

Sets (Class-11th maths)
Chapter - 1

Set: a collection of well defined objects.

For example,

not well defined
↓
Criteria

Set

Not Set.

- ✓ collection of rivers of india
- ✓ ——— vowels of english alphabet
- ✓ collection of students who got ~~mark~~ marks above 90%.

→ collection of large river
x

→ collection of good students in class 11th
x

Set of vowels of english alphabet = $A = \{a, e, i, o, u\}$

Curly braces

Name of set

(Generally capital letter)

objects / members / elements
of sets.

(Generally in small letter)

Famous Sets

their Name

✓ Set of natural numbers = $N = \{1, 2, 3, \dots\}$

↓

Curly Braces

✓ Set of whole numbers = $W = \{0, 1, 2, 3, 4, \dots\}$

✓ Set of integers = $I = Z = \{\dots -3, -2, -1, 0, 1, 2, \dots\}$

↓

इन्में से कोई सा भी आ सकता है।

✓ Set of rational numbers = $Q = \{-2, 0, 3, \frac{7}{5}, \dots\}$

$$\text{rational numbers} = \frac{p}{q} \quad \left(\begin{array}{l} p, q \Rightarrow \text{integers} \\ q \neq 0 \end{array} \right)$$

✓ Set of irrational numbers = $Q' = I$

✓ $= \{\sqrt{2}, \sqrt{3}, \pi, e, 2^{\frac{1}{3}}, \dots\}$

etc.

their decimal expansion has non terminating non repeating pattern

✓ Set of real numbers = R = it has both rational as well as irrational numbers.

✓ Set of positive integers = $\mathbb{Z}^+$ or $\mathbb{I}^+$
= $\{1, 2, 3, \dots\}$

✓ Set of positive real numbers = $\mathbb{R}^+$

✓ $\mathbb{R}_0$ = Set of real number except zero.

✓ $\mathbb{I}_0$ or $\mathbb{Z}_0$ = Set of integers except zero
= $\{\dots, -3, -2, -1, 1, 2, 3, 4, \dots\}$

Useful symbols: Let $A = \{a, e, i, o, u\}$

Symbol meaning

$\in$

belongs to

for example

e belongs to A

$\Rightarrow e \in A$

Also $i \in A, o \in A, u \in A, a \in A$

$\notin$

does not belong to

for example

$b \notin A, x \notin A$

$\exists$

there exists

: or |

such that

$\forall$

for all values of

we will use these
symbols later.

Representation of a Set:

- (i) Roster (Tabular) Form
- (ii) Set Builder (Property) Form

(in both forms we use curly braces)

(i) Roster Form:

write all the elements in curly braces (no need to repeat the elements & order does not matter)

For example:

✓ Set of natural numbers less than 5

$$\begin{array}{c} \uparrow \\ \text{B} = \{1, 2, 3, 4\} = \{2, 1, 3, 4\} = \{4, 3, 1, 2\} \\ \text{name of set} \quad \leftarrow \text{all are same} \rightarrow \end{array}$$

$$\text{But } B = \{1, 2, \overbrace{3, 3}^{\text{repetition}}, \overbrace{4, 4, 4}^{\text{repetition}}\}$$

So repetition does not make sense.

✓ Set of letters of word "MATTER"

$$= \{M, A, T, E, R\}$$

(do not repeat 'T' in set)

(ii) Set Builder form

We do not write each element in this form, we write only property.

