# Logic

## # Propositional logic →

proposition → A declarative sentence to which we can assign one and only one of the truth value: i.e. true (T) or false (F).

ex. 1) London is a city. → T
2) $2 \times 3 = 5$ → F.
} Propositions.

3) $x + 2 = 3$
4) 15th August is independence day.
} Not propositions.

* Law of excluded middle →
(assumptions) If a proposition is not true, then it is false. Similarly, if a proposition is not false, then it is true.

* Law of contradiction →
A proposition cannot be simultaneously true and false.

* Types of propositions →

1) Atomic propositions (simple statements) →
A proposition which cannot be divided further into two or more propositions is said to be "atomic".
Usually, atomic propositions are denoted by 'p', 'q', 't' and so on.

2) Compound proposition (Statement Formula / propositional func$^n$) →

Two or more atomic propositions can be combined using connectives to form a "compound proposition".

not $\sim$ $\neg$

and $\wedge$

or $\vee$

implies $\rightarrow$

iff $\leftrightarrow$

* Connectives →

1) Negation (Not) →

If 'P' is a proposition then

not P written as '$\sim$P'. is a proposition whose truth value is "false" if P has truth value "true" and viceversa.

Truth table of '$\neg$P' →

| P | $\sim$q |
|---|---|
| T | F |
| F | T |

2) Disjunction (or) →

If p and q are any two propositions then "p OR q" written as "$p \vee q$" is a proposition whose truth value is "false" iff both p and q have truth values false

Truth table of $p \vee q$ →

| p | q | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

$p \vee q$ is true if at least one of the two propositions is true.

⊛ Disjunctive syllogism / elimination rule → (Rules of inference)

If $\{p \vee q$ is true and p is "false"$\}$.

then q is true

3) Conjunction (and) →

If p and q are any two propositions, then "p and q" written as "$p \wedge q$" is a proposition whose truth value is true only when both p and q have truth value "true"

Truth table of $p \wedge q$ →

| p | q | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

If $p \wedge q$ is false, then at least one of the two propositions is false.

* Conjunctive Syllogism →

If $\left\{ p \wedge q \text{ is false and } p \text{ is true} \right\}$, then q is false.

4) Implication / Condition (conditional) →

If p and q are any two propositions, then "p implies q" (or) "if p then q" written as "$p \to q$" is a proposition whose truth value is false only when p has truth value "true" and q has truth value "false".

Truth table of $p \to q$ is

| p | q | $p \to q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

If $p \to q$ is true, then either p is false or q is true or both too.

In $p \to q$, p is called "antecedent / hyposis" and q is called "consequent / conclusive".

*Note → 1) whenever p is false, then $p \to q$ is true always

2) whenever q is true, then $p \to q$ is also true always

3) a) The converse of $(p \to q)$ is $(q \to p)$ and viceversa.

b) The inverse of $(p \to q)$ is $(\bar{p} \to \bar{q})$ / $(\sim p \to \sim q)$.

c) The contrapositive of $(p \to q)$ is $(\sim q \to \sim p)$

4) $(p \to q)$ is $\cong (\sim q \to \sim p)$
T F T F

5) Biconditional / Biimplication (iff) →

If p and q are any two propositions then "p iff q" written as "$p \leftrightarrow q$" is a proposition whose truth value is true only when both p and q have same truth values.

Truth table for $p \leftrightarrow q$.

| p | q | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

$p \leftrightarrow q$ is false iff both p and q have diff. truth values.

$$(p \leftrightarrow q) \cong ((p \to q) \wedge (q \to p))$$

$$\sim (p \leftrightarrow q) \cong ((p \wedge \sim q) \vee (\sim p \wedge q))$$

$$\cong (p \oplus q) \leftarrow (\text{xor})$$

# Tautology →

An atomic proposition cannot be tautology. So, a compound proposition which is always true is called as "tautology".

ex. → 1) $p \vee (\sim p)$

2) $p \vee (p \to q)$

3) $p \to (p \vee q)$

# Contradiction →

A compound proposition which is always false is called a "contradiction."

ex. → $p \wedge (\sim p) \cong F$

# Contigency →

A compound proposition which is neither a tautology nor a contradiction. is called "contigency."

ex → $p \to q$ , $p \vee q$, $p \wedge q$, $p \leftrightarrow q$.

# Satisfiable Function & Satisfiability →

A compound proposition which is not a contradiction is called " satisfiable funcⁿ".

⊛ A satisfiable funcⁿ can be a tautology also.

Note → Every contigency is satisfiable but a satisfiable funcⁿ can be a tautology also.

# Equivalences →

Let P, Q, R be any compound propositions, then

i) Double negation

$\sim(\sim p) \Leftrightarrow p$

2) Commutative laws →

$$(p \vee q) \Leftrightarrow (q \vee p)$$

$$(p \wedge q) \Leftrightarrow (q \wedge p)$$

$$(p \leftrightarrow q) \Leftrightarrow (q \leftrightarrow p)$$

3) Associative laws →

$$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r).$$

$$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r).$$

4) Distributive laws →

$$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$$

$$p + (q \cdot r) \Leftrightarrow (p+q) \cdot (p+r)$$

$$p \cdot (q+r) \Leftrightarrow (p \cdot q) + (p \cdot r)$$

⊛ Principle of duality →

The dual of any formula in boolean algebra can be obtained by replacing →

i) '+' with '·' and viceversa

and

ii) '0' with '1' and viceversa.

5) De-Morgan's laws →

i) $\sim(p \vee q) \Leftrightarrow (\sim p \wedge \sim q)$

(or) $(\overline{p+q}) \Leftrightarrow (\bar{p} \cdot \bar{q})$

ii) $\sim(p \wedge q) \Leftrightarrow (\sim p \vee \sim q)$

(or) $(\overline{p \cdot q}) \Leftrightarrow (\bar{p} + \bar{q})$

6) Idempotent laws →

i) $(p \vee p) \Leftrightarrow p$ (or) $p+p \Leftrightarrow p$

ii) $(p \wedge p) \Leftrightarrow p$ (or) $p \cdot p \Leftrightarrow p$

7) Absorption laws →

i) $p \vee (p \wedge q) \Leftrightarrow p$

ii) $p \wedge (p \vee q) \Leftrightarrow p$

8) More identities →

i) $p \vee T \Leftrightarrow T$ (or) $p+1 \Leftrightarrow 1$

ii) $p \wedge T \Leftrightarrow p$ (or) $p \cdot 1 \Leftrightarrow p$

iii) $p \wedge F \Leftrightarrow F$ (or) $p \cdot 0 \Leftrightarrow 0$

iv) $p \vee F \Leftrightarrow p$ (or) $p+0 \Leftrightarrow p$

v) $p \wedge \sim p \Leftrightarrow F$ (or) $p \cdot \bar{p} \Leftrightarrow 0$

vi) $p \vee \sim p \Leftrightarrow T$ (or) $p+\bar{p} \Leftrightarrow 1$

9) $(p \rightarrow q) \Leftrightarrow (\sim p \vee q)$

10) $\sim(p \rightarrow q) \Leftrightarrow (p \wedge \sim q)$

11) $(p \rightarrow q) \Leftrightarrow (\sim q \rightarrow \sim p)$

12) $(p \leftrightarrow q) \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

13) $\sim(p \leftrightarrow q) \Leftrightarrow (p \wedge \sim q) \vee (\sim p \wedge q) \rightarrow$ EX-OR

14) $p \rightarrow (q \rightarrow R) \Leftrightarrow (p \wedge q) \rightarrow R$

LHS $\Rightarrow \sim p \vee (\sim q \vee R)$ by (Eq. 9) equivalence.

