CBSE Class 11 Mathematics
Sample Papers 10 (2020-21)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
ii. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
iii. Both Part A and Part B have choices.

Part – A:

i. It consists of two sections- I and II.
ii. Section I comprises of 16 very short answer type questions.
iii. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

i. It consists of three sections- III, IV and V.
ii. Section III comprises of 10 questions of 2 marks each.
iii. Section IV comprises of 7 questions of 3 marks each.
iv. Section V comprises of 3 questions of 5 marks each.
v. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part - A Section - I

1. If A = {1, 2, 3, 4, 5}, B = {4, 5, 6 ,7, 8}, C = {7, 8, 9, 10, 11} and D = {10, 11, 12, 13, 14}. Find: $A \cup B \cup D$.

OR

Is A = {x : |x| = 7, x∈N} a singleton set?

2. Name the octants in which the following points lie
(1, 2, 3), (4, -2, 3), (4, -2, -5), (-4, 2, -5), (-4, 2, 5), (-4, 2, 5), (-3, -1, 6), (2, -4, -7)
3. Find the value of cot (585°)

OR

Prove that: $\cos\frac{7\pi}{12} + \cos\frac{\pi}{12} = \sin\frac{5\pi}{12} - \sin\frac{\pi}{12}$.

4. Find the product of complex numbers (2 + 9i), (11 + 3i).
5. Write the value of $({}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5)$.

OR

How many five - digit numbers can be formed with the digits 5, 4, 3, 5, 3?

6. Find the GM between the numbers -6.3 and -2.8.
7. Find the domain of the real function: f(x) = $\frac{x^2-x+1}{x^2-5x+4}$

OR

If R = {(x, y): x, y ∈ Z, $x^2 + y^2 \leq 4$} is a relation defined on the set Z of integers, then write domain of R.

8. Find the equation of a circle with centre (h, k) and touching the y-axis.
9. A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is 12.

OR

If A and B are two mutually exclusive events such that P(A) = (1/2) and P(B) = (1/3), find P(A or B).

10. A pair of dice is rolled. If the outcome is a doublet, a coin is tossed. Determine the total number of elementary events associated to this experiment.
11. The centroid of a triangle ABC is at the point (1, 1, 1). If the coordinates of A and B are (3, – 5, 7) and (–1, 7, – 6), respectively, find the coordinates of the point C.

12. Find the number of arrangements of the letters of the word ALGEBRA without altering the relative positions of the vowels and the consonants.
13. Prove that $\sin 40^\circ \cos 20^\circ + \cos 40^\circ \sin 20^\circ = \frac{\sqrt{3}}{2}$
14. Prove the identities: $\cot^4 x + \cot^2 x = \operatorname{cosec}^4 x - \operatorname{cosec}^2 x$.
15. Express of the angle in radian: -270°
16. Find the domain of of the following real valued functions $f(x) = \frac{x-1}{x-3}$.

Section - II

17. **Read the Case study given below and attempt any 4 sub parts:**

In drilling world's deepest hole, the Kola Superdeep Borehole, the deepest manmade hole on Earth and deepest artificial point on Earth, as a result of a scientific drilling project, it was found that the temperature T in degree Celsius, x km below the surface of Earth, was given by:

$T = 30 + 25 (x - 3), 3 < x < 15.$

If the required temperature lies between 200^0 C and 300^0 C, then

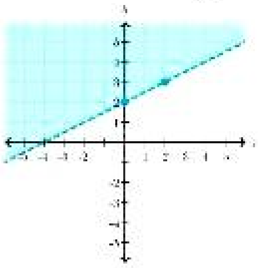
i. the depth, x will lie between
   a. 9 km and 13 km
   b. 9.8 km and 13.8 km
   c. 9.5 km and 13.5 km
   d. 10 km and 14 km

ii. Solve for x. $-9x+2 > 18$ OR $13x+15 \le -4$
   a. $\mathbf{x} \le \frac{-19}{13}$
   b. $x < \frac{-16}{13}$

c. $\frac{-16}{13} < x < \frac{-19}{13}$

d. There are no solution.

iii. **Find the inequality represented by the graph**



a. $y \leq \frac{1}{2}x+2$

b. $y > \frac{1}{2}x+2$

c. $y \geq \frac{1}{2}x+2$

d. $y < \frac{1}{2}x+2$

iv. If $|x| < 5$ then the value of x lies in the interval

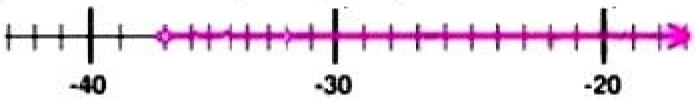
a. $(-\infty, -5)$

b. $(\infty, 5)$

c. $(-5, \infty)$

d. $(-5, 5)$

v. Graph the following inequality on the number line: $x > -32$

a. 

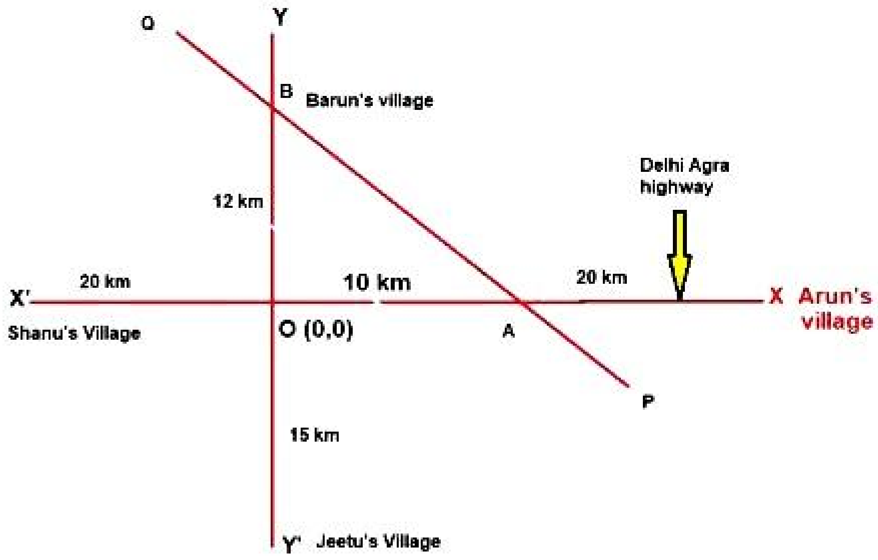
b. 

c. 

d. 

