Squares and Square Roots

Square of a Number

The square of a number is the product of the number with itself. Thus for a given number 'a', $a^2 = a \times a$.
Thus to find the square of a number, we have to multiply the number with itself.

Perfect Square of a Number

A natural number is said to be its perfect square, if it can be written as the square of the factors of natural number. Some of the numbers which are perfect squares are:
Squares From 1^2 TO 9^2

1 Squared = 1^2 = 1×1 = 1
2 Squared = 2^2 = 2×2 = 4
3 Squared = 3^2 = 3×3 = 9
4 Squared = 4^2 = 4×4 = 16
5 Squared = 5^2 = 5×5 = 25
6 Squared = 6^2 = 6×6 = 36
7 Squared = 7^2 = 7×7 = 49
8 Squared = 8^2 = 8×8 = 64
9 Squared = 9^2 = 9×9 = 81

The perfect squares are the squares of the whole numbers.

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	etc
Their Perfect Sures:	1	4	9	16	25	36	49	64	81	100	110	144	169	196	225	…

Properties of Square Number

A Perfect Square Number can only end with Digits 0,1,4,5, 6 and 9,

- If the last digit of a number is 0, its square ends with 00 and the preceding digits must also form a square.
- If the last digit of a number is 1 or 9, its square ends with 1 and the number formed by its preceding digits must be divisible by four.
- If the last digit of a number is 2 or 8, its square ends with 4 and the preceding digits must be even.
- If the last digit of a number is 3 or 7, its square ends with 9 and the number formed by its preceding digits must be divisible by four.
- If the last digit of a number is 4 or 6, its square ends with 6 and the preceding digits must be odd.
- If the last digit of a number is 5, its square ends with 5 and the preceding digits must be 2.
- A square number cannot be a perfect number.

Methods to Find Square of a Number

- If the number is in the form of m5, where m represents the preceding digits, its square is n25, where $n = m \times (m+1)$ and represents digits before 25.
  For example the square of 65 can be calculated by $n = 6 \times (6+1) = 42$ Thus square of 65 is equal to 4225.
- If the number is in the form of mO where m represents the preceding digits, its square is n00 where $n = m^2$.
  For example the square of 70 is 4900. where n = 49 & m = 7.

- If the number has two digits and it is in the form of5m where m represents the unit digit, its square is AABB where AA = 25 + m and $BB = m^2$
  Example: Calculate the square of 57.
- AA = 25 + 7 = 32 and BB = 49, it means $57^2 = 3249$.
- Squares of even numbers are even,
  since $(2n)^2 = 4n^2$
- Squares of odd numbers are odd, since $(2n+1)^2 = 4(n^2+n+1)$. It follows that square roots of even square numbers are even and square roots of odd square numbers are odd.

Pythagorean Triplet

A Pythagorean triplet consists of three positive integers a, b, and c, such that
$a^2 + b^2 = c^2$.
Pythagorean theorem states that, in any right triangle, the sum of squares of base and height is equal to the square of its hypotenuse. Pythagorean triplets describe a relation among three sides of a right triangle. For every natural number n > 1, we have the Pythagorean triplet is given by $(2n, n^2-1, n^2+1)$

Illustrative EXAMPLE

Let n = 3, then the corresponding Pythagorean triplet is obtained as
$2n = 2\times 3 = 6$
$n^2 - 1 = 3^2 - 1 = 8$
$n^2 + 1 = 3^2 + 1 = 10$
Hence 6, 8, 10 are Pythagorean triplets.

Square of Negative Numbers

Square of a negative number is always positive. Some example of square of negative numbers are given below:
$(-a)^2 = -a \times -a = a^2$

Numbers between Square Numbers

In general there are 2n non-perfect square numbers between the squares of any two numbers n and n + 1.
For example between 5 and 6 the number of numbers is
$6^2 - 5^2 = 36 - 25 = 11$
Thus there are 10 non square numbers which is one less than the difference of the square of two numbers which is equal to 2n i.e. $2\times 5 = 10$.