By reading this property, we can find elements.

$$A = \{x : \text{Property of } x\}$$

↑
name of set

Such that

We read the above expression as — "A is the set of all values of x such that x follows a property"

For example,

Roster Form

$$A = \{1, 2\}$$

Set builder form.

$$A = \{x : x \text{ is natural no. less than } 3\}$$

$$A = \{x : x \text{ is the root of } x^2 - 3x + 2 = 0\}$$

$$A = \{x : x \text{ is positive factor of } 2\}$$

All are same

By the above example we can say that
A set can be represented in set builder
form in many ways but ~~all~~ all
ways gives same set.

we use ' x ' for support, in actual ' n '
is not an element of set but value of
' x ' is an element of set.

eg. Convert $A = \left\{ x : x = \frac{n}{n+1}, n \in \mathbb{N}, n \leq 4 \right\}$ Set builder form.

in roster form

Answer.

we have formula for
values of x .

$$x = \frac{n}{n+1}$$

Here n is a natural number ($\because n \in \mathbb{N}$)
and it is less than or equal to 4.

it means $n = 1, 2, 3, 4$

$$x = \frac{n}{n+1} \rightarrow n=1 \Rightarrow x = \frac{1}{2}$$

$$\rightarrow n=2 \Rightarrow x = \frac{2}{3}$$

$$\rightarrow n=3 \Rightarrow x = \frac{3}{4}$$

$$\rightarrow n=4 \Rightarrow x = \frac{4}{5}$$

values of $x = \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$.

$$\therefore A = \left\{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5} \right\} = \text{Roster Form.}$$

Exercise [1.1]

Empty set: Set which has no element.

$$\hookrightarrow \phi = \{ \} = \text{empty/null/void set.}$$

General name = phi (φ)

e.g. Set of persons living on sun = $\{ \} = \phi$

Note: do not write ϕ in bracket
like $\{ \phi \}$ $\leftarrow$ this is wrong.

$\updownarrow$
(it means there is an element ϕ
in the set)

this is not empty set.

Singleton Set : which has exactly one element

Example:

Set of even prime number $= \{2\}$

"2 is the only prime number which is even also."

Finite Set: having finite number of (Countable) elements.

Infinite Set: having infinite (∞) number of elements.

Examples:

$A = \{1, 2, 3, 4, 5\} \rightarrow$ Finite set
(no. of elements = 5)

$B = \{1, 2, 3, 4, \dots\} \rightarrow$ Infinite set
(no. of elements = ∞)

Cardinal number of a set $A = n(A) =$ no. of elements in set A .

Example

$$A = \{1, 2, 3\} \rightarrow n(A) = 3$$
$$B = \{a, e, i, o, u\} \rightarrow n(B) = 5$$
$$\phi = \{\} \rightarrow n(\phi) = 0$$

$$C = \{1, 2, 3, \dots\} \rightarrow n(C) = \infty$$

= Not Defined

Equal Sets : Two sets A & B are said to be equal sets if there all elements are same and written as $A=B$.

Example.

$$A = \{1, 2, 3\} = B = \{3, 2, 1\}$$

$$A = \{x, a\} = B = \{a, x\}$$

$$\text{But } A = \{1, 2, 3\}, B = \{a, b, c\} \Rightarrow A \neq B$$

$$A = \{1, 2\}, B = \{1, 2, 3\} \Rightarrow A \neq B$$

Extra

Equivalent Sets : two sets A & B

are said to be equivalent if their no. of elements are equal (~~from~~ their elements may or may not be same) written as $\rightarrow A \sim B$

e.g.

$$A = \{1, 2, 3\}, B = \{a, b, c\} \rightarrow A \neq B \text{ but } A \sim B$$

$$A = \{1, 2, 3\}, B = \{1, 2\} \rightarrow A \neq B \text{ \& } A \not\sim B$$

$$A = \{1, 2, 3\}, B = \{1, 2, 3\} \rightarrow A = B \text{ \& } A \sim B$$

All equal sets are equivalent but all equivalent sets may not be equal.

Subsets :

X = Set of all students in your school.

Y = Set of all students in your class 11th.

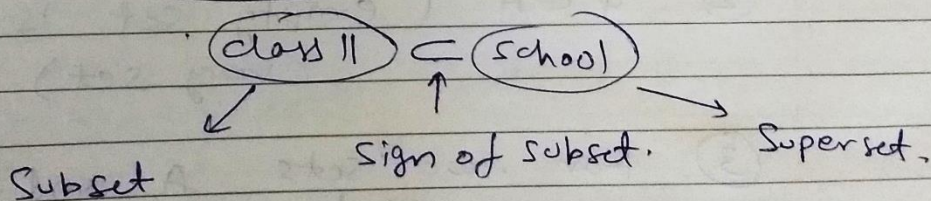
Clearly Y is smaller set and X is bigger set.