18. **Read the Case study given below and attempt any 4 sub parts:**

villages of Shanu and Arun's are 50km apart and are situated on Delhi Agra highway as shown in the following picture. Another highway YY' crosses Agra Delhi highway at O(0,0). A small local road PQ crosses both the highways at pints A and B such that OA=10 km and OB =12 km. Also, the villages of Barun and Jeetu are on the smaller high way YY'. Barun's village B is 12km from O and that of Jeetu is 15 km from O.



Now answer the following questions:

i. What are the coordinates of A?

   a. (10, 0)
   b. (10, 12)
   c. (0,10)
   d. (0,15)

ii. What is the equation of line AB?

   a. 5x + 6y = 60
   b. 6x + 5y = 60
   c. x = 10
   d. y = 12

iii. What is the distance of AB from O(0, 0)?

   a. 60 km
   b. 60/√61 km
   c. √61 km
   d. 60 km

iv. What is the slope of line AB?

   a. 6/5
   b. 5/6
   c. -6/5
   d. 10/12

v. What is the length of line AB?

   a. √61 km

b. 12 km

c. 10 km

d. 2√ 61 km

Part - B Section - III

19. Write the set in roster form: C = {x : x is a two-digit number such that the sum of its digits is 9}.

20. Let R = {(x, y): x + 3y = 12, x ∈ N and y ∈ N}.

    i. Write R in roster form.

    ii. Find dom (R) and range (R).

OR

If f(x) = cos $\left[\pi^2\right]x + \cos\left[-\pi^2\right]x$ where (x) denotes the greatest integer less than or equal to x then write the value of f(x).

21. Prove that: 3 sin $\frac{\pi}{6}$ sec $\frac{\pi}{3}$ - 4 sin $\frac{5\pi}{6}$ cot $\frac{\pi}{4}$ = 1.

22. Solve the quadratic equation by using the general expressions for the roots of a quadratic equation: $2x^2 + 3ix + 2 = 0$

23. Solve the quadratic equation: $ix^2 - 4x - 4i = 0$.

OR

Show that $\left|\frac{z-2}{z-3}\right|$ = 2 represents a circle. Find its centre and radius.

24. Find the domain and the range of the real function: f(x) = $\sqrt{x-3}$

25. Evaluate: $\lim\limits_{n\to\infty} \frac{n^2}{1+2+3+...+n}$.

26. Find the derivative from the first principle: $\frac{1}{\sqrt{x+2}}$.

27. Let A and B are sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X. Show that A = B.

    [Hints A = A ∩ (A ∪ X) , B = B ∩ (B ∪ X) and use Distributive law]

28. Express $\left(\frac{3-\sqrt{-16}}{1-\sqrt{-9}}\right)$ in the form (a + ib).

OR

Solve: $13x^2 + 7x + 1 = 0$.

Section - IV

29. Discuss the existence of the limits: $\lim_{x \to 0} \frac{1}{|x|}$.

30. From a well-shuffled deck of 52 cards, 4 cards are drawn at random. What is the probability that all the drawn cards are of the same colour?

31. The side of a given square is 10 cm. The midpoints of its sides are joined to form a new square. Again, the midpoints of the sides of this new square are joined to form another square. This process is continued indefinitely. Find the sum of the perimeters of the squares.

OR

Find the sum of the terms of an infinite decreasing G.P. in which all the terms are positive, the first term is 4, and the difference between the third and fifth term is equal to $\frac{32}{81}$.

32. A man running a racecourse notes that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m. Find the equation of the path traced by the man.

33. Find the sum of all the numbers that can be formed with the digits 2, 3, 4, 5 taken all at a time.

34. For any two sets A and B, prove that $A \cup B = A \cap B \Leftrightarrow A = B$

OR

There are 210 members in a club. 100 of them drink tea and 65 drink tea but not coffee, each member drinks tea or coffee. Find how many drink coffee, How many drink coffee, but not tea.

35. Find the range of the function $f(x) = \frac{|x+4|}{x+4}$.

Section - V

36. If A is the arithmetic mean and G_1, G_2 be two geometric means between any two number, then prove that $2A = \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$..

OR

The ratio of the A.M. and G.M. of two positive numbers a and b is $m : n$. Show that

$$a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$$

37. Calculate mean, variance and standard deviation of the following frequency distribution:

Class:	0-10	10-20	20-30	30-40	40-50	50-60
Frequency:	11	29	18	4	5	3

OR

While calculating the mean and variance of 10 readings, a student wrongly used the reading 52 for the correct reading 25. He obtained the mean and variance as 45 and 16 respectively. Find the correct mean and the variance.

38. A small manufacturing firm produces two types of gadgets A and B, which are first processed in the foundry and then sent to another machine for finishing. The number of man-hours of labour required in each shop for the production of each unit of A and B and the number of man-hours for the firm available per week are as follows:

	Foundry	**Machine shop**
Man-hours for 1 unit of gadget A	10	5
Man-hours for 1 unit of gadget B	6	4
Firm's capacity per week in man-hours	1000	600

Formulate it in the form of linear inequations. Draw the graph showing the solution of all these inequations.

OR

A man wants to cut three lengths from a single piece of board of length 91cm. The second length is to be 3cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5cm longer than the second?

[Hint: If x is the length of the shortest board, then x , (x + 3) and 2x are the lengths of the second and third piece, respectively. Thus, $x + (x + 3) + 2x \leq 91$ and $2x \geq (x + 3) + 5$].

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Solution

Part - A Section - I

1. Here,it is given that

   A = {1, 2, 3, 4, 5}, B = {4, 5, 6 ,7, 8), C = {7, 8, 9, 10, 11} and D = {10, 11, 12, 13, 14}

   $A \cup B \cup D$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}.

   OR

   We have, A = {x : |x| = 7, x∈N} = {7}

   $[\because |x|=7 \Rightarrow x=\pm 7, \text{ but } x \in N]$

   So, A is a singleton set.

2. Point (1, 2, 3) lies in Ist Octant.

   Point (4, - 2, 3) lies in IVth Octant. Point (4, -2, -5) lies in VIIIth Octant.

   Point (4, 2, -5) lies in Vth Octant. Point (-4, 2, -5) lies in VIth octant.

   Point (-4, 2, 5) lies in IInd Octant. Point (-3, -1, 6) lies in VIIIth octant.

   Point (2, -4, -7) lies in VIIIth Octant.

3. Let y = cot (585°), then

   y = cot (585°) $= \cot(90^\circ \times 6 + 45^\circ)$

   = cot 45° = 1

   OR

   To prove $\cos\frac{7\pi}{12} + \cos\frac{\pi}{12} = \sin\frac{5\pi}{12} - \sin\frac{\pi}{12}$

   $105^\circ = \frac{7\pi}{12}, 15^\circ = \frac{\pi}{12}, 75^\circ = \frac{5\pi}{12}, 15^\circ = \frac{\pi}{12}$

   Now take LHS

   $\cos 105^o + \cos 15^o$

   $= \cos (90^o + 15^o) + \cos(90^o - 75^o)$

   $= -\sin 15^o + \sin 75^o$ [As $\cos(90^o + A) = -\sin A$ and $\cos(90^o - B) = \sin B$]

   $= \sin 75^o - \sin 15^o$

   = RHS

Hence proved.

