CGPET 2018

Solved Paper

Question 1

The capacity of a parallel plate capacitor increases with the

Options:

A. decrease of its area

B. increase of its distance

C. increase of its area

D. None of these

Answer: C

Solution:

Solution:
For parallel plate capacitor,
$C = \frac{\varepsilon A}{d} \text{or} C \propto A$
Hence, capacity of a parallel plate capacitor increases with the increase of its area

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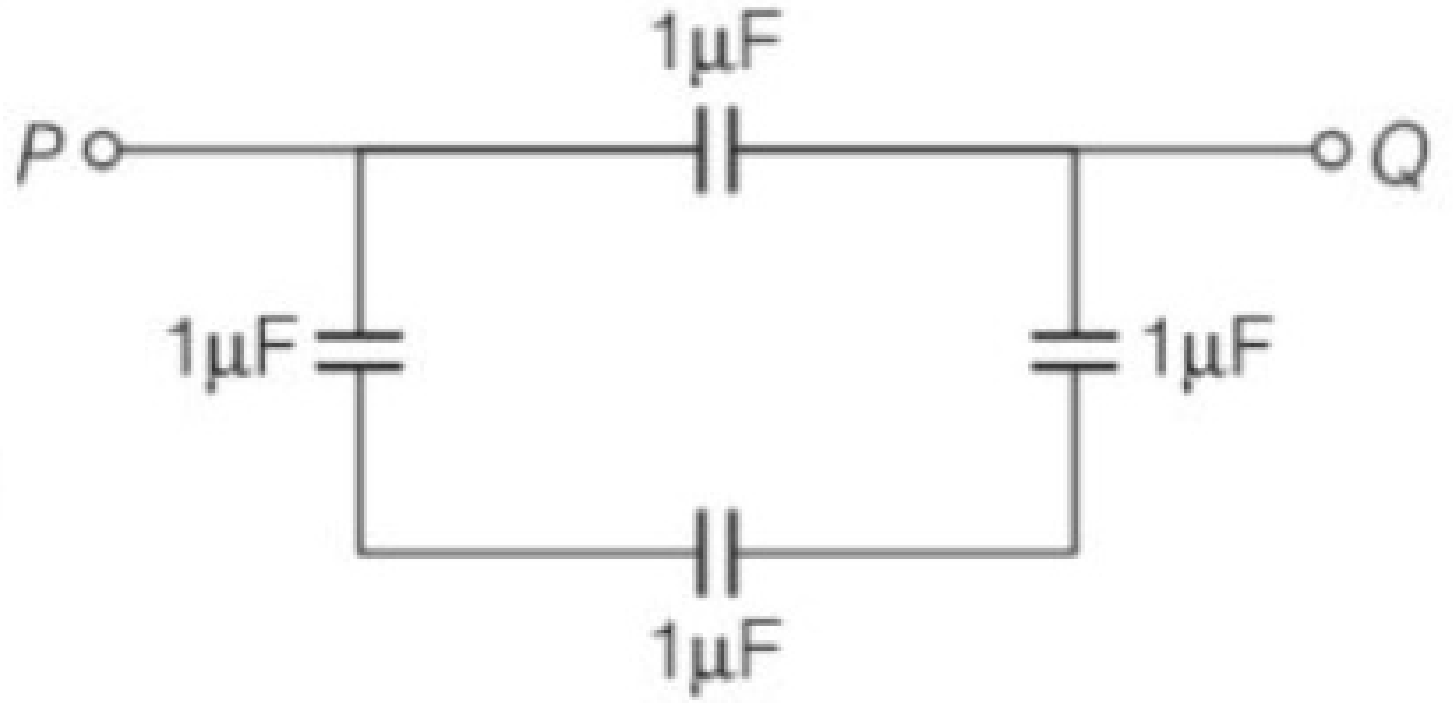
Question 2

Four capacitors are connected as shown in the figure below. The equivalent capacitance between the points P and Q is



Options:

A. $4\mu F$

B. $\frac{1}{4}\mu F$

C. $\frac{3}{4}\mu F$

D. $\frac{4}{3}\mu F$

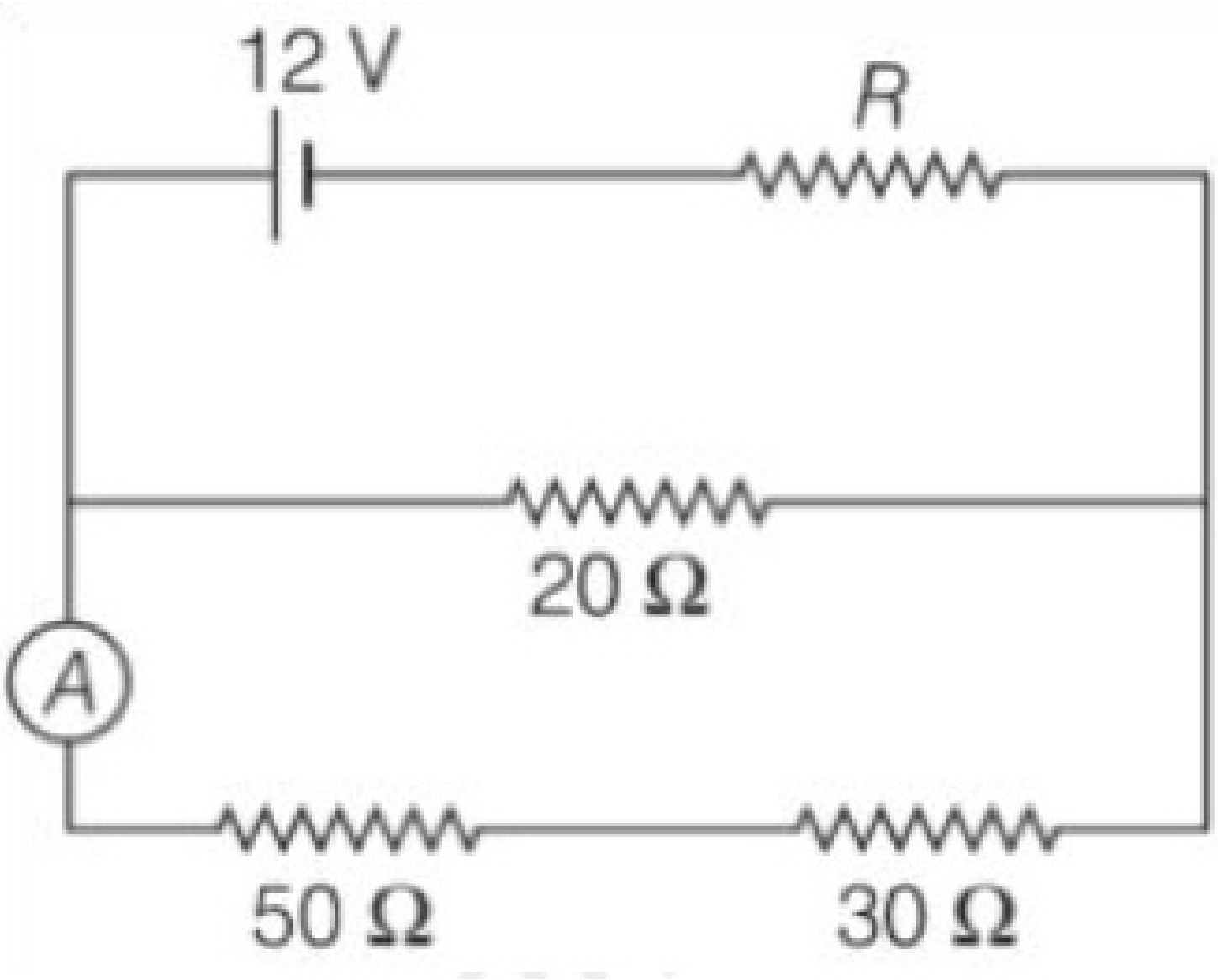
Answer: D

Solution:

Solution:
According to given figure, three $1\mu F$ capacitors are connected in series combination. So, their resultant capacitance,
$\frac{1}{C_1} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 3$ or $C_1 = \frac{1}{3}\mu F$
Now, capacitor C_1 is connected in parallel combination with $1\mu F$ capacitor. Hence, equivalent capacitance between the P and Q,
$C_{PQ} = C_1 + 1 = \frac{1}{3} + 1 = \frac{4}{3}\mu F$

Question 3

In the circuit shown in the figure below, the ammeter A, assumed to have negligible resistance, reads $0.1A$. The value of R is



Options:

A. 6Ω

B. 8Ω

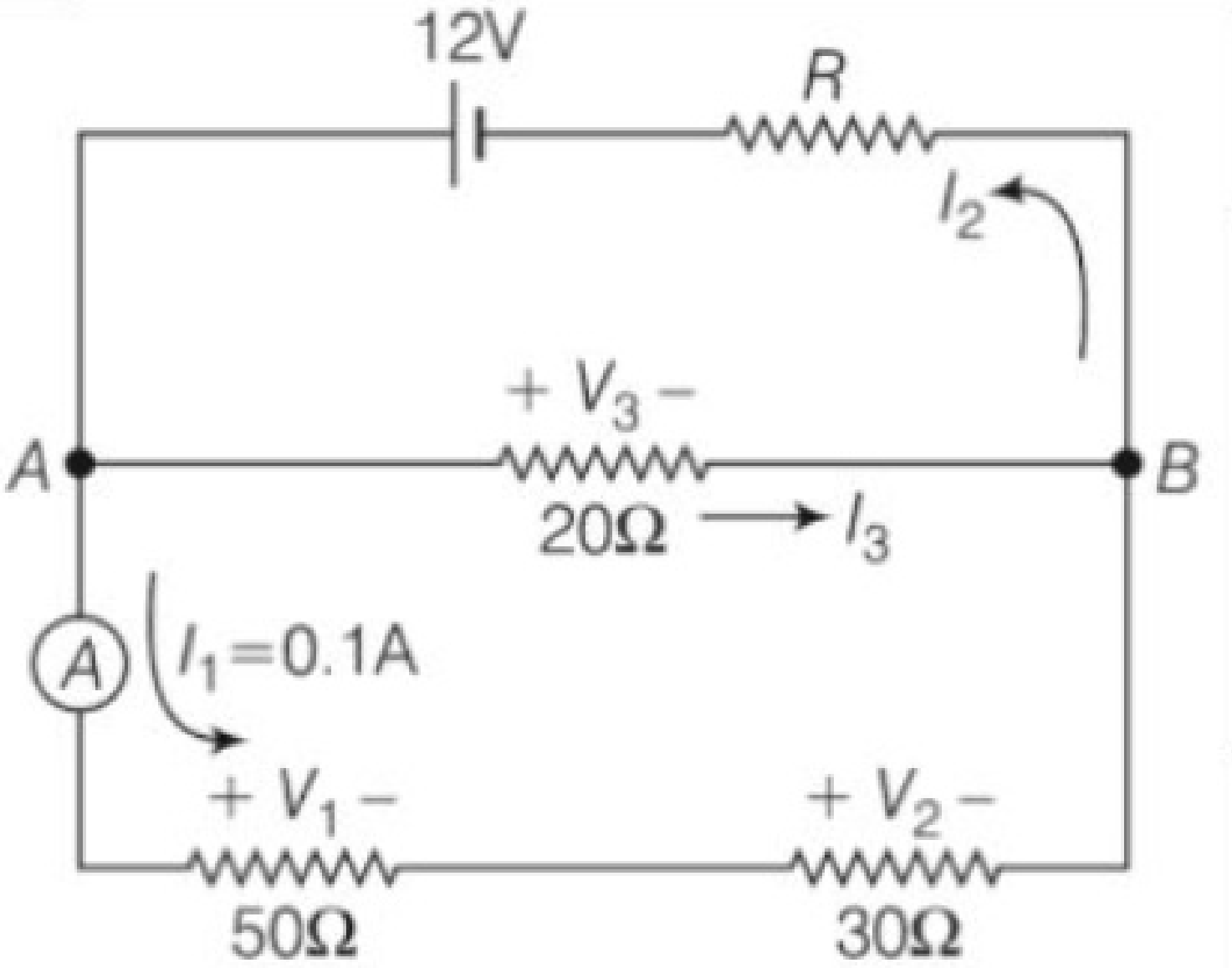
C. 16Ω

D. 20Ω

Answer: B

Solution:

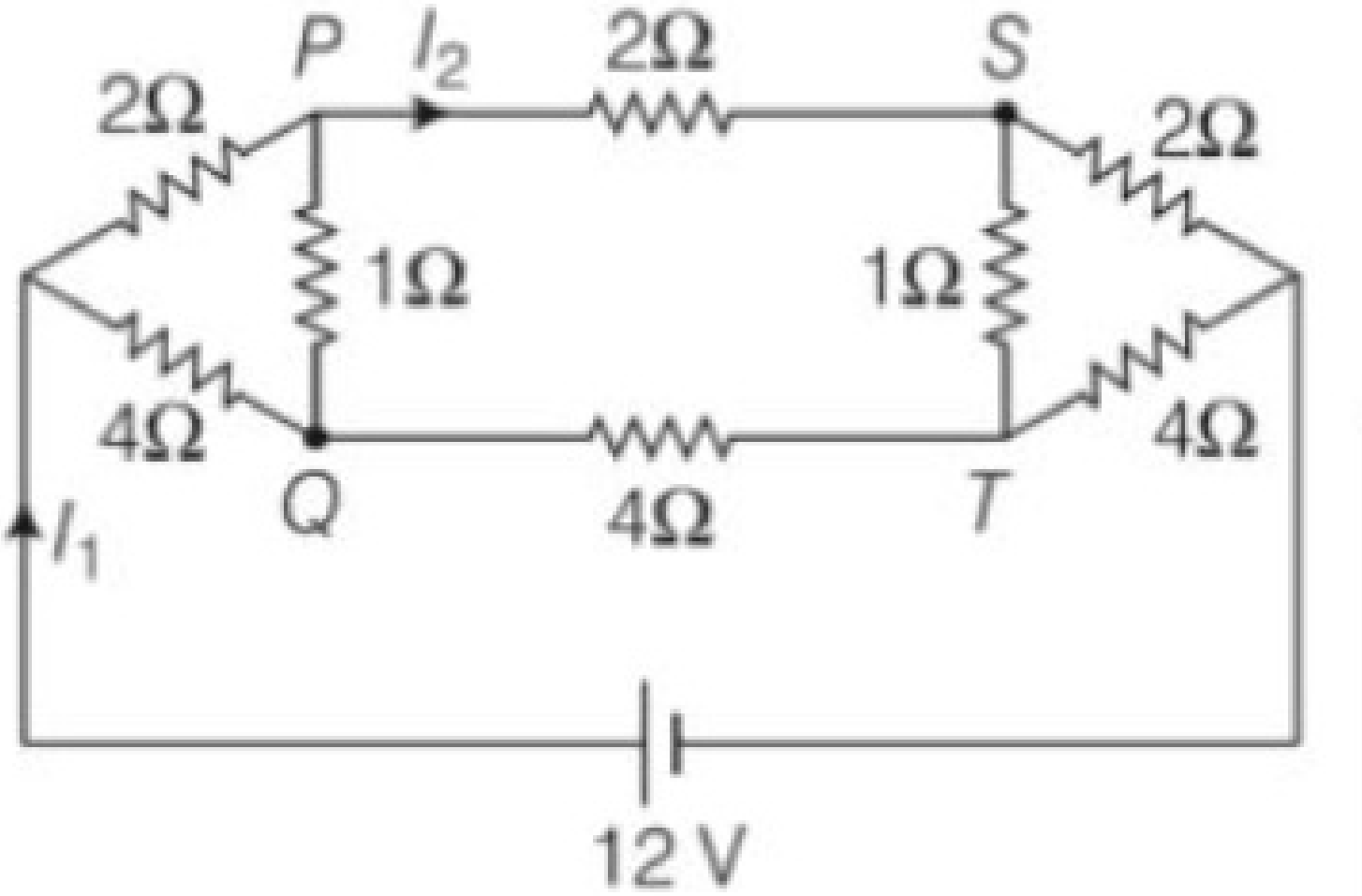
Solution:
In the circuit shown in the figure below.



So, voltage across 50Ω and 30Ω resistor,
$V_1 = I_1 \times 50 = 0.1 \times 50 = 5V$
$V_2 = I_1 \times 30 = 0.1 \times 30 = 3V$
$\because$ Three branches are connected in parallel combination, therefore voltage across each branch will be equal.
$\therefore V_1 + V_2 = V_3$
$\Rightarrow 5 + 3 = V_3$
$\Rightarrow V_3 = 8V$
Applying KCL at point B,
$I_2 = I_1 + I_3 = 0.1 + \frac{2}{5} = 0.5A$
Hence, $V_1 + V_2 = 12 - I_2R$
$\Rightarrow R = \frac{12 - 8}{0.5} = 8\Omega$

Question 4

For the resistance network shown in the figure below, the correct option is



Options:

A. the current through PQ is not zero

B. the potential at S is less than that at Q

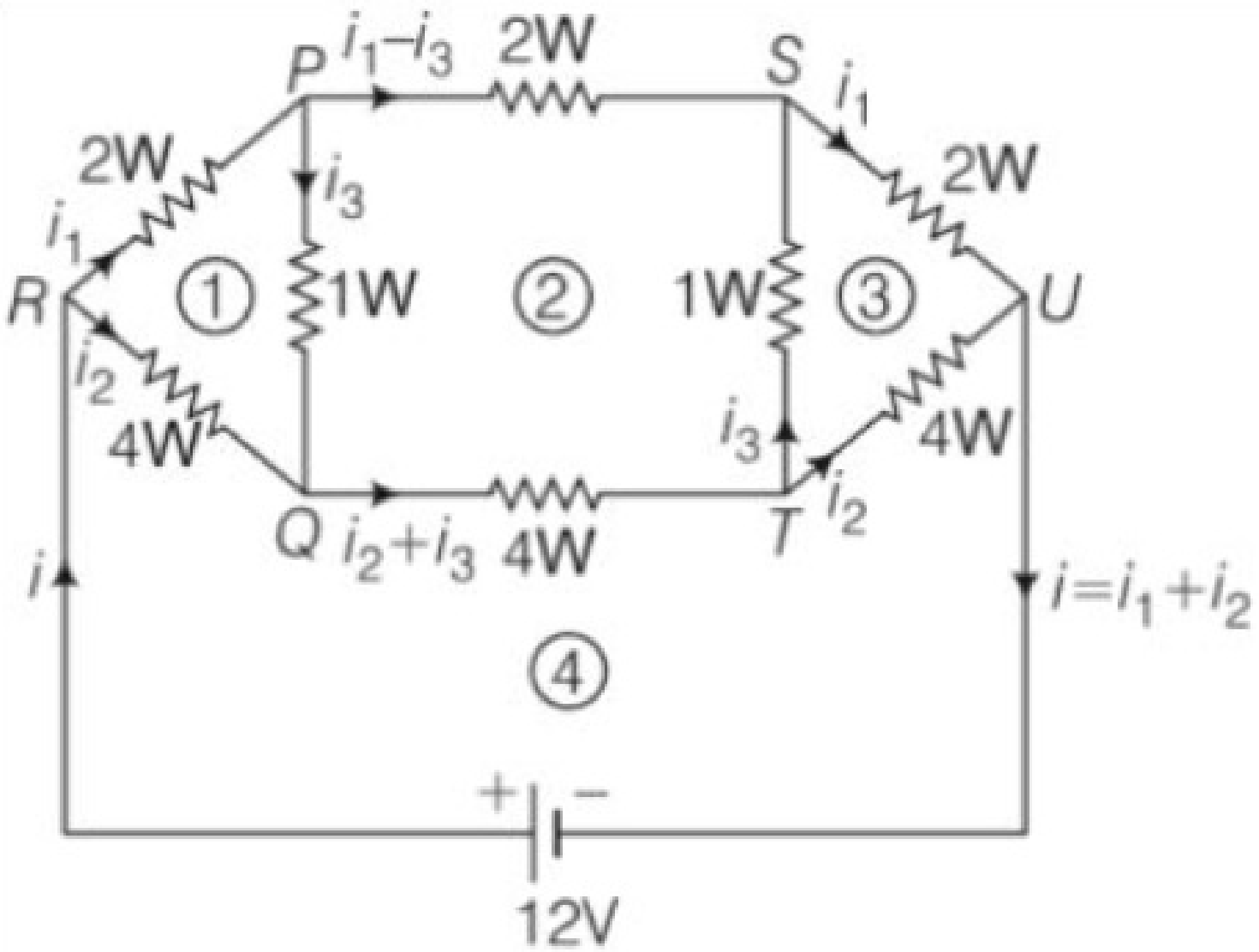
C. $I_1 = 2A$

D. $I_2 = 3A$

Answer: B

Solution:

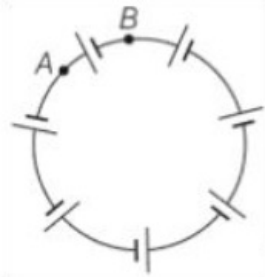
Solution:



By Kirchhoff's loop rule;
In loop (1),
$-2i_1 - i_3 + 4i_2 = 0$
or $4i_2 = 2i_1 + i_3$ .....(i)
In loop(2),
$-2(i_1 - i_3) + 1(i_3) + 4(i_2 + i_3) + i_3 = 0$
$\Rightarrow -2i_1 + 2i_3 + i_3 + 4i_2 + 4i_3 + i_3 = 0$
$\Rightarrow 4i_2 + 8i_3 = 2i_1$ .....(ii)
In loop (4),
$-4i_2 - 4(i_2 + i_3) - 4i_2 + 12 = 0$
or $12i_2 + 4i_3 = 12$ .....(iii)
From Eqs. (i) and (ii), we get
$4i_2 = 4i_2 + 8i_3 + i_3$
$\Rightarrow 9i_3 = 0$ or $i_3 = 0$ .....(iv)
From Eq. (iv), option (a) is incorrect.
From Eqs. (iii) and (iv), we get
$12i_2 + 4 \times 0 = 12$ or $i_2 = 1A$
Now, from Eq. (i), we get
$4 \times 1 = 2i_1 + 0$
$\Rightarrow i_1 = 2A$
Hence, $I_1 = i_1 + i_2 = 2 + 1 = 3A$ .....(v)
and $I_2 = i_1 - i_3 = 2 - 0 = 2A$ .....(iv)
From Eqs. (v) and (vi), option (c) and (d) are incorrect.
Now,
$V_P - V_Q = 0 \times 1 = 0 \Rightarrow V_p = V_Q$
and $V_P - V_S = 2 \times 2 = 4$
$\therefore V_P > V_s$
or $V_Q > V_s$ .....(vii)
From Eq. (vii), option (b) is correct.