Students in Y are already in X but students in X (whole school) may or may not be in Y .

in this situation Y is subset of X

& X is super set of Y .

For Example, class 11 is subset of School



Definition of Subset:

a set ' A ' is said to be a subset of a set B if each element of ' A ' is also an element of ' B '. written as $A \subset B$

you can understand this definition by replacing ' A ' by class 11th, ' B ' by 'School'.

eg. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$$A_1 = \{1, 2\} \rightarrow A_1 \subset A$$

$$A_2 = \{2, 7, 8, 9, 10\} \rightarrow A_2 \subset A$$

$$A_3 = \{7, 8, 15, 2\} \rightarrow A_3 \not\subset A$$

$$\because 15 \in A_3$$

$$\text{but } 15 \notin A$$

(A_3 is not subset of A)

Note:

① $A \subset A$ (each set is subset of itself)

② $\phi \subset A$ (empty set is subset of every set)

③ For two ^Nsets A and B
equal

$A = B$ so we can say
that $A \subset B$ & $B \subset A$.

$$A = B \iff A \subset B \text{ and } B \subset A$$

④

If A is subset of B but there are some elements in B which ~~is~~ are not in A . then ' A ' is proper subset of B (in other words $A \subset B$ but $A \neq B$)

Here B is super set of ' A '.

Other wise ' A ' will be improper subset.

Example:

Let $B = \{1, 2, 3\}$

Subsets of B

$$A_1 = \phi$$

$$A_2 = \{1\}$$

$$A_3 = \{2\}$$

$$A_4 = \{3\}$$

$$A_5 = \{1, 2\}$$

$$A_6 = \{2, 3\}$$

$$A_7 = \{3, 1\}$$

$$A_8 = \{1, 2, 3\}$$

Proper subsets of B .

improper subsets of B .

Because $A_8 = B$.

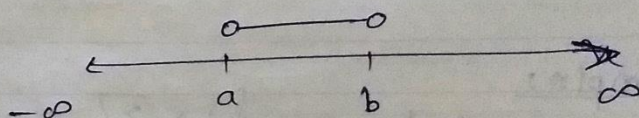
Intervals as subsets of ' $\mathbb{R}$ '

Real numbers occur continuously on number line.

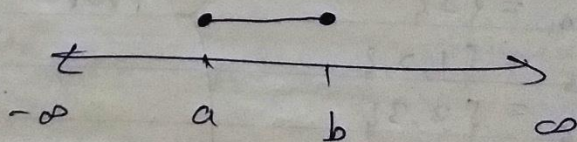
we use intervals to understand inequality.

~~# If~~ $a < x < b$, then $x \in (a, b)$

Diagram illustrating the interval (a, b) on a number line. The number line is marked with $-\infty$, a , b , and ∞ . Above the line, there are two arrows: one pointing left from a labeled "smaller no." and one pointing right from b labeled "larger". Above the interval (a, b) , there are two arrows: one pointing left from b labeled "smaller" and one pointing right from a labeled "larger".



~~# If~~ $a \leq x \leq b$, then $x \in [a, b]$



Note. Understand 1st case

x is not equal to a & b , x is just between them. In the form of intervals we write $x \in (a, b)$. Here we use small bracket.