Square Root Methods

We know that addition is inverse of subtraction, multiplication is inverse of division, and similarly square root is the inverse of square of a number. There are different methods to find the square root of a number. We can find the square root either by finding the factors or by repeated subtraction or by long division method.

By Prime Factorization Methods

In this method we find the prime factors of the given number and then pair them up. We multiply the numbers taking one from each pair and get the square root of the required number. It is useful only in case the numbers have perfect pairs of prime factors.

Illustrative EXAMPLE

$\sqrt{256} = \sqrt{(2\times2)\times(2\times2)\times(2\times2)\times(2\times2)}$

There are four such order pairs within the square root of the number, therefore, the square root of the above number is $2\times2\times2\times2=16$

Repeated Subtraction Method

In this method we subtract the successive odd numbers from the given number starting from 1 till we get the result zero. The number of steps required to reduce the given number to zero will be the square root of the given number.

Illustrative EXAMPLE

Take the number 64

Solution:

$64-1=63 \Rightarrow 63-3=60 \Rightarrow 60-5=55 \Rightarrow 55-7=48 \Rightarrow 48-9=39 \Rightarrow 39-11=28 \Rightarrow 28-13=15 \Rightarrow 15-15=0$ There are eight steps required to reduce the number to 0. Therefore, square root of 64 is 8.

Here are some more Squares and Square Roots

——— *Square* ⟶ ⟵ *Square Root* ———		
4		16
5		25
6		36

Finding Square Root of a Number by Division Method

We can find the square root by division method. The division method involves dividing the larger number by the smaller number and again the divisor with the remainder of the previous one and continue till the remainder becomes zero.

$\sqrt{4096}$ =

	64
6	4096
6	36
124	496
4	496
128	0

Therefore, square root of 4096 is 64.

Adding and Subtracting Square Roots

Find the square root of $21\frac{2797}{3364}$.

(a) $\frac{289}{58}$ (b) $\frac{271}{58}$

(c) $\frac{281}{58}$ (d) $\frac{291}{58}$ (e) None of these

Answer: (b)

Find the square root 0.000529.

(a) 0.023 (b) 0.027
(c) 0.033 (d) 0.037
(e) None of these

Answer: (a)

If $\sqrt{2}=1.414, \sqrt{3}=1.732$ and $\sqrt{5}=2.236$, then find the value of $\sqrt{\frac{800}{45}}$.

(a) 5.214 (b) 4.216
(c) 4.214 (d) 5.216
(e) None of these

Answer: (b)

Simplify: $\frac{\sqrt{1024}-\sqrt{324}}{\sqrt{441+\sqrt{196}}}$

(a) $\frac{2}{5}$ (b) $\sqrt{\frac{2}{5}}$
(c) $\sqrt{\frac{8}{5}}$ (d) $\sqrt{\frac{4}{25}}$
(e) None of these

Answer: (a)

A rectangular garden is such that its length is twice the breath and its perimeter is equal to the perimetre of the square field whose area is given as 5184 m2. The area of the rectangular field is:

(a) $5608\,\mathrm{m}^2$ (b) $4608\,\mathrm{m}^2$
(c) $3608\,\mathrm{m}^2$ (d) $2408\,\mathrm{m}^2$
(e) None of these

Answer: (b)

If $\sqrt{4096}=64$ then find the value of $\sqrt{4096}+\sqrt{4093}+\sqrt{0.004096}$.

(a) 70.646
(b) 60.464
(c) 70.464
(d) 60.646
(e) None of these

Answer: (c)

Find the value of 'y' such that $\sqrt{188+\sqrt{53+\sqrt{y}}}=14$.

(a) 121 (b) 11
(c) 1331 (d) 161
(e) None of these

Answer: (a)

If $\sqrt{1+\frac{25}{144}}=1+\frac{p}{12}$, then find the value of p for which this is satisfied.