Question 5

Seven identical cells each of emf E and internal resistance r are connected as shown in the figure below. The potential difference between A and B is



Options:

A. $7E$

B. E

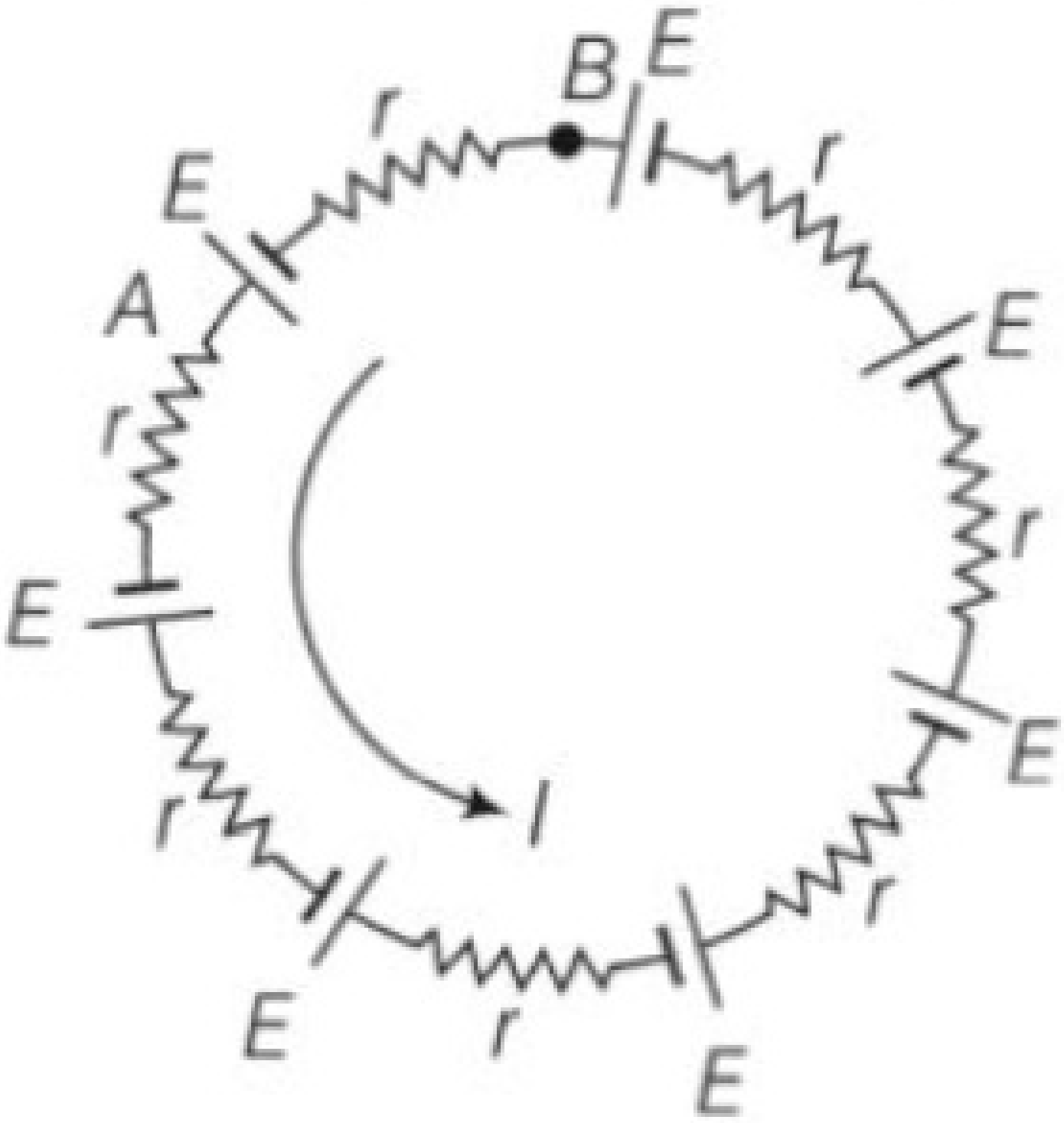
C. $6E$

D. zero

Answer: D

Solution:

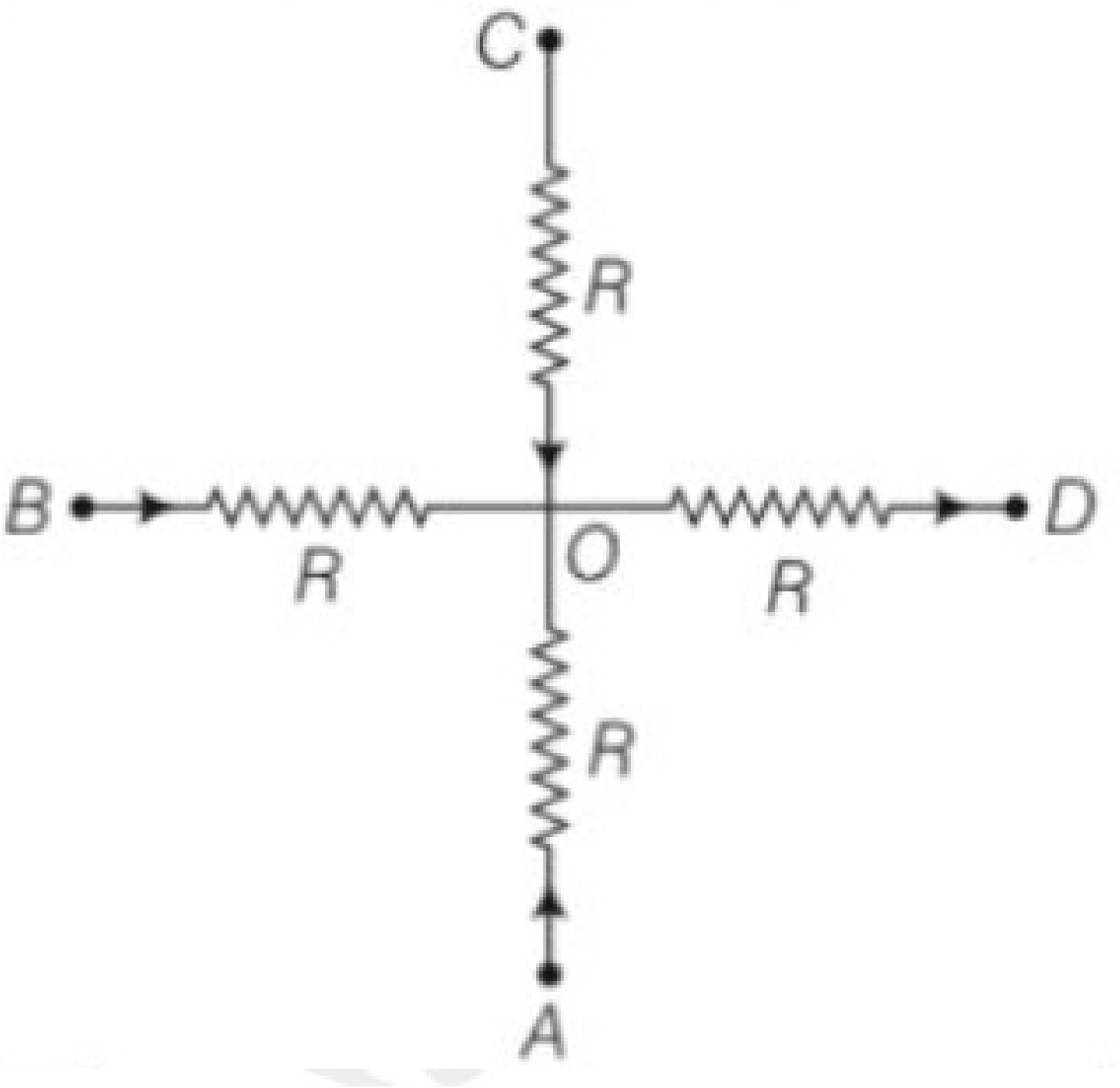
Solution:
$\therefore$ Seven identical cells each of $emfE$ and internal resistance r are connected as shown in the figure below.



$\because\ V_{AB} = E - Ir$ .....(i)
Applying KVL in the loop,
$-E + Ir - E + Ir - E + Ir - E$
$+Ir - E + Ir - E + Ir - E + Ir = 0$
$\Rightarrow E = Ir$ .....(ii)
From Eqs. (i) and (ii), we get
$V_{AB} = Ir - Ir = 0$

Question 6

In the network shown in the figure below, the points A, B and C are at same potential. The potential difference between A and D is 40 V. The potential difference between A and O is



Options:

A. $10V$

B. $15V$

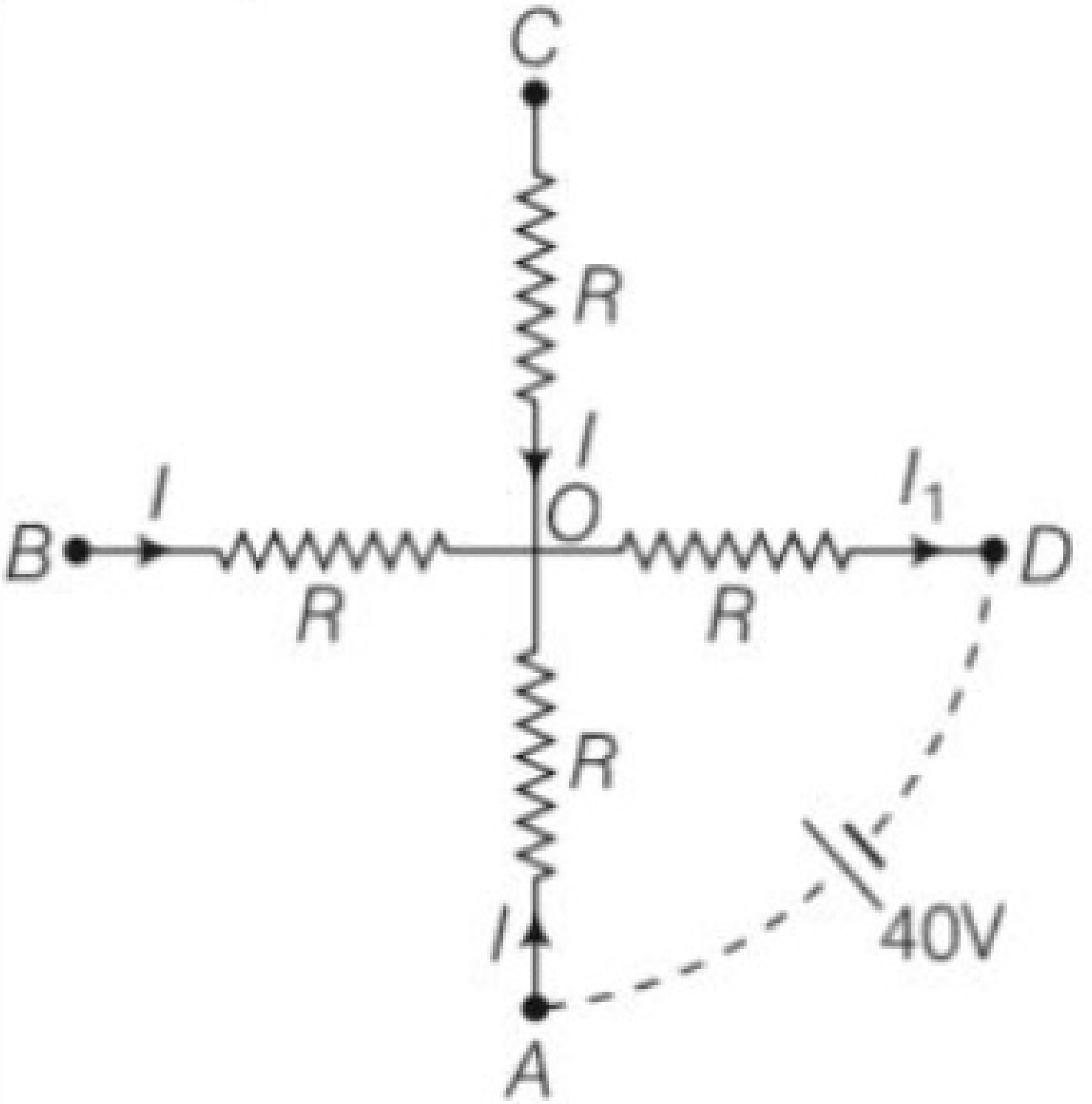
C. $30V$

D. $20V$

Answer: A

Solution:

Solution:
As shown in figure below,
$V_A = V_B = V_C, V_A - V_D = V_{AD} = 40V$
So, current in branch BO, AO and CO will be equal.



Applying KCL at point O,
$IR + IR + IR = I_1R$
$\Rightarrow I_1 = 3I$
Applying KVL in AOD loop,
$-40 + IR + I_1R = 0$
$\Rightarrow 4IR = 40$
$\Rightarrow IR = 10 \text{or} V_{AO} = 10V[\because V_{AO} = IR]$

Question 7

The intensity of magnetic field due to current / in a long straight wire is proportional to

Options:

A. I

B. I^2

C. $\sqrt{I}$

D. $\frac{1}{I}$

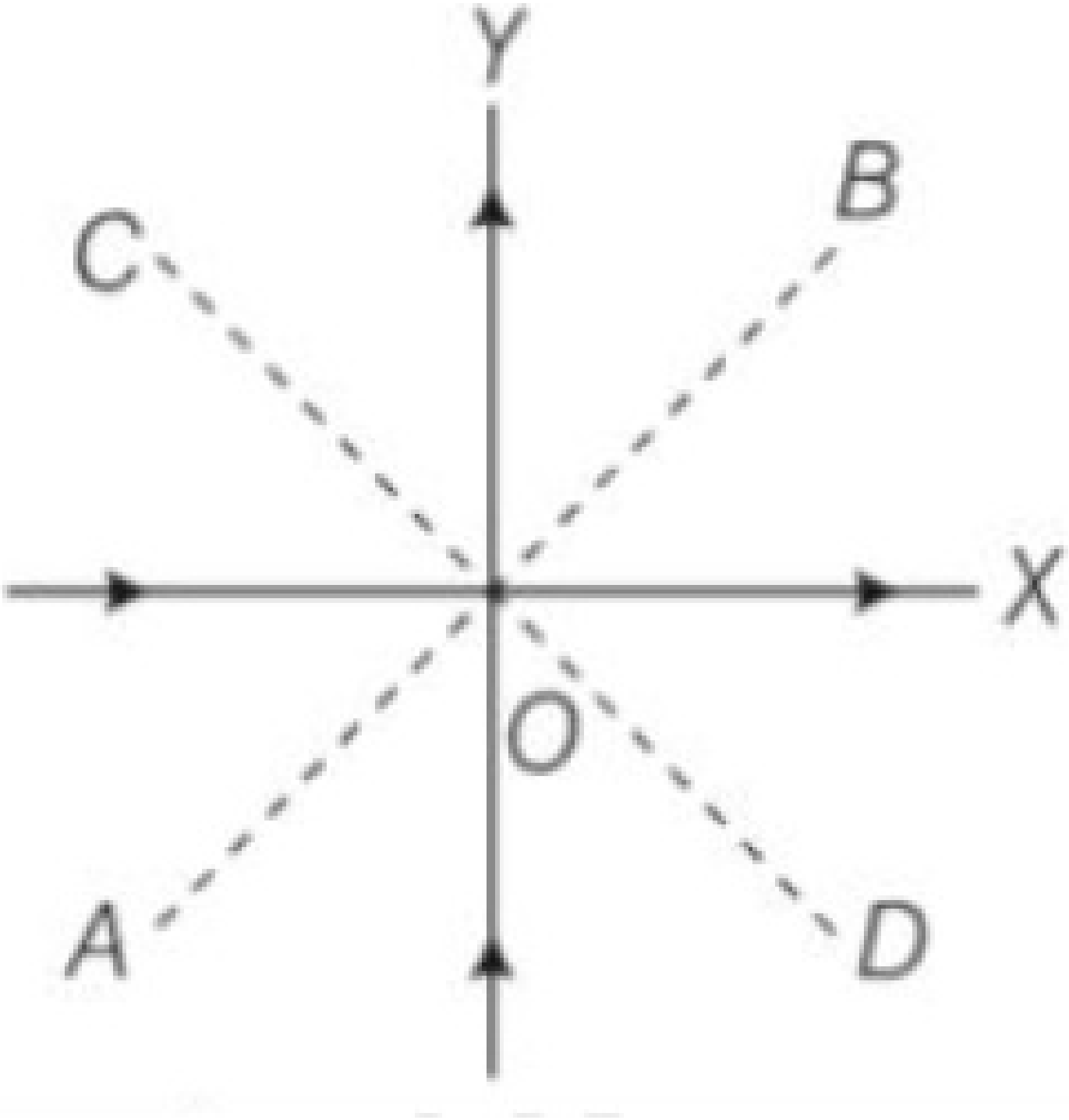
Answer: A

Solution:

Solution:
The intensity of magnetic field due to current I in a long straight wire,
$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{R}$
or $B \propto I$

Question 8

Two thin metallic wires lie on X and Y -axes and both carry the same current as shown in the figure below. AB and CD are the two lines making angles 45° with the axes and the origin of axes is O. The magnetic field will be zero



Options:

A. on the line AB

B. on the line CD

C. only on part OB of the line AB

D. only on part OC of the line CD

Answer: C

Solution:

Solution:
According to Maxwell's right hand rule, direction of magnetic field due to $+X$ -axis will be outward and due to $+Y$ -axis will be inward. So, the magnetic field will cancel out each other on the line OB.

Question 9

A moving electric charge produces

Options:

A. electric and magnetic fields

B. only the electric field

C. only the magnetic field

D. None of the above

Answer: A

Solution:

Solution:
A moving electric charge produces electric and magnetic fields.

Question 10

An α -particle moving with velocity $5 \times 10^5 \hat{i}\, m/s$ enters in a magnetic field $3\hat{i} + 2\hat{j}$ tesla. The force experienced by the particle is

Options:

A. $2.3 \times 10^{-13} \hat{i}\, N$

B. $3.2 \times 10^{-13} \hat{k}\, N$

C. $5.2 \times 10^{-12} \hat{k}\, N$

D. $2.5 \times 10^{-13} \hat{j}\, N$

Answer: B

Solution:

Solution:
$\therefore$ Force on moving charge in a uniform magnetic field,
$F = q(v \times B)$
$\because$ Charge of an α -particle $= 2 \times$ Charge of a proton
$\therefore F = 2 \times 1.6 \times 10^{-19}[5 \times 10^5 \hat{i} \times (3\hat{i} + 2\hat{j})]$
$F = 2 \times 1.6 \times 10^{-19}(10^6 \hat{k})$
Hence, the force experienced by the particle,
$F = 3.2 \times 10^{-13} \hat{k}\, N$

Question 11

A current-carrying short coil behaves like a short magnet. If the area of the coil is A and magnetic moment is M, the current in the coil is

Options:

A. $\frac{M}{A}$

B. $\frac{A}{M}$

C. MA

D. A^2M

Answer: A

Solution:

Solution:
∴ Magnetic dipole moment of current loop,
$M = NiA$
Here, $N = 1$
Hence, $i = \frac{M}{A}$

Question 12

The area of a coil is A. The coil is placed in a magnetic field. The magnetic field changes from B_0 to $4B_0$ in time t. The induced emf in the coil will be

Options:

A. $\frac{3AB_0}{t}$

B. $\frac{4AB_0}{t}$

C. $\frac{3B_0}{At}$

D. $\frac{4B_0}{At}$

Answer: A

Solution:

Solution:
According to second law of Faraday,
∴ induced emf of the coil,

$|e| = \frac{\Delta\phi}{\Delta t}$ .....(i)
$\because \phi = BA$
$\therefore |e| = \frac{A\Delta B}{\Delta t} = A\frac{(4B_0 - B_0)}{t} = \frac{3AB_0}{t}$

Question 13

A transformer is used to glow a $140W - 240V$ bulb at $240VAC$. If current in the primary coil is $0.7A$, then the efficiency of the transformer is

Options:

A. 16.7 %

B. 50 %

C. 83.3 %

D. 25 %

Answer: C

Solution:

Solution:
Given,
$i_i = 0.7A, V_i = 240V, P_0 = 140W$
$\therefore$ Efficiency of transformer, $\eta = \frac{P_0}{P_i} \times 100$
or $\eta = \frac{140}{V_i i_i} \times 100 = \frac{140}{240 \times 0.7} \times 100$
or $\eta = 83.3\%$

Question 14

The dimension of $\frac{1}{\mu_0 \varepsilon_0}$, where symbols have their usual meanings, are

Options:

A. $[L^{-2}T^2]$

B. [LT ^{-1}]

C. $[L^2T^{-2}]$

D. $[L^{-1}T]$

Answer: C

Solution:

Solution:

$\therefore c^2 = \frac{1}{\mu_0 \varepsilon_0}$, so its dimension is $[L^2 T^{-2}]$

Question 15

A car moves at a speed of $36 km/h$ on a level road. The coefficient of friction between the car tyres and the road is 0.8. The car negotiates a curve of radius R. If $g = 10 ms^{-2}$, the car will skid (or slip) while negotiating the curve, if the value of R is

Options:

A. $20 m$

B. $14 m$

C. $12 m$

D. $50 m$

Answer: C

Solution:

Solution:
Given, $v = 36 km/h = 10 m/s$,
$\mu_s = 0.8, g = 10 ms^{-2}$
For a safe turn without sliding, the velocity of a car,
$v \leq \sqrt{\mu_s R g}$
$\therefore R \geq \frac{v^2}{\mu_s g}$
$\Rightarrow \quad R \geq \frac{100}{0.8 \times 10}$
Or $R \geq 12.5 m$
Hence, option (c), for $R = 12 m$, the car will skid (or slip) while negotiating the curve.

Question 16

A projectile thrown at an angle of 30° with the horizontal has a range R_1 and attains a maximum height h_1. Another projectile thrown with the same speed, at an angle of 30° with the vertical has a range R_2 and attains a maximum height h_2, then

Options:

A. $R_2 = R_1, h_1 = h_2$

B. $R_2 = 2R_1, h_1 = 3h_2$

C. $R_2 = R_1, h_2 = 3h_1$

D. $R_2 = 3R_1, h_1 = 3h_2$

Answer: C

Solution:

Solution:

Height of projectile, $H = \frac{u^2}{2g}\sin^2\theta$ .....(i)

Range of projectile, $R = \frac{u^2 \sin 2\theta}{g}$ .....(ii)

Therefore, $R_1 = \frac{u^2 \sin(2 \times 30°)}{g} = \frac{\sqrt{3}}{2} \cdot \frac{u^2}{g}$

and $R_2 = \frac{u^2 \sin 2(90° - 30°)}{g} = \frac{\sqrt{3}}{2}\frac{u^2}{g}$

$\therefore h_1 = \frac{u^2 \sin^2 30°}{2g} = \frac{1}{4}\left[\frac{u^2}{2g}\right]$

and $h_2 = \frac{u^2 \sin^2 60°}{2g} = \frac{3}{4}\left[\frac{u^2}{2g}\right]$

Hence, from above result,

$R_2 = R_1, h_2 = 3h_1$

Question 17

The moment of inertia of a uniform horizontal solid cylinder of mass M about an axis passing through its edge and perpendicular to the axis of the cylinder, when its length is 6 times its radius R, is

Options:

A. $\frac{49MR^2}{4}$

B. $\frac{39MR^2}{4}$

C. $\frac{36MR^2}{11}$

D. $\frac{3MR^2}{2}$

Answer: A

Solution:

Solution:

The moment of inertia of a uniform horizontal solid cylinder about an axis passing through its edge and perpendicular to the axis,

$I = M\left[\frac{L^2}{3} + \frac{R^2}{4}\right]$

Given, $L = 6R$

$\therefore I = M\left[\frac{36R^2}{3} + \frac{R^2}{4}\right] = \frac{49}{4}MR^2$

Question 18

Two springs of force constants k_1 and k_2 are stretched by the same force. The ratio of potential energies stored in them is

Options:

A. $k_1 : k_2$

B. $k_2 : k_1$

C. $\sqrt{k_1} : \sqrt{k_2}$

D. $k_2^2 : k_1^2$

Answer: B

Solution:

Solution:
$\therefore$ Potential energy stored in stretched spring,
$PE = \frac{1}{2}kx^2$
$\therefore PE_1 = \frac{1}{2}k_1x_1^2$ and $PE_2 = \frac{1}{2}k_2x_2^2$
$\therefore$ Ratio of PE, $\frac{PE_1}{PE_2} = \frac{k_1}{k_2}.\frac{x_1^2}{x_2^2}$ .....(i)
Given, $F_1 = F_2$
$k_1x_1 = k_2x_2$ or $\frac{x_1}{x_2} = \frac{k_2}{k_1}$ .....(ii)
$\therefore$ From Eqs. (i) and (ii), we get
$\frac{PE_1}{PE_2} = \frac{k_1}{k_2}\left(\frac{k_2^2}{k_1^2}\right)$
or $PE_1 : PE_2 = k_2 : k_1$

Question 19

A policeman fires a bullet of mass $50g$ with a speed of $200m/s$, in a wooden piece of thickness $2.0cm$. The bullet leaves the wooden piece with only 10% of its initial kinetic energy. By how much percentage has the speed of the bullet been reduced?

Options:

A. 74.2%

B. 40%

C. 90%

D. 68.4%

Answer: D

Solution:

Solution:
Kinetic energy, when bullet leaves the wooden piece,
$(KE)_f = (KE)_i \times \frac{10}{100}$
or $\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 \times \frac{10}{100}$

or $v_f = \frac{v_i}{\sqrt{10}}$
Hence, reduced speed of bullet
$= \frac{v_i - v_f}{v_i} \times 100$
$= \frac{v_i - \frac{v_i}{\sqrt{10}}}{v_i} \times 100 \approx 68.4\,\%$

Question 20

Two bodies of masses m and $4m$ are placed at a distance r. The gravitational potential a a point on the line joining them, where the gravitational field is zero, is

Options:

A. zero

B. $-\frac{4Gm}{r}$

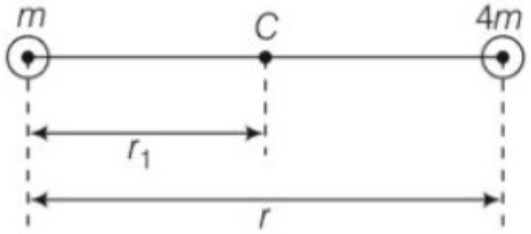
C. $-\frac{6Gm}{r}$

D. $-\frac{9Gm}{r}$

Answer: D

Solution:

Solution:
According to question,



Let gravitational field is zero at point C.
$\therefore I_1 = I_2$
$\Rightarrow \frac{Gm}{r_1^2} = \frac{G(4m)}{(r - r_1)^2}$ or $r - r_1 = 2r_1$
$\Rightarrow r = 3r_1 \text{or} r_1 = \frac{r}{3}$
$\therefore$ Gravitational potential at point C cause of mass M,
$V = -\frac{GM}{r}$
$\therefore$ For $M = m, V_1 = -\frac{Gm}{r_1}$
or $V_1 = \frac{-Gm}{r/3} = \frac{-3Gm}{r}$
For $M = 4m, V_2 = \frac{-G(4m)}{(r - r_1)} = \frac{-G(4m)}{r - r/3}$
or $V_2 = \frac{-6Gm}{r}$
Hence, net gravitational potential at point C,
$V_{\text{net}} = V_1 + V_2 = \frac{-3Gm}{r} + (\frac{-6Gm}{r}) = -\frac{9Gm}{r}$

Question 21

The radius of the Earth is R and g is the acceleration due to gravity on its surface. What should be the angular speed of the Earth so that bodies lying on the equator may appear weightless?

Options:

A. $\sqrt{\frac{2g}{R}}$

B. $\sqrt{\frac{g}{R}}$

C. $\sqrt{\frac{g}{2R}}$

D. $2\sqrt{\frac{g}{R}}$

Answer: B

Solution:

Solution:
$\therefore$ Expression for variation of g with latitude,
$g' = g - R_e\omega^2\cos^2\lambda$
$\because R_e = R$ and for equator $\lambda = 0°$.
$\therefore g' = g - R\omega^2$
Multiply with m on both sides, we get
$mg' = mg - mR\omega^2$
$\because$ For weightless, $mg' = 0$
$\therefore 0 = mg - mR\omega^2$
$\therefore$ Angular speed of Earth, $\omega = \sqrt{\frac{g}{R}}$.

