Chapter : 10. DIFFERENTIATION

Exercise : 10A

Question: 1

Differentiate eac

Solution:

__**Formulae**__ **:**

• $\frac{d}{dx}(\sin x) = \cos x$

• $\frac{d}{dx}(kx) = k$

Let,

y = sin 4x

and u = 4x

therefore, y = sin u

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\sin u).\frac{d}{dx}(4x)$

$= \cos u . 4$ .... $\left(\because \frac{d}{dx}(\sin x) = \cos x \;\&\; \frac{d}{dx}(kx) = k\right)$

= cos 4x . 4

= 4 cos 4x

Question: 2

Differentiate eac

Solution:

__**Formulae**__ **:**

• $\frac{d}{dx}(\cos x) = -\sin x$

• $\frac{d}{dx}(kx) = k$

Let,

y = cos 5x

and u = 5x

therefore, y= cos u

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\cos u).\frac{d}{dx}(5x)$

$= -\sin u . 5$ .... $\left(\because \frac{d}{dx}(\cos x) = -\sin x \;\&\; \frac{d}{dx}(kx) = k\right)$

$= - \sin 5x \,.\, 5$

$= - 5 \sin 5x$

Question: 3

Differentiate eac

Solution:

<u>**Formulae**</u> **:**

- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(kx) = k$

Let,

$y = \tan 3x$

and $u = 3x$

therefore, $y = \tan u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\tan u) \cdot \frac{d}{dx}(3x)$

$= \sec^2 u \,.3$ .... $\left(\because \frac{d}{dx}(\tan x) = \sec^2 x \;\&\; \frac{d}{dx}(kx) = k\right)$

$= \sec^2 3x \,.\, 3$

$= 3 \sec^2 3x$

Question: 4

Differentiate eac

Solution:

<u>**Formulae**</u> **:**

- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

$y = \cos x^3$

and $u = x^3$

therefore, $y = \cos u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\cos u) \cdot \frac{d}{dx}(x^3)$

$= -\sin u \,.3x^2$ .... $\left(\because \frac{d}{dx}(\cos x) = -\sin x \;\&\; \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$= - \sin x^3 \,.\, 3x^2$

$= - 3x^2 \sin x^3$

Question: 5

Differentiate eac

Solution:

Formulae :

- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
- $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

$y = \cot^2 x$

and $u = \cot x$

therefore, $y = u^2$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(u^2).\frac{d}{dx}(\cot x)$

$= 2u.(-\operatorname{cosec}^2 x)$ .... $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x\right)$

$= 2 \cot x .(- \operatorname{cosec}^2 x)$

$= - 2\cot x . \operatorname{cosec}^2 x$

Question: 6

Differentiate eac

Solution:

Formulae :

- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

$y = \tan^3 x$

and $u = \tan x$

therefore, $y = u^3$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(u^3).\frac{d}{dx}(\tan x)$

$= 3u^2 .\sec^2 x$ .... $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(\tan x) = \sec^2 x\right)$

$= 3 \tan^2 x .(\sec^2 x)$

$= 3 \tan^2 x . \sec^2 x$

Question: 7

Differentiate eac

Solution:

Formulae :

- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

Let,

$y = \cot\sqrt{x}$

and $u = \sqrt{x}$

therefore, y= cot u

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\cot u).\frac{d}{dx}(\sqrt{x})$

$= -\operatorname{cosec}^2 u\ .\frac{1}{2\sqrt{x}}$ .... $\left(\because \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x \,\&\, \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}\right)$

$= -\operatorname{cosec}^2\sqrt{x}\,.\frac{1}{2\sqrt{x}}$

$= \frac{-1}{2\sqrt{x}}\operatorname{cosec}^2\sqrt{x}$

Question: 8

Differentiate eac

Solution:

Formulae :

- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

Let,

$y = \sqrt{\tan x}$

and u = tan x

therefore, $y = \sqrt{u}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\sqrt{u}).\frac{d}{dx}(\tan x)$

$= \frac{1}{2\sqrt{u}}\,.\sec^2 x$ .... $\left(\because \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \,\&\, \frac{d}{dx}(\tan x) = \sec^2 x\right)$

$= \frac{1}{2\sqrt{\tan x}}\,.\sec^2 x$

$= \frac{\sec^2 x}{2\sqrt{\tan x}}$

Question: 9

Differentiate eac

Solution:

Formulae :

• $\frac{d}{dx}(x^n) = n.x^{n-1}$

• $\frac{d}{dx}(kx) = k$

• $\frac{d}{dx}(k) = 0$

• $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = (5+7x)^6$

and $u = (5+7x)$

therefore, $y = u^6$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ ………… By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(u^6) . \frac{d}{dx}(5 + 7x)$

$= 6.(u)^5 . \left(\frac{d}{dx}(5) + \frac{d}{dx}(7x)\right)$ ………… $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= 6.(5+7x)^5.(0+7)$ ………… $\left(\because \frac{d}{dx}(k) = 0 \;\&\; \frac{d}{dx}(kx) = k\right)$

$= 42.(5+7x)^5$

Question: 10

Differentiate eac

Solution:

Formulae :

• $\frac{d}{dx}(x^n) = n.x^{n-1}$

• $\frac{d}{dx}(kx) = k$

• $\frac{d}{dx}(k) = 0$

• $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = (3-4x)^5$

and $u = (3-4x)$

therefore, $y = u^5$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ ………… By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(u^5).\frac{d}{dx}(3-4x)$

$= 5.(u)^4 .\left(\frac{d}{dx}(3) + \frac{d}{dx}(-4x)\right)$ ………… $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \text{ \& } \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= 5. (3\text{-}4x)^4. (0\text{-}4)$ ………… $\left(\because \frac{d}{dx}(k) = 0 \text{ \& } \frac{d}{dx}(kx) = k\right)$

$= -20\ (3\text{-}4x)^4$

Question: 11

Differentiate eac

Solution:

<u>Formulae</u> :

- $\frac{d}{dx}(x^n) = n.x^{n-1}$
- $\frac{d}{dx}(kx) = k$
- $\frac{d}{dx}(k) = 0$
- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = (2x^2 - 3x + 4)^5$

and $u = (2x^2 - 3x + 4)$

therefore, $y = u^5$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ ………… By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(u^5).\frac{d}{dx}(2x^2 - 3x + 4)$

$= 5.(u)^4 .\left(\frac{d}{dx}(2x^2) + \frac{d}{dx}(-3x) + \frac{d}{dx}(4)\right)$ ………… $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \text{ \& } \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= 5. (2x^2 - 3x + 4)^4. (4x\text{-}3\text{+}0)$ ………… $\left(\because \frac{d}{dx}(kx) = k \text{ \& } \frac{d}{dx}(k) = 0\right)$

$= 5. (2x^2 - 3x + 4)^4\ (4x\text{-}3)$

Question: 12

Differentiate eac

Solution:

<u>Formulae</u> :

- $\frac{d}{dx}(x^n) = n.x^{n-1}$
- $\frac{d}{dx}(kx) = k$
- $\frac{d}{dx}(k) = 0$
- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = (ax^2 + bx + c)^6$

and $u = (ax^2 + bx + c)$

therefore, $y = u^6$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ ………… By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(u^6) \cdot \frac{d}{dx}(ax^2 + bx + c)$

$= 6 \cdot (u)^5 \cdot \left(\frac{d}{dx}(ax^2) + \frac{d}{dx}(bx) + \frac{d}{dx}(c) \right)$

$= 6 \cdot (ax^2 + bx + c)^5 \cdot \frac{d}{dx}(ax^2 + bx + c)$ ………… $\left(\because \frac{d}{dx}(x^n) = n \cdot x^{n-1} \;\&\; \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx} \right)$

$= 6 \cdot (ax^2 + bx + c)^5 \cdot (2ax+b+0)$ ………… $\left(\because \frac{d}{dx}(kx) = k \;\&\; \frac{d}{dx}(k) = 0 \right)$

Question: 13

Differentiate eac

Solution:

<u>Formulae</u> :

- $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$
- $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$
- $\frac{d}{dx}(kx) = k$
- $\frac{d}{dx}(k) = 0$
- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = \frac{1}{(x^2 - 3x + 5)^3}$

Let, $u = (x^2-3x+5)^3$

Therefore, $y = \frac{1}{u}$

For $u = (x^2-3x+5)^3$

Let, $v = (x^2-3x+5)$

Therefore, $u = (v)^3$

Therefore, $y = \frac{1}{v^3}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$ ………… By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}\left(\frac{1}{u}\right) \cdot \frac{d}{dv}(v)^3 \cdot \frac{d}{dx}(x^2 - 3x + 5)$

$$= \frac{-1}{u^2} .3v^2.\left(\frac{d}{dx}(x^2) + \frac{d}{dx}(-3x) + \frac{d}{dx}(5)\right)$$

$$\text{............} \left(\because \frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}, \frac{d}{dx}(x^n) = n.x^{n-1} \,\&\, \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$$

$$= \frac{-1}{(x^2-3x+5)^6}.3(x^2-3x+5)^2.(2x-3+0) \text{ .... } \left(\because \frac{d}{dx}(kx) = k \,\&\, \frac{d}{dx}(k) = 0\right)$$

$$= \frac{-3}{(x^2-3x+5)^4}.(2x-3)$$

$$= \frac{-3(2x-3)}{(x^2-3x+5)^4}$$

Question: 14

Differentiate eac

Solution:

<u>**Formulae**</u> **:**

- $\frac{d}{dx}(x^n) = n.x^{n-1}$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}(k) = 0$
- $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{(v)^2}$

Let,

$$y = \sqrt{\frac{a^2-x^2}{a^2+x^2}}$$

and $u = \frac{a^2-x^2}{a^2+x^2}$

$\therefore y = \sqrt{u}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\sqrt{u}).\frac{d}{dx}\left(\frac{a^2-x^2}{a^2+x^2}\right)$$

$$= \frac{1}{2\sqrt{u}}\left(\frac{(a^2+x^2).\frac{d}{dx}(a^2-x^2) - (a^2-x^2).\frac{d}{dx}(a^2+x^2)}{(a^2+x^2)^2}\right) \text{ .... } \left(\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{(v)^2} \,\&\, \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}\right)$$

$$= \frac{1}{2\sqrt{\frac{a^2-x^2}{a^2+x^2}}}\left(\frac{(a^2+x^2).(-2x) - (a^2-x^2).(2x)}{(a^2+x^2)^2}\right) \text{ .... } \left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \,\&\, \frac{d}{dx}(k) = 0\right)$$

$$= \frac{\sqrt{a^2+x^2}}{2\sqrt{a^2-x^2}}.(2x)\left(\frac{-a^2-x^2-a^2+x^2}{(a^2+x^2)^2}\right)$$

$$= \frac{(a^2+x^2)^{1/2}}{2(a^2-x^2)^{1/2}}.(2x).\frac{-2a^2}{(a^2+x^2)^2}$$

$$= \frac{-2a^2x}{(a^2-x^2)^{1/2}(a^2+x^2)^{2-\frac{1}{2}}}$$

$= \frac{-2a^2x}{(a^2-x^2)^{1/2}.(a^2+x^2)^{3/2}}$

Question: 15

Differentiate eac

Solution:

Formulae :

• $1 - \sin^2x = \cos^2x$

• $\frac{d}{dx}(\sec x) = \sec x.\tan x$

• $\frac{d}{dx}(\tan x) = \sec^2x$

Let,

$y = \sqrt{\frac{1+\sin x}{1-\sin x}}$

Multiplying numerator and denominator by (1+sin x),

$\therefore y = \sqrt{\frac{1+\sin x}{1-\sin x}.\frac{1+\sin x}{1+\sin x}}$

$= \sqrt{\frac{(1+\sin x)^2}{1-\sin^2x}}$

$= \sqrt{\frac{(1+\sin x)^2}{\cos^2x}}$ .... $(1 - \sin^2x = \cos^2x)$

$= \frac{1+\sin x}{\cos x}$

$= \frac{1}{\cos x}+\frac{\sin x}{\cos x}$

$y = \sec x + \tan x$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sec x + \tan x)$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sec x) + \frac{d}{dx}(\tan x)$

$= \sec x.\tan x + \sec^2x$ .... $\left(\because \frac{d}{dx}(\sec x) = \sec x.\tan x \;\&\; \frac{d}{dx}(\tan x) = \sec^2x\right)$

$= \sec x\,(\tan x + \sec x)$

Question: 16

Differentiate eac

Solution:

Formulae :

• $\frac{d}{dx}(\cos x) = -\sin x$

• $\frac{d}{dx}(x^n) = n.x^{n-1}$

• 2 sin x. cos x = sin 2x

Let,

y = $\cos^2 x^3$

and u = x^3

therefore, y= $\cos^2 u$

let, v = cos u

therefore, y= v^2

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^2).\frac{d}{du}(\cos u).\frac{d}{dx}(x^3)$

$= 2\,v\,.(-\sin u).3x^2$ .... $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1}\ \&\ \frac{d}{dx}(\cos x) = -\sin x\right)$

$= -2\cos u\,.\sin u\,.3x^2$

$= -\sin 2u\,.3x^2$ .... $(\because 2\sin x.\cos x = \sin 2x)$

$= -\sin 2x^3\,.3x^2$

Question: 17

Differentiate eac

Solution:

<u>Formulae</u> :

• $\frac{d}{dx}(\sec x) = \sec x.\tan x$

• $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

y = $\sec^3 (x^2+1)$

and u = x^2+1

therefore, y= $\sec^3 u$

let, v = sec u

therefore, y= v^3

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^3).\frac{d}{du}(\sec u).\frac{d}{dx}(x^2+1)$

$= 3\,v^2\,.(\sec u.\tan u).2x$ .... $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1}\ \&\ \frac{d}{dx}(\sec x) = \sec x.\tan x\right)$

$= 3\sec^2 u\,.(\sec u.\tan u).2x$

$= 6x.\sec^3 u\,.\tan u$

$= 6x.\sec^3(x^2+1)\,.\tan(x^2+1)$

Question: 18

Differentiate eac

Solution:

Formulae :

- $\frac{d}{dx}(\cos x) = -\sin x$

- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

- $\frac{d}{dx}(kx) = k$

Let,

$y = \sqrt{\cos 3x}$

and u = 3x

therefore, $y = \sqrt{\cos u}$

let, v = cos u

therefore, $y = \sqrt{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{dv}(\sqrt{v}) \cdot \frac{d}{du}(\cos u) \cdot \frac{d}{dx}(3x)$

$= \frac{1}{2\sqrt{v}} \cdot (-\sin u) \cdot 3$ .... $\left(\because \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}, \frac{d}{dx}(\cos x) = -\sin x \;\&\; \frac{d}{dx}(kx) = k\right)$

$= \frac{-3}{2} \cdot \frac{\sin u}{\sqrt{\cos u}}$

$= \frac{-3}{2} \cdot \frac{\sin 3x}{\sqrt{\cos 3x}}$

Question: 19

Differentiate eac

Solution:

Formulae :

- $\frac{d}{dx}(\sin x) = \cos x$

- $\frac{d}{dx}(x^n) = n.x^{n-1}$

- $\frac{d}{dx}(kx) = k$

Let,

$y = \sqrt[3]{\sin 2x}$

and u = 2x

therefore, $y = \sqrt[3]{\sin u}$

let, v = sin u

therefore, $y = \sqrt[3]{v} = v^{3/2}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ ………… By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{dv}\left(v^{1/3}\right).\frac{d}{du}(\sin u).\frac{d}{dx}(2x)$

$= \frac{1}{3}v^{-2/3}.(\cos u).2$ ………… $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1}, \frac{d}{dx}(\sin x) = \cos x \;\&\; \frac{d}{dx}(kx) = k\right)$

$= \frac{2}{3}\frac{\cos u}{v^{2/3}}$.

$= \frac{2}{3}\frac{\cos u}{(\sin u)^{2/3}}$

$= \frac{2}{3}\frac{\cos 2x}{(\sin 2x)^{2/3}}$

Question: 20

Differentiate eac

Solution:

Formulae :

- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
- $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}(k) = 0$
- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = \sqrt{1+\cot x}$

and u = 1+ cot x

therefore, $y = \sqrt{u}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ ………… By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}\left(\sqrt{u}\right).\frac{d}{dx}(1+\cot x)$

$= \frac{1}{2\sqrt{u}}.\left(\frac{d}{dx}(1) + \frac{d}{dx}(\cot x)\right)$ ………… $\left(\because \frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}} \;\&\; \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= \frac{1}{2\sqrt{1+\cot x}}.(0 - \operatorname{cosec}^2 x)$.

$= \frac{-1}{2}\frac{\operatorname{cosec}^2 x}{\sqrt{1+\cot x}}$

Question: 21

Differentiate eac

Solution:

Formulae :

- $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x.\cot x$
- $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

$y = \operatorname{cosec}^3 \frac{1}{x^2}$

and $u = \frac{1}{x^2}$

therefore, y= cosec^3 u

let, v = cosec u

therefore, y= v^3

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^3).\frac{d}{du}(\operatorname{cosec} u).\frac{d}{dx}\left(\frac{1}{x^2}\right)$

$= 3v^2 \;.(-\operatorname{cosec} u.\cot u).\frac{d}{dx}(x^{-2})$

.... $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x.\cot x\right)$

$= 3\operatorname{cosec}^2 u \;.(-\operatorname{cosec} u.\cot u).(-2x^{-3})$

$= 3\operatorname{cosec}^3 u.\cot u \;(2\frac{1}{x^3})$

$= \frac{6}{x^3}.\operatorname{cosec}^3\left(\frac{1}{x^2}\right).\cot\left(\frac{1}{x^2}\right)$

Question: 22

Differentiate eac

Solution:

<u>Formulae</u> :

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

$y = \sqrt{\sin x^3}$

and u = x^3

therefore, $y = \sqrt{\sin u}$

let, v = sin u

therefore, $y = \sqrt{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv} . \frac{dv}{du} . \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{dv}(\sqrt{v}) . \frac{d}{du}(\sin u) . \frac{d}{dx}(x^3)$

$= \frac{1}{2\sqrt{v}} . (\cos u) . 3x^2$

.... $\left(\because \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}, \frac{d}{dx}(x^n) = n.x^{n-1} \ \& \ \frac{d}{dx}(\sin x) = \cos x\right)$

$= \frac{1}{2\sqrt{\sin u}} . (\cos u) . 3x^2$

$= \frac{3}{2}x^2 . \frac{\cos x^3}{\sqrt{\sin x^3}}$

Question: 23

Differentiate eac

Solution:

<u>Formulae</u> :

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}(kx) = k$
- $\frac{d}{dx}(u.v) = u.\frac{d}{dx}(v) + v.\frac{d}{dx}(u)$

Let,

$y = \sqrt{x.\sin x}$

and u = x. sin x

therefore, $y = \sqrt{u}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\sqrt{u}) . \frac{d}{dx}(x.\sin x)$

$= \frac{1}{2\sqrt{u}} . \left(x.\frac{d}{dx}(\sin x) + \sin x . \frac{d}{dx}(x)\right)$

.... $\left(\because \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \ \& \ \frac{d}{dx}(u.v) = u.\frac{d}{dx}(v) + v.\frac{d}{dx}(u)\right)$

$= \frac{1}{2\sqrt{x.\sin x}} . (x.(\cos x) + \sin x.(1))$ .... $\left(\because \frac{d}{dx}(kx) = k \ \& \ \frac{d}{dx}(\sin x) = \cos x\right)$

$= \frac{(x.\cos x + \sin x)}{2\sqrt{x.\sin x}}$

Question: 24

Differentiate eac

Solution:

Formulae :

- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

Let,

$y = \sqrt{\cot\sqrt{x}}$

And $u = \sqrt{x}$

therefore, $y = \sqrt{\cot u}$

let, v = cot u

therefore, $y = \sqrt{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$ ………… By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(\sqrt{v}) \cdot \frac{d}{du}(\cot u) \cdot \frac{d}{dx}(\sqrt{x})$$

$$= \frac{1}{2\sqrt{v}} \cdot (-\operatorname{cosec}^2 u) \cdot \frac{1}{2\sqrt{x}}$$

………… $\left(\because \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \ \& \ \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2\right)$

$$= \frac{1}{2\sqrt{\cot u}} \cdot (-\operatorname{cosec}^2 u) \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{\cot\sqrt{x}}} \cdot \left(-\operatorname{cosec}^2\sqrt{x}\right) \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{-\operatorname{cosec}^2\sqrt{x}}{4\sqrt{x}\sqrt{\cot\sqrt{x}}}$$

Question: 25

Differentiate eac

Solution:

Formulae :

- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
- $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

$y = \cot^3 x^2$

and $u = x^2$

therefore, $y = \cot^3 u$

let, v = cot u

therefore, $y = v^3$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^3).\frac{d}{du}(\cot u).\frac{d}{dx}(x^2)$

$= 3v^2 .(-\mathrm{cosec}^2 u).2x$ .... $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x\right)$

$= 3\cot^2 u .(-\mathrm{cosec}^2 u).2x$

$= -6x.\cot^2 u .\mathrm{cosec}^2 u$

$= -6x.\cot^2(x^2) .\mathrm{cosec}^2(x^2)$

Question: 26

Differentiate eac

Solution:

Formulae :

- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = \cos\left(\sin\sqrt{ax+b}\right)$

and u = ax + b

therefore, $y = \cos\left(\sin\sqrt{u}\right)$

let, $v = \sqrt{u}$

therefore, $y = \cos(\sin v)$

let, $w = \sin v$

therefore, $y = \cos w$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dw}.\frac{dw}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{dw}(\cos w).\frac{d}{dv}(\sin v).\frac{d}{du}(\sqrt{u}).\frac{d}{dx}(ax + b)$

$= (-\sin w).(\cos v).\left(\frac{1}{2\sqrt{u}}\right).\left(\frac{d}{dx}(ax) + \frac{d}{dx}(b)\right)$

.... $\left(\because \frac{d}{dx}(\cos x) = -\sin x , \frac{d}{dx}(\sin x) = \cos x, \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \;\&\; \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= (-\sin(\sin v)).(\cos\sqrt{u}).\left(\frac{1}{2\sqrt{ax + b}}\right).(a+0)$

$= (-\sin(\sin\sqrt{u})).(\cos\sqrt{ax + b}).\left(\frac{1}{2\sqrt{ax + b}}\right).(a)$

$$= \left(\frac{-a.\cos\sqrt{ax + b}}{2\sqrt{ax + b}}\right).\left(\sin\left(\sin\sqrt{ax + b}\right)\right)$$

Question: 27

Differentiate eac

Solution:

<u>Formulae</u> :

- $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x . \cot x$
- $\frac{d}{dx}(x^n) = n.x^{n-1}$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = \sqrt{\operatorname{cosec}(x^3 + 1)}$

and $u = x^3 + 1$

therefore, $y = \sqrt{\operatorname{cosec} u}$

let, $v = \operatorname{cosec} u$

therefore, $y = \sqrt{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(\sqrt{v}).\frac{d}{du}(\operatorname{cosec} u).\frac{d}{dx}(x^3 + 1)$$

$$= \frac{1}{2\sqrt{v}}.(-\operatorname{cosec} u.\cot u).\left(\frac{d}{dx}(x^3) + \frac{d}{dx}(1)\right)$$

.... $\left(\because \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x . \cot x , \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \& \frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$$= \frac{1}{2\sqrt{\operatorname{cosec} u}}.(-\operatorname{cosec}(x^3 + 1).\cot(x^3 + 1)).(3x^2 + 0)$$

.... $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$$= \frac{1}{2\sqrt{\operatorname{cosec}(x^3 + 1)}}.(-\operatorname{cosec}(x^3 + 1).\cot(x^3 + 1)).(3x^2)$$

$$= \frac{-3x^2}{2}.\sqrt{\operatorname{cosec}(x^3 + 1)}.\cot(x^3 + 1)$$

Question: 28

Differentiate eac

Solution:

<u>Formulae</u> :

- $(2\sin a . \cos b) = \sin(a + b) + \sin(a - b)$

• $\frac{d}{dx}(\sin x) = \cos x$

• $\frac{d}{dx}(kx) = k$

• $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

y = sin 5x. cos 3x

$$y = \frac{1}{2}(2\sin 5x.\cos 3x)$$

$$y = \frac{1}{2}(\sin(5x+3x) + \sin(5x-3x)) \text{ .... } (\because (2\sin a.\cos b) = \sin(a+b) + \sin(a-b))$$

$$y = \frac{1}{2}(\sin(8x) + \sin(2x))$$

Differentiating above equation w.r.t. x,

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}(\sin(8x) + \sin(2x))\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\left(\frac{d}{dx}\sin 8x + \frac{d}{dx}\sin 2x\right) \text{ .... } \left(\because \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$$

$$= \frac{1}{2}(8\cos 8x + 2\cos 2x) \text{ .... } \left(\because \frac{d}{dx}(\sin x) = \cos x \ \& \ \frac{d}{dx}(kx) = k\right)$$

$$= 4\cos 8x + \cos 2x$$

Question: 29

Differentiate eac

Solution:

<u>Formulae</u> :

• $(2\sin a.\sin b) = \cos(a-b) - \cos(a+b)$

• $\frac{d}{dx}(\cos x) = -\sin x$

• $\frac{d}{dx}(kx) = k$

• $\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$

Let,

y = sin 2x. sin x

$$y = \frac{1}{2}(2\sin 2x.\sin x)$$

$$y = \frac{1}{2}(\cos(2x-x) - \cos(2x+x)) \text{ .... } (\because (2\sin a.\sin b) = \cos(a-b) - \cos(a+b))$$

$$y = \frac{1}{2}(\cos x - \cos 3x)$$

Differentiating above equation w.r.t. x,

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}(\cos x - \cos 3x)\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\left(\frac{d}{dx}\cos x - \frac{d}{dx}\cos 3x\right) \text{ .... } \left(\because \frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}\right)$$

$= \frac{1}{2}(-\sin x + 3\sin 3x)$ .... $\left(\because \frac{d}{dx}(\cos x) = -\sin x \ \& \ \frac{d}{dx}(kx) = k\right)$

$= \frac{3}{2}\sin 3x - \frac{1}{2}\sin x$

Question: 30

Differentiate eac

Solution:

<u>Formulae</u> :

- $(2\cos a . \cos b) = \cos(a+b) + \cos(a-b)$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(kx) = k$
- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

y = cos 4x. cos 2x

$y = \frac{1}{2}(2\cos 4x . \cos 2x)$

$y = \frac{1}{2}(\cos(4x+2x) + \cos(4x-2x))$ .... $(\because (2\cos a . \cos b) = \cos(a+b) + \cos(a-b))$

$y = \frac{1}{2}(\cos 6x + \cos 2x)$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}(\cos 6x + \cos 2x)\right)$

$\therefore \frac{dy}{dx} = \frac{1}{2}\left(\frac{d}{dx}\cos 6x + \frac{d}{dx}\cos 2x\right)$ .... $\left(\because \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= \frac{1}{2}(-6\sin 6x - 2\sin 2x)$ .... $\left(\because \frac{d}{dx}(\cos x) = -\sin x \ \& \ \frac{d}{dx}(kx) = k\right)$

= -3 sin 6x – sin 2x

= - (3 sin 6x + sin 2x)

Question: 31

Find

Solution:

<u>Formulae</u> :

- $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$
- $\frac{1+\tan^2 x}{1-\tan^2 x} = \cos 2x$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $1 + \tan^2 x = \sec^2 x$

Given,

$y = \sin\left(\frac{1+x^2}{1-x^2}\right)$

Put x = tan a

Therefore, $\frac{dx}{da} = \sec^2 a$ .... eq (1)

$y = \sin\left(\frac{1+\tan^2 a}{1-\tan^2 a}\right)$

y = sin (cos 2a) .... $\left(\because \frac{1+\tan^2 x}{1-\tan^2 x} = \cos 2x\right)$

Differentiating above equation w.r.t. a ,

$\frac{dy}{da} = \frac{d}{da}(\sin(\cos 2a))$

$= (\cos(\cos 2a))\frac{d}{da}(\cos 2a)$ .... $\left(\because \frac{d}{dx}(\sin x) = \cos x\right)$

$= (\cos(\cos 2a)).(-\sin 2a).\frac{d}{da}(2a)$ .... $\left(\because \frac{d}{dx}(\cos x) = -\sin x\right)$

$= (-2\sin 2a).(\cos(\cos 2a))$

$= -2\left(\frac{2\tan a}{1+\tan^2 a}\right).\left(\cos\left(\frac{1+\tan^2 a}{1-\tan^2 a}\right)\right)$ .... $\left(\because \frac{1+\tan^2 x}{1-\tan^2 x} = \cos 2x \ \& \ \frac{2\tan x}{1+\tan^2 x} = \sin 2x\right)$

But, x = tan a

$\frac{dy}{da} = -2\left(\frac{2x}{1+x^2}\right).\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right)$

$\frac{dy}{da} = \left(\frac{-4x}{1+x^2}\right).\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right)$ .... eq (2)

Now,

$\frac{dy}{dx} = \frac{dy}{da}.\frac{da}{dx}$............ By chain rule

$\therefore \frac{dy}{dx} = \left(\frac{-4x}{1+x^2}\right).\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right).\frac{1}{\sec^2 a}$ .... from eq (1) & eq (2)

$= \left(\frac{-4x}{1+x^2}\right).\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right).\frac{1}{1+\tan^2 a}$ .... $(\because 1+\tan^2 x = \sec^2 x)$

$= \left(\frac{-4x}{1+x^2}\right).\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right).\frac{1}{1+x^2}$ .... $(\because x = \tan a)$

$\therefore \frac{dy}{dx} = \frac{-4x}{(1+x^2)^2}.\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right)$

Question: 32

Find

Solution:

<u>Formulae</u> :

- $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{(v)^2}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$

• $\frac{d}{dx}(x^n) = n.x^{n-1}$

Given,

$$y = \frac{\sin x + x^2}{\cot 2x}$$

Differentiating above equation w.r.t. x ,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin x + x^2}{\cot 2x}\right)$$

$$= \frac{\cot 2x.\frac{d}{dx}(\sin x + x^2) - (\sin x + x^2).\frac{d}{dx}(\cot 2x)}{(\cot 2x)^2} \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{(v)^2}\right)$$

$$= \frac{\cot 2x.(\cos 2x + 2x) - (\sin x + x^2).(-2\operatorname{cosec}^2 2x)}{(\cot 2x)^2}$$

$$\ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}(\sin x) = \cos x, \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2\right)$$

$$= \frac{(\cos 2x + 2x)}{\cot 2x} - \frac{(\sin x + x^2).(-2\operatorname{cosec}^2 2x)}{(\cot 2x)^2}$$

$$= \tan 2x.(\cos 2x + 2x) + \frac{(\sin x + x^2).\left(\frac{2}{\sin^2 x}\right)}{\frac{\cos^2 x}{\sin^2 x}}$$

$$= \tan 2x.(\cos 2x + 2x) + \frac{2(\sin x + x^2)}{\cos^2 x}$$

$$= \tan 2x.(\cos 2x + 2x) + 2\sec^2 2x.(\sin x + x^2)$$

$$\therefore \frac{dy}{dx} = \tan 2x.(\cos 2x + 2x) + 2\sec^2 2x.(\sin x + x^2)$$

Question: 33

If <

Solution:

<u>Formulae</u> :

• $\frac{\sin x}{\cos x} = \tan x$

• $\frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4} - x\right)$

• $\frac{d}{dx}(\tan x) = \sec^2 x$

• $\frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}$

• $\frac{d}{dx}(kx) = k$

• $\frac{d}{dx}(k) = 0$

• $\tan^2 x + 1 = \sec^2 x$

Given,

$$y = \frac{(\cos x - \sin x)}{(\cos x + \sin x)}$$

Dividing numerator and denominator by cosx,

$$y = \frac{\left(1 - \frac{\sin x}{\cos x}\right)}{\left(1 + \frac{\sin x}{\cos x}\right)}$$

$$y = \frac{1 - \tan x}{1 + \tan x} \text{............} \left(\because \frac{\sin x}{\cos x} = \tan x\right)$$

$$y = \tan\left(\frac{\pi}{4} - x\right) \text{............} \left(\because \frac{1 - \tan x}{1 + \tan x} = \tan\left(\frac{\pi}{4} - x\right)\right)$$

Differentiating above equation w.r.t. x ,

$$\frac{dy}{dx} = \frac{d}{dx} \tan\left(\frac{\pi}{4} - x\right)$$

$$= \sec^2\left(\frac{\pi}{4} - x\right) . \frac{d}{dx}\left(\frac{\pi}{4} - x\right) \text{............} \left(\because \frac{d}{dx}(\tan x) = \sec^2 x\right)$$

$$= \sec^2\left(\frac{\pi}{4} - x\right) . \left(\frac{d}{dx}\left(\frac{\pi}{4}\right) - \frac{d}{dx}(x)\right) \text{............} \left(\because \frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}\right)$$

$$= \sec^2\left(x + \frac{\pi}{4}\right) . (0 - 1) \text{............} \left(\because \frac{d}{dx}(kx) = k \;\&\; \frac{d}{dx}(k) = 0\right)$$

$$= -\sec^2\left(x + \frac{\pi}{4}\right)$$

$$\therefore \frac{dy}{dx} = -\sec^2\left(x + \frac{\pi}{4}\right)$$

Now,

$$\frac{dy}{dx} + y^2 + 1 = -\sec^2\left(x + \frac{\pi}{4}\right) + \left(\tan^2\left(x + \frac{\pi}{4}\right) + 1\right)$$

$$= -\sec^2\left(x + \frac{\pi}{4}\right) + \left(\sec^2\left(x + \frac{\pi}{4}\right)\right) \text{............} (\because \tan^2 x + 1 = \sec^2 x)$$

$$= 0$$

$$\therefore \frac{dy}{dx} + y^2 + 1 = 0$$

Hence Proved.

Question: 34

If <

Solution:

<u>**Formulae**</u> **:**

- $\frac{\sin x}{\cos x} = \tan x$
- $\frac{1 + \tan x}{1 - \tan x} = \tan\left(x + \frac{\pi}{4}\right)$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$
- $\frac{d}{dx}(kx) = k$
- $\frac{d}{dx}(k) = 0$

Given,

$$y = \frac{(\cos x + \sin x)}{(\cos x - \sin x)}$$

Dividing numerator and denominator by cosx,

$$y = \frac{\left(1 + \frac{\sin x}{\cos x}\right)}{\left(1 - \frac{\sin x}{\cos x}\right)}$$

$$y = \frac{1 + \tan x}{1 - \tan x} \ldots\ldots\ldots\ldots \left(\because \frac{\sin x}{\cos x} = \tan x\right)$$

$$y = \tan\left(x + \frac{\pi}{4}\right) \ldots\ldots\ldots\ldots \left(\because \frac{1 + \tan x}{1 - \tan x} = \tan\left(x + \frac{\pi}{4}\right)\right)$$

Differentiating above equation w.r.t. x ,

$$\frac{dy}{dx} = \frac{d}{dx} \tan\left(x + \frac{\pi}{4}\right)$$

$$= \sec^2\left(x + \frac{\pi}{4}\right) . \frac{d}{dx}\left(x + \frac{\pi}{4}\right) \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx} (\tan x) = \sec^2 x\right)$$

$$= \sec^2\left(x + \frac{\pi}{4}\right) . \left(\frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{\pi}{4}\right)\right) \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx} (u + v) = \frac{du}{dx} + \frac{dv}{dx}\right)$$

$$= \sec^2\left(x + \frac{\pi}{4}\right) . (1 + 0) \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx} (kx) = k \;\&\; \frac{d}{dx} (k) = 0\right)$$

$$= \sec^2\left(x + \frac{\pi}{4}\right)$$

$$\therefore \frac{dy}{dx} = \sec^2\left(x + \frac{\pi}{4}\right)$$

Hence Proved.