$\Rightarrow (\sim p \vee \sim q) \vee R$ By associative

$\Rightarrow (p \wedge q) \rightarrow R$ (eq. 9)

$=$ R.H.S.

Q.1. The compound proposition $(p \vee q) \vee \sim p$ is a

✓a) Tautology

b) Contradiction

c) Contingency

d) $\Leftrightarrow q$

e) $\Leftrightarrow p$

$p \to (q \to R)$
$T \quad F$

$\to (q \vee p) \vee \sim p$ By $E_2$ Commutative

$\Rightarrow q \vee (p \vee \sim p)$ by $E_3$ Associative

$\Rightarrow q \vee T$

$\Rightarrow \boxed{T}$

Q. 2. The compound proposition $\sim(p \vee q) \vee (\sim p \wedge q) \vee p$ is

same options. $\boxed{\text{Tautology}}$

$\Rightarrow (\sim p \wedge \sim q) \vee (\sim p \wedge q) \vee p$ De Morgan's

$\Rightarrow \sim p \wedge (\sim q \vee q) \vee p$

$\Rightarrow \{\sim p \wedge T\} \vee p$

$\Rightarrow \sim p \vee p \Rightarrow \boxed{T}$

(or) The given formula is equivalent to

$(\overline{p+q}) + (\bar{p} \cdot q) + p$

$\Rightarrow (\bar{p}\,\bar{q}) + (\bar{p}\,q) + p$ De Morgan's

$\Rightarrow \bar{p} \cdot (\bar{q} + q) + p$

$\Rightarrow \bar{p} \cdot 1 + p \Rightarrow \bar{p} + p \Rightarrow \boxed{1}$

Q.3. The statement formula

$((p \to q) \leftrightarrow (\sim p \vee q)) \wedge s$ is

a) Tautology

b) $\Leftrightarrow p$

c) $\Leftrightarrow q$

✓d) $\Leftrightarrow s$

Q.4. Consider the following statement formulae

i) $p \vee \sim(p \wedge q)$

ii) $(p \wedge q) \wedge (\sim p \vee \sim q)$

which of the following options is true?

✓a) $S_1$ is a tautology and $S_2$ is a contradict$^n$.

b) $S_1$ is a contradiction and $S_2$ is a taantology.

c) Both are valid.

d) Both $S_1$ and $S_2$ are not satisfiable.

$\Rightarrow$ $S_2$ : $(p \wedge q) \wedge \sim (p \wedge q)$.

Contradiction.

$S_1$ : $p \vee \sim (p \wedge q) \Rightarrow p \vee (\sim p \vee \sim q)$

$\Rightarrow (p \vee \sim p) \vee \sim q$

$\Rightarrow$ Tautology.

Q.5. The propositional function

$(\sim p \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$ is

$\Rightarrow$

$\Rightarrow (\sim p \wedge \sim q \wedge \sim R) \vee R \wedge (Q \vee P)$

$\Rightarrow (\sim (p \vee Q) \wedge R) \vee (R \wedge (Q \vee P))$

$\Rightarrow R \wedge ((p \vee Q) \vee \sim (p \vee Q)) \Rightarrow R \wedge T$

$\Rightarrow \boxed{R}$

a) $\Leftrightarrow P$

b) $\Leftrightarrow Q$

✓c) $\Leftrightarrow R$

d) a tautology

Q.6. The propositional function

$\{((P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R))\} \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$

→ ✓a) tautology

b) contradiction.

c) contingency d) none of these.

1st term →

→ $(P \vee Q) \wedge (P \vee (Q \wedge R))$

⇒ $P \vee (Q \wedge (Q \wedge R))$

$P \vee (Q \wedge R)$.

2nd term →

$(\sim P \wedge \sim Q) \Rightarrow \sim (P \vee Q)$

V

3rd term → $(\sim P \wedge \sim R)$

$\sim P \wedge (\sim Q \vee \sim R)$.

∴ total → $P \vee (Q \wedge R) \vee (\sim P \wedge (\sim Q \vee \sim R))$.

⇒ $P \vee (Q \wedge R) \vee \overline{(P \vee (Q \wedge R))}$

→ $\boxed{T}$

Tautology.

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Q.7. The compound proposition

is $(P \to (Q \vee R)) \to ((P \wedge Q) \to R)$ is

a) satisfiable but not valid (i.e. contingency).

b) valid (tautology).

c) Not satisfiable (contradiction)

d) valid but not satisfiable. (not possible).

→ when $P \to T$, $Q \to T$ and $R \to F$, the given formula is <u>false</u>. Hence, it is not <u>valid</u>.

when $P \to T$, $Q \to F$, $R \to F$, $P \to (Q \vee R)$ is false. So, whole formula <u>true</u>.

∴ The given formula is satisfiable but not valid

Q. 8. The statement formula is

$$(a \to (b \to c)) \to ((a \to b) \to (a \to c))$$ is

→ tautology i.e. <u>Valid</u>

Here, the given can be false only in one case when $((a \to b) \to (a \to c))$ is false and

$(a \to (b \to c))$ is true. But when earlier is false, later also becomes false. $\boxed{\text{tautology}}$

Clurne's method $\rightarrow$

Case-1 $\rightarrow$ when $a$ is true.

$$(T \rightarrow (b \rightarrow c)) \rightarrow ((T \rightarrow b) \rightarrow (T \rightarrow c))$$

$$\Rightarrow (b \rightarrow c) \rightarrow (b \rightarrow c)$$

$$\Rightarrow T$$

Case-2 $\rightarrow$ when $a$ is false.

$$(F \rightarrow (b \rightarrow c)) \rightarrow ((F \rightarrow b) \rightarrow (F \rightarrow c))$$

$$\Rightarrow T \rightarrow (T \rightarrow T)$$

$$\Rightarrow T$$

$\therefore$ Both the cases yield T, $\therefore$ given formula is valid.

Q. 9. The statement formula

$$((p \rightarrow \sim p) \wedge (\sim p \rightarrow p)) \text{ is}$$

$\sim p \vee \sim p$

F

$$\Rightarrow (\sim p \vee \sim p) \wedge (p \vee p)$$

$\sim p \vee \sim p$

F $\wedge$

$$\Rightarrow \sim p \wedge (p \vee p).$$

$$\Rightarrow \sim p \wedge p \rightarrow \boxed{\text{Conhadiction}}$$

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Q.10 No. of nonequivalent propositional functions (different truth tables) possible with n atomic propositions is → ?

a) $2n$  b) $n^2$  ✓c) $2^{(2^n)}$  d) $2^{(n^2)}$

→

$f(x_1, x_2, x_3, \dots, x_n)$

*Note → A set of connectives is said to be <u>functionally complete</u> if any statement formula can be written using those connectives.