Question 22

A rubber (eraser) $3cm \times 1cm \times 8cm$ is clamped at one end with $8cm$ edge vertical. A horizontal force of $2.4N$ is applied at the free end (the top face). If the shear modulus of the rubber is $1.6 \times 10^5 Nm^{-2}$, then the horizontal displacement of the top face will be

Options:

A. $1mm$

B. $2mm$

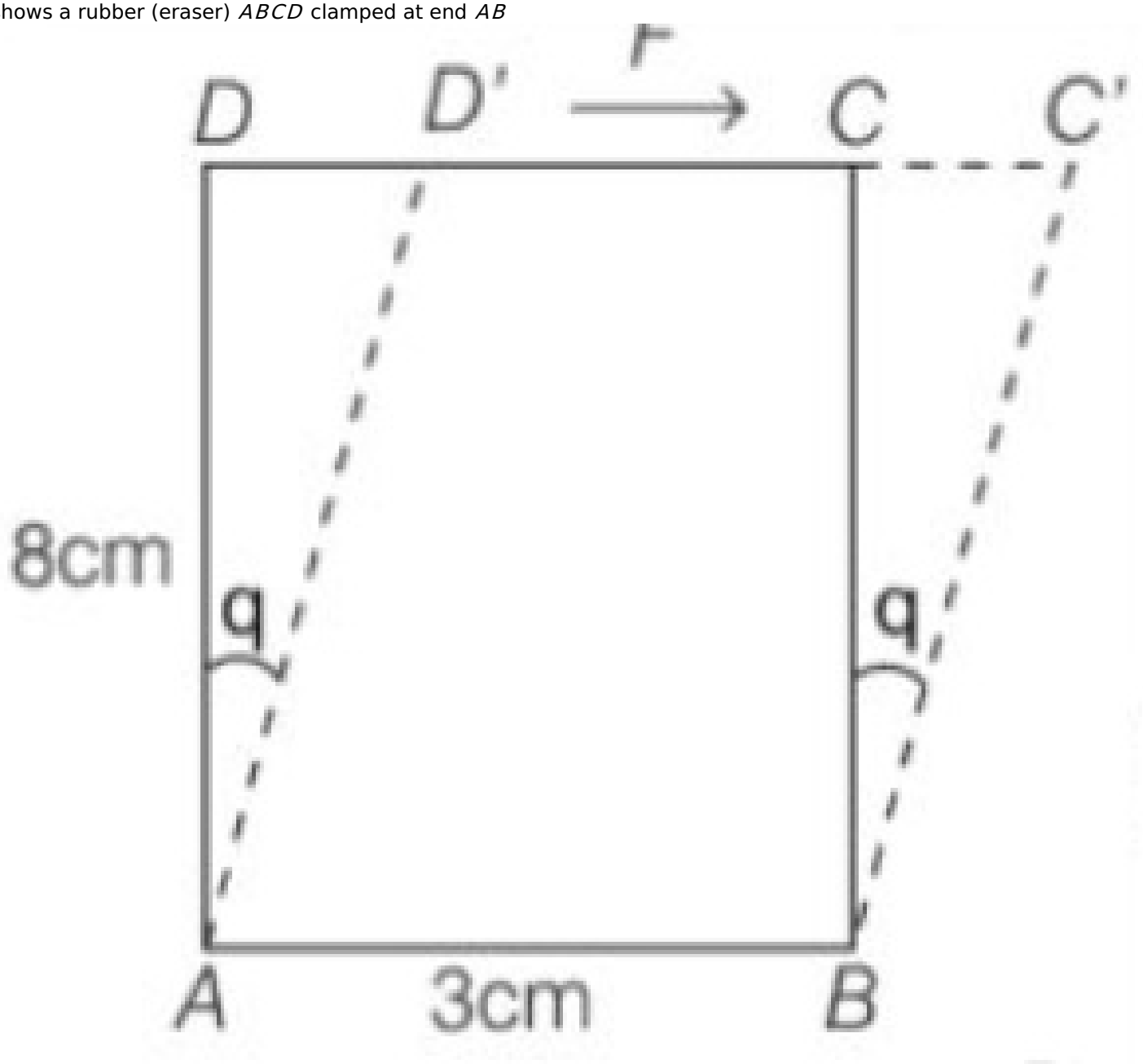
C. $3mm$

D. $4mm$

Answer: D

Solution:

Solution:
Figure shows a rubber (eraser) $ABCD$ clamped at end AB



Shear angle is given by
$\tan\theta = \frac{DD'}{AD}$
Since, θ is small, $\tan\theta \approx \theta = \frac{DD'}{AD}$
Area of top face $= 3cm \times 1cm = 3 \times 10^{-4} m^2$
therefore Shearing stress $= \frac{F}{A} = \frac{2.4}{3 \times 10^{-4}}$
$= 8 \times 10^3 Nm^{-2}$
$\therefore$ Shearing angle, $\theta = \frac{\text{Shearing stress}}{\text{Shearing modulus}}$
$= \frac{8 \times 10^3}{1.6 \times 10^5} = 0.05 rad$
or $\frac{DD'}{AD} = 0.05$
Or $DD' = 0.05 \times AD$
$= 0.05 \times 8cm = 4mm$

Question 23

Water rises up in a glass capillary upto a height of $9cm$, while mercury falls down by $3.4cm$ in the same capillary. Assume the angles of contact for water-glass and mercury-glass are 0° and 135°, respectively. The ratio of surface tensions of mercury and water is (Take,

$\cos 135° = -0.71$)

Options:

A. 8.1:1

B. 3.6:1

C. 7.2:1

D. 8.9:1

Answer: C

Solution:

Solution:
We know that,
$T = \frac{rh\rho g}{2\cos\theta}$
For water, $T_w = \frac{rh_w\rho_w g}{2\cos\theta_w}$
For mercury, $T_m = \frac{rh_n\rho_m g}{2\cos\theta_m}$
$\therefore \frac{T_m}{T_w} = \left(\frac{h_m}{h_w}\right)\left(\frac{\rho_m}{\rho_w}\right)\left(\frac{\cos\theta_w}{\cos\theta_m}\right)$
$= \left(\frac{-3.4\times10^{-2}}{9\times10^{-2}}\right)\left(\frac{13.6\times10^3}{10^3}\right)\ \left(\frac{\cos 0°}{\cos 135°}\right)$
or $\frac{T_m}{T_w} = \frac{7.2}{1}$ or $T_m : T_w = 7.2:1$

Question 24

There are only 5 molecules per cubic cm on average in a region of space, where the temperature is $3K$. What will be the pressure of such fluid gas? (Take, Boltzmann constant, $k = 1.38\times10^{-23}J/K$)

Options:

A. $7.0\times10^{-12}N/m^2$

B. $1.5\times10^{-14}N/m^2$

C. $1.0\times10^{-15}N/m^2$

D. $2.070\times10^{-16}N/m^2$

Answer: D

Solution:

Solution:
Ideal gas equation for n number of molecules,
$pV = nkT$ .....(i)
Given, $\frac{n}{V} = 5\times10^6 mol/m^3$
$T = 3K$
Hence, from Eq. (i), we get

$p = (\frac{n}{V})kT = 5 \times 10^6 \times 1.38 \times 10^{-23} \times 3$
$p = 2.070 \times 10^{-16} N/m^2$

Question 25

An insulated box containing a diatomic gas of molar mass M is moving with a velocity y The box is suddenly stopped. The resulting change in temperature is (where, R is the gas constant)

Options:

A. $\frac{Mv^2}{2R}$

B. $\frac{Mv^2}{3R}$

C. $\frac{Mv^2}{5R}$

D. $\frac{2Mv^2}{5R}$

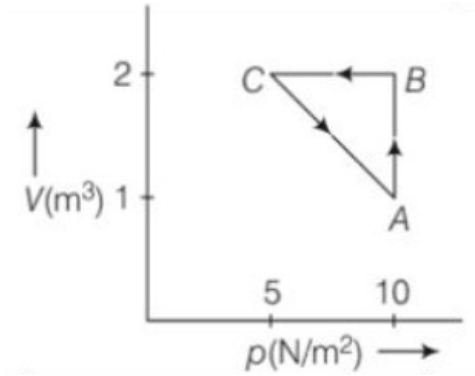
Answer: C

Solution:

Solution:
For diatomic gas, $f = 5$
Change in total internal energy of an diatomic ideal gas,
$\Delta U = \frac{1}{2} fR\Delta T = \frac{5}{2} R\Delta T$ .....(i)
Change in kinetic energy of box containing gas of molar mass M ,
$\Delta E = \frac{1}{2} Mv^2$
According to question,
$\Delta E = \Delta U$
$\therefore \frac{1}{2} Mv^2 = \frac{5}{2} R\Delta T$
Hence, resultant change in temperature, $\Delta T = \frac{Mv^2}{5R}$

Question 26

An ideal gas is taken through the cycle $A \rightarrow B \rightarrow C \rightarrow A$, as shown in the figure below. If the net heat supplied to the gas is 5J, then the work done by the gas in the process $C \rightarrow A$ is



Options:

A. $-5J$

B. $-10J$

C. $-15J$

D. $-20J$

Answer: A

Solution:

Solution:
In this cyclic process, $\Delta U = 0$
$\therefore \Delta Q = \Delta W$
$\Delta W = W_{A \to B} + W_{B \to C} + W_{C \to A}$
$W_{B \to C} = 0$ (volume remains constant)
$W_{A \to B} = p(V_2 - V_1)$ [pressure remains constant]
$= 10(2 - 1) = 10J$
Therefore, $5 = 10 + W_{C \to A}$
$W_{C \to A} = -5J$

Question 27

The volume of a gas is reduced adiabatically to $\frac{1}{4}$ of its volume at $27°C$. If γ = 1.4, then thenew temperature will be

Options:

A. $300 \times (2)^{0.4} K$

B. $300 \times (4)^{1.4} K$

C. $300 \times (4)^{0.4} K$

D. $300 \times (2)^{1.4} K$

Answer: C

Solution:

Solution:
$\therefore$ Relation between temperature and volume for adiabatic process,

$TV^{\gamma-1}$ = constant
According to question,
$T_1 V_1^{\gamma-1} = k$
$\Rightarrow 300 V_1^{(1.4-1)} = k$ .....(i)
$T_2 V_2^{\gamma-1} = k$
$\Rightarrow T_2\left(\frac{V_1}{4}\right)^{(1.4-1)} = k$ .....(ii)
From Eqs. (i) and (ii), we get
$\left(\frac{300}{T_2}\right)(4)^{(1.4-1)} = 1$
$\Rightarrow T_2 = 300 \times (4)^{0.4} K$

Question 28

A piece of metal and a piece of wood are kept at a temperature of $45°C$. On touching the two with hand,

Options:

A. the two will appear equally hot

B. the piece of wood will appear hotter than the piece of metal

C. the piece of metal will appear hotter than the piece of wood

D. the distinction in hotness of two pieces will not be possible

Answer: C

Solution:

Solution:
$\therefore$ Metal is a good conductor of heat and wood is a insulator of heat.
Cause of this, the piece of metal will appear hotter than the piece of wood.

Question 29

If the temperature of a hot body is increased by 50%, then the amount of radiation emitted by it increases approximately by

Options:

A. 400%

B. 225%

C. 250%

D. 500%

Answer: A

Solution:

Solution:
According to Stefan's law,
$E = \sigma T^4$
If temperature of hot body is increased by $50\,\%$, then percentage increase in amount of radiation
$= \frac{E_2 - E_1}{E_1} \times 100$
$= \frac{\sigma(\frac{150}{100}T_1)^4 - \sigma(T_1)^4}{\sigma T_1^4} \times 100$
$= 406.25\,\% \approx 400\,\%$

Question 30

A body of mass $1kg$ is executing simple harmonic motion given by $y = 6.0\cos(100t + \frac{\pi}{4})\,cm$, where t is in second. What is the maximum kinetic energy?

Options:

A. $18J$

B. $9J$

C. $36J$

D. None of these

Answer: A

Solution:

Solution:
Given, $y = 6.0\cos(100t + \frac{\pi}{4})\,cm$
Here, $a = 6 \times 10^{-2}m, \omega = 100rad/s$
$\therefore$ Maximum kinetic energy in simple harmonic motion,
$K_{max} = \frac{1}{2}m\omega^2a^2$
$= \frac{1}{2} \times 1 \times (100)^2 \times (6 \times 10^{-2})^2 \; = 18J$

Question 31

A simple pendulum is oscillating in a lift. If the lift starts moving upwards with uniform acceleration, the period

Options:

A. will remain unaffected

B. will be shorter

C. will be longer

D. may be shorter or longer depending on the magnitude of acceleration

Answer: B

Solution:

Solution:
The period of oscillation of simple pendulum,
$T_1 = 2\pi\sqrt{\frac{l}{g}}$ .....(i)
When the lift starts accelerating upwards with uniform acceleration, the period,
$T_2 = 2\pi\sqrt{\frac{l}{g+a}}$ .....(ii)
From Eqs. (i) and (ii), we get
$T_1 > T_2$
Hence, the period will be shorter.

Question 32

A car, sounding a horn of frequency 1000Hz, is moving directly towards a huge wall at a speed of 15m/s. If the speed of sound is 340m/s, then the frequency of echo heard by the driver is

Options:

A. 1046Hz

B. 954Hz

C. 1092Hz

D. 908Hz

Answer: C

Solution:

Solution:
Given, in this condition, $v_0 = v_s = 15m/s$
Virtual frequency of sound source moving towards a huge wall,
$n' = (\frac{v}{v - v_s})n$ .....(i)
The frequency of echo heard by driver,
$n'' = (\frac{v + v_0}{v})n'$
From Eqs. (i) and (ii), we get
$n'' = (\frac{v + v_0}{v - v_s})n$
$= (\frac{340 + 15}{340 - 15}) \times 1000$
$\approx 1092Hz$

Question 33

Light follows wave nature because

Options:

A. light travels in a straight line

B. light exhibits the phenomena of reflection and refraction

C. light exhibits the phenomenon of interference

D. light causes the phenomenon of photoelectric effect

Answer: C

Solution:

Solution:
Light follows wave nature because light exhibits the phenomenon of interference.

Question 34

In a Young's double slit experiment, the spacing between the two slits is $0.1mm$. If the screen is kept at a distance of $1.0m$ from the slit and the wavelength of light is $5000Å$, then the fringe width will be

Options:

A. $1.0cm$

B. $1.5cm$

C. $0.5cm$

D. $2.0cm$

Answer: C

Solution:

Solution:
The fringe width, $W = \frac{D\lambda}{d}$
Here, $D = 1.0m, d = 0.1mm = 0.1 \times 10^{-3}m$,
$\lambda = 5000Å = 5000 \times 10^{-10}m$
$\therefore W = \frac{1 \times 5000 \times 10^{-10}}{0.1 \times 10^{-3}}$
$= 5 \times 10^{-3}m = 0.5cm$

Question 35

A real, inverted and equal in size image is formed by

Options:

A. a concave mirror

B. a convex mirror

C. a plane mirror

D. None of these

Answer: A

Solution:

Solution:
A concave mirror formed a real, inverted and equal in size image.

Question 36

If the refracting angle of a prism is 60° and minimum deviation is 30°, then the angle of incidence will be

Options:

A. 30°

B. 45°

C. 60°

D. 90°

Answer: B

Solution:

Solution:
Given, $}A = 60°, \delta_m = 30°$
$\therefore$ Incident angle of prism, $i = \frac{\delta_m + A}{2}$
$\Rightarrow i = \frac{30° + 60°}{2} = 45°$

Question 37

The spectrum formed by an ordinary tubelight is a /an

Options:

A. line spectrum

B. continuous spectrum

C. absorption spectrum

D. band spectrum

Answer: A

Solution:

Solution:
Line spectrum is formed by an ordinary tubelight.

Question 38

The photoelectric effect proves that

Options:

A. light travels in the form of quanta

B. light travels in the form of transverse waves

C. velocity of light is infinite

D. None of the above

Answer: A

Solution:

Solution:
The photoelectric effect proves that light travels in the form of quanta.

Question 39

The magnifying power of a simple microscope is (when final image is formed at $D = 25cm$ from eye)

Options:

A. $\frac{D}{f}$

B. $1 + \frac{D}{f}$

C. $1 + \frac{f}{D}$

D. $1 - \frac{D}{f}$

Answer: B

Solution:

Solution:
The magnifying power of a simple microscope (when final image is formed at $D = 25cm$ from eye) is
$M = 1 + \frac{D}{f}$

Question 40

A monochromatic light of frequency v is incident on emitter having threshold frequency v_0. The kinetic energy of ejected electron will be

Options:

A. hv

B. $h(v - v_0)$

C. hv_0

D. $h(v + v_0)$

Answer: B

Solution:

Solution:
According to Einstein's photoelectric equation,
$K_{max} = hv - W_0$
where, K_{max} is maximum kinetic energy of ejected electron, v is the frequency of incident photon, h is Planck's constant and W_0 is the work function.
As, $W_0 = hv_0$
Hence, $K_{max} = hv - hv_0 = h(v - v_0)$

Question 41

Which of the following is conserved in Bohr model of an atom?

Options:

A. Angular momentum

B. Angular velocity

C. Torque

D. All of these

Answer: D

Solution:

Solution:
In Bohr model of an atom, angular momentum, angular velocity and torque is conserved.

Question 42

Which of the following wavelengths falls in X-ray region?

Options:

A. 10000Å

B. 1000Å

C. 1Å

D. 10^{-2}Å

Answer: C

Solution:

Solution:
The wavelength range of X-rays is from $1 \times 10^{-13} m$ to $3 \times 10^{-8} m$
Hence, option (c) is correct.

Question 43

The most suitable element for nuclear fission is the element with atomic number near

Options:

A. 11

B. 21

C. 52

D. 92

Answer: D

Solution:

Solution:
The most suitable element for nuclear fission is uranium and its atomic number is 92 .

Question 44

The bonding in germanium crystal (semiconductor) is

Options:

A. metallic

B. ionic

C. van der Waals' type

D. covalent

Answer: D

Solution:

Solution:
∵ Germanium is the semiconductor material, therefore the bonding in germanium crystal is covalent.

Question 45

A junction diode has a resistance of 25Ω when forward biased and 2500Ω when reversed biased. The current in the diode for the arrangement as shown in the figure below will be

Options:

A. $\frac{1}{15}A$

B. $\frac{1}{7}A$

C. $\frac{1}{25}A$

D. $\frac{1}{480}A$

Answer: B

Solution:

Solution:
Junction diode is in forward biased.

$\therefore I = (\frac{5-0}{25+10}) = \frac{5}{35}$ or $I = \frac{1}{7}A$

Question 46

If a magnet of pole strength m is divided into four parts such that the length and width of each small part is half that of initial one, then the pole strength of each part will be

Options:

A. $\frac{m}{4}$

B. $\frac{m}{2}$

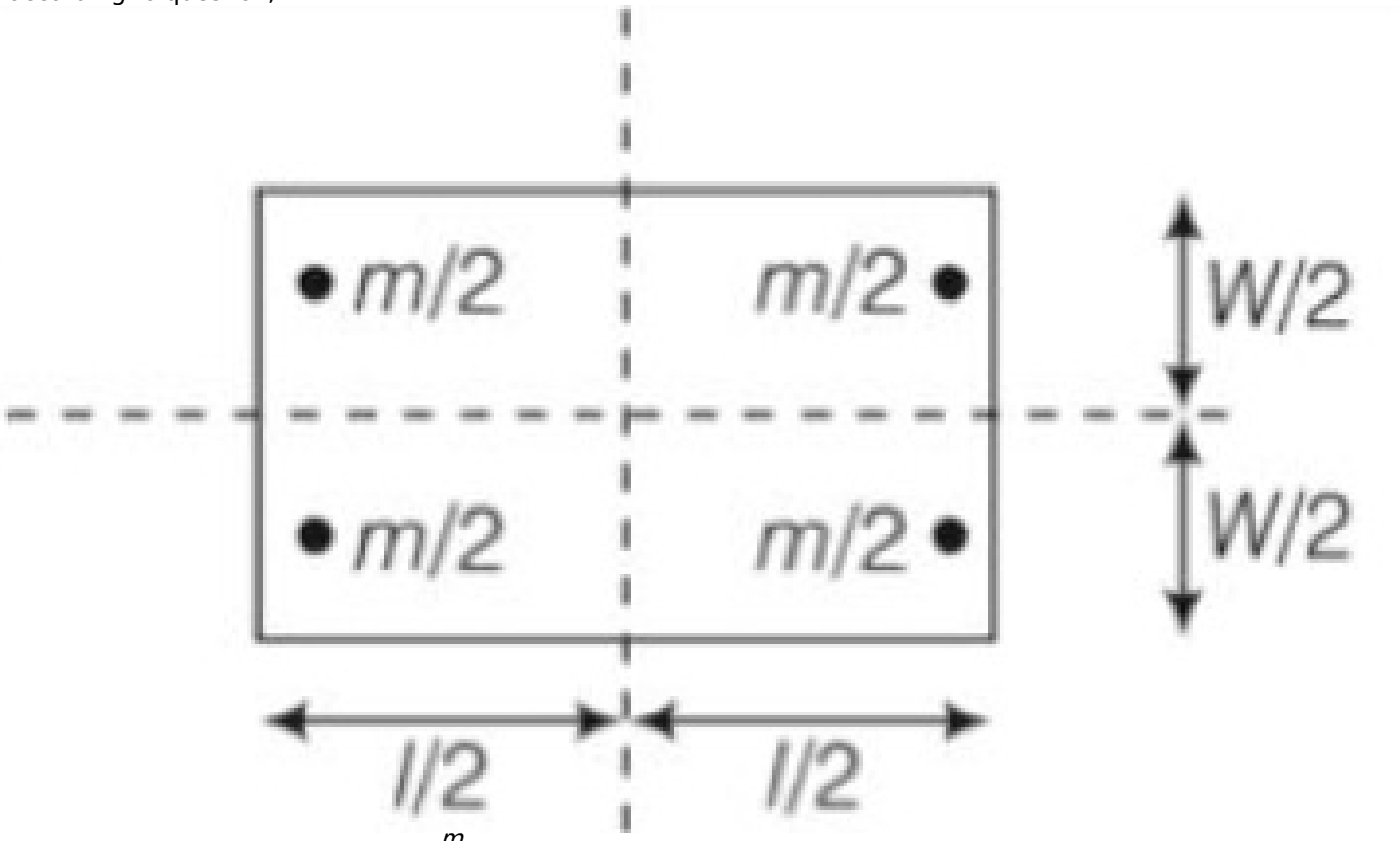
C. $\frac{m}{8}$

D. $4m$

Answer: B

Solution:

Solution:
When a magnet of pole strength m is divided into two parts along its length, then pole strength of each magnet will becomes half.
Now, according to question,



Hence, pole strength of each part will be $\frac{m}{2}$.

Question 47

The North pole of Earth's magnet is near the geographical

Options:

A. South

B. East

C. West

D. North

Answer: A

Solution:

Solution:
The North pole of Earth's magnet is near the geographical South and the South pole of Earth's magnet is near the geographical North.

Question 48

A pendulum bob of mass $30.7 \times 10^{-6} kg$ and carrying a charge $2 \times 10^{-8} C$ is at rest in a horizontal uniform electric field of $20000 V/m$. The tension in the thread of this pendulum is (Take, $g = 9.8 m/s^2$)

Options:

A. $3 \times 10^{-4} N$

B. $4 \times 10^{-4} N$

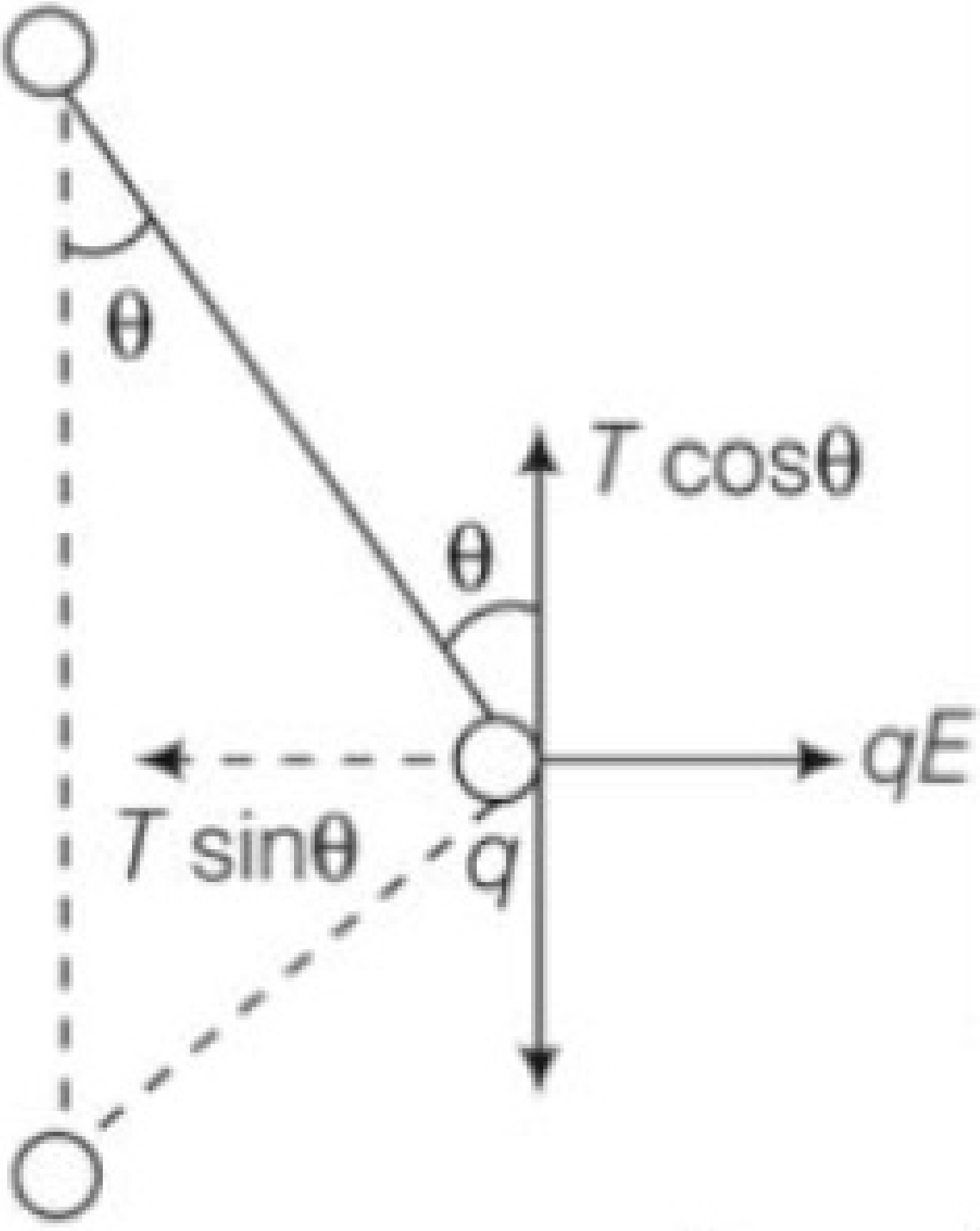
C. $5 \times 10^{-4} N$

D. $6 \times 10^{-4} N$

Answer: C

Solution:

Solution:



Given, mass, $m = 30.7 \times 10^{-6} kg$
Charge, $q = 2 \times 10^{-8} C$
Electric field, $E = 2 \times 10^{4} V / m$
Let T is the tension in the string and θ is angle of string with vertical.
In equilibrium,
$T \sin\theta = qE$ .....(i)
$T \cos\theta = mg$ .....(ii)
From Eqs. (i) and (ii), we get

$$\tan\theta = \frac{qE}{mg} = \frac{2 \times 10^{-8} \times 2 \times 10^{4}}{30.7 \times 10^{-6} \times 9.8} = 1.329$$

$\Rightarrow \theta = \tan^{-1} 1.329$
$\therefore \theta = 53.05°$

Hence, tension, $T = \frac{qE}{\sin\theta} = \frac{2 \times 10^{-8} \times 2 \times 10^{4}}{\sin 53.05°}$
$= 5 \times 10^{-4} N$

Question 49

If the electric flux entering and leaving an enclosed surface respectively are ϕ_1 and ϕ_2, then the electric charge inside the surface will be

Options:

A. $(\phi_1 + \phi_2)\varepsilon_0$

B. $(\phi_1 - \phi_2)\varepsilon_0$

C. $(\phi_1 + \phi_2) / \varepsilon_0$

D. None of these

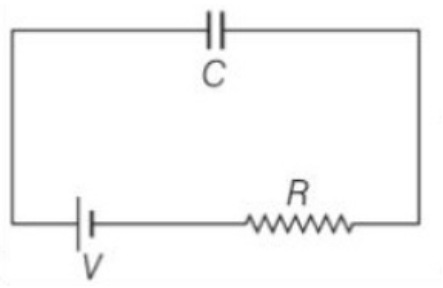
Answer: D

Solution:

Solution:
The total flux linked with a closed surface called Gaussian surface, is $\frac{1}{\varepsilon_0}$ times the charge enclosed by the closed surface, i.e.
$\phi_{net} = \frac{1}{\varepsilon_0}(Q_{enc})$
$(-\phi_1 + \phi_2) = \frac{1}{\varepsilon_0}(Q_{enc})$
$(\phi_2 - \phi_1) = \frac{1}{\varepsilon_0}(Q_{enc})$
$Q_{enc} = (\phi_2 - \phi_1)\varepsilon_0$
So, no option is correct.