Exercise : 10B

Question: 1

Differentiate eac

Solution:

(i) Let $y = e^{4x}$ $z = 4x$

Formula : $\frac{d(e^x)}{dx} = e^x$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= (e^{4x}) \times 4$$

$$= 4e^{4x}$$

(ii) Let $y = e^{-5x}$ $z = -5x$

Formula : $\frac{d(e^x)}{dx} = e^x$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= (e^{-5x}) \times (-5)$$

$$= -5e^{-5x}$$

(iii) Let $y = (e)^{x^3}$ $z = x^3$

Formula : $\frac{d(e^x)}{dx} = e^x , \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= \left((e)^{x^3}\right) \times 3x^2$

$= 3x^2 (e)^{x^3}$

Question: 2

Differentiate eac

Solution:

(i) Let y =$e^{2/x}$ z = 2/x

Formula : $\frac{d(e^x)}{dx} = e^x , \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= \left(e^{2/x}\right) \times (\frac{-2}{x^2})$

$= \frac{-2}{x^2} \times e^{\frac{2}{x}}$

(ii) Let y =$e^{\sqrt{x}}$ z = $\sqrt{x}$

Formula : $\frac{d(e^x)}{dx} = e^x , \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= \left(e^{\sqrt{x}}\right) \times \left(\frac{1}{2} \times x^{-0.5}\right) = \left(e^{\sqrt{x}}\right) \times \left(\frac{1}{2 \times \sqrt{x}}\right)$

$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

(iii) Let y =$e^{-2\sqrt{x}}$ z = -2$\sqrt{x}$

Formula : $\frac{d(e^x)}{dx} = e^x , \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= \left(e^{-2\sqrt{x}}\right) \times \left(-2 \times \frac{1}{2} \times x^{-0.5}\right) = \left(e^{-2\sqrt{x}}\right) \times \left(\frac{-1}{\sqrt{x}}\right)$

$= \frac{-e^{-2\sqrt{x}}}{\sqrt{x}}$

Question: 3

Differentiate eac

Solution:

(i) Let y $= e^{\cot x}$ z $= \cot x$

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$= (e^{\cot x}) \times (-\operatorname{cosec}^2 x)$

$= -\operatorname{cosec}^2 x \, e^{\cot x}$

(ii) Let y $= e^{-\sin 2x}$ z $= -\sin 2x$

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(\sin x)}{dx} = \cos x$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$= \left(e^{-\sin 2x}\right) \times (-\cos 2x \times 2)$

$= (-2\cos 2x)\, e^{-\sin 2x}$

(iii) Let y $= e^{\sqrt{\sin x}}$ z $= \sqrt{\sin x}$

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(\sin x)}{dx} = \cos x$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$= \left(e^{\sqrt{\sin x}}\right) \times \left(\frac{1}{2} \times (\sin x)^{-0.5} \times \cos x\right) = \left(e^{\sqrt{\sin x}}\right) \times \left(\frac{1 \times \cos x}{2\sqrt{\sin x}}\right)$

$= \frac{\cos x}{2\sqrt{\sin x}} e^{\sqrt{\sin x}}$

Question: 4

Differentiate eac

Solution:

(i) Let y =tan(log x) z = log x

Formula : $\frac{d(\tan x)}{dx} = \sec^2 x, \frac{d(\log x)}{dx} = 1/_x$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$= (\sec^2 \log x) \times \left(\frac{1}{x}\right)$

$= \frac{\sec^2 (\log x)}{x}$

(ii) Let y = log (sec x) z = sec x

Formula : $\frac{d(\sec x)}{dx} = \sec x \times \tan x, \frac{d(\log x)}{dx} = 1/_x$

According to chain rule of differentiation

$^{dy}/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= \left(\frac{1}{\sec x}\right)(\sec x \times \tan x)$

$= \tan x$

(iii) Let y = log (sin (x/2)) z = sin (x/2)

Formula : $\frac{d(\sin x)}{dx} = \cos x, \frac{d(\log x)}{dx} = {}^{1}/_{x}$

According to chain rule of differentiation

$^{dy}/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= \left(\frac{1}{\sin (x/2)}\right)\left(\cos (x/2) \times \frac{1}{2}\right)$

$= \frac{1}{2} \times \cot (x/2)$

Question: 5

Differentiate eac

Solution:

(i) Let y = $\log_3 x$

Formula : $\log_a b = \frac{\log b}{\log a}, \frac{d(\log x)}{dx} = {}^{1}/_{x}$

Therefore y =$\frac{\log x}{\log 3}$

According to chain rule of differentiation

$^{dy}/_{dx} = \frac{dy}{dx}$

$= \left(\frac{1}{\log 3}\right)\left(\frac{1}{x}\right)$

$= \frac{1}{x(\log 3)}$

(ii) Let y = 2^{-x} z = -x

Formula : $\frac{d(a^x)}{dx} = a^x (\log a)$

According to chain rule of differentiation

$^{dy}/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= (2^{-x}) \times (\log 2)(-1)$

$= -2^{-x}(\log 2)$

(iii) Let y = 3^{x+2} z = x

Therefore Y = $3^2 \times 3^x$

Formula : $\frac{d(a^x)}{dx} = a^x (\log a)$

According to chain rule of differentiation

$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= 9(3^x) \times (\log 3)$

Question: 6

Differentiate eac

Solution:

(i) Let y = $\log(x + \frac{1}{x})$ z = $x + \frac{1}{x}$

Formula : $\frac{d(\log x)}{dx} = \frac{1}{x}, \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= \left(\frac{1}{x + \frac{1}{x}}\right) \times (1 - \frac{1}{x^2})$

$= \left(\frac{x}{x^2 + 1}\right) \times (\frac{x^2 - 1}{x^2})$

$= \left(\frac{x^2 - 1}{x(x^2 + 1)}\right)$

(ii) Let y = log (sin (3x)) z = sin (3x)

Formula : $\frac{d(\sin x)}{dx} = \cos x, \frac{d(\log x)}{dx} = 1/_x$

According to chain rule of differentiation

$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= \left(\frac{1}{\sin(3x)}\right)(\cos(3x) \times 3)$

$= 3 \times \cot(3x)$

(iii) Let y = $\log(x + \sqrt{1 + x^2})$ z = $x + \sqrt{1 + x^2}$

Formula : $\frac{d(\log x)}{dx} = \frac{1}{x}, \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= \left(\frac{1}{x + \sqrt{1 + x^2}}\right) \times (1 + \frac{1}{2}(1 + x^2)^{-0.5}\ 2x)$

$= \left(\frac{1}{x + \sqrt{1 + x^2}}\right) \times (1 + \frac{x}{1}(1 + x^2)^{-0.5})$

$= \left(\frac{1}{x + \sqrt{1 + x^2}}\right) \times (1 + \frac{x}{\sqrt{1 + x^2}})$

$= \left(\frac{1}{x + \sqrt{1 + x^2}}\right) \times (\frac{\sqrt{1 + x^2} + x}{\sqrt{1 + x^2}})$

$= \left(\frac{1}{\sqrt{1+x^2}}\right)$

Question: 7

Differentiate eac

Solution:

Let y = $e^{\sqrt{x}} \log x$, z = $e^{\sqrt{x}}$ and w = log (x)

Formula : $\frac{d(e^x)}{dx} = e^x$, $\frac{d(\log x)}{dx} = \frac{1}{x}$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\log(x) \times (e^{\sqrt{x}}) \times \frac{1}{2\sqrt{x}}] + [e^{\sqrt{x}} \times \frac{1}{x}]$$

$$= e^{\sqrt{x}} \times [\frac{\log(x)}{2\sqrt{x}} + \frac{1}{x}]$$

$$= e^{\sqrt{x}} \times [\frac{\sqrt{x}\log(x)}{2x} + \frac{2}{2x}]$$

$$= e^{\sqrt{x}} \times [\frac{2 + \sqrt{x}\log(x)}{2x}]$$

Question: 8

Differentiate eac

Solution:

Let y = $\log \sin \sqrt{1+x^2}$, z = $\sin \sqrt{1+x^2}$

Formula : $\frac{d(\sin x)}{dx} = \cos x$, $\frac{d(\log x)}{dx} = \frac{1}{x}$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= [\frac{1}{\sin\sqrt{1+x^2}}] \times [\cos\sqrt{1+x^2}] \times [\frac{1}{2} \times \frac{1}{\sqrt{1+x^2}} \times 2x]$$

$$= [\cot\sqrt{1+x^2}] \times [\frac{1}{1} \times \frac{1}{\sqrt{1+x^2}} \times x]$$

$$= \frac{x}{\sqrt{x^2+1}} \cot\sqrt{x^2+1}$$

Question: 9

Differentiate eac

Solution:

Let y = e^{2x} sin 3x, z = e^{2x} and w = sin 3x

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\sin x)}{dx} = \cos x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$= [\sin 3x \times (2 \times e^{2x})] + [e^{2x} \times 3\cos 3x]$

$= e^{2x} \times [2\sin 3x + 3\cos 3x]$

Question: 10

Differentiate eac

Solution:

Let y = e^{3x} cos 2x, z = e^{3x} and w = cos 2x

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cos x)}{dx} = -\sin x$

According to product rule of differentiation

$^{dy}/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$

$= [\cos 2x \times (3 \times e^{3x})] + [e^{3x} \times (-2\sin 2x)]$

$= e^{3x} \times [3\cos 2x - 2\sin 2x]$

Question: 11

Differentiate eac

Solution:

Let y = e^{-5x} cot 4x , z = e^{-5x} and w = cot 4x

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

According to product rule of differentiation

$^{dy}/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$

$= [\cot 4x \times (-5e^{-5x})] + [e^{-5x} \times (-4\operatorname{cosec}^2 4x)]$

$= -e^{-5x} \times [5\cot 4x + 4\operatorname{cosec}^2 4x]$

Question: 12

Differentiate eac

Solution:

Let y = e^x log (sin 2x), z = e^x and w = log (sin 2x)

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(\log x)}{dx} = \frac{1}{x}$ and $\frac{d(\sin x)}{dx} = \cos x$

According to product rule of differentiation

$^{dy}/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$

$= [\log(\sin 2x) \times (e^x)] + [e^x \times \frac{1}{\sin 2x} \times 2\cos 2x]$

$= e^x \times [\log(\sin 2x) + \frac{2\cos 2x}{\sin 2x}]$

$= e^x \times [\log(\sin 2x) + 2\cot 2x]$

Question: 13

Differentiate eac

Solution:

Let y = $\log(\operatorname{cosec} x - \cot x)$, z =$(\operatorname{cosec} x - \cot x)$

Formula :

$$\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cot x \,, \frac{d(\log x)}{dx} = \frac{1}{x} \text{ and } \frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= [\frac{1}{(\operatorname{cosec} x - \cot x)}] \times [-\operatorname{cosec} x \cot x - (-\operatorname{cosec}^2 x)]$$

$$= [\frac{1}{(\operatorname{cosec} x - \cot x)}] \times [-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x]$$

$$= [\frac{1}{(\operatorname{cosec} x - \cot x)}] \times [\operatorname{cosec} x(\operatorname{cosec} x - \cot x)]$$

= cosec x

Question: 14

Differentiate eac

Solution:

Let y = $\log(\sec \frac{x}{2} + \tan \frac{x}{2})$, z =$(\sec \frac{x}{2} + \tan \frac{x}{2})$

Formula :

$$\frac{d(\sec x)}{dx} = \sec x \tan x \,, \frac{d(\log x)}{dx} = \frac{1}{x} \text{ and } \frac{d(\tan x)}{dx} = \sec^2 x$$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= [\frac{1}{(\sec\frac{x}{2} + \tan \frac{x}{2})}] \times [(\sec \frac{x}{2} \tan \frac{x}{2} \times \frac{1}{2}) + (\sec^2 \frac{x}{2} \times \frac{1}{2})]$$

$$= [\frac{1}{(\sec\frac{x}{2} + \tan \frac{x}{2})}] \times [\frac{1}{2} \sec \frac{x}{2} (\sec\frac{x}{2} + \tan \frac{x}{2})]$$

$$= \frac{1}{2} \sec \frac{x}{2}$$

Question: 15

Differentiate eac

Solution:

Let y = $\sqrt{\frac{1+e^x}{1-e^x}}$, u =$1 + e^x$, v =$1 - e^x$, z=$\frac{1+e^x}{1-e^x}$

Formula : $\frac{d(e^x)}{dx} = e^x$

According to quotient rule of differentiation

If z = $\frac{u}{v}$

$$dz/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1-e^x)\times(e^x) - (1+e^x)\times(-e^x)}{(1-e^x)^2}$$

$$= \frac{e^x - e^{2x} + e^x + e^{2x}}{(1-e^x)^2}$$

$$= \frac{2e^x}{(1-e^x)^2}$$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz}\times\frac{dz}{dx}$$

$$= \left[\frac{1}{2}\times\left(\frac{1+e^x}{1-e^x}\right)^{\frac{1}{2}-1}\right]\times\left[\frac{2e^x}{(1-e^x)^2}\right]$$

$$= \left[\frac{e^x}{1}\times\left(\frac{1+e^x}{1}\right)^{-\frac{1}{2}}\right]\times\left[\frac{1}{(1-e^x)^{2-\frac{1}{2}}}\right]$$

$$= \left[\frac{e^x}{(1+e^x)^{\frac{1}{2}}\times(1-e^x)^{2-\frac{1}{2}}}\right]$$

$$= \left[\frac{e^x}{(1+e^x)^{\frac{1}{2}}\times(1-e^x)^{\frac{1}{2}}\times(1-e^x)^1}\right]$$

$$= \left[\frac{e^x}{((1+e^x)(1-e^x))^{\frac{1}{2}}\times(1-e^x)^1}\right]$$

$$= \frac{e^x}{(1-e^x)\sqrt{1-e^{2x}}}$$

Question: 16

Differentiate eac

Solution:

Let $y = \frac{e^x+e^{-x}}{e^x-e^{-x}}$, $u = e^x + e^{-x}$, $v = e^x - e^{-x}$

Formula : $\frac{d(e^x)}{dx} = e^x$

According to quotient rule of differentiation

If $y = \frac{u}{v}$

$$dy/_{dx} = \frac{v\times\frac{du}{dx} - u\times\frac{dv}{dx}}{v^2}$$

$$= \frac{(e^x - e^{-x})\times(e^x - e^{-x}) - (e^x + e^{-x})\times(e^x + e^{-x})}{(e^x - e^{-x})^2}$$

$$= \frac{(e^x - e^{-x})^2 - (e^x + e^{-x})^2}{(e^x - e^{-x})^2}$$

$$= \frac{(e^x - e^{-x} + e^x + e^{-x})(e^x - e^{-x} - e^x - e^{-x})}{(e^x - e^{-x})^2}$$

$(a^2 - b^2 = (a-b)(a+b))$

$$= \frac{(2\,e^{x})\,(-2e^{-x})}{(e^{x}-e^{-x})^{2}}$$

$$= \frac{-4}{(e^{x}-e^{-x})^{2}}$$

Question: 17

Differentiate eac

Solution:

Let y = $xe^{\sqrt{\sin x}}$, z = x and w = $e^{\sqrt{\sin x}}$

Formula : $\frac{d(e^{x})}{dx} = e^{x}, \frac{d(\sin x)}{dx} = \cos x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\,e^{\sqrt{\sin x}} \times (1)] + [x \times e^{\sqrt{\sin x}} \times \frac{1}{2} \times \frac{1}{\sqrt{\sin x}} \times \cos x\,]$$

$$= e^{\sqrt{\sin x}} \times [1 + \frac{x\cos x}{2\sqrt{\sin x}}]$$

Question: 18

Differentiate eac

Solution:

Let y = $e^{\sin x}\sin e^{x}$, z = $e^{\sin x}$ and w = $\sin e^{x}$

Formula : $\frac{d(e^{x})}{dx} = e^{x}, \frac{d(\sin x)}{dx} = \cos x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\,\sin e^{x} \times \left(e^{\sin x} \times \cos x\right)] + [e^{\sin x} \times \cos e^{x} \times e^{x}\,]$$

$$= e^{\sin x}[\,(\sin e^{x} \times \cos x) + (\cos e^{x} \times e^{x})\,]$$

$$= e^{\sin x}(e^{x}\cos e^{x} + \cos x \sin e^{x})$$

Question: 19

Differentiate eac

Solution:

Let y = $e^{\sqrt{1-x^{2}}}\tan x$, z = $e^{\sqrt{1-x^{2}}}$ and w = $\tan x$

Formula : $\frac{d(e^{x})}{dx} = e^{x}, \frac{d(\tan x)}{dx} = \sec^{2} x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\,\tan x \times \left(e^{\sqrt{1-x^{2}}} \times \frac{1}{2} \times \frac{1}{\sqrt{1-x^{2}}} \times (-2x)\right)] + [e^{\sqrt{1-x^{2}}} \times \sec^{2} x\,]$$

$$= e^{\sqrt{1-x^{2}}} \times \left[\sec^{2} x - \frac{x\tan x}{\sqrt{1-x^{2}}}\right]$$

Question: 20

Differentiate eac

Solution:

Let $y = \frac{e^x}{1+\cos x}$, $u = e^x$, $v = 1 + \cos x$

Formula: $\frac{d(e^x)}{dx} = e^x, \frac{d(\cos x)}{dx} = -\sin x$

According to quotient rule of differentiation

If $y = \frac{u}{v}$

$$dy/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1 + \cos x) \times (e^x) - (e^x) \times (-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{e^x(1 + \cos x + \sin x)}{(1 + \cos x)^2}$$

Question: 21

Differentiate eac

Solution:

Let $y = x^3 e^x \cos x$, $z = x^3$ and $w = e^x \cos x$

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cos x)}{dx} = -\sin x$

$$\frac{dw}{dx} = [\cos x \times (e^x)] + [e^x \times (-\sin x)] = e^x[\cos x - \sin x]$$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [e^x \cos x \times (3x^2)] + [x^3 \times (e^x[\cos x - \sin x])]$$

$$= e^x x^2 \times [3 \cos x + x \cos x - x \sin x]$$

$$= e^x x^2 (x \cos x - x \sin x + 3 \cos x)$$

Question: 22

Differentiate eac

Solution:

Let $y = e^{x \cos x}$, $z = x\cos x$

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cos x)}{dx} = -\sin x$

$\frac{dz}{dx} = [\cos x \times (1)] + [x \times (-\sin x)] = [\cos x - x \sin x]$ (Using product rule)

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= [e^{x\cos x}] \times [\cos x - x \sin x]$$

$= e^{x\cos x}(\cos x - x\sin x)$

Exercise : 10C

Question: 1

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}(kx) = k$

Answer :

Let,

$y = \cos^{-1}2x$

and u = 2x

therefore, $y = \cos^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\cos^{-1}u).\frac{d}{dx}(2x)$

$= \frac{-1}{\sqrt{1-u^2}}.2$

.... $\left(\because \frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx}(kx) = k\right)$

$= \frac{-2}{\sqrt{1-(2x)^2}}$

$= \frac{-2}{\sqrt{1-4x^2}}$

$\therefore \frac{dy}{dx} = \frac{-2}{\sqrt{1-4x^2}}$

Question: 2

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}(x^n) = n.x^{n-1}$

Answer :

Let,

$y = \tan^{-1}x^2$

and u = x^2

therefore, $y = \tan^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\tan^{-1}u) . \frac{d}{dx}(x^2)$$

$$= \frac{1}{1+u^2} . 2x$$

.... $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \ \& \ \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$$= \frac{2x}{1+(x^2)^2}$$

$$= \frac{2x}{1+x^4}$$

$\therefore \frac{dy}{dx} = \frac{2x}{1+x^4}$

Question: 3

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

ii) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

Answer :

Let,

$y = \sec^{-1}\sqrt{x}$

and $u = \sqrt{x}$

therefore, $y = \sec^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\sec^{-1}u) . \frac{d}{dx}(\sqrt{x})$$

$$= \frac{1}{u\sqrt{u^2-1}} . \frac{1}{2\sqrt{x}}$$

.... $\left(\because \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}} \ \& \ \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}\right)$

$$= \frac{1}{\sqrt{x}\sqrt{(\sqrt{x})^2-1}} . \left(\frac{1}{2\sqrt{x}}\right)$$

$$= \frac{1}{2\sqrt{x}.\sqrt{x}\sqrt{x-1}}$$

$$= \frac{1}{2x\sqrt{x-1}}$$

$\therefore \frac{dy}{dx} = \frac{1}{2x\sqrt{x-1}}$

Question: 4

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}\left(\sin^{-1}x\right) = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}(kx) = k$

Answer :

Let,

$y = \sin^{-1}\left(\frac{x}{a}\right)$

and $u = \frac{x}{a}$

therefore, $y = \sin^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}\left(\sin^{-1}u\right) \cdot \frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{1}{\sqrt{1-u^2}} \cdot \frac{1}{a}$$

.... $\left(\because \frac{d}{dx}\left(\sin^{-1}x\right) = \frac{1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx}(kx) = k\right)$

$$= \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}} \cdot \frac{1}{a}$$

$$= \frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}} \cdot \frac{1}{a}$$

$$= \frac{a}{\sqrt{a^2-x^2}} \cdot \frac{1}{a}$$

$$= \frac{1}{\sqrt{a^2-x^2}}$$

$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{a^2-x^2}}$

Question: 5

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}(\log x) = \frac{1}{x}$

Answer :

Let,

$y = \tan^{-1}(\log x)$

and u = log x

therefore, $y = \tan^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\tan^{-1}u) \cdot \frac{d}{dx}(\log x)$$

$$= \frac{1}{1+u^2} \cdot \frac{1}{x}$$

.... $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \;\&\; \frac{d}{dx}(\log x) = \frac{1}{x}\right)$

$$= \frac{1}{1+(\log x)^2} \cdot \frac{1}{x}$$

$$= \frac{1}{x\{1+(\log x)^2\}}$$

$\therefore \frac{dy}{dx} = \frac{1}{x\{1+(\log x)^2\}}$

Question: 6

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

ii) $\frac{d}{dx}(e^x) = e^x$

Answer :

Let,

$y = \cot^{-1}(e^x)$

and u = e^x

therefore, $y = \cot^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\cot^{-1}u) \cdot \frac{d}{dx}(e^x)$$

$$= \frac{-1}{1+u^2} \cdot e^x$$

.... $\left(\because \frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2} \text{ \& } \frac{d}{dx}(e^x) = e^x\right)$

$$= \frac{-1}{1+(e^x)^2}.e^x$$

$$= \frac{-e^x}{1+e^{2x}}$$

$\therefore \frac{dy}{dx} = \frac{-e^x}{1+e^{2x}}$

Question: 7

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}(\log x) = \frac{1}{x}$

ii) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

Answer :

Let,

$y = \log(\tan^{-1})$

and u = $\tan^{-1}x$

therefore, $y = \log u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\log u).\frac{d}{dx}(\tan^{-1}x)$$

$$= \frac{1}{u}.\frac{1}{1+x^2}$$

.... $\left(\because \frac{d}{dx}(\log x) = \frac{1}{x} \text{ \& } \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}\right)$

$$= \frac{1}{\tan^{-1}x}.\frac{1}{1+x^2}$$

$$= \frac{1}{(1+x^2).\tan^{-1}x}$$

$\therefore \frac{dy}{dx} = \frac{1}{(1+x^2).\tan^{-1}x}$

Question: 8

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

ii) $\frac{d}{dx}(x^n) = n.x^{n-1}$

Answer :

Let,

$y = \cot^{-1}(x^3)$

and $u = x^3$

therefore, $y = \cot^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\cot^{-1}u) \cdot \frac{d}{dx}(x^3)$$

$$= \frac{-1}{1+u^2} \cdot 3x^2$$

.... $\left(\because \frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2} \;\&\; \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$$= \frac{-1}{1+(x^3)^2} \cdot 3x^2$$

$$= \frac{-3x^2}{1+x^6}$$

$\therefore \frac{dy}{dx} = \frac{-3x^2}{1+x^6}$

Question: 9

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}(\cos x) = -\sin x$

iii) $\sin^2 x + \cos^2 x = 1$

Answer :

Let,

$y = \sin^{-1}(\cos x)$

and $u = \cos x$

therefore, $y = \sin^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\sin^{-1}u) \cdot \frac{d}{dx}(\cos x)$$

$$= \frac{1}{\sqrt{1-u^2}} \cdot (-\sin x)$$

.... $\left(\because \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx}(\cos x) = -\sin x\right)$

$= \frac{1}{\sqrt{1-(\cos x)^2}}.(-\sin x)$

$= \frac{1}{\sqrt{\sin^2 x}}.(-\sin x)$ ....($\because \sin^2 x + \cos^2 x = 1$)

$= \frac{1}{\sin x}.(-\sin x)$

$= -1$

$\therefore \frac{dy}{dx} = -1$

Question: 10

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\ \frac{du}{dx}$

ii) $\frac{d}{dx}(u+v) = \frac{du}{dx} + \ \frac{dv}{dx}$

iii) $\frac{d}{dx}\ (\tan^{-1}x) = \frac{1}{1+x^2}$

iv) $\frac{d}{dx}\ (k) = 0$

v) $\frac{d}{dx}\ (x^n) = n.x^{n-1}$

Answer :

Let,

$y = (1+x^2)\tan^{-1}x$

Let, $u = (1+x^2)$ and $v=\tan^{-1}x$

therefore, y=u.v

$\therefore \frac{dy}{dx} = (1+x^2).\frac{d}{dx}(\tan^{-1}x) + (\tan^{-1}u).\frac{d}{dx}(1+x^2)$

.........$\left(\because \frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\ \frac{du}{dx}\right)$

$= (1+x^2)\ .\frac{1}{1+x^2} + (\tan^{-1}x)\left\{\frac{d}{dx}(1) + \ \frac{d}{dx}(x^2)\right\}$

.... $\left(\because \frac{d}{dx}\ (\tan^{-1}x) = \frac{1}{1+x^2}\ \&\ \frac{d}{dx}(u+v) = \frac{du}{dx} + \ \frac{dv}{dx}\right)$

$= 1 + (\tan^{-1}x)(0+2x)$

.... $\left(\because \frac{d}{dx}\ (k) = 0\ \&\ \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$= 1 + 2x\tan^{-1}x$

$\therefore \frac{dy}{dx} = 1 + 2x\ tan^{-1}x$

Question: 11

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$

iii) $1 + \cot^2 x = \mathrm{cosec}^2 x$

Answer :

Let,

$y = \tan^{-1}(\cot x)$

and u = cot x

therefore, $y = \tan^{-1} u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\tan^{-1}u) \cdot \frac{d}{dx}(\cot x)$$

$$= \frac{1}{1+u^2} \cdot (-\mathrm{cosec}^2 x)$$

.... $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \ \& \ \frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x\right)$

$$= \frac{-\mathrm{cosec}^2 x}{1+(\cot x)^2}$$

$= \frac{-\mathrm{cosec}^2 x}{\mathrm{cosec}^2 x}$ ....$(\because 1 + \cot^2 x = \mathrm{cosec}^2 x)$

= -1

$\therefore \frac{dy}{dx} = -1$

Question: 12

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}(\log x) = \frac{1}{x}$

ii) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

iii) $\frac{d}{dx}(x^n) = n.x^{n-1}$

Answer :

Let,

$y = \log(\sin^{-1}x^4)$

and u = x^4

therefore, $y = \log(\sin^{-1}u)$

let, $v = \sin^{-1}u$

therefore, y= log v

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(\log v).\frac{d}{du}(\sin^{-1}u).\frac{d}{dx}(x^4)$$

$$= \frac{1}{v}.\left(\frac{1}{\sqrt{1-u^2}}\right).4x^3$$

.... $\left(\because \frac{d}{dx}(\log x) = \frac{1}{x}, \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$$= \frac{1}{\sin^{-1}u}.\left(\frac{1}{\sqrt{1-(x^4)^2}}\right).4x^3$$

$$= \frac{1}{\sin^{-1}x^4}.\left(\frac{1}{\sqrt{1-x^8}}\right).4x^3$$

$$= \frac{4x^3}{\sin^{-1}x^4 . \sqrt{1-x^8}}$$

$$\therefore \frac{dy}{dx} = \frac{4x^3}{\sin^{-1}x^4 . \sqrt{1-x^8}}$$

Question: 13

Differentiate eac

Solution:

<u>Formulae</u> :

i) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

ii) $\frac{d}{dx}(x^n) = n.x^{n-1}$

<u>Answer</u> :

Let,

$y = (\cot^{-1}x^2)^3$

and $u = x^2$

therefore, $y = (\cot^{-1}u)^3$

let, $v = \cot^{-1}u$

therefore, y= v^3

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^3).\frac{d}{du}(\cot^{-1}u).\frac{d}{dx}(x^2)$$

$$= 3v^2.\left(\frac{-1}{1+u^2}\right).2x$$

.... $\left(\because \frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2} \;\&\; \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$$= 3(\cot^{-1}u)^2 . \left(\frac{-1}{1+(x^2)^2}\right). 2x$$

$$= \left(\cot^{-1}(x^2)\right)^2 . \frac{-6x}{1+(x^2)^2}$$

$$= \frac{-6x\left(\cot^{-1}(x^2)\right)^2}{1+x^4}$$

$$\therefore \frac{dy}{dx} = \frac{-6x\left(cot^{-1}(x^2)\right)^2}{1+x^4}$$

Question: 14

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

iii) $\frac{d}{dx}(\cos x) = -\sin x$

Answer :

Let,

$y = \tan^{-1}(\cos\sqrt{x})$

and $u = \sqrt{x}$

therefore, $y = \tan^{-1}(\cos u)$

let, $v = \cos u$

therefore, $y = \tan^{-1}v$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(\tan^{-1}v).\frac{d}{du}(\cos u).\frac{d}{dx}(\sqrt{x})$$

$$= \frac{1}{1+v^2} . (-\sin u). \frac{1}{2\sqrt{x}}$$

.... $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}, \frac{d}{dx}(\cos x) = -\sin x \;\&\; \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}\right)$

$$= \frac{1}{1+(\cos u)^2} . (-\sin\sqrt{x}). \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{1+\left(\cos\sqrt{x}\right)^2} . (-\sin\sqrt{x}). \frac{1}{2\sqrt{x}}$$

$$= \frac{-\sin\sqrt{x}}{2\sqrt{x}\left(1+\left(\cos\sqrt{x}\right)^2\right)}$$

$\therefore \frac{dy}{dx} = \frac{-\sin\sqrt{x}}{2\sqrt{x}\left(1+\left(\cos\sqrt{x}\right)^2\right)}$

Question: 15

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}(\tan x) = \sec^2 x$

Answer :

Let,

$y = \tan(\sin^{-1}x)$

and $u = \sin^{-1}x$

therefore, $y = \tan u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\tan u).\frac{d}{dx}(\sin^{-1}x)$

$= \sec^2 u.\frac{1}{\sqrt{1-x^2}}$

.... $\left(\because \frac{d}{dx}(\tan x) = \sec^2 x \,\&\, \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}\right)$

$= \sec^2(\sin^{-1}x).\frac{1}{\sqrt{1-x^2}}$

$= \frac{\sec^2(\sin^{-1}x)}{\sqrt{1-x^2}}$

$\therefore \frac{dy}{dx} = \frac{\sec^2(\sin^{-1}x)}{\sqrt{1-x^2}}$

Question: 16

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

iii) $\frac{d}{dx}(e^x) = e^x$

Answer :

Let,

$y = e^{\tan^{-1}\sqrt{x}}$

and $u = \sqrt{x}$

therefore, $y = e^{\tan^{-1}u}$

let, $v = \tan^{-1}u$

therefore, $y = e^{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{dv}(e^{v}).\frac{d}{du}(\tan^{-1}u).\frac{d}{dx}(\sqrt{x})$

$= e^{v}.\left(\frac{1}{1+u^2}\right).\frac{1}{2\sqrt{x}}$

.... $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}, \frac{d}{dx}(e^{x}) = e^{x} \;\&\; \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}\right)$

$= e^{\tan^{-1}u}.\left(\frac{1}{1+(\sqrt{x})^2}\right).\frac{1}{2\sqrt{x}}$

$= e^{\tan^{-1}\sqrt{x}}.\left(\frac{1}{1+x}\right).\frac{1}{2\sqrt{x}}$

$= \frac{e^{\tan^{-1}\sqrt{x}}}{2\sqrt{x}\,(1+x)}$

$\therefore \frac{dy}{dx} = \frac{e^{\tan^{-1}\sqrt{x}}}{2\sqrt{x}\,(1+x)}$

Question: 17

Differentiate eac

Solution:

Formulae :

i) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

iii) $\frac{d}{dx}(x^{n}) = n.x^{n-1}$

Answer :

Let,

$y = \sqrt{\sin^{-1}x^2}$

and $u = x^2$

therefore, $y = \sqrt{\sin^{-1}u}$

let, $v = \sin^{-1}u$

therefore, $y = \sqrt{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv} . \frac{dv}{du} . \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{dv} (\sqrt{v}) . \frac{d}{du} (\sin^{-1} u) . \frac{d}{dx} (x^2)$

$= \frac{1}{2\sqrt{v}} . \left(\frac{1}{\sqrt{1-u^2}}\right) . 2x$

.... $\left(\because \frac{d}{dx} (\sqrt{x}) = \frac{1}{2\sqrt{x}}, \frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx} (x^n) = n.x^{n-1}\right)$

$= \frac{1}{2\sqrt{\sin^{-1} u}} . \left(\frac{1}{\sqrt{1-(x^2)^2}}\right) . 2x$

$= \frac{1}{\sqrt{\sin^{-1} (x^2)}} . \left(\frac{1}{\sqrt{1-x^4}}\right) . x$

$= \frac{x}{\sqrt{\sin^{-1} (x^2)} \left(\sqrt{1-x^4}\right)}$

$\therefore \frac{dy}{dx} = \frac{x}{\sqrt{\sin^{-1} (x^2)} \left(\sqrt{1-x^4}\right)}$

Question: 18

If <

Solution:

Given : $y = \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$

To Prove : $\frac{dy}{dx} = -2$

Formulae :

i) $\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx} (\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

iii) $\frac{d}{dx} (\cos x) = -\sin x$

iv) $\frac{d}{dx} (\sin x) = \cos x$

v) $\sin^2 x + \cos^2 x = 1$

vi) $\frac{d}{dx} (u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Answer :

Given equation,

$y = \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$

Let $s = \sin^{-1}(\cos x)$ & $t = \cos^{-1}(\sin x)$

Therefore, y = s + t ....eq(1)

I. For $\sin^{-1}(\cos x)$

let $u = \cos x$

therefore, $s = \sin^{-1} u$

Differentiating above equation w.r.t. x,

$\therefore \frac{ds}{dx} = \frac{ds}{du} . \frac{du}{dx}$ .... By chain rule

$\therefore \frac{ds}{dx} = \frac{d}{du} (\sin^{-1} u) . \frac{d}{dx} (\cos x)$

$= \frac{1}{\sqrt{1-u^2}} . (-\sin x)$

.... $\left(\because \frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx} (\cos x) = -\sin x\right)$

$= \frac{1}{\sqrt{1-(\cos x)^2}} . (-\sin x)$

$= \frac{1}{\sqrt{\sin^2 x}} . (-\sin x)$ ....$(\because \sin^2 x + \cos^2 x = 1)$

$= \frac{1}{\sin x} . (-\sin x)$

$= -1$

$\therefore \frac{ds}{dx} = -1$ ....eq(2)

II. <u>For $\cos^{-1}(\sin x)$</u>

let $u = \sin x$

therefore, $t = \cos^{-1} u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dt}{dx} = \frac{dt}{du} . \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dt}{dx} = \frac{d}{du} (\cos^{-1} u) . \frac{d}{dx} (\sin x)$

$= \frac{-1}{\sqrt{1-u^2}} . (\cos x)$

.... $\left(\because \frac{d}{dx} (\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx} (\sin x) = \cos x\right)$

$= \frac{-1}{\sqrt{1-(\sin x)^2}} . (\cos x)$

$= \frac{-1}{\sqrt{\cos^2 x}} . (\cos x)$ ....$(\because \sin^2 x + \cos^2 x = 1)$

$= \frac{-1}{\cos x} . (\cos x)$

$= -1$

$\therefore \frac{dt}{dx} = -1$ ....eq(2)

Differentiating eq(1) w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{d}{dx} (s + t)$

$= \frac{ds}{dx} + \frac{dt}{dx}$ ....$\left(\because \frac{d}{dx} (u + v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

= -1 -1 ....from eq(2) and eq(3)

= -2

$\therefore \frac{dy}{dx} = -2$

Hence proved !!!