Q.11. Which of the following sets of connectives is not functionally complete ?

a) $\{\sim, \vee\}$  b) $\{\sim, \wedge\}$  c) $\{\sim, \rightarrow\}$  ✓d) $\{\vee, \wedge\}$

e) $\{\rightarrow, F\}$

→ a) $\{\sim, \vee\}$ (eq. to NOR gate) → 1) $p \wedge q$

⇒ $\sim(\sim p \vee \sim q)$.

2) $p \rightarrow q$.

⇒ $(\sim p \vee q)$

3) $p \leftrightarrow q$

⇒ $(p \rightarrow q) \wedge (q \rightarrow p)$

⇒ $(\sim p \vee q) \wedge (\sim q \vee p)$

⇒ $\sim\{\sim(\sim p \vee q) \vee \sim(\sim q \vee$

We can imply any operation using only $\{\sim, \vee\}$.
So, it is functionally complete.

b) $\{\sim, \wedge\}$ → equivalent to NAND gate.

So, it is also functionally complete.

c) $\{\sim, \rightarrow\}$

→ i) $(p \vee q)$

⇒ $(\sim p \rightarrow q)$

NOT is there and we can perform OR operation. So, c) is also functionally complete.

d) $\{\wedge, \vee\}$

→ We cannot perform NOT operation using $\{\vee, \wedge\}$. So, it is not functionally complete.

e) $\{\rightarrow, F\}$

→ i) $\sim p$

⇒ $(p \rightarrow F)$

⇒ $(\sim p \vee F)$

⇒ $\sim p$.

We can perform NOT operation & → is there. So, This set is functionally complete.

Q.12 Which of the following is not valid ?

a) $(p \wedge q) \Longrightarrow (p \leftrightarrow q)$

$(T \wedge T) \Rightarrow (T \leftrightarrow T)$ always true.

The statement is valid because AND operation is TRUE in only one case for which the

T ↔ T    T

F ↔ F    T.

biconditional is also true. Hence, it is a tautological implication.

b) $(p \leftrightarrow q) \Rightarrow (q \rightarrow p)$

$T \leftrightarrow T \Rightarrow T \rightarrow T$

$F \leftrightarrow F \Rightarrow F \rightarrow F$    always.

The statement is valid because whenever biconditional is true, the conditional on the other side is also true.

c) $(p \leftrightarrow \sim q) \Rightarrow (p \rightarrow q)$

p    ~q    p    q

$T \leftrightarrow T$    $T \rightarrow F$ ✓ F.

$F \leftrightarrow F$    $F \rightarrow T$

for one case, when $(p \leftrightarrow \sim q)$ is true, $(p \rightarrow q)$ is coming fal

This statement is not valid because when p is T and q is F, the biconditional is true, but the conditionals is false.

d) $\sim (p \rightarrow q) \Rightarrow \sim q$.

it will be false only when $T \rightarrow F$

$\sim q \rightarrow F$    $q \rightarrow T$

whenever q is True, $\sim (p \rightarrow q)$ can not be true, so, the statement is valid.

Q.13. which of the following is not a tautology?

a) $p \rightarrow (p \rightarrow q)$

$\rightarrow$ $T \rightarrow F$ , then it will become false.

$(p \rightarrow q) \rightarrow F$ when $p = T$ $q = F$ , possible.

$\therefore$ $p \rightarrow (p \rightarrow q)$ gives False for $p = T$ , $q = F$.

$\therefore$ It is not a tautology.

The statement is not a tautology , because when $p = T$ and $q = F$ , given statement is false.

b) $q \rightarrow (p \rightarrow q)$

$\rightarrow$ $T \rightarrow T$ always.

The given statement is a tautology because whenever $q = T$, $p \rightarrow q$ is always true.

c) $(p \wedge (p \rightarrow q)) \rightarrow q$

to check whether tautology or not , we have to check for the False condition i.e. $T \rightarrow F$.

$\therefore$ $p \wedge (p \rightarrow q)$ but whenever L.H.S is T,
$T \wedge (T \rightarrow T)$ right side becomes T always.
$T$.

$\therefore$ the given stmt is a tautology.

$(p \vee q)$ T.
$\sim p$
T p F. (T)

T → F
p = F. q = F
T ∧ ? F

d) $(\sim p \wedge (p \vee q)) \rightarrow q$ (Disjunctive syllogism)

→ T. F.

$(\sim p \wedge (p \vee q))$

$\sim p = T$ $p = F$.

$T \wedge (F \vee F) \rightarrow F$ always. ∴ T → F cond^n not possible.

∴ the given statement is a tautology.

Q.14. which of the foll. arguments is not valid. ?

a) The conclusion R follows from the premises / hypothesis.

$\{P \rightarrow Q, Q \rightarrow R, P\}$

1) $P \rightarrow Q$
2) $Q \rightarrow R$
3) $P$.

∴ R. ✓ is valid.

4) From 1) and 2), $P \rightarrow R$ by the rule of transitivity

5) $P \rightarrow R$
P
∴ R by modus-ponnens.

The argument is valid.

b) The conclusion $\sim p$ follows from the premises $\{p \to q, q \to R, \sim R\}$.

→ 1) $p \to q$

2) $q \to R$

3) $\sim R$.

from 1) and 2) by the rule of transitivity,

4) $p \to R$

3) $\sim R$.

$\therefore \sim p$. by modus-tonnens.

∴ argument is valid.

c) The conclusion $\sim R$ follows from the premises $\{(p \to q), (q \to R), \sim p\}$.

→ 1) $p \to q$

2) $q \to R$

3) $\sim p$

4) $p \to R$ from 1, 2.

3) $\sim p$

nothing ; R can be anything.

∴ the given argument is not valid.

$(p \to (q \to R))$ $p \wedge q$. $p = T$ $q = T$. $\frac{p \wedge q}{T \quad T}$. $T \to (T \to R)$ 

$T \to$ $R$

We cannot derive the conclusion from the premise by any rules of inference. ∴ argument is not valid.

* If the given conclusion $\sim R$ on the premises to be valid, we can write in the form of tautological implication as →

$$\underbrace{\{(p \to q), (q \to R), \sim p\}}_{T} \Rightarrow \underbrace{\sim R}_{F}$$

this condition is satisfied. Hence, the above tautological implication is not valid.

d) The conclusion R follows from the premises

$\{p \to (q \to R), (p \wedge q)\}$

→ 1) $p \to (q \to R)$

2) $p \wedge q$ ∴ $p = q = T$.

3) $T \to (T \to R)$

∴ $R$

∴ the argument is valid.