Question 50

As shown in the figure below, if a capacitor C is charged by connecting it with resistance R , then energy given by the battery will be



Options:

A. $\frac{1}{2}CV^2$

B. more than $\frac{1}{2}CV^2$

C. less than $\frac{1}{2}CV^2$

D. zero

Answer: B

Solution:

Solution:
$\because$ A capacitor and a resistor are connected in series combination, so energy given by battery will be more than the $\frac{1}{2}CV^2$.

Question 51

Which of the following statements are correct?
1. 0.90g butane diol gives 224mL H_2 at STP when reacts with Na.
2. Optically active 2 -iodobutane reacts with NaI to form racemic mixture.
3. 2 -methyl butane is optically active.
4. C_4H_8 shows 7 -isomer.

Options:

A. 2,3

B. 2,4

C. 1,2

D. 1,3

Answer: C

Solution:

Solution:
Formula of butane diol is $C_4H_{10}O_2$.
$\therefore$ It's molecular mass $= (4 \times 12) + (10 \times 1) + (2 \times 16) = 90$
Given, mass of butane diol $(w) = 0.9g$
$\therefore n = \frac{w}{m} = \frac{0.9}{90} = 0.01mol$
where, $n =$ number of moles
At STP, $n = \frac{\text{Volume of gas}(V)}{22.4(L)}$
$\therefore V = n \times 22.4L$
or $V = n \times 22400(mL)$
or $V = 0.01 \times 22400$ or $V = 224mL$
$\therefore (1)$ is the correct statement.
2. When 2 -iodobutane reacts with NaI, it gives alkyl di-iodide in which iodine atoms are attached with the 2° -carbon. Thus, due to steric-hindrance it will undergo S_N1 reaction. Hence, we get racemic mixture.
$\therefore$ Statement (2) is correct.
3. Structure of 2 -methyl butane is as follows:

$$\overset{1}{C}H_3 - \underset{\underset{CH_3}{|}}{\overset{2}{C}}H - \overset{3}{C}H_2 - \overset{4}{C}H_3$$

$\because$ It has two similar groups, present on C_2 carbon atom. It is not optically active.
$\therefore$ Statement (3) is wrong.

4. C_4H_8 does not show 7 -isomers, as it has only four carbon atoms.
∴ Statement (4) is wrong.
∴ Option (c) is the correct option.

Question 52

The correct order of octane number is

Options:

A. cycloalkanes < alkenes < alkanes < aromatic hydrocarbons

B. alkenes < alkanes < aromatic hydrocarbons < cycloalkanes

C. alkanes < aromatic hydrocarbons < cycloalkanes < alkenes

D. alkanes < alkenes < cycloalkanes < aromatic hydrocarbons

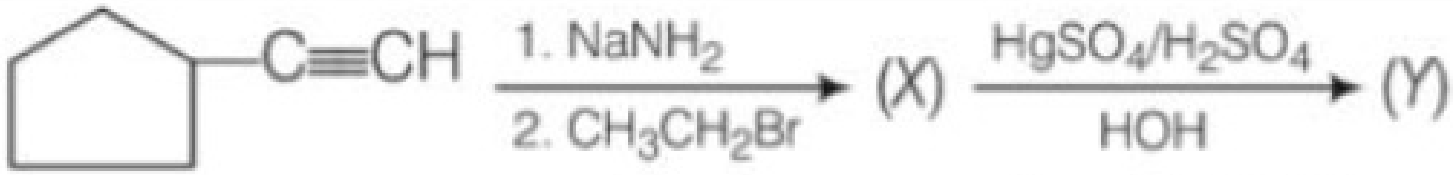
Answer: D

Solution:

Solution:
Octane number indicates the anti-knock properties of fuels. Higher the octane number, more difficult the auto-ignition. Octane number of straight chain alkane are lower than that of branch chain alkanes, while aromatic hydrocarbons show higher value of octane number. The correct order of octane number is as follows:
Alkane < alkenes < cycloalkanes < aromatic hydrocarbons
Thus, (*d*) is the correct option.

Question 53

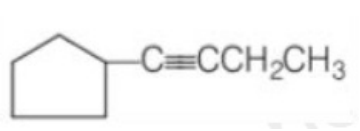
Consider the following reaction:



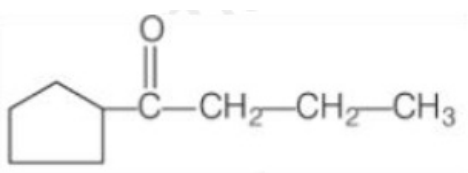
The product (*Y*) is

Options:

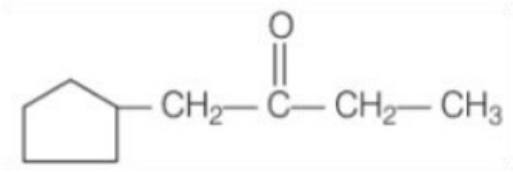
A.



B.



C.

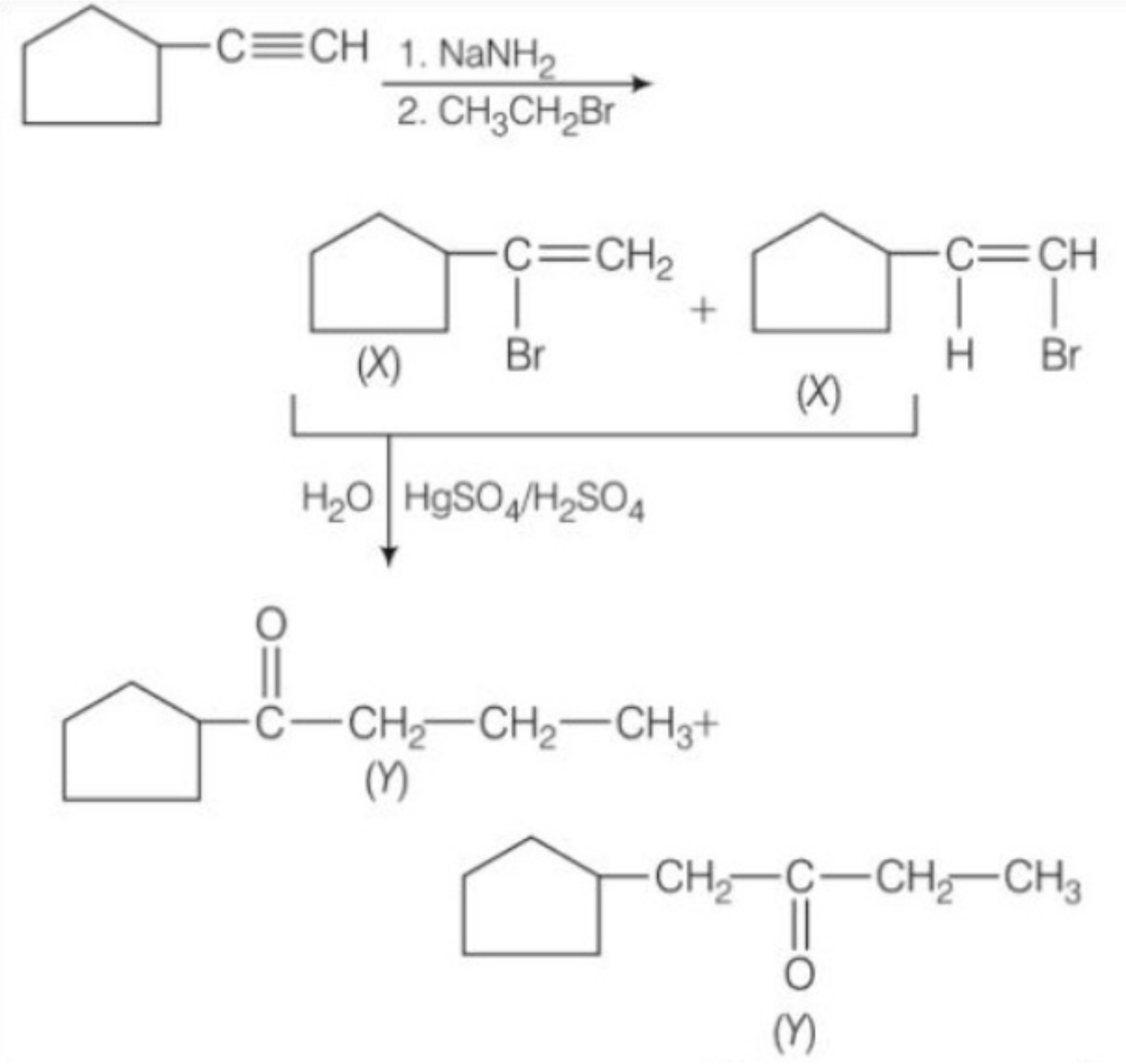


D. Both (b) and (c)

Answer: D

Solution:

Solution:



∴ Option (*d*) is the correct option.

Question 54

Consider the following statements:
1. Toluene may be oxidised to benzaldehyde by the use of CrO_2Cl_2
2. Nitration of benzene is electrophilic addition reaction.
3. The function of anhydrous $AlCl_3$ in Friedel-Craft reaction is to absorb water.
4. Nitration of benzene is carried out by a mixture of conc. HNO_3 and conc. H_2SO_4.
The wrong statements are

Options:

A. 1 and 2

B. 2 and 3

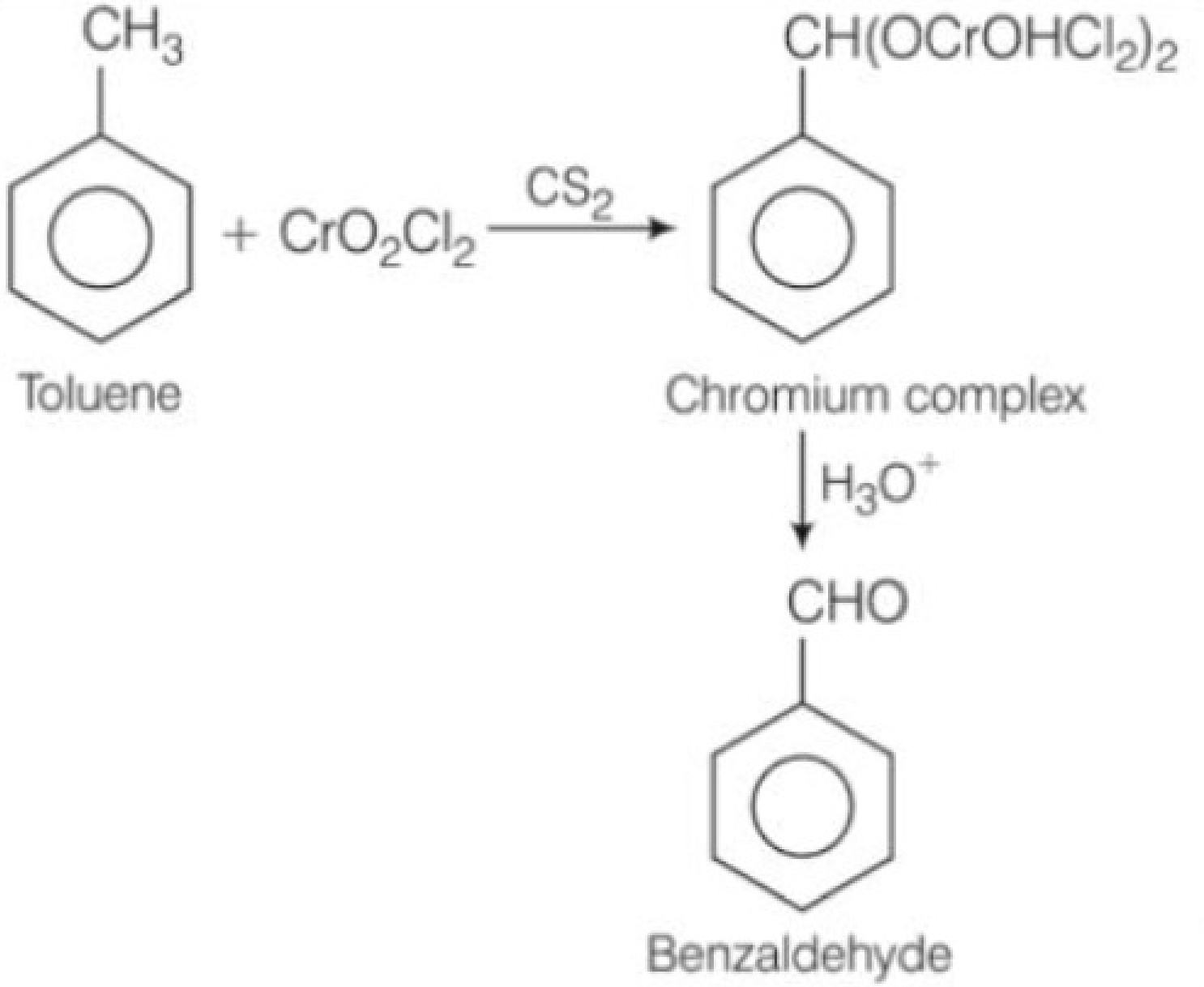
C. 1 and 3

D. 3 and 4

Answer: B

Solution:

Solution:
1. The given reaction is an Etard reaction, in which toluene reacts with chromyl chloride (CrO_2Cl_2), to give benzaldehyde.



Hence, statement (1) is correct answer.
2. Nitration of benzene is an example of electrophilic substitution, in which $-NO_2$ group is replaced by hydrogen atoms, attached to the ring. Hence, statement (2) is wrong.
3. $AlCl_3$ is a strong Lewis acid, in Friedel-Craft reaction, it will keep the halide part away from the aromatic ring by making a complex $[AlCl_3 - X]$
Thus, the given statement is wrong.
4. Nitration of benzene is carried out by a mixture of HNO_3 (conc.) and H_2SO_4 (conc.).
(i) H_2SO_4 (conc.) $+ HNO_3$ (conc.) $\longrightarrow [H_2NO_3]^+ . [HSO_4]^-$
(ii) $[H_2NO_3]^+ \longrightarrow NO_2^+ + H_2O$
In this reaction, HNO_3 (conc.) behaves as a base.
Thus, statement (iv) is correct.
Hence, statements (2) and (3) are wrong.
Therefore, option (b) is the correct answer.

Question 55

The correct order of basicity of amines in water is

Options:

A. $(CH_3)_3N > (CH_3)_2NH > CH_3NH_2$

B. $CH_3NH_2 > (CH_3)_2NH > (CH_3)_3N$

C. $(CH_3)_2NH > (CH_3)_3N > CH_3NH_2$

D. $(CH_3)_3N > CH_3NH_2 > (CH_3)_2NH$

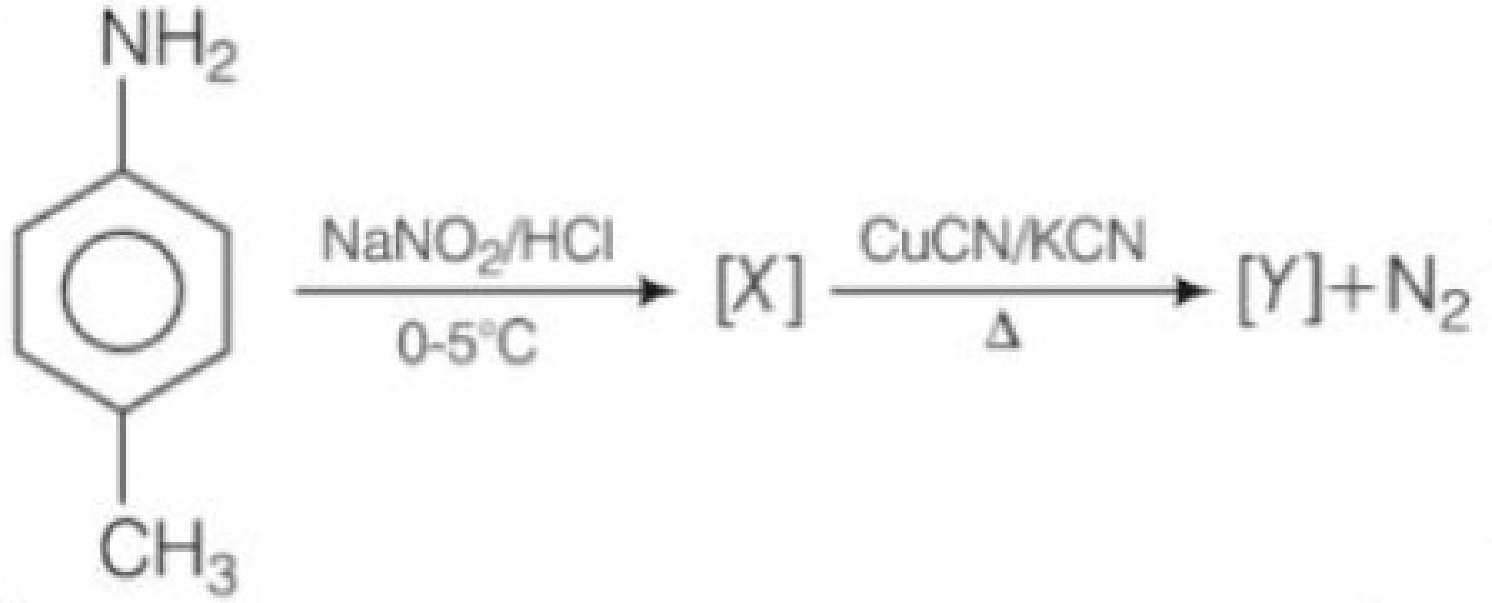
Answer: C

Solution:

Solution:
More be the ability of nitrogen atom to donate the lone pair of electrons over it, more be the basic nature of the given amine.
The basic strength of alkyl amines in the aqueous state is decided by three factors-Inductive effect, solvation effect and steric hinderance of alkyl group.
In water, amines form hydrogen bonds with Water.
As 3° amines are less protonated in aqueous solution, the correct order of basic nature of amines in water is
2° amine > 3° amine > 1° amine.
Hence, order of basic nature of amines:
$(CH_3)_2NH > (CH_3)_3N > CH_3 - NH_2$
Therefore (c) is the correct option.

Question 56

In the reaction

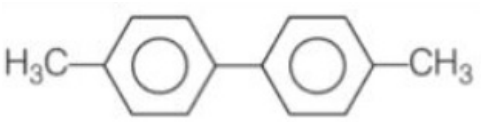


the product [Y] is

Options:

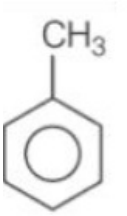
A.

B.



C.

D.



Answer: C

Solution:

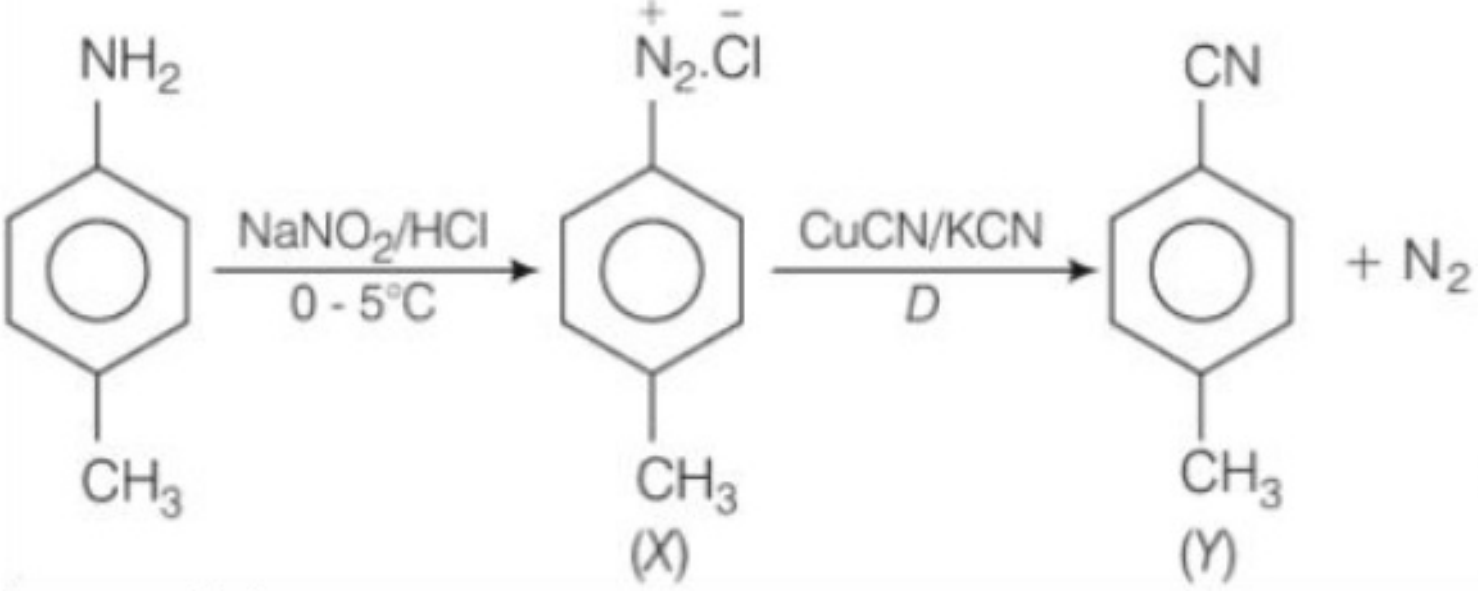
Solution:

(i)  gives benzene diazonium salt with

$NaNO_2 / HCl$ at low temperature (i.e. $0 - 5°C$)

(ii) The diazonium salt thus formed, when react

with CuCN/KCN, it will give  as product.



Hence, (c) is the correct option.

Question 57

In the reaction

$CH_3NH_2 + CHCl_3 + KOH \longrightarrow [X] + KCl + H_2O$

the compound (X) is

Options:

A. $CH_3\overset{+}{N} \equiv \overset{-}{C}$

B. $CH_3 - NH - CH_3$

C. $CH_3 - C \equiv N$

D. $CH_3 - \overset{+}{N} \equiv \overset{-}{C}$

Answer: A

Solution:

Solution:
When a primary amine like methylamine (CH_3NH_2) react with chloroform ($CHCl_3$) in the presence of KOH, it will form iso-cyanide having same number of carbon atoms, as in alkyl group of amine. i.e. methyl amine. This reaction is known as carbylamine reaction.

$$\underset{\text{Methylamine}}{CH_3NH_2} + CHCl_3 + 3KOH \longrightarrow \underset{\text{Methyl isocyanide}}{CH_3 - NC} + 3KCl + 3H_2O$$

Hence, (a) is the correct option.

Question 58

The strongest acid amongst the following compounds is

Options:

A. CH_3COOH

B. $HCOOH$

C. $CH_3CH_2CH(Cl)COOH$

D. $ClCH_2CH_2CH_2COOH$

Answer: C

Solution:

Solution:
Presence of electron withdrawing groups (like $Cl, -NO_2$ etc.) increases the acidic strength of carboxylic acids by stabilising the conjugate base through delocalisation of the negative charge by inductive or resonance effect while, presence of electron releasing group (like alkyl) decreases the acidic nature. Hence, (c) is the correct option.

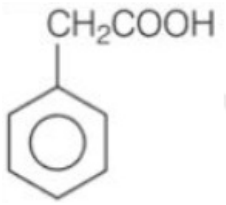
Question 59

Which one of the following carboxylic acids decarboxylate easily?

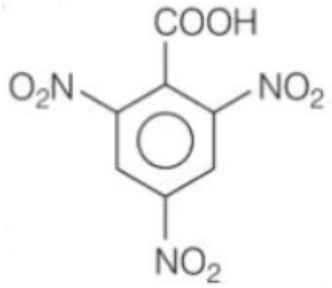
Options:

A.

B.



C.



D. $n-C_4H_9COOH$

Answer: C

Solution:

Solution:
More stronger be the acid, more chances to give up the proton (H^+) by an acid and becomes carboxylate ion. The process of releasing the proton and to become a carboxylate ion is known as decarboxylate process.
If electron withdrawing groups (such as $-NO_2$, $-Cl$ etc) are attached to the benzene ring at ortho and para positions, they will increase the acidic strength of the acid means, it will release the protons easily. So, $2,4,6$ -trinitrobenzoic acid will decarboxylate easily.
Thus, (c) is the correct option.

Question 60

Phenacetin is an example of

Options:

A. antibiotic

B. antimalarial

C. antipyretic

D. antiseptic

Answer: C

Solution:

Solution:
Phenacetin is also known as acetophenetidin.
It is used to lower down the temperature of the human body, thus is an antipyretic by nature.
Hence, (c) is the correct option.

Question 61

Glycerol is added to soap. What is its function?

Options:

A. As a filler

B. To increase lathering

C. To prevent rapid drying

D. Acts as an antiseptic

Answer: C

Solution:

Solution:
Glycerol will keep the moisture intact when added to soap, thus is used to prevent the rapid drying of skin.
Hence, (c) is the correct option.

Question 62

Which of the following is not a condensation polymer?

Options:

A. Nylon-6, 6

B. Nylon-6

C. Dacron

D. Buna-S

Answer: D

Solution:

Solution:
(a) Nylon-6, 6 is the polymer of hexamethylene diamine and adipic acid. It is made by the condensation of given monomers.
(b) Nylon-6 is also a condensation polymer of caprolactum. It is a polyamide.
(c) Dacron is also a condensation polymer of ethylene glycol and terephthalic acid. It is an example of polyester.

(d) Buna-S is an addition polymer of 1,3 , butadiene and styrene.
Hence, only (d) is not a condensation polymer.
$\therefore (d)$ is the correct option.

Question 63

Which of the following rubbers is not a polydiene?