Question: 19

Prove that<

Solution:

<u>To Prove</u> : $\frac{d}{dx}\{2x\tan^{-1}x - \log(1+x^2)\} = 2\tan^{-1}x$

<u>Formulae</u> :

i) $\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$

ii) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

iii) $\frac{d}{dx}(kx) = k$

iv) $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

v) $\frac{d}{dx}(kx) = 0$

vi) $\frac{d}{dx}(x^n) = n.x^{n-1}$

vii) $\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$

<u>Answer</u> :

Let,

$y = 2x\tan^{-1}x - \log(1+x^2)$

Let $s = 2x\tan^{-1}x$ & $t = \log(1+x^2)$

Therefore, y = s - t ....eq(1)

I. <u>For $2x\tan^{-1}x$</u>

let $u = 2x$ & $v = \tan^{-1}x$

therefore, $s = u.v$

Differentiating above equation w.r.t. x,

$\therefore \frac{ds}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ....$\left(\because \frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}\right)$

$$\therefore \frac{ds}{dx} = 2x\frac{d}{dx}(\tan^{-1}x) + \tan^{-1}x\frac{d}{dx}(2x)$$

$$= 2x.\frac{1}{1+x^2} + \tan^{-1}x.2$$

.... $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \text{ \& } \frac{d}{dx}(kx) = k\right)$

$$= \frac{2x}{1+x^2} + 2\tan^{-1}x$$

$\therefore \frac{ds}{dx} = \frac{2x}{1+x^2} + 2\tan^{-1}x$ ....eq(2)

II. For $\log(1+x^2)$

let $u = (1+x^2)$

therefore, $t = \log u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dt}{dx} = \frac{dt}{du} \cdot \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dt}{dx} = \frac{d}{du}(\log u) \cdot \frac{d}{dx}(1+x^2)$

$= \frac{1}{u} \cdot \left(\frac{d}{dx}(1) + \frac{d}{dx}(x^2)\right)$ .... $\left(\because \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= \frac{1}{(1+x^2)} \cdot (0+2x)$

.... $\left(\because \frac{d}{dx}(k) = 0 \;\&\; \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$= \frac{2x}{1+x^2}$

$\therefore \frac{dt}{dx} = \frac{2x}{1+x^2}$ ....eq(3)

Differentiating eq(1) w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{d}{dx}(s-t)$

$= \frac{ds}{dx} - \frac{dt}{dx}$ ....$\left(\because \frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}\right)$

$= \frac{2x}{1+x^2} + 2\tan^{-1}x - \frac{2x}{1+x^2}$ ....from eq(2) and eq(3)

$= 2\tan^{-1}x$

$\therefore \frac{dy}{dx} = 2\tan^{-1}x$

Hence proved !!!

Exercise : 10D

Question: 1

Differentiate eac

Solution:

To find: Value of $\sin^{-1}\left\{\sqrt{\frac{1-\cos x}{2}}\right\}$

Formula used: (i) $\cos\theta = 2\sin^2\frac{\theta}{2}$

We have, $\sin^{-1}\left\{\sqrt{\frac{1-\cos x}{2}}\right\}$

$\Rightarrow \sin^{-1}\left\{\sqrt{\frac{2\sin^2\frac{x}{2}}{2}}\right\}$

$\Rightarrow \sin^{-1}\left\{\sqrt{\sin^2\frac{x}{2}}\right\}$

$\Rightarrow \sin^{-1}\left\{\sin\frac{x}{2}\right\}$

$\Rightarrow \frac{x}{2}$

Now, we can see that $\sin^{-1}\left\{\sqrt{\frac{1-\cos x}{2}}\right\} = \frac{x}{2}$

Now differentiating ,

$\Rightarrow \frac{d\left(\frac{x}{2}\right)}{dx}$

$\Rightarrow \frac{1}{2}\frac{dx}{dx}$

$\Rightarrow \frac{1}{2}$

Ans) $\frac{1}{2}$

Question: 2

Differentiate eac

Solution:

To find: Value of $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

Formula used: (i) $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

We have, $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sin x}{2\cos^2\frac{x}{2}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)$

$\Rightarrow \tan^{-1}\left(\tan\frac{x}{2}\right)$

$\Rightarrow \frac{x}{2}$

Now, we can see that $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right) = \frac{x}{2}$

Now differentiating ,

$\Rightarrow \frac{d\left(\frac{x}{2}\right)}{dx}$

$\Rightarrow \frac{1}{2}\frac{dx}{dx}$

$\Rightarrow \frac{1}{2}$

Ans) $\frac{1}{2}$

Question: 3

Differentiate eac

Solution:

To find: Value of $\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

Formula used: (i) $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

We have, $\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

$\Rightarrow \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{\sin x}\right)$

$\Rightarrow \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$

$\Rightarrow \cot^{-1}\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right)$

$\Rightarrow \cot^{-1}\left(\cot\frac{x}{2}\right)$

$\Rightarrow \frac{x}{2}$

Now, we can see that $\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right) = \frac{x}{2}$

Now differentiating ,

$\Rightarrow \frac{d\left(\frac{x}{2}\right)}{dx}$

$\Rightarrow \frac{1}{2}\frac{dx}{dx}$

$\Rightarrow \frac{1}{2}$

Ans) $\frac{1}{2}$

Question: 4

Differentiate eac

Solution:

To find: Value of $\cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$

Formula used: (i) $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

We have, $\cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$

$\Rightarrow \cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\sqrt{\frac{1+\cos x}{1+\cos x}}\right)$

$$\Rightarrow \cot^{-1}\left(\sqrt{\frac{(1+\cos x)^2}{1-\cos^2 x}}\right)$$

$$\Rightarrow \cot^{-1}\left(\sqrt{\frac{(1+\cos x)^2}{\sin^2 x}}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow \frac{x}{2}$$

Now, we can see that $\cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right) = \frac{x}{2}$

Now differentiating ,

$$\Rightarrow \frac{d\left(\frac{x}{2}\right)}{dx}$$

$$\Rightarrow \frac{1}{2}\frac{dx}{dx}$$

$$\Rightarrow \frac{1}{2}$$

Ans) $\frac{1}{2}$

Question: 5

Differentiate eac

Solution:

To find: Value of $\tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$

Formula used: (i) $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$

We have, $\tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$

Dividing numerator and denominator by cosx

$$\Rightarrow \tan^{-1}\left(\frac{\frac{\cos x+\sin x}{\cos x}}{\frac{\cos x-\sin x}{\cos x}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan x\tan\frac{\pi}{4}}\right)$$

$\Rightarrow \tan^{-1}\left(\tan\left(\frac{\pi}{4}+x\right)\right)$

$\Rightarrow \frac{\pi}{4}+x$

Now, we can see that $\tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right) = \frac{\pi}{4}+x$

Now differentiating ,

$\Rightarrow \frac{d\left(\frac{\pi}{4}+x\right)}{dx}$

$\Rightarrow \frac{d\left(\frac{\pi}{4}\right)}{dx}+\frac{dx}{dx}$

$\Rightarrow 0 + 1$

$\Rightarrow 1$

Ans) 1

Question: 6

Differentiate eac

Solution:

To find: Value of $\cot^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

Formula used: (i) $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$

We have, $\cot^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

Dividing numerator and denominator by cosx

$\Rightarrow \cot^{-1}\left(\frac{\frac{\cos x-\sin x}{\cos x}}{\frac{\cos x+\sin x}{\cos x}}\right)$

$\Rightarrow \cot^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$

$\Rightarrow \cot^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan x\tan\frac{\pi}{4}}\right)$

$\Rightarrow \cot^{-1}\left(\tan\left(\frac{\pi}{4}-x\right)\right)$

$\Rightarrow \cot^{-1}\left(\cot\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-x\right)\right)\right)$

$\Rightarrow \cot^{-1}\left(\cot\left(\frac{\pi}{4}+x\right)\right)$

$\Rightarrow \frac{\pi}{4}+x$

Now, we can see that $\cot^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right) = \frac{\pi}{4}+x$

Now differentiating ,

$\Rightarrow \frac{d\left(\frac{\pi}{4}+x\right)}{dx}$

$\Rightarrow \frac{d\left(\frac{\pi}{4}\right)}{dx} + \frac{dx}{dx}$

$\Rightarrow 0 + 1$

$\Rightarrow 1$

Ans) 1

Question: 7

Differentiate eac

Solution:

To find: Value of $\cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\right)$

Formula used: (i) $1 - \cos\theta = 2\sin^2\frac{\theta}{2}$

(ii) $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

We have, $\cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\right)$

$$\Rightarrow \cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{2\sin^2\frac{3x}{2}}}\right)$$

$$\Rightarrow \cot^{-1}\left(\sqrt{\frac{2\cos^2\frac{3x}{2}}{2\sin^2\frac{3x}{2}}}\right)$$

$$\Rightarrow \cot^{-1}\left(\sqrt{\cot^2\left(\frac{3x}{2}\right)}\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\left(\frac{3x}{2}\right)\right)$$

$$\Rightarrow \frac{3x}{2}$$

Now, we can see that $\cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\right) = \frac{3x}{2}$

Now differentiating ,

$\Rightarrow \frac{d\left(\frac{3x}{2}\right)}{dx}$

$\Rightarrow \frac{3}{2}\frac{dx}{dx}$

$\Rightarrow \frac{3}{2}$

Ans) $\frac{3}{2}$

Question: 8

Differentiate eac

Solution:

To find: Value of $\sec^{-1}\left(\frac{1+\tan^2 x}{1-\tan^2 x}\right)$

Formula used: (i) cos $2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

We have, $\sec^{-1}\left(\frac{1+\tan^2 x}{1-\tan^2 x}\right)$

Dividing numerator and denominator by $1+\tan^2 x$

$$\Rightarrow \sec^{-1}\left(\frac{\left(\frac{1+tan^2 x}{1+tan^2 x}\right)}{\left(\frac{1-tan^2 x}{1+tan^2 x}\right)}\right)$$

$$\Rightarrow \sec^{-1}\left(\frac{1}{\left(\frac{1-tan^2 x}{1+tan^2 x}\right)}\right)$$

$$\Rightarrow \sec^{-1}\left(\frac{1}{\cos 2x}\right)$$

$$\Rightarrow \sec^{-1}(\sec 2x)$$

$\Rightarrow 2x$

Now, we can see that $\sec^{-1}\left(\frac{1+\tan^2 x}{1-\tan^2 x}\right) = 2x$

Now differentiating ,

$$\Rightarrow \frac{d(2x)}{dx}$$

$$\Rightarrow 2\frac{dx}{dx}$$

$\Rightarrow 2$

Ans) 2

Question: 9

Differentiate eac

Solution:

To find: Value of $\sin^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$

Formula used: (i) cos $2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

We have, $\sin^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$

$$\Rightarrow \sin^{-1}(\cos 2x)$$

$$\Rightarrow \sin^{-1}\left(\sin\left(\frac{\pi}{2}-2x\right)\right)$$

$\Rightarrow \frac{\pi}{2} - 2x$

Now, we can see that $\sin^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right) = \frac{\pi}{2} - 2x$

Now differentiating ,

$\Rightarrow \frac{d\left(\frac{\pi}{2} - 2x\right)}{dx}$

$\Rightarrow \frac{d\left(\frac{\pi}{2}\right)}{dx} - 2\frac{dx}{dx}$

⇒ 0 - 2

⇒ -2

Ans) -2

Question: 10

Differentiate eac

Solution:

To find: Value of $\operatorname{cosec}^{-1}\left(\frac{1+\tan^2 x}{2\tan x}\right)$

Formula used: (i) sin $2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

We have, $\operatorname{cosec}^{-1}\left(\frac{1+\tan^2 x}{2\tan x}\right)$

Dividing Numerator and Denominator with $1+\tan^2 x$

$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{\left(\frac{1+\tan^2 x}{1+\tan^2 x}\right)}{\left(\frac{2\tan x}{1+\tan^2 x}\right)}\right)$

$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{(1)}{\left(\frac{2\tan x}{1+\tan^2 x}\right)}\right)$

$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{1}{\sin 2x}\right)$

$\Rightarrow \operatorname{cosec}^{-1}(\operatorname{cosec} 2x)$

⇒ 2x

Now, we can see that $\operatorname{cosec}^{-1}\left(\frac{1+\tan^2 x}{2\tan x}\right) = 2x$

Now differentiating ,

$\Rightarrow \frac{d(2x)}{dx}$

$\Rightarrow 2\frac{dx}{dx}$

⇒ 2

Ans) 2

Question: 11

Differentiate eac

Solution:

To find: Value of $\cot^{-1}(\text{cosec } x + \cot x)$

Formula used: (i) $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

We have, $\cot^{-1}(\text{cosec } x + \cot x)$

$$\Rightarrow \cot^{-1}\left(\frac{1}{\sin x} + \frac{\cos x}{\sin x}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{\sin x}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow \frac{x}{2}$$

Now, we can see that $\cot^{-1}(\text{cosec } x + \cot x) = \frac{x}{2}$

Now differentiating ,

$$\Rightarrow \frac{d\left(\frac{x}{2}\right)}{dx}$$

$$\Rightarrow \frac{1}{2}\frac{dx}{dx}$$

$$\Rightarrow \frac{1}{2}$$

Ans) $\frac{1}{2}$

Question: 12

Differentiate eac

Solution:

To find: Value of $\tan^{-1}(\cot x) + \cot^{-1}(\tan x)$

The formula used: (i) $\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$

(ii) $\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

We have, $\tan^{-1}(\cot x) + \cot^{-1}(\tan x)$

$$\Rightarrow \tan^{-1}\left[\tan\left(\frac{\pi}{2} - x\right)\right] + \cot^{-1}\left[\cot\left(\frac{\pi}{2} - x\right)\right]$$

$\Rightarrow \left(\frac{\pi}{2} - x\right) + \left(\frac{\pi}{2} - x\right)$

$\Rightarrow \pi - 2x$

Now, we can see that $\tan^{-1}(\cot x) + \cot^{-1}(\tan x) = \pi - 2x$

Now differentiating ,

$\Rightarrow \frac{d(\pi - 2x)}{dx}$

$\Rightarrow \frac{d\pi}{dx} - \frac{d2x}{dx}$

$\Rightarrow -2$

Ans) -2

Question: 13

Differentiate eac

Solution:

To find: Value of $\sin^{-1}\{\sqrt{1-x^2}\}$

The formula used: (i) $\cos \theta = \sin \left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\{\sqrt{1-x^2}\}$

$\Rightarrow$ Putting x = cosθ

$\theta = \cos^{-1}x$ ... (i)

Putting x = cosθ in the equation

$\Rightarrow \sin^{-1}\{\sqrt{1-\cos^2\theta}\}$

$\Rightarrow \sin^{-1}\left(\sqrt{\sin^2\theta}\right)$

$\Rightarrow \sin^{-1}(\sin\theta)$

$\Rightarrow \theta$

$\Rightarrow \frac{d\theta}{dx}$

$\Rightarrow \frac{d(\cos^{-1}x)}{dx}$ [From (i)]

$\Rightarrow -\frac{1}{\sqrt{1-x^2}}$

Ans) $-\frac{1}{\sqrt{1-x^2}}$

Question: 14

Differentiate eac

Solution:

To find: Value of $\sin^{-1}\left(\sqrt{\frac{1-x}{2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\left(\sqrt{\frac{1-x}{2}}\right)$

⇒ Putting x = cosθ

$\theta = \cos^{-1}x$ ... (i)

Putting x = cosθ in the equation

$\Rightarrow \sin^{-1}\left(\sqrt{\frac{1-\cos\theta}{2}}\right)$

$\Rightarrow \sin^{-1}\left(\sqrt{\sin^2\frac{\theta}{2}}\right)$

$\Rightarrow \sin^{-1}\left(\sin\frac{\theta}{2}\right)$

$\Rightarrow \frac{\theta}{2}$

Now, we can see that $\sin^{-1}\left(\sqrt{\frac{1-x}{2}}\right) = \frac{\theta}{2}$

$\Rightarrow \theta = \cos^{-1}x$

$\Rightarrow \frac{d\left(\frac{\theta}{2}\right)}{dx}$

$\Rightarrow \frac{d\left(\frac{\cos^{-1}x}{2}\right)}{dx}$

$\Rightarrow -\frac{1}{2\sqrt{1-x^2}}$

Ans) $-\frac{1}{2\sqrt{1-x^2}}$

Question: 15

Differentiate eac

Solution:

To find: Value of $\cos^{-1}\left(\sqrt{\frac{1+x}{2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\cos^{-1}\left(\sqrt{\frac{1+x}{2}}\right)$

⇒ Putting x = cosθ

$\theta = \cos^{-1}x$ ... (i)

Putting x = cosθ in the equation

$\Rightarrow \cos^{-1}\left(\sqrt{\frac{1+cos\theta}{2}}\right)$

$\Rightarrow \cos^{-1}\left(\sqrt{\cos^2\frac{\theta}{2}}\right)$

$\Rightarrow \cos^{-1}\left(\cos\frac{\theta}{2}\right)$

$\Rightarrow \frac{\theta}{2}$

Now, we can see that $\cos^{-1}\left(\sqrt{\frac{1+x}{2}}\right) = \frac{\theta}{2}$

$\Rightarrow \theta = \cos^{-1}x$

$\Rightarrow \frac{d\left(\frac{\theta}{2}\right)}{dx}$

$\Rightarrow \frac{d\left(\frac{\cos^{-1}x}{2}\right)}{dx}$

$\Rightarrow -\frac{1}{2\sqrt{1-x^2}}$

Ans) $-\frac{1}{2\sqrt{1-x^2}}$

Question: 16

Differentiate eac

Solution:

To find: Value of $\cos^{-1}\left(\sqrt{1-x^2}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\cos^{-1}\left(\sqrt{1-x^2}\right)$

$\Rightarrow$ Putting x = sinθ

θ = $\sin^{-1}x$ ... (i)

Putting x = sinθ in the equation

$\Rightarrow \cos^{-1}\left(\sqrt{1-(sin\theta)^2}\right)$

$\Rightarrow \cos^{-1}\left(\sqrt{1-\sin^2\theta}\right)$

$\Rightarrow \cos^{-1}(\cos\theta)$

$\Rightarrow \theta$

Now, we can see that $\cos^{-1}\left(\sqrt{1-x^2}\right) = \theta$

$\Rightarrow \theta = \sin^{-1}x$

$\Rightarrow \frac{d(\theta)}{dx}$

$\Rightarrow \frac{d(\sin^{-1}x)}{dx}$

$\Rightarrow \frac{1}{\sqrt{1-x^2}}$

Ans) $\frac{1}{\sqrt{1-x^2}}$

Question: 17

Differentiate eac

Solution:

To find: Value of $\sin^{-1}(2x\sqrt{1-x^2})$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}(2x\sqrt{1-x^2})$

⇒ Putting x = sinθ

θ = $\sin^{-1}$x … (i)

Putting x = sinθ in the equation

$\Rightarrow \sin^{-1}\left(2\sin\theta\sqrt{1-(\sin\theta)^2}\right)$

$\Rightarrow \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$\Rightarrow \sin^{-1}(2\sin\theta\cos\theta)$

$\Rightarrow \sin^{-1}(\sin 2\theta)$

⇒ 2θ

⇒ $2\sin^{-1}$x

Now, we can see that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x$

Now Differentiating

$\Rightarrow \frac{d2\theta}{dx} = \frac{d(2\sin^{-1}x)}{dx}$

$\Rightarrow 2\frac{d(\theta)}{dx}$

$\Rightarrow 2\frac{d(\sin^{-1}x)}{dx}$

$\Rightarrow 2\frac{1}{\sqrt{1-x^2}}$

Ans) $\frac{2}{\sqrt{1-x^2}}$

Question: 18

Differentiate eac

Solution:

To find: Value of $\sin^{-1}(3x - 4x^3)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}(3x - 4x^3)$

⇒ Putting x = sinθ

θ = $\sin^{-1}x$ ... (i)

Putting x = sinθ in the equation

$\Rightarrow \sin^{-1}(3\sin\theta - 4(\sin\theta)^3)$

$\Rightarrow \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$\Rightarrow \sin^{-1}(\sin 3\theta)$

⇒ 3θ

Now, we can see that $\sin^{-1}(3x - 4x^3) = 3\theta$

Now Differentiating

$$\Rightarrow \frac{d3\theta}{dx} = \frac{d(3\sin^{-1}x)}{dx}$$

$$\Rightarrow 3\frac{d(\sin^{-1}x)}{dx}$$

$$\Rightarrow 3\frac{1}{\sqrt{1-x^2}}$$

Ans) $\frac{3}{\sqrt{1-x^2}}$

Question: 19

Differentiate eac

Solution:

To find: Value of $\sin^{-1}(1 - 2x^2)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}(1 - 2x^2)$

⇒ Putting x = sinθ

θ = $\sin^{-1}x$ ... (i)

Putting x = sinθ in the equation

$\Rightarrow \sin^{-1}\left(1 - 2(\sin\theta)^2\right)$

$\Rightarrow \sin^{-1}(1 - 2\sin^2\theta)$

$\Rightarrow \sin^{-1}(\cos 2\theta)$

$\Rightarrow \sin^{-1}\left(\sin\left(\frac{\pi}{2} - 2\theta\right)\right)$

$\Rightarrow \frac{\pi}{2}-2\theta$

Now, we can see that $\sin^{-1}\left(1 - 2x^2\right) = \frac{\pi}{2}-2\theta$

Now Differentiating

$$\Rightarrow \frac{d\left(\frac{\pi}{2}-2\theta\right)}{dx} = \frac{d\left(\frac{\pi}{2}\right)}{dx} - \frac{d2\theta}{dx}$$

$$\Rightarrow 0 - \frac{d2\theta}{dx}$$

$$\Rightarrow -2\frac{d\sin^{-1}x}{dx}$$

$$\Rightarrow \frac{-2}{\sqrt{1-x^2}}$$

Ans) $\frac{-2}{\sqrt{1-x^2}}$

Question: 20

Differentiate eac

Solution:

To find: Value of $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

⇒ Putting x = sinθ

θ = $\sin^{-1}x$ ... (i)

Putting x = sinθ in the equation

$$\Rightarrow \sec^{-1}\left(\frac{1}{\sqrt{1-(\sin\theta)^2}}\right)$$

$$\Rightarrow \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$$

$$\Rightarrow \sec^{-1}\left(\frac{1}{\sqrt{\cos^2\theta}}\right)$$

$$\Rightarrow \sec^{-1}\left(\frac{1}{\cos\theta}\right)$$

$$\Rightarrow \sec^{-1}(\sec\theta)$$

⇒ θ

Now, we can see that $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \theta$

Now Differentiating

$\Rightarrow \frac{d\theta}{dx}$

$\Rightarrow \frac{d(\sin^{-1}x)}{dx}$

$\Rightarrow \frac{1}{\sqrt{1-x^2}}$

Ans) $\frac{1}{\sqrt{1-x^2}}$

Question: 21

Differentiate eac

Solution:

To find: Value of $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

⇒ Putting x = sinθ

θ = $\sin^{-1}x$ ... (i)

Putting x = sinθ in the equation

$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-(\sin\theta)^2}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos^2\theta}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$

$\Rightarrow \tan^{-1}(\tan\theta)$

⇒ θ

Now, we can see that $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \theta$

Now Differentiating

$\Rightarrow \frac{d\theta}{dx}$

$\Rightarrow \frac{d(\sin^{-1}x)}{dx}$

$\Rightarrow \frac{1}{\sqrt{1-x^2}}$

Ans) $\frac{1}{\sqrt{1-x^2}}$

Question: 22

Differentiate eac

Solution:

To find: Value of $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2}-\theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$

⇒ Putting x = sinθ

$\theta = \sin^{-1}x$ ... (i)

Putting x = sinθ in the equation

$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{1+\sqrt{1-(\sin\theta)^2}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{1+\sqrt{1-\sin^2\theta}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{1+\sqrt{\cos^2\theta}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right)$

$\Rightarrow \tan^{-1}\left(\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right)$

$\Rightarrow \tan^{-1}\left(\tan\frac{\theta}{2}\right)$

$\Rightarrow \frac{\theta}{2}$

Now, we can see that $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right) = \frac{\theta}{2}$

Now Differentiating

$\Rightarrow \frac{d\left(\frac{\theta}{2}\right)}{dx}$

$\Rightarrow \frac{1}{2}\frac{d(\theta)}{dx}$

$\Rightarrow \frac{1}{2}\frac{d\sin^{-1}x}{dx}$

$\Rightarrow \frac{1}{2\sqrt{1-x^2}}$

Ans) $\frac{1}{2\sqrt{1-x^2}}$

Question: 23

Differentiate eac

Solution:

To find: Value of $\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

⇒ Putting x = sinθ

θ = $\sin^{-1}x$ ... (i)

Putting x = sinθ in the equation

$$\Rightarrow \cot^{-1}\left(\frac{\sqrt{1-(\sin\theta)^2}}{\sin\theta}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\sqrt{\cos^2\theta}}{\sin\theta}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$$

$$\Rightarrow \cot^{-1}(\cot\theta)$$

$$\Rightarrow \theta$$

Now, we can see that $\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \theta$

Now Differentiating

$$\Rightarrow \frac{d(\theta)}{dx}$$

$$\Rightarrow \frac{d(\sin^{-1}x)}{dx}$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}}$$

Ans) $\frac{1}{\sqrt{1-x^2}}$

Question: 24

Differentiate eac

Solution:

To find: Value of $\sec^{-1}\left(\frac{1}{1-2x^2}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sec^{-1}\left(\frac{1}{1 - 2x^2}\right)$

⇒ Putting x = sinθ

θ = $\sin^{-1}$x ... (i)

Putting x = sinθ in the equation

$\Rightarrow \sec^{-1}\left(\frac{1}{1 - 2(\sin\theta)^2}\right)$

$\Rightarrow \sec^{-1}\left(\frac{1}{1 - 2\sin^2\theta}\right)$

$\Rightarrow \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

$\Rightarrow \sec^{-1}(\sec 2\theta)$

$\Rightarrow 2\theta$

Now, we can see that $\sec^{-1}\left(\frac{1}{1 - 2x^2}\right) = 2\theta$

Now Differentiating

$\Rightarrow \frac{d(2\theta)}{dx}$

$\Rightarrow 2\frac{d(\sin^{-1}x)}{dx}$

$\Rightarrow \frac{2}{\sqrt{1-x^2}}$

Ans) $\frac{2}{\sqrt{1-x^2}}$

Question: 25

Differentiate eac

Solution:

To find: Value of $\sin^{-1}\left(\frac{1}{\sqrt{1 + x^2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\left(\frac{1}{\sqrt{1 + x^2}}\right)$

⇒ Putting x = cotθ

θ = $\cot^{-1}$x ... (i)

Putting x = cotθ in the equation

$\Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{1 + (\cot\theta)^2}}\right)$

$\Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{1+\cot^2\theta}}\right)$

$\Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{\operatorname{cosec}^2\theta}}\right)$

$\Rightarrow \sin^{-1}\left(\frac{1}{\operatorname{cosec}\theta}\right)$

$\Rightarrow \sin^{-1}(\sin\theta)$

$\Rightarrow \theta$

Now, we can see that $\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \theta$

Now Differentiating

$\Rightarrow \frac{d(\theta)}{dx}$

$\Rightarrow \frac{d(\cot^{-1}x)}{dx}$

$\Rightarrow -\frac{1}{1+x^2}$

Ans) $-\frac{1}{1+x^2}$

Question: 26

Differentiate eac

Solution:

To find: Value of $\tan^{-1}\left(\frac{1+x}{1-x}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{1+x}{1-x}\right)$

$\Rightarrow$ Putting $x = \tan\theta$

$\theta = \tan^{-1}x$ ... (i)

Putting $x = \tan\theta$ in the equation

$\Rightarrow \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}\right)$

$\Rightarrow \tan^{-1}\left(\tan\frac{\pi}{4}+\theta\right)$

$\Rightarrow \frac{\pi}{4} + \theta$

Now, we can see that $\tan^{-1}\left(\frac{1+x}{1-x}\right) = \frac{\pi}{4} + \theta$

Now Differentiating

$\Rightarrow \dfrac{d\left(\frac{\pi}{4}+\theta\right)}{dx}$

$\Rightarrow \dfrac{d\left(\frac{\pi}{4}\right)}{dx}+\dfrac{d(\theta)}{dx}$

$\Rightarrow 0+\dfrac{d(\theta)}{dx}$

$\Rightarrow \dfrac{d(\tan^{-1}x)}{dx}$

$\Rightarrow \dfrac{1}{1+x^2}$

Ans) $\dfrac{1}{1+x^2}$

Question: 27

Differentiate eac

Solution:

To find: Value of $\cot^{-1}\left(\frac{1+x}{1-x}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2}-\theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\cot^{-1}\left(\frac{1+x}{1-x}\right)$

⇒ Putting x = tanθ

θ = $\tan^{-1}x$ … (i)

Putting x = tanθ in the equation

$\Rightarrow \cot^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$

$\Rightarrow \cot^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}\right)$

$\Rightarrow \cot^{-1}\left(\tan\frac{\pi}{4}+\theta\right)$

$\Rightarrow \cot^{-1}\left(\cot\left(\frac{\pi}{2}-\left(\frac{\pi}{4}+\theta\right)\right)\right)$

$\Rightarrow \cot^{-1}\left(\cot\left(\frac{\pi}{2}-\left(\frac{\pi}{4}+\theta\right)\right)\right)$

$\Rightarrow \cot^{-1}\left(\cot\left(\frac{\pi}{4}-\theta\right)\right)$

$\Rightarrow \frac{\pi}{4}-\theta$

Now, we can see that $\cot^{-1}\left(\frac{1+x}{1-x}\right) = \frac{\pi}{4}-\theta$

Now Differentiating

$\Rightarrow \dfrac{d\left(\frac{\pi}{4}-\theta\right)}{dx}$

$\Rightarrow \frac{d\left(\frac{\pi}{4}\right)}{dx} - \frac{d(\theta)}{dx}$

$\Rightarrow 0 - \frac{d(\theta)}{dx}$

$\Rightarrow -\frac{d(\tan^{-1}x)}{dx}$

$\Rightarrow -\frac{1}{1+x^2}$

Ans) $-\frac{1}{1+x^2}$

Question: 28

Differentiate eac

Solution:

To find: Value of $\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

⇒ Putting x = tanθ

$\theta = \tan^{-1}x$ ... (i)

Putting x = tanθ in the equation

$\Rightarrow \tan^{-1}\left(\frac{3\tan\theta - (\tan\theta)^3}{1 - 3(\tan\theta)^2}\right)$

$\Rightarrow \tan^{-1}(\tan 3\theta)$

$\Rightarrow \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$

$\Rightarrow \tan^{-1}(\tan 3\theta)$

$\Rightarrow 3\theta$

Now, we can see that $\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) = 3\theta$

Now Differentiating

$\Rightarrow \frac{d(3\theta)}{dx}$

$\Rightarrow 3\frac{d(\tan^{-1}x)}{dx}$

$\Rightarrow \frac{3}{1+x^2}$

Ans) $\frac{3}{1+x^2}$

Question: 29

Differentiate eac

Solution:

To find: Value of $\operatorname{cosec}^{-1}\left(\frac{1+x^2}{2x}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\operatorname{cosec}^{-1}\left(\frac{1+x^2}{2x}\right)$

⇒ Putting x = tanθ

θ = $\tan^{-1}x$ ... (i)

Putting x = tanθ in the equation

$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{1+(\tan\theta)^2}{2\tan\theta}\right)$

$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{1+\tan^2\theta}{2\tan\theta}\right)$

$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{1}{\sin 2\theta}\right)$

$\Rightarrow \operatorname{cosec}^{-1}(\operatorname{cosec}2\theta)$

$\Rightarrow 2\theta$

Now, we can see that $\operatorname{cosec}^{-1}\left(\frac{1+x^2}{2x}\right) = 2\theta$

Now Differentiating

$\Rightarrow \frac{d(2\theta)}{dx}$

$\Rightarrow 2\frac{d(\tan^{-1}x)}{dx}$

$\Rightarrow \frac{2}{1+x^2}$

Ans) $\frac{2}{1+x^2}$

Question: 30

Differentiate eac

Solution:

To find: Value of $\sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

⇒ Putting x = tanθ

θ = $\tan^{-1}x$ ... (i)