4. $(2 + 9i).(11 + 3i) = 2 \times 11 + 2 \times 3i + 11 \times 9i + 9 \times 3i^2$

$= 22 + 6i + 99i - 27$ [$\because i^2 = -1$]

$= -5 + 105i$

5. $\Rightarrow {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$

$\Rightarrow {}^6C_2 + {}^6C_4 + 1$ ......[As ${}^5C_5 = 1$]

$\Rightarrow 15 + 15 + 1 = 31$

OR

To find: Number of 5 - digit numbers that can be formed

2 numbers are of 1 kind, and 2 are of another kind

Total number of permutations $= \frac{5!}{2!2!} = 30$

30 number can be formed

6. Given: -6.3 and -2.8

Geometric mean between a and b = $\sqrt{ab}$

Geometric mean of two numbers = $\sqrt{ab}$

$= \sqrt{-6.3 \times -2.8} = \pm 4.2$

Therefore, the required geometric mean between -6.3 and -2.8 is -4.2.

7. The given function is,

$f(x) = \frac{x^2 - x + 1}{x^2 - 5x + 4}$

Clearly, f(x) is defined for all real values of x except those at which

$x^2 - 5x + 4 = 0$.

But, $x^2 - 5x + 4 = 0$

$\Rightarrow (x - 1)(x - 4) = 0$

$\Rightarrow x = 1$ or $x = 4$.

$\therefore$ dom (f) = R - {1, 4}.

OR

We have given that, relation R is defined on Z of integers

And $R = \{(x,y) : x, y \in Z, x^2 + y^2 \leq 4\}$

$= \{(-2, 0),(-1, 0),(0, 0),(1, 0),(2, 0),(0, -2),(0, -1),(0, 1),(0, 2),(1, 1),(-1, -1),(1, -1),(-1, 1)\}$

Now we know,Domain is the set which consist all first elements of ordered pairs in relation R.

So,Domain(R) = {-2, -1, 0, 1, 2}

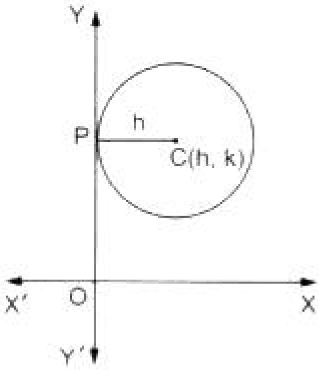
8. Given: a circle with centre (h, k) and touching the y-axis.

If a circle is touching only y-axis then,

radius of the circle = | x coordinate of centre | = h.

So, the required equation is $(x - h)^2 + (y - k)^2 = h^2$

$\Rightarrow x^2 + y^2 - 2hx - 2ky + k^2 = 0$



9. The coin with 1 marked on one face and 6 on the other face.

The coin and die are tossed together.

∴ S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

⇒ smpale space n(S) = 12

Let B be the event having a sum of the numbers is 12

therefore, B = {(6, 6)}

⇒ n (B) = 1

Thus, $P(B) = \frac{1}{12}$

OR

Given : A and B are mutually exclusive events

$P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$

To find : P(A or B)

Formula used : P(A or B) = P(A) + P(B) - P(A and B)

For mutually exclusive events A and B, P(A and B) = 0

Substituting in the above formula we get,

$P(A \text{ or } B) = \frac{1}{2} + \frac{1}{3} - 0$

$P(A \text{ or } B) = \frac{5}{6}$.

10. A pair of dice is rolled, then, No. of elementary events are 6^2 = 36

Now, If outcomes are doublet means (1, 1)(2, 2)(3, 3)(4, 4)(5, 5)(6, 6), then a coin is tossed.

If the coin is tossed then no. of sample spaces is 2

So, The total no. of elementary events including doublet = 6 × 2 = 12

Thus, The Total number of elementary events is 30 + 12 = 42 events.

11. Let the coordinates of C be (x, y, z) and the coordinates of the centroid G be (1, 1, 1). Then

$\frac{x+3-1}{3} = 1, \Rightarrow$ x = 1

$\frac{y-5+7}{3} = 1, \Rightarrow$ y = 1

$\frac{z+7-6}{3} = 1, \Rightarrow$ z= 2

Hence, coordinates of C are (1, 1, 2).

12. To find: number of arrangements without changing the relative position

The following table shows where the vowels and consonants can be placed

Consonants can be placed in the blank places

vowel, __, __, __, vowel, __, __, vowel

There are 3 spaces for vowels

There are 3 vowels out of which 2 are alike

Vowels can be placed in $\frac{3!}{2!}$ = 3 ways

There are 4 consonants, and they can be placed in 4! = 24 ways

Total number of arrangements = 24×3 = 72 ways

72 arrangements can be made

13. sin 40° cos 20° + cos 40° sin 20° = sin (40° + 20°)

= sin 60° $= \frac{\sqrt{3}}{2}$.

14. LHS = $\cot^4 x + \cot^2 x = (\cot^2 x)^2 + \cot^2 x$

$= (\operatorname{cosec}^2 x - 1)^2 + (\operatorname{cosec}^2 x - 1)$ [$\because 1 + \cot^2 x = \operatorname{cosec}^2 x$]

$= \operatorname{cosec}^4 x - 2 \operatorname{cosec}^2 x + 1 + \operatorname{cosec}^2 x - 1 = \operatorname{cosec}^4 x - \operatorname{cosec}^2 x$ = RHS

15. We know that, Angle in radians = Angle in degrees $\times \frac{\pi}{180}$

Therefore, Angle in radians = $-270 \times \frac{\pi}{180} = -\frac{3\pi}{2}$

16. We have $f(x) = \frac{2x-3}{x^2-3x+2}$

We observe that f (x) is a rational function of x as $\frac{x-1}{x-3}$ is a rational expression.

Clearly, f(x) assumes real values for all x except for the values of x for which x - 3 = 0 i.e. x = 3.

Hence,Domain (f) = R - {3}.