Q.15 which of the following arguments is not valid?

a) $[a \to b;\ c \to d;\ a \vee c] \Rightarrow [b \vee d]$

→ (constructive dilemma)

1) $a \to b$

2) $c \to d$

3) $a \vee c$ (by eq. 9)

$\sim a \to c$

from 3) and 2), by transitivity,

6) $\sim a \to c$

$c \to d$

$\therefore (\sim a \to d)$

5) $(a \to b) = (\sim b \to \sim a)$ (contrapositive)

6) $\sim b \to \sim a$ (Rule of transitivity).

$\sim a \to d$

$\therefore (\sim b \to d)$.

7) $(\sim b \to d) = (b \vee d)$

$\therefore \{(a \to b), (c \to d), a \vee c\} \Rightarrow (b \vee d)$. is valid

b) $\{p \to (R \to S), \sim R \to \sim p, p\} \Rightarrow S$

→ 1) $p \to (R \to S)$

2) $\sim R \to \sim p$

3) $p$

$\sim d \to T$
$d \to F$
$a \to F$
$b \to T$

$a \to F$ $b \to T$

$, \sim d$ $d \to F$ $a \to F$

$a \to F$ $b \to T$ $b \to T$ $c \to T$

from 1) & 3), by modus-ponnens,

4) $p \to (R \to s)$

$$\frac{P}{\therefore R \to S}$$

5) $\sim R \to \sim p$

$$\frac{p}{\therefore R}$$ by modus-tollens.

6) $$\frac{\begin{matrix} R \to s \\ R \end{matrix}}{\therefore s}$$ by modus-ponnens.

$\therefore$ argument is valid.

c) $\{ a \vee b, b \to c, a \to d, \sim d \} \Rightarrow c$

$\to$ 1) $a \vee b$

2) $b \to c$

3) $a \to d$

4) $\sim d$.

5) $$\frac{\begin{matrix} a \to d \\ \sim d \end{matrix}}{\sim a.}$$ 3 and 4 by modus-tonnens.

$a \to F \quad b \to T$

$T \to (F \to c)$

6) $a \vee b$ from 1) + 5) by Disjunctive syllogism.

$$\frac{\begin{array}{c} a \vee b \\ \sim a \end{array}}{b}$$

7) by modus-ponens.

$$\frac{\begin{array}{c} b \to c \\ b \end{array}}{\therefore c}$$

$\therefore$ the given argument is valid.

d) $\{(\sim a \wedge b) \wedge (b \to (a \to c))\} \Rightarrow \sim c$

$\to$ 1) $\sim a \wedge b$

2) $b \to (a \to c)$

3) $$\frac{\sim a \wedge b}{\therefore \sim a \ \therefore b}$$

4) $$\frac{\begin{array}{c} b \to (a \to c) \\ b \end{array}}{\therefore (a \to c)}$$

5) $$\frac{\begin{array}{c} a \to c \\ \sim a \end{array}}{\therefore \; ?}$$ (fallacy.) we cant say.

$\therefore$ the argument is invalid.

Q.16. which of the following is not valid?

a) $(a \wedge (a \to c) \wedge (b \vee \sim c)) \to b$

→ this argument / statement is valid if the foll. argument is valid.

1) $a$

2) $a \to c$

3) $b \vee \sim c$
$\therefore b$

4) $a \to c$
$a$
$\therefore c$    1 & 2.

5) $b \vee \sim c$
$c$
$\therefore b$    3 and 4. Disjunctive syllogism.

∴ the given argument is valid.

b) $[(\sim p \to \sim R) \wedge \sim S \wedge (p \to w) \wedge (R \vee S)] \to w$

→ 1) $\sim p \to \sim R \Rightarrow R \to p$ (contrapositive)

2) $\sim S$

3) $p \to w$

4) $R \vee S$

5) from 1) and 3) $R \to P$
$p \to w$
$\therefore R \to w$    transitivity rule

6) $R \vee S$ 2) and 4)

$\sim S$

$\overline{\phantom{xx}}$

$R$

7) $R \rightarrow W$

$R$

$\overline{\phantom{xx}}$

$\therefore W$

$\therefore$ the given statement is valid.

c) $\big(\sim R \rightarrow (S \rightarrow \sim p)\big) \wedge (\sim R \vee W) \wedge (\sim Q \rightarrow S) \wedge \sim W \big) \rightarrow (p \rightarrow Q)$

$\rightarrow$ 1) $\sim R \rightarrow (S \rightarrow \sim p)$

2) $\sim R \vee W$

3) $\sim Q \rightarrow S$

4) $\sim W$

5) $\sim R \vee W$ from 2) & 4).

$\sim W$

$\therefore \sim R$

6) $\sim R \rightarrow (S \rightarrow \sim p)$ from 1) and 5)

$\sim R$ by modus ponnens.

$\therefore (S \rightarrow \sim p)$

7) $\sim Q \rightarrow S$ from 3) & 6) by transitivity rule

$S \rightarrow \sim p$

$\therefore \sim Q \rightarrow \sim p$.

$p \to q$. $\sim\sim$. $(w \to f)$ $\sim p \to T$. $(R \to F)$

$T \to (\overset{F}{s} \to \sim p)$ $\overset{\sim p}{T}$ $(p \to F)$

$C \to F$. $F \to a$ $a \to$ $b \to T$.

c) $\sim q \to \sim p = p \to q$ by contrapositive.

∴ Given statement is valid.

d) $\left( ((b \wedge c) \to a) \wedge (\sim b \vee a) \right) \to c$

→ 1) $(b \wedge c) \to a$

2) $(\sim b \vee a) \Rightarrow b \to a$

3) No relation can be found. So, put some truth values.

$$\underbrace{\left( ((b \wedge c) \to a) \wedge (\sim b \vee a) \right)}_{T} \to \underbrace{c}_{F}$$

we get a condn $T \to F$ ∴ So, the given statement is not valid.

# Conditional Proof (C.P.) :

If a set of premises $\{P_1, P_2, \cdots, P_n \text{ and } (\wedge) Q\} \Rightarrow R$ — ①

then $\{P_1, P_2, \ldots, P_n\} \Rightarrow (Q \rightarrow R)$ — ②

Note: To apply C.P. Rule for an argument which is given in form ②,

i) first convert the argument into form ① and prove the argument ①.

ii) then by C.P. rule, the given argument is also valid.

# Indirect Proof (Proof by contradiction):

To apply this rule, first we assume that the argument is not valid. So, we take negation of the conclusion as a new premise.

When this new premise is combined with other premises, if we get any contradiction, then the argument is valid.

# Inconsistency :

A set of premises $(P_1, P_2, \ldots, P_n)$ is said to inconsistent if all the premises cannot be true simultaneously.

i.e. the conjunction of all the premises is a contradiction.

$\{P_1 \wedge P_2 \wedge \cdots \wedge P_n\} \Leftrightarrow F$

*Note :-* In any argument, if the premises are inconsistent then the argument is valid (always).

$$\underbrace{\{P_1 \wedge P_2 \wedge \dots \wedge P_n\}}_{F \text{ (always)}} \Rightarrow Q.$$

$\therefore$ always valid.