Putting x = tanθ in the equation

$\Rightarrow \sec^{-1}\left(\frac{1+(\tan\theta)^2}{1-(\tan\theta)^2}\right)$

$\Rightarrow \sec^{-1}\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)$

$\Rightarrow \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

$\Rightarrow \sec^{-1}(\sec 2\theta)$

$\Rightarrow 2\theta$

Now, we can see that $\sec^{-1}\left(\frac{1+x^2}{1-x^2}\right) = 2\theta$

Now Differentiating

$\Rightarrow \frac{d(2\theta)}{dx}$

$\Rightarrow 2\frac{d(\tan^{-1}x)}{dx}$

$\Rightarrow \frac{2}{1+x^2}$

Ans) $\frac{2}{1+x^2}$

Question: 31

Differentiate eac

Solution:

To find: Value of $\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

⇒ Putting x = tanθ

θ = $\tan^{-1}x$ ... (i)

Putting x = tanθ in the equation

$\Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{1+(\tan\theta)^2}}\right)$

$\Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{1+\tan^2\theta}}\right)$

$\Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{\sec^2\theta}}\right)$

$\Rightarrow \sin^{-1}\left(\frac{1}{\sec\theta}\right)$

$\Rightarrow \sin^{-1}(\cos\theta)$

$\Rightarrow \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \theta\right)\right)$

$\Rightarrow \frac{\pi}{2} - \theta$

Now, we can see that $\sin^{-1}\left(\frac{1}{\sqrt{1 + x^2}}\right) = \frac{\pi}{2} - \theta$

Now Differentiating

$\Rightarrow \frac{d\left(\frac{\pi}{2} - \theta\right)}{dx}$

$\Rightarrow \frac{d\left(\frac{\pi}{2}\right)}{dx} - \frac{d(\theta)}{dx}$

$\Rightarrow 0 - \frac{d(\tan^{-1}x)}{dx}$

$\Rightarrow -\frac{1}{1+x^2}$

Ans) $-\frac{1}{1+x^2}$

Question: 32

Differentiate eac

Solution:

To find: Value of $\sec^{-1}\left(\frac{x^2 + 1}{x^2 - 1}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sec^{-1}\left(\frac{x^2 + 1}{x^2 - 1}\right)$

⇒ Putting x = tanθ

θ = $\tan^{-1}x$ ... (i)

Putting x = tanθ in the equation

$\Rightarrow \sec^{-1}\left(\frac{(\tan\theta)^2 + 1}{(\tan\theta)^2 - 1}\right)$

$\Rightarrow \sec^{-1}\left(\frac{\tan^2\theta + 1}{\tan^2\theta - 1}\right)$

$\Rightarrow \sec^{-1}\left[-\left(\frac{1 + \tan^2\theta}{1 - \tan^2\theta}\right)\right]$

$\Rightarrow \pi - \sec^{-1}\left(\frac{1 + \tan^2\theta}{1 - \tan^2\theta}\right)$

$\Rightarrow \pi - \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

$\Rightarrow \pi - \sec^{-1}(\sec 2\theta)$

$\Rightarrow \pi - 2\theta$

$\Rightarrow \pi - 2\tan^{-1}x$

Now, we can see that $\sec^{-1}\left(\frac{x^2 + 1}{x^2 - 1}\right) = \pi - 2\tan^{-1}x$

Now Differentiating

$\Rightarrow \frac{d(\pi - 2\tan^{-1}x)}{dx}$

$\Rightarrow \frac{d(\pi)}{dx} - \frac{d(2\tan^{-1}x)}{dx}$

$\Rightarrow 0\text{-}2\frac{d(\tan^{-1}x)}{dx}$

$\Rightarrow -\frac{2}{1+x^2}$

Ans) $-\frac{1}{1+x^2}$

Question: 33

Differentiate eac

Solution:

To find: Value of $\cos^{-1}\left(\frac{1 - x^{2n}}{1 + x^{2n}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\cos^{-1}\left(\frac{1 - x^{2n}}{1 + x^{2n}}\right)$

$\Rightarrow \cos^{-1}\left(\frac{1 - (x^n)^2}{1 + (x^n)^2}\right)$

$\Rightarrow$ Putting $x^n = \tan\theta$

$\theta = \tan^{-1}(x^n)$ ... (i)

Putting $x^n = \tan\theta$ in the equation

$\Rightarrow \cos^{-1}\left(\frac{1 - \tan^2\theta}{1 + \tan^2\theta}\right)$

$\Rightarrow \cos^{-1}(\cos 2\theta)$

$\Rightarrow 2\theta$

$\Rightarrow 2\tan^{-1}(x^n)$

Now, we can see that $\cos^{-1}\left(\frac{1 - x^{2n}}{1 + x^{2n}}\right) = 2\tan^{-1}(x^n)$

Now Differentiating

$\Rightarrow \frac{d(2\tan^{-1}(x^n))}{dx}$

$\Rightarrow 2\frac{d(\tan^{-1}(x^n))}{dx^n}\frac{dx^n}{dx}$

$\Rightarrow 2\frac{1}{1+(x^n)^2}nx^{n-1}$

$\Rightarrow \frac{2nx^{n-1}}{1+x^{2n}}$

Ans) $\frac{2nx^{n-1}}{1+x^{2n}}$

Question: 34

Differentiate eac

Solution:

To find: Value of $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$

⇒ Putting x = asinθ

$\sin\theta = \frac{x}{a}$

$\theta = \sin^{-1}\left(\frac{x}{a}\right)$ ... (i)

Putting x = asinθ in the equation

$$\Rightarrow \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2-(a\sin\theta)^2}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2\left(1-\sin^2\theta\right)}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$$

$$\Rightarrow \tan^{-1}(\tan\theta)$$

$$\Rightarrow \theta$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{a}\right)$$

Now, we can see that $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right) = \sin^{-1}\left(\frac{x}{a}\right)$

Now Differentiating

$$\Rightarrow \frac{d\left(\sin^{-1}\left(\frac{x}{a}\right)\right)}{dx}$$

$$\Rightarrow \frac{d\left(\sin^{-1}\left(\frac{x}{a}\right)\right)}{d\left(\frac{x}{a}\right)}\frac{d\left(\frac{x}{a}\right)}{dx}$$

$$\Rightarrow \left(\frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\right)\frac{1}{a}$$

$$\Rightarrow \left(\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\right)\frac{1}{a}$$

$$\Rightarrow \left(\frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}}\right)\frac{1}{a}$$

$$\Rightarrow \left(\frac{a}{\sqrt{a^2-x^2}}\right)\frac{1}{a}$$

$$\Rightarrow \frac{1}{\sqrt{a^2-x^2}}$$

Ans) $\frac{1}{\sqrt{a^2-x^2}}$

Question: 35

Differentiate eac

Solution:

To find: Value of $\sin^{-1}\left\{2ax\sqrt{1-a^2x^2}\right\}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2}-\theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\left\{2ax\sqrt{1-a^2x^2}\right\}$

⇒ Putting ax = sinθ

$\theta = \sin^{-1}(ax)$ ... (i)

Putting ax = sinθ in the equation

$\Rightarrow \sin^{-1}\left\{2\sin\theta\sqrt{1-(\sin\theta)^2}\right\}$

$\Rightarrow \sin^{-1}\left\{2\sin\theta\sqrt{1-\sin^2\theta}\right\}$

$\Rightarrow \sin^{-1}\{2\sin\theta\cos\theta\}$

$\Rightarrow \sin^{-1}\{\sin 2\theta\}$

$\Rightarrow 2\theta$

$\Rightarrow 2\sin^{-1}(ax)$

Now, we can see that $\sin^{-1}\left\{2ax\sqrt{1-a^2x^2}\right\} = 2\sin^{-1}(ax)$

Now Differentiating

$$\Rightarrow \frac{d(2\sin^{-1}(ax))}{dx}$$

$$\Rightarrow 2\frac{d(\sin^{-1}(ax))}{dax}\frac{dax}{dx}$$

$$\Rightarrow \left(2\frac{1}{\sqrt{1-(ax)^2}}\right)a$$

$$\Rightarrow \left(\frac{2a}{\sqrt{1-a^2x^2}}\right)$$

Ans) $\frac{2a}{\sqrt{1-a^2x^2}}$

Question: 36

Differentiate eac

Solution:

To find: Value of $\tan^{-1}\left\{\frac{\sqrt{1+a^2x^2}-1}{ax}\right\}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left\{\frac{\sqrt{1+a^2x^2}-1}{ax}\right\}$

⇒ Putting ax = tanθ

$\theta = \tan^{-1}(ax)$ … (i)

Putting ax = tanθ in the equation

$$\Rightarrow \tan^{-1}\left\{\frac{\sqrt{1+(\tan\theta)^2}-1}{\tan\theta}\right\}$$

$$\Rightarrow \tan^{-1}\left\{\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right\}$$

$$\Rightarrow \tan^{-1}\left\{\frac{\sec\theta-1}{\tan\theta}\right\}$$

$$\Rightarrow \tan^{-1}\left\{\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right\}$$

$$\Rightarrow \tan^{-1}\left\{\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right\}$$

$$\Rightarrow \tan^{-1}\left\{\frac{1-\cos\theta}{\sin\theta}\right\}$$

$$\Rightarrow \tan^{-1}\left\{\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right\}$$

$$\Rightarrow \tan^{-1}\left\{\tan\frac{\theta}{2}\right\}$$

$$\Rightarrow \frac{\theta}{2}$$

$$\Rightarrow \frac{\tan^{-1}(ax)}{2}$$

Now, we can see that $\tan^{-1}\left\{\frac{\sqrt{1+a^2x^2}-1}{ax}\right\} = \frac{\tan^{-1}(ax)}{2}$

Now Differentiating

$$\Rightarrow \frac{d\left(\frac{\tan^{-1}(ax)}{2}\right)}{dx}$$

$$\Rightarrow \frac{1}{2}\frac{d(\tan^{-1}(ax))}{dax}\frac{dax}{dx}$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{1+(ax)^2}\right)a$$

$$\Rightarrow \frac{a}{2(1+a^2x^2)}$$

Ans) $\frac{a}{2(1+a^2x^2)}$

Question: 37

Differentiate eac

Solution:

To find: Value of $\sin^{-1}\left\{\frac{x^2}{\sqrt{x^4+a^4}}\right\}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\left\{\frac{x^2}{\sqrt{x^4+a^4}}\right\}$

$\Rightarrow$ Putting $x^2 = a^2\cot\theta$

$\theta = \cot^{-1}\left(\frac{x^2}{a^2}\right)$ ... (i)

Putting $x^2 = a^2\cot\theta$ in the equation

$$\Rightarrow \sin^{-1}\left\{\frac{a^2\cot\theta}{\sqrt{(a^2\cot\theta)^2+a^4}}\right\}$$

$$\Rightarrow \sin^{-1}\left\{\frac{a^2\cot\theta}{\sqrt{a^4\cot^2\theta+a^4}}\right\}$$

$$\Rightarrow \sin^{-1}\left\{\frac{a^2\cot\theta}{\sqrt{a^4(\cot^2\theta+1)}}\right\}$$

$\Rightarrow \sin^{-1}\left\{\frac{a^2 \cot\theta}{a^2 cosec\theta}\right\}$

$\Rightarrow \sin^{-1}\{\cos\theta\}$

$\Rightarrow \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - \theta\right)\right\}$

$\Rightarrow \frac{\pi}{2} - \theta$

$\Rightarrow \frac{\pi}{2} - \cot^{-1}\left(\frac{x^2}{a^2}\right)$

Now, we can see that $\sin^{-1}\left\{\frac{x^2}{\sqrt{x^4+a^4}}\right\} = \frac{\pi}{2} - \cot^{-1}\left(\frac{x^2}{a^2}\right)$

Now Differentiating

$$\Rightarrow \frac{d\left(\frac{\pi}{2} - \cot^{-1}\left(\frac{x^2}{a^2}\right)\right)}{dx}$$

$$\Rightarrow \frac{d\left(\frac{\pi}{2}\right)}{dx} - \frac{d\left(\cot^{-1}\left(\frac{x^2}{a^2}\right)\right)}{dx}$$

$$\Rightarrow 0 - \frac{d\left(\cot^{-1}\left(\frac{x^2}{a^2}\right)\right)}{d\frac{x^2}{a^2}} \frac{d\frac{x^2}{a^2}}{dx}$$

$$\Rightarrow \left(\frac{1}{1+\left(\frac{x^2}{a^2}\right)^2}\right)\frac{1}{a^2}2x$$

$$\Rightarrow \left(\frac{a^4}{a^4+x^4}\right)\frac{1}{a^2}2x$$

$$\Rightarrow \left(\frac{2a^2x}{a^4+x^4}\right)$$

Ans) $\frac{2a^2x}{a^4+x^4}$

Question: 38

Differentiate eac

Solution:

To find: Value of $\tan^{-1}\left\{\frac{e^{2x}+1}{e^{2x}-1}\right\}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left\{\frac{e^{2x}+1}{e^{2x}-1}\right\}$

$\Rightarrow \tan^{-1}\left\{\frac{1+e^{2x}}{-(1-e^{2x})}\right\}$

$-\tan^{-1}\left\{\frac{1+e^{2x}}{1-e^{2x}}\right\}$

Putting $e^{2x} = \tan\theta$

$\theta = \tan^{-1}(e^{2x})$ … (i)

Putting $e^{2x} = \tan\theta$ in the equation

$\Rightarrow -\tan^{-1}\left\{\frac{1+\tan\theta}{1-\tan\theta}\right\}$

$\Rightarrow -\tan^{-1}\left\{\frac{tan\frac{\pi}{4}+\tan\theta}{1-tan\frac{\pi}{4}\tan\theta}\right\}$

$\Rightarrow -\tan^{-1}\left\{\tan\left(\frac{\pi}{4}+\theta\right)\right\}$

$\Rightarrow -\left(\frac{\pi}{4}+\theta\right)$

$\Rightarrow -\frac{\pi}{4}-\theta$

$\Rightarrow -\frac{\pi}{4}-\tan^{-1}(e^{2x})$

Now, we can see that $\tan^{-1}\left\{\frac{e^{2x}+1}{e^{2x}-1}\right\} = -\frac{\pi}{4}-\tan^{-1}(e^{2x})$

Now Differentiating

$\Rightarrow \frac{d\left(-\frac{\pi}{4}-\tan^{-1}(e^{2x})\right)}{dx}$

$\Rightarrow \frac{d\left(-\frac{\pi}{4}\right)}{dx}-\frac{d(\tan^{-1}(e^{2x}))}{dx}$

$\Rightarrow 0-\frac{d(\tan^{-1}(e^{2x}))}{de^{2x}}\frac{de^{2x}}{d2x}\frac{d2x}{dx}$

$\Rightarrow -\left(\frac{1}{1+(e^{2x})^2}\right)e^{2x}2$

$\Rightarrow -\left(\frac{2e^{2x}}{1+e^{4x}}\right)$

$\Rightarrow \frac{-2e^{2x}}{1+e^{4x}}$

Ans) $\frac{-2e^{2x}}{1+e^{4x}}$

Question: 39

Differentiate eac

Solution:

To find: Value of $\cos^{-1}(2x)+2\cos^{-1}\sqrt{1-4x^2}$

The formula used: (i) $\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$

(ii) $\frac{d(cos^{-1}x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$

We have, $\cos^{-1}(2x)+2\cos^{-1}\sqrt{1-4x^2}$

Putting $2x = \cos\theta$

$\theta = \cos^{-1}(2x)$ … (i)

Putting $e^{2x} = \tan\theta$ in the equation

$\Rightarrow \cos^{-1}(\cos\theta) + 2\cos^{-1}\sqrt{1-(cos\theta)^2}$

$\Rightarrow \cos^{-1}(\cos\theta) + 2\cos^{-1}\sqrt{1-\cos^2\theta}$

$\Rightarrow \theta + 2\cos^{-1}\sqrt{\sin^2\theta}$

$\Rightarrow \theta + 2\cos^{-1}(\sin\theta)$

$\Rightarrow \theta + 2\cos^{-1}\left(\cos\left(\frac{\pi}{2}-\theta\right)\right)$

$\Rightarrow \theta + 2\left(\frac{\pi}{2}-\theta\right)$

$\Rightarrow \pi - \theta$

$\Rightarrow \pi - \cos^{-1}(2x)$

Now, we can see that $\cos^{-1}(2x) + 2\cos^{-1}\sqrt{1-4x^2} = \pi - \cos^{-1}(2x)$

Now Differentiating

$\Rightarrow \frac{d(\pi - \cos^{-1}(2x))}{dx}$

$\Rightarrow \frac{d(\pi)}{dx} - \frac{d(\cos^{-1}(2x))}{dx}$

$\Rightarrow 0 - \frac{d(\cos^{-1}(2x))}{d2x}\frac{d2x}{dx}$

$\Rightarrow \left(\frac{1}{\sqrt{1-(2x)^2}}\right)2$

$\Rightarrow \left(\frac{2}{\sqrt{1-4x^2}}\right)$

Ans) $\frac{2}{\sqrt{1-4x^2}}$

Question: 40

Differentiate eac

Solution:

To find: Value of $\tan^{-1}\left\{\frac{a-x}{1+ax}\right\}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$

We have, $\tan^{-1}\left\{\frac{a-x}{1+ax}\right\}$

$\Rightarrow \tan^{-1}a - \tan^{-1}x$

Now Differentiating

$\Rightarrow \frac{d(\tan^{-1}a - \tan^{-1}x)}{dx}$

$$\Rightarrow \frac{d(\tan^{-1}a)}{dx} - \frac{d(\tan^{-1}x)}{dx}$$

$$\Rightarrow 0 - \frac{1}{1+x^2}$$

Ans) $-\frac{1}{1+x^2}$

Question: 41

Differentiate eac

Solution:

To find: Value of $\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{\frac{3}{2}}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{\frac{3}{2}}}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{\sqrt{x}-x}{1+x\sqrt{x}}\right)$$

$$\Rightarrow \tan^{-1}\sqrt{x} - \tan^{-1}x$$

Now Differentiating

$$\Rightarrow \frac{d\left(\tan^{-1}\sqrt{x} - \tan^{-1}x\right)}{dx}$$

$$\Rightarrow \frac{d\left(\tan^{-1}\sqrt{x}\right)}{dx} - \frac{d\left(\tan^{-1}x\right)}{dx}$$

$$\Rightarrow \frac{d\left(\tan^{-1}\sqrt{x}\right)}{d\sqrt{x}}\frac{d\sqrt{x}}{x} - \frac{d\left(\tan^{-1}x\right)}{dx}$$

$$\Rightarrow \frac{1}{1+(\sqrt{x})^2}\frac{1}{2\sqrt{x}} - \frac{1}{1+x^2}$$

$$\Rightarrow \frac{1}{2\sqrt{x}(1+x)} - \frac{1}{1+x^2}$$

Ans) $\frac{1}{2\sqrt{x}(1+x)} - \frac{1}{1+x^2}$

Question: 42

Differentiate eac

Solution:

To find: Value of $\tan^{-1}\left(\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{x}\sqrt{a}}\right)$

$\Rightarrow \tan^{-1}\sqrt{a} + \tan^{-1}\sqrt{x}$

Now Differentiating

$\Rightarrow \frac{d\left(tan^{-1}\sqrt{a} + tan^{-1}\sqrt{x}\right)}{dx}$

$\Rightarrow \frac{d\left(\tan^{-1}\sqrt{a}\right)}{dx} - \frac{d\left(\tan^{-1}\sqrt{x}\right)}{dx}$

$\Rightarrow 0 - \frac{d\left(\tan^{-1}\sqrt{x}\right)}{d\sqrt{x}}\frac{d\sqrt{x}}{x}$

$\Rightarrow -\frac{1}{1+\left(\sqrt{x}\right)^2}\frac{1}{2\sqrt{x}}$

$\Rightarrow -\frac{1}{2\sqrt{x}(1+x)}$

Ans) $-\frac{1}{2\sqrt{x}(1+x)}$

Question: 43

Differentiate eac

Solution:

Given: Value of $\tan^{-1}\left(\frac{3-2x}{1+6x}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{3-2x}{1+6x}\right)$

$\Rightarrow \tan^{-1}\left(\frac{3-2x}{1+3\times 2x}\right)$

$\Rightarrow \tan^{-1}3 - \tan^{-1}2x$

Now Differentiating

$\Rightarrow \frac{d(\tan^{-1}3 - \tan^{-1}2x)}{dx}$

$\Rightarrow 0 - \frac{d(\tan^{-1}2x)}{d2x}\frac{d2x}{dx}$

$\Rightarrow -\frac{1}{1+(2x)^2}2$

$\Rightarrow -\frac{2}{1+4x^2}$

Ans) $-\frac{2}{1+4x^2}$

Question: 44

Differentiate eac

Solution:

Given: Value of $\tan^{-1}\left(\frac{5x}{1-6x^2}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{5x}{1-6x^2}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{3x+2x}{1-3x\times 2x}\right)$$

$$\Rightarrow \tan^{-1}3x + \tan^{-1}2x$$

Now Differentiating

$$\Rightarrow \frac{d(\tan^{-1}3x \mp \tan^{-1}2x)}{dx}$$

$$\Rightarrow \frac{d(\tan^{-1}3x)}{d3x}\frac{d3x}{dx} + \frac{d(\tan^{-1}2x)}{d2x}\frac{d2x}{dx}$$

$$\Rightarrow \frac{1}{1+(3x)^2}3 + \frac{1}{1+(2x)^2}2$$

$$\Rightarrow \frac{3}{1+9x^2} + \frac{2}{1+4x^2}$$

Ans) $\frac{3}{1+9x^2} + \frac{2}{1+4x^2}$

Question: 45

Differentiate eac

Solution:

Given: Value of $\tan^{-1}\left(\frac{2x}{1+15x^2}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{2x}{1+15x^2}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{5x-3x}{1+5x\times 3x}\right)$$

$$\Rightarrow \tan^{-1}5x - \tan^{-1}3x$$

Now Differentiating

$$\Rightarrow \frac{d(\tan^{-1}5x - \tan^{-1}3x)}{dx}$$

$$\Rightarrow \frac{d(tan^{-1}5x)}{d5x}\frac{d5x}{dx} - \frac{d(tan^{-1}3x)}{d3x}\frac{d3x}{dx}$$

$$\Rightarrow \frac{1}{1+(5x)^2}5 + \frac{1}{1+(3x)^2}3$$

$$\Rightarrow \frac{5}{1+25x^2} + \frac{3}{1+9x^2}$$

Ans) $\frac{5}{1+25x^2}+\frac{3}{1+9x^2}$

Question: 46

Differentiate eac

Solution:

Given: Value of $\tan^{-1}\left(\frac{ax-b}{bx+a}\right)$

To Prove: $\frac{dy}{dx} = \frac{1}{1+x^2}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{ax-b}{bx+a}\right)$

Dividing numerator and denominator with a

$$\Rightarrow \tan^{-1}\left(\frac{\frac{ax-b}{a}}{\frac{bx+a}{a}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{x-\frac{b}{a}}{1+\frac{b}{a}x}\right)$$

$$\Rightarrow \tan^{-1}x - \tan^{-1}\left(\frac{b}{a}\right)$$

Now Differentiating

$$\Rightarrow \frac{d\left(\tan^{-1}x - \tan^{-1}\left(\frac{b}{a}\right)\right)}{dx}$$

$$\Rightarrow \frac{d(tan^{-1}x)}{dx} - \frac{d\left(tan^{-1}\left(\frac{b}{a}\right)\right)}{dx}$$

$$\Rightarrow \frac{1}{1+x^2}+0$$

Ans) $\frac{1}{1+x^2}$

Question: 47

Differentiate eac

Solution:

Given: Value of $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)+\sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

To Prove: $\frac{dy}{dx} = \frac{4}{(1+x^2)}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

Putting x = tanθ

$\theta = \tan^{-1}x$

Dividing numerator and denominator with a

$$\Rightarrow \sin^{-1}\left(\frac{2\tan\theta}{1+(\tan\theta)^2}\right)+\sec^{-1}\left(\frac{1+(\tan\theta)^2}{1-(\tan\theta)^2}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)+\sec^{-1}\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)$$

$$\Rightarrow \sin^{-1}(\sin 2\theta)+\sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$$

$$\Rightarrow \sin^{-1}(\sin 2\theta)+\sec^{-1}(\sec 2\theta)$$

$\Rightarrow 2\theta+2\theta$

$\Rightarrow 4\theta$

$\Rightarrow 4\tan^{-1}x$

Now Differentiating

$$\Rightarrow \frac{d(4\tan^{-1}x)}{dx}$$

$$\Rightarrow 4\frac{1}{1+x^2}$$

Ans) $\frac{4}{1+x^2}$

Question: 48

Differentiate eac

Solution:

Given: Value of $y = \sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$

To Prove: $\frac{dy}{dx} = 0$

Formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2}-\theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$

$$\Rightarrow \cos^{-1}\left(\frac{x-1}{x+1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$$

$\Rightarrow \frac{\pi}{2}$

Now Differentiating

$$\Rightarrow \frac{d\left(\frac{\pi}{2}\right)}{dx}$$

$\Rightarrow 0$

Ans) $\frac{4}{1+x^2}$

Question: 49

Differentiate eac

Solution:

Given: Value of $y = \sin\left\{2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right\}$

To Prove: $\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}$

Formula used: (i) $\frac{d(\cos^{-1}x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$

Let x = cosθ

θ = $\cos^{-1}$x

Putting x = cosθ in equation

$$\Rightarrow \sin\left\{2\tan^{-1}\left(\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right)\right\}$$

$$\Rightarrow \sin\left\{2\tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\right)\right\}$$

$$\Rightarrow \sin\left\{2\tan^{-1}\left(\sqrt{\tan^2\frac{\theta}{2}}\right)\right\}$$

$$\Rightarrow \sin\left\{2\tan^{-1}\left(\tan\frac{\theta}{2}\right)\right\}$$

$$\Rightarrow \sin\left\{2\frac{\theta}{2}\right\}$$

$$\Rightarrow \sin\theta$$

$$\Rightarrow \sin(\cos^{-1}x)$$

Now Differentiating

$$\Rightarrow \frac{d(\sin(\cos^{-1}x))}{dx}$$

$$\Rightarrow \frac{d(\sin(\cos^{-1}x))}{d\cos^{-1}x}\frac{d\cos^{-1}x}{dx}$$

$$\Rightarrow -\cos(\cos^{-1}x)\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow -\frac{x}{\sqrt{1-x^2}}$$

Ans) $\frac{4}{1+x^2}$

Question: 50

Differentiate eac

Solution:

Given: Value of $y = \tan^{-1}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

To Prove: $\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\cos^{-1}x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$

Let x = cos2θ

$2\theta = \cos^{-1}x$

$\theta = \frac{1}{2}\cos^{-1}x$

Putting x = cos2θ

$$y = \tan^{-1}\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}$$

$$y = \tan^{-1}\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}$$

$$y = \tan^{-1}\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}$$

$$y = \tan^{-1}\frac{\sqrt{2}(\cos\theta-\sin\theta)}{\sqrt{2}(\cos\theta+\sin\theta)}$$

Dividing by cosθ in the numerator and denominator

$$y = \tan^{-1}\frac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\cos\theta+\sin\theta}{\cos\theta}}$$

$$y = \tan^{-1}\frac{1-\tan\theta}{1+\tan\theta}$$

$$y = \tan^{-1}\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}$$

$$y = \tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right)$$

$$y = \frac{\pi}{4}-\theta$$

$$y = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$$

Now Differentiating

$$\Rightarrow \frac{d\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x\right)}{dx}$$

$$\Rightarrow \frac{d\left(\frac{\pi}{4}\right)}{dx} - \frac{1}{2}\frac{d\cos^{-1}x}{dx}$$

$$\Rightarrow \frac{1}{2}\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{1}{2\sqrt{1-x^2}}$$

Ans) $\frac{1}{2\sqrt{1-x^2}}$

Question: 51

Differentiate eac

Solution:

Given: Value of $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

To find: $\frac{dy}{dx}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$

$$y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$$

$$y = \sin^{-1}\left(\frac{2^x.2}{1+\left(2^2\right)^x}\right)$$

$$y = \sin^{-1}\left(\frac{2^x.2}{1+\left(2^x\right)^2}\right)$$

Let $2^x = \tan\theta$

$\theta = \tan^{-1}(2^x)$

Putting $2^x = \tan\theta$

$$y = \sin^{-1}\left(\frac{\tan\theta.2}{1+(\tan\theta)^2}\right)$$

$$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$y = \sin^{-1}(\sin 2\theta)$$

$$y = 2\theta$$

$$y = 2\tan^{-1}(2^x)$$

Now Differentiating

$$\Rightarrow \frac{d(2\tan^{-1}(2^x))}{dx}$$

$$\Rightarrow 2\frac{d(\tan^{-1}(2^x))}{d2^x}\frac{d2^x}{dx}$$

$$\Rightarrow 2\frac{1}{1+(2^x)^2}.2^x\log 2$$

$$\Rightarrow \frac{2^{1+x}\log 2}{1+4^x}.$$

Ans) $\frac{2^{1+x}\log 2}{1+4^x}$

Exercise : 10E

Question: 1

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to the chain rule of differentiation

$$d(y^2)/_{dx} = \frac{d(y^2)}{dy} \times \frac{dy}{dx} = 2y \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(x^2)}{dx} + \frac{d(y^2)}{dx} = \frac{d(4)}{dx}$$

$$2x + 2y \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

Question: 2

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to the chain rule of differentiation

$$\frac{d(y^2/_{b^2})}{dx} = \frac{d(y^2/_{b^2})}{dy} \times \frac{dy}{dx} = \frac{2y}{b^2} \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(x^2/_{a^2})}{dx} + \frac{d(y^2/_{b^2})}{dx} = \frac{d(1)}{dx}$$

$$\frac{2x}{a^2} + \frac{2y}{b^2} \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-\frac{2x}{a^2}}{\frac{2y}{b^2}}$$

$$\frac{dy}{dx} = \frac{-b^2 x}{a^2 y}$$

Question: 3

Find , when:

<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to the chain rule of differentiation

$$\frac{d(\sqrt{y})}{dx} = \frac{d(\sqrt{y})}{dy} \times \frac{dy}{dx} = \frac{1}{2\sqrt{y}} \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(\sqrt{x})}{dx} + \frac{d(\sqrt{y})}{dx} = \frac{d(\sqrt{a})}{dx}$$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-\frac{1}{2\sqrt{x}}}{\frac{1}{2\sqrt{y}}}$$

$$\frac{dy}{dx} = \frac{-2\sqrt{y}}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{-\sqrt{y}}{\sqrt{x}}$$

Question: 4

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to the chain rule of differentiation

$$\frac{d(y^{2/3})}{dx} = \frac{d(y^{2/3})}{dy} \times \frac{dy}{dx} = \frac{2}{3y^{1/3}} \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(x^{2/3})}{dx} + \frac{d(y^{2/3})}{dx} = \frac{d(a^{2/3})}{dx}$$

$$\frac{2}{3\,x^{1/3}} + \frac{2}{3y^{1/3}} \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-\frac{2}{3x^{1/3}}}{\frac{2}{3y^{1/3}}}$$

$$\frac{dy}{dx} = \frac{-y^{1/3}}{x^{1/3}}$$

Question: 5

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xd(y)}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d(xy)}{dx} = \frac{d(c^2)}{dx}$$

$$x \times \frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

$$\frac{dy}{dx} = \frac{-xy}{x^2}$$

$$\frac{dy}{dx} = \frac{-c^2}{x^2}$$

Question: 6

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to chain rule of differentiation

$$\frac{d(y^2)}{dx} = \frac{d(y^2)}{dy} \times \frac{dy}{dx} = 2y \times \frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xd(y)}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d(x^2)}{dx} + \frac{d(y^2)}{dx} - 3\frac{d(xy)}{dx} = \frac{d(1)}{dx}$$

$$2x + 2y \times \frac{dy}{dx} - 3(x \times \frac{dy}{dx} + y) = 0$$

$$(2y - 3x)\frac{dy}{dx} + 2x - 3y = 0$$

$$\frac{dy}{dx} = \frac{-(2x - 3y)}{2y - 3x}$$

$$\frac{dy}{dx} = \frac{2x - 3y}{3x - 2y}$$

Question: 7

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to chain rule of differentiation

$$\frac{d(y^2)}{dx} = \frac{d(y^2)}{dy} \times \frac{dy}{dx} = 2y \times \frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xd(y)}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d(xy^2)}{dx} - \frac{d(x^2y)}{dx} = \frac{d(5)}{dx}$$

$$x \times \frac{d(y^2)}{dx} + y^2 - [x^2 \times \frac{d(y)}{dx} + y \times 2x] = 0$$

$$x \times (2y \times \frac{dy}{dx}) + y^2 - [x^2 \times \frac{d(y)}{dx} + y \times 2x] = 0$$

$$2xy\frac{dy}{dx} - x^2\frac{dy}{dx} + y^2 - 2xy = 0$$

$$\frac{dy}{dx} = \frac{2xy - y^2 dy}{2xy - x^2 dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$$

Question: 8

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to chain rule of differentiation

$$\frac{d(y^2)}{dx} = \frac{d(y^2)}{dy} \times \frac{dy}{dx} = 2y \times \frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xd(y)}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d((x^2 + y^2)^2)}{dx} = \frac{d(xy)}{dx}$$

$$2(x^2 + y^2) \times \frac{d(x^2 + y^2)}{dx} = [x \times \frac{d(y)}{dx} + y]$$

$$2(x^2+y^2)\times[2x+2y\times\frac{dy}{dx}]=[x\times\frac{d(y)}{dx}+y]$$

$$4x(x^2+y^2)+4y(x^2+y^2)\frac{dy}{dx}=x\frac{dy}{dx}+y$$

$$\frac{dy}{dx}[4y(x^2+y^2)-x]=y-4x(x^2+y^2)$$

$$\frac{dy}{dx}=\frac{y-4x(x^2+y^2)}{[4y(x^2+y^2)-x]}$$

$$\frac{dy}{dx}=\frac{y-4x^3-4xy^2}{4y^3+4x^2y-x}$$

Question: 9

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx}=n\times x^{(n-1)}, \frac{d(\log x)}{dx}=\frac{1}{x}$

According to chain rule of differentiation

$$\frac{d(y^2)}{dx}=\frac{d(y^2)}{dy}\times\frac{dy}{dx}=2y\times\frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(xy)}{dx}=\frac{xd(y)}{dx}+\frac{yd(x)}{dx}=x\times\frac{dy}{dx}+y$$

