15. Quadrilaterals

Exercise 15
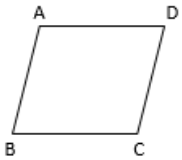
1. Question

Fill in the blanks:

(i) A quadrilateral has ....sides.

(ii) A quadrilateral has............... angles.

(iii) A quadrilateral has............... vertices, no three of which are................

(iv) A quadrilateral has ....diagonals.

(v) A diagonal of a quadrilateral is a line segment that joins two.................. vertices of the quadrilateral.

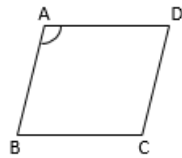(vi) The sum of the angles of a quadrilateral is ....

Answer

(i) Four

AB, BC, CD and DA are four sides of this quadrilateral

A quadrilateral is polygon having four sides and four corners.

(ii) four
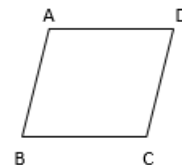

∠A, ∠B, ∠C and ∠D are four angles of this quadrilateral

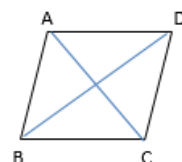A quadrilateral is polygon having four sides and four corners.

(iii) Four, collinear

A, B, C and D are the four vertices of this quadrilateral.

In quadrilateral, no three out of four vertices are collinear. If all the vertices are collinear then we will get a line segment and if three out of four vertices is collinear, we will get a triangle.
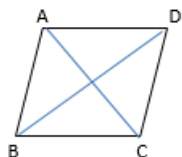
(iv) two

A diagonal is a line segment that joins two opposite vertices of the quadrilateral.

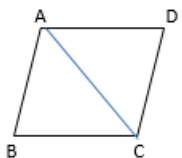AC and BD are the two diagonals of the quadrilateral ABCD.

(v) opposite

A diagonal is a line segment that joins two opposite vertices of the quadrilateral.

AC and BD are the two diagonals of the quadrilateral ABCD.

(vi) 360⁰

ABCD is a quadrilateral and AC is a diagonal. Now, we get two triangles

Δ ABC and Δ ACD.

As we know that sum of angles of triangle is $180°$

*So, sum of two triangles will be $180° \times 2 = 360°$*

i.e., *the sum of the angles of a quadrilateral is 360°*

2. Question

In the adjoining figure, ABCD is a quadrilateral.

(i) How many pairs of adjacent sides are there? Name them.

(ii) How many pairs of opposite sides are there? Name them.

(iii) How many pairs of adjacent angles are there? Name them.

(iv) How many pairs of opposite angles are there? Name them.

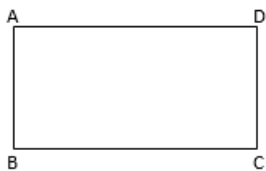(v) How many diagonals are there? Name them.
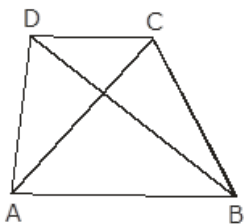
Answer

(i) four; (AB, BC), (BC, CD), (CD, DA), (DA, AB)

When two sides of quadrilateral have same end point, they are called as Adjacent Sides.

(AB, BC), (BC, CD), (CD, DA), (DA, AB) are the adjacent sides of this quadrilateral.
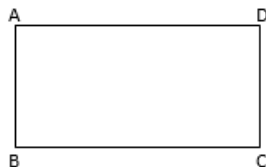
(ii) two; (AB, DC), (AD, BC)

Two sides of quadrilateral who do have same end point are called as Opposite Sides.

(AB, DC), (AD, BC) are the opposite sides of this quadrilateral.

(iii)

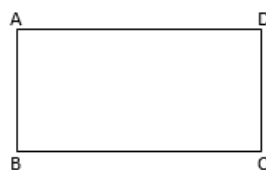*four*; $(\angle A, \angle B)$, $(\angle B, \angle C)$, $(\angle C, \angle D)$, $(\angle D, \angle A)$

When two angles of quadrilateral share the common arm it is called as Adjacent angles of the quadrilateral.

($\angle$A, $\angle$B), ($\angle$B, $\angle$C), ($\angle$C, $\angle$D) and ($\angle$D, $\angle$A) are adjacent angles of this quadrilateral.

(iv)

two; $(\angle A, \angle C)$, $(\angle B, \angle D)$

When two angles of quadrilateral are not adjacent angles then it is called as opposite angles of the quadrilateral.

($\angle$ A, $\angle$C) and ($\angle$B, $\angle$D) are opposite angles of this quadrilateral.
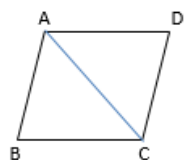
(v) two; (AC, BD)

A diagonal is a line segment that joins two opposite vertices of the quadrilateral.