Q.17: which of the following arguments is not valid ?

a) $\{P \to (Q \to S), (\sim R \vee P), Q\} \Rightarrow (R \to S)$

→ This argument is valid by C.P. Rule if the following argument is valid.

1) $P \to (Q \to S)$

2) $(\sim R \vee P)$

3) $Q$

4) $R$ ← * new premise to apply C.P.

5) $\sim R \vee P$ 2) & 4) by disjunctive syllogism.

$$\frac{R}{\therefore P}$$

6.) $P \to (Q \to S)$ 1) & 5) by modus-ponnens.

$$\frac{P}{\therefore (Q \to S)}$$

7) $Q \to S$ 3) & 6) by modus-ponnens.
$Q$
$\therefore S$

$\therefore$ the argument is valid.

and $\therefore$ the given argument is also valid by C.P Rule.

b) $\{(p \to Q), \sim(p \wedge Q)\} \Rightarrow \sim p$

1) $p \to Q$

negation of $(\sim p)$

2) $\sim(p \wedge Q)$

3) $p$. ← new premise to apply I.P.

4) $p \to Q$ 1) and 3)
$p$ modus-ponnens.
$Q$.

5) $(p \wedge Q)$ by 3) and 4)

Here, ② and ⑤ contradict each other. Hence, the given argument is valid.

$p \to F$



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Q.18. which of the following is not valid ?

a) $(\sim(a \wedge b) \wedge (b \vee c) \wedge (c \to d)) \to (a \to d)$

$\to$ 1) $\sim(a \wedge b)$

2) $(b \vee c)$

3) $(c \to d)$

4) $\dfrac{a}{\therefore d}$ * new premise to apply C.P. rule

the given formula is valid by C.P rule if the following argument is valid.

5) $\dfrac{\sim(a \wedge b) \quad a}{\therefore \sim b}$ 1) and 4) by conjuctive syllogism.

6) $\dfrac{b \vee c \quad \sim b}{\therefore c}$ 2) and 5) by disjunctive syllogism.

7) $\dfrac{c \to d \quad c}{\boxed{\therefore d}}$ ✓ 3) and 6) by modus ponnens.

$\therefore$ the given statement is valid.

$b \to T$
$d \to F$.

b) $((a \vee b) \to (c \wedge d)) \to (b \to d)$.

→ This formula is valid by C.P rule if the following argument is valid.

1) $(a \vee b) \to (c \wedge d)$

2) ~~$c \wedge d$~~ *

3) $\frac{b.}{\therefore d}$ * new premise to apply C.P.

4) $\frac{b}{\therefore (a \vee b)}$

5) $(a \vee b) \to (c \wedge d)$ 1 and 4 modus ponnens.
$\frac{(a \vee b)}{\therefore (c \wedge d)}$

6) $\frac{c \wedge d}{\therefore d}$

∴ the given statement is valid.

c) $\{(a \vee b) \to c, a\} \Rightarrow \{c \to b\}$

→ 1) $(a \vee b) \to c$

2) $a$

3) $\frac{c.}{\therefore b}$ by (C.P Rule).

$x \to F$ $\quad ((\sim a \vee b) \to (c \wedge d)) \to (b \to d)$ $\quad b \to T \quad d \to F$

$T \to \boxed{y \to T}$ $\quad (T \to F) \quad F \to F \quad F. \quad T \to F \quad \sim y \to F.$

$(((a \vee b) \to c) \wedge a) \to (c \to b)$

$T \quad T \to F) \to$

$c \wedge f \quad (F \to F)$

The given argument is not valid because when $a \to T$, $c \to T$, $b \to F$, the premises are true but conclusion is false.

d) $((\sim x \to y) \wedge (\sim x \wedge \sim y)) \to x$

$\Rightarrow$ 1) $\sim x \to y = x \vee y$.

2) $\sim x \wedge \sim y = \sim (x \vee y)$

$\Rightarrow (x \vee y) \wedge \sim (x \vee y) \to x$

$\Rightarrow F \to x$

$\Rightarrow T$

$\therefore$ the given statement is valid.

Q.19. Which of the following is not valid?

a) If today is David's birthday then today is 2nd of July.
Today is 2nd of July.
$\therefore$ Today is David's birthday.

$\rightarrow$ p: Today is David's birthday.

q: Today is 2nd of July.

$p \to q$
$q$
$\therefore p$ (not valid.)

∴ the given statement is not valid.

b) If Canada is a country then London is a city.

London is not a city

---

∴ Canada is not a country.

→ p: Canada is a country.

q: London is a city.

$p \to q$

$\sim q$

$\therefore \sim p$

∴ the given statement is valid.

c) If A works hard then B or C will enjoy themselves

If B enjoys himself, then A will not work hard.

If C enjoys himself, then D will not enjoy himself

---

∴ If A works hard then D will not enjoy himself.

→ 1) $a \to (b \vee c)$

2) $b \to (\sim a)$

3) $c \to \sim d$

$\therefore (a \to \sim d)$

then it will be valid.

$a \to (b \vee c)$. $(a \to (b \vee c)) \wedge$
F → T
$(b \to \sim a) \wedge$ $(a \to T) \to (d\ T)$
$(c \to \sim d)$ → $A \to \sim d$

4) $a$ ← new premise to apply C.P.
5) $(b \vee c)$ 1 and 4. M.P.
6) $b \to (\sim a)$ from 2) & 4).

$$\frac{a}{\sim b}$$ by modus - tollens.

7) $\dfrac{\begin{array}{c} b \vee c \\ \sim b \end{array}}{c}$ 5) & 6) by disjunctive syllogism

8) $\dfrac{\begin{array}{c} c \to \sim d \\ c \end{array}}{\sim d}$ 3) & 7) by modus ponens.

∴ the given statement is valid.

d) If John misses many classes (P), then he fails high school (q).
If john fails highschool, then he is uneducated (s).
If John read a lot of books (t), then John is not uneducated (~s).

John missed many classes and read a lot of books

∴ John is educated.

→ 1) $p \to q$
2) $q \to s$
3) $t \to \sim s$
4) $p \wedge t$
∴ W

5) $p \to s$    1) & 2) transitivity rule.

6) $p$    4) simp.

7) $t$    4) simp.

8) $s$    5) & 6) Modus-ponnens.

9) $\sim s$    3) & 7) modus-ponnens.

8) & 9) contradict each other.

∴ the given statement / argument is valid.

Q.20. A binary relation $*$ is defined by the following truth table

| p | q | p * q |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | F |
| F | F | T |

which of the following is equivalent to $(p \wedge q)$

a) $\sim (p * q)$

b) $(p * \sim q)$

✓c) $\sim (\sim p * q)$

d) $\sim (p * \sim q)$.

for option c)

| $p$ | $q$ | $\sim p$ | $q$ | $\sim p * q$ | $\sim(\sim p * q)$ |
|---|---|---|---|---|---|
| T | T | F | T | F | T |
| T | F | F | F | T | F |
| F | T | T | T | T | F |
| F | F | T | F | T | F |

$= (p \wedge q)$

$\therefore$ C is correct.