Options:

A. Polyisoprene

B. Polychloroprene

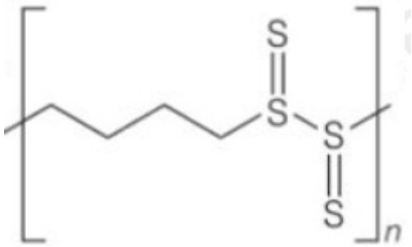
C. Thiokol rubber

D. Nitrile rubber

Answer: C

Solution:

Solution:
Thiokol rubber is not a polydiene. It is in fact a polysulphide having structure as follows:



Hence, (c) is the correct option.

Question 64

The synthesis of each molecule of glucose in photosynthesis involves

Options:

A. 6 molecules of ATP

B. 8 molecules of ATP

C. 10 molecules of ATP

D. 18 molecules of ATP

Answer: D

Solution:

Solution:
Photosynthesis is the process carried out by the plants to convert light energy into chemical energy, which later on

released to other organisms from the plant. Synthesis of each molecule of glucose in photosynthesis involves 18 ATP units. The reaction occurs during photosynthesis is as follows
$6CO_2 + 18ATP + 12NADPH \longrightarrow C_6H_{12}O_6 + 18P + 12NADP^+ + 18ADP.6H_2O$

Question 65

DNA and RNA are chiral molecules due to the presence of

Options:

A. chiral base

B. phosphate-ester unit

C. D-sugar component

D. L-sugar component

Answer: C

Solution:

Solution:
DNA and RNA have deoxyribose and ribose sugar, respectively. Both are chiral molecules due to presence of deoxyribose or D-ribose sugar components.
Hence, (c) is the correct option.

Question 66

A particle of mass nearly equal to proton is moving with a velocity nearly equal to the velocity of light. The wavelength of wave associated with it is

Options:

A. inversely proportional to its velocity

B. inversely proportional to its energy

C. (a) is wrong while (b) is correct

D. Both (a) and (b) are correct

Answer: A

Solution:

Solution:
According to the relation,
$\lambda = \frac{h}{mv}$
where, $\lambda =$ wavelength,
$h =$ Planck's constant,
$m =$ mass of the moving particle and

$v =$ velocity of the moving particle.
Given, mass of the particle (m) = mass of proton m_p) and velocity of particle (v) = velocity of light (c).
Thus, according to the relation
$\lambda = \frac{h}{mv} = \frac{h}{m_p . c}$
Hence, wavelength (λ) is inversely proportional to the velocity.
$\therefore$ Hence, (a) is the correct answer.

Question 67

An atom has an electron in fifth orbit, in s -orbital, with anticlockwise spin. Its magnetic quantum number is

Options:

A. 5

B. 0

C. Both (a) and (b) are wrong

D. Both (a) and (b) are correct

Answer: B

Solution:

Solution:
As the electron is present in 5 th orbit in s -subshell and value of l for s -subshell = 0
Values of magnetic quantum number (m_s) for s -subshell can be calculated as follows:
$m_s = -1 . . . . . 0 + ... + l$
$l = 1, m_s = -1, 0, +1 . . . . . .$ so on.
Hence, value of magnetic quantum number of the electron in s -subshell is also zero.
Hence, (b) is the correct option.

Question 68

If $0.05g$ of urea is dissolved in $5g$ of water, then

Options:

A. its molarity will be greater than molality

B. its molality will be greater than molarity

C. molarity and molality will be same

D. its normality will be $50 / 60$

Answer: B

Solution:

Solution:

Molarity $= \frac{\text{number of moles of solute}}{\text{volume of solution (in L)}}$
and molality $= \frac{\text{number of moles of solute}}{\text{mass of solvent (in kg)}}$
$\because$ Mass of solute (urea) and solvent (water) remain unchanged, during the calculations of molarity and molality. Thus, volume of solution contain less water molecules.
($\because$ In case of volume of solution, it has both solute and solvent).
Thus, molality will be greater than that of molarity.

Question 69

100mL brandy contains 40mL ethanol. The mole fraction of water is

Options:

A. 0.6

B. 0.667

C. 0.26

D. 0.425

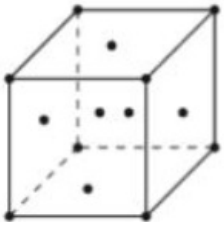
Answer: A

Solution:

Solution:
$\therefore$ Volume of water $= 100 - 40 = 60mL$ andvolume fraction $\propto$ mole fraction
Volume fraction of water $= \frac{\text{volume of water}}{\text{total volume}}$
$= \frac{60}{100} = 0.6$
$\therefore$ (a) is the correct option.

Question 70

The lattice point perunit cell in the figure



is

Options:

A. 14

B. $\frac{14}{6}$

C. 4

D. $\frac{8}{2}+\frac{6}{3}$

Answer: C

Solution:

Solution:
Lattice points per unit cell for the given figure are as follows:
(i) We have eight corners and each corner is shared by eight other unit cells, the lattice point at corner has share towards one unit cell is $8 \times \frac{1}{8}=1$
(ii) There are six faces in a cubic unit cell and we have one sphere at the center of each face. Each face is common between two unit-cells.
Thus, contribution of each sphere per unit cell is $6 \times \frac{1}{2}=3$
Hence, there are $1+3=4$ lattice points per unit cell in the given figure.
$\therefore (c)$ is the correct option.

Question 71

The radius of sodium atom is $1.857 \times 10^{-8} cm$. If sodium crystal is a body centred, then the edge length of unit cell is

Options:

A. 3.81Å

B. 4.29Å

C. 2.37Å

D. None of these

Answer: B

Solution:

Solution:
Edge length (a) and radius of sphere (r) in a body centered cubic unit cell are related with each other in the following manner.
Given, $\sqrt{3}a=4r$
$r=1.857 \times 10^{-8} cm$
Thus, $a=\frac{4 \times 1.857 \times 10^{-8}}{\sqrt{3}}$
or $a=\frac{4 \times 1.857 \times 10^{-8}}{1.73}$
$a=4.29$Å
Hence, (b) is the correct option.

Question 72

Select the correct option

Options:

A. $1Bq = 3.7C$

B. $1C = 3.7$ Bq

C. $1C = 37rd$

D. $1rd = MBq$

Answer: D

Solution:

Solution:
Becquerel is the SI unit of radioactivity. One becquerel is defined as one kilo-becquerel (i.e. 10^3Bq), while mega-becquerel is 10^6Bq and is written as MBq. It is equivalent to 1 -rutherford (rd).
$1Bq = 2.7 \times 10^{-11}C$
and $1C = 3.7 \times 10Bq = 37GBq$
Hence, (d) is the correct option.

Question 73

In nuclear transmutation $_8O^{16}(A, B)_7N^{13}$ A and B are respectively

Options:

A. p, α

B. α, p

C. p, n

D. d, p

Answer: A

Solution:

Solution:
On bombardment of $_1^1H$ proton, an α -particle will comes out as follows:
$_8O^{16} + {}_1H^1 \rightarrow {}_7N^{13} + {}_2He^4$
Sum of mass number
Reactant side $= 16 + 1 = 17$
Product side $= 13 + 4 = 17$
Sum of atomic number
Reactant side $= 8 + 1 = 9$
Product side $= 7 + 2 = 9$
As bombarding particle, $_1H^1$ is can be written as proton (p) and ejecting particle can be written as (α). The whole reaction can be summarised as follows:
$_8O^{16}(p, \alpha), N^{13}$
Hence, (a) is the correct option.

Question 74

A metal halide has molar concentration of 0.000011mol/L in saturated state. If its $K_{sp} = 39.5 \times 10^{-20}$, then the halide is

Options:

A. M_2X_4

B. MX_4

C. M_2X_6

D. MX_3

Answer: D

Solution:

Solution:
Key concept First calculate K_{sp} of all the metal halides, then compare it with already given value of K_{sp}
Given,
molar concentration, i.e. solubility (S)
$= 0.000011 mol/L$
$K_{sp} = 39.5 \times 10^{-20}$
(a) $M_2X_4 \rightleftharpoons 2M^+ + 4X^-$
$K_{sp} = (2S)^2 \times (4S)^4 = 1024(S)^6$
$= 18.14 \times 10^{-28}$
(b) $MX_4 \rightleftharpoons M^+ + 4X^-$
$K_{sp} = (S) \times (4S)^4 = 256(S)^5$
$= 42.68 \times 10^{-24}$
$(c) M_2X_6 \rightleftharpoons 2M^+ + 6X^-$
$K_{sp} = (2S)^2 \times (6S)^6$
$= 186624(S)^8$
$= 40 \times 10^{-36}$
$(d) MX_3 \rightleftharpoons M^+ + 3X^-$
$K_{sp} = (S) \times (3S)^3 = 27(S)^4$
$= 39.5 \times 10^{-20}$
$\therefore$ The correct formula of the metal halide is MX_3.

Question 75

In a 100mL of buffer solution of acetic acid and sodium acetate, both are 0.1mol. In this buffer, 10mL of 0.01$mole$ of HCl is added. The pH of the buffer

Options:

A. will be increased by 1

B. will be decreased by 1

C. first increases and then decreases

D. no change

Answer: D

Solution:

Solution:
The solutions which resist the change in pH even on small addition of an acid or base are called buffer solutions.
As the given solution of acetic acid and sodium acetate is a buffer solution, the addition of $10mL$ of $0.01mol/HCl$ solution will not change the pH of the buffer solution.
Hence, option (d) is the correct answer.

Question 76

Consider the following reaction :
$CaO(s) + CO_2(g) \rightarrow CaCO_3(s)$
The reaction is spontaneous at low temperature because

Options:

A. ΔH is negative,ΔS is positive ,ΔG is negative

B. ΔH is negative ,ΔS is negative, ΔG is negative

C. ΔH is positive, ΔS is negative , ΔG is positive

D. ΔH is positive, ΔS is positive , ΔG is negative

Answer: B

Solution:

Solution:
For the given reaction.
$CaO(s) + CO_2(g) \rightarrow CaCO_3(s)$
At low temperature,
the value of $\Delta n = 0 - 1 = (-)1$
$\because \Delta n =$ moles of gaseous products $-$ moles of gaseous reactants
For the reaction contain gaseous species.
$\Delta H = \Delta E + \Delta nRT$
where, $\Delta H =$ enthalpy change
$\Delta E =$ internal energy change
$\Delta n =$ change in number of moles
$R =$ gas constant
$T =$ temperature in Kelvin.
$\because \Delta n = (-)1$
$\Delta H = \Delta E - RT$
means $\Delta H < \Delta E$ (negative) and of exothermic nature.
For the reaction to occur spontaneously,
$\Delta G = \Delta H - T\Delta S$, value of ΔG must be negative. At low temperature for ΔG and ΔH to be negative.
$\because \Delta n = (-)1$
So, ΔS must be negative, as number of gaseous moles in product side are less than of reactant side.
Hence, the correct set is
$\Delta H =$ negative
$\Delta S =$ negative
$\Delta G =$ negative
$\therefore (b)$ is the correct option.

Question 77

The heat of neutralisation of hydrogen cyanide (HCN) is $-2.9kcal/mol^{-1}$. This shows that heat of dissociation of HCN is

Options:

A. $+13.7 kcal/mol^{-1}$

B. $+10.8 kcal/mol^{-1}$

C. -13.7 kcal mol^{-1}

D. -13.7 kcal mol^{-1}

Answer: B

Solution:

Solution:
When a strong acid will react with a strong base, nearly 13.7 kcal mol $^{-1}$ heat comes out from the reaction.
But when a weak acid (like HCN) go for neutralisation, some heat is used up to dissociate the HCN molecules.
$\because$ Given, only $2.9 kcal//mol$ heat is coming out, means.
$13.7 - 2.9 = +10.8 kcal//mol$ heat is used upto dissociates the HCN molecules.
$\therefore (b)$ is the correct option.

Question 78

$n =$ order of reaction

Options:

A. $n = 2, t_{1/2} = \frac{1}{a}$

B. $n = 1, t_{1/2} = \frac{1}{x \times a}$

C. $n = 1, t_{1/2} = \frac{0.693}{k}$

D. None of these

Answer: C

Solution:

Solution:
For the given plot, log $t_{1/2}$ is constant for allconcentration terms (a), means the reaction is of first order i.e. $n = 1$
where, $n =$ order of reaction
For first order,
$t_{1/2} = \frac{2.303}{k} . \log \frac{a}{a - x}$
$t_{1/2} = \frac{2.303}{k} \log \frac{a}{a/2}$
where, $k =$ rate constant
$a =$ initial concentration
$x =$ concentration at $t = t_{1/2}$
$t_{1/2} =$ Half-life of the reaction
$t_{1/2} = \frac{2.303}{k} \log 2$
or, $t_{1/2} = \frac{2.303 \times 0.3010}{k}$
$t_{1/2} = \frac{0.693}{k}$
Hence, (c) is the correct answer.

Question 79

Half-life period for a first order reaction is 10 minutes. How much time is required to change the concentration of the reactants $0.08M$ to $0.01M$?

Options:

A. 20 min

B. $30min$

C. $40min$

D. 50 min

Answer: B

Solution:

Solution:
Given,
Half-life of the reaction $(t_{1/2}) = 10min$
Initital concentration $(a) = 0.08M$
Concentration at time 't' i.e. $(a - x) = 0.01M$
As per relation of first order

$$\frac{t}{t_{1/2}} = \frac{\log[a/a-x]_1}{\log[a/a-x]_2} = \frac{\log\frac{0.08}{0.01}}{\log 2}$$

$$\frac{t}{10} = \frac{\log 8}{\log 2} = \frac{3\log 2}{\log 2} = \frac{3 \times 0.3010}{0.3010}$$

$t = 3 \times 10 = 30\,\text{min}$.
Hence, (b) is the correct option.

Question 80

The weight of Cu deposited, when 2 Faraday of electricity is passed will be

Options:

A. $31.75g$

B. $23.85g$

C. $63.5g$

D. $125.67g$

Answer: C

Solution:

Solution:

Copper (Cu) show $(+)2$ charge in its stable ionic-state and has molecular mass $= 63.5$
Also, 1 Faraday $(F) =$ charge of one mole of electron.
Therefore, on passing $2F$ electricity we get one mole of copper (Cu) means $63.5g$ of copper.
Hence, option (c) is the correct answer.

Question 81

Calculate the standard free energy change for the reaction $2Ag + 2H^+ \longrightarrow H_2 + 2Ag^+$ $E°$ for $Ag^+ + e^- \longrightarrow Ag$ is $0.80V$.

Options:

A. $+308.8kJ$

B. $+154.4kJ$

C. $-308.8kJ$

D. $-154.5kJ$

Answer: B

Solution:

Solution:
For the reaction,
$2Ag + 2H^+ \longrightarrow H_2 + 2Ag^+$
Standard free energy change $(\Delta G°) = -nFE°$
where, $n =$ number of electrons change.
$F =$ charge over one mole of electrons i.e. $96500C$.
$E° =$ standard electrode potential
$\because$ In the reaction, $Ag \longrightarrow Ag^+$ i.e. oxidation of silver takes place and $n = 2$
$\therefore \Delta G° = -2 \times 96500 \times [(-)0.8]$
$= 154400J/mol = 154.4kJ/mol$
$\therefore$ Hence, (b) is the correct option.

Question 82

Which of the following processes is responsible for the digestion of fats in the intestines?

Options:

A. Electroosmosis

B. Demulsification

C. Electrophoresis

D. Emulsification

Answer: D

Solution:

Solution:
Digestion is the process to breakdown of large insoluble molecules into water soluble molecules.
As fat, is insoluble in water, so by emulsification, it should be made soluble in water.
Hence, (d) is the correct option.

Question 83

The nitride ion in lithium nitride is composed of

Options:

A. 7 protons +7 electrons

B. 10 protons +7 electrons

C. 7 protons +10 electrons

D. 10 protons +10 electrons

Answer: B

Solution:

Solution:
The formula of lithium nitride is Li_3N, where N contains $(-)3$ charge.
Thus, formula of nitride-ion $= N^{3-}$
$\because$ For a neutral atom,
atomic number = number of protons
and number of protons = number of electrons
Also, atomic number of nitrogen $(N) = 7$
$\therefore$ It has 7 protons and due to $(-)3$ charge,
it has $7 + 3 = 10$ electrons.
$\therefore$ Hence, (c) is the correct answer.

Question 84

Fusion mixture is

Options:

A. $K_2CO_3 + Na_2CO_3$

B. $KHSO_4 + NaHSO_4$

C. $Na_2CO_3 + KNO_3$

D. $KHSO_4 + NaSO_4$

Answer: A

Solution:

Solution:
The mixture of sodium carbonate (Na_2CO_3) and potassium carbonate (K_2CO_3) is known as fusion mixture. It is used for

elemental analysis of inorganic compounds.
$\therefore$ Hence, (a) is the correct answer.

Question 85

Corundum is

Options:

A. SrO_2

B. Al_2O_3

C. $CaCl_2$

D. Cu_2Cl_2

Answer: B

Solution:

Solution:
Term 'corundum' is used for aluminium oxide (Al_2O_3)
$\therefore$ Hence, (b) is the correct option.

Question 86

Mercury is the only metal which is liquid at $0°C$. This is due to

Options:

A. very high ionisation energy and weak metallic bond

B. low ionisation potential

C. high atomic weight

D. high vapour pressure

Answer: A

Solution:

Solution:
Mercury is liquid at $0°C$ due to very high ionisation energy and weak metallic bonds. The high ionisation energy makes atoms of mercury stable in liquid state and weak metallic bonds allows atoms to remain loose i.e. to exist in liquid state.
$\therefore$ Hence, (a) is the correct option.

Question 87

Which of the following indicates the correct variation in eletronegativities?

Options:

A. $F > N < O > C$

B. $F > N > O > C$

C. $F < N < O < C$

D. $F > N > O < C$

Answer: A

Solution:

Solution:
All given elements belong to same period and electronegativity increases as we move from left to right in a period,
Thus, order of electronegativity will be
$F > O > N > C$ or $F > N < O > C$
$\therefore$ Hence, (a) is the correct option.

Question 88

In the isoelectronic species, the ionic radii of N^{3-}, O^{2-} and F^{-} are respectively given by

Options:

A. $1.36, 1.40, 1.71$

B. $1.36, 1.71, 1.40$

C. $1.71, 1.40, 1.36$

D. $1.71, 1.36, 1.10$

Answer: C

Solution:

Solution:
For isoelectronic species, ionic radii $\propto$ charge on the ion (species).
Thus, N^{3-} shows maximum ionic radius and F^{-} shows minimum ionic radius. Hence, correct order is $1.71, 1.40, 1.36$
$\therefore$ Hence, (c) is the correct option.

Question 89

Pink colour of non-stoichiometric LiCl is due to

Options:

A. Cl^- ion in the lattice

B. Li^+ ion in the lattice

C. electrons in the lattice

D. Both the ions in the lattice

Answer: C

Solution:

Solution:
Unpaired electrons causes F-centers in the crystal lattice, which are responsible for the colour of the lattice.
$\therefore$ Hence, (c) is the correct option.

Question 90

Match List-I with List-II and select the correct answer using the codes given below:

List I	List II
(A) Hydrolith	1. KCNS
(B) Rhodamine	2. $Al_2(SO_4)_3.18H_2O$
(C) Hair salt	3. $BaSO_4.ZnS$
(D) Lithopone	4. CaH_2

Options:

A. A-1, B-3, C-4, D-2

B. A-4, B-1, C-2, D-3

C. A-4, B-2, C-1, D-3

D. A-1, B-4, C-2, D-3

Answer: B

Solution:

Solution:
(A) Hydrolith → It is CaH_2.
(B) Rhodamine → It belongs to the compounds having - CN group thus, it is KCNS.
(C) Hair-salt → It is hydrated aluminium sulphate i.e. $Al_2(SO_4)_3.18H_2O$.
(D) Lithopone → It is $BaSO_4.ZnS$
∴ Hence, (b) is the correct option.

Question 91

Excess of acidic solution of KI to mixed with $25mL\ H_2O_2$. Liberated I_2 requires $20mL$ of$0.3N\ Na_2S_2O_3$ solution. The volume strength of H_2O_2 is

Options:

A. 1.344 volume

B. 2.688 volume

C. 1.5 volume

D. 2.5 volume

Answer: A

Solution:

Solution:
Meq. of $H_2O_2 \equiv$ Meq. of $Na_2S_2O_3$
∴ If $'N'$ be the normality of H_2O_2. Thus,
$N \times 25 = 0.3 \times 20$
$\Rightarrow N = 0.24N$
(volume strength) $N \times 5.6 = 0.24 \times 5.6 = 1.33$ volume.
∴ Hence, (a) is correct option.

Question 92

Match List-I with List-II and select the correct answer using the codes given below:

List I	List II
(A) Electronic configuration of americium.	1. Corrosive Sublimate
(B) BM of $Sm^{3+}, La^{3+}, Gd^{3+}$	2. $[Rn]5f^7 7s^2$
(C) $TiCl_4 \xrightarrow{H_2O}$ Amphoteric white solid	3. $5.9, 0.0, 7.94$
(D) $Hg_2Cl_2 \xrightarrow{\text{aqua} - \text{regia}}$ Compound used in antiseptic and dressing skin	4. TiO_2

Options:

A. A-1, B-2, C-3, D-4

B. A-1, B-4, C-2, D-3

C. A-2, B-4, C-1, D-3

D. A-2, B-3, C-4, D-1

Answer: D

Solution:

Solution:
A. Electronic configuration of americium $\rightarrow$ $[Rn]5f^7.7s^2$
B. Bm of Sm^{3+}, La^{3+} and $Gd^{3+} \rightarrow 5.9, 0.0, 7.94$ as these have 5,0 and 7 unpaired electrons in their outermost shell. (Element belongs to f-block).
C. $TiCl_4 \xrightarrow{H_2O\, TiO_2}$ i.e. on hydrolysis $TiCl_4$ give TiO_2
D. $Hg_2Cl_2 \xrightarrow{\text{aqua} - \text{regia}}$ Corrosive sublimate
$\therefore$ Hence, (d) is the correct option.

Question 93

Which of the following statements are wrong?
1. $Zn(OH)_2ZnCrO_4 . H_2O$ has yellow colour.
2. $K_2Cr_2O_7 + H_2O_2$ gives K_2CrO_4.
3. Mn_3O_4, Mn_2O_3 and MnO_2 are amphoteric in nature.
4. Nichrome contains 25% Ni.

Options:

A. 1, 2, 4

B. 1, 3, 4

C. 1,3

D. 1,4

Answer: B

Solution:

Solution:
1. Zinc and its ion does not contain any unpaired electron, thus, does not show any colour in its compounds.
$\therefore$ It is a wrong statement.
2. $K_2Cr_2O_7 + H_2O_2$ gives K_2CrO_4 as follows:
$2K_2Cr_2O_7 + H_2O_2 \rightarrow 2K_2CrO_4 + H_2O + Cr_2O_3 + 4[O]$
Hence, statement is correct.
3. The higher oxidation states of Mn are acidic in nature and lower oxidation states are basic in nature e.g. Mn_2O_3.
Thus, statement is incorrect.
4. Composition of nichrome is: Iron $\rightarrow 21-25\,\%$
Chromium $\rightarrow 14-18\,\%$
Nickel (Ni) $\rightarrow$ about $55\,\%$ - $56\,\%$
rest $1\,\%$ are other elements.
$\therefore$ Statement is wrong
$\therefore$ Hence, (b) is the correct answer.

Question 94

Match List-I with List-II and select the correct answer using the codes given below:

List-I	List-II
(A) $HCl \xrightarrow{CuCl_2} Cl_2$	1. Reppe synthesis
(B) Ethene $\xrightarrow{PdCl_2} CH_3CHO$	2. Fenton reagent
(C) Alcohol $\xrightarrow[H_2O_2]{FeSO_4}$ Aldehyde	3. Deacon process
(D) Alkyne $\xrightarrow[\text{Complex}]{Ni}$ Benzene	4. Wacker process

Options:

A. A-3, B-4, C-2, D-1

B. A-3, B-1, C-2, D-4

C. A-2, B-1, C-3, D-4

D. A-2, B-4, C-1, D-3

Answer: A

Solution:

Solution:
A. HCl $\xrightarrow{CuCl_2} Cl_2$ is known as Deacon process. This process is used during the manufacture of alkali's by Leblanc

process.
B. Ethene $\xrightarrow{PdCl_2} CH_3CHO$,
the above process, is known as Wacker process, which refers to the oxidation of ethene to acetaldehye in the presence of $PbCl_2$.
C. Alcohol $\xrightarrow[H_2O_2]{FeSO_4}$ Aldehyde
Fenton reagent is a solution of H_2O_2 with $FeSO_4$ and is used as catalyst to oxidise alcohols to aldehyde.
D. Alkyne $\xrightarrow[\text{Complex}]{Ni}$ Benzene, it is the reaction used to convert alkyne into benzene. In this reaction, Ni is used as catalyst along with carbon monoxide (CO), which is used for carboxylation. The reaction is known as Reppe synthesis.
$\therefore$ Hence, (a) is the correct option.

Question 95

Dipole moment will be zero in which of the following complexes?
1. $[Ni(CN)_4]^{2-}$
2. cis $-[Pt(NH_3)_2Cl_2]$
3. trans $-[Pt(NH_3)_2Cl_2]$
4. $[Ag(CN)_2]$
Select the correct answer using the codes given below.

Options:

A. 1, 2, 3

B. 2, 3, 4

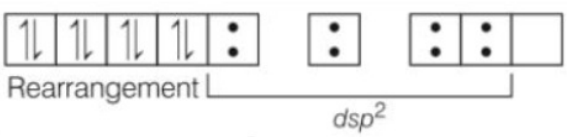
C. 3,4

D. 1, 3, 4

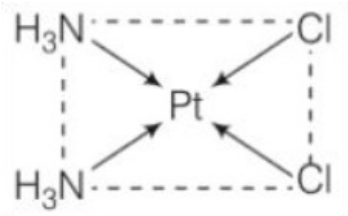
Answer: D

Solution:

Solution:
$[Ni(CN)_4]^{2-}$

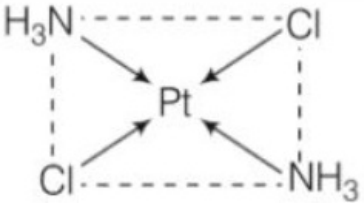
Atom/ion complex	Configuration	Oxidation state of metal	Type of hybridisation	Geometry (Shape)
$Ni^{2+}\left(d^8\right)$		+2	...	...
$\left[Ni(CN)_4\right]^{2-}$		+2	dsp^2	Square planar

$\because$ Has square planar geometry with no unpaired electron.
$\therefore$ Has zero dipole -moment.
2. cis $[Pt(NH_3)_2Cl_2]$
Structure of cis $[Pt(NH_3)_2Cl_2]$



$\therefore$ Both NH_3 and Cl ligands are in the same plane as shown in the structure. Thus, they give cis - $[Pt(NH_3)_2Cl_2]$ and show a net dipole moment.