AC and BD are the two diagonals of the quadrilateral ABCD.

3. Question

Prove that the sum of the angles of a quadrilateral is 360°.

Answer

ABCD is a quadrilateral and AC is a diagonal. Now, we get two triangles

Δ ABC and Δ ACD.

As we know that sum of angles of triangle is 18$0°$

So, sum of two triangles will be 180° × 2 = 360°

i.e. the sum of the angles of a quadrilateral is 360°

4. Question

The three angles of a quadrilateral are 76°, 54° and 108°. Find the measure of the fourth angle.

Answer

Let $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are the four angles of quadrilateral.

As we know that, Sum of all four angles of quadrilateral is 360°.

$\angle A = 76°$

$\angle B = 54°$

$\angle C = 108°$

So, $\angle D = 360^o – (\angle A + \angle B + \angle C)$

$= 360^o – (76° + 54° + 108°)$

$= 122^o$

So, fourth angle of quadrilateral will be 122°.

5. Question

The angles of a quadrilateral are in the ratio 3 : 5 : 7 : 9. Find the measure of each of these angles.

Answer

Let x be the common multiple.

As per question,

$\angle A = 3x$

$\angle B = 5x$

$\angle C = 7x$

$\angle D = 9x$

As we know that, Sum of all four angles of quadrilateral is 360°.

$\angle A + \angle B + \angle C + \angle D = 360°$

$3x + 5x + 7x + 9x = 360°$

$24x = 360°$

$X = 360/24$

$= 15°$

$\angle A = 3 \times 15° = 45°$

$\angle B = 5 \times 15° = 75°$

$\angle C = 7 \times 15° = 105°$

$\angle D = 9 \times 15° = 135°$

So, Angles of quadrilateral are 45°, 75°, 105° and 135°.

6. Question

A quadrilateral has three acute angles, each measuring 75°. Find the measure of the fourth angle.

Answer

Three angles are acute angle and each measuring is 75° means

$\angle A = \angle B = \angle C = 75°$

(Acute angle is angle whose measuring is greater than 0 and less than 90.)

As we know that, Sum of all four angles of quadrilateral is 360°.

$\angle A + \angle B + \angle C + \angle D = 360°$

$75° + 75° + 75° + \angle D = 360°$

$\angle D = 360° - (75° + 75° + 75°)$

$= 360° - 225°$

$= 135°$

So, fourth angle of quadrilateral is 135°.

7. Question

Three angles of a quadrilateral are equal and the measure of the fourth angle is 120°. Find the measure of each of the equal angles.

Answer

Let x be the common angle of quadrilateral.

As per question,

$\angle A = \angle B = \angle C = x$

$\angle D = 120°$

As we know that, Sum of all four angles of quadrilateral is 360°.

$\angle A + \angle B + \angle C + \angle D = 360°$

$x + x + x + 120° = 360°$

$3x = 360° - 120°$

$3x = 240°$

$X = 240 / 3$

$= 80°$

$\angle A = \angle B = \angle C = 80°$

So, Three Angles of quadrilateral whose measuring's are equal is 80°.

8. Question

Two angles of a quadrilateral measure 85° and 75° respectively. The other two angles are equal. Find the measure of each of these equal angles.

Answer

Let x be the common angle of quadrilateral.

As per question,

$\angle A = 85°$

$\angle B = 75°$

$\angle C = \angle D = x$

As we know that, Sum of all four angles of quadrilateral is 360°.

$\angle A + \angle B + \angle C + \angle D = 360°$

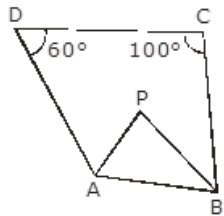$85° + 75° + x + x = 360°$

$2x = 360° - (85° + 75°)$

2x = 200°

X = 200 / 2

= 100°

∠C = ∠D = 100°

So, Two angles of quadrilateral whose measuring's are equal is 100°.

9. Question

In the adjacent figure, the bisectors of $\angle A$ and $\angle B$ meet in a point P. D

If $\angle C$ =100° and $\angle D$ = 60°, find the measure of $\angle APB$.

Answer

As we know that, Sum of all four angles of quadrilateral is 360°.

∠A + ∠B + ∠C + ∠D = 360°

∠A + ∠B + 100° + 60° = 360°

∠A + ∠B = 360° - 160°

= 200°

Now, according to question bisector of $\angle A$ and $\angle B$ meet in a point P and forms the triangle

PAB.

So,

1/2 ∠A + 1/2 ∠B = 200°/2

= 100°

As we know that, sum of all angles of triangle is 180°.

$\angle BAP + \angle ABP + \angle APB = 180°$

1/2 ∠A + 1/2 ∠B + ∠APB = 180°

100° + ∠APB = 180°

∠APB = 180° - 100°

= 80°

So, ∠APB = 80°