(OR) $(p * q) \Leftrightarrow (p \vee \sim q)$.

Q.21. A binary operator $*$ is defined by foll. truth table

| $p$ | $q$ | $p * q$ |
|---|---|---|
| T | T | F |
| T | F | F |
| F | T | T |
| F | F | F. |

$(p \vee q) = ?$

a) $\sim(p * q)$

b) $(p * \sim q)$

c) $\sim(\sim p * q)$

✓d) $\sim(p * \sim q)$

$\rightarrow \quad (p * q) \Rightarrow (\sim p \wedge q)$.

$(p \vee q) \Rightarrow \sim(\sim p \wedge \sim q)$.

$\Rightarrow \boxed{\sim(p * \sim q)}$

# First Order Logic (Predicate Logic) :-

Consider,

John is a politician.

subject predicate

let

j : John

p : is a politician.

So, we can denote the above statement as $p(j)$.

$\therefore$ $p(j)$ : John is a politician.

Consider,

Dhoni is a sportsman.

D : Dhoni

S : is a sportsman.

$\therefore$ $S(D)$ : Dhoni is a sportsman.

If $x$ and $y$ are any two persons, then

$p(x)$ : $x$ is a politician.
$S(y)$ : $y$ is a sportsman.

} not propositions because we cant assign any truth values.

Consider,

$x$ is a friend of $y$.

F : is a friend of.

$\therefore$ $F(x,y)$ : $x$ is a friend of $y$.

Let G denote the predicate " is greater than". and let $x$ and $y$ be any two nos.

then $G(x,y)$ : $x$ is greater than $y$. } not a proposition.

Consider,

B: is in between.

then $B(x,y,z)$ : $y$ is in between $x$ and $z$.

* Let $p(x)$: $x$ is a politician.

$S(x)$: $x$ is a sportsman.

where $x$ is any person.

1) $\sim p(x)$: $x$ is not a politician.

2) $p(x) \vee s(x)$ ; Either $x$ is a politician or $x$ is a sportsman.

(inclusive OR).

3) $p(x) \wedge s(x)$; $x$ is a politician cum sportsman.

2) $\exists x\, p(x)$ : At least one is true.

3) $\forall x \sim p(x)$ : All are false.

4) $\exists x \sim p(x)$ : At least one is false.

5) $\sim [\forall x\, p(x)]$ : At least one is false

(or)

Not All are true.

$\Leftrightarrow \exists x \sim p(x)$

6) $\sim [\exists x\, p(x)]$ : none is true

(or)

: All are false.

$\Leftrightarrow \forall x \sim p(x)$.

7) $\sim [\forall x \sim p(x)]$ : At least one is true

(or)

Not all are false.

$\Leftrightarrow \exists x\, p(x)$.

8) $\sim [\exists x \sim p(x)]$ : All are true

(or)

none is false.

$\Leftrightarrow \forall x\, p(x)$

$\sim \forall x \{ p(x) \rightarrow q(x) \}$
$\sim p(x) \vee q(x)$
$p(x) \wedge \sim q(x)$

* Note :- To negate a statement formula in first order logic, we have to replace $\exists x$ with $\forall x$, $\forall x$ with $\exists x$ and finally, we have to negate the scope of the quantifiers.

Q.1. ex → The negation of

$\exists x \{\sim p(x) \wedge q(x)\}$ is

$\rightarrow \sim \{\exists x \{\sim p(x) \wedge q(x)\}\}$

$\Rightarrow \forall x \{\sim(\sim p(x) \wedge q(x))\}$

$\Rightarrow \forall x \{ p(x) \vee \sim q(x)\}$. (by De-Morgan's law)

Q.2. The negation of

$\forall x \exists y [p(x,y) \rightarrow \{q(x,y) \wedge R(x,y)\}]$ is

$\rightarrow \exists x \forall y [\sim\{p(x,y) \rightarrow \{q(x,y) \wedge R(x,y)\}\}]$

$\Rightarrow \exists x \forall y [p(x,y) \wedge \sim\{q(x,y) \wedge R(x,y)\}]$

$\Rightarrow \exists x \forall y [p(x,y) \wedge \{\sim q(x,y) \vee \sim R(x,y)\}]$.

Q.3. The statement formula

$\exists x \{p(x) \wedge \sim q(x)\}$ is equivalent to

Q. 3. which of the following is not valid ?

a) $\forall x \exists y\ p(x,y) \Rightarrow \exists x \exists y\ p(x,y)$

→ Valid.

b) $\exists x \forall y\ p(x,y) \Rightarrow \forall x \exists y\ p(y,x)$

→ Valid.

c) $\exists x \forall y\ p(x,y) \Rightarrow \forall x \exists y\ p(x,y)$.

→ not valid.

* All from relationship diagram.

d) $\forall x \forall y\ p(x,y) \Rightarrow \forall x \exists y\ p(x,y)$

→ valid.

Q. 4. Let $G(x,y)$ denote a predicate "$x$ is greater than $y$".

$G(x,y)$: $x$ is greater than $y$.

where $x$ and $y$ are any positive integers.

Consider the statement.

"For any positive integer, there exists a greater positive integer."

which of the following first order logic sentences correctly represent the above statemen

a) $\forall x \exists y\, G(x,y)$

→ $\{1, 2, 3, \cdots, \infty\}$

1 is not greater than any positive integer.

✓ b) $\forall y \exists x\, G(x,y)$.

→ For every positive integer there exists a great positive integer.

c) $\exists x \forall y\, G(x,y)$.

→ There exists a largest +ve integer.

d) $\exists y \forall x\, G(x,y)$.

→ There exists a smallest +ve integer.

(1 ≯ 1) ∈ False.

Q.5. Let $F(x,y)$: $x$ is father of $y$

where $x$ and $y$ any two persons.

which of the foll. statements is true ?

$\sim(\forall x \exists y$



a) $\forall x \exists y \; F(x,y)$

→ Everybody is father of someone.

↑ False.

b) $\exists x \forall y \; F(x,y)$.

→ There is someone who is father of everybody

↑ False.

c) $\exists y \forall x \; F(x,y)$

→ There exists of someone who is child of everybody. ↑ False

✓d) $\forall y \exists x \; F(x,y)$

→ Everyone is ~~father~~ child of someone.

↑ True

Q.6. Let $L(x,y)$ : $x$ likes $y$.

Universe of discourse is set of all people

Consider the statement.

"There is someone whom noone likes".

Y $\exists y \forall x \sim L(x,y)$.

$\sim L(x,y)$ : $x$ does not like $y$.

which of the following first order logic sentence correctly represent the above sentence?

a) $\forall x \exists y \sim L(x,y)$. ⊗.