For number line we use like
 $\circ \text{---} \circ$ (both side hollow)

understand II case.

x is equal to a & b and also ' x ' lies between them. So we use square bracket. For number line, we use $\bullet$ — $\bullet$, both side filled.

let's do more cases:

$$\# \boxed{a < x \leq b} \Rightarrow \boxed{x \in (a, b]} \Rightarrow \text{Number line diagram: } \leftarrow \begin{array}{cc} \bullet & \bullet \\ | & | \\ a & b \end{array} \rightarrow$$

$$\# \boxed{a \leq x < b} \Rightarrow \boxed{x \in [a, b)} \Rightarrow \text{Number line diagram: } \leftarrow \begin{array}{cc} \bullet & \circ \\ | & | \\ a & b \end{array} \rightarrow$$

$$\# \boxed{a < x} \Rightarrow \boxed{x \in (a, \infty)} \Rightarrow \text{Number line diagram: } \leftarrow \begin{array}{cc} \circ & \rightarrow \\ | & | \\ a & \infty \end{array} \rightarrow$$

$a < x < \infty$

Because each number is less than ' ∞ '

$$\# \boxed{x \leq b} \Rightarrow \boxed{x \in (-\infty, b]} \Rightarrow \text{Number line diagram: } \leftarrow \begin{array}{cc} \bullet & \rightarrow \\ | & | \\ -\infty & b \end{array} \rightarrow$$

$-\infty < x \leq b$

because each number is greater than $(-\infty)$

Also we can not include (∞) or $(-\infty)$ because they are not defined.

(P(A)) Power set: is the collection (set) of subsets of any set.

Example: $A = \{1, 2\}$

Subsets of $A = \phi, \{1\}, \{2\}, \{1, 2\}$

Collection of Subsets of A

$$= \{ \phi, \{1\}, \{2\}, \{1, 2\} \} = P(A)$$

Apply extra

Curly bracket

(Because inner sets are just ~~set~~ elements of power set)

~~NOTE~~

Note, If number of elements in A is m then number of subsets $= 2^m$.

So If $n(A) = m$, then $n(P(A)) = 2^m$.
power set = set of subsets

Universal Set: $(U) \rightarrow (\mathbb{U})$

A set which has all elements ~~is~~ discussed in any problem.

For example

If in a problem there are A, B & C sets.

Let $A = \{1, 2, 3\}$, $B = \{1, 2, 3\}$, $C = \{7, 8\}$

Someone can assume $U = \{1, 2, 3, 7, 8\}$ $\leftarrow$ It has all elements of A, B, C.
 $\uparrow$
Universal set

Someone can assume $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Someone can assume $U = \{1, 2, 3, \dots\} = \mathbb{N}$
(Natural no.)

Venn Diagram: Understand sets using diagrams.

Generally Universal set is rectangle and nothing goes beyond this rectangle.

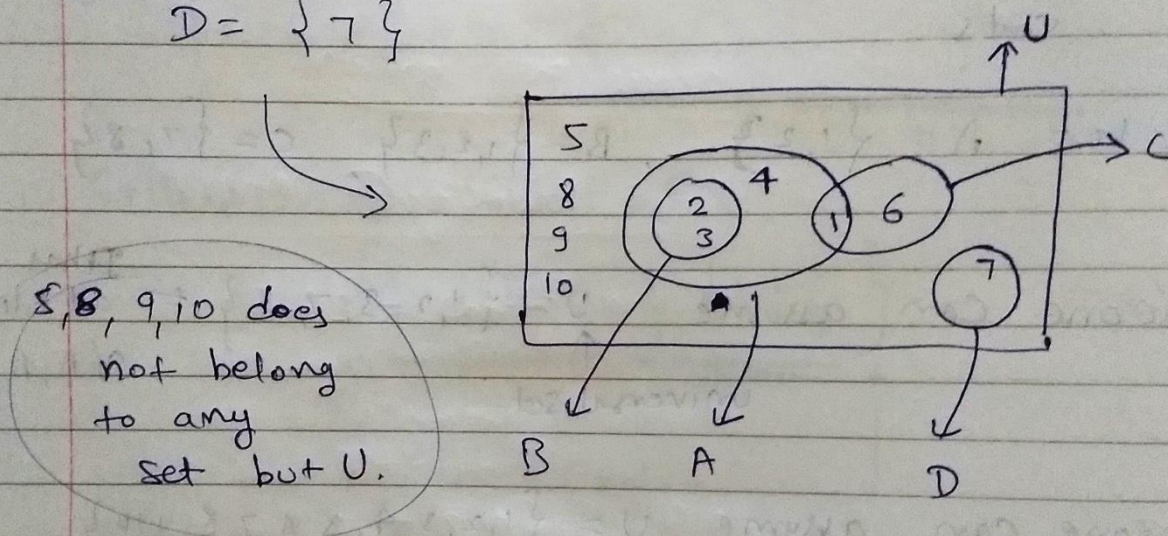
All other sets are circles.

