developed in the body is directly proportional to the strain of the body.

i.e., Stress ∝ Strain

Stress = K Strain

$$K = \frac{\text{Stress}}{\text{Strain}}$$

where, K = a constant, called modulus of elasticity

Excess pressure at the bottom layer by water column of height h is

$P = h\rho g$ = 3000 × 1000 × 10

or $P = 3 \times 10^7$ N m^{-2} = stress

Bulk modulus, $B = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{P}{\Delta V / V}$

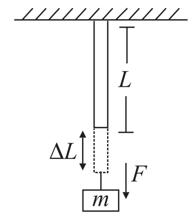
∴ Fractional compression,

$$\frac{\Delta V}{V} = \frac{P}{B} = \frac{3 \times 10^7}{2.2 \times 10^9} = 1.36 \times 10^{-2} \text{ or } 1.36\%$$

21. The relative change in configuration of a body on applying deforming force is called strain of the body. Different types of strain are

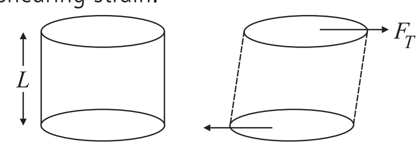
Longitudinal strain : The ratio of change in length to the original length of the body is called longitudinal strain.

$$\varepsilon_L = \frac{\Delta L}{L}$$



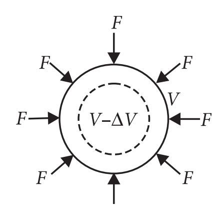
Tangential strain or Shearing strain : The ratio of relative displacement between the opposite faces of the cylinder to the separation between two faces of the cylinder is called tangential strain or shearing strain.

$$\varepsilon_T = \frac{\Delta x}{L} = \tan\theta$$



Volumetric strain :

The ratio of change in volume per unit original volume of the body on applying the normal deforming force to surface is called the volumetric strain.



$$\varepsilon_V = \frac{\Delta V}{V}$$

22. When a body is subjected to a deforming force the restoring force developed per unit area is known as stress.

The magnitude of stress, $S = \dfrac{F}{A}$.

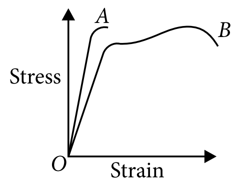
Its SI unit is N m^2 and dimension is [$ML^{-1} T^1$]. Different types of stress are

Longitudinal stress : The restoring force acting per unit area along the length of the body is called longitudinal stress.

Tensile stress : When a cylinder is stretched by two equal forces applied normal to its cross-sectional area, the restoring force per unit area developed in this case is called tensile stress.

Compressive stress : When a cylinder compressed under the action of deforming force, the restoring force per unit area is called compressive stress.

23. Stress-strain curve of a brittle and ductile materials can be shown in the given figure. In the figure curve A represents brittle material and B represents ductile material.



Difference between ductile and brittle materials are :

	Ductile Material	Brittle Material
(i)	Ductile materials can withstand large strain before specimen rupture	Brittle materials fracture at much lower strains.
(ii)	Slope of stress-strain curve is smaller for ductile materials within elastic limit.	Slope of stress strain curve is greater for brittle materials.
(iii)	Ductile materials exhibits large yielding.	Brittle materials fails suddenly without much warning.
(iv)	Steel and aluminium falls under this class	Glass and cast iron falls under this class.

24. (a) Wire of material A with larger plastic region is more ductile.

(b) For given strain, larger stress is required for A than that for B.

$\therefore$ A is stronger than B.

(c) Material with smaller plastic region is more brittle, therefore B is more brittle than A.

25. Here, $r = 10$ mm $= 0.01$ m, $l = 1$ m,
$F = 100$ kN $= 10^5$ N, $Y = 2.0 \times 10^{11}$ N m^{-2}

(a) Stress $= \dfrac{F}{A} = \dfrac{F}{\pi r^2} = \dfrac{10^5}{(3.14)\times(0.01)^2}$
$= 3.18 \times 10^8$ N m^{-2}

(b) As $Y = \dfrac{F}{A}\cdot\dfrac{l}{\Delta l}$

$\therefore$ Elongation, $\Delta l = \dfrac{F}{A}\cdot\dfrac{l}{Y} = \dfrac{3.18\times10^8\times1}{2.0\times10^{11}}$
$= 1.59 \times 10^{-3}$ m $= 1.59$ mm

(c) Strain $= \dfrac{\Delta l}{l} = \dfrac{1.59\times10^{-3}\text{m}}{1\text{m}}$
$= 1.59 \times 10^{-3} = 0.16\%$

26. $P =$ 100 atm $= 100 \times 1.013 \times 10^5$ Pa
Initial volume, $V = 100$ litre $= 100 \times 10^{-3}$ m^3
Final volume, $V_1 = 100.5$ litre $= 100.5 \times 10^{-3}$ m^3
$\therefore$ ΔV = change in volume $= V_1 - V$
$= (100.5 - 100) \times 10^{-3}$ m^3 $= 0.5 \times 10^{-3}$ m^3

As we know, $B_w = \dfrac{P}{\Delta V/V}$

$\therefore$ $B_w = \dfrac{100\times1.013\times10^5\times100\times10^{-3}}{0.5\times10^{-3}}$

or $B_w = 2.026 \times 10^9$ Pa

Also we know that the bulk modulus of air at S.T.P. is given by
$B_{air} = 10^5$ Pa

$\therefore$ $\dfrac{B_w}{B_{air}} = \dfrac{2.026\times10^9}{10^5} = 2.026 \times 10^4 = 20260$

The ratio is too large. It means gases are highly compressible whereas liquids are almost incompressible.