→ Everybody hates someone.
not same.

b) $\sim [\forall x \exists y \; L(x,y)]$ ⊗.

→ $\exists x \forall y \sim L(x,y)$

There exists someone whol hates everyone.
not same.

✓ c) $\sim [\forall y \exists x \; L(x,y)]$

→ $\exists y \forall x \sim L(x,y)$
same.

There exists some y, whom everybody hates.
same

✓ d) $\exists x \forall y \sim L(y,x)$
same.

$\exists x\ (G(x) \wedge \sim C(x))$.

3.7. which of the following statements is/are true if the universe of discourse is set of all integers ?

S1) $\forall x\ \exists y\ (x+y=5)$

True

$y = 5-x$

S2) $\exists y\ \forall x\ (x+y=0)$

False

* It means, there exists some $y$ for which $x+y=0$ for all $x$.

* $y$ is fixed. For any fixed set of integers, the given statement is not true.

S3) $\forall y\ \exists x\ (x \cdot y)=1$

False.

* For it to true, we have to take

$x = \frac{1}{y}$ but then $x$ is no more an integer.

* Also, for integer $y=0$, no integer can satisfy the eqn.

some $x$ for which $x \cdot y^2 = x$

for all int.

S4) $\exists x \forall y \, (x^2 \cdot y = y)$.

True

* for $x = \pm 1$, $x^2 \cdot y = y$ always for all.

Q-8. Let $G(x)$: $x$ is a graph

and $C(x)$ : $x$ is connected.

Consider the statement

" Not every graph is connected. "

which of the foll. first-order logic sentences does not represent the above statement ?

a) $\sim [\forall x \, G(x) \rightarrow C(x)]$

$\sim$ ( All graphs are connected )

same

b) $\exists x [G(x) \wedge \sim C(x)]$

* Some graphs are not connected.

same

c) $\sim [\forall x \{\sim G(x) \vee C(x)\}]$

Same

✓ d) $\forall x \{G(x) \rightarrow \sim C(x)\}$.

$\uparrow$ not same

* Every graph is not connected.

Q.9. Let $G(x)$: $x$ is gold ornament.

$P(x)$: $x$ is platinum ornament.

$V(x)$: $x$ is valuable ornament.

Consider the statement:

"Gold and Platinum ornaments are valuable"

which of the following first-order logic sentences is most appropriate one to represent above st.?

a) $\forall x \{V(x) \rightarrow (G(x) \wedge P(x))\}$

$\uparrow$ not same

$\rightarrow$ All valuable ornaments are gold and platinum

cannot use $\rightarrow$ in $\exists x$.

b) $\exists x \{G(x) \wedge P(x) \rightarrow P(x)\}$

$\uparrow$ not same

c) $\forall x \{(G(x) \wedge P(x)) \rightarrow V(x)\}$ $\leftarrow$ not same

✓ d) $\forall x \{(G(x) \vee P(x)) \rightarrow V(x)\}$

$\uparrow$ Same

# Rules of inference for quantified propositions :

1) Universal instantiation → (specification). (U.S.)

$$\frac{\forall x\ p(x)}{\therefore p(a)}$$

Valid for any element 'a' in the universe of discourse.

2) Existential specification (instantiation) → (E.S.)

$$\frac{\exists x\ P(x)}{\therefore p(c)}$$

valid for some elements 'c' in the universe of discourse.

3) Existential Generalization → (E.G.)

If p(c) is valid/true for some element in the univers of discourse.

$$\frac{\qquad}{\therefore \exists x\ p(x).}$$

4) Universal Generalization (U.G.) →

If p(a) is true for all elements in the universe of discourse

$$\frac{\qquad}{\therefore \forall x\ p(x).}$$

# Equivalences →

1) $\forall x \{p(x) \wedge Q(x)\} \Leftrightarrow \{\forall x\, p(x)\} \wedge \{\forall x\, Q(x)\}$

* **Note** → If we replace $\forall x$ with $\exists x$, the above statement does not hold good.

i.e. $\exists x\{p(x) \wedge Q(x)\} \not\Leftrightarrow \exists x\, p(x) \wedge \exists x R(x)$.

It's a tautological implication i.e.

$$\exists x\{p(x) \wedge Q(x)\} \Rightarrow \{\exists x\, p(x)\} \wedge \{\exists x\, R(x)\}$$

↑

tautological implication.

2) $\exists x \{p(x) \vee Q(x)\} \Leftrightarrow \{\exists x\, p(x)\} \vee \{\exists x\, Q(x)\}$.

* **Note** → If we replace $\exists x$ with $\forall x$, then these above statement does not hold good.

i.e. $\forall x \{p(x) \vee Q(x)\} \not\Leftrightarrow \{\forall x\, p(x)\} \vee \{\forall x\, Q(x)\}$

It is a tautological implication i.e.

$$\forall x\{p(x) \vee Q(x)\} \; \underset{\Leftarrow}{\not\Rightarrow} \; \{\forall x\, p(x)\} \vee \{\forall x\, Q(x)\}$$

$16 \to 10$ $\forall$ $p(x)$. $\forall x\, (p(x) \vee q(x))$

Q.10. which of the following is not valid ?

a) $\forall x \{p(x) \to q(x)\} \Rightarrow \{\forall x\, p(x) \to \forall x\, q(x).\}$

Valid.

* This formula is valid by C.P Rule if the following argument is valid.

1) $\forall x \{p(x) \to q(x)\}$

2) $\forall x\, p(x)$. * new premise to apply C.P.

$\therefore \forall x\, q(x)$

* If all are universal quantifiers, we can treat it like ordinary argument.

$\therefore$ by modus ponnens, we can get the answer →

3) $p(a) \to q(a)$

4) $p(a)$ by modus ponnens.

$\therefore q(a)$

$\therefore$ we can write $\forall x\, q(x)$.

$\therefore$ The given statement is valid.

b) $\{\forall x\ p(x) \rightarrow \forall x\ \varphi(x)\} \Rightarrow \{\forall x\ \{p(x) \rightarrow \varphi(x)\}\}$

not valid.

for $U = \{\text{Modi}, \text{Dhoni}\}$.

$p(x)$: $x$ is a politician.

$\varphi(x)$: $x$ is a sportsman.

then $\underbrace{\{\underbrace{\forall x\ p(x)}_{T} \rightarrow \underbrace{\forall x\ \varphi(x)}_{T}\}}_{\text{true}}$

False

and $\underbrace{\forall x\ \{p(x) \rightarrow \varphi(x)\}}_{T}$

false

$\therefore$ not valid.

$\{\exists x \sim p(x) \rightarrow \exists x\ \varphi(x)\}$

c) $\forall x\ \{p(x) \vee \varphi(x)\} \Rightarrow \{\forall x\ p(x)\} \vee \{\exists x\ \varphi(x)\}$.

* The given formula is valid by C.P rule if the foll: argument is valid.

1) $\forall x\ \{p(x) \vee \varphi(x)\}$

2) $\exists x \sim p(x)$ * new premise to apply C.P

$\therefore \exists x\ \varphi(x)$

3) $\sim p(a)$ from 2) by E.S.