Therefore ,

$$\frac{d(x^2)}{dx}+\frac{d(y^2)}{dx}=\frac{d(\log xy)}{dx}$$

$$2x+2y\frac{d(y)}{dx}=[\frac{1}{xy}\frac{d(xy)}{dx}]$$

$$2x+2y\frac{d(y)}{dx}=\frac{1}{xy}(x\frac{dy}{dx}+y)$$

$$2x+2y\frac{dy}{dx}=\frac{1}{y}\frac{dy}{dx}+\frac{1}{x}$$

$$\frac{dy}{dx}\left[2y-\frac{1}{y}\right]=\frac{1}{x}-2x$$

$$\frac{dy}{dx}(\frac{2y^2-1}{y})=\frac{1-2x^2}{x}$$

$$\frac{dy}{dx}=\frac{y(1-2x^2)}{x(2y^2-1)}$$

Question: 10

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to the chain rule of differentiation

$$\frac{d(y^n)}{dx} = \frac{d(y^n)}{dy} \times \frac{dy}{dx} = ny^{n-1} \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(x^n)}{dx} + \frac{d(y^n)}{dx} = \frac{d(a^n)}{dx}$$

$$nx^{n-1} + ny^{n-1} \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-nx^{n-1}}{ny^{n-1}}$$

$$\frac{dy}{dx} = \frac{-x^{n-1}}{y^{n-1}}$$

Question: 11

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\sin x)}{dx} = \cos x$, $\frac{d(\cos x)}{dx} = -\sin x$

According to the chain rule of differentiation

$$\frac{d(\sin 2y)}{dx} = \frac{d(\sin 2y)}{dy} \times \frac{dy}{dx} = 2\cos 2y \times \frac{dy}{dx}$$

According to the product rule of differentiation

$$\frac{d(x\sin 2y)}{dx} = \frac{xd(\sin 2y)}{dx} + \frac{\sin 2y\, d(x)}{dx} = x \times \frac{d(\sin 2y)}{dx} + \sin 2y$$

Therefore ,

$$\frac{d(x\sin 2y)}{dx} = \frac{d(y\cos 2x)}{dx}$$

$$x \times \frac{d(\sin 2y)}{dx} + \sin 2y = \cos 2x \times \frac{d(y)}{dx} + y(-2\sin 2x)$$

$$x \times 2\cos 2y \times \frac{dy}{dx} + \sin 2y = \cos 2x \times \frac{d(y)}{dx} + y(-2\sin 2x)$$

$$\frac{dy}{dx}[2x\cos 2y - \cos 2x] = -2y\sin 2x - \sin 2y$$

$$\frac{dy}{dx} = \frac{-(2y\sin 2x + \sin 2y)}{2x\cos 2y - \cos 2x}$$

$$\frac{dy}{dx} = \frac{(2y\sin 2x + \sin 2y)}{\cos 2x - 2x\cos 2y}$$

Question: 12

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\sin x)}{dx} = \cos x$, $\frac{d(\cos x)}{dx} = -\sin x$

According to chain rule of differentiation

$$\frac{d(\cos y)}{dx} = \frac{d(\cos y)}{dy} \times \frac{dy}{dx} = -\sin y \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(\sin^2 x)}{dx} + \frac{d(2\cos y)}{dx} + \frac{d(xy)}{dx} = 0$$

$$2\sin x \times \frac{d(\sin x)}{dx} + 2\left(-\sin y \times \frac{dy}{dx}\right) + x \times \frac{dy}{dx} + y = 0$$

$$2\sin x \times \cos x + y = 2\left(\sin y \times \frac{dy}{dx}\right) - x \times \frac{dy}{dx}$$

$$\frac{dy}{dx}[2\sin y - x] = \sin 2x + y$$

$$\frac{dy}{dx} = \frac{\sin 2x + y}{2x \cos 2y - \cos 2x}$$

$$\frac{dy}{dx} = \frac{\sin 2x + y}{2\sin y - x}$$

Question: 13

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\sec x)}{dx} = \sec x \tan x$, $\frac{d(\tan x)}{dx} = \sec^2 x$

According to product rule of differentiation

$$\frac{d(x^2 y)}{dx} = \frac{x^2 \mathrm{dy}}{dx} + \frac{y d(x^2)}{dx} = x^2 \frac{dy}{dx} + 2xy$$

Therefore ,

$$\frac{d(y\sec x)}{dx} + \frac{d(\tan x)}{dx} + \frac{d(x^2 y)}{dx} = 0$$

$$\sec x \times \frac{d(y)}{dx} + y\sec x \tan x + \sec^2 \mathrm{x} + x^2 \frac{dy}{dx} + 2xy = 0$$

$$\frac{dy}{dx}[x^2 + \sec x] = -(y\sec x \tan x + \sec^2 \mathrm{x} + 2\mathrm{xy})$$

$$\frac{dy}{dx} = \frac{-(y\sec x \tan x + \sec^2 \mathrm{x} + 2\mathrm{xy})}{x^2 + \sec x}$$

Question: 14

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\cot x)}{dx} = -cosec^2 x$

According to chain rule of differentiation

$$\frac{d(\cot xy)}{dx} = -\text{cosec}^2\, \text{xy} \times \frac{d(xy)}{dx}$$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xdy}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d(\cot xy)}{dx} + \frac{d(xy)}{dx} = \frac{dy}{dx}$$

$$-\text{cosec}^2\, \text{xy} \times \frac{d(xy)}{dx} + \frac{d(xy)}{dx} = \frac{dy}{dx}$$

$$\frac{d(xy)}{dx}[-\text{cosec}^2\, \text{xy} + 1] = \frac{dy}{dx}$$

$$[x\frac{dy}{dx} + y][-\cot^2 \text{xy}] = \frac{dy}{dx} \text{ (Since, } 1 - \text{cosec}^2\, \text{xy} = -\cot^2 \text{xy)}$$

$$x\frac{dy}{dx}(-\cot^2 \text{xy}) - y\cot^2 \text{xy} = \frac{dy}{dx}$$

$$\frac{dy}{dx}[-\text{x}\cot^2 \text{xy} - 1] = y\cot^2 \text{xy}$$

$$\frac{dy}{dx} = \frac{-\,y\cot^2 \text{xy}}{\text{x}\cot^2 \text{xy} + 1}$$

Question: 15

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\tan x)}{dx} = sec^2 x$, $\frac{d(\cos x)}{dx} = -sin\ x$

According to chain rule of differentiation

$$\frac{d(y^2)}{dx} = \frac{d(y^2)}{dy} \times \frac{dy}{dx} = 2y \times \frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(y\tan x)}{dx} = \text{ysec}^2\, \text{x} + \tan x \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(y\tan x)}{dx} - \frac{d(y^2\cos x)}{dx} + \frac{d(2x)}{dx} = 0$$

$$\text{ysec}^2\, \text{x} + \tan x \times \frac{dy}{dx} - \cos x\frac{d(y^2)}{dx} - y^2(-\sin x) + 2 = 0$$

$$\text{ysec}^2\, \text{x} + \tan x \times \frac{dy}{dx} - \cos x\left(2y\frac{dy}{dx}\right) + y^2(\sin x) + 2 = 0$$

$$\text{ysec}^2\, \text{x} + \frac{dy}{dx}[\tan x - 2y\, cos\ x] + y^2(\sin x) + 2 = 0$$

$$\mathrm{ysec}^2 \mathrm{x} + y^2(\sin x) + 2 = \frac{dy}{dx}[2y \, cos \, x - \tan x]$$

$$\frac{dy}{dx} = \frac{\mathrm{ysec}^2 \mathrm{x} + y^2 \sin x + 2}{2y \, cos \, x - \tan x}$$

Question: 16

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$, $\frac{d(\log x)}{dx} = \frac{1}{x}$

According to chain rule of differentiation

$$\frac{d(\sin^{-1} y)}{dx} = \frac{d(\sin^{-1} y)}{dy} \times \frac{dy}{dx} = \frac{1}{\sqrt{1-y^2}} \times \frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(e^x \log y)}{dx} = e^x \log y + e^x \times \frac{d(\log y)}{dx} = e^x \log y + e^x \times \frac{1}{y} \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(e^x \log y)}{dx} = \frac{d(\sin^{-1} x)}{dx} + \frac{d(\sin^{-1} y)}{dx}$$

$$e^x \log y + e^x \frac{1}{y}\frac{dy}{dx} = \frac{1}{\sqrt{1-\mathrm{x}^2}} + \frac{1}{\sqrt{1-\mathrm{y}^2}} \times \frac{dy}{dx}$$

$$\frac{dy}{dx}[e^x \frac{1}{y} - \frac{1}{\sqrt{1-\mathrm{y}^2}}] = \frac{1}{\sqrt{1-\mathrm{x}^2}} - e^x \log y$$

$$\frac{dy}{dx}[\frac{e^x\sqrt{1-\mathrm{y}^2} - y}{y\sqrt{1-\mathrm{y}^2}}] = \frac{1-(e^x \log y \sqrt{1-\mathrm{x}^2})}{\sqrt{1-\mathrm{x}^2}}$$

$$\frac{dy}{dx} = \frac{y\sqrt{1-\mathrm{y}^2}}{e^x\sqrt{1-\mathrm{y}^2} - y} \times \frac{1-(e^x \log y \sqrt{1-\mathrm{x}^2})}{\sqrt{1-\mathrm{x}^2}}$$

$$\frac{dy}{dx} = y \times \sqrt{\frac{1-\mathrm{y}^2}{1-\mathrm{x}^2}} \times \frac{1-(e^x \log y \sqrt{1-\mathrm{x}^2})}{(e^x\sqrt{1-\mathrm{y}^2}) - y}$$

Question: 17

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$, $\frac{d(\log x)}{dx} = \frac{1}{x}$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xd(y)}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d(xy \times \log x + y)}{dx} = \frac{d(1)}{dx}$$

$$\log x + y \times \frac{d(xy)}{dx} + xy \times \frac{d(\log x + y)}{dx} = \frac{d(1)}{dx}$$

$$\log x + y\left[x\frac{dy}{dx} + y\right] + xy[\frac{1}{x+y} \times \left(1 + \frac{dy}{dx}\right)] = 0$$

$$\frac{dy}{dx}[x \times \log x + y] + y \times \log(x+y) + \frac{xy}{x+y}\left(1 + \frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx}\left(x\log(x+y) + \frac{xy}{x+y}\right) = -\left(y\log(x+y) + \frac{xy}{x+y}\right)$$

$$\frac{dy}{dx}[(x^2 + xy)\log(x+y) + xy] = -[(y^2 + xy)\log(x+y) + xy]$$

$\frac{dy}{dx} = \frac{-y^2\log(x+y) - xy\log(x+y) - xy}{x[(x+y)\log(x+y)+y]} \times \frac{x}{x}$ (Multiply and divide by x)

$$\frac{dy}{dx} = \frac{-y\,xy\log(x+y) - x\,xy\log(x+y) - x^2y}{x^2[(x+y)\log(x+y) + y]}$$

$$\frac{dy}{dx} = \frac{-y\,(1) - x\,(1) - x^2y}{x^2[(x+y)\log(x+y) + y]}$$

$$\frac{dy}{dx} = \frac{-\left(x + y + x^2y\right)}{x^2\{y + (x+y)\log(x+y)\}}$$

Question: 18

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\tan x)}{dx} = sec^2 x$

Therefore ,

$$\frac{d(\tan(x+y))}{dx} + \frac{d(\tan(x-y))}{dx} = \frac{d(1)}{dx}$$

$$\sec^2(x+y)[1 + \frac{dy}{dx}] + \sec^2(x-y)[1 - \frac{dy}{dx}] = 0$$

$$\sec^2(x+y) + \sec^2(x+y)\frac{dy}{dx} + \sec^2(x-y) - \sec^2(x-y)\frac{dy}{dx} = 0$$

$$\sec^2(x+y) + \sec^2(x-y) = \frac{dy}{dx}[\sec^2(x-y) - \sec^2(x+y)]$$

$$\frac{dy}{dx} = \frac{\sec^2(x+y) + \sec^2(x-y)}{\sec^2(x+y) - \sec^2(x-y)}$$

Question: 19

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$, $\frac{d(\log x)}{dx} = \frac{1}{x}$

According to quotient rule of differentiation

$$\frac{d(^y/_x)}{dx} = \frac{\frac{xd(y)}{dx} - \frac{yd(x)}{dx}}{x^2} = \frac{x\frac{dy}{dx} - y}{x^2}$$

Therefore ,

$$\frac{d(\log\sqrt{x^2+y^2})}{dx} = \frac{d(\tan^{-1}\frac{y}{x})}{dx}$$

$$\frac{1}{\sqrt{x^2+y^2}} \times \frac{d(\sqrt{x^2+y^2})}{dx} = \frac{1}{1+(\frac{y}{x})^2} \times \frac{d(\frac{y}{x})}{dx}$$

$$\frac{1}{\sqrt{x^2+y^2}} \times \frac{1}{2\sqrt{x^2+y^2}} \times [2x + 2y\frac{d(y)}{dx}] = \frac{1}{1+(\frac{y}{x})^2} \times \frac{x\frac{dy}{dx} - y}{x^2}$$

$$\frac{1}{x^2+y^2} \times [x + y\frac{dy}{dx}] = \frac{x^2}{x^2+y^2} \times \frac{x\frac{dy}{dx} - y}{x^2}$$

$$x + y\frac{dy}{dx} = x\frac{dy}{dx} - y$$

$$\frac{dy}{dx}[x - y] = x + y$$

$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

Question: 20

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\sin x)}{dx} = \cos x$

According to chain rule of differentiation

$$\frac{d(\sin y)}{dx} = \frac{d(\sin y)}{dy} \times \frac{dy}{dx} = \cos y \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(y)}{dx} = \frac{d(x\sin y)}{dx}$$

$$\frac{dy}{dx} = x\frac{d(\sin y)}{dx} + \sin y$$

$$\frac{dy}{dx} = x\cos y\frac{dy}{dx} + \sin y$$

$$\frac{dy}{dx}[1 - x\cos y] = \sin y$$

$$\frac{dy}{dx} = \frac{\sin y}{1 - x\cos y}$$

Question: 21

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\tan x)}{dx} = sec^2 x$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xdy}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d(xy)}{dx} = \frac{d(\tan xy)}{dx}$$

$$x\frac{dy}{dx} + y = sec^2(xy) \times \frac{d(xy)}{dx}$$

$$x\frac{dy}{dx} + y = sec^2(xy) \times [x\frac{dy}{dx} + y]$$

$$\frac{dy}{dx}[x - xsec^2(xy)] = ysec^2(xy) - y$$

$$x\frac{dy}{dx}(1 - \sec^2 \text{xy}) = y(sec^2(xy) - 1)$$

$$\frac{dy}{dx} = \frac{-y(1 - sec^2(xy))}{x(1 - \sec^2 \text{xy})}$$

$$\frac{dy}{dx} = \frac{-\text{y}}{\text{x}}$$

Question: 22

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$,$\frac{d(log\ x)}{dx} = \frac{1}{x}$

According to product rule of differentiation

$$\frac{d(y \log x)}{dx} = \frac{\log x\, d(y)}{dx} + \frac{y\, d(\log x)}{dx} = \log x \times \frac{dy}{dx} + \frac{y}{x}$$

Therefore ,

$$\frac{d(y \times \log x)}{dx} = \frac{d(x - y)}{dx}$$

$$\log x \times \frac{d(y)}{dx} + \frac{y}{x} = 1 - \frac{d(y)}{dx}$$

$$\frac{dy}{dx}[\log x\ + 1] = 1 - \frac{y}{x}$$

$$\frac{dy}{dx}[(1 + \log x)^2] = 1 - \frac{y}{x}(1 + \log x)$$

(Multiply by 1+log x on both sides)

$$\frac{dy}{dx}[(1+\log x)^2] = 1+\log x - \frac{y}{x} - \frac{y}{x}\log x$$

$$\frac{dy}{dx}[(1+\log x)^2] = 1+\log x - \frac{y}{x} - \frac{(x-y)}{x}$$ (y log x = x - y)

$$\frac{dy}{dx}[(1+\log x)^2] = 1+\log x - \frac{y}{x} - 1 + \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$$

Question: 23

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\cos x)}{dx} = -\sin x$

According to chain rule of differentiation

$$\frac{d(\cos y)}{dx} = \frac{d(\cos y)}{dy} \times \frac{dy}{dx} = -\sin y \times \frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(x\cos{(y+a)})}{dx} = x\frac{d(\cos y + a)}{dx} + \cos(y+a)$$

Therefore ,

$$\frac{d(\cos y)}{dx} = \frac{d(x\cos(y+a))}{dx}$$

$$-\sin y\frac{dy}{dx} = x\frac{d(\cos(y+a))}{dx} + \cos(y+a)$$

$$-\sin y\frac{dy}{dx} = x(-\sin(y+a)\frac{dy}{dx}) + \cos(y+a)$$

$$\frac{dy}{dx}[-\sin y + x\sin{(y+a)}] = \cos(y+a)$$

$$\frac{dy}{dx} = \frac{\cos(y+a)}{x\sin(y+a) - \sin y}$$

$$\frac{dy}{dx} = \frac{\cos^2(y+a)}{x\cos(y+a)\sin(y+a) - \cos(y+a)\sin y}$$

(Multiply and divide by cos (y+a))

$$\frac{dy}{dx} = \frac{\cos^2(y+a)}{\sin(y+a)\cos y - \cos(y+a)\sin y}$$ (Since cos y = x cos (y + a))

$$\frac{dy}{dx} = \frac{\cos^2(y+a)}{\sin(y+a-y)}$$ (Formula sin(a-b)=sin a cos b – cos a sin b)

$$\frac{dy}{dx} = \frac{\cos^2(y+a)}{\sin a}$$

Question: 24

Find , when:
<

Solution:

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\cos^{-1}x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$

According to the chain rule of differentiation

$$\frac{d(y^2)}{dx} = \frac{d(y^2)}{dy} \times \frac{dy}{dx} = 2y \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(\cos^{-1}\frac{x^2-y^2}{x^2+y^2})}{dx} = \frac{d(\tan^{-1}a)}{dx}$$

$$-\frac{1}{\sqrt{1-(\frac{x^2-y^2}{x^2+y^2})^2}} \times \frac{d(\frac{x^2-y^2}{x^2+y^2})}{dx} = 0$$

$$\frac{d(\frac{x^2-y^2}{x^2+y^2})}{dx} = 0$$

$$\frac{x^2+y^2\left[\frac{d(x^2-y^2)}{dx}\right]-(x^2-y^2)[\frac{d(x^2+y^2)}{dx}]}{(x^2+y^2)^2} = 0$$

$$x^2+y^2\left[\frac{d(x^2-y^2)}{dx}\right]-(x^2-y^2)[\frac{d(x^2+y^2)}{dx}] = 0$$

$$(x^2+y^2)\left(2x-2y\frac{dy}{dx}\right)-(x^2-y^2)(2x+2y\frac{dy}{dx}) = 0$$

$$(x^2+y^2)\left(x-y\frac{dy}{dx}\right) = (x^2-y^2)(x+y\frac{dy}{dx})$$

$$\frac{dy}{dx}[-x^2y-y^3-x^2y+y^3] = x^3-xy^2-x^3-xy^2$$

$$\frac{dy}{dx}[-2x^2y] = -2xy^2$$

$$\frac{dy}{dx} = \frac{-2xy^2}{-2yx^2}$$

$$\frac{dy}{dx} = \frac{y}{x}$$

Exercise : 10F

Question: 1

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = \frac{lnx}{x}\ \{\ln(x^m) = m(lnx)\}$$

Now differentiating both sides by x we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{x\left(\frac{1}{x}\right) - lnx(1)}{x^2}$$

$$\frac{dy}{dx} = \frac{1 - lnx}{x^2} \times y \left\{ divide\ rule\ \frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \right\}$$

$$\frac{dy}{dx} = \frac{1 - lnx}{x^2} \times x^{\frac{1}{x}} \{y = x^{\frac{1}{x}}\}$$

Question: 2

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = \sqrt{x}\, lnx$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \sqrt{x}\left(\frac{1}{x}\right) + lnx\left(\frac{1}{2\sqrt{x}}\right) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x}}\left(1 + \frac{lnx}{2}\right) \times y$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x}} \times \left(1 + \frac{lnx}{2}\right) \times \left(x^{\sqrt{x}}\right)$$

Question: 3

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = x \ln(lnx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = x\left(\frac{1}{lnx} \times \frac{1}{x}\right) + \ln(lnx) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{1}{lnx} + \ln(lnx)\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{1}{lnx} + \ln(lnx\ )\right) \times (lnx)^x$$

Question: 4

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = sinx\ lnx$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = sinx \times \frac{1}{x} + \ lnx \times cosx \ \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(sinx \times \frac{1}{x} + lnx \times cosx\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{sinx}{x} + cosx(lnx)\right) \times x^{sinx}$$

Question: 5

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = \cos^{-1} x \, lnx$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \cos^{-1} x \times \left(\frac{1}{x}\right) + lnx \times \left(-\frac{1}{\sqrt{1-x^2}}\right) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \cos^{-1} x \times \left(\frac{1}{x}\right) + lnx \times \left(-\frac{1}{\sqrt{1-x^2}}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{\cos^{-1} x}{x} - \frac{lnx}{\sqrt{1-x^2}}\right) \times x^{(\cos^{-1} x)}$$

Question: 6

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = \left(\frac{1}{x}\right) \ln(tanx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(\frac{1}{x}\right) \times \left(\frac{1}{tanx} \times \sec^2 x\right) + \ln(tanx) \times \left(-\frac{1}{x^2}\right) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{\sec^2 x}{x \times tanx} - \frac{\ln(tanx)}{x^2}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{\sec^2 x}{x \times tanx} - \frac{\ln(tanx)}{x^2}\right) \times tanx^{\frac{1}{x}}$$

Question: 7

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (cosx)\ln(sinx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (cosx) \times \left(\frac{1}{sinx} \times cosx\right)$$
$$+ \ln(sinx) \times (-sinx) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{\cos^2 x}{sinx} - sinx(\ln(sinx))\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{\cos^2 x}{sinx} - sinx(\ln(sinx))\right) \times sinx^{cosx}$$

Question: 8

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$lny = (sinx)\ln(lnx\ )$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (sinx) \times \left(\frac{1}{lnx} \times \frac{1}{x}\right)$$
$$+ \ln(lnx) \times (cosx) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{\sin x}{x \times lnx} - cosx(\ln(lnx))\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{\sin x}{x \times lnx} - cosx(\ln(lnx))\right) \times lnx^{sinx}$$

Question: 9

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$lny = (lnx)\ln(cosx\ )$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (lnx) \times \left(\frac{1}{cosx} \times (-sinx)\right)$$
$$+ \ln(cosx) \times \left(\frac{1}{x}\right) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(-\frac{sin\ x \times lnx}{cosx} + \frac{(\ln cosx)}{x}\right) \times y$$

$$\frac{dy}{dx} = \left(-\frac{sin\ x \times lnx}{cosx} + \frac{(\ln cosx)}{x}\right) \times cosx^{lnx}$$

Question: 10

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$lny = (sinx)\ln(tanx\ )$

Now differentiating both sides by x, we get,

$$\frac{1}{y}\times\frac{dy}{dx}=(sinx)\times\left(\frac{1}{tanx}\times(\sec^2 x)\right)$$
$$+\ln(tanx)\times(cosx)\left\{product\ rule,\frac{d(uv)}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx}=\left(\frac{sin\,x\times\sec^2 x}{tanx}+\ln(tanx)\,cosx\right)\times y$$

$$\frac{dy}{dx}=\left(\frac{sin\,x\times\sec^2 x}{tanx}+\ln(tanx)\,cosx\right)\times tanx^{sinx}$$

Question: 11

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$lny=(cosx)\ln(cosx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y}\times\frac{dy}{dx}=(cosx)\times\left(\frac{1}{cosx}\times(-sinx)\right)$$
$$+\ln(cosx)\times(-sinx)\left\{product\ rule,\frac{d(uv)}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx}=(-sinx-\ln(cosx)\,sinx)\times y$$

$$\frac{dy}{dx}=(-sinx-\ln(cosx)\,sinx)\times cosx^{cosx}$$

Question: 12

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$lny=(\cot x)\ln(\tan x)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y}\times\frac{dy}{dx}=(cotx)\times\left(\frac{1}{tanx}\times(-\sec^2 x)\right)$$
$$+\ln(tanx)\times(-cosec^2x)\left\{product\ rule,\frac{d(uv)}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx}=(-cosec^2x-\ln(tanx)cosec^2x)\times y$$

$$\frac{dy}{dx}=-cosec^2x\times(1+\ln(cosx))\times tanx^{\cot x}$$

Question: 13

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$lny=(sin2x)\ln(x)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (sin2x) \times \left(\frac{1}{x}\right) + \ln(x) \times (cos2x \times 2) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{sin2x}{x} + 2\cos 2x \times \ln x\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{sin2x}{x} + 2\cos 2x \times \ln x\right) \times x^{\sin 2x}$$

Question: 14

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (x)\ln(\sin^{-1} x)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (x) \times \left(\frac{1}{\sin^{-1} x} \times \frac{1}{\sqrt{1-x^2}}\right) + \ln(\sin^{-1} x) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{x}{\sin^{-1} x \times \sqrt{1-x^2}} \times \ln \sin^{-1} x\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{x}{\sin^{-1} x \times \sqrt{1-x^2}} \times \ln \sin^{-1} x\right) \times \sin^{-1} x^x$$

Question: 15

Find

Solution:

Here, the argument of the sinusoidal function has exponent as x itself.

For that, we will consider $x^x = u$ for simplicity.

$$y = \sin u$$

Differentiating both the sides,

$\frac{dy}{dx} = \cos u \times \frac{du}{dx}$. ....(1)

Now we have to find $\frac{du}{dx}$, where $u = x^x$

take log both the sides

$$\ln u = x \ln x$$

Now differentiating both sides by x, we get,

$$\frac{1}{u} \times \frac{du}{dx} = x\left(\frac{1}{x}\right) + \ln x$$

$$\frac{du}{dx} = (1 + \ln x) \times u$$

$$\frac{du}{dx} = (1 + \ln x) \times x^x$$

Substituting the value in equation 1,

$$\frac{dy}{dx} = cosx\ (1 + \ln x\ ) \times x^{x}$$

Question: 16

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (2x - 3)\ln(3x - 5)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (2x - 3) \times \left(\frac{1}{3x - 5} \times 3\right) + \ln(3x - 5) \times 2\ \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{2x - 3}{3x - 5} \times 3 + \ln(3x - 5) \times 2\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{2x - 3}{3x - 5} \times 3 + 2 \times \ln 3x - 5\ \right) \times (3x - 5)^{2x-3}$$

Question: 17

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = 3\ln(x + 1) +\ 4\ln(x + 2) + 5\ln(x + 3)\ \{\ln(mn) = \ln n + \ln m\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{3}{x + 1} + \frac{4}{x + 2} + \frac{5}{x + 3}$$

$$\frac{dy}{dx} = \left(\frac{3}{x + 1} + \frac{4}{x + 2} + \frac{5}{x + 3}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{3}{x + 1} + \frac{4}{x + 2} + \frac{5}{x + 3}\right) \times (x + 1)^{3}(x + 2)^{4}(x + 3)^{5}$$

Question: 18

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = \frac{1}{2}(\ln(x - 1) + \ln(x - 2) - \ln(x - 3) - \ln(x - 4) - \ln(x - 5))$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x - 1} + \frac{1}{x - 2} - \frac{1}{x - 3} - \frac{1}{x - 4} - \frac{1}{x - 5}\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x - 1} + \frac{1}{x - 2} - \frac{1}{x - 3} - \frac{1}{x - 4} - \frac{1}{x - 5}\right) \times y$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right) \times \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

Question: 19

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 3\ln(2-x) + 5\ln(3+2x)$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = 3\left(\frac{-1}{2-x}\right) + 5\left(\frac{1}{3+2x} \times 2\right)$$

$$\frac{dy}{dx} = \left(\frac{3}{x-2} + \frac{10}{3+2x}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{3}{x-2} + \frac{10}{3+2x}\right) \times (2-x)^3(3+2x)^5$$

Question: 20

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = \ln(cos x) + \ln(\cos 2x) + \ln\cos 3x$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{1}{\cos x} \times (-\sin x) + \frac{1}{\cos 2x} \times (-2\sin 2x) + \frac{1}{\cos 3x}(-3\sin 3x)$$

$$\frac{dy}{dx} = \left(\frac{-\sin x}{\cos x} - \frac{2\sin 2x}{\cos 2x} - \frac{3\sin 3x}{\cos 3x}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{-\sin x}{\cos x} - \frac{2\sin 2x}{\cos 2x} - \frac{3\sin 3x}{\cos 3x}\right) \times \cos x \cos 2x \cos 3x$$

$$\frac{dy}{dx} = (-\tan x - 2\tan 2x - 3\tan 3x) \times \cos x \cos 2x \cos 3x$$

Question: 21

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 5\ln(x) + \frac{1}{2}\ln(x+4) - 2\ln(2x+3)$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{5}{x} + \frac{1}{2(x+4)} - \frac{4}{2x+3}$$

$$\frac{dy}{dx} = \left(\frac{5}{x} + \frac{1}{2(x+4)} - \frac{4}{2x+3}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{5}{x} + \frac{1}{2(x+4)} - \frac{4}{2x+3}\right) \times \frac{x^5\sqrt{x+4}}{(2x+3)^2}$$

Question: 22

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 2\ln(x+1) + \frac{1}{2}\ln(x-1) - 3\ln(x+4) - x$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{2}{x+1} + \frac{1}{2(x-1)} - \frac{3}{x+4} - 1$$

$$\frac{dy}{dx} = \left(\frac{2}{x+1} + \frac{1}{2(x-1)} - \frac{3}{x+4} - 1\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{2}{x+1} + \frac{1}{2(x-1)} - \frac{3}{x+4} - 1\right) \times \frac{(x+1)^2\sqrt{x-1}}{(x+4)^3.e^x}$$

Question: 23

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 2\ln(3x+5) + \frac{1}{2}\ln(x) - \frac{1}{2}\ln(x+1)$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{2}{3x+5} + \frac{1}{2x} - \frac{1}{2(x+1)}$$

$$\frac{dy}{dx} = \left(\frac{2}{3x+5} + \frac{1}{2x} - \frac{1}{2(x+1)}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{2}{3x+5} + \frac{1}{2x} - \frac{1}{2(x+1)}\right) \times \frac{(3x+5)^2\sqrt{x}}{\sqrt{x+1}}$$

Question: 24

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 2\ln(x) + \frac{1}{2}\ln(x+1) - \frac{3}{2}\ln(x^2+1)$$

$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{2}{x} + \frac{1}{2(x+1)} - \frac{3}{2(x^2+1)} \times 2x$$

$$\frac{dy}{dx} = \left(\frac{2}{x} + \frac{1}{2(x+1)} - \frac{6x}{2(x^2+1)}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{2}{x} + \frac{1}{2(x+1)} - \frac{6x}{2(x^2+1)}\right) \times \frac{(x)^2\sqrt{x+1}}{(1+x^2)^{\frac{3}{2}}}$$

Question: 25

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = \frac{1}{2}(\ln(x-2) + \ln(2x-3) + \ln(3x-4))$$

$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x-2} + \frac{2}{2x-3} + \frac{3}{3x-4}\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x-2} + \frac{2}{2x-3} + \frac{3}{3x-4}\right) \times y$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x-2} + \frac{2}{2x-3} + \frac{3}{3x-4}\right) \times \sqrt{(x-2)(2x-3)(3x-4)}$$

Question: 26

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = (\ln(\sin 2x) + \ln(\sin 3x) + \ln(\sin 4x))$$

$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(\frac{2}{\sin 2x} + \frac{3}{\sin 3x} + \frac{4}{\sin 4x}\right)$$

$$\frac{dy}{dx} = \left(\frac{2}{\sin 2x} + \frac{3}{\sin 3x} + \frac{4}{\sin 4x}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{2}{\sin 2x} + \frac{3}{\sin 3x} + \frac{4}{\sin 4x}\right) \times \sin 2x \sin 3x \sin 4x$$

Question: 27

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$\ln y = 3\ln x + \ln \sin x - x$

$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(\frac{1}{\sin x} \times \cos x + \frac{3}{x} - 1\right)$$

$$\frac{dy}{dx} = \left(\frac{\cos x}{\sin x} + \frac{3}{x} - 1\right) \times y$$

$$\frac{dy}{dx} = \left(\cot x + \frac{3}{x} - 1\right) \times \frac{x^3 \sin x}{e^x}$$

Question: 28

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$\ln y = x + \ln(\ln x) - 2\ln x$

$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(1 + \frac{1}{x \ln x} - \frac{2}{x}\right)$$

$$\frac{dy}{dx} = \left(1 + \frac{1}{x \ln x} - \frac{2}{x}\right) \times y$$

$$\frac{dy}{dx} = \left(1 + \frac{1}{x \ln x} - \frac{2}{x}\right) \times \frac{e^x \log x}{x^2}$$

Question: 29

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$\ln y = \ln \cos^{-1} x + \ln(x) - \frac{1}{2}\ln(1 - x^2)$

$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(-\frac{1}{\sqrt{1 - x^2}} + \frac{1}{x} + \frac{2x}{2(1 - x^2)}\right)$$

$$\frac{dy}{dx} = \left(-\frac{1}{\sqrt{1 - x^2}} + \frac{1}{x} + \frac{2x}{2(1 - x^2)}\right) \times y$$

$$\frac{dy}{dx} = \left(-\frac{1}{\sqrt{1 - x^2}} + \frac{1}{x} + \frac{x}{(1 - x^2)}\right) \times \frac{x cos^{-1} x}{\sqrt{1 - x^2}}$$

Question: 30

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$\ln y = \ln(1+x) + \ln(1+x^2) + \ln(1+x^4) + \ln(1+x^6)$

$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{6x^5}{1+x^6}\right)$$

$$\frac{dy}{dx} = \left(\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{6x^5}{1+x^6}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{6x^5}{1+x^6}\right) \times (1+x)(1+x^2)(1+x^4)(1+x^6)$$

Question: 31

Find

Solution:

simply taking log both sides would not help more.

For that let us assume $u = x^x$ and $v = 2^{\sin x}$

$$\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$$

$u = x^x$

Take log both sides

$\ln u = x \ln x$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = x\left(\frac{1}{x}\right) + \ln x$$

$$\frac{du}{dx} = (1 + \ln x) \times u$$

$\frac{du}{dx} = (1 + \ln x) \times x^x$ ....(1)

$v = 2^{\sin x}$

Take log both sides,

$\ln v = \sin x . \ln 2$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \sin x(0) + \ln 2 . \cos x$$

$$\frac{dv}{dx} = \ln 2 . \cos x \times v$$

$\frac{dv}{dx} = \ln 2 . \cos x \times 2^{\sin x}$ ......(2)

$$\frac{dy}{dx} = (1 + \ln x) \times x^x - \ln 2 . \cos x \times 2^{\sin x}$$

Question: 32

Find

Solution:

simply taking log both sides would not help more.