Section - II

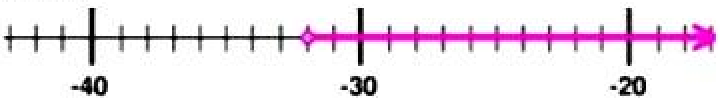
17. i. (b) 9.8 km and 13.8 km

  ii. (a)

  $\frac{-19}{13} \leq \mathbf{x}$

  iii. (b) y > $\frac{1}{2}$x+2

  iv. (d) (-5, 5)

  v. (d) 

18. i. (a) (10, 0)

  ii. (b) 6x + 5y = 60

  iii. (b) 60/√ 61 km

  iv. (c) -6/5

  v. (d) 2√61 km

Part - B Section - III

19. We have,

  9 = 0 + 9, Numbers can be 09, 90

  9 = 1 + 8, Numbers can be 18, 81

  9 = 2 + 7, Numbers can be 27, 72

  9 = 3 + 6, Numbers can be 36, 63

  9 = 4 + 5, Numbers can be 45, 54

  9 = 5 + 4, Numbers can be 54, 45

  The elements of this set are 18, 27, 36, 45, 54, 63, 72, 81 and 90 and

  Therefore, C = {18, 27, 36, 45, 54, 63, 72, 81, 90}

20. Here we have, A = {(x, y): x + 3y = 12, x ∈ N and y ∈ N}

  i. Roster form of R,

  R = {{3, 3}, {6, 2}, {9, 1}}

  ii. Domain or R = {3, 6, 9}

  Range of R = {1, 2, 3}

OR

Given , in the question,

If π^2 = 9.8596

So, we have $[\pi 2] = 9$ and $[-\pi 2] = -10$

f(x) = cos 18x+ cos (-10)x

= cos 18x + cos 10x

$= 2\cos\left(\frac{18x+10x}{2}\right)\cos\left(\frac{18x-10x}{2}\right)$

$= f(\pi) = 2\cos 14\pi \cos 4\pi$

$f(\pi) = 2\cos 14\pi \cos 4\pi$

= $2 \times 1 \times 1$

Therefore, $f(\pi) = 2$

21. **LHS** $= 3sin\ \frac{\pi}{6} \times sec\ \frac{\pi}{3} - 4sin\ \frac{5\pi}{6} \times cot\ \frac{\pi}{4}$

= $3 \times \frac{1}{2} \times 2 - 4\sin\left(\pi - \frac{\pi}{6}\right) \times \cot\frac{\pi}{4}$

= $3 - 4\sin\frac{\pi}{6} \times 1$ [$\because \sin(\pi - \theta) = \sin\theta$]

$= 3 - 4 \times \frac{1}{2} = 3 - 2 = 1 =$ **RHS**

$\therefore$ **LHS** = **RHS**

Hence proved.

22. Given: $2x^2 + 3ix + 2 = 0$ ...(i)

On comparing Eq. (i) with $ax^2 + bx + c = 0$, we get

a = 2, b = 3i and c = 2

$\because \alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$

$\therefore \alpha = \frac{-3i+\sqrt{(3i)^2-4\times 2\times 2}}{2\times 2}$

= $\frac{-3i+\sqrt{-9-16}}{4}$ [$\because i^2 = -1$]

= $\frac{-3i+\sqrt{-25}}{4} = \frac{-3i+5i}{4} = \frac{i}{2}$

and $\beta = \frac{-3i-\sqrt{(3i)^2-4\times 2\times 2}}{2\times 2}$

= $\frac{-3i-\sqrt{-9-16}}{4}$

= $\frac{-3i-\sqrt{-25}}{4} = \frac{-3i-5i}{4} = -2i$

23. given equation

$ix^2 - 4x - 4i = 0$

$\Rightarrow i(x^2 + 4ix - 4) = 0$

$\Rightarrow (x^2 + 4ix - 4) = 0$

$\Rightarrow (x + 2i)^2 = 0$

$\Rightarrow$ x + 2i = 0

$\Rightarrow$ x = -2i

So, the roots of the given quadratic equation are -2i and -2i.

OR

We have $\left|\frac{z-2}{z-3}\right| = 2$

putting = x + iy, we get

$\left|\frac{x+iy-2}{x+iy-3}\right| = 2$

$\frac{|x+iy-2|}{|x+iy-3|} = 2$

$\Rightarrow |x - 2 + iy| = 2|x - 3 + iy| \Rightarrow \sqrt{(x-2)^2+y^2} = 2\sqrt{(x-3)^2+y^2}$

Squaring both the sides, we get

$\Rightarrow x^2 - 4x + 4 + y^2 = 4(x^2 - 6x + 9 + y^2) \Rightarrow 3x^2 + 3y^2 - 20x + 32 = 0$

$\Rightarrow x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0 \Rightarrow \left(x - \frac{10}{3}\right)^2 + y^2 + \frac{32}{3} - \frac{100}{9} = 0$

$\Rightarrow \left(x - \frac{10}{3}\right)^2 + (y - 0)2 = \frac{4}{9}$

Hence, the centre of the circle is $\left(\frac{10}{3}, 0\right)$ and radius is $\frac{2}{3}$.

24. Here we are given that, $f(x) = \sqrt{x-3}$

Clearly, f(x) is defined for all real values of x for which $x - 3 \geq 0$, i.e., $x \geq 3$

$\therefore$ dom (f) = $[3, \infty)$

Also, $x \geq 3 \Rightarrow f(x) = \sqrt{x-3} \geq 0$

$\therefore$ range (f) = $[0, \infty)$

Hence, dom (f) = $[3, \infty)$ and range (f) = $[0, \infty)$

25. $\lim_{n\to\infty}\left[\frac{n^2}{1+2+3.\ldots..n}\right]$

It is of the form $\frac{\infty}{\infty}$

$\Rightarrow \lim_{n\to\infty}\left[\frac{n^2}{n\frac{(n+1)}{2}}\right]$

$= \lim_{n\to\infty}\left[\frac{2n}{n+1}\right]$

Dividing the numerator and the denominator by n:

$\lim_{n\to\infty}\frac{2}{1+\frac{1}{n}}$

= 2

$\therefore \lim\limits_{n\to\infty} \frac{n^2}{1+2+3+\ldots+n} = 2$

26. Let $f(x) = \frac{1}{\sqrt{x+2}}$

We need to find the derivative of f(x) i.e. f'(x)

We know that by using first principle,

$f'(x) = \lim_{h\to0} \frac{f(x+h)-f(x)}{h}$ ...(i)

$f(x) = \frac{1}{\sqrt{x+2}}$

$f(x+h) = \frac{1}{\sqrt{x+h+2}}$

Put values in (i), we get

$f\text{ (x) } = \lim_{h\to0} \frac{\frac{1}{\sqrt{x+h+2}} - \frac{1}{\sqrt{x+2}}}{h}$