~~4) $p(a) \vee \phi(a)$~~
4) $p(a) \vee \phi(a)$ from 1) by U.S.

5) $\phi(a)$ from 3) and 4) by disjunctive syllogism.

6) $\exists x\, \phi(x)$. from 5) by E.G.

$\therefore$ valid.

d) $\forall x \{p(x) \vee \phi(x)\} \Rightarrow \{\forall x\, p(x)\} \vee \{\forall x\, \phi(x)\}$

$\rightarrow$ not valid

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Q.11. which of the following arguments is not valid?

a) $\{\forall x\{p(x) \rightarrow \phi(x)\}, \exists y\, p(y)\} \Rightarrow \exists z\, \phi(z)$

valid

1) $\forall x\{p(x) \rightarrow \phi(x)\}$

2) $\exists y\, p(y)$.

3) $p(a)$ by E.S. from ②

4) $p(a) \rightarrow \phi(a)$ from ① by U.S.

5) $\phi(a)$ from 3) and 4) by modus-ponens

6) $\therefore \exists z\, Q(z)$ E.G. from 5)

$\therefore$ the given argument is valid.

b) $\{\exists x\, P(x), \exists x\, Q(x)\} \Rightarrow \exists x\{P(x) \wedge Q(x)\}$

not valid

→ left side denotes → there exists some politician and there exists some sportsman.

Right side denotes → there exists some who is both politician and sportsman.

$\therefore$ LHS $\neq$ RHS

$\therefore$ Not valid.

c) $\{\{\forall x\, \forall y\, \{P(x,y) \rightarrow Q(x,y)\}\}, \sim Q(a,b)\} \Rightarrow \sim P(a,b)$.

1) $\forall x\, \forall y\, \{P(x,y) \rightarrow Q(x,y)\}$

2) $\sim Q(a,b)$

3) $\forall y\, \{P(a,y) \rightarrow Q(a,y)\}$ from 1), by U.S.

4) $P(a,b) \rightarrow Q(a,b)$ from 3) by U.S.

5) $\sim P(a,b)$. from 3) and 2) and 4) by modus-tonnens.

$\therefore$ the given argument is valid.

$\neg f(x) \; ((\ldots$
$\exists x \; (( C(x) \wedge P(x)).$

d) $\underbrace{\forall x \{ P(x) \wedge Q(x) \}}_{A} \rightarrow \underbrace{\exists y \{ R(y) \rightarrow W(y) \}}_{B}$

$\forall y \{ R(y) \wedge \sim W(y) \} \rightarrow (\sim B)$

$\therefore \; \exists x \{ P(x) \rightarrow \sim Q(x) \} \rightarrow (\sim A)$

$\rightarrow$ 1) $A \rightarrow B$

2) $\sim B$

$\therefore \sim A.$

$\therefore \; \exists x \{ P(x) \rightarrow \sim Q(x) \}.$

$\therefore$ the given argument is valid.

Q.12. which of the following arguments is not valid ?

a) All computer science graduates are ~~logical thinkers~~ people.

Some computer science graduates are logical thinkers.

---

$\therefore$ Some people are logical thinkers.

$\rightarrow$ Let $C(x)$ : $x$ is a C.S. graduate

$P(x)$ : $x$ is a person.

$L(x)$ : $x$ is a logical thinker.

first stmt → $\forall x\{c(x) \rightarrow p(x)\}$ 1)

Second stmt → $\exists x\{c(x) \wedge L(x)\}$ 2)

conclusion → $\exists x\{p(x) \wedge L(x)\}$ 3)

4) $c(a) \wedge L(a)$ from 2) by E.S.

5) $c(a)$ and $L(a)$ from 4) by simplificat$^n$

6) $c(a) \rightarrow p(a)$ from 1) by U.S.

7) $p(a)$ from 6) & 5) by modus-ponens.

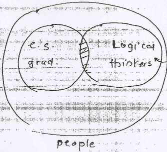
8) $p(a) \wedge L(a)$ from 7) & 5) by conjunction.

9) $\therefore \exists x\{p(x) \wedge L(x)\}$ from 8) by E.G.

∴ the given statement/argument is valid.

(OR)

We can solve it by Venn diagrams.




logical thinker need not be person. It might be a mic.

b) i) Every dog likes people or hates cats.

ii) Rover is a dog.

iii) Rover likes cats.

∴ iv) Some dogs likes people.

→ Let $D(x)$ : $x$ is a dog.

$L(x)$ : $x$ likes people.

$H(x)$ : $x$ hates cats.

a : Rover.

i) → $\forall x \{D(x) \rightarrow \{L(x) \vee H(x)\}\}$ 1)

ii) → $D(a)$ 2)

iii) → $\sim H(a)$ 3)

∴ iv) $\exists x \{D(x) \wedge L(x)\}$

4) $D(a) \rightarrow \{L(a) \vee H(a)\}$ from 1), U.S.

5) $L(a) \vee H(a)$ from 4) and 2), by modus-ponens.

6) $L(a)$ from 5) & 3) by disjunctive syllogism.

7) $D(a) \wedge L(a)$ from 2) and 6), by conjunct$^n$.

$\therefore \exists x\,(D(x) \wedge L(x))$ by E.G.

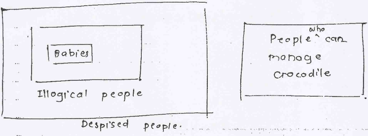
$\therefore$ the given statement is valid.

C) i) Babies are illogical.

ii) Nobody is despised who can manage a crocodiles.

iii) Illogical people are despised.

$\therefore$ iv) Babies cannot manage crocodiles.




(A) (B)

(A) and (B) are disjoint sets.

$\therefore$ the given statement is valid.

(OR)

$B(x)$ : $x$ is a baby

$I(x)$: $x$ is illogical.

$D(x)$ : $x$ is despised.

$M(x)$: $x$ can manage crocodiles.

i) $\forall x \{ B(x) \rightarrow I(x) \}$ 1)

ii) $\forall x \{ M(x) \rightarrow \sim D(x) \}$. 2)

iii) $\forall x \{ I(x) \rightarrow D(x) \}$ 3)

$\therefore$ iv) $\forall x \{ B(x) \rightarrow \sim M(x) \}$.

4) $D(x) \rightarrow \sim M(x)$ from 2).

by contrapositive.

5) $B(x) \rightarrow \sim M(x)$ from 1), 3) & 2)

by transitivity.

$\therefore \forall x \{ B(x) \rightarrow \sim M(x) \}$.

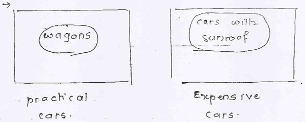
$\therefore$ the given argument is valid.

d) i) No practical car is expensive.

ii) Cars with sun roof are expensive

iii) All wagons are practical cars.

∴ iv) Some wagons have sunroofs.




∴ No wagon may have sun roof.

∴ the argument is not valid.