For that let us assume $u = (\ln x)^x$ and $v = x^{\ln x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$u = (\ln x\,)^x$

Take log both sides

$\ln u = x \ln(\ln x)$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = x\left(\frac{1}{x} \times \frac{1}{\ln x}\right) + \ln(\ln x)$$

$$\frac{du}{dx} = \left(\frac{1}{\ln x} + \ln(\ln x\right) \times u$$

$\frac{du}{dx} = \left(\frac{1}{\ln x} + \ln(\ln x\right) \times (\ln x)^x$ ....(1)

$v = x^{\ln x}$

Take log both sides,

$\ln v = \ln x . \ln x$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = 2. \ln x \times \frac{1}{x}$$

$$\frac{dv}{dx} = \frac{2. \ln x}{x} \times v$$

$\frac{dv}{dx} = \frac{2.\ln x}{x} \times x^{\ln x}$ ......(2)

$$\frac{dy}{dx} = \left(\frac{1}{\ln x} + \ln(\ln x\right) \times (\ln x)^x + \frac{2. \ln x}{x} \times x^{\ln x}$$

Question: 33

Find

Solution:

simply taking log both sides would not help more.

For that let us assume $u = x^{\sin x}$ and $v = \sin x^{\cos x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$u = (x)^{\sin x}$

Take log both sides

$\ln u = \sin x \ln(x)$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \sin x \left(\frac{1}{x}\right) + \ln(x) \times \cos x$$

$$\frac{du}{dx} = \left(\frac{\sin x}{x} + \ln(x) \times \cos x\right) \times u$$

$\frac{du}{dx} = \left(\frac{\sin x}{x} + \ln(x) \times \cos x\right) \times (x^{\sin x} + (\sin x)^{\cos x})$ ....(1)

$v = (\sin x)^{\cos x}$

Take log both sides,

$\ln v = \cos\ x \ln(\sin x)$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \cos x(\frac{1}{\sin x} \times \cos x)$$

$$\frac{dv}{dx} = \frac{\cos^2 x}{\sin x} \times v$$

$\frac{dv}{dx} = \frac{\cos^2 x}{\sin x} \times (\sin x)^{\cos x}$ ......(2)

$$\frac{dy}{dx} = \frac{\cos^2 x}{\sin x} \times (\sin x)^{\cos x} + \left(\frac{\sin x}{x} + \ln(x) \times \cos x\right) \times (x^{\sin x} + (\sin x)^{\cos x})$$

Question: 34

Find

Solution:

simply taking log both sides would not help more.

For that let us assume $u = (x.\cos x)^x$ and $v = (x.\sin x)^{\frac{1}{x}}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$u = (x.\cos x)^x$

Take log both sides

$\ln u = x\,(\ln(x) + \ln\cos x)$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = x\left(\frac{1}{x} - \frac{\sin x}{\cos x}\right) + \ (\ln(x) + \ln\cos x)$$

$$\frac{du}{dx} = \left(x\left(\frac{1}{x} - \frac{\sin x}{\cos x}\right) + \ (\ln(x) + \ln\cos x)\ \right) \times u$$

$\frac{du}{dx} = \left(x\left(\frac{1}{x} - \frac{\sin x}{\cos x}\right) + \ (\ln(x) + \ln\cos x)\ \right) \times ((x.\cos x)^x)$ ....(1)

$v = (x.\sin x)^{\frac{1}{x}}$

Take log both sides,

$$\ln v = \frac{1}{x} \times (\ln x + \ln\sin x)$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \frac{1}{x}\left(\frac{1}{x} + \frac{\cos x}{\sin x}\right) - \frac{1}{x^2} \times (\ln x + \ln\sin x)$$

$$\frac{dv}{dx} = \left(\frac{1}{x}\left(\frac{1}{x} + \frac{\cos x}{\sin x}\right) - \frac{1}{x^2} \times (\ln x + \ln\sin x)\right) \times v$$

$$\frac{dv}{dx} = \left(\frac{1}{x}\left(\frac{1}{x} + \frac{\cos x}{\sin x}\right) - \frac{1}{x^2} \times (\ln x + \ln \sin x)\right) \times (x.\sin x)^{\frac{1}{x}} \ldots\ldots(2)$$

$$\frac{dy}{dx} = \left(\frac{1}{x}\left(\frac{1}{x} + \frac{\cos x}{\sin x}\right) - \frac{1}{x^2} \times (\ln x + \ln \sin x)\right) \times (x.\sin x)^{\frac{1}{x}} + \left(x\left(\frac{1}{x} - \frac{\sin x}{\cos x}\right) + (\ln(x) + \ln \cos x)\right) \times ((x.\cos x)^x)$$

Question: 35

Find

Solution:

simply taking log both sides would not help more.

For that let us assume $u = (\sin x)^x$ and $v = \sin^{-1}\sqrt{x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$u = (\sin x)^x$

Take log both sides

$\ln u = x . \ln \sin x$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = x\left(\frac{\cos x}{\sin x}\right) + \ln \sin x$$

$$\frac{du}{dx} = \left(x\left(\frac{\cos x}{\sin x}\right) + \ln \sin x\right) \times u$$

$$\frac{du}{dx} = \left(x\left(\frac{\cos x}{\sin x}\right) + \ln \sin x\right) \times ((\sin x)^x) \ldots\ldots\ldots.(1)$$

for v we do not have to take log just simply differentiate it,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \times \frac{1}{2\sqrt{x}} \ldots\ldots..(2)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \times \frac{1}{2\sqrt{x}} + \left(x\left(\frac{\cos x}{\sin x}\right) + \ln \sin x\right) \times ((\sin x)^x)$$

Question: 36

Find

Solution:

simply taking log both sides would not help more.

For that let us assume $u = (x)^{x.\cos x}$ and $v = \frac{x^2+1}{x^2-1}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$u = (x)^{x.\cos x}$

Take log both sides

$\ln u = x . \cos x . \ln x$

Here there are three terms to differentiate for this; we can take two term as one and then apply product rule, I am taking $x. \ln x$ as a single term

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \cos x\left(x\left(\frac{1}{x}\right) + \ln x\right) + x.\ln x(-\sin x)$$

$$\frac{du}{dx} = (\cos x(1 + \ln x) - x.\ln x.\sin x) \times u$$

$$\frac{du}{dx} = (\cos x(1 + \ln x) - x.\ln x.\sin x) \times (x.\cos x.\ln x) \text{..........(1)}$$

for v we do not have to take log just simply differentiate it,

$$\frac{dv}{dx} = \frac{(x^2 - 1)(2x) - (x^2 + 1)(2x)}{(x^2 - 1)^2}$$

$$\frac{dy}{dx} = \frac{2x(-2)}{(x^2 - 1)^2} \text{........(2)}$$

$$\frac{dy}{dx} = (\cos x(1 + \ln x) - x.\ln x.\sin x) \times (x.\cos x.\ln x) + \frac{2x(-2)}{(x^2 - 1)^2}$$

Question: 37

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = x + 3.\ln \sin x + 4 \ln \cos x$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(\frac{3.\cos x}{\sin x} - \frac{4 \sin x}{\cos x} + 1\right)$$

$$\frac{dy}{dx} = \left(\frac{3.\cos x}{\sin x} - \frac{4 \sin x}{\cos x} + 1\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{3.\cos x}{\sin x} - \frac{4 \sin x}{\cos x} + 1\right) \times e^x.\sin^3 x.\cos^4 x$$

Question: 38

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = x.\ln 2 + 3x + \ln \sin 4x$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \ln 2 + 3 + \frac{\cos 4x}{\sin 4x} \times 4$$

$$\frac{dy}{dx} = \left(\ln 2 + 3 + \frac{\cos 4x}{\sin 4x} \times 4\right) \times y$$

$$\frac{dy}{dx} = \left(\ln 2 + 3 + \frac{\cos 4x}{\sin 4x} \times 4\right) \times e^{3x}.\sin 4x.2^x$$

Question: 39

Find

Solution:

Here we need to take log both the sides to get that differentiation simple.

$\ln y = x.\ln x + 2x + 5$

$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = 1 + \ln x + 2$$

$$\frac{dy}{dx} = (\ln x + 3) \times y$$

$$\frac{dy}{dx} = (\ln x + 3) \times x^x.e^{2x+5}$$

Question: 40

Find

Solution:

Here, we need to take log both the sides to get that differentiation simple.

$\ln y = 5.\ln(2x + 5) + 7.\ln(3x - 5) + 3.\ln(5x - 1)$

$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{5 \times 2}{2x + 5} + \frac{7 \times 3}{3x - 5} + \frac{3 \times 5}{5x - 1}$$

$$\frac{dy}{dx} = \left(\frac{10}{2x + 5} + \frac{21}{3x - 5} + \frac{15}{5x - 1}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{10}{2x + 5} + \frac{21}{3x - 5} + \frac{15}{5x - 1}\right) \times (2x + 5)^5(3x - 5)^7(5x - 1)^3$$

Question: 41

Find

Solution:

. So the equation given is implicit, we will just take log both sides

$y.\ln(\cos x) = x.\ln(\cos y)$

Now differentiate it with respect to x and consider $\frac{dy}{dx} = y'$

$$y\left(\frac{-\sin x}{\cos x}\right) + \ln\cos x.y' = x\left(\frac{-\sin y}{\cos y} \times y'\right) + \ln\cos y$$

Taking y' one side, we get

$y'(\ln\cos x + x.\tan x) = \ln\cos y + y.\tan x$

$$y' = \frac{\ln\cos y + y.\tan x}{\ln\cos x + x.\tan x}$$

Question: 42

Find

Solution:

. So the equation given is implicit, we will just take log both sides

$$y.\ln(\tan x) = x.\ln(\tan y)$$

Now differentiate it with respect to x and consider $\frac{dy}{dx} = y'$

$$y\left(\frac{\sec^2 x}{\tan x}\right) + \ln\tan x . y' = x\left(\frac{\sec^2 y}{\tan y} \times y'\right) + \ln\tan y$$

Taking y’ one side, we get

$$y'\left(\ln\tan x + \frac{x}{\sin y . \cos y}\right) = \ln\tan y + \frac{y}{\sin x . \cos x}$$

$$y' = \frac{\sin 2\mathrm{x} . \ln\tan y + 2y}{\sin 2\mathrm{y} . \ln\tan x + 2x}$$

Question: 43

Find

Solution:

we can write this equation as,

$$y = e^{x ln(\ln x)} + e^{\ln x . \ln x}$$

Differentiate

$$y' = (\ln x)^x \left(x\left(\frac{1}{\ln x} \times \frac{1}{x}\right) + \ln(ln\, x)\right) + x^{\ln x}\left(2 . \frac{\ln x}{x}\right)$$

$$y' = (\ln x)^x \left(\frac{1}{\ln x} + \ln(ln\, x)\right) + x^{\ln x}\left(\frac{2\ln x}{x}\right)$$

Question: 44

If <

Solution:

differentiate the given y to get the result,

$$\frac{dy}{dx} = \frac{\sqrt{1-x^2}\left(\frac{1}{\sqrt{1-x^2}}\right) - \sin^{-1} x\left(\frac{-x}{\sqrt{1-x^2}}\right)}{\left(\sqrt{1-x^2}\right)^2}$$

$$\frac{dy}{dx} = \frac{1 + \frac{x . \sin^{-1} x}{\sqrt{1-x^2}}}{1-x^2}$$

$$\frac{dy}{dx}(1-x^2) = 1 + xy$$

Question: 45

If <

Solution:

differentiate the given y to get the result,

$$\frac{dy}{dx} = \frac{1 + \frac{dy}{dx}}{2\sqrt{x+y}}$$

$$let\,, \frac{dy}{dx} = y'$$

$y' = \frac{1+y\prime}{2\sqrt{x+y}}$ {taking y’ one side}

$$y'\left(2\sqrt{x+y}-1\right) = 1$$

$$\frac{dy}{dx} = \frac{1}{2y-1}$$

Question: 46

If <

Solution:

taking log both sides,

$$a\ln x + b\ln y = (a+b).\ln(x+y)$$

differentiating both sides,

$$\frac{a}{x} + \frac{b}{y} \times y' = \frac{a+b}{x+y} \times (1+y')$$

Take y’ one side,

$$y'\left(\frac{b}{y} - \frac{a+b}{x+y}\right) = \frac{a+b}{x+y} - \frac{a}{x}$$

$$y' = \frac{ax+bx-(ax+ay)}{x.(x+y)} \times \frac{y.(x+y)}{bx+by-(ay+by)}$$

$$y' = \frac{bx-ay}{x} \times \frac{y}{bx-ay}$$

$$y' = \frac{y}{x}$$

Question: 47

If <

Solution:

differentiate both sides,

$$x^x(1+\ln x) + y^x\left(\frac{x}{y} \times y' + \ln y\right) = 0$$

Taking y’ one side,

$$y' = \left(\frac{x^x(1+\ln x)}{y^x} - \ln y\right) \times \frac{y}{x}$$

$$y' = \frac{x^x(1+\ln x) - y^x.\ln y}{x.y^{x-1}}$$

Question: 48

If <

Solution:

differentiate both sides,

$$y' = e^{\sin x}(\cos x) + (\tan x)^x\left(x\left(\frac{\sec^2 x}{\tan x}\right) + \ln\tan x\right)$$

$$y' = e^{\sin x}(\cos x) + (\tan x)^x(2x.cosec2x + \ln\tan x)$$

Question: 49

If <

Solution:

differentiate both sides,

$$y' = \frac{1}{x+\sqrt{1+x^2}} \times \left(1+\frac{x}{\sqrt{1+x^2}}\right)$$

$$y' = \frac{1}{x+\sqrt{1+x^2}} \times \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}$$

$$y' = \frac{1}{\sqrt{1+x^2}}$$

Question: 50

If <

Solution:

differentiate both sides,

$$y' = \frac{1}{\sin\left(\sqrt{1+x^2}\right)} \times \cos\left(\sqrt{1+x^2}\right) \times \frac{x}{\sqrt{1+x^2}}$$

$$y' = \frac{\left(\cot\left(\sqrt{1+x^2}\right).x\right)}{\sqrt{1+x^2}}$$

Question: 51

If <

Solution:

differentiate both sides,

$$y' = \sqrt{\frac{1+\cos x}{1-\cos x}} \times \frac{(1+\cos x)(\sin x)-(1-\cos x)(-\sin x)}{(1+\cos x)^2}$$

$$y' = \sqrt{\frac{1+\cos x}{1-\cos x}} \times \frac{(\sin x+\sin x.\cos x)+(\sin x-\sin x.\cos x)}{(1+\cos x)^2}$$

$$y' = \sqrt{\frac{1+\cos x}{1-\cos x}} \times \frac{2\sin x}{(1+\cos x)^2}$$

$$y' = \sqrt{\frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}}} \times \frac{4\sin\frac{x}{2}.\cos\frac{x}{2}}{(1+\cos x)^2}$$

$$y' = 4\cos\frac{x}{2} \times \frac{\cos\frac{x}{2}}{4\cos^4\frac{x}{2}}$$

$$y' = \frac{1}{\cos\frac{x}{2}}$$

$$y' = \sec^2\frac{x}{2}$$

Question: 52

If

Solution:

differentiate both sides,

$$y' = \frac{1}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)} \times \sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right) \times \frac{1}{2}$$

$$y' = \frac{1}{2 \times \sin\left(\frac{\pi}{4}+\frac{x}{2}\right).\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}$$

$$y' = \frac{1}{\sin\left(\frac{\pi}{2}+x\right)}$$

$$y' = \sec x$$

Question: 53

If <

Solution:

differentiate both sides,

$$y' = \frac{1}{2} \times \sqrt{\frac{1+\sin 2x}{1-\sin 2x}} \times \frac{(1+\sin 2x)(-2\cos 2x)-(1-\sin 2x)(2\cos 2x)}{(1+\sin 2x)^2}$$

$$y' = \frac{1}{2} \times \sqrt{\frac{1}{1-\sin^2 2x}} \times \frac{2\cos 2x(-1-\sin 2x - 1 + \sin 2x)}{1+\sin 2x}$$

$$y' = \frac{1}{2} \times \frac{-4}{1+\sin 2x}$$

$$y' = \frac{-2}{(\cos x + \sin x)^2}$$

$$y' = \frac{-1}{\left(\frac{\cos x}{\sqrt{2}} + \frac{\sin x}{\sqrt{2}}\right)^2}$$

$$y' = \frac{-1}{\cos^2\left(\frac{\pi}{4}+x\right)}$$

$$\frac{dy}{dx} + \sec^2\left(\frac{\pi}{4}+x\right) = 0$$

Question: 54

If <

Solution:

differentiate both sides,

$$y' = \sqrt{\frac{1-e^{2x}}{1+\cos^2 x}} \times \frac{(1-e^{2x})(-2\cos x.\sin x)-(1+\cos^2 x)(-2e^{2x})}{(1-e^{2x})^2} \times \frac{1}{2} \times \sqrt{\frac{1-e^{2x}}{1+\cos^2 x}}$$

$$y' = \sqrt{\frac{1-e^{2x}}{\cos 2x}} \times \frac{(e^{2x}-1)(\sin 2x)+2e^{2x}(\cos 2x)}{(1-e^{2x})^2} \times \frac{1}{2} \times \sqrt{\frac{1-e^{2x}}{\cos 2x}}$$

$$y' = \frac{(e^{2x} - 1)\tan 2x + 2e^{2x}}{2.(1 - e^{2x})}$$

$$y' = \frac{e^{2x}}{(1 - e^{2x})} - \frac{\sin x.\cos x}{(1 + \cos^2 x)}$$

Question: 55

If

Solution:

simply taking log both sides would not help more.

For that let us assume $u = (x)^{\cos x}$ and $v = (\sin x)^{\tan x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (x)^{\cos x}$$

Take log both sides

$$\ln u = \cos x.\ln x$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \cos x\left(\frac{1}{x}\right) + \ln x(-\sin x)$$

$$\frac{du}{dx} = \left(\frac{\cos x}{x} - \ln x.\sin x\right) \times u$$

$$\frac{du}{dx} = \left(\frac{\cos x}{x} - \ln x.\sin x\right) \times (x^{\cos x}) \text{ ....(1)}$$

$$v = (\sin x)^{\tan x}$$

Take log both sides,

$$\ln v = \tan x \times (\ln \sin x)$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \tan x\left(\frac{\cos x}{\sin x}\right) + \ln \sin x(\sec^2 x)$$

$$\frac{dv}{dx} = \left(\tan x\left(\frac{\cos x}{\sin x}\right) + \ln \sin x(\sec^2 x)\right) \times v$$

$$\frac{dv}{dx} = (1 + \ln \sin x(\sec^2 x)) \times (\sin x)^{\tan x} \text{ ......(2)}$$

$$\frac{dy}{dx} = (1 + \ln \sin x(\sec^2 x)) \times (\sin x)^{\tan x} + \left(\frac{\cos x}{x} - \ln x.\sin x\right) \times (x^{\cos x})$$

Question: 56

If <

Solution:

simply taking log both sides would not help more.

For that let us assume $u = (\sin x)^{\cos x}$ and $v = (\cos x)^{\sin x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (\sin x)^{\cos x}$$

Take log both sides

$\ln u = \cos x . \ln \sin x$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \cos x \left(\frac{\cos x}{\sin x}\right) + \ln(\sin x)(-\sin x)$$

$$\frac{du}{dx} = \left(\frac{\cos^2 x}{\sin x} - \ln \sin x . \sin x\right) \times u$$

$$\frac{du}{dx} = \left(\frac{\cos^2 x}{\sin x} - \ln \sin x . \sin x\right) \times (\sin x^{\cos x}) \text{ ....(1)}$$

$v = (\cos x)^{\sin x}$

Take log both sides,

$\ln v = \sin x \times (\ln \cos x)$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \sin x \left(\frac{-\sin x}{\cos x}\right) + \ln \cos x (\cos x)$$

$$\frac{dv}{dx} = \left(\sin x \left(\frac{-\sin x}{\cos x}\right) + \ln \cos x (\cos x)\right) \times v$$

$$\frac{dv}{dx} = \left(\sin x \left(\frac{-\sin x}{\cos x}\right) + \ln \cos x (\cos x)\right) \times (\cos x)^{\sin x} \text{ ......(2)}$$

$$\frac{dy}{dx} = \left(\sin x \left(\frac{-\sin x}{\cos x}\right) + \ln \cos x (\cos x)\right) \times (\cos x)^{\sin x} + \left(\frac{\cos^2 x}{\sin x} - \ln \sin x . \sin x\right) \times (\sin x^{\cos x})$$

Question: 57

If

Solution:

simply taking log both sides would not help more.

For that let us assume $u = (\tan x)^{\cot x}$ and $v = (\cot x)^{\tan x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$u = (\tan x)^{\cot x}$

Take log both sides

$\ln u = \cot x . \ln \tan x$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \cot x \left(\frac{\sec^2 x}{\tan x}\right) + \ln(\tan x)(-cosec^2 x)$$

$$\frac{du}{dx} = \left(\cot x \left(\frac{\sec^2 x}{\tan x}\right) - \ln(\tan x)(cosec^2 x)\right) \times u$$

$$\frac{du}{dx} = (cosec^2 x(1 - \ln(\tan x))) \times (\tan x)^{\cot x} \text{ ....(1)}$$

$v = (\cot x)^{\tan x}$

Take log both sides,

$\ln v = \tan x \times (\ln \cot x)$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \tan x \left(\frac{-cosec^2 x}{\cot x} \right) + \ln \cot x (\sec^2 x)$$

$$\frac{dv}{dx} = (\sec^2 x (\ln \cot x - 1)) \times v$$

$\frac{dv}{dx} = (\sec^2 x (\ln \cot x - 1)) \times (\cot x)^{\tan x}$ ......(2)

$$\frac{dy}{dx} = (\sec^2 x (\ln \cot x - 1)) \times (\cot x)^{\tan x} + (cosec^2 x (1 - \ln(\tan x))) \times (\tan x)^{\cot x}$$

Question: 58

If <

Solution:

simply taking log both sides would not help more.

For that let us assume $u = (x)^{\cos x}$ and $v = (\cos x)^x$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$u = (x)^{\cos x}$

Take log both sides

$\ln u = \cos x . \ln x$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \cos x \left(\frac{1}{x} \right) + \ln(x)(-\sin x)$$

$$\frac{du}{dx} = \left(\cos x \left(\frac{1}{x} \right) + \ln(x)(-\sin x) \right) \times u$$

$\frac{du}{dx} = \left(\cos x \left(\frac{1}{x} \right) + \ln(x)(-\sin x) \right) \times (x)^{\cos x}$ ....(1)

$v = (\cos x)^x$

Take log both sides,

$\ln v = x \times (\ln \cos x)$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = x \left(\frac{-\sin x}{\cos x} \right) + \ln \cos x . 1$$

$$\frac{dv}{dx} = \left(x \left(\frac{-\sin x}{\cos x} \right) + \ln \cos x . 1 \right) \times v$$

$\frac{dv}{dx} = \left(x \left(\frac{-\sin x}{\cos x} \right) + \ln \cos x . 1 \right) \times (\cos x)^x$ ......(2)

$$\frac{dy}{dx} = \left(x \left(\frac{-\sin x}{\cos x} \right) + \ln \cos x . 1 \right) \times (\cos x)^x + \left(\cos x \left(\frac{1}{x} \right) + \ln(x)(-\sin x) \right) \times (x)^{\cos x}$$

Question: 59

If <

Solution:

simply taking log both sides would not help more.

For that let us assume $u = (x)^{\ln x}$ and $v = (\ln x)^{x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (x)^{\ln x}$$

Take log both sides

$$\ln u = \ln x . \ln x$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = 2 \ln x \left(\frac{1}{x}\right)$$

$$\frac{du}{dx} = \left(2 \ln x \left(\frac{1}{x}\right)\right) \times u$$

$$\frac{du}{dx} = \left(2 \ln x \left(\frac{1}{x}\right)\right) \times (x)^{\ln x} \ \ldots\ldots\ldots.(1)$$

$$v = (\ln x)^{x}$$

Take log both sides,

$$\ln v = x \times (\ln x)$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \mathrm{x}\left(\frac{1}{x}\right) + \ln x$$

$$\frac{dv}{dx} = (1 + \ln x) \times v$$

$$\frac{dv}{dx} = (1 + \ln x) \times (\ln x)^{x} \ \ldots\ldots(2)$$

$$\frac{dy}{dx} = (1 + \ln x) \times (\ln x)^{x} + \left(2 \ln x \left(\frac{1}{x}\right)\right) \times (x)^{\ln x}$$

Question: 60

If <

Solution:

equality is not given but we may assume that it is equal to 0.

We can also write this equation as

$$y - e^{(x^2-3).\ln x} + e^{x^2.\ln(x-3)} = 0$$

Now differentiating it,

$$y' - x^{x^2-3}\left(\frac{x^2-3}{x} + \ln x . 2x\right) + (x-3)^{x^2} . \left(\frac{x^2}{x-3} + \ln(x-3) . 2x\right) = 0$$

$$y' = x^{x^2-3}\left(\frac{x^2-3}{x} + \ln x . 2x\right) - (x-3)^{x^2} . \left(\frac{x^2}{x-3} + \ln(x-3) . 2x\right)$$

Question: 61

If <

Solution:

take log both the side,

$$\ln f(x) = (2+3x).\ln\left(\frac{3+x}{1+x}\right)$$

Now differentiate it,

$$\frac{1}{f(x)} \times f'(x) = (2+3x)\left(\frac{1+x}{3+x}\right)\left(\frac{1+x-(3+x)}{(1+x)^2}\right) + \ln\left(\frac{3+x}{1+x}\right).3$$

$$f'(x) = \left(\frac{(2+3x)(-2)}{(3+x)(1+x)} + 3.\ln\left(\frac{3+x}{1+x}\right)\right) \times f(x)$$

To get f'(0) we need to find f(0),

Putting x=0 in f

$$f(0) = \left(\frac{3}{1}\right)^2$$

$$f(0) = 9$$

Now put x=0 in f'(x),

$$f'(0) = \left(\left(\frac{2\times(-2)}{3}\right) + 3\ln 3\right) \times 9$$

$$f'(0) = 9\left(3\ln 3 - \frac{4}{3}\right)$$

Question: 62

If <

Solution:

we can write this equation as,

$$y = e^{x\, ln(\sin x)} + \sin^{-1}\sqrt{x}$$

Differentiate it,

$$y' = (\sin x)^x\left(\frac{x \times \cos x}{\sin x} + \ln(\sin x)\right) + \frac{1}{\sqrt{1-\sqrt{x}^2}} \times \frac{1}{2\sqrt{x}}$$

$$y' = (\sin x)^x(x.\cot x + \ln \sin x) + \frac{1}{2\sqrt{x}.\sqrt{1-x}}$$

Question: 63

If <

Solution:

simply differentiate both sides,

$$2(x^2+y^2)(2x+2y.y') = x.y' + y$$

Take y' one side

$$4x^3 + 4x^2.y.y' + 4y^2.x + 4y^3.y' = x.y' + y$$

$$y'(4x^2.y + 4y^3 - x) = y - 4x^3 - 4y^2x$$

$$y' = \frac{y - 4x^3 - 4y^2x}{4x^2.y + 4y^3 - x}$$

Question: 64

Solution:

we can write this as,

$$y = e^{\cot x . \ln x} + \frac{2x^2 - 3}{x^2 + x + 2}$$

Differentiate ,

$$y' = x^{\cot x}\left(\frac{\cot x}{x} + \ln x(-cosec^2 x\ )\right) + \frac{(x^2 + x + 2)(4x) - (2x^2 - 3)(2x + 1)}{(x^2 + x + 2)^2}$$

$$y' = x^{\cot x}\left(\frac{\cot x}{x} + \ln x(-cosec^2 x\ )\right) + \frac{2x^2 + 14x + 3}{(x^2 + x + 2)^2}$$

Question: 65

Find , when:
<

Solution:

Differentiate it,

$$y' = \frac{1}{1 + \frac{a^2}{x^2}} \times \left(-\frac{a}{x^2}\right) + \sqrt{\frac{x + a}{x - a}} \times \frac{1}{2} \times \sqrt{\frac{x + a}{x - a}} \times \frac{(x + a) - (x - a)}{(x + a)^2}$$

$$y' = \frac{-a}{x^2 + a^2} + \frac{x - a}{2(x + a)} \times \frac{2a}{(x - a)^2}$$

$$y' = -\frac{a}{(x^2 + a^2)} + \frac{a}{x^2 - a^2}$$

$$y' = \frac{ax^2 + a^3 - ax^2 + a^3}{x^4 - a^4}$$

$$y' = \frac{2a^3}{x^4 - a^4}$$

Question: 66

If <

Solution:

taking log both sides,

$$m \ln x + n \ln y = (m + n).\ln(x + y)$$

differentiating both sides,

$$\frac{m}{x} + \frac{n}{y} \times y' = \frac{m + n}{x + y} \times (1 + y')$$

Take y’ one side,

$$y'\left(\frac{n}{y} - \frac{m + n}{x + y}\right) = \frac{m + n}{x + y} - \frac{m}{x}$$

$$y' = \frac{mx + nx - (mx + my)}{x.(x + y)} \times \frac{y.(x + y)}{nx + ny - (my + ny)}$$

$y' = \frac{nx - my}{x} \times \frac{y}{nx - my}$

$y' = \frac{y}{x}$

Exercise : 10G

Question: 1

If

To prove : $\frac{dy}{dx} = \frac{y^2 \cot x}{1 - y\log \sin x}$

Formula used : $\log a = \log b^m$

$\log a = m \log b$

$\frac{d(\log y)}{dx} = \frac{1}{y}\frac{dy}{dx}$

$\frac{d(\sin x)}{dx} = \cos x$

If u and v are functions of x,then $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

The CHAIN RULE states that the derivative of f(g(x)) is f’(g(x)).g’(x)

$y = (\sin x)^y$

taking log on both sides

$\log y = \log (\sin x)^y$

$\log y = y \log (\sin x)$

Differentiating both sides with respect to x

$\frac{d(\log y)}{dx} = \frac{d[y \log(\sin x)]}{dx}$

$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log(\sin x) + y\frac{d \log(\sin x)}{dx}$

$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log(\sin x) + y\frac{1}{\sin x} \times \frac{d(\sin x)}{dx}$

$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log(\sin x) + y\frac{\cos x}{\sin x}$

$(\frac{1}{y} - \log \sin x)\frac{dy}{dx} = y\cot x$

$\frac{1 - y\log \sin x}{y}\frac{dy}{dx} = y\cot x$

$$\frac{dy}{dx} = \frac{y^2 \cot x}{1 - y\log \sin x}$$

$$\frac{dy}{dx} = \frac{y^2 \cot x}{1 - y\log \sin x}$$

Question: 2

If <

Solution:

Given : $y = (\cos x)^{(\cos x)^{(\cos x)^{(\cos x)\ldots\ldots\ldots\infty}}}$

To prove : $\frac{dy}{dx} = \frac{-y^2 \tan x}{1-y\log \cos x}$

Formula used : $\log a = \log b^m$

$\log a = m \log b$

$\frac{d(\log y)}{dx} = \frac{1}{y}\frac{dy}{dx}$

$\frac{d(\cos x)}{dx}$ = - sinx

If u and v are functions of x,then $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

The CHAIN RULE states that the derivative of f(g(x)) is f'(g(x)).g'(x)

Given that y = $(\cos x)^y$

taking log on both sides

log y = log $(\cos x)^y$

log y = y log $(\cos x)$

Differentiating both sides with respect to x

$\frac{d(\log y)}{dx} = \frac{d[y\log(\cos x)]}{dx}$

$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log(\cos x) + y\frac{d\log(\cos x)}{dx}$

$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log(\cos x) + y\frac{1}{\cos x} \times \frac{d(\cos x)}{dx}$

$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log(\cos x) + y\frac{-\sin x}{\cos x}$

$(\frac{1}{y} - \log\cos x)\frac{dy}{dx}$ = - ytan x

$$\frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y\log \cos x}$$

$$\frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y\log \cos x}$$

Question: 3

If <

Solution:

Given : $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots\dots\dots.\infty}}}$

To prove : $\frac{dy}{dx} = \frac{1}{2y-1}$

Formula used : $\log a = \log b^m$

$\log a = m \log b$

$\frac{d(\log y)}{dx} = \frac{1}{y}\frac{dy}{dx}$

$\frac{dx}{dx}$ = 1

If u and v are functions of x,then $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

The CHAIN RULE states that the derivative of f(g(x)) is f’(g(x)).g’(x)

$y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots \ldots \ldots . \infty}}}$.