(a) (-1, 25) (b) (1, -25)
(c) (-1, -25) (d) (1, 25)
(e) None of these

Answer: (b)

Find the value of y such that $\sqrt{1+\sqrt{1-\frac{2176}{2401}}}=1+\frac{y}{7}$.

(a) (1, -15) (b) (-1, -15)
(c) (1, 15) (d) (-1, 15)
(e) None of these

Answer: (a)

Operation of Multiplication on Square Roots

We use the fact that the product of two radicals is the same as the radical of the product and vice versa.

Illustrative EXAMPLE

we have, $\sqrt{a}\times\sqrt{a}=a^{\frac{1}{2}}\times a^{\frac{1}{2}}=a^{\frac{1}{2}+\frac{1}{2}}=a^1$

$\therefore \quad \sqrt{3}\times\sqrt{3}=3$

Also, $\sqrt{3}\times\sqrt{5}=\sqrt{3\times5}=\sqrt{15}$

Problems Related to Navigation

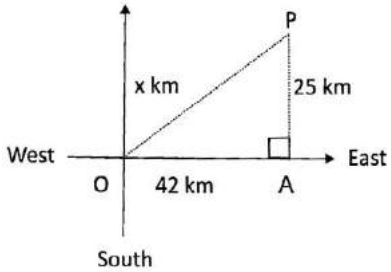
A ship sails 42 km due east and then 25 km due north. How far is the ship from its starting position when it completes this voyage?

Solution:

Let the distance of the ship from its starting point be x km. From the figure given below the distance is OP
Thus ΔOPA is a right triangle with right angle at A.
Hence, by Pythagoras' Theorem,

$OP^2=OA^2+AP^2$

$\Rightarrow x^2 = 42^2 + 25^2 \Rightarrow x^2 = 2389 \Rightarrow x = \sqrt{2389} = 48.88 km$

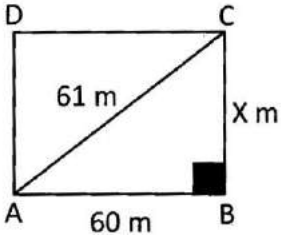


So, the ship is about 48.88 km far from the starting point.

Illustrative EXAMPLE

The length of the diagonal of a rectangular paddock is 61 m and the length of one side is 60 m. Find:
(a) The width of the paddock.
(b) The length of the fencing needed to enclose the paddock.

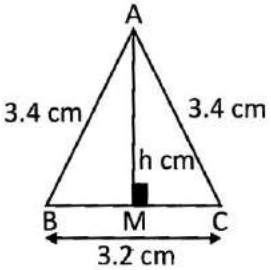
Solution:
(a) Let the width of the paddock be X
By Pythagoras' Theorem, from the diagram given below.



$x^2 = 61^2 - 60^2$
$x^2 = 3721 - 3600$
$x^2 = 121$
$x = \sqrt{121}$
$x = 11m$
So/the width of the paddock is 11 m.
(b) Now, Perimeter $= 2(\mathrm{I} + \mathrm{w})$
$= 2(60 + 11) = 2 \times 71 = 142$
So, the length of the fence required to enclose the paddock is 142 m.

Illustrative EXAMPLE

Use the information given in the diagram to find:
(a) Height of the triangle
(b) The area of the triangle



Solution:

(a) By Pythagoras' Theorem in ΔAMC,

By symmetry, M is the midpoint of BC;

$\therefore\ MC \frac{1}{2} BC = \frac{3.2}{2} = 1.6cm$

$h^2 + 1.6^2 = 3.4^2$

$h^2 + 2.56 = 11.56$

$h^2 = 11.56 - 2.56$

$h^2 = 9$

$h = \sqrt{9} = 3$

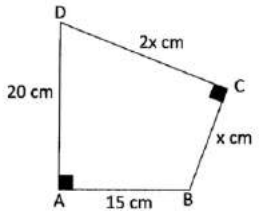
(b) Area of $\Delta ABC = \frac{Base \times Height}{2}$

$= \frac{3.2 \times 3}{2} = 4.8cm$

So, the area of ΔABC is $4.8\ cm^2$

Illustrative EXAMPLE

Use the information given in the diagram to find the value of x.