For example.

$$\text{Let } U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{1, 2, 3, 4\} \quad B = \{2, 3\} \quad C = \{1, 6\}$$

$$D = \{7\}$$



Operations on sets:

- ① Union of sets
- ② Intersection of sets
- ③ Complement of sets
- ④ Difference of sets

} each operation gives new set.

① Union of Sets

$$A \cup B \Rightarrow A \text{ union } B$$

$A \cup B$ is the set containing elements which belong to either A or B or both.

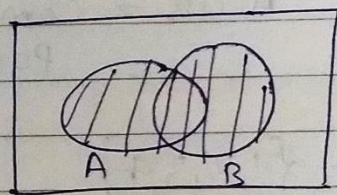
For examples:

$$A = \{1, 2, 3\}, \quad B = \{3, 4, 5\}$$

$$\therefore A \cup B = \{1, 2, 3, 4, 5\}$$

By Venn Diagram

$A \cup B$



$\rightarrow A \cup B$

$$\text{So } A \cup B = \{x : \underline{x \in A} \text{ or } \underline{x \in B}\}$$

notice that we used word 'or'

Properties

① $A \cup A = A$

② $A \cup U = U = \text{universal set}$
 ↑
 universal set

③ $A \cup \phi = A$

④ $A \cup B = B \cup A \rightarrow (\text{Commutative})$

⑤ $(A \cup B) \cup C = A \cup (B \cup C) \rightarrow (\text{Associative property})$

② Intersection of sets:

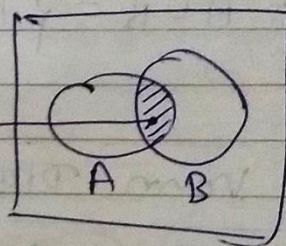
$\{A \cap B \Rightarrow A \text{ intersection } B\}$ = set of common elements.

$$A \cap B = \{x: x \in A \text{ and } x \in B\}$$

notice the word 'and'

By Venn Diagram

$A \cap B$ = Common Portion



e.g. $A = \{1, 2, 3, 4\}$
 $B = \{1, 3, 7, 8, 9\}$

$A \cap B = \{1, 3\} \Rightarrow$ Because 1 & 3 are common in Both A and B

Properties:

$$\begin{cases} A \cap A = A \\ A \cap U = A \\ A \cap \phi = \phi \end{cases}$$

$$\begin{cases} A \cap B = B \cap A \\ (A \cap B) \cap C = A \cap (B \cap C) \end{cases}$$

Commutative
Associative

Distributive Law:

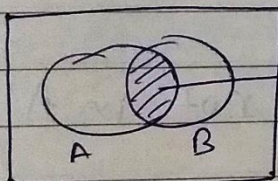
$$① (A \cup B) \cap C = \cancel{A \cap B} (A \cap C) \cup (B \cap C)$$

understand it like $(A+B) \times C = A \times C + B \times C$

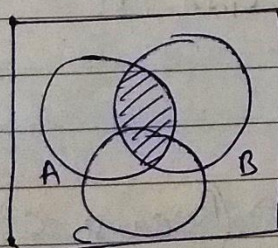
$$② (A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