3. trans $[Pt(NH_3)_2Cl_2]$ due to presence of Cl and NH_3 ligands, the compound is stable but in the case, pair of both the ligands (i.e. NH_3 and Cl) are in the opposite plane and direction, thus net dipole moment becomes zero.



4. As structure of $[Ag(CN)_2]$ is linear, in which two $-$ CN groups are present on the opposite side of the central atom silver (Ag). Hence, show zero dipole moment.

$\therefore$ Hence, (d) is the correct answer.

Question 96

Which of the following complexes shows paramagnetic?

Options:

A. $[Zn(NH_3)_3]^{2+}$

B. $[PdCl_2(PPH_3)_2]$

C. $[NiCl_2(PPH_3)_2]$

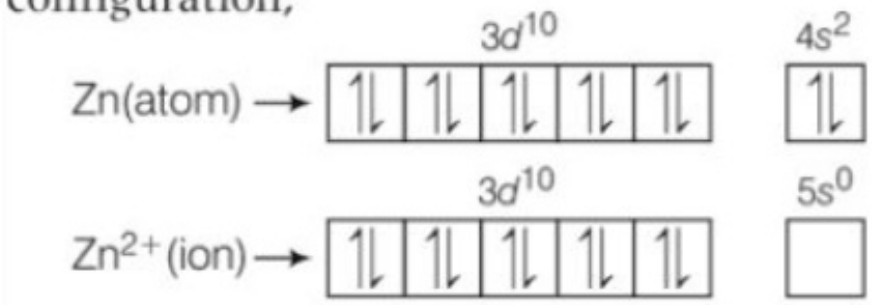
D. $[Co(en)_3]^{3+}$

Answer: C

Solution:

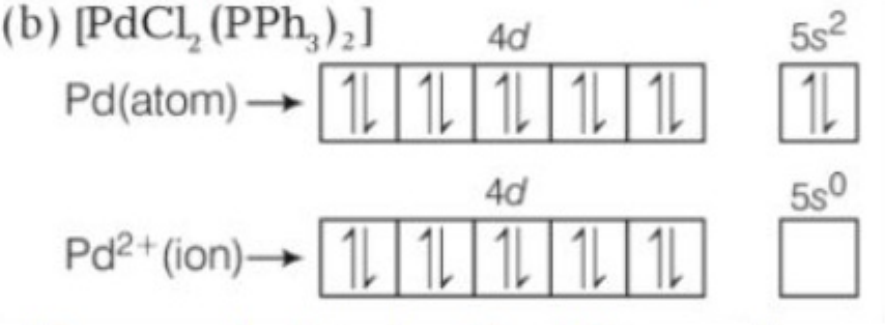
Solution:

(a) $\left[Zn\left(NH_3\right)_2\right]^{2+}$ outermost of d and s configuration,



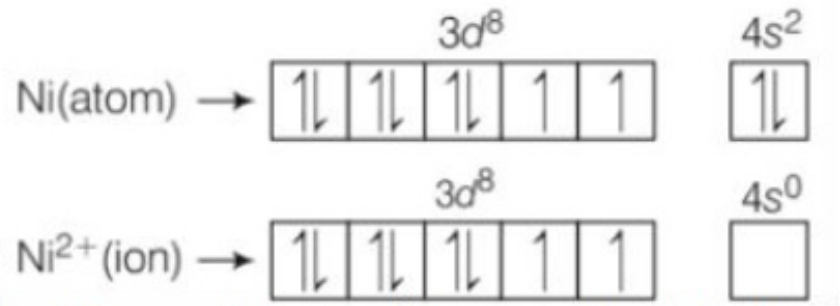
∵ Has no unpaired electron, thus is diamagnetic.

(b) $[PdCl_2(PPh_3)_2]$



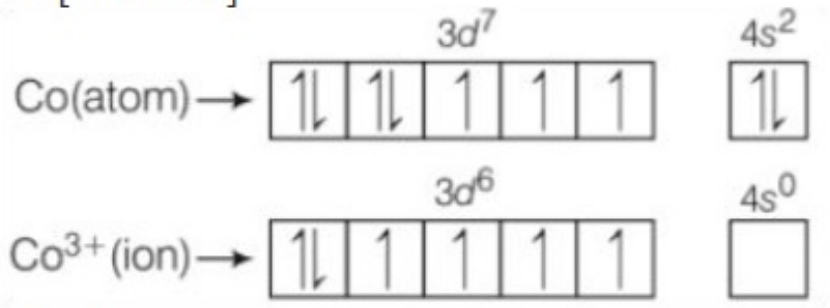
∵ Has no unpaired electron, thus is diamagnetic.

(c) $\left[NiCl_2\left(PPh_3\right)_2\right]$



As it has weak ligands with two unpaired electrons, thus it will show paramagnetic nature.

(d) $\left[Co\left(en\right)_3\right]^{3+}$



∵ (en) is a strong field ligand, all the unpaired electrons (i.e. 4) will go for pairing and we have no unpaired electron in the complex.
Hence, is diamagnetic
∴ (c) is the correct option.

Question 97

Which of the following complex compounds shows optical isomerism?

Options:

A. cis $-[Co(en)_2Cl_2]NO_2$

B. $[Co(H_2O)_6]^2$

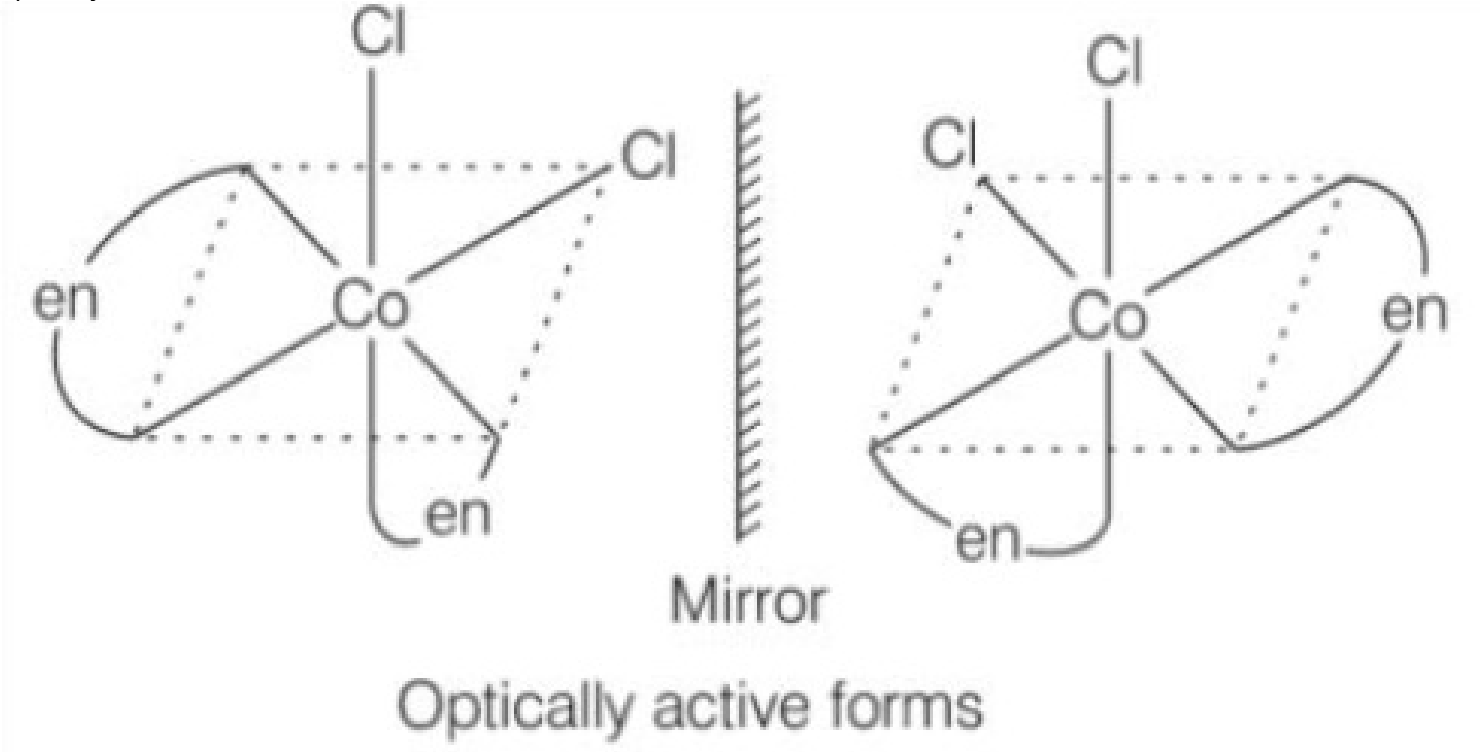
C. $[Co(CN)_6]^{4-}$

D. $K_3[Fe(ox)_3]$

Answer: A

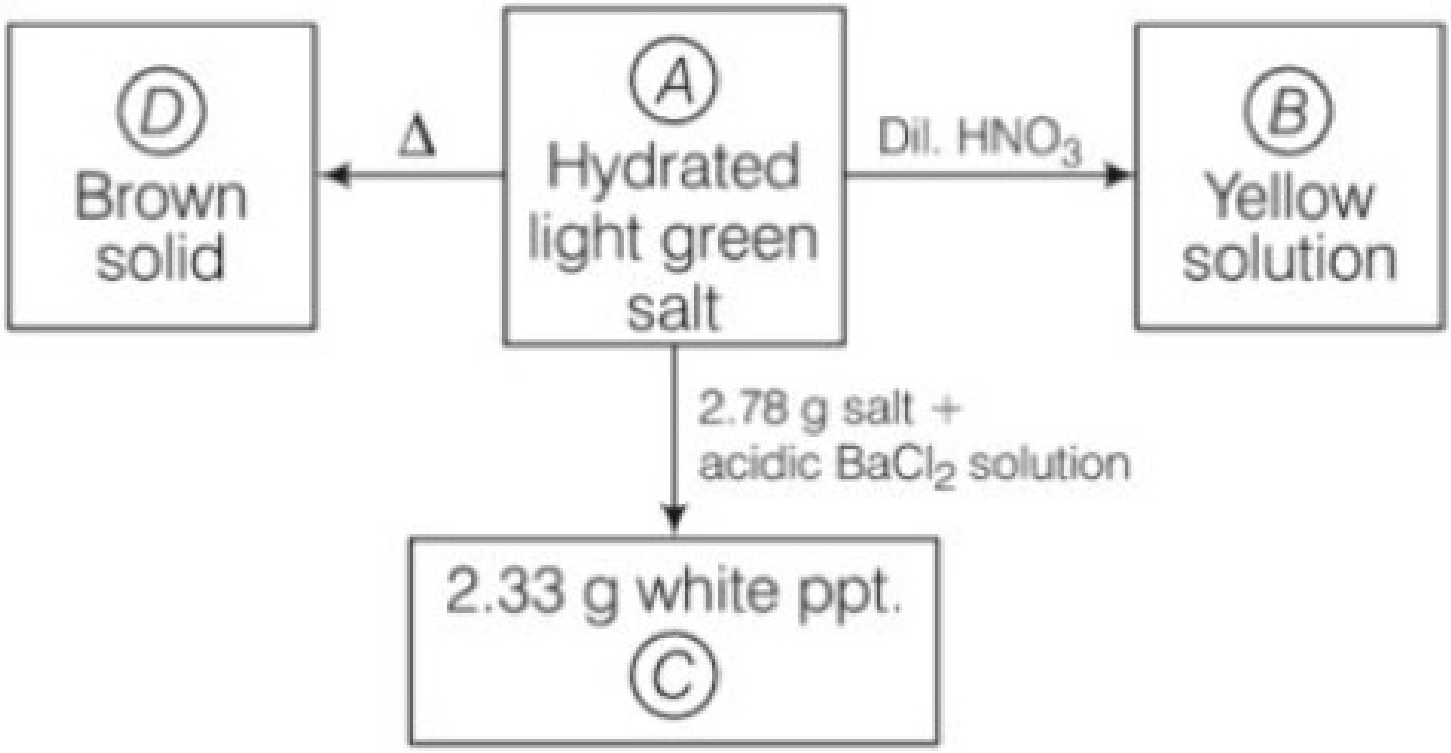
Solution:

Solution:
(cis $-Co[en]_2Cl_2)\ NO_2$. cis-dichloridobis(ethylenediamine) cobalt (III) ion, $[Co(en)_2Cl_2]^+$, the cis-form of the complex is optically active and exists as follows:



(b) and (c).
$\because [M(A)_6]^{n+ \text{or} n-}$ type compounds like $[Co(H_2O)_6]^{2+}$ and $[Co(CN)_6]^{4-}$, do not show optical isomerism.
Due to symmetry, where $m =$ central element $(A) =$ monodentate ligand.
(d) $\because [M(AA)_3]^{n-}$ type of complex, do not show optical isomerism.
Where, (AA) is a bidentate ligand
$\therefore (a)$ is the correct option.

Question 98



What are A, B, C, D ?

Options:

A. $Fe_2(SO_4)_3 FeCl_2, BaSO_4, Fe_3O_4$

B. $Fe_2SO_4 . 7H_2O, Fe(NO_3)_3, BaSO_4, Fe_2O_3$

C. $CoSO_4 4H_2O, CoCl_3, BaSO_4, Co_2O_3$

D. $PbSO_4, PbCl_2, FeCl_2, Pb_2O_3$

Answer: B

Solution:

Solution:
(a) Among the given options:
$\because FeSO_4 . 7H_2O$ is of light green colour.
$\therefore FeSO_4 . 7H_2O$ is (A)
(b) $3FeSO_4.7H_2O + \text{dil.} 4HNO_3 \longrightarrow Fe(NO_3)_3 + Fe_2(SO_4)_3 + NO + 23H_2O$
The yellow solution is due to $Fe(NO_3)_3$.
Note : In options, $FeCl_3$ was given which was an error and thus, has replaced by $Fe(NO_3)_3$.
(c) $FeSO_4 . 7H_2O$ gives white ppt. with $BaCl_2$.
$FeSO_4 . 7KH_2O + BaCl_2 \longrightarrow FeCl_2 + BaSO_4 + 7KH_2O$
$\therefore (c)$ is $(BaSO_4)$
(d) When $FeSO_4 . 7H_2O$ is heated, it looses its water and give brown mass of $Fe_2O_3, SO_2(g)$ and $SO_3(g)$ as follows.
$2FeSO_4 \xrightarrow{\Delta} Fe_2O_3(s) + SO_2(g) + SO_3(g)$
$\therefore (d)$ is (Fe_2O_3)
$\therefore (b)$ is the correct option.

Question 99

How many grams of cyclohexanol is required to produce 20 g cyclohexane, if \% yield is 54% ?

Options:

A. 88

B. 66

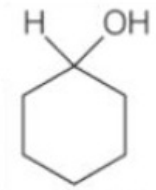
C. 22

D. 44

Answer: D

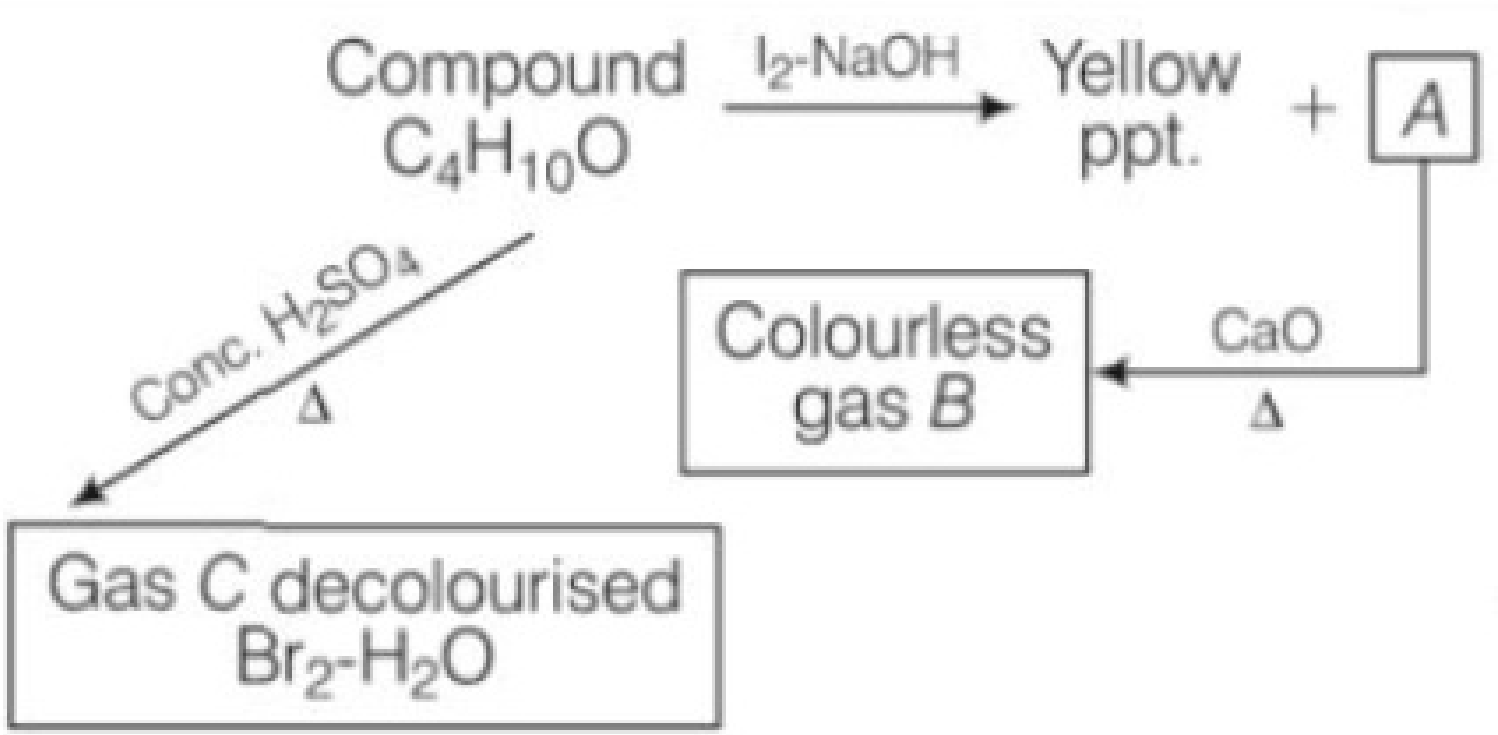
Solution:

Solution:
Formula of cyclohexanol is C_6H_{11}. OH i.e.



Molecular mass of $C_6H_{11}.OH = 100$
Molecular mass of cyclohexane (C_6H_{12}) = 84
$\because$ For $84g$ of cyclohexane, we need $= 100g$ of material $C_6H_{11}.(OH)$
$\therefore$ For $20g$ of cyclohexane, we need $= \frac{100 \times 20}{84}$
$= 23.80g$ of $C_6H_{11}(OH)$
$\because 54g$ of $C_6H_{11}(OH)$ get from $= 100g$ of $C_6H_{11}(OH)$
$\therefore 23.8g$ of $C_6H_{11}(OH)$ get from $= \frac{100 \times 23.8}{54} = 44\,\%$
$\therefore$ Hence, (d) is the correct answer.

Question 100



What are A, B and C compounds?

Options:

A. CH_3CH_2COONa, ethane, butene-2

B. CH_3COONa, ethane, butene-1

C. $CHI_3, HC \equiv CH$ butene-2

D. C_6H_5COONa, C_6H_6, ethene

Answer: A

Solution:

Solution:

(i) $C_4H_{10}O \xrightarrow{I_2 - NaOH} \underset{\text{Yellow ppt . of iodoform}}{CH_3I} + \underset{(A)}{CH_3CH_2COONa}$

(ii) $CH_3CH_2COONa \xrightarrow[\Delta]{CaO} \underset{\text{(ethane colourless gas)}}{CH_3 - CH_3}$

It is a decarboxylation reaction, which give an alkane with one carbon-atom less, as in CH_3CH_2COONa i.e. $CH_3 - CH_3$(ethane) .

(iii) $C_4H_{10}O_1$ i.e. C_4H_9OH. When treated with H_2SO_4 (conc.), it will produce butene-2 as follows:

$C_4H_9OH \xrightarrow[\Delta]{H_2SO_4(\text{conc})} H_3C - HC = CH - CH_3$

2-butene is an unsaturated compound.

Thus, will decolourise bromine water,

Hence, A, B, C are respectively

$CH_3CH_2COONa - (A)$

$CH_3 - CH_3(\text{ethane}) - (B)$

$CH_3 - CH = CH - CH_3$2-(butene) $- (C)$

$\therefore (a)$ is the correct answer.

Question 101

The solution of the differential equation $x\frac{dy}{dx} = y\left(\log y - \log x + 1\right)$ will be

Options:

A. $y = xe^{Cx}$

B. $y + xe^{Cx} = 0$

C. $y + e^x = 0$

D. None of these

Answer: A

Solution:

Solution:

Given differential equation,

$x\frac{dy}{dx} = y(\log y - \log x + 1)$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}(\log y - \log x + 1)$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}(\log \frac{y}{x} + 1)$ .....(i)

It is homogeneous differential equation.

Now, put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ .....(ii)
From Eqs. (i) and (ii), we get
$v + x\frac{dv}{dx} = v(\log v + 1)$
$\Rightarrow \frac{dv}{v \log v} = \frac{dx}{x}$
By integrating both the sides, we have
$\log(\log v) = \log x + \log C$
$\Rightarrow \log(\log \frac{y}{x}) = \log Cx$
$\Rightarrow \log \frac{y}{x} = Cx$
$\Rightarrow y = xe^{\alpha}$

Question 102

Solution of the differential equation $x\frac{dy}{dx} = y - x\tan(\frac{y}{x})$ will be

Options:

A. $x\sin(\frac{x}{y}) + C = 0$

B. $x\sin y + C = 0$

C. $x\sin(\frac{y}{x}) = C$

D. None of these

Answer: C

Solution:

Solution:
Given differential equation
$x\frac{dy}{dx} = y - x\tan(\frac{y}{x})$
$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \tan(\frac{y}{x})$ .....(i)
which is homogeneous differential equation.
Now, put $\frac{y}{x} = v$
$\Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ .....(ii)
From Eqs. (i) and (ii), we get
$\Rightarrow v + x\frac{dv}{dx} = v - \tan v$
$\Rightarrow x\frac{dv}{dx} = -\tan v \Rightarrow \frac{dv}{\tan v} = -\frac{dx}{x}$
$\Rightarrow \cot v\, dv = -\frac{1}{x}dx$
On integrating both the sides, we get
$\log(\sin v) = -\log x + \log C$
$\Rightarrow \log(\sin \frac{y}{x}) + \log x = \log C$
$\Rightarrow \log(x.\sin \frac{y}{x}) = \log C$
$\Rightarrow x.\sin(\frac{y}{x}) = C$

Question 103

The order and degree of the differential equation $(1+3\frac{dy}{dx})^{2/3} = 4\frac{d^3y}{dx^3}$ will be

Options:

A. $1, \frac{2}{3}$

B. 3,1

C. 3,3

D. 1,2

Answer: C

Solution:

Solution:
$(1+3\frac{dy}{dx})^{2/3} = 4\frac{d^3y}{dx^3}$
For the order and degree of the given differential equation, cube both the sides,
$(1+\frac{3dy}{dx})^{2} = \left(4.\frac{d^3y}{dx^3}\right)^3$
Hence, order = 3, degree = 3

Question 104

If A and B are two events such that $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{5}$, then $P(\frac{\overline{B}}{\overline{A}})$ will be

Options:

A. $\frac{37}{40}$

B. $\frac{37}{45}$

C. $\frac{23}{40}$

D. $\frac{27}{40}$

Answer: A

Solution:

Solution:
We know that
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow P(A \cup B) = \frac{1}{3} + \frac{1}{4} - \frac{1}{5} = \frac{23}{60}$ [$\because P(A) = \frac{1}{3}, P(B) = \frac{1}{4}, P(A \cap B) = \frac{1}{5}$]

Now, $P\left(\frac{\overline{B}}{\overline{A}}\right) = \frac{1 - P(A \cup B)}{P(\overline{A})} = \frac{1 - \frac{23}{60}}{1 - \frac{1}{3}}$ [$\because P(\overline{A}) = 1 - P(A)$]

$= \frac{37}{60} \times \frac{3}{2} = \frac{37}{40}$

Question 105

One letter from the word POSSESSIVE is chosen at random, then the probability of its being S will be

Options:

A. $\frac{3}{10}$

B. $\frac{4}{10}$

C. $\frac{3}{6}$

D. $\frac{4}{6}$

Answer: B

Solution:

Solution:
There are total 10 letters in the word POSSESSIVE. Out of which 4 are S.