$= \lim_{h\to0} \frac{\frac{\sqrt{x+2}-\sqrt{x+h+2}}{(\sqrt{x+h+2})(\sqrt{x+2})}}{h}$

Now rationalizing the numerator by multiplying and divide by the conjugate of

$\sqrt{x+2} - \sqrt{x+h+2}$

$= \lim_{h\to0} \frac{\sqrt{x+2}-\sqrt{x+h+2}}{h(\sqrt{x+h+2})(\sqrt{x+2})} \times \frac{\sqrt{x+2}+\sqrt{x+h+2}}{\sqrt{x+2}+\sqrt{x+h+2}}$

$= \lim_{h\to0} \frac{(\sqrt{x+2})^2-(\sqrt{x+h+2})^2}{h(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2}+\sqrt{x+h+2})}$

$= \lim_{h\to0} \frac{x+2-x-11-2}{h(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2}+\sqrt{x+h+2})}$

$= \frac{-1}{(\sqrt{x+0+2})(\sqrt{x+2})(\sqrt{x+2}+\sqrt{x+0+2})}$

$= \frac{-1}{(\sqrt{x+2})^2(2\sqrt{x+2})} = \frac{-1}{2(\sqrt{x+2})^3}$

Hence $f'(x) = \frac{-1}{2(\sqrt{x+2})^3}$

27. Here $A \cup X = B \cup X$ for some X

$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$

$\Rightarrow A = (A \cap B) \cup (A \cap X)$ $[\because A \cap (A \cup X) = A]$

$\Rightarrow A = (A \cap B) \cup \phi [\because A \cap X = \phi]$

$\Rightarrow A = A \cap B \ldots$ (i)

$\Rightarrow A \subset B$

Also $A \cup X = B \cup X$

$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$

$\Rightarrow (B \cap A) \cup (B \cap X) = B [\because B \cap (B \cup X) = B]$

$\Rightarrow (B \cap A) \cup \phi = B [\because B \cap X = \phi]$

$\Rightarrow B \cap A = B$

$\Rightarrow B \cap A...$ (ii)

From (i) and (ii), we have

A = B.

28. We have, $\frac{3-\sqrt{-16}}{1-\sqrt{-9}}$

$\frac{3-\sqrt{-16}}{1-\sqrt{-9}} = \frac{3-4i}{1-3i}$ [$\sqrt{-1}$ = i]

$\frac{3-\sqrt{-16}}{1-\sqrt{-9}} = \frac{3-4i}{1-3i} \times \frac{1+3i}{1+3i}$ [multiply and divide by 1 + 3i]

$\frac{3-\sqrt{-16}}{1-\sqrt{-9}} = \frac{3+9i-4i-12i^2}{(1)^2-(3i)^2}$

$\frac{3-\sqrt{-16}}{1-\sqrt{-9}} = \frac{15+5i}{1+9} = \frac{15}{10} + \frac{5i}{10} = \frac{3}{2} + \frac{1}{2}i$

Hence, $\frac{3-\sqrt{-16}}{1-\sqrt{-9}} = \frac{3}{2} + \frac{i}{2}$

OR

Given: $13x^2 + 7x + 1 = 0$

Comparing $13x^2 + 7x + 1 = 0$ with the general form of the quadratic equation $ax^2 + bx + c = 0$,

we get a = 13, b = 7 and c = 1.

Substituting these values in $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$, we get

$\alpha = \frac{-7+\sqrt{49}-4\times13\times1}{2\times13}$ and $\beta = \frac{-7-\sqrt{49-4\times13\times1}}{2\times13}$

$\Rightarrow \alpha = \frac{-7+\sqrt{49}-52}{26}$ and $\beta = \frac{-7-\sqrt{49-52}}{26}$

$\Rightarrow \alpha = \frac{-7+\sqrt{-3}}{26}$ and $\beta = \frac{-7-\sqrt{-3}}{26}$

$\Rightarrow \alpha = \frac{-7+i\sqrt{3}}{26}$ and $\beta = \frac{-7-i\sqrt{3}}{26}$

$\Rightarrow \alpha = -\frac{7}{26} + \frac{\sqrt{3}}{26}i$ and $\beta = -\frac{7}{26} - \frac{\sqrt{3}}{26}i$

Hence, the roots of the equation are $-\frac{7}{26} \pm \frac{\sqrt{3}}{26}i$

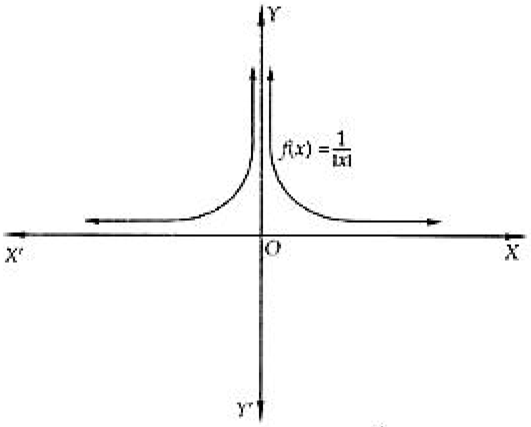
Section - IV

29. The graph of f(x) = $\frac{1}{|x|}$ is shown in Fig.

We observe that as x approaches to 0 from the LHS i.e. x is negative and very close to zero, then |x| is close to zero and is positive.

Consequently, $\frac{1}{|x|}$ is large and positive.

$\therefore \lim_{x\to0^-} \frac{1}{|x|} \to -\infty$



Similarly, when x approaches to 0 from RHS i,e, x is positive and close to 0, then |x| is close to zero and is positive.

Consequently, $\frac{1}{|x|}$

$\therefore \lim_{x \to 0^+} \frac{1}{|x|} \to \infty$

Now, $\lim_{x \to 0^-} \frac{1}{|x|} \neq \lim_{x \to 0^+} \frac{1}{|x|}$.

Therefore, $\lim_{x \to 0} \frac{1}{|x|}$ does not exist.

30. We have to find the probability that all the drawn cards are of the same colour

Out of 52 cards, four cards can be randomly chosen in $^{52}C_4$ ways.