$y = \sqrt{x + y}$

squaring on both sides

$y^2 = x + y$

Differentiating with respect to x

$2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$(2y - 1)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y-1}$

$\frac{dy}{dx} = \frac{1}{2y-1}$

Question: 4

If

~~**Solution:**~~

~~Given :~~ $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \ldots \ldots \ldots . \infty}}}$

~~To prove :~~ $\frac{dy}{dx} = \frac{\sin x}{2y-1}$

~~Formula used :~~ $\log a = \log b^m$

$\log a = m \log b$

$\frac{d(\log y)}{dx} = \frac{1}{y}\frac{dy}{dx}$

$\frac{d(\cos x)}{dx} = -\sin x$

~~If u and v are functions of x,then~~ $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

~~The CHAIN RULE states that the derivative of f(g(x)) is f’(g(x)).g’(x)~~

$y = \sqrt{\cos x + y}$

~~squaring on both sides~~

$y^2 = \cos x + y$

~~Differentiating with respect to x~~

$2y\frac{dy}{dx} = -\sin x + \frac{dy}{dx}$

$(2y - 1)\frac{dy}{dx} = -\sin x$

$\frac{dy}{dx} = \frac{-\sin x}{2y-1} = \frac{\sin x}{1-2y}$

$\frac{dy}{dx} = \frac{\sin x}{1-2y}$

$\frac{dy}{dx} = \frac{\sin x}{1-2y}$

Question: 5

If <

Solution:

Given : $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \ldots \ldots \ldots \infty}}}$

To prove : $\frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$

Formula used : $\log a = \log b^m$

$\log a = m \log b$

$\frac{d(\log y)}{dx} = \frac{1}{y}\frac{dy}{dx}$

$\frac{d(\tan x)}{dx} = \sec^2 x$

If u and v are functions of x,then $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

The CHAIN RULE states that the derivative of f(g(x)) is f'(g(x)).g'(x)

$y = \sqrt{\tan x + y}$

squaring on both sides

$y^2 = \tan x + y$

Differentiating with respect to x

$2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$

$(2y-1)\frac{dy}{dx} = \sec^2 x$

$\frac{dy}{dx} = \frac{\sec^2 x}{2y-1} = \frac{\sec^2 x}{2y-1}$

$\frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$

$\frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$

Question: 6

If <

Solution:

Given : $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots \ldots \ldots \infty}}}$

To show : $(2y-1).\frac{dy}{dx} = \frac{1}{x}$

Formula used : $\log a = \log b^m$

$\log a = m \log b$

$\frac{d(\log y)}{dx} = \frac{1}{y}\frac{dy}{dx}$

If u and v are functions of x,then $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

The CHAIN RULE states that the derivative of f(g(x)) is f'(g(x)).g'(x)

$y = \sqrt{\log x + y}$

squaring on both sides

$y^2 = \log x + y$

Differentiating with respect to x

$2y\frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$

$(2y - 1)\frac{dy}{dx} = \frac{1}{x}$

$(2y - 1)\frac{dy}{dx} = \frac{1}{x}$

Question: 7

If <

Solution:

Given : $y = a^{x^{a^{x \cdots\cdots \infty}}}$

To show : $\frac{dy}{dx} = \frac{y^2 (\log y)}{x[1 - y(\log x)(\log y)]}$

Formula used : $\log a = \log b^m$

$\log a = m \log b$

$$\frac{d(\log y)}{dx} = \frac{1}{y}\frac{dy}{dx}$$

If u and v are functions of x,then $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

The CHAIN RULE states that the derivative of f(g(x)) is f'(g(x)).g'(x)

$y = a^{x^y}$

taking log on both sides

$\log y = \log a^{x^y}$

$\log y = x^y . \log a$

taking log on both sides

$\log(\log y) = \log(x^y . \log a)$

$\log(\log y) = y.\log x + \log(\log a)$

Differentiating both sides with respect to x

$\frac{d(\log[\log y])}{dx} = \frac{d(y.\log x)}{dx} + 0$ (as differentiation of $\log(\log a)$ [constant] is zero)

$\frac{1}{\log y}\frac{d \log y}{dx} = \log x\frac{dy}{dx} + y.\frac{d \log x}{dx}$

$\frac{1}{\log y}.\frac{1}{y}\frac{dy}{dx} = \log x\frac{dy}{dx} + y.\frac{1}{x}$

$\left(\frac{1}{\log y}.\frac{1}{y} - \log x\right)\frac{dy}{dx} = \frac{y}{x}$

$\left(\frac{1-y(\log x)(\log y)}{y(\log y)}\right)\frac{dy}{dx}=\frac{y}{x}$

$\frac{dy}{dx}=\frac{y^2(\log y)}{x[1-y(\log x)(\log y)]}$.

$$\frac{dy}{dx}=\frac{y^2(\log y)}{x[1-y(\log x)(\log y)]}$$

~~Question: 8~~

~~If~~

~~Solution:~~

~~Given :~~ $y=x+\cfrac{1}{x+\cfrac{1}{x+\cfrac{1}{x\ldots\infty}}}$,

~~To show :~~ $\frac{dy}{dx}=\frac{y}{(2y-x)}$

~~Formula used :~~ $\log a=\log b^m$

$\log a=m\log b$

$$\frac{d(\log y)}{dx}=\frac{1}{y}\frac{dy}{dx}$$

~~If u and v are functions of x,then~~ $\frac{d(uv)}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$

~~The CHAIN RULE states that the derivative of f(g(x)) is f'(g(x)).g'(x)~~

$y=x+\frac{1}{y}$

$y^2=xy+1$

~~Differentiating with respect to x~~

$\frac{d(y^2)}{dx}=\frac{d(xy)}{dx}+0$ ~~(as differentiation of constant is zero)~~

$2y.\frac{dy}{dx}=x.\frac{dy}{dx}+y$

$(2y-x)\frac{dy}{dx}=y$

$\frac{dy}{dx}=\frac{y}{(2y-x)}$

$\frac{dy}{dx}=\frac{y}{(2y-x)}$

~~Exercise : 10H~~

~~Question: 1~~

~~Differentiate~~

~~Solution:~~

~~Given : Let u =~~ x^6 ~~and v =~~ $\frac{1}{\sqrt{x}}$

~~To differentiate :~~ x^6 ~~with respect to~~ $(1/\sqrt{x})$.

~~Formula used :~~ $\frac{d(x^n)}{dx}=n.x^{n-1}$

~~The CHAIN RULE states that the derivative of f(g(x)) is f'(g(x)).g'(x)~~

~~Let u =~~ x^6 ~~and v =~~ $\frac{1}{\sqrt{x}}$

~~Differentiating u with respect to x~~

$\frac{du}{dx} = 6x^5$

~~Differentiating v with respect to x~~

$\frac{dv}{dx} = \frac{-1}{2}\, x^{-\frac{3}{2}}$

$\frac{du}{dv} = \frac{du}{dx} \Big/ \frac{dv}{dx}$

$\frac{du}{dv} = 6x^5 \Big/ \frac{-1}{2}\, x^{-\frac{3}{2}}$

$\frac{du}{dv} = -12x^{5+\frac{3}{2}}$

$\frac{du}{dv} = -12x^{\frac{13}{2}}$

~~Ans.~~ $-12x^{13/2}$

~~Question: 2~~

~~Differentia~~

~~Solution:~~

~~Given : Let u = log x and v = cot x~~

~~To differentiate : log xwith respect to cot x~~

~~Formula used :~~ $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

$$\frac{d(\log x)}{dx} = \frac{1}{x}$$

~~The CHAIN RULE states that the derivative of f(g(x)) is f'(g(x)).g'(x)~~

~~Let u = log x and v = cot x~~

~~Differentiating u with respect to x~~

$\frac{du}{dx} = \frac{1}{x}$

~~Differentiating v with respect to x~~

$\frac{dv}{dx} = -\operatorname{cosec}^2 x$

$\frac{du}{dv} = \frac{du}{dx} \Big/ \frac{dv}{dx}$

$\frac{du}{dv} = \frac{\frac{1}{x}}{-\operatorname{cosec}^2 x}$

$\frac{du}{dv} = \frac{-1}{x \operatorname{cosec}^2 x}$

~~Question: 3~~

~~Differentia~~

~~Solution:~~

~~Given : Let u =~~ $e^{\sin x}$ ~~and v = cos x~~

~~To differentiate :~~ $e^{\sin x}$ ~~with respect to cos x~~

~~Formula used :~~ $\frac{d(e^x)}{dx} = e^x$

$\frac{d(\cos x)}{dx} = -\sin x$

~~The CHAIN RULE states that the derivative of f(g(x)) is f'(g(x)).g'(x)~~

~~Let u =~~ $e^{\sin x}$ ~~and v = cos x~~

~~Differentiating u with respect to x~~

$\frac{du}{dx} = \frac{d(e^{\sin x})}{dx} = \cos x.\ e^{\sin x}$

~~Differentiating v with respect to x~~

$\frac{dv}{dx} = -\sin x$

$\frac{du}{dv} = \frac{du}{dx} \Big/ \frac{dv}{dx}$

$\frac{du}{dv} = \frac{\cos x.\ e^{\sin x}}{-\sin x}$

$\frac{du}{dv} = -\ e^{\sin x}.\cot x$

~~Ans.~~ $-e^{\sin x} \cot x$

~~Question: 4~~

~~Differentia~~

~~Solution:~~

~~Given : Let u =~~ $\tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}$ ~~and v =~~ $\cos^{-1} x^2$.

~~To differentiate :~~ $\tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}$ ~~with respect to~~ $\cos^{-1} x^2$.

~~Formula used :~~ $\frac{d(x^n)}{dx} = n.x^{n-1}$

~~The CHAIN RULE states that the derivative of f(g(x)) is f'(g(x)).g'(x)~~

$\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+\ x^2}$

$\frac{d(\cos^{-1} x)}{dx} = \frac{-1}{\sqrt{1-\ x^2}}$

$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{vdu-udv}{v^2}$

~~Let u =~~ $\tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}$ ~~and v =~~ $\cos^{-1} x^2$.

~~Differentiating u with respect to x~~

$$\frac{du}{dx} = \frac{d\left(\tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}\right)}{dx} = \frac{1}{1+\frac{1-x^2}{1+x^2}} \cdot \frac{d\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)}{dx}$$

$$\frac{du}{dx} = \frac{1+x^2}{1+x^2+1-x^2} \cdot \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{-1}{2}} \cdot \frac{-2x(1+x^2)-2x(1-x^2)}{(1+x^2)^2}$$

$$\frac{du}{dx} = \frac{1+x^2}{2} \cdot \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{-1}{2}} \cdot \frac{-2x-2x^3-2x+2x^3}{(1+x^2)^2} = \frac{1+x^2}{2} \cdot \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{-1}{2}} \cdot \frac{-4x}{(1+x^2)^2}$$

$$\frac{du}{dx} = \left(\frac{1-x^2}{1+x^2}\right)^{\frac{-1}{2}} \cdot \frac{-x}{(1+x^2)} = \sqrt{\frac{1+x^2}{1-x^2}} \cdot \frac{-x}{(1+x^2)} = \frac{-x}{\sqrt{(1-x^2)(1+x^2)}} = \frac{-x}{\sqrt{1-x^4}}$$

$$\frac{du}{dx} = \frac{-x}{\sqrt{1-x^4}}$$

~~Differentiating v with respect to x~~

$$\frac{dv}{dx} = -\frac{1}{\sqrt{1-(x^2)^2}} \cdot \frac{d(x^2)}{dx} = \frac{-2x}{\sqrt{1-x^4}}$$

$$\frac{dv}{dx} = \frac{-2x}{\sqrt{1-x^4}}$$

$$\frac{du}{dv} = \frac{du}{dx} \Big/ \frac{dv}{dx}$$

$$\frac{du}{dv} = \frac{\frac{-x}{\sqrt{1-x^4}}}{\frac{-2x}{\sqrt{1-x^4}}} = \frac{1}{2}$$

$$\frac{du}{dv} = \frac{1}{2}$$

~~Ans.~~ $\frac{1}{2}$

~~Question: 5~~

~~Differentia~~

~~Solution:~~

~~Given : Let u =~~ $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ ~~and v =~~ $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

~~To differentiate :~~ $\tan^{-1}\frac{2x}{1-x^2}$ ~~with respect to~~ $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

~~Formula used :~~ $\frac{d(x^n)}{dx} = n.x^{n-1}$

$$\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$$

$$\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$$

~~The CHAIN RULE states that the derivative of f(g(x)) is f'(g(x)).g'(x)~~

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{vdu-udv}{v^2}$$

~~Let u =~~ $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ ~~and v =~~ $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

~~Differentiating u with respect to x~~

$$\frac{du}{dx} = \frac{d(\tan^{-1}\frac{2x}{1-x^2})}{dx} = \frac{1}{1+(\frac{2x}{1-x^2})^2} \cdot \frac{d\left(\frac{2x}{1-x^2}\right)}{dx} = \frac{1}{1+\frac{4x^2}{1+x^4-2x^2}} \cdot \frac{2\left(1-x^2\right)+2x(2x)}{(1-x^2)^2}$$

$$\frac{du}{dx} = \frac{(1-x^2)^2}{1+x^4-2x^2+4x^2} \cdot \frac{2-2x^2+4x^2}{(1-x^2)^2} = \frac{(1-x^2)^2}{1+x^4+2x^2} \cdot \frac{2+2x^2}{(1-x^2)^2} = \frac{2(1+x^2)}{(1+x^2)^2} = \frac{2}{(1+x^2)}$$

$$\frac{du}{dx} = \frac{2}{(1+x^2)}$$

~~Differentiating v with respect to x~~

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-(\frac{2x}{1+x^2})^2}} \cdot \frac{d(\frac{2x}{1+x^2})}{dx} = \frac{1+x^2}{\sqrt{1+x^4+2x^2-4x^2}} \cdot \frac{2\left(1+x^2\right)-2x(2x)}{(1+x^2)^2}$$

$$\frac{dv}{dx} = \frac{1+x^2}{\sqrt{1+x^4-2x^2}} \cdot \frac{2+2x^2-4x^2}{(1+x^2)^2} = \frac{1+x^2}{\sqrt{(1-x^2)^2}} \cdot \frac{2-2x^2}{(1+x^2)^2} = \frac{1+x^2}{1-x^2} \cdot \frac{2(1-x^2)}{(1+x^2)^2} = \frac{2}{1+x^2}$$

$$\frac{dv}{dx} = \frac{2}{1+x^2}$$

$$\frac{du}{dv} = \frac{du}{dx} \Big/ \frac{dv}{dx}$$

$$\frac{du}{dv} = \frac{\frac{2}{(1+x^2)}}{\frac{2}{(1+x^2)}} = 1$$

$$\frac{du}{dv} = 1$$

~~Ans. 1~~

~~Question: 6~~

~~Differentia~~

~~Solution:~~

~~Given : Let u =~~ $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ ~~and v =~~ $\cos^{-1}(2x^2-1)$.

~~To differentiate :~~ $\tan^{-1}\frac{x}{\sqrt{1-x^2}}$ ~~with respect to~~ $\cos^{-1}(2x^2-1)$

~~Formula used :~~ $\frac{d\,(x^n)}{dx} = n.x^{n-1}$

$$\frac{d\,(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$$

$$\frac{d\,(\cos^{-1}x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

~~The CHAIN RULE states that the derivative of f(g(x)) is f'(g(x)).g'(x)~~

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{vdu-udv}{v^2}$$

~~Let u =~~ $\tan^{-1}\frac{x}{\sqrt{1-x^2}}$ ~~and v =~~ $\cos^{-1}(2x^2-1)$

~~Differentiating u with respect to x~~

$$\frac{du}{dx} = \frac{d(\tan^{-1}\frac{x}{\sqrt{1-x^2}})}{dx} = \frac{1}{1+(\frac{x}{\sqrt{1-x^2}})^2} \cdot \frac{d\left(\frac{x}{\sqrt{1-x^2}}\right)}{dx} = \frac{1}{1+\frac{x^2}{1-x^2}} \cdot \frac{1\left(\sqrt{1-x^2}\right)+x\left(\frac{-2x}{2\sqrt{1-x^2}}\right)}{1-x^2}$$

$$\frac{du}{dx} = \frac{1-x^2}{1-x^2+x^2} \cdot \frac{\sqrt{1-x^2} - \frac{-x^2}{\sqrt{1-x^2}}}{1-x^2} = (1-x^2) \cdot \frac{\frac{1-x^2+x^2}{\sqrt{1-x^2}}}{(1-x^2)^2} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$$

~~Differentiating v with respect to x~~

$$\frac{dv}{dx} = \frac{d\,[\cos^{-1}(2x^2-1)]}{dx} = \frac{-1}{\sqrt{1-(2x^2-1)^2}} \cdot \frac{d(2x^2-1)}{dx} = \frac{-1}{\sqrt{1-4x^4-1+4x^2}} \cdot 4x$$

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{4x^2-4x^4}} = \frac{-4x}{2x\sqrt{1-x^2}} = \frac{-2}{\sqrt{1-x^2}}$$

$$\frac{dv}{dx} = \frac{-2}{\sqrt{1-x^2}}$$

$$\frac{du}{dv} = \frac{du}{dx} \Big/ \frac{dv}{dx}$$

$$\frac{du}{dv} = \frac{\frac{1}{\sqrt{1-x^2}}}{\frac{-2}{\sqrt{1-x^2}}} = \frac{-1}{2}$$

$$\frac{du}{dv} = \frac{-1}{2}$$

~~Ans.~~ $\frac{-1}{2}$

~~Question: 7~~

~~Differentia~~

~~Solution:~~

~~Given : Let u =~~ $\sin^3 x$ ~~and v =~~ $\cos^3 x$

~~To differentiate :~~ $\sin^3 x$ ~~with respect to~~ $\cos^3 x$

~~Formula used :~~ $\frac{d(x^n)}{dx} = n.x^{n-1}$

$$\frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

~~The CHAIN RULE states that the derivative of f(g(x)) is f'(g(x)).g'(x)~~

~~Let u =~~ $\sin^3 x$ ~~and v =~~ $\cos^3 x$

~~Differentiating u with respect to x~~

$$\frac{du}{dx} = 3\sin^2 x . \frac{d(\sin x)}{dx} = 3\sin^2 x \cos x$$

$$\frac{du}{dx} = 3\sin^2 x \cos x$$

~~Differentiating v with respect to x~~

$$\frac{dv}{dx} = 3\cos^2 x . \frac{d(\cos x)}{dx} = -3\cos^2 x \sin x$$

$$\frac{dv}{dx} = -3\cos^2 x \sin x$$

$$\frac{du}{dv} = \frac{du}{dx} \Big/ \frac{dv}{dx}$$

$$\frac{du}{dv} = \frac{3\sin^2 x\cos x}{-3\cos^2 x\sin x} = \frac{\sin x}{-\cos x} = -\tan x$$

$$\frac{du}{dv} = -\tan x$$

Ans. $-\tan x$

Question: 8

Differentia

Solution:

Given : Let u = $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ and v = $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

To differentiate : $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

Formula used : $\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} = \tan 3\theta$

$$\frac{d(x^n)}{dx} = n.x^{n-1}$$

$$\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$$

$$\frac{d(\cos^{-1}x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

The CHAIN RULE states that the derivative of f(g(x)) is f'(g(x)).g'(x)

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{vdu-udv}{v^2}$$

Let u = $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ and v = $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

Differentiating u with respect to x

$$\frac{du}{dx} = \frac{d\cos^{-1}\frac{1-x^2}{1+x^2}}{dx} = \frac{-1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{d\left(\frac{1-x^2}{1+x^2}\right)}{dx} = \frac{-(1+x^2)}{\sqrt{(1+x^2)^2-(1-x^2)^2}} \cdot \frac{-2x(1+x^2)-2x(1-x^2)}{(1+x^2)^2}$$

$$\frac{du}{dx} = \frac{-(1+x^2)}{\sqrt{1+x^4+2x^2-1-x^4+2x^2}} \cdot \frac{-2x-2x^3-2x+2x^3}{(1+x^2)^2} = \frac{-(1+x^2)}{\sqrt{4x^2}} \cdot \frac{-4x}{(1+x^2)^2} = \frac{+2}{1+x^2}$$

$$\frac{du}{dx} = \frac{+2}{1+x^2}$$

For v = $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

Let x = tan θ

$$\tan^{-1}\frac{3x-x^3}{1-3x^2} = \tan^{-1}\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta} = \tan^{-1}(\tan 3\theta) = 3\theta = 3\tan^{-1}x$$

$$\tan^{-1}\frac{3x-x^3}{1-3x^2} = 3\tan^{-1}x$$

Differentiating v with respect to x ,

$$\frac{dv}{dx} = \frac{d(3\tan^{-1}x)}{dx} = \frac{3}{1+x^2}$$

$$\frac{dv}{dx} = \frac{3}{1+x^2}$$

$$\frac{du}{dv} = \frac{du}{dx} \Big/ \frac{dv}{dx}$$

$$\frac{du}{dv} = \frac{\frac{+2}{1+x^2}}{\frac{+3}{1+x^2}} = \frac{2}{3}$$

$$\frac{du}{dv} = \frac{2}{3}$$

Ans. $\frac{2}{3}$

Question: 9

Differentia

Solution:

Given : Let u = $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and v = $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

To differentiate : $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Formula used : $\frac{d(x^n)}{dx} = n.x^{n-1}$

$$\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$$

$$\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$$

The CHAIN RULE states that the derivative of f(g(x)) is f'(g(x)).g'(x)

Let u = $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and v = $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Put x = cot θ or θ = $\cot^{-1} x$ in u

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = \tan^{-1}\frac{\sqrt{1+\cot^2\theta}-1}{\cot\theta} = \tan^{-1}\frac{\operatorname{cosec}\theta-1}{\cot\theta}$$

$$\tan^{-1}\frac{\operatorname{cosec}\theta-1}{\cot\theta} = \tan^{-1}\frac{\frac{1}{\sin\theta}-1}{\cot\theta} = \tan^{-1}\frac{\frac{1-\sin\theta}{\sin\theta}}{\cot\theta} = \tan^{-1}\frac{\frac{1-\sin\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}}$$

$$\tan^{-1}\frac{\frac{1-\sin\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}} = \tan^{-1}\frac{1-\sin\theta}{\cos\theta}$$

We know that $1-\sin\theta = \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} - 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ and $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$

$$1-\sin\theta = \left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)^2$$

Substituting the above values in $\tan^{-1}\frac{1-\sin\theta}{\cos\theta}$, we get

$$\tan^{-1}\frac{1-\sin\theta}{\cos\theta} = \tan^{-1}\frac{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2}{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}} = \tan^{-1}\frac{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2}{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}$$

$\tan^{-1}\frac{1-\sin\theta}{\cos\theta}=\tan^{-1}\frac{(\cos\frac{\theta}{2}-\sin\frac{\theta}{2})}{(\cos\frac{\theta}{2}+\sin\frac{\theta}{2})}$

~~Dividing by $\cos\frac{\theta}{2}$ on numerator and denominator,we get~~

$\tan^{-1}\frac{(\cos\frac{\theta}{2}-\sin\frac{\theta}{2})}{(\cos\frac{\theta}{2}+\sin\frac{\theta}{2})}=\tan^{-1}\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}=\tan^{-1}\tan(\frac{\pi}{4}-\frac{\theta}{2})=\frac{\pi}{4}-\frac{\theta}{2}$

$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}=\frac{\pi}{4}-\frac{\theta}{2}=\frac{\pi}{4}-\frac{\cot^{-1}x}{2}$

~~Differentiating u with respect to x~~

$\frac{d(\tan^{-1}\frac{\sqrt{1+x^2}-1}{x})}{dx}=\frac{d(\frac{\pi}{4}-\frac{\cot^{-1}x}{2})}{dx}=\frac{1}{2(1+x^2)}$

$\frac{du}{dx}=\frac{1}{2(1+x^2)}$

$v=\sin^{-1}\frac{2x}{1+x^2}$

~~Put $x = \tan\theta$~~

$V=\sin^{-1}\frac{2x}{1+x^2}=\sin^{-1}\frac{2\tan\theta}{1+\tan^2\theta}=\sin^{-1}\frac{2\frac{\sin\theta}{\cos\theta}}{\sec^2\theta}=\sin^{-1}\frac{2\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}=\sin^{-1}(2\sin\theta\cos\theta)$

$V=\sin^{-1}\frac{2x}{1+x^2}=\sin^{-1}(2\sin\theta\cos\theta)=\sin^{-1}(\sin 2\theta)=2\theta=2\tan^{-1}x$

$V=\sin^{-1}\frac{2x}{1+x^2}=2\tan^{-1}x$

~~Differentiating v with respect to x~~

$\frac{dv}{dx}=\frac{2}{1+x^2}$

$\frac{du}{dv}=\frac{du}{dx}\Big/\frac{dv}{dx}$

$\frac{du}{dv}=\frac{\frac{1}{2(1+x^2)}}{\frac{2}{1+x^2}}=\frac{1}{4}$

$\frac{du}{dv}=\frac{1}{4}$

~~Ans.~~ $\frac{1}{4}$

~~Question: 10~~

~~Differentia~~

~~Solution:~~

~~Given : Let u =~~ $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ ~~and v =~~ $\cos^{-1}(2x\sqrt{1-x^2})$

~~To differentiate :~~ $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ ~~with respect to~~ $\cos^{-1}(2x\sqrt{1-x^2})$

~~Formula used :~~ $\frac{d(x^n)}{dx}=n.x^{n-1}$

$$\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$$

$$\frac{d(\cos^{-1}x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

~~The CHAIN RULE states that the derivative of f(g(x)) is f'(g(x)).g'(x)~~

~~Let u =~~ $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ ~~and v =~~ $\cos^{-1}(2x\sqrt{1-x^2})$

~~Substitute x = cosθ in u~~

$u = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \tan^{-1}\left(\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}\right) = \tan^{-1}\left(\frac{\sqrt{\sin^2\theta}}{\cos\theta}\right)$

$u = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right) = \tan^{-1}(\tan\theta) = \theta$

$u = \theta = \cos^{-1}x$

~~Differentiating u with respect to x~~

$\frac{du}{dx} = \frac{-1}{\sqrt{1-x^2}}$

~~Substitute x = sinθ in v ,~~

$v = \cos^{-1}(2x\sqrt{1-x^2}) = \cos^{-1}(2\sin\theta\sqrt{1-\sin^2\theta}) = \cos^{-1}(2\sin\theta\sqrt{\cos^2\theta})$

$v = \cos^{-1}(2\sin\theta\sqrt{\cos^2\theta}) = \cos^{-1}(2\sin\theta.\cos\theta) = \cos^{-1}(\sin2\theta)$

$v = \cos^{-1}(\sin2\theta) = \cos^{-1}(\cos[\frac{\pi}{2} - 2\theta]) = \frac{\pi}{2} - 2\theta$

$v = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2\sin^{-1}x$

$v = \frac{\pi}{2} - 2\sin^{-1}x$

~~Differentiating v with respect to x~~

$\frac{dv}{dx} = \frac{-2}{\sqrt{1-x^2}}$

$\frac{du}{dv} = \frac{du}{dx}\Big/\frac{dv}{dx}$

$\frac{du}{dv} = \frac{\frac{-1}{\sqrt{1-x^2}}}{\frac{-2}{\sqrt{1-x^2}}} = \frac{1}{2}$

~~Ans.~~ $-\frac{1}{2}$

~~Exercise : 10I~~

~~Question: 1~~

~~Find~~

~~Solution:~~

~~Theorem: y and x are given in a different variable that is t. We can find~~ $\frac{dy}{dx}$ ~~by finding~~ $\frac{dy}{dt}$ ~~and~~ $\frac{dx}{dt}$ ~~and then dividing them to get the required thing.~~

$\frac{dy}{dt} = \frac{d(2at)}{dt}$

$= 2a$. ......(1)

$\frac{dx}{dt} = \frac{d(at^2)}{dt}$

$= 2at$ ......(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{2a}{2at}$

$= \frac{1}{t}$

Question: 2

Find

Solution:

y and x are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{dbsin\theta}{d\theta}$ ($\frac{dsin\theta}{d\theta}$=cosθ)

= bcosθ. ......(1)

$\frac{dx}{d\theta} = \frac{d(acos\theta)}{d\theta}$ ($\frac{dcos\theta}{d\theta} = -sin\theta$)

= -asinθ ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{bcos\theta}{-asin\theta}$ ($\frac{cos\theta}{sin\theta}$ = cotθ)

$= \frac{-bcot\theta}{a}$.

Question: 3

Find

Solution:

Theorem: y and x are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{dbsin^2\theta}{d\theta}$

= b× 2sinθ × cosθ (using the chain rule $\frac{dsin^2\theta}{d\theta}$ = 2sinθ× $\frac{dsin\theta}{d\theta}$ = 2sinθ × cosθ)

= 2bsinθcosθ . ...(1)

$\frac{dx}{d\theta} = \frac{dacos^2\theta}{d\theta}$

= a × (2cosθ)× (-sinθ) (using chain rule $\frac{dcos^2\theta}{d\theta}$ = 2cosθ× $\frac{dcos\theta}{d\theta}$ = 2 cosθ × (-sinθ))

= -2asinθcosθ.

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{2bsin\theta cos\theta}{-2asin\theta cos\theta}$

$= \frac{-b}{a}$.

Question: 4

Find

Solution:

Theorem: y and x are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{dasin^3\theta}{d\theta}$

$= a \times 3\sin^2\theta \times \cos\theta$ (using the chain rule $\frac{dsin^3\theta}{d\theta} = 3\sin^2\theta \times \frac{dsin\theta}{d\theta} = 2\sin^2\theta \times \cos\theta$)

$= 3a\sin^2\theta\cos\theta$ . ...(1)

$$\frac{dx}{d\theta} = \frac{dacos^3\theta}{d\theta}$$

$= a \times (3\cos^2\theta) \times (-\sin\theta)$ (using chain rule $\frac{dcos^2\theta}{d\theta} = 2\cos\theta \times \frac{dcos\theta}{d\theta} = 2\cos\theta \times (-\sin\theta)$)

$= -3a\sin\theta\cos^2\theta$.

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{3\,a\sin^2\theta\cos\theta}{-3a\sin\theta\cos^2\theta}$

$= \frac{-sin\theta}{cos\theta}$.

$= -\tan\theta$

Question: 5

Find

Solution:

Theorem: y and x are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{da(\theta+sin\theta)}{d\theta}$

$= a \times (1+\cos\theta)$ ....(1)

$$\frac{dx}{d\theta} = \frac{da(1-cos\theta)}{d\theta}$$

$= a\sin\theta$. ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{a(1+co\theta)}{asin\theta}$

$= \frac{1+cos\theta}{sin\theta}$.

$= \frac{2\cos^2(\theta/2)}{2\sin(\theta/2)\cos(\theta/2)}$ ($1+\cos\theta = 2\cos^2\theta/2$ and $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$)

$= \cot(\theta/2)$

Question: 6

Find

Solution:

Theorem: y and x are given in a different variable that is t. We can find $\frac{dy}{dx}$ by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.

$\frac{dy}{dt} = \frac{dbsint}{dt}$

$= bcost$ ....(1)

$$\frac{dx}{dt} = \frac{d(a\,logt)}{dt}$$

$= \frac{a}{t}$ ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{b\,cost}{a/t}$

$= \frac{bt\,cost}{a}$.

Question: 7

Find

Solution:

Theorem: y and x are given in a different variable that is t . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.

$\frac{dy}{dt} = \frac{d(e^t+sint)}{dt}$

$= e^t + cost$ ....(1) $(\frac{de^t}{dt} = e^t)$

$$\frac{dx}{dt} = \frac{d(logt + \ cost)}{dt}$$

$= \frac{1}{t} - sint.$ ....(2) $(\frac{d\,logt}{dt} = \frac{1}{t})$

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{e^t+cost}{\frac{1}{t}-sint}$

$= \frac{t(e^t+cost)}{1-tsint}$.

Question: 8

Find

Solution:

Theorem: y and x are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{d(sin\theta+sin2\theta)}{d\theta}$

$= cos\theta + cos2\theta\times2$ ....(1) $(using\ chain\ rule\ \frac{d\,sin2\theta}{d\theta} = cos2\theta \times \frac{d\,2\theta}{d\theta})$

$$\frac{dx}{d\theta} = \frac{d(cos\theta + cos2\theta)}{d\theta}$$

$= -sin\theta - 2sin2\theta$ ....(2) $(using\ chain\ rule\ \frac{d\,cos2\theta}{d\theta} = sin2\theta \times \frac{d\,2\theta}{d\theta})$

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{cos\theta+2cos2\theta}{-(sin\theta+2sin2\theta)}$

Question: 9

Find

Solution:

Theorem: y and x are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dx}{d\theta} = \frac{d\sqrt{sin2\theta}}{d\theta}$

$= \frac{2cos2\theta}{2\sqrt{sin2\theta}}$ $(using\ chain\ rule\ \frac{d\sqrt{sin2\theta}}{d\theta} = \frac{1}{2\sqrt{sin2\theta}} \times \frac{d\,sin2\theta}{d\theta})$

$\frac{dx}{d\theta} = \frac{cos2\theta}{\sqrt{sin2\theta}}$ ....(1)

$$\frac{dy}{d\theta} = \frac{d(\sqrt{cos2\theta})}{d\theta}$$

$= \frac{-2sin2\theta}{2\sqrt{cos2\theta}}$ $(using\ chain\ rule\ \frac{d\sqrt{sin2\theta}}{d\theta} = \frac{1}{2\sqrt{sin2\theta}} \times \frac{d\,sin2\theta}{d\theta})$

$= \frac{-sin2\theta}{\sqrt{cos2\theta}}$ ....(2)

Dividing (2) and (2), we get

$\frac{dy}{dx} = -\frac{sin2\theta/\sqrt{cos2\theta}}{cos2\theta/\sqrt{sin2\theta}}$

$= -\frac{\sqrt{\sin^3 2\theta}}{\sqrt{\cos^3 2\theta}}$

$= -(\tan2\theta)^{3/2}$

Question: 10

Find

Solution:

Theorem: y and x are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{d\,e^{\theta}(sin\theta - cos\theta)}{d\theta}$

$= e^{\theta}(cos\theta + sin\theta) + (sin\theta - cos\theta)e^{\theta}$ ....(1) {by using product rule, $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$}

$$\frac{dx}{d\theta} = \frac{d\ e^{\theta}(sin\theta + cos\theta)}{d\theta}$$

$= e^{\theta}(cos\theta - sin\theta) + e^{\theta}(sin\theta + cos\theta)$ ....(2) {by using product rule, $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$}

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{e^{\theta}(2sin\theta)}{e^{\theta}(2cos\theta)}$

$= \tan\theta$.

Question: 11 `

Find

Solution:

Theorem: y and x are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{d\,a(sin\theta - \theta cos\theta)}{d\theta}$

= a(cosθ - (θ sinθ + cosθ)) {by using product rule, $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ while differentiating θcosθ }

= a(θsinθ) ....(1)

$\frac{dx}{d\theta} = \frac{d\ a(cos\theta + \theta sin\theta)}{d\theta}$

= a(sinθ +θ cosθ +sinθ) {by using product rule, $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ while differentiating θcosθ }

= a× θcosθ ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{a \times \theta sin\theta}{a \times \theta cos\theta}$

= tanθ ANS

Question: 12

by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.

$\frac{dy}{dt} = \frac{d\frac{3at^2}{(1+t^2)}}{dt}$

$= \frac{(1+t^2)6at - 3at^2(2t)}{(1+t^2)^2}$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ }

$= \frac{6at + 6at^3 - 6at^3}{(1+t^2)^2}$

$= \frac{6at}{(1+t^2)^2}$ ....(1)

$\frac{dx}{dt} = \frac{d\ (\frac{3at}{1+t^2})}{d\theta}$

$= \frac{(1+t^2)3a - 3at(2t)}{(1+t^2)^2}$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ }

$= \frac{3a + 3at^2 - 6at^2}{(1+t^2)^2}$

$= \frac{3a - 3at^2}{(1+t^2)^2}$ ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{6at/(1+t^2)^2}{3a(1-t^2)/(1+t^2)^2}$

$= \frac{2t}{(1-t^2)}$

Question: 13

by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.

$\frac{dy}{dt} = \frac{d\frac{2t}{(1+t^2)}}{dt}$

$= \frac{(1+t^2)2 - 2t(2t)}{(1+t^2)^2}$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ }

$= \frac{2 + 2t^2 - 4t^2}{(1+t^2)^2}$

$= \frac{2-2t^2}{(1+t^2)^2}$ ....(1)

$$\frac{dx}{dt} = \frac{d\left(\frac{1-t^2}{1+t^2}\right)}{d\theta}$$

$= \frac{(1+t^2)(-2t)-(1-t^2)(2t)}{(1+t^2)^2}$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ }

$= \frac{-2t-2t^3-2t+2t^3}{(1+t^2)^2}$

$= \frac{-4t}{(1+t^2)^2}$ ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{2-2t^2/\left(1+t^2\right)^2}{-4t/(1+t^2)^2}$

$= \frac{t^2-1}{(2t)}$

Question: 14

by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.