$\therefore$ Required probability $= \frac{{}^4C_1}{{}^{10}C_1} = \frac{4}{10}$

Question 106

If $\frac{1+3P}{3}, \frac{1-P}{4}$ and $\frac{1-2P}{2}$ are the probabilities of three mutually exclusive events, then the set containing all the possible values of P is

Options:

A. $\frac{1}{3} \leq P \leq \frac{1}{2}$

B. $\frac{1}{2} < P < \frac{1}{2}$

C. $\frac{1}{2} \leq P \leq \frac{2}{3}$

D. $\frac{1}{2} < P < \frac{2}{3}$

Answer: A

Solution:

Solution:
Since,
$(\frac{1+3P}{3}), (\frac{1-P}{4})$ and $(\frac{1-2P}{2})$ are probability of three events.
Hence,
$0 \leq \frac{1+3P}{3} \leq 1, 0 \leq \frac{1-P}{4} \leq 1$ and $0 \leq \frac{1-2P}{2} \leq 1$
$\Rightarrow -1 \leq 3P \leq 2, -3 \leq P \leq 1$ and $-1 \leq 2P \leq 1$
$\Rightarrow -\frac{1}{3} \leq P \leq \frac{2}{3}, -3 \leq P \leq 1$ and $-\frac{1}{2} \leq P \leq 2$
$\Rightarrow \frac{1+3P}{3}, \frac{1-P}{4}$ and $\frac{1-2P}{2}$ are the probabilities of three mutually exclusive events.
Hence,
$0 \leq \frac{1+3P}{3} + \frac{1-P}{4} + \frac{1-2P}{2} \leq 1$
$\Rightarrow 0 \leq 4 + 12P + 3 - 3P + 6 - 12P \leq 12 \ = \frac{1}{3} \leq P \leq \frac{13}{3}$
Hence, the required value of P.
Maximum $\{-\frac{1}{3}, -3, -\frac{1}{2}, \frac{1}{3}\} \leq P \leq$ Minimum $\{\frac{2}{3}, 1, \frac{1}{2}, \frac{13}{3}\}$
$\Rightarrow \frac{1}{3} \leq P \leq \frac{1}{2}$

Question 107

From a pack of cards, two cards are drawn one-by-one. Then, the probability that first card is an ace while the second card is coloured (before drawing the second card, the first card is not replaced in the pack of cards) will be

Options:

A. $\frac{1}{26}$

B. $\frac{5}{52}$

C. $\frac{5}{221}$

D. $\frac{4}{13}$

Answer: C

Solution:

Solution:
Probability of getting on ace $P(E_1) = \frac{4}{52} = \frac{1}{13}$
$\therefore P\left(\frac{E_2}{E_1}\right) = \frac{15}{51} = \frac{5}{17}$
$\therefore P(E_1 \cap E_2) = P(E_1) \cdot P\left(\frac{E_2}{E_1}\right)$
$= \frac{1}{13} \cdot \frac{5}{17} = \frac{5}{221}$

Question 108

One root of the equation, $x^3 - 3x - 5 = 0$ lies between 2 and 2.5. Using Newton-Raphson method, the value of that root will be

Options:

A. 2.25

B. 2.33

C. 2.35

D. 2.45

Answer: B

Solution:

Solution:
Let $f(x) = x^3 - 3x - 5 = 0$
Then, $f'(x) = 3x^2 - 3$
Using Newton Raphson formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 - 3x_n - 5}{3x_n^2 - 3}$$

$$\Rightarrow x_{n+1} = \frac{2x_n^3 + 5}{3x_n^2 - 3} \text{ .....(i)}$$

Given, one root is between 2 and 2.5. So taking
$x_0 = 2, n = 0$ in Eq. (i).

$$x_1 = \frac{2x_0^3 + 5}{3x_0^2 - 3} = \frac{2(2)^3 + 5}{3(2)^2 - 3} = \frac{21}{9} = 2.33$$

Question 109

Taking two sub-intervals and using Simpson's $\frac{1}{3}rd$ rule, the value of $\int_0^1 \frac{dx}{1+x}$ will be

Options:

A. $\frac{17}{24}$

B. $\frac{17}{36}$

C. $\frac{25}{36}$

D. $\frac{17}{25}$

Answer: C

Solution:

Solution:
$\int_0^1 \frac{dx}{1+x}$
Taking two sub-interval of range $[0,1]$ of integral,
$h = \frac{1-0}{2} = \frac{1}{2}$

x	$y = \frac{1}{1+x}$
$x_0 = 0$	$y_0 = 1$
$x_0 + h = \frac{1}{2}$	$y_1 = \frac{2}{3}$
$x_0 + 2h = 1$	$y_2 = \frac{1}{2}$

Using Simpson's $\frac{1}{3}$ rd rule.
$\int_0^1 y dx = \frac{h}{3}[y_0 + 4y_1 + y_2]$
$\int_0^1 \frac{dx}{1+x} = \frac{1}{3.2}[1 + 4(\frac{2}{3}) + \frac{1}{2}]$
$= \frac{1}{6}[1 + \frac{8}{3} + \frac{1}{2}] = \frac{1}{6} \cdot \frac{25}{6} = \frac{25}{36}$

Question 110

Using trapezoidal rule and taking $n = 4$, the approximate value of integral $\int_1^9 x^2 dx$ is $2[\frac{1}{2}(1+9^2) + \alpha^2 + \beta^2 + 7^2]$, then

Options:

A. $\alpha = 1, \beta = 3$

B. $\alpha = 2, \beta = 4$

C. $\alpha = 3, \beta = 5$

D. $\alpha = 4, \beta = 6$

Answer: C

Solution:

Solution:
Given, $n = 4$,
We have,
$\Delta x = \frac{9-1}{4} = \frac{8}{4} = 2$
We compute the values of $y_0, y_1, y_2, \ldots\ldots\ldots, y_4$

x	1	3	5	7	9
$y = x^2$	1	9	25	49	81

Therefore,
$\int_1^9 x^2 dx \approx \frac{2}{2}[1 + 3^2 + 5^2 + 7^2 + 9^2]$
$\approx 2[\frac{1}{2}\{(1+9^2) + 3^2 + 5^2 + 7^2\}]$
But we have given that,

$\int_1^9 x^2 dx \approx 2[\frac{1}{2}(1 + 9^2) + \alpha^2 + \beta^2 + 7^2]$
By comparison, we get that $\alpha = 3$ and $\beta = 5$

Question 111

Which one of the following is not an operating system?

Options:

A. Unix

B. Linux

C. Mac

D. C_{++}

Answer: D

Solution:

Solution:
Unix, Linux, Mac are operating systems. Hence, C_{++} is not operating system.

Question 112

Which one of the following is a basis for computers?

Options:

A. Abstract algebra

B. Linear algebra

C. Boolean algebra

D. None of these

Answer: C

Solution:

Solution:
Boolean algebra is a basis for computers.

Question 113

Which one of the following is used to transfer files in Transmission

Control Protocol (TCP)?

Options:

A. Numbers

B. Alphabets

C. Packets

D. None of these

Answer: C

Solution:

Solution:
'Packets' is used to transfer files in Transmission Control Protocol (TCP).

Question 114

If $e^{i\,\text{theta}} = \cos\text{theta} + i\sin\text{theta}$, then the value of frac $(1+i)^2\,1-i$ is

Options:

A. $\sqrt{2}e^{i\pi/4}$

B. $\sqrt{2}e^{-i\pi/4}$

C. $\sqrt{2}e^{3i\pi/4}$

D. $\sqrt{2}e^{-3i\pi/4}$

Answer: C

Solution:

Solution:
Given, $e^{i\theta} = \cos\theta + i\sin\theta$
Then, $\frac{(1+i)^2}{1-i} = \frac{(1+i^2+2i)}{(1-i)} \times \frac{(1+i)}{(1+i)}$
$\frac{1+i^2+2i+i+i^3+2i^2}{1+1}$
$= \frac{1-1+3i-i+2(-1)}{2}$
$= \frac{2i-2}{2} = \frac{2(i-1)}{2} = i-1$
$= -1+i = \sqrt{2}.e^{3i\pi/4}$
Since, $\sqrt{2}e^{3i\pi/4} \Rightarrow \sqrt{2}(\cos\frac{3\pi}{4} + i\sin\frac{\pi}{4})$
$\Rightarrow \sqrt{2}(-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}})$
$\Rightarrow \frac{\sqrt{2}}{\sqrt{2}}(-1+i) = -1+i$

Question 115

If for complex numbers z_1 and z_2, arg $(\frac{z_1}{z_2}) = 0$, then $|z_1 - z_2|$ is equal to$

Options:

A. $|z_1| - |z_2|$

B. $||z_1| - |z_2||$

C. $|z_1| + |z_2|$

D. $||z_1| + |z_2||$

Answer: B

Solution:

Solution:
We know that $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos(\theta_1 - \theta_2)$
where, $\theta_1 = \arg(z_1)$ and $\theta_2 = \arg(z_2)$
$\because \arg(\frac{z_1}{z_2}) = 0$
$\Rightarrow \arg(z_1) - \arg(z_2) = 0$
$\theta_1 - \theta_2 = 0$
$\therefore |z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2|z_1||z_2| = (|z_1| - |z_2|)^2$
$|z_1 - z_2| = ||z_1| - |z_2||$

Question 116

Arithmetic mean progression of two numbers is 6 more than the geometric mean progression of the numbers. The ratio of numbers is 9:1, then numbers are

Options:

A. 27,3

B. 16,3

C. 15,4

D. None of these

Answer: A

Solution:

Solution:
Let a and b be numbers.
Now, Arithmetic mean $= \frac{a+b}{2}$
and Geometric mean $= \sqrt{ab}$
According to question, Arithmetic mean = Geometric mean +6
$\Rightarrow \frac{a+b}{2} = \sqrt{ab} + 6$ .....(i)
and $\frac{a}{b} = \frac{9}{1}$

$\Rightarrow a = 9b$ .....(ii)
From Eqs. (i) and (ii), we get
$\frac{9b + b}{2} = \sqrt{9b^2} + 6$
$\Rightarrow 5b = 3b + 6$
$\Rightarrow 2b = 6$
$\Rightarrow b = 3$
$\because a = 9b = 9 \times 3 = 27$

Question 117

The geometric mean of roots of the equation $2x^2 + 3x + 16 = 0$ is

Options:

A. 4

B. 3

C. $2\sqrt{2}$

D. 1

Answer: C

Solution:

Solution:
Given, Equation
$2x^2 + 3x + 16 = 0$ .....(i)
Let α and β be the roots of the given equation.
Then, product of roots $(\alpha\beta) = \frac{16}{2} = 8[\because \alpha\beta = \frac{c}{a}]$
$\therefore$ Geometric mean $= \sqrt{\alpha\beta} = \sqrt{8} = 2\sqrt{2}$

Question 118

If the equation, $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ $a, b, c \in R$, has equal roots, then

Options:

A. $a = b \neq c$

B. $a + b\omega + c\omega^2 = 0$

C. $a \neq b = c$

D. None of these

Answer: D

Solution:

Solution:

$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$
$=3x^2-2(a+b+c)x+(ab+bc+ca)=0$
$\because$ Roots are equal
$\therefore B^2-4AC=0$
$4(a+b+c)^2-4(3)(ab+bc+ca)=0$
$\Rightarrow 4\{(a+b+c)^2-3(ab+bc+ca)\}=0$
$\Rightarrow 4\{a^2+b^2+c^2-ab-bc-ca\}=0$
$\Rightarrow 2\{(a-b)^2+(b-c)^2+(c-a)^2\}=0$
$\Rightarrow (a-b)^2+(b-c)^2+(c-a)^2=0$
Hence, two roots are always equal $a=b$ or $b=c$ or $c=a$.

Question 119

Two finite sets have m and n elements. The total number of subsets of the first set is 48 more than the total number of subsets of the second. The values of m and n respectively, are

Options:

A. 6,2

B. 5,3

C. 7,5

D. 6,4

Answer: D

Solution:

Solution:
Given, two finite sets have m and n elements.
Then, according to question,
$2^m=2^n+48$
$\Rightarrow 2^m-2^n=48$
$\Rightarrow 2^n(2^{m-n}-1)=2^4(2^2-1)$
On comparing both sides
$n=4 \text{and} m-n=2$
$\Rightarrow m-4=2$
$\Rightarrow m=6$
$\therefore$ Values of m and $n=6,4$

Question 120

If $A=\begin{bmatrix}2 & +1\\ -1 & 2\end{bmatrix}$ and I is the unit matrix of order 2, then A^2 is equal to

Options:

A. $2A-3I$

B. $4A+5I$

C. $4A-5I$

D. None of these

Answer: C

Solution:

Solution:

Given, $A = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$

$\therefore A^2 = A \,.\, A$

$= \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4-1 & 2+2 \\ -2-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$ .....(i)

Now, $4A - 5I = \quad = \begin{bmatrix} 8 & 4 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$ .....(ii)

therefore From Eqs. (i) and (ii), we have

$A^2 = 4A - 5I$

Question 121

The system of linear equations $x + y + z = 2$, $3x - y + 2z = 6$ and $3x + y + z = -18$ has

Options:

A. no solution

B. zero solution as the only solution

C. a unique solution

D. an infinite number of solutions

Answer: C

Solution:

Solution:

Given, system of linear equations is

$x + y + z = 2$,

$3x - y + 2z = 6$

and $3x + y + z = -18$

We can write system of equations in this form as

$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -18 \end{bmatrix}$

Now , $\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = 1(-1-2) - 1(3-6) + 1(3+3)$

$= -3 + 3 + 6 = 6$

Since, $\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix} \neq 0$

Hence, a unique solution is exist.

Question 122

The equation $\sin x + \cos x = 2$ has

Options:

A. only one solution

B. no solution

C. infinite number of solutions

D. two solutions

Answer: B

Solution:

Solution:
$\because \sin x + \cos x = 2$
We know that,
$-\sqrt{2} \leq \sin x + \cos x \leq \sqrt{2}$
$\therefore \sin x + \cos x \neq 2$
So, given equation has no solution.

Question 123

If $a = 2, b = 3, c = 5$ in ΔABC, then $\angle C =$

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{8}$

D. None of these

Answer: D

Solution:

Solution:
Given, $a = 2, b = 3, c = 5 \because$ We know that,
$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
$= \frac{(2)^2 + (3)^2 - (5)^2}{2 \times 2 \times 3}$
$= \frac{4 + 9 - 25}{12}$
$\cos C = -\frac{12}{12} = -1$
$\therefore \angle C = \pi$

which is not possible.

Question 124

$\tan^{-1}(\frac{3}{5}) + \cos^{-1}(\frac{4}{5}) =$

Options:

A. $\tan^{-1}(\frac{27}{11})$

B. $\sin^{-1}(\frac{11}{27})$

C. $\tan^{-1}(\frac{11}{27})$

D. None of these

Answer: A

Solution:

Solution:
Given,

$$\tan^{-1}(\frac{3}{5}) + \cos^{-1}(\frac{4}{5}) = \tan^{-1}(\frac{3}{5}) + \tan^{-1}\left(\frac{\sqrt{1-\frac{16}{25}}}{\frac{4}{5}}\right)$$

$$= \tan^{-1}(\frac{3}{5}) + \tan^{-1}(\frac{3}{4}) = \tan^{-1}\left[\frac{\frac{3}{5}+\frac{3}{4}}{1-(\frac{3}{5})(\frac{3}{4})}\right]$$

$$= \tan^{-1}\left[\frac{\frac{12+15}{20}}{\frac{11}{20}}\right] = \tan^{-1}(\frac{27}{11})$$

Question 125

The angle of elevation of a tower at a point distant $20\sqrt{3}m$ from its base is 30°. Then, height of the tower (in meter) is

Options:

A. 20

B. 15

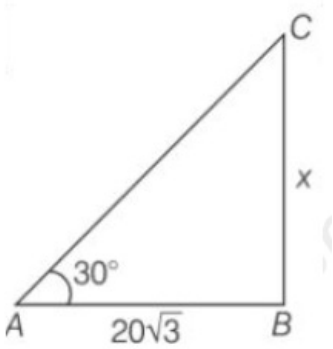
C. 30

D. 18

Answer: A

Solution:

Solution:
Let BC be the tower of height x m and A is a point from its base at a distance of $20\sqrt{3}m$.



Now, in ΔABC

$\tan 30° = \frac{BC}{AB}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{20\sqrt{3}}$

$x = 20m$

Hence, height of tower, $BC = 20m$

Question 126

The orthocentre of the triangle formed by the lines $x = 2, y = 3$ and $3x + 2y = 6$ is at the point

Options:

A. $(2, 0)$

B. $(2, 3)$

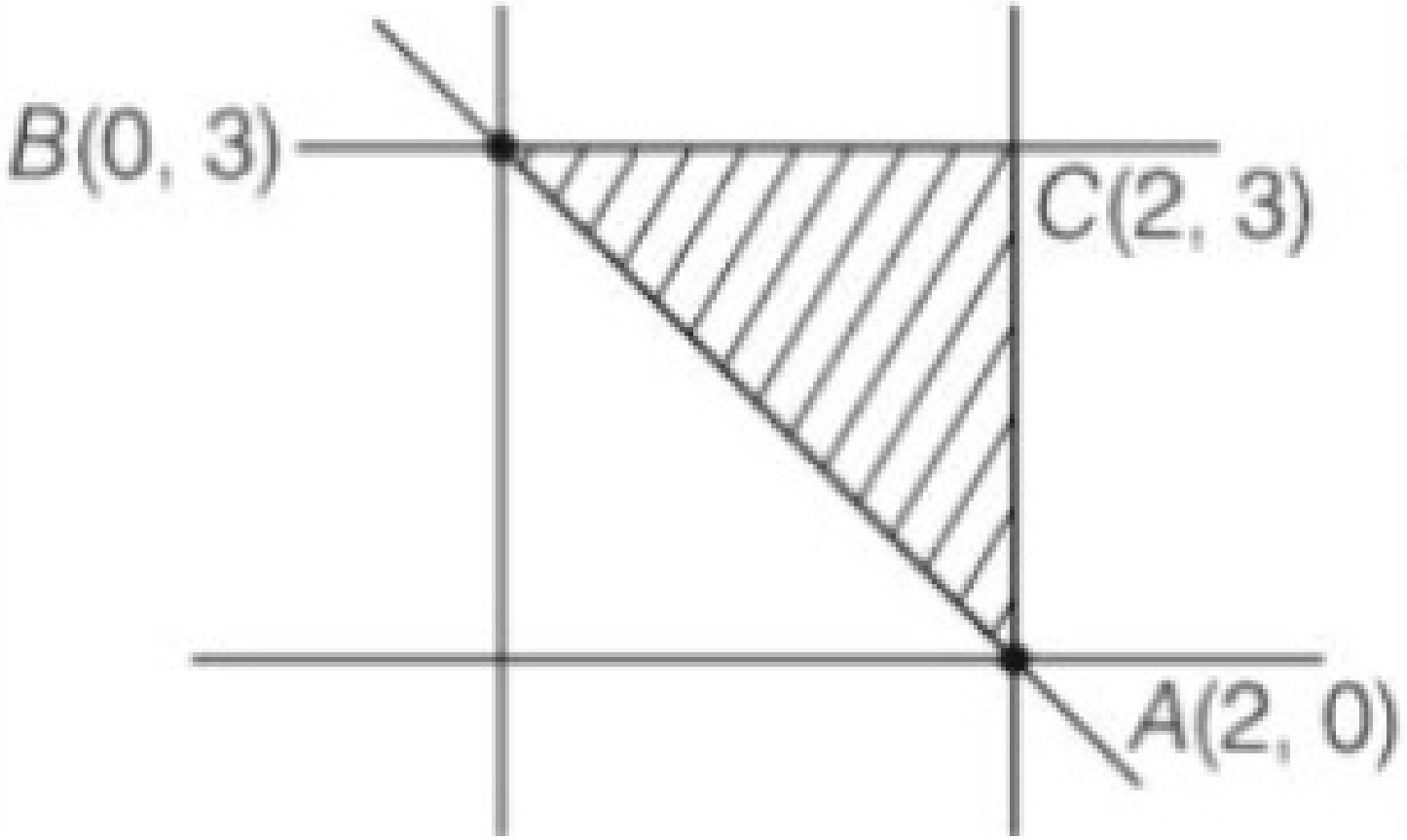
C. $(0, 3)$

D. None of these

Answer: B

Solution:

Solution:
Given, equation of lines are
$x = 2, y = 3$ and $3x + 2y = 6$



Since, ΔABC is right angled at C. Now, orthocentre of δABC will the vertex at which right angle is forming.
$\therefore$ Orthocentre of ΔABC is $(2, 3)$.

Question 127

Which of the following lines is a normal to the circle $x^2 + y^2 - 2x - 10y + 6 = 0$?

Options:

A. $3x - 8y = 9$

B. $x + y = 3$

C. $2x + y = 7$

D. None of these

Answer: C

Solution:

Solution:
Given equation of circle
$x^2 + y^2 - 2x - 10y + 6 = 0$
$\therefore$ Centre $= (-g, -f) = (1, 5)$
Since, normal to the circle always passes through the centre.
$\therefore 2x + y = 7$ is equation of normal as it passes through $(1, 5)$.

Question 128

If the tangents and normals at the extremities of a focal chord of a parabola intersect at (x_1, y_1) and (x_2, y_2) respectively, then

Options:

A. $x_1 = y_2$

B. $x_1 = x_2$

C. $y_1 = x_2$

D. $y_1 = y_2$

Answer: D

Solution:

Solution:
Let $P(at_1^2, 2at_1)$ and $Q(at_2^2, at_2)$ be a focal chord of the parabola $y^2 = 4ax$. The tangents at P and Q intersect at $(at_1t_2, a(t_1 + t_2))$
$\therefore x_1 = at_1t_2$ and $y_1 = a(t_1 + t_2)$
$\Rightarrow x_1 = -a$ and $y_1 = a(t_1 + t_2)$ [$\because PQ$ is a focal chord $\Rightarrow t_1t_2 = -1$]
The normals at P and Q intersect at
$[2a + a(t_1^2 + t_2^2 + t_1t_2), -at_1t_2(t_1 + t_2)]$
$\therefore x_2 = 2a + a(t_1^2 + t_2^2 + t_1t_2)$ and $y_2 = -at_1t_2(t_1 + t_2)$
$\Rightarrow x_2 = a + a(t_1^2 + t_2^2)$ and $y_2 = a(t_1 + t_2)$
$\therefore y_1 = y_2$

Question 129

The equation of a diameter conjugate to a diameter $y = \frac{b}{a}x$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is

Options:

A. $y = -\frac{a}{b}x$

B. $x = -\frac{a}{b}y$

C. $y = \frac{a}{b}x$

D. $x = \frac{a}{b}y$

Answer: B

Solution:

Solution:
Two diameters $y = m_1x$ and $y = m_2x$ are conjugate diameters if $m_1m_2 = -\frac{b^2}{a^2}$
Hence, equation a conjugate diameter is

$y = -\frac{b}{a}x \text{ or } x = -\frac{a}{b}y$
There is no option correct.

Question 130

If the line OP of length r makes an angle α with X -axis and lies in the XZ -plane, then coordinates of P are

Options:

A. $(r\cos\alpha, 0, r\sin\alpha)$

B. $(0, 0, r\cos\alpha)$

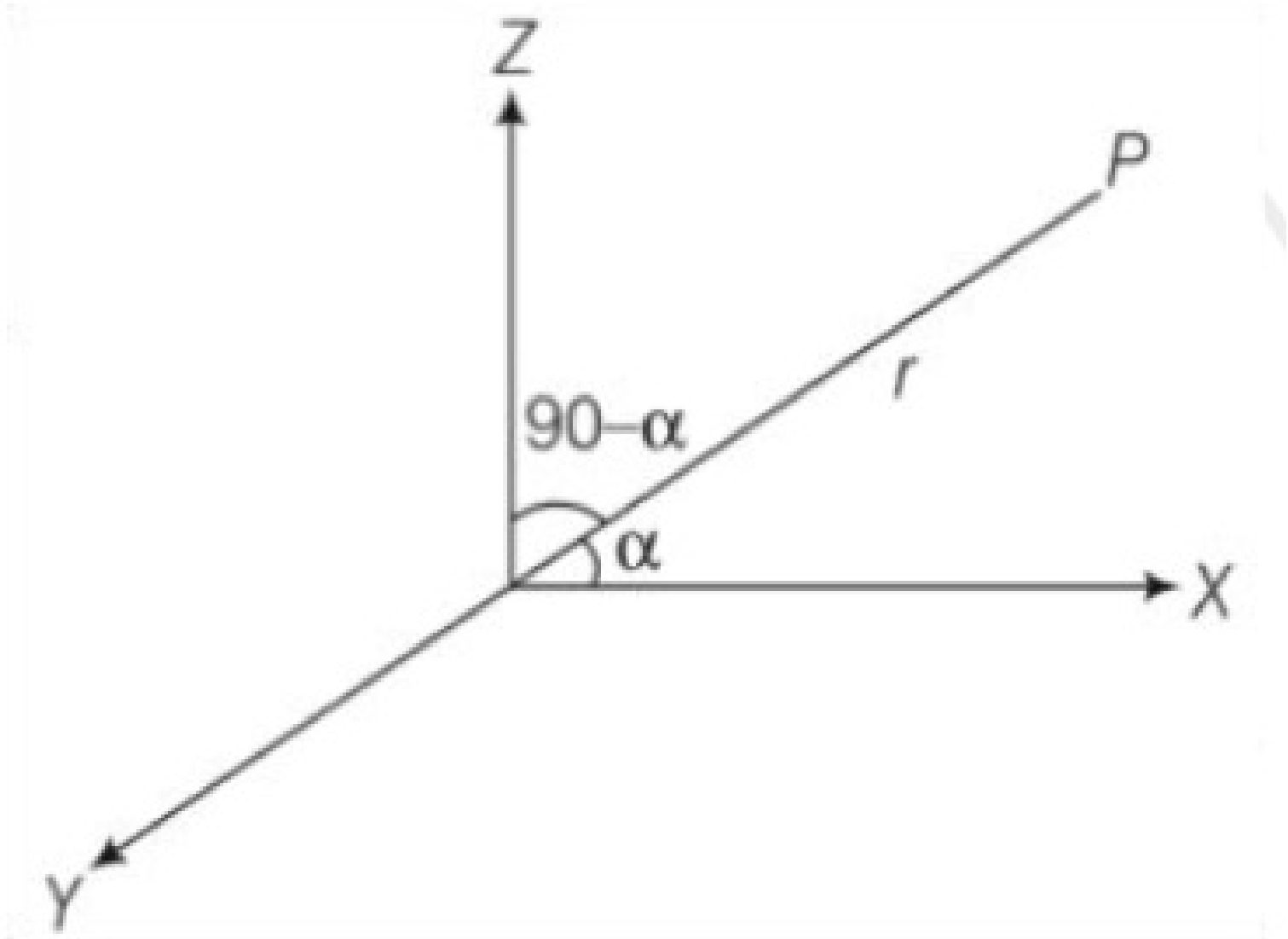
C. $(0, 0, r\sin\alpha)$

D. None of these

Answer: A

Solution:

Solution:
The line OP of length r makes an angle α with X -axis and lies in the XZ -plane.