$\therefore$ n(S) = $^{52}C_4$

Let A = event where the four cards drawn are red

and B = event where the four cards drawn are black

Then, n(A) = ^{26}C4 and n(B) = $^{26}C_4$

$\Rightarrow$ P(A) = $\frac{^{25}C_4}{^{52}C_4}$ and PB = $\frac{^{25}C_4}{^{52}C_4}$

A and B are mutually exclusive events.

i.e. P $(A \cap B)$ = 0

By addition theorem, we have:

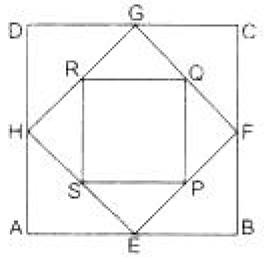
P $(A \cup B)$ = P(A) + P (B) – P$(A \cap B)$

$= \frac{26}{\sqrt[3]{2}C_4} + \frac{25C_4}{652C_4}$ - 0

$= \frac{46}{17 \times 49} + \frac{46}{17 \times 49}$

$= 2 \times \frac{46}{17 \times 49} = \frac{92}{838}$

Hence, the probability that all the drawn cards are of the same colour is $\frac{92}{833}$.

31. 

Suppose ABCD be the given square with each side equal to 10 cm. Suppose E, F, G, H be the midpoints of the sides AB, BC, CD and DA respectively. Suppose P, Q, R, S be the midpoints of the sides EF, FG, GH and HE respectively.

$\therefore$ BE = BF = 5 cm

$\Rightarrow$ EF = $\sqrt{BE^2 + BF^2}$ = $\sqrt{25+25}$ cm = $\sqrt{50}$ cm = $5\sqrt{2}$ cm

$\therefore$ FQ = FP = $\frac{1}{2}$ EF = $\frac{5\sqrt{2}}{2}$ cm = $\frac{5}{\sqrt{2}}$ cm

$\Rightarrow$ PQ = $\sqrt{FP^2 + FQ^2}$ = $\sqrt{\frac{25}{2} + \frac{25}{2}}$ cm = $\sqrt{25}$ cm = 5 cm.

Therefore, the sides of the squares are 10 cm, $5\sqrt{2}$ cm, 5 cm, ...

Sum of perimeters of the squares formed

= (40 + $20\sqrt{2}$ + 20 + ...) cm

= $\frac{40}{\left(1-\frac{1}{\sqrt{2}}\right)}$ cm = $\frac{40\sqrt{2}}{(\sqrt{2}-1)} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}$ cm = (80 + $40\sqrt{2}$) cm

OR

Let r be the common ratio of the given G.P.

$\therefore$ a = 4

Sum of the geometric infinite series;

$S_\infty = 4 + 4r + 4r^2 + \ldots \infty$

Now, $S_\infty = \frac{4}{1-r}$ .... (i)

The difference between the third and fifth term is $\frac{32}{81}$

$a_3 - a_5 = \frac{32}{81}$

$\Rightarrow 4r^2 - 4r^4 = \frac{32}{81}$

$\Rightarrow 4\left(r^2 - r^4\right) = \frac{32}{81}$

$\Rightarrow$ $81r^4 - 81r^2 + 8 = 0$ .... (ii)

Now, suppose $r^2 = y$

Let us put this in (ii)

$\therefore$ $81r^4 - 81r^2 + 8 = 0$

$\Rightarrow 81y^2 - 81y + 8 = 0$

$\Rightarrow 81y^2 - 72y - 9y + 8 = 0$

$\Rightarrow 9y(9y - 1) -8(9y - 1) = 0$

$\Rightarrow (9y - 8)(9y - 1)$

$\Rightarrow y = \frac{1}{9}, \frac{8}{9}$

Substituting $y = r^2$, we obtain $r = \frac{1}{3}$ and $\frac{2\sqrt{2}}{3}$

Putting $r = \frac{1}{3}$ and $\frac{2\sqrt{2}}{3}$ in (i);

$S_\infty = \frac{4}{1-\frac{1}{3}} = \frac{12}{2} = 6$

Similarly, $S_\infty = \frac{4}{1-\frac{2\sqrt{2}}{3}} = \frac{12}{3-2\sqrt{2}}$

$\therefore S_\infty = 6, \frac{12}{3-2\sqrt{2}}$

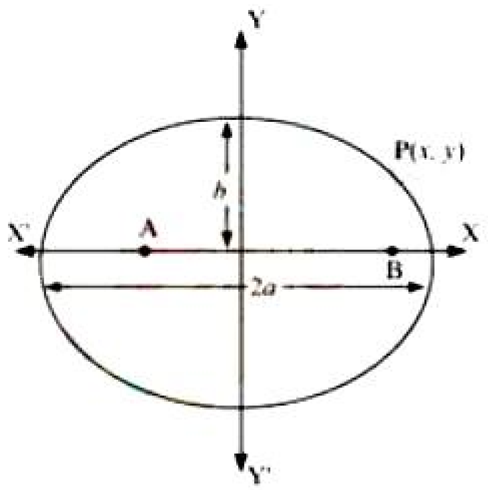
32. Let A and B be the positions of the two flag posts and P(x, y) be the position of the man.

Accordingly, PA + PB = 10

We know that if a point moves in-plane in such a way that the sum of its distance from two fixed points is constant, then the path is an ellipse and this constant value is equal to the length of the major axis of the ellipse.

Therefore, the path described by the man is an ellipse where the length of the major axis is 10m, while points A and B are the foci.

Taking the origin of the coordinate plane as the center of the ellipse, while taking the major axis along the x-axis,



The equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a is the semi-major axis.

Accordingly, 2a = 10 $\Rightarrow$ a = 5

Distance between the foci = 2ae = 2c = 8

$\Rightarrow$ c = 4

On using the relation, c = $\sqrt{a^2 - b^2}$, we get,

4 = $\sqrt{25 - b^2}$

$\Rightarrow$ 16 = 25 – b^2

$\Rightarrow$ b^2 = 25 -1 6 = 9

$\Rightarrow$ b = 3

Put value of a and b in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1 \Rightarrow \frac{x^2}{25} + \frac{y^2}{9} = 1$.

33. The total number of numbers formed with the digits 2, 3, 4, 5 taken all at a time = Number of the arrangement of 4 digits, taken all at a time = ${}^4P_4$ = 4! = 24.

To find the sum of these 24 numbers, we will find the sum of digits at unit's, ten's, hundred's and thousand's places in all these numbers.

Consider the digits in the unit's places in all these numbers. Each of the digits 2, 3, 4, 5 occurs in 3! (= 6) times in the unit's place.

So, total for the digits in the unit's place in all the numbers = (2 + 3 + 4 + 5) × 3! = 84

Since each of the digits 2, 3, 4, 5 occurs 3! times in any one of the remaining places.

So, the sum of the digits in the ten's, hundred's and thousand's places in all the numbers = (2+ 3 + 4 + 5) × 3! = 84.