Solution:

Join BD of the quadrilateral to form the right-angled triangles ΔABD and ΔBCD.

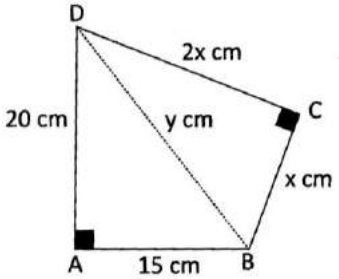
Let BD = y cm.

By Pythagoras' Theorem in ΔABD

$y^2 = 15^2 + 20^2$

$= 225 + 400$

$= 625 \Rightarrow y = \sqrt{625} = 25$



By Pythagoras' Theorem in ΔBCD

$x^2 + (2x)^2 = y^2 \qquad (\because y = 25)$

$x^2 + 4x^2 = 25^2$

$5x^2 = 625 \qquad \Rightarrow \qquad \frac{5x^2}{5} = \frac{625}{5}$

$x^2 = 125 \qquad \Rightarrow \qquad x = \sqrt{125}$

$x = \sqrt{25 \times 5} \qquad \Rightarrow \qquad x = 5\sqrt{5}$

You Must KNOW

- There are two square roots of 1 namely 1 and i.
- The binary form of the digit 9 is 1001.
- Every positive number has two square roots one is positive and other is negative.
- The square root of negative number is not real.

SUMMARY

- The product of a number with itself is called its square.
- The square of an even number is even and the square of an odd number is odd.
- Every natural number n can be written in the form of $(n+1)^2 - n^2 = (n+1) + n$.
- If m, n, p are natural number such that $m^2 + n^2 = P^2$ then (m, n, p) is called a Pythagorean triplet.
- For any natural number m, where, m > 1, the Pythagorean triplet is given by $(2m, m^2 - 1, m^2 + 1)$.
- The square root of a number n is the product of numbers obtained as a factor of the given number obtained in pairs.
- The different methods of obtaining the square root of the given numbers are prime factorization method, repeated subtraction method, and long division method.

Commonly Asked QUESTIONS

The sum of the numbers $1+3+5+7+9+11+13+15+17+19+21+23$ is equal to:

(a) 144
(b) 212
(c) 221
(d) 112
(e) None of these

Answer: (a)
Explanation
There are 11 odd numbers in the series, therefore, the sum is equal to $11^2 = 121$.

The number of odd numbers whose sum results 81 is:

(a) 5
(b) 6
(c) 8
(d) 9
(e) None of these
Answer: (d)
Explanation
81 an be written as 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17.

The Pythagorean triplet whose smallest number is 14 is:

(a) 28, 195 & 197 (b) 14, 196 & 198
(c) 14, 197 & 163 (d) 14, 16 & 20
(e) None of these

Answer: (a)

Which one among the following is the perfect square?

(a) 130321 (b) 21296
(c) 36501 (d) 27648
(e) None of these

Answer: (a)

The number 16777216 is the square of which one of the following numbers?

(a) 4276 (b) 4096
(c) 4086 (d) 5006
(e) None of these

Answer: (b)

What is the value of $\sqrt{522756}$?

(a) 1232
(b) 2434
(c) 1324
(d) 1426
(e) 1234

Answer: (e)
Explanation
From the prime factorization of $\sqrt{1522756}$,we get 1234.

What is the value of $\sqrt{3018+\sqrt{36+\sqrt{169}}}$?

(a) 55
(b) 25
(c) 35
(d) 65
(e) 45

Answer: (a)
Explanation
Since, $\sqrt{3018+\sqrt{36+\sqrt{169}}}=55$

Find the square root of 15876.