understand it like $(a+b) \times c = a.c + b.c$

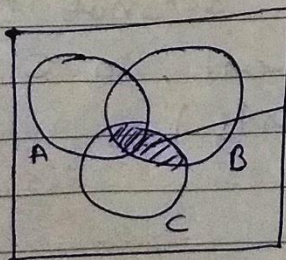
Examples



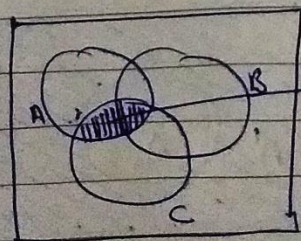
$\rightarrow A \cap B$



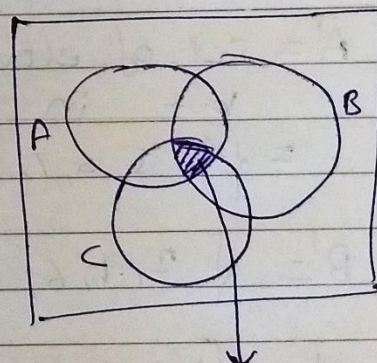
$A \cap B$



$\rightarrow B \cap C$

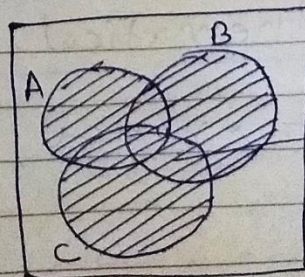


$\rightarrow A \cap C$



$A \cap B \cap C$

(Common portion of all)



$A \cup B \cup C$

③ Difference of sets: $A - B \Rightarrow A \text{ minus } B$

$A - B =$ set of elements of only A
which does not belong to B

For example,

$$A = \{1, 2, 3, 4, 5\} \quad B = \{3, 4, 5, 6, 7\}$$

$$\Rightarrow A - B = \{1, 2, 3, 4, 5\} - \{3, 4, 5, 6, 7\}$$

Here 3, 4, 5 also lies in 'B'
so we remove them.

$$\Rightarrow A - B = \{1, 2\} \quad \text{we write elements of only 'A'}$$

forget about remaining elements of B, they will not come in $A - B$.

$$\begin{aligned} \text{Now } B - A &= \{ \underline{3, 4, 5, 6, 7} \} - \{ \underline{1, 2, 3, 4, 5} \} \\ &= \{ 6, 7 \} \end{aligned}$$

← came ↓ forget.

One more Example.

$$A = \{1, 2, 3\} \quad B = \{2, 3\}$$

$$A - B = \{1, \underline{2}, \underline{3}\} - \{\underline{2}, \underline{3}\} = \{1\}$$

↓
Come

$$\text{But } B - A = \{\underline{2}, \underline{3}\} - \{\underline{1}, \underline{2}, \underline{3}\} = \phi = \{\}$$

↑
Every element is removed

④ Complement of a set $A' = A^c = \bar{A}$
representation

Definition = set of elements which are not in A

example. Let $U = \{1, 2, 3, \dots, 9, 10\}$
 $A = \{1, 2, 3\}$

$$B = \{1, 3, 5, 7, 9\}$$

$$C = \{ \} = \phi$$

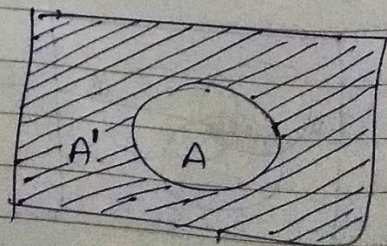
A' = set of elements which are not in 'A' but in universal set
 $= \{4, 5, 6, 7, 8, 9, 10\} \leftarrow$ outside A.

$$B' = \{2, 4, 6, 8, 10\}$$

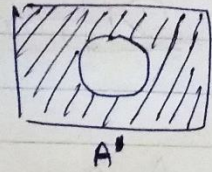
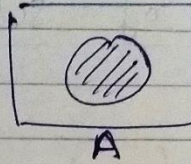
$C' = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \rightarrow$ (Because 'C' does not have any element, so 'C' has all elements)