Hence, the sum of all the numbers = 84 (10° + 101 + 102 +103) = 93324

34. Let $A \cup B = A \cap B$

Then, we have to prove that A = B

Let $x \in A \Rightarrow x \in A \cap B$

$\Rightarrow x \in A \cap B \ (\because A \cup B = A \cap B)$

$\Rightarrow x \in A and x \in B$

$\Rightarrow x \in B$

$\therefore A \subseteq B \ldots$ (i)

Let $x \in B \Rightarrow x \in A \Rightarrow x \in A \cup B$

$\Rightarrow x \in A \cap B \ (\because A \cup B = A \cap B)$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in A$

$\therefore B \subseteq A \ldots$ (ii)

From (i) and (ii), we get

A = B

Thus $A \cup B = A \cap B \Rightarrow A = B...$ (iii)

Now let A = B

Thus, we have to prove that $A \cup B = A \cap B$

$\therefore A \cup B = A$ and $A \cap B = A$

$\Rightarrow A \cup B = A \cap B$

Thus A = B $\Rightarrow A \cup B = A \cap B...$ (iv)

From (iii) and (iv), we get $A \cup B = A \cap B \Leftrightarrow A = B.$

OR

n (T) = 100

n (T - C) = 65

$n(T \cup C) = 210$

n (T - C) = n (T) - n $(T \cap C)$

65 = 100 - n $(T \cap C)$

$n(T \cap C) = 35$

$n(T \cup C)$ = n (T) + n (C) - n $(T \cap C)$

210 = 100 + n (C) - 35

n (C) = 145.

Now,

$n(C - T) = n(C) - n(C \cap T)$

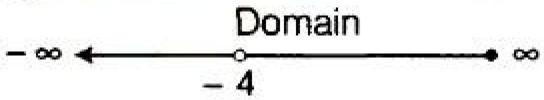
n(C-T) = 145-35

n(C-T) = 110

35. Given, $f(x) = \frac{|x+4|}{x+4}$

Clearly, for f to be defined, the denominator $x + 4 \neq 0$ i.e., $x \neq -4.$

$\therefore$ the domain of f is the set of all real numbers excluding -4.



Now, consider the two cases

When $x + 4 > 0$ i.e., $x > -4$

Then, $|x + 4| = x + 4,$

$\Rightarrow f(x) = \frac{|x+4|}{x+4} = \frac{x+4}{x+4} = 1$ for all $x > -4$

When $x + 4 < 0$ i.e., $x < -4$

Then, $|x+4| = -(x+4)$

$\therefore \quad f(x) = \frac{|x+4|}{x+4} = \frac{-(x+4)}{(x+4)} = -1$ for all $x < -4$

$f(x) = \begin{cases} 1, & \text{if } x > -4 \\ -1, & \text{if } x < -4 \end{cases}$

$\therefore$ The range of f is $\{-1, 1\}$.

Section - V

36. let the two numbers be x and y

$\therefore A = \frac{x+y}{2}$ ....(i)

If G_1 and G_2 be the geometric means between x and y then x, G_1, G_2, y are in G.P.

Then $y = xr^{4-1}$ [$\because a_n = ar^{n-1}$]

$\Rightarrow y = xr^3 \Rightarrow \frac{y}{x} = r^3$

$\Rightarrow r = \left(\frac{y}{x}\right)^{1/3}$

Now $G_1 = xr = x\left(\frac{y}{x}\right)^{13} \left[\because r = \left(\frac{y}{x}\right)^{1/3}\right]$

and $G_2 = x\sigma^2 = x\left(\frac{y}{x}\right)^{2/3}$

$\therefore$ from RHS $\frac{G_1^2}{G_2} + \frac{G_2^2}{G_1} = \frac{x^2\left(\frac{y}{x}\right)^{2/3}}{x\left(\frac{y}{x}\right)^{2/3}} + \frac{x^2\left(\frac{y}{x}\right)^{4/3}}{x\left(\frac{y}{x}\right)^{1/3}}$

$= x + x\left(\frac{y}{x}\right)^{\frac{4}{3}-\frac{1}{3}} = x + x\left(\frac{y}{x}\right)$

= x + y = 2A LHS. [using eq. (i)]

$\therefore$ LHS = RHS Hence proved.

OR

Given: $\frac{a+b}{2} : \sqrt{ab} = m : n$

$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$

By componendo and dividendo,

$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{m+n}{m-n}$

$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{m+n}{m-n}$

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{m+n}}{\sqrt{m-n}}$

Again by componendo and dividendo,

$$\frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$$

$$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$$

$$\Rightarrow \frac{a}{b} = \frac{(\sqrt{m+n}+\sqrt{m-n})^2}{(\sqrt{m+n}-\sqrt{m-n})^2}$$

$$\Rightarrow \frac{a}{b} = \frac{m+n+m-n+2\sqrt{(m+n)(m-n)}}{m+n+m-n-2\sqrt{(m+n)(m-n)}}$$

$$\Rightarrow \frac{a}{b} = \frac{2m+2\sqrt{(m+n)(m-n)}}{2m-2\sqrt{(m+n)(m-n)}}$$

$$\Rightarrow \frac{a}{b} = \frac{m+\sqrt{(m+n)(m-n)}}{m-\sqrt{(m+n)(m-n)}}$$

Therefore, $a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$

37.

class interval	f	Mid-value, x_i	$u_i = \frac{x_i - 35}{10}$	$f_i u_i$	u_i^2	$f_i u_i^2$
0-10	11	5	-3	-33	9	99
10-20	29	15	-2	-58	4	116
20-30	18	25	-1	-18	1	18
30-40	4	35	0	0	0	0
40-50	5	45	1	5	1	5
50-60	3	55	2	6	4	12
	$N = \Sigma f_i = 70$			$\Sigma f_i u_i = -98$		$\Sigma f_i u_i^2 = 250$

N = 70, $\Sigma f_i u_i = -98$ $\Sigma f_i u_i^2 = 250$, A = 35 and h = 10

Mean = A + h $\left(\frac{1}{N}\Sigma f_i u_i\right)$ = 35 + 10 $\left(\frac{-98}{70}\right)$ = 21

Var (X) = $h^2 \left\{\left(\frac{1}{N}\sum f_i u_i^2\right) - \left(\frac{1}{N}\sum f_i u_i\right)^2\right\}$ = 100

$\left\{\left(\frac{1}{70} \times 250\right) - \left(\frac{1}{70} \times (-98)\right)^2\right\}$ = 100 {3.57 - 1.96} = 161

SD = $\sqrt{\text{var}(X)} = \sqrt{161}$ = 12.69

OR

To find: the correct mean and the variance.