Coordinates in Y -axis is zero.

and X -axis respect coordinates $= r\cos\alpha$
and Z -axis respect coordinates $= r\cos(90-\alpha) = r\sin\alpha$
Hence, coordinates of $P = (r\cos\alpha, 0 r\sin\alpha)$

Question 131

The equation of the plane passing through the line of intersection of the planes $x+y+z=6$ and $2x+3y+4z+5=0$ and passing through $(1,1,1)$ is

Options:

A. $x+y+z=3$

B. $2x+3y+4z=9$

C. $20x+23y+26z=69$

D. $23x+20y+z=96$

Answer: C

Solution:

Solution:
Equations of the given planes $x+y+z-6=0$ .....(i)
and $2x+3y+4z+5=0$ .....(ii)
The equation of the plane passing through the line of intersection of the plane Eqs. (i) and (ii).
$x+y+z-6+\lambda(2x+3y+4z+5)=0$ ..... (iii)
$\because$ Plane passing through the point $(1,1,1)$.
Hence, $1+1+1-6+\lambda[2(1)+3(1)+4(1)+5]=0$
$\Rightarrow -3+14\lambda=0$[from Eq. (iii)]
$\Rightarrow \lambda = \frac{3}{14}$
Put the value of λ in Eq. (iii).
$x+y+z-6+\frac{3}{14}(2x+3y+4z+5)=0$
$\Rightarrow 14x+14y+14z-84+6x+9y+12z+15=0$
$\Rightarrow 20x+23y+26z=69$

Question 132

If $P(1,2,3)$ is the given point and PN is the length of perpendicular from P on the given line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$, then $PN=$

Options:

A. 6

B. 7

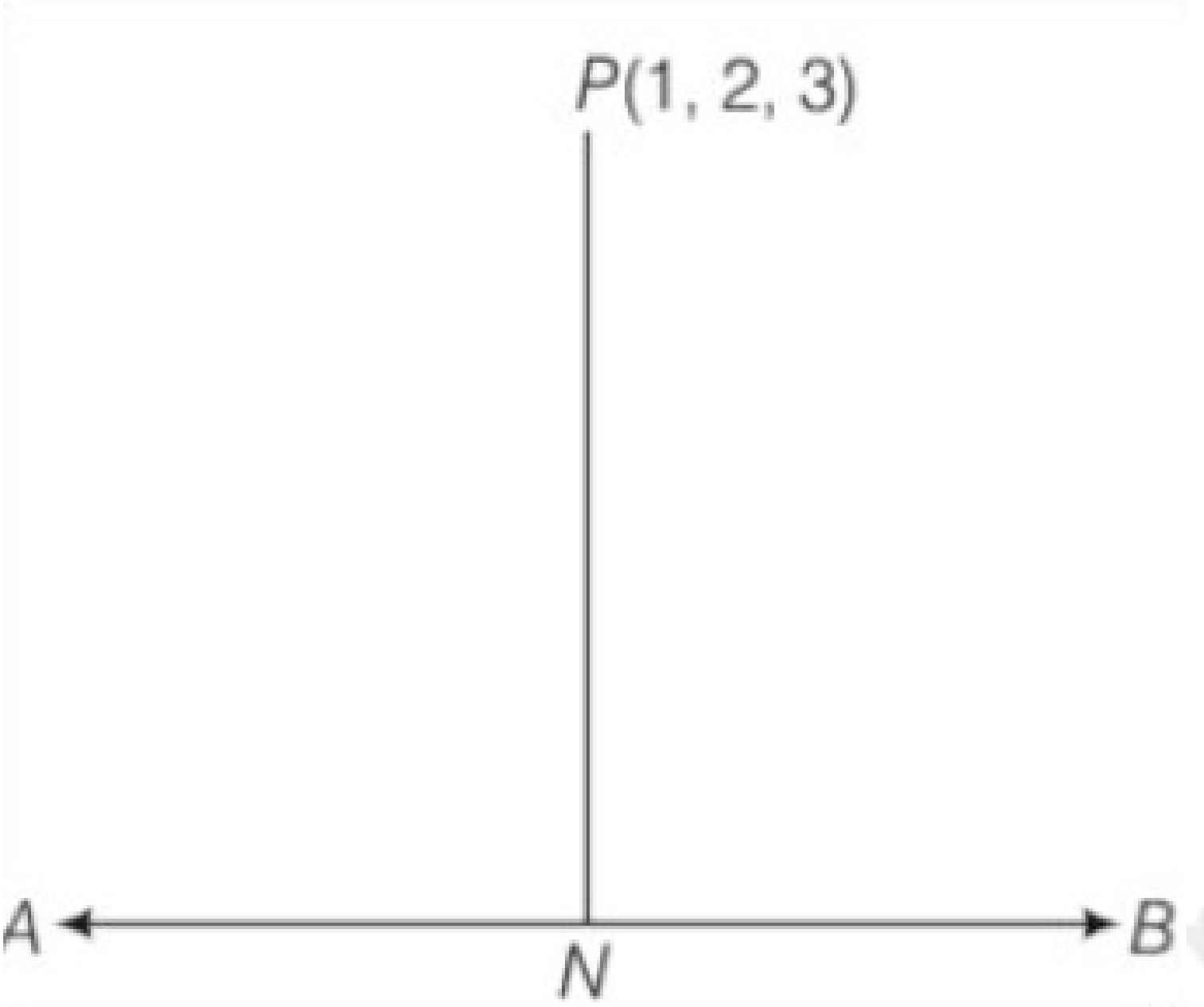
C. 5

D. 4

Answer: B

Solution:

Solution:
Let given point be $P = (1, 2, 3)$ and equation of line as
$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = \lambda$ (let)
$\Rightarrow x = 3\lambda + 6, y = 2\lambda + 7, \ z = -2\lambda + 7$



Coordinates of $N = (3\lambda + 6, 2\lambda + 7, -2\lambda + 7)$ .....(i)
Since, PN is perpendicular from P on the line AB. Then, direction ratio of
$PN = 3\lambda + 6 - 1, 2\lambda + 7 - 2, \ -2\lambda + 7 - 3$
$= 3\lambda + 5, 2\lambda + 5, -2\lambda + 4$
and direction ratio of given line $= < 3, 2, -2 >$
$\because PN \perp AB$
$\therefore 3(3\lambda + 5) + 2(2\lambda + 5) + (-2)\ (-2\lambda + 4) = 0$ [$\because a_1a_2 + b_1b_2 + c_1c_2 = 0$]
$= 9\lambda + 15 + 4\lambda + 10 + 4\lambda - 8 = 0$
$\Rightarrow 17\lambda + 17 = 0$
$\Rightarrow \lambda = -1$
Put the value of lambda in Eq. (i), we get
$= (-3 + 6, -2 + 7, 2 + 7) \ = (3, 5, 9)$
Hence, length of perpendicular from the given point.
$PN = \sqrt{(3-1)^2 + (5-2)^2 + (9-3)^2}$
$= \sqrt{(2)^2 + (3)^2 + (6)^2}$
$= \sqrt{4 + 9 + 36} = \sqrt{49} = 7$ units

Question 133

The lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x-1}{-2}=\frac{y-2}{-4}=\frac{z-3}{-6}$ are

Options:

A. skew

B. parallel

C. intersecting

D. coincident

Answer: B

Solution:

Solution:
Given, line (I) $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and Line (II) $\frac{x-1}{-2}=\frac{y-2}{-4}=\frac{z-3}{-6}$
Direction ratio of line (I) $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ is $(1,2,3)$ .and direction ratio of line (II)
$\frac{x-1}{-2}=\frac{y-2}{-4}=\frac{z-3}{-6}$ is $(-2,-4,-6)=(1,2,3)$
Hence, direction ratio are equal of both given lines. So, lines are parallel.

Question 134

The points having position vectors $60\hat{i}+3\hat{j}, 40\hat{i}-8\hat{j}, a\hat{i}-52\hat{j}$, are collinear, if a =

Options:

A. -20

B. -40

C. 20

D. 40

Answer: B

Solution:

Solution:
We have,
$\overrightarrow{OA}=60\hat{i}+3\hat{j}$
$\overrightarrow{OB}=40\hat{i}-8\hat{j}$
$\overrightarrow{OC}=a\hat{i}-52\hat{j}$
Now, $\overrightarrow{AB}=-20\hat{i}-11\hat{j}$
$\overrightarrow{AC}=(a-60)\hat{i}-55\hat{j}$
$\because$ Vector are collinear.

$\therefore \overrightarrow{AB} = \lambda \overrightarrow{AC}$
$\Rightarrow \frac{-20}{a-60} = \frac{-11}{-55}$
$\Rightarrow \frac{-20}{a-60} = \frac{1}{5}$
$\Rightarrow a - 60 = -100$
$\Rightarrow a = -100 + 60$
$\Rightarrow a = -40$

Question 135

$(\hat{i} + \hat{j}) \times (\hat{j} + \hat{k}) . (\hat{k} + \hat{i}) =$

Options:

A. $[\hat{i}\hat{j}\hat{k}]$

B. $(\hat{i} \times \hat{j}) \times \hat{k}$

C. 1

D. 2

Answer: D

Solution:

Solution:
We have, $(\hat{i} + \hat{j}) \times (\hat{j} + \hat{k}) . (\hat{k} + \hat{i})$
$= \hat{i} \times (\hat{j} + \hat{k}) . (\hat{k} + \hat{i}) + \hat{j} \times (\hat{j} + \hat{k}) . (\hat{k} + \hat{i})$
$= \hat{i} \times \hat{j} . \hat{k} + 0 + \hat{j} \times \hat{k} . \hat{i}$
$= [\hat{i}\ \hat{j}\ \hat{k}] + [\hat{j}\ \hat{k}\ \hat{i}]$
$= 1 + 1$ { $\because [\hat{i}\ \hat{j}\ \hat{k}] = 1$}
$= 2$

Question 136

If $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $|\vec{c}| = 3$, then$[\vec{a} - \vec{b}\ \ \vec{b} - \vec{c}\ \ \vec{c} - \vec{a}] =$

Options:

A. 0

B. 1

C. 3

D. 6

Answer: A

Solution:

Solution:

$[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}]$

$\{(\vec{a}-\vec{b})\times(\vec{b}-\vec{c}).\vec{c}-\vec{a}\}$

$=(\vec{a}\times\vec{b}-\vec{a}\times\vec{c}-\vec{b}\times\vec{b}+\vec{b}\times\vec{c}).(\vec{c}-\vec{a})$

$=(\vec{a}\times\vec{b}).\vec{c}-(\vec{a}\times\vec{b}).\vec{a}+(\vec{c}\times\vec{a})\vec{c}-(\vec{c}\times\vec{a}).\vec{a}+(\vec{b}\times\vec{c}).\vec{c}-(\vec{b}\times\vec{c}).\vec{a}$

$=[\vec{a}\ \vec{b}\ \vec{c}]-[\vec{a}\ \vec{b}\ \vec{a}]+[\vec{c}\ \vec{a}\ \vec{c}]-[\vec{c}\ \vec{a}\ \vec{a}]+[\vec{b}\ \vec{c}\ \vec{c}]-[\vec{b}\ \vec{c}\ \vec{a}]=0$

Question 137

If the volume of a tetrahedron having vertices with position vectors $\hat{i}-6\hat{j}+10\hat{k}, -\hat{i}-3\hat{j}+7\hat{k}$, $5\hat{i}-7\hat{j}+\lambda\hat{k}$ and $7\hat{i}-4\hat{j}+7\hat{k}$ is 11 units, then λ =

Options:

A. 3

B. -3

C. -1

D. None of these

Answer: D

Solution:

Solution:

Given,

$A=\hat{i}-6\hat{j}+10\hat{k}$, $B=-\hat{i}-3\hat{j}+7\hat{k}$,

$C=5\hat{i}-7\hat{j}+\lambda\hat{k}$, $D=7\hat{i}-4\hat{j}+7\hat{k}$

Then,

$\vec{a}=\vec{AB}=-2\hat{i}+3\hat{j}-3\hat{k}$

$\vec{b}=\vec{AC}=4\hat{i}-\hat{j}+(\lambda-10)\hat{k}$

$\vec{c}=\vec{AD}=6\hat{i}+2\hat{j}-3\hat{k}$

$\therefore$ Volume of tetrahedron $=\frac{1}{6}[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

$$\therefore 11=\frac{1}{6}\begin{vmatrix}-2 & 3 & -3\\ 4 & -1 & (\lambda-10)\\ 6 & 2 & -3\end{vmatrix}$$

$\Rightarrow 11=\frac{1}{6}[-2\{3-2(\lambda-10)\}-3(-12-6(\lambda-10))-3(8+6)]$

$\Rightarrow 11=\frac{1}{6}[-2\{23-2\lambda\}-3(48-6\lambda)-48]$

$\Rightarrow 66=[-46+4\lambda-144+18\lambda-48]$

$\Rightarrow 66=[-238+22\lambda]$

$\Rightarrow 22\lambda=304$

$\Rightarrow \lambda=\frac{304}{22}=\frac{152}{11}$ unit

Question 138

The function $f(x)=\tan\pi x-x+[x]$ has period

Options:

A. 1

B. π

C. 2π

D. None of these

Answer: D

Solution:

Solution:
Given function is $f(x) = \tan \pi x - x + [x]$
Since, $[x]$ is not periodic function.
$\therefore f(x)$ is a non-periodic function.

Question 139

If $f(x) = 2\sin x, g(x) = \cos^2 x$, then $(f+g)\frac{\pi}{3} =$

Options:

A. $\frac{2}{\sqrt{3}}$

B. $\sqrt{3} + \frac{1}{4}$

C. 1

D. $\frac{\sqrt{3}}{2}$

Answer: B

Solution:

Solution:
We have, $f(x) = 2\sin x$
$g(x) = \cos^2 x$
$\because$ We know that $(f+g)(x) = f(x) + g(x)$
$\therefore (f+g)\frac{\pi}{3} = f(\frac{\pi}{3}) + g(\frac{\pi}{3})$
$= 2\sin(\frac{\pi}{3}) + \cos^2(\frac{\pi}{3}) \quad = 2 . \frac{\sqrt{3}}{2} + (\frac{1}{2})^2 = \sqrt{3} + \frac{1}{4}$

Question 140

$\lim_{x \to 1} \frac{\log x}{x-1}$ is equal to

Options:

A. -1

B. 0

C. e

D. 1

Answer: D

Solution:

Solution:
Given,
$\lim_{x \to 1} \frac{\log x}{x-1}$
$= \lim_{x \to 1} \frac{1/x}{1}$ [using L' Hospital Rule]
$= 1$

Question 141

If $y = \sin^{-1}(\frac{1-x}{1+x})$, then differential coefficient W.r.t. $\sqrt{x}$ is

Options:

A. $\frac{-2}{1+x}$

B. $\sqrt{x}$

C. $\frac{2}{\sqrt{x}}$

D. 1

Answer: A

Solution:

Solution:
We have, $y = \sin^{-1}(\frac{1-x}{1+x})$
$y = \frac{\pi}{2} - \cos^{-1}(\frac{1-x}{1+x})$
$y = \frac{\pi}{2} - \cos^{-1}\left[\frac{1-(\sqrt{x})^2}{1+(\sqrt{x})^2}\right]$
$y = \frac{\pi}{2} - 2\tan^{-1}\sqrt{x}$ .....(i)
On differentiating Eq. (i) w.r.t x, we get
$\frac{dy}{dx} = 0 - \frac{2}{1+x} \cdot \frac{1}{2\sqrt{x}} = \frac{-1}{(1+x)\sqrt{x}}$ .....(ii)
Now, let $u = \sqrt{x}$
On differentiating w.r.t. x, we have
$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ .....(iii)
Now,
$\frac{dy}{du} = \frac{\frac{dy}{dx}}{\frac{du}{dx}} = \frac{-\frac{1}{(1+x)\sqrt{x}}}{\frac{1}{2\sqrt{x}}}$
$= \frac{-2}{1+x}$

Question 142

If $f(x) = |x - 2|$, then

Options:

A. $\lim_{x \to 2^+} f(x) \neq 0$

B. $\lim_{x \to 2^-} f(x) \neq 0$

C. $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$

D. $f(x)$ is continuous at $x = 2$

Answer: D

Solution:

Solution:

$\because f(x) = |x - 2|$

$f(x) = \begin{cases} 2 - x, & \text{if} x < 2 \\ x - 2, & \text{if} x \geq 2 \end{cases}$

$\therefore \lim_{x \to 2^+} f(x) = \lim_{x \to 2} (x - 2)$

$= 2 - 2 = 0$

and $\lim_{x \to 2^-} f(x) = \lim_{x \to 2} (2 - x)$

$= 2 - 2 = 0$

Also, $f(2) = |2 - 2| = 0$

Since,

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) = f(2)$

$\therefore f(x)$ is continuous at $x = 2$

Question 143

If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx} =$

Options:

A. $2^{x-y}\frac{2^y - 1}{2^x - 1}$

B. $2^{x-y}\frac{2^y - 1}{1 - 2^x}$

C. $\frac{2^x + 2^y}{2^x - 2^y}$

D. None of these

Answer: D

Solution:

Solution:
Given,
$2^x + 2^y = 2^{x+y}$.....(i)
On differentiating, w.r.t. x, we get
$2^x \ln 2 + 2^y \frac{dy}{dx} \ln 2 = 2^{x+y} \ln 2(1 + \frac{dy}{dx})$
$\Rightarrow [2^x - 2^{x+y}] \ln 2 = (2^{x+y}\frac{dy}{dx} - 2^y \frac{dy}{dx}) \ln 2$
$\Rightarrow 2^x - 2^{x+y} = 2^{x+y}\frac{dy}{dx} - 2^y \frac{dy}{dx}$
$\Rightarrow 2^x - 2^{x+y} = \frac{dy}{dx}(2^{x+y} - 2^y)$
$\Rightarrow \frac{2^x(1-2^y)}{2^y(2^x-1)} = \frac{dy}{dx}$
$\Rightarrow \frac{dy}{dx} = \frac{2^x(1-2^y)}{2^y(2^x-1)}$ .....(ii)
$\Rightarrow \frac{dy}{dx} = \frac{2^x - 2^{x+y}}{2^{x+y} - 2^y}$
$= \frac{2^x - 2^x - 2^y}{2^y(2^x-1)}$ [by Eq. (i)]
$= \frac{-2^y}{2^y(2^x-1)} = \frac{1}{1-2^x}$

Question 144

For the function $x + \frac{1}{x}, x \in [1, 3]$, the value of c for the mean value theorem is

Options:

A. 1

B. 2

C. $\sqrt{3}$

D. None of these

Answer: C

Solution:

Solution:
Given
$f(x) = x + \frac{1}{x}$
$\therefore f'(x) = 1 - \frac{1}{x^2}$
$\Rightarrow f'(c) = 1 - \frac{1}{c^2}$
Now, according to mean value theorem,
$f'(c) = \frac{f(b) - f(a)}{b - a}$
$\therefore 1 - \frac{1}{c^2} = \frac{\frac{10}{3} - 2}{3 - 1}$
$\Rightarrow 1 - \frac{1}{c^2} = \frac{2}{3}$
$\Rightarrow c^2 = 3$
$\Rightarrow c = \pm\sqrt{3}$
$\Rightarrow c = \sqrt{3}$ [$\because c \in (1, 3)$]

Question 145

Rolle's theorem is true for the function $f(x) = x^2 - 4$ in the interval

Options:

A. $[-2, 0]$

B. $[-2, 2]$

C. $[0, \frac{1}{2}]$

D. $[0, 2]$

Answer: B

Solution:

Solution:
Given function is $f(x) = x^2 - 4$
For Rolle's theorem $f(a) = f(b)$
Let $a = -2$ then $f(a) = (-2)^2 - 4 = 4 - 4 = 0$
and $b = 2$, then $f(b) = (2)^2 - 4 = 4 - 4 = 0$
Hence, $f(-2) = f(2)$
$\therefore$ Required interval is $[-2, 2]$.

Question 146

$\int e^x(1 + \tan x) \sec x \, dx =$

Options:

A. $e^x \sec x$

B. $e^x \cos x$

C. $e^x \cot x$

D. $e^x \tan x$

Answer: A

Solution:

Solution:
Given, $\int e^x . (1 + \tan x) . \sec x \, dx$
$\int e^x . \sec x \, dx + \int e^x . \tan x . \sec x \, dx$
$= e^x . \sec x - \int e^x . \sec x . \tan x \, dx + \int e^x . \sec x . \tan x \, dx$
$= e^x \sec x + C$

Question 147

$$\int_{-1}^{1} \log \frac{2-x}{2+x}\, dx =$$

Options:

A. 2

B. 1

C. -1

D. 0

Answer: D

Solution:

Solution:
Let $f(x) = \log(\frac{2-x}{2+x})$
$\therefore f(-x) = \log(\frac{2-(-x)}{2+(-x)})$
$= \log \frac{2+x}{2-x})$
$= \log[(\frac{2-x}{2+x})^{-1}]$
$= -\log(\frac{2-x}{2+x})$
$= -f(x)$
$\therefore f(x)$ is odd function.
$\therefore \int_{-1}^{1} \log \frac{2-x}{2+x}\, dx = 0$

Question 148

The area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is

Options:

A. $\frac{4}{3}$ sq unit

B. $\frac{32}{3}$ sq unit

C. $\frac{9}{8}$ sq unit

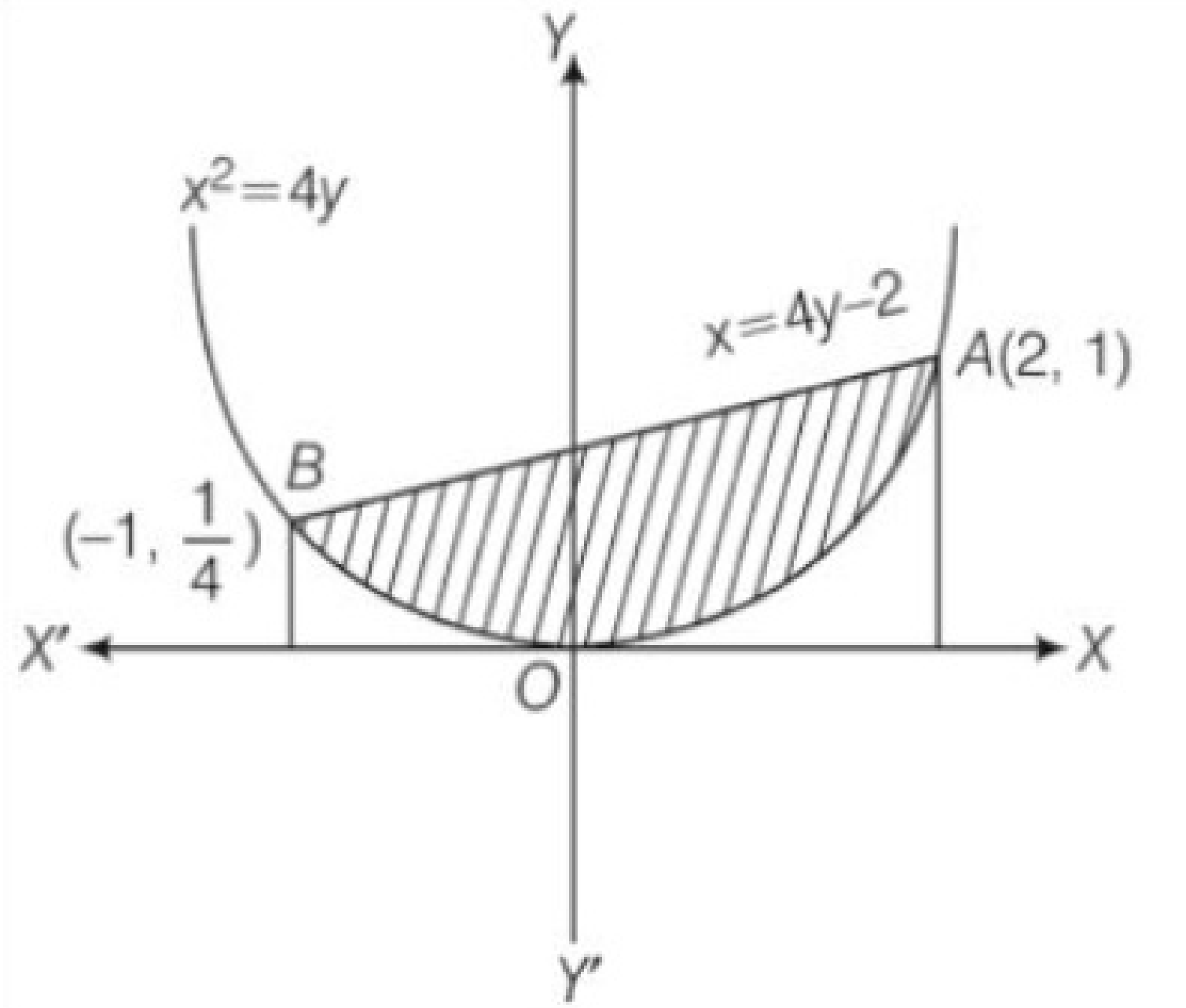
D. $\frac{8}{3}$ sq unit

Answer: C

Solution:

Solution:
Given,
$x^2 = 4y$ .....(i)
and $x = 4y - 2$ .....(ii)

By solving Eqs. (i) and (ii), intersection point is $A(2,1)$ and $B(-1,\frac{1}{4})$.



$\therefore$ Required area = Shaded region

$$= \left[\int_{-1}^{2} y\,.\,dx_{(x=4y-2)}\right] - \left[\int_{-1}^{2} y\,.\,dx_{(x^2=4y)}\right]$$

$$= \int_{-1}^{2} \frac{1}{4}(x+2)\,dx - \int_{-1}^{2} \frac{1}{4}x^2 dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2}+2x\right]_{-1}^{2} - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{9}{8} \text{ sq unit}$$

Question 149

$\int \tan^{-1} x\,dx =$

Options:

A. $x\tan^{-1}x + \frac{1}{2}\log(1+x^2)$

B. $x\tan^{-1}x - \frac{1}{2}\log(1+x^2)$

C. $(x-1)\tan^{-1}x$

D. $x\tan^{-1}x - \log x$

Answer: B

Solution:

Solution:
Given,
$\int \tan^{-1}x\,dx = \int 1.\tan^{-1}x\,dx$
$= x.\tan^{-1}x - \int \frac{x}{1+x^2}dx + C$
$= x.\tan^{-1}x - \frac{1}{2}\int \frac{2x}{1+x^2}dx + C$
$= x.\tan^{-1}x - \frac{1}{2}\log(1+x^2) + C$

Question 150

The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ will be

Options:

A. $e^y = e^x + \frac{x^3}{3} + C$

B. $e^y = e^x + 2x + C$

C. $e^y = e^x + x^3 + C$

D. $y = e^x + C$

Answer: A

Solution:

Solution:
Given,
$\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$
$\Rightarrow \frac{dy}{dx} = e^{-y}(e^x + x^2)$
$\Rightarrow e^y dy = (x^2 + e^x)\,dx$
On integrating both sides, we get
$\Rightarrow e^y = \frac{x^3}{3} + e^x + C$