Mathematical Definition:

$$A' = \{x : x \in U, x \notin A\}$$



Properties :



① $(A')' = A$

② $A \cup A' = U = \text{universal set}$
↑
union

③ $A \cap A' = \phi = \text{empty set}$

④ $U' = \phi \quad \& \quad \phi' = U$

⑤ De Morgan's Law:

$(A \cup B)' = A' \cap B'$

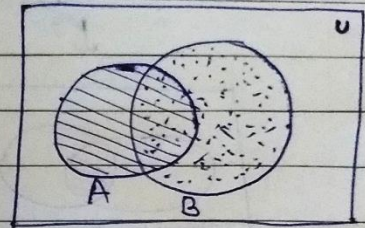
~~Sign~~ Sign change.

$(A \cap B)' = A' \cup B'$

Practical problems on Sets:

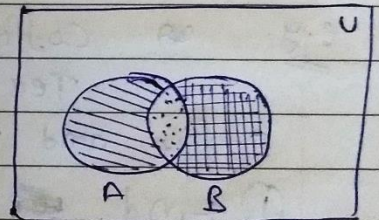
①

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$



②

$$n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$$

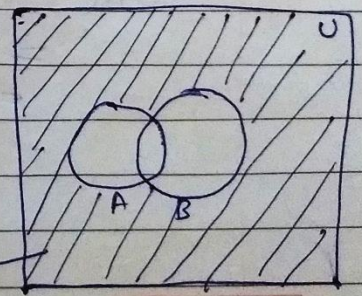


③

$$n(\overline{A \cup B}) = n(U) - n(A \cup B)$$

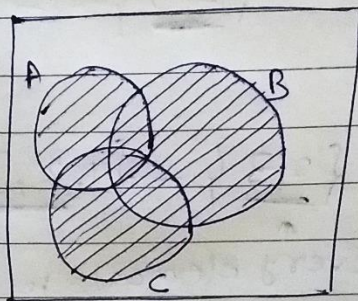


$\overline{A \cup B}$



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④ $n(A \cup B \cup C) = n(A) + n(B) + n(C)$
 $- n(A \cap B) - n(B \cap C) - n(C \cap A)$
 $+ n(A \cap B \cap C)$



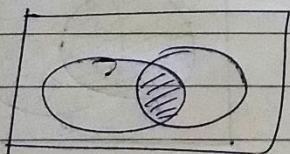
Here



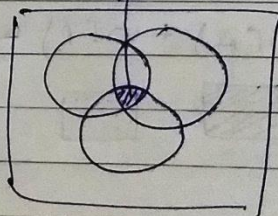
→ $A \cup B \cup C$.

Note: to solve practical problems on sets, always start with the most common portion.

$n(A \cap B)$



$n(A \cap B \cap C)$



e.g. $\left. \begin{array}{l} \text{Coffee पीने वाले} = n(C) = 15 \\ \text{Tea पीने वाले} = n(T) = 10 \\ \text{Coffee and Tea पीने वाले} = n(C \cap T) = 6 \end{array} \right\} \text{Let}$

① Find ~~the~~ no. of persons who drink at least one of coffee & tea

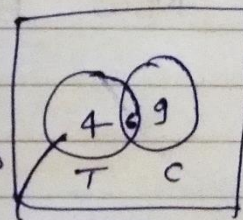
$$= n(C \cup T) = n(C) + n(T) - n(C \cap T)$$

$$\Rightarrow n(C \cup T) = 15 + 10 - 6$$

$$= 19$$

② Find who drink only Tea not Coffee

1st method by Venn Diagram
(start by labelling 6 in $C \cap T$)



Ans = 4

2nd method

we know that

$$n(A) = n(A-B) + n(A \cap B)$$

$$\Rightarrow n(T) = n(T-C) + n(C \cap T)$$

$$\Rightarrow 10 = n(T-C) + 6$$

$$\Rightarrow n(T-C) = 4 = \text{Tea only not coffee.}$$