27. (a) Young's modulus,

$$Y = \frac{Mgl}{\pi r^2.\Delta l} = \frac{Mgl}{\pi\left(\frac{D}{2}\right)^2.\Delta l} = \frac{4Mgl}{\pi D^2.\Delta l}$$

where D is the diameter of the wire.

Elongation, $\Delta l = \dfrac{4\,Mgl}{\pi\, D^2 Y}$ *i.e.*, $\Delta l \propto \dfrac{1}{D^2}$

Clearly, if the diameter becomes half, the elongation will increase four times.

(b) Also load, $Mg = \dfrac{\pi D^2.\Delta l.Y}{4l}$ *i.e.*, $Mg \propto D^2$

Clearly, if the diameter becomes half, the wire can support 1/4 times the original load.

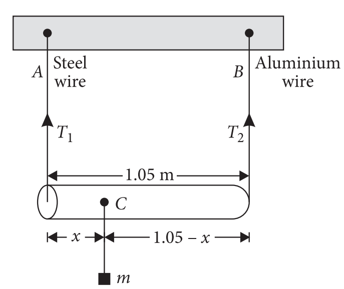
28. Suppose the mass m is suspended at distance x from the wire A. Let T_1 and T_2 be the tensions in the steel and aluminium wires respectively.

(a) Stress in steel wire $= \dfrac{T_1}{A_1}$

Stress in aluminium wire $= \dfrac{T_2}{A_2}$

As both the stresses are equal, so

$$\frac{T_1}{A_1} = \frac{T_2}{A_2} \text{ or } \frac{T_1}{T_2} = \frac{A_1}{A_2} = \frac{1.0\text{ mm}^2}{2.0\text{ mm}^2} = \frac{1}{2}$$



Now the moments about point C are equal because the system is in equilibrium.

$\therefore \quad T_1 x = T_2\,(1.05 - x)$

or $\quad \dfrac{T_1}{T_2} = \dfrac{1.05 - x}{x}$ or $\dfrac{1}{2} = \dfrac{1.05 - x}{x}$ or $x = 2.10 - 2x$

or $\quad x = 0.7$ m (from steel wire)

(b) $\quad \text{Strain} = \dfrac{\text{Stress}}{\text{Young's modulus}}$

$\therefore \quad$ Strain in steel wire $= \dfrac{T_1 / A_1}{Y_1} = \dfrac{T_1}{A_1 Y_1}$

Strain in aluminium wire $= \dfrac{T_2}{A_2 Y_2}$

For the two strains to be equal,

$$\frac{T_1}{A_1 Y_1} = \frac{T_2}{A_2 Y_2}$$

or $\quad \dfrac{T_1}{T_2} = \dfrac{A_1 Y_1}{A_2 Y_2} = \dfrac{1.0\ \text{mm}^2 \times 200 \times 10^9\ \text{Pa}}{2.0\ \text{mm}^2 \times 70 \times 10^9\ \text{Pa}} = \dfrac{10}{7}$

Again, $T_1 x = T_2\,(1.05 - x)$

or $\quad \dfrac{T_1}{T_2} = \dfrac{1.05 - x}{x}$ or $\dfrac{10}{7} = \dfrac{1.05 - x}{x}$

or $\quad 10x = 7.35 - 7x$ or $x = \dfrac{7.35}{17} = 0.43\ \text{m}$

(from steel wire)

29. (a) $Y = 2 \times 10^{10}$ N m^{-2}, $L = 50$ cm $= 0.5$ m

$A = 1$ mm$^2 = 10^{-6}$ m^2

Extension, $\Delta l = 1$ mm $= 10^{-3}$ m,

We know, $Y = \dfrac{F/A}{\Delta l / L} = \dfrac{FL}{A\Delta l}$ $\quad$ or $\quad F = \dfrac{YA\Delta l}{L}$

$$= \frac{2 \times 10^{10} \times 10^{-6} \times 10^{-3}}{0.5} = 40\ \text{N}$$

Work done in stretching the wire = Average force × Extension

$$= \frac{1}{2} F \times \Delta l = \frac{1}{2} \times 40 \times 10^{-3} = 2 \times 10^{-2}\ \text{J}$$

(b) Let the length and the area of cross-section of the wire be L and A respectively.

$\therefore \quad$ Weight of the wire = Volume × Density × g

= Length × Area × Density × g = $LA\rho g$

$\therefore \quad$ Stress $= \dfrac{LA\rho g}{A} = L\rho g$

Given :

Breaking stress $= 5 \times 10^7$ N m^{-2}

$\rho = 3 \times 10^3$ kg/m^3

Breaking stress = Stress due to its own weight

$\therefore \quad 5 \times 10^7 = L\rho g$

or $\quad L = \dfrac{5 \times 10^7}{3 \times 10^3 \times 10} = 1.67 \times 10^3$ m $= 1.67$ km

30. (a) The stress-strain curves for rubber differ from the stress-strain curve for a metal in following respects:

(i) Hooke's law is not obeyed even for small stresses.

(ii) There is no permanent set (residual strain) even for large stresses.

(iii) There is large elastic regain for both types of rubber.

(iv) Neither material retraces the curve during unloading. Thus both materials exhibit elastic hysteresis.

(b) The area of the hysteresis loop is proportional to the energy dissipated by the material as heat when the material undergoes loading and unloading. A material for which the hysteresis loop has larger area would absorb more energy when subjected to vibrations. Therefore to absorb vibrations, we would prefer rubber *B*.

(c) In car tyre, the energy dissipation must be minimised to avoid excessive heating of the car tyre. As rubber *A* has smaller hysteresis loop area (and hence smaller energy loss), so it is preferred over *B* for a car tyre.