As per given criteria,

Number of reading, n=10

Mean of the given readings before correction, $\bar{x} = 45$

But we know,

$$\bar{x} = \frac{\sum x_i}{n}$$

Substituting the corresponding values, we get

$$45 = \frac{\sum x_i}{10}$$

$$\Rightarrow \sum x_i = 45 \times 10 = 450$$

It is said one reading 25 was wrongly taken as 52,

So $\sum x_i = 450 - 52 + 25 = 423$

So the correct mean after correction is

$$\bar{x} = \frac{\sum x_i}{n} = \frac{423}{10} = 42.3$$

Also given the variance of the 10 readings is 16 before correction,

i.e., $\sigma^2 = 16$

But we know

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

Substituting the corresponding values, we get

$$16 = \frac{\sum x_i^2}{10} - (45)^2$$

$$\Rightarrow 16 = \frac{\sum x_i^2}{10} - 2025$$

$$\Rightarrow 16 + 2025 = \frac{\sum x_i^2}{10}$$

$$\Rightarrow \frac{\Sigma x_i^2}{10} = 2041$$

$$\Rightarrow \sum x_i^2 = 20410$$

It is said one reading 25 was wrongly taken as 52, so

$$\Rightarrow \sum x_i^2 = 20410 - (52)^2 + (25)^2$$

$$\Rightarrow \sum x_i^2 = 20410 - 2704 + 625$$

$$\Rightarrow \sum x_i^2 = 18331$$

So the correct variance after correction is

$$\sigma^2 = \frac{18331}{10} - \left(\frac{423}{10}\right)^2$$

$\sigma^2 = 1833.1 - (42.3)^2 = 1833.1 - 1789.29$

σ^2 = 43.81

Hence the corrected mean and variance is 42.3 and 43.81 respectively.

38. According to the question , we can state,

Suppose that the firm manufactures x units of A and y units of B.

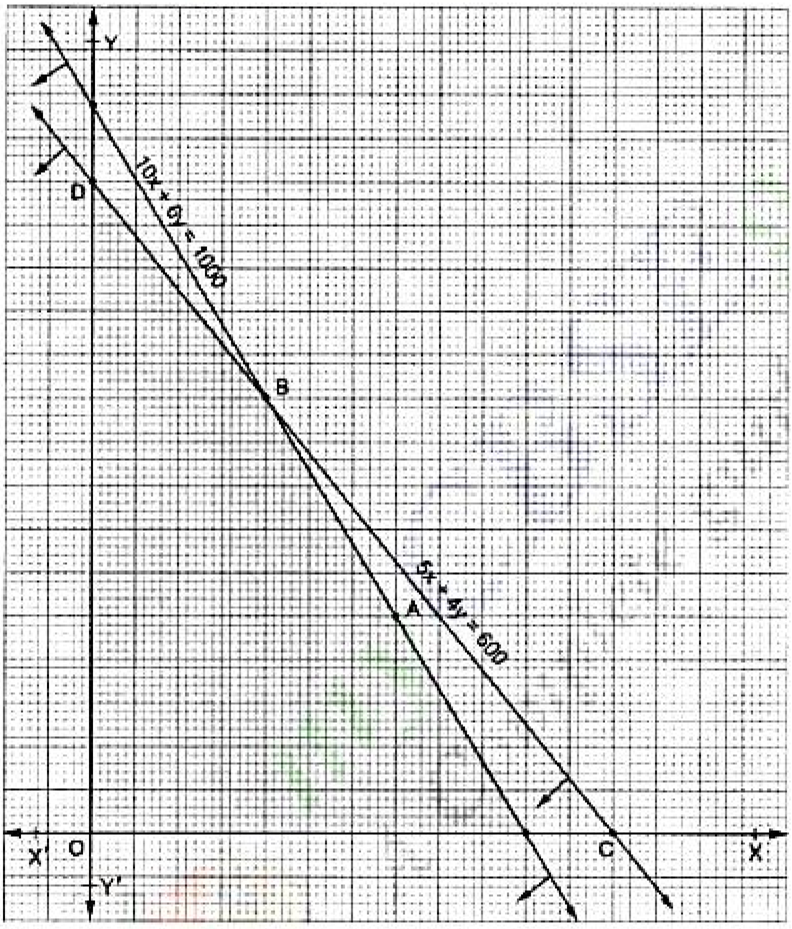
$\therefore$ number of man-hours for A and 8 in the foundry = 10x + 6y,

and number of man-hours for A and B in the machine shop = 5x + 4y.

$\therefore$ 10x + 6y $\leq$ 1000 and 5x + 4y $\leq$ 600.

Clearly, x $\geq$ 0 and y $\geq$ 0 [$\because$ number of units cannot be negative].

Scale: 1 small division = 2 units.



i. First we draw the graph of 10x + 6y $\leq$ 1000.

Consider the equation, 10x + 6y = 1000.

10x + 6y = 1000 $\Rightarrow$ 5x + 3y = 500.

The values of (x, y) satisfying 5x + 3y = 500 are:

x	70	40
y	50	100

Plot the points A(70, 50) and B(40,100) on a graph paper and join them by the thick line AB.

Consider (0, 0). It does not lie on 5x + 3y = 500.

Also, (0, 0) satisfies 10x + 6y $\leq$ 1000.

Therefore, the line AB and the part of the plane separated by AB, containing (0, 0), represent the graph of 10x + 6y $\leq$ 1000

ii. Now, we draw the graph of 5x + 4y $\leq$ 600.

Consider the line, 5x + 4y = 600.

5x + 4y = 600 $\Rightarrow \frac{x}{120} + \frac{y}{150} = 1$

This line meets the axes at C(120, 0) and D(0,150).

Plot these points on the same graph paper as above and join them by the thick line CD.

Now, consider (0,0). It does not lie on 5x + 4y = 600.

Also, (0, 0) satisfies the inequation 5x + 4y $\leq$ 600.

So, the line CD and that part of the plane separated by CD, containing (0, 0), represent the graph of 5x + 4y $\leq$ 600.

x $\geq$ 0 is represented by the y-axis and the part of the plane on its right side.

y > 0 is represented by the x-axis and the plane above the x-axis.

Clearly, the shaded region together with its boundary represents the solution set of the given inequations.

OR

Let the length of the shortest board be x cm

Then length of the second board = (x + 3) cm

length of the third board = 2x cm

Now $x + (x+3) + 2x \leqslant 91$ and $2x \geqslant (x+3) + 5$

$\Rightarrow 4x + 3 \leqslant 91$ and $2x - (x+3) \geqslant 5$

$\Rightarrow 4x \leqslant 91 - 3$ and $2x - x - 3 \geqslant 5$

$\Rightarrow 4x \leqslant 88$ and $x \geqslant 5 + 3$

$\Rightarrow x \leqslant 22$ and $x \geqslant 8$

Thus minimum length of shortest board is 8 cm and maximum length is 22 cm.