(a) 126 (b) 144
(c) 184 (d) 156
(e) None of these

Answer: (a)
Explanation

	126
1	15876
1	1
22	58
2	44
246	1476
6	1476
252	0

Find the square root of 17424.

(a) 132 (b) 124
(c) 142 (d) 172
(e) None of these

Answer: (a)

Simplify: $\sqrt{\frac{1183}{2023}}$

(a) $\frac{17}{13}$ (b) $\frac{1}{13}$
(c) $\frac{13}{17}$ (d) $\frac{1}{17}$
(e) $7\sqrt{2}$

Answer: (c)

Simplify: $\sqrt{3\frac{33}{289}}$

(a) $\sqrt{\frac{3}{17}}$ (b) $\sqrt{\frac{30}{17}}$
(c) $\sqrt{\frac{33}{17}}$ (d) $\sqrt{\frac{11}{17}}$
(e) None of these
Answer: (b)

Simplify: $\sqrt{10.0489}$

(a) 3.27 (b) 3.07

(c) 3.17 (d) 3.47

(e) None of these

Answer: (c)

Find the value of $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{5}}}}}$ **?**

(a) $2^{\frac{1}{31}}$ (b) $2^{\frac{1}{32}}$

(c) $2^{\frac{31}{32}}$ (d) $2^{\frac{30}{31}}$

(e) None of these

Answer: (c)

Simplify the given expression and find the value of the expression

$$\sqrt{\frac{0.256\times 0.081\times 4.356}{1.5625\times 0.0121\times 129.6\times 64}}$$

(a) 1.096 (b) 0,024

(c) 2.196 (d) 4.096

(e) None of these

Answer: (b)

If $a=\frac{\sqrt{2}+1}{\sqrt{2}-1}$ **and** $b=\frac{\sqrt{2}-1}{\sqrt{2}+1}$ **then find the value of** $\frac{a^2+ab+b^2}{a^2-ab+b^2}$

(a) $32-4\sqrt{2}$ (b) $32+4\sqrt{2}$

(c) 0 (d) $\frac{7}{5}$

(e) None of these

Answer: (d)

Find the value of $\sqrt{6+\sqrt{6+\sqrt{6+---}}}$

(a) (-3, 2) (b) (-3, -2)

(c) (3, 2) (d) (3, -2)

(e) None of these

Answer: (d)

A group of students in a class collects Rs. 9216. The amount contributed by each student is equivalent to the number of students in the class. Find the number of students in the class.

(a) 66 (b) 48
(c) 96 (d) 36
(e) None of these
Answer: (c)

Self Evaluation TEST

Duration 10 Minutes

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1. **In an auditorium the number of rows is equal to the number of chairs in each row. If the capacity of the auditorium is 2025. The number of chairs in each row is:**

(a) 40 (b) 45
(c) 47 (d) 42
(e) None of these

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2. **A teacher wants to draw up his 16160 students in the form of a complete square, and finds that he has 31 students left over. The number of students in the front row is:**

(a) 121 (b) 127
(c) 141 (d) 189
(e) None of these

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3. **The smallest number by which 396 must be multiplied so that the product becomes a perfect square.**

(a) 10 (b) 11
(c) 13 (d) 9
(e) None of these

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4. **Find the least square number divisible by each one of 8, 9 and 10.**

(a) 360 (b) 36
(c) 3600 (d) 36000
(e) None of these

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5. **Find the cost of fencing around a square field whose area is 9 hectares, if the rate of fencing the field is Rs 3.50 per metre:**

(a) Rs. 4000 (b) Rs.4100
(c) Rs. 4200 (d) Rs.4600
(e) None of these

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6. **The least number must be subtracted from 7250 to get a perfect square is:**

(a) 20
(b) 15
(c) 12
(d) 25
(e) None of these

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7. **Find the 4 digit greatest number which is a perfect square?**

(a) 9999 (b) 9801
(c) 9832 (d) 9888
(e) None of these

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8. **What is the least number that must be added to 5607 to make the sum, a perfect square?**

(a) 12 (b) 15
(c) 18 (d) 19
(e) None of these

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9. **What is the least number that should be added to the smallest 6 digit number to make it a perfect square?**

(a) 389 (b) 489
(c) 289 (d) 139
(e) 548

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10. **Find the value of the given expression** $2+\frac{1}{\sqrt{2}}+\frac{1}{2+\sqrt{2}}+\frac{1}{\sqrt{2}-2}$ **?**

(a) $2+\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$
(c) $2-\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$
(e) None of these

Answers – Self Evaluation Test																			
1.	B	**2.**	B	**3.**	B	**4.**	C	**5.**	C	**6.**	D	**7.**	B	**8.**	C	**9.**	B	**10.**	C

Self Evaluation Test
SOLUTIONS

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1. Let the number of chairs in each row be x.
Then the number of rows = x
Total number of chairs in the auditorium $= (x \times x) = x^2$
But the capacity of the auditorium = 2025
$x^2 = 2025 \ \therefore \ x = \sqrt{2025}$
$x = \sqrt{\underline{5\times5}\times\underline{3\times3}\times\underline{3\times3}} = 5\times3\times3 = 45$

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2. The number of extra students left after forming the square is 31, so the number of students used in forming the square is 16160 - 31 = 16129.

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3. $396 = 2\times2\times3\times3\times11$
2 and 3 are paired but
11 is unpaired, so if we multiply the above given number by 11 then each factors of the number becomes in pair and hence it becomes a perfect square.

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4. The L . C. M of 8, 9, 10 is 360
$360 = 2\times\underline{2\times2}\times\underline{3\times3}\times5$
Now, if we multiply the above given number by 2 and 5 it becomes a perfect square. Therefore, required number is 3600.

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5. Area of the square field $= (9\times10000)\text{m}^2 = 90000\ \text{m}^2$
Length of each side of the field $= \sqrt{90000} =$ 300 m
Perimeter of the field $= (4\times300) = 1200\ \text{m}$
Cost of fencing $= \text{Rs.}(1200\times3.50) = \text{Rs. } 4200$

6.

	85
8	7250
8	-64
165	0850
	-825
	025

After long division of the above given number, we get remainder 25.
Therefore, the number which is to be subtracted from the above given number to get a perfect square is 25.
Thus if 25 is subtracted from 7250 the resultant number is 7225 and it is a perfect square. Hence 25 is least number which must be subtracted from 7250 to get a perfect square.

7. The greatest number of four digit is 9999, but it is not a perfect square. Now solve the number by long division method to get required number.

	99
9	9999
9	-81
189	1899
	-1701
	0198

The required number is 9999 - remainder = 9999 -198 = 9801. If 198 is subtracted from 9999 then it forms a perfect square, so 9801 is correct answer.

8.

	74
7	5907
7	-49
144	707
	-576
	131

From above calculation we get quotient 74, but the square of 74 is less than the above given number. Therefore, the number which is to be added to the given number is obtained as:

$= (75)^2 - \text{given number} = 5625 - 5607 = 18$.

So required number to be added = 18.

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9. The least number of six digits = 100000

Now, To find the least number which must be added to the given number to reduce it to perfect square can be obtained as below:

	316
3	100000
3	-9
61	100
1	-61
626	3900
	-3756
	144

Therefore, the perfect square number before $100000 = (316)^2 = 99856$

We know that $(316)^2 < 100000 < (317)^2$

The least number to be added $= (317)^2 - 100000 = 489$.

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10. On rationalizing the denominator of each terms the expression reduces to

$$2 + \frac{\sqrt{2}}{2} + \frac{2-\sqrt{2}}{2} - \frac{\sqrt{2}+2}{2} = \frac{4+\sqrt{2}+2-\sqrt{2}-\sqrt{2}-2}{2}$$

$$= 2 - \frac{1}{\sqrt{2}}$$