Let us assume u= $\frac{t}{\sqrt{(1+t^2)}}$

$\frac{dy}{dt} = \frac{d\sin^{-1}(u)}{dt}$

$= \frac{1}{\sqrt{(1-u^2)}} \times \frac{du}{dt}$

$= \frac{1}{\sqrt{(1-u^2)}} \times \frac{\sqrt{1+t^2}\times 1-t(2t/2\sqrt{(1+t^2)})}{\left(\sqrt{1+t^2}\right)^2}$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ }

Putting value of u

$= \frac{\sqrt{(1+t^2)}}{1} \times \frac{1}{(1+t^2)^{\left(\frac{3}{2}\right)}}$

$= \frac{1}{1+t^2}$ ....(1)

Let assume v= $\frac{1}{\sqrt{(1+t^2)}}$

$$\frac{dx}{dt} = \frac{d\left(\cos^{-1} v\right)}{dv} \times \frac{dv}{dt}$$

$= \frac{-1}{\sqrt{(1-v^2)}} \times \left(\frac{-1}{\left(\sqrt{1+t^2}\right)^2}\right) \times \frac{2t}{2\sqrt{(1+t^2)}}$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ }

Putting value of v

$= \frac{t\sqrt{(1+t^2)}}{t\times(1+t^2)^{\frac{3}{2}}}$

$= \frac{\sqrt{(1+t^2)}}{(1+t^2)^{\frac{3}{2}}}$

$= \frac{1}{(1+t^2)}$ ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{1}{1+t^2} \times \frac{(1+t^2)}{1}$

$= 1$

~~Question: 15~~

~~If <~~

~~Solution:~~

~~Theorem: y and x are given in a different variable that is t . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.~~

$\frac{dy}{dt} = \frac{d(sint - 2\sin^3 t)}{dt}$

$= cost - 6\,sin^2 t \times cost$ ~~...........(1)~~ (using chain rule)

$$\frac{dx}{dt} = \frac{d(2cost - 2\cos^3 t)}{dt}$$

$= -2sint + 6cos^2 t \times sint$ ~~................(2)~~ (using chain rule)

~~Dividing (1) and (2), we get~~

$\frac{dy}{dx} = \frac{cost(1-6\sin^2 t)}{2sint\,(3\cos^2 t-1)}$

$= \frac{t(e^t+cost)}{1-tsint}$.

~~Question: 16~~

~~If <~~

~~Solution:~~

~~Theorem: y and x are given in a different variable that is t . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.~~

$\frac{dy}{dt} = \frac{d(3+2logt)/t}{dt}$

$= \frac{t\left(\frac{2}{t}\right)-(3+2logt)\times 1}{t^2}$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$}

$= -\frac{1+2logt}{t^2}$ ~~...........(1)~~

$$\frac{dx}{dt} = \frac{d(1+logt)/t^2}{dt}$$

$= \frac{t^2\left(\frac{1}{t}\right)-(2t+2t\ logt)}{t^4}$ ~~{by using divide rule,~~ $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ~~}~~

$= -\frac{2logt+1}{t^3}$ ~~................(2)~~

~~Dividing (1) and (2), we get~~

$\frac{dy}{dx} = \frac{-(1+2\ logt)/t^2}{-(1+2\ logt)/t^3}$

~~= t.~~

~~Question: 17~~

~~If <~~

~~Solution:~~

~~Theorem: y and x are given in a different variable that is θ . We can find~~ $\frac{dy}{dx}$ ~~by finding~~ $\frac{dy}{d\theta}$ ~~and~~ $\frac{dx}{d\theta}$ ~~and then dividing them to get the required thing.~~

$\frac{dy}{d\theta} = \frac{d\, a(1-cos\theta)}{d\theta}$

~~= asinθ ....(1)~~

$$\frac{dx}{d\theta} = \frac{d\ a(\theta - sin\theta\ )}{d\theta}$$

~~= a(1-cosθ) ....(2)~~

~~Dividing (1) and (2), we get~~

$\frac{dy}{dx} = \frac{asin\theta}{a\times(1-cos\theta)}$

~~Putting θ= π/2~~

$= \frac{\sin(\pi/2)}{1-\cos(\pi/2)}$

~~= 1.~~

~~Question: 18~~

~~If <~~

~~Solution:~~

~~Theorem: y and x are given in a different variable that is θ . We can find~~ $\frac{dy}{dx}$ ~~by finding~~ $\frac{dy}{d\theta}$ ~~and~~ $\frac{dx}{d\theta}$ ~~and then dividing them to get the required thing.~~

$\frac{dy}{d\theta} = \frac{d\ (2sin\theta - sin2\theta)}{d\theta}$

~~= 2cosθ – 2cos2θ ....(1)~~

$$\frac{dx}{d\theta} = \frac{d\ (2cos\theta - cos2\theta)}{d\theta}$$

~~= -2sinθ + 2sin2θ ....(2)~~

~~Dividing (1) and (2), we get~~

$\frac{dy}{dx} = \frac{2cos\theta - 2cos2\theta}{2sin2\theta - 2sin\theta}$

$= \frac{cos\theta - cos2\theta}{sin2\theta - sin\theta}$

$= \frac{cos\theta - (2\cos^2\theta - 1\,)}{2sin\theta cos\theta - sin\theta}$ $\{sin2t = 2sintcost\}\ \{cos2t = 2\cos^2 t - 1\}$

~~By factorising numerator, we get~~

$= \frac{(1-cos\theta)(cos\theta + \frac{1}{2})}{2sin\theta(cos\theta - \frac{1}{2})}$

$= \frac{1-cos\theta}{2sin\theta} \times \frac{cos\theta + \frac{1}{2}}{cos\theta - \frac{1}{2}}\ \{\frac{1-cos\theta}{sin\theta} = \tan(\frac{\theta}{2})\}$

$= \frac{\tan(\frac{\theta}{2})}{1} \times \frac{(2(1-\tan^2(\frac{\theta}{2})) + (1+\tan^2(\frac{\theta}{2}\,))}{2(1-\tan^2(\frac{\theta}{2})) - (1+\tan^2(\frac{\theta}{2}))}$

~~Foe simplicity let's take θ/2 as x.~~

$= \frac{tanx}{2} \times \frac{2 - 2\tan^2 x + 1 + \tan^2 x}{2 - 2\tan^2 x - 1 - \tan^2 x}$

$= \frac{tanx}{2} \times \frac{3 - \tan^2 x}{1 - 3\tan^2 x}$

$= \frac{3tanx - \tan^3 x}{1 - 3\tan^2 x} \; \frac{3tanx - \tan^3 x}{1 - 3\tan^2 x} = \tan 3x$

$= \frac{tan3x}{2} \; x = \frac{\theta}{2}$

$= \frac{\tan\left(\frac{3\theta}{2}\right)}{2}.$

~~**Question: 19**~~

~~If <~~

~~**Solution:**~~

~~Theorem: y and x are given in a different variable that is t . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.~~

$\frac{dx}{dt} = \frac{d\left(\frac{(\sin^3 t)}{(\sqrt{cos2t})}\right))}{dt}$

$= \frac{\sqrt{cos2t}(3\sin^2 t \times cost) - \sin^3 t\left(\frac{(-sin2t)}{\sqrt{cos2t}}\right))}{cos2t}$ ~~{by using divide rule,~~ $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ~~}~~

$= \frac{cos2t \times (3\sin^2 t \times cost) + \sin^3 t \times (2sintcost)}{(cos2t)^{\frac{3}{2}}}$ $\{sin2t = 2sintcost\}$

$= \frac{\sin^2 t \; cost(3cos2t + 2\sin^2 t)}{(cos2t)^{\frac{3}{2}}}$ $\{cos2t = 1 - 2\sin^2 t\}$

$= \frac{\sin^2 t \; cost(3 - 4\sin^2 t)}{(cos2t)^{\frac{3}{2}}}$

$= \frac{sint \cos t \; (3sint - 4\sin^3 t)}{2(cos2t)^{\frac{3}{2}}}$ $\{sin3t = 3sint - 4\sin^3 t\}$

$= \frac{sin2t \times sin3t}{(cos2t)^{\frac{3}{2}}}$ ~~.........(1)~~

$$\frac{dy}{dt} = \frac{d\frac{\cos^3 t}{\sqrt{cos2t}}}{dv}$$

$= \frac{\sqrt{cos2t}(3\cos^2 t \times (-sint) - \cos^3 t\left(\frac{(-sin2t)}{\sqrt{cos2t}}\right))}{cos2t}$ ~~{by using divide rule,~~ $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ~~}~~

$= \frac{cos2t \times (-3\cos^2 t \times sint) + \cos^3 t \times (2sintcost)}{(cos2t)^{\frac{3}{2}}}$ $\{sin2t = 2sintcost\}$

$= \frac{\cos^2 t \; sint(-3cos2t + 2\cos^2 t)}{(cos2t)^{\frac{3}{2}}}$ $\{cos2t = 2\cos^2 t - 1\}$

$= \frac{cos^2 t \; sint(3 - 4\cos^2 t)}{(cos2t)^{\frac{3}{2}}}$

$= \frac{sint \cos t \; (3cost - 4\cos^3 t)}{(cos2t)^{\frac{3}{2}}}$ $\{cos3t = 4\cos^3 t - 3cost\}$

$= -\frac{sin2t \times cos3t}{2(cos2t)^{\frac{3}{2}}}$ ~~.........(1)~~

~~Dividing (1) and (2), we get~~

$\frac{dy}{dx} = \frac{-\frac{sin2t \times cos3t}{2(cos2t)^{\frac{3}{2}}}}{\frac{sin2t \times sin3t}{(cos2t)^{\frac{3}{2}}}}$

$= -cot3t$

Question: 20

If <

Solution:

here we have to find the double derivative, so to find double derivative we will just differentiate the first derivative once again with a similar method.

Theorem: *y* and *x* are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$$\frac{dy}{d\theta} = \frac{d\,(2sin\theta - sin2\theta)}{d\theta}$$

= 2cosθ – 2cos2θ ....(1)

$$\frac{dx}{d\theta} = \frac{d\,(2cos\theta - cos2\theta)}{d\theta}$$

= -2sinθ + 2sin2θ ....(2)

Dividing (1) and (2), we get

$$\frac{dy}{dx} = \frac{cos\theta - cos2\theta}{sin2\theta - sin\theta}$$

$= \tan(\frac{3\theta}{2})$ {as shown in question no. 18}

Let $\frac{dy}{dx} = f'$

$$\frac{d^2\,y}{dx^2} = f''$$

⇒ To find *f''* we will differentiate *f'* with θ and then divide with equation (2).

$$\frac{d\frac{dy}{dx}}{d\theta} = \frac{d\tan(\frac{3\theta}{2})}{d\theta}$$

$$= \frac{\sec^2(\frac{3\theta}{2})}{1} \times \frac{3}{2}$$

Now divide by equation (2).

$$\frac{d^2y}{dx^2} = \frac{3\sec^2(\frac{3\theta}{2})}{4} \times \frac{1}{(sin2\theta - sin\theta)}$$

Putting θ = π/2

$$\frac{d^2y}{dx^2} = \frac{3}{4} \times (-2)$$

$$= -\frac{3}{2}.$$

Question: 21

If <

Solution:

here we have to find the double derivative, so to find double derivative we will just differentiate the first derivative once again with a similar method.

Theorem: *y* and *x* are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{d\ a(1+cos\theta)}{d\theta}$

$= -asin\theta$ ....(1)

$$\frac{dx}{d\theta} = \frac{d\ a(\theta - sin\theta\ )}{d\theta}$$

$= a(1-cos\theta\ )$ ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{-asin\theta}{a\times(1-cos\theta)}$

$= \frac{-2\sin(\frac{\theta}{2})\cos(\frac{\theta}{2})}{2\sin^2\frac{\theta}{2}}$ $\{sin2t = 2sintcost\}$ $\{cos2t = 1 - 2\sin^2 t\}$

$= -\cot(\theta/2)$

⇒ To find *f''* we will differentiate *f'* with θ and then divide with equation (2).

$\frac{d\frac{dy}{dx}}{d\theta} = \frac{cosec^2(\frac{\theta}{2})}{2} \times \frac{1}{a(1-cos\theta)}$

$= \frac{-1}{2a\sin^2(\frac{\theta}{2})\times(2\sin^2(\frac{\theta}{2}))}$ $\left\{1 - cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)\right\}$ $\{cosec^2\theta = \frac{1}{\sin^2\theta}\}$

$= \frac{1}{4a} cosec^4\left(\frac{\theta}{2}\right)$.

Exercise : 10J

Question: 1

Find the second d

Solution:

(i) x^{11}

Differentiating with respect to x

$f'(x) = 11x^{11-1}$

$f'(x)=11x^{10}$

Differentiating with respect to x

$f''(x) = 110x^{10-1}$

$f''(x)= 110x^9$

(ii) 5^x

Differentiating with respect to x

$f'(x)= 5^x \log_e 5$ [Formula: $a^x = a^x \log_e a$]

Differentiating with respect to x

$f''(x)= \log_e 5 . 5^x \log_e 5$

$= 5^x(\log_e 5)^2$

(iii) tan x

Differentiating with respect to x

$f'(x)= \sec^2 x$

Differentiating with respect to x

~~$f''(x) = 2\sec x \cdot \sec x \tan x$~~

~~$= 2\sec^2 x \tan x$~~

~~(iv) $\cos^{-1} x$~~

~~Differentiating with respect to x~~

$$f'(x) = \frac{-1}{\sqrt{1-x^2}}$$

~~Differentiating with respect to x~~

$$f''(x) = \frac{-1}{2} \times \frac{-1}{(1-x^2)^{\frac{3}{2}}} \times -2x$$

~~$=$~~ $\frac{-x}{(1-x^2)^{\frac{3}{2}}}$

~~**Question: 2**~~

~~Find the second d~~

~~**Solution:**~~

~~Differentiating with respect to x~~

~~$f'(x) = \sin x + x\cos x$~~

~~Differentiating with respect to x~~

~~$f''(x) = \cos x + \cos x - x\sin x$~~

~~$= -\sin x + 2\cos x$~~

~~(ii) $e^{2x} \cos 3x$~~

~~Differentiating with respect to x~~

~~$f'(x) = 2e^{2x}\cos 3x + e^{2x}(-\sin 3x).3$~~

~~$= 2e^{2x}\cos 3x - 3e^{2x}\sin 3x$~~

~~Differentiating with respect to x~~

~~$f''(x) = 2.2e^{2x}\cos 3x + 2e^{2x}(-\sin 3x).3 - 3.2e^{2x}\sin 3x - 3e^{2x}\cos 3x.3$~~

~~$= 4e^{2x}\cos 3x - 6e^{2x}\sin 3x - 6e^{2x}\sin 3x - 9e^{2x}\cos 3x$~~

~~$= -12e^{2x}\sin 3x - 5e^{2x}\cos 3x$~~

~~(iii) $x^3 \log x$~~

~~Differentiating with respect to x~~

~~$f'(x) =$~~ $3x^2 \log x + \frac{x^3}{x}$

~~$f'(x) = 3x^2 \log x + x^2$~~

~~Differentiating with respect to x~~

$$f''(x) = 6x \log x + \frac{3x^2}{x} + 2x$$

~~$= 6x \log x + 3x + 2x$~~

~~$= 6x \log x + 5x$~~

~~**Question: 3**~~

~~If $y = x + \tan x$, $\Rightarrow \tan x = y - x$.... (i)~~

~~Differentiating with respect to x~~

$$\frac{dy}{dx} = 1 + \sec^2 x$$

~~Differentiating with respect to x~~

$$\frac{d^2y}{dx^2} = 2\sec x\ .\sec x \tan x$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2\sec^2 x\ \tan x$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{2\tan x}{\cos^2 x}$$

$\Rightarrow \cos^2 x\ \frac{d^2y}{dx^2} = 2\tan x$ ~~[putting value of tan x from (i)]~~

$$\Rightarrow \cos^2 x\ \frac{d^2y}{dx^2}\ = 2y - 2x$$

$$\Rightarrow \cos^2 x\ \frac{d^2y}{dx^2} - 2y + 2x = 0$$

~~Question: 4~~

~~If <~~

~~Solution:~~

~~Differentiating with respect to x~~

$$\frac{dy}{dx} = 2\cos x - 3\sin x$$

~~Differentiating with respect to x~~

$$\frac{d^2y}{dx^2} = \ -2\sin x - 3\cos x$$

$$\frac{d^2y}{dx^2} = \ -y$$

$$\frac{d^2y}{dx^2} + \ y = 0$$

~~Hence Proved~~

~~Question: 5~~

~~If <~~

~~Solution:~~

~~Differentiating with respect to x~~

$$y_1\ = -\ 3\sin(\log x)\frac{1}{x}\ + 4\cos(\log x)\ \frac{1}{x}$$

$\Rightarrow y_1 = \frac{-3\sin(\log x)\ +4\cos(\log x)}{x}$ ~~[we can also write this as xy_1 = -3sin(log x) +4cos(log x)~~

~~Differentiating with respect to x~~

$$y_2 = \frac{x\left(-3\cos(\log x)\frac{1}{x} - 4\sin(\log x\ )\frac{1}{x}\right) -\ (-3\sin(\log x)\ \ + 4\cos(\log x))}{x^2}$$

$$\Rightarrow x^2y_2 = \frac{-x}{x}\ (3\cos(\log x) - 4\sin(\log x)) - (y_1x)$$

$$\Rightarrow x^2y_2 = \ -y - xy_1$$

$\Rightarrow x^2y_2 + xy_1 + y = 0$

Hence Proved

Question: 6

If <

Solution:

Differentiating with respect to x

$$\frac{dy}{dx}\frac{dy}{dx} = -e^{-x}\cos xx + e^{-x}(-\sin xx)$$

$$\Rightarrow \frac{dy}{dx} = -ee^{-xx}\cos x - e^{-x}\sin x$$

$$\Rightarrow \frac{dy}{dx} = -e^{-x}(\cos x + \sin x)$$

Differentiating with respect to x

$$\frac{d^2y}{dx^2} = e^{-x}(\cos x + \sin x) - e^{-x}(-\sin x + \cos x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = e^{-x}(\cos x + \sin x - (-\sin x) - \cos x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = e^{-x}(\sin x + \sin x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2e^{-x}\sin x$$

Hence proved

Question: 7

If <

Solution:

Differentiating with respect to x

$$\frac{dy}{dx} = \sec x\tan x - \sec^2 x$$

Differentiating with respect to x

$$\frac{d^2y}{dx^2} = \sec x\ \tan x \times \tan x + \sec x \times \sec^2 x - 2\sec x \times \sec x\tan x$$

$$\Rightarrow \frac{d^2y}{dx^2} = \sec x\tan^2 x + \sec^3 x - 2\sec^2 x\tan x$$

$$\Rightarrow \frac{d^2y}{dx^2} = \sec x(\tan^2 x + \sec^2 x - 2\sec x\tan x)$$

$$\Rightarrow \frac{1}{\sec x}\frac{d^2y}{dx^2} = (\sec x - \tan x)^2$$

$$\Rightarrow \cos x\frac{d^2y}{dx^2} = y^2$$

Hence Proved

Question: 8

If <

Solution:

$$\frac{dy}{dx} = -\operatorname{cosec} x\cot x - \operatorname{cosec}^2 x$$

~~Differentiating with respect to x~~

$$\frac{d^2y}{dx^2} = \operatorname{cosec} x \cot^2 x + \operatorname{cosec}^3 x + 2\operatorname{cosec} x \times \operatorname{cosec} x \cot x$$

$$\Rightarrow \frac{d^2y}{dx^2} = \operatorname{cosec} x\,(\cot^2 x + \operatorname{cosec}^2 x + 2\operatorname{cosec} x \cot x)$$

$$\Rightarrow \frac{1}{\operatorname{cosec} x}\frac{d^2y}{dx^2} = (\cot x + \operatorname{cosec} x)^2$$

$$\Rightarrow \sin x \frac{d^2y}{dx^2} = y^2$$

$$\Rightarrow \sin x \frac{d^2y}{dx^2} - y^2 = 0$$

~~Hence proved~~

~~Question: 9~~

~~If <~~

~~Solution:~~

~~Differentiating with respect to x~~

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\Rightarrow (1+x^2)\frac{dy}{dx} = 1$$

~~Differentiating with respect to x~~

$$(1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$$

~~Hence Proved~~

~~Question: 10~~

~~If <~~

~~Solution:~~

~~Differentiating with respect to x~~

$$\frac{dy}{dx} = \cos(\sin x)\,\cos x$$

~~Differentiating with respect to x~~

$$\frac{d^2y}{dx^2} = -\sin(\sin x)\cos x\,\cos x - \sin x\cos(\sin x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -y\cos^2 x - \sin x\,\frac{\frac{dy}{dx}}{\cos x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -y\cos^2 x - \tan x\frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} + y\cos^2 x + \tan x\frac{dy}{dx} = 0$$

~~Hence Proved~~

~~Question: 11~~

~~If <~~

~~Solution:~~

~~Differentiating with respect to x~~

$y_1 = -a\sin(\log x)\frac{1}{x}$ ~~[can also be written as $xy_1 = a\sin(\log x)$]~~

~~Differentiating with respect to x~~

$$y_2 = \frac{-x\,a\cos(\log x)\frac{1}{x} + a\sin(\log x)}{x^2}$$

$\Rightarrow x^2y_2 = -y - xy_1$

$\Rightarrow x^2y_2 + xy_1 + y = 0$

~~Hence Proved~~

~~Question: 12~~

~~Find the se~~

~~Solution:~~

~~Differentiating with respect to x~~

$$\frac{dy}{dx} = 3e^{3x}\sin 4x + 4e^{3x}\cos 4x$$

~~Differentiating with respect to x~~

$\Rightarrow \frac{d^2y}{dx^2} = 9e^{3x}\sin 4x + 12e^{3x}\cos 4x + 12e^{3x}\cos 4x - 16e^{3x}\sin 4x$

$\Rightarrow \frac{d^2y}{dx^2} = 24e^{3x}\cos 4x - 7e^{3x}\sin x$

$\Rightarrow \frac{d^2y}{dx^2} = e^{3x}(24\cos x - 7\sin x)$

~~Question: 13~~

~~Find the se~~

~~Solution:~~

$$y = \frac{1}{2}[\sin(5x + 3x) + \sin(5x - 3x)]$$

$$y = \frac{1}{2}\sin 8x + \frac{1}{2}\sin 2x$$

~~Differentiating with respect to x~~

$$\frac{dy}{dx} = \frac{8}{2}\cos 8x + \frac{2}{2}\cos 2x$$

$\Rightarrow \frac{dy}{dx} = 4\cos 8x + \cos 2x$

~~Differentiating with respect to x~~

$$\frac{d^2y}{dx^2} = -32\sin 8x - 2\sin 2x$$

~~Hence Proved~~

~~Question: 14~~

~~If Differentiating with respect to x~~

$$\frac{dy}{dx} = \sec^2 x\, e^{\tan x}$$

$\Rightarrow \frac{1}{\sec^2 x}\frac{dy}{dx} = e^{\tan x}$

$\Rightarrow \cos^2 x \frac{dy}{dx} = e^{\tan x}$

~~Differentiating with respect to x~~

$$(\cos^2 x)\frac{d^2y}{dx^2} - (2\cos x \sin x)\frac{dy}{dx} = \sec^2 x\, e^{\tan x}$$

$\Rightarrow (\cos^2 x)\frac{d^2y}{dx^2} - \sin 2x \frac{dy}{dx} = \frac{dy}{dx}$

$\Rightarrow (\cos^2 x)\frac{d^2y}{dx^2} - \sin 2x \frac{dy}{dx} - \frac{dy}{dx} = 0$

$\Rightarrow (\cos^2 x)\frac{d^2y}{dx^2} - (\sin 2x + 1)\frac{dy}{dx} = 0$

~~Hence Proved~~

~~Question: 15~~

~~If <~~

~~Solution:~~

~~Differentiating with respect to x~~

$$\frac{dy}{dx} = \frac{\frac{1}{x} \times x - \log x}{x^2}$$

$\Rightarrow \frac{dy}{dx} = \frac{1 - \log x}{x^2}$

~~Differentiating with respect to x~~

$$\frac{d^2y}{dx^2} = \frac{\frac{-1}{x} \times x^2 - 2x(1 - \log x)}{x^4}$$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-x - 2x(1 - \log x)}{x^4}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-1 - 2 + 2\log x}{x^3}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{(2\log x - 3)}{x^3}$

~~Hence proved~~

~~Question: 16~~

~~If <~~

~~Solution:~~

~~Differentiating with respect to x~~

$$\frac{dy}{dx} = ae^{ax}\cos bx - be^{ax}\sin bx$$

$$be^{ax}\sin bx = ae^{ax}\cos bx - \frac{dy}{dx}$$

~~Differentiating with respect to x~~

$$\frac{d^2y}{dx^2} = a^2e^{ax}\cos bx - abe^{ax}\sin bx - abe^{ax}\sin bx - b^2e^{ax}\cos bx$$

$\Rightarrow \frac{d^2y}{dx^2} = a^2e^{ax}\cos bx - 2abe^{ax}\sin bx - b^2e^{ax}\cos bx$

$\Rightarrow \frac{d^2y}{dx^2} = a^2e^{ax}\cos bx - 2a\left(ae^{ax}\cos bx - \frac{dy}{dx}\right) - b^2e^{ax}\cos bx$

$\Rightarrow \frac{d^2y}{dx^2} = a^2e^{ax}\cos bx - 2a^2e^{ax}\cos bx + 2a\frac{dy}{dx} - b^2e^{ax}\cos bx \Rightarrow$

$\frac{d^2y}{dx^2} = -a^2e^{ax}\cos bx - b^2e^{ax}\cos bx + 2a\frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = -(a^2+b^2)(e^{ax}\cos bx) + 2a\frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = -(a^2+b^2)y + 2a\frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2+b^2)y = 0$

Hence Proved

Question: 17

If <

Solution:

Taking log on both sides

$\log y = a\cos^{-1}x \log e$

$\log y = a\cos^{-1}x$

Differentiating with respect to x

$$\frac{1}{y}\frac{dy}{dx} = \frac{-a}{\sqrt{1-x^2}}$$

$\Rightarrow \frac{dy}{dx} = \frac{-ae^{a\cos^{-1}x}}{\sqrt{1-x^2}}$

Differentiating with respect to x

$$\frac{d^2y}{dx^2} = \frac{\frac{a^2e^{a\cos^{-1}x}}{\sqrt{1-x^2}} \times \sqrt{1-x^2} - ae^{\cos^{-1}x} \times \frac{2x}{2\sqrt{1-x^2}}}{(1-x^2)}$$

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} = a^2e^{\cos^{-1}x} - \frac{axe^{\cos^{-1}x}}{\sqrt{1-x^2}}$

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} = a^2y + x\frac{dy}{dx}$

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - a^2y - x\frac{dy}{dx} = 0$

Hence Proved

Question: 18

If <

Solution:

Differentiating with t

$$\frac{dx}{dt} = 2at \; \frac{dy}{dt} = 2a$$

$$\frac{dy}{dt} \div \frac{dx}{dt} = \frac{dy}{dx}$$

$\Rightarrow \frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$

Differentiating with respect to x

$$\frac{d^2y}{dx^2} = \frac{-1}{t^2}\frac{dt}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-1}{4} \times \frac{1}{2at}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-1}{4} \times \frac{1}{4a}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-1}{16a}$$

~~Question: 19~~

~~If <~~

~~Solution:~~

~~Differentiating with respect to θ~~

$$\frac{dx}{d\theta} = a(1 - \cos\theta)\ \frac{dy}{d\theta} = a\sin\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{a\sin\theta}{a(1-\cos\theta)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin\theta}{1-\cos\theta}$$

~~Differentiating with respect to x~~

$$\frac{d^2y}{dx^2} = \frac{\cos\theta(1 - \cos\theta) - \sin^2\theta}{(1 - \cos\theta)^2} \times \frac{d\theta}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{\cos\theta - \cos^2\theta - \sin^2\theta}{(1-\cos\theta)^2} \times \frac{1}{a(1-\cos\theta)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{\cos\theta - 1}{(1-\cos\theta)^2} \times \frac{1}{a(1-\cos\theta)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-(1-\cos\theta)}{(1-\cos\theta)^2} \times \frac{1}{a(1-\cos\theta)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-1}{a(1-\cos\theta)^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-1}{a\left(1-(-1)\right)^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-1}{4a}$$

~~Question: 20~~

~~If <~~

~~Solution:~~

~~Differentiating with respect to~~

$$\frac{dy}{dx} = \cos(\log x)\ \frac{1}{x}$$

$$\Rightarrow x\frac{dy}{dx} = \cos(\log x)$$

~~Differentiating with respect to x~~

$$\frac{d^2y}{dx^2} = \frac{-\sin(\log x)\frac{1}{x}x - \cos(\log x)}{x^2}$$

$\Rightarrow x^2 \frac{d^2y}{dx^2} = -\sin(\log x) - \cos(\log x)$

$\Rightarrow x^2 \frac{d^2y}{dx^2} = -y - x\frac{dy}{dx}$

$\Rightarrow x^2 \frac{d^2y}{dx^2} + y + x\frac{dy}{dx} = 0$

~~Hence Proved~~

~~Question: 21~~

~~If <~~

~~Solution:~~

$\sqrt{1-x^2}\, y = \sin^{-1} x$

~~Differentiating with respect to x~~

$$\sqrt{1-x^2}\frac{dy}{dx} - \frac{2xy}{2\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$$

$\Rightarrow (1-x^2)\frac{dy}{dx} - xy = 1$

~~Differentiating with respect to x~~

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - x\frac{dy}{dx} - y = 0$$

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$

~~Hence Proved~~

~~Question: 22~~

~~If <~~

~~Solution:~~

$y = e^x \sin x$

~~Differentiating with respect to x~~

$$\frac{dy}{dx} = e^x \sin x + e^x \cos x$$

$$\left[e^x \cos x = \frac{dy}{dx} - e^x \sin x\right]$$

~~Differentiating with respect to x~~

$$\frac{d^2y}{dx^2} = e^x \sin x + e^x \cos x + e^x \cos x - e^x \sin x$$

$\Rightarrow \frac{d^2y}{dx^2} = 2e^x \cos x$

$\Rightarrow \frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 2e^x \sin x$

$\Rightarrow \frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 2y$

$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

~~Question: 23~~

~~If~~

$$= a\left(-\sin\theta + \frac{1}{\sin\theta}\right)$$

$$= a\left(\frac{-\sin^2\theta + 1}{\sin\theta}\right)$$

$$= \frac{a\cos^2\theta}{\sin\theta}$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$$

$$\frac{dy}{dx} = a\cos\theta \times \frac{\sin\theta}{a\cos^2\theta}$$

$$\frac{dy}{dx} = \tan\theta$$

~~Differentiating with respect to x~~

$$\frac{d^2y}{dx^2} = \sec^2\theta\frac{d\theta}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \left(\sqrt{2}\right)^2 \times \frac{\sin\theta}{a\cos^2\theta}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2 \times \frac{\frac{1}{\sqrt{2}}}{a\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{2\sqrt{2}}{a}$$

~~**Question: 24**~~

~~If~~

$$= -\sin t + \frac{1}{\sin t}$$

$$= \frac{-\sin^2 t + 1}{\sin t}$$

$$= \frac{\cos^2 t}{\sin t}$$

$$\frac{dy}{dt} = \cos t$$

~~Differentiating with respect to t~~

$\Rightarrow \frac{d^2y}{dt^2} = -\sin t$ ~~[Putting t = π /4]~~

$$\Rightarrow \frac{d^2y}{dt^2} = -\frac{1}{\sqrt{2}}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dx}{dt}$$

$$\Rightarrow \frac{dy}{dx} = \cos t \times \frac{\sin t}{\cos^2 t}$$

$$\Rightarrow \frac{dy}{dx} = \tan t$$

~~Differentiating with respect to x~~

$\frac{d^2y}{dx^2} = \sec^2 t\frac{dt}{dx}$ ~~[Putting t = π /4]~~

$$\Rightarrow \frac{d^2y}{dx^2} = (\sqrt{2}) \times \frac{\sin t}{\cos^2 t}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2 \times \frac{\frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2\sqrt{2}$$

~~**Question: 25**~~

~~If <~~

~~**Solution:**~~

~~$y = x^x$~~

~~Taking log on both sides~~

~~log y = x log x~~

~~Differentiating with respect to x~~

$$\frac{1}{y}\frac{dy}{dx} = 1 + \log x \ldots (i)$$

$$\Rightarrow \frac{dy}{dx} = y(1 + \log x)$$

~~Differentiating with respect to x~~

$$\frac{d^2y}{dx^2} = \frac{y}{x} + (1 + \log x)\frac{dy}{dx}$$ ~~[putting value of (1 + log x) from (i)]~~

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{y}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{y}{x} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 = 0$$

~~Hence Proved~~

~~**Question: 26**~~

~~If <~~

~~**Solution:**~~

$$y = (\cot^{-1} x)^2$$

~~Differentiating with respect to x~~

$$\frac{dy}{dx} = \frac{-2\cot^{-1} x}{1 + x^2}$$

$$\Rightarrow -2\cot^{-1} x = (1 + x^2)\frac{dy}{dx}$$

~~Differentiating with respect to x~~

$$\frac{d^2y}{dx^2} = \frac{2 + 4x\cot^{-1} x}{(1 + x^2)^2}$$

$$\Rightarrow (1 + x^2)^2 \frac{d^2y}{dx^2} - 4x\cot^{-1} x = 2$$

$$\Rightarrow (1 + x^2)^2 \frac{d^2y}{dx^2} - 2x\left(-(1 + x^2)\frac{dy}{dx}\right) = 2$$

$$\Rightarrow (1 + x^2)\frac{d^2y}{dx^2} + 2x(1 + x^2)\frac{dy}{dx} = 2$$

~~Hence Proved~~

~~**Question: 27**~~

~~If <~~

~~**Solution:**~~

~~Differentiating with respect to x~~

$$\frac{dy}{dx} = m\left\{x + \sqrt{x^2+1}\right\}^{m-1}\left(1 + \frac{2x}{2\sqrt{x^2+1}}\right)$$

$$\Rightarrow \frac{dy}{dx} = m\{x + \sqrt{x^2+1}\}^{m-1}\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right)$$

$$\Rightarrow \frac{dy}{dx} = m\frac{\{x+\sqrt{x^2+1}\}^m}{\sqrt{x^2+1}}$$

$$\Rightarrow \frac{dy}{dx} = m\frac{y}{\sqrt{x^2+1}}$$

$$\left[\frac{dy}{dx}\sqrt{x^2+1} = my\right]$$

~~Differentiating with respect to x~~

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{m\frac{dy}{dx}\sqrt{1+x^2} - \frac{2xmy}{2\sqrt{x^2+1}}}{(1+x^2)}$$

$$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} = m^2y - x\frac{dy}{dx}$$

$$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$$

~~Hence Proved~~

~~**Question: 28**~~

~~If <~~

~~**Solution:**~~

$$\frac{dy}{dx} = \frac{1 + \frac{2x}{2\sqrt{x^2+a^2}}}{x + \sqrt{x^2+a^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2\sqrt{x^2+a^2}+2x}{2\sqrt{x^2+a^2}} \times \frac{1}{x+\sqrt{x^2+a^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{x^2+a^2}}$$

~~Differentiating with respect to x~~

$$\frac{d^2y}{dx^2} = \frac{-2x}{2(x^2+a^2)\sqrt{x^2+a^2}}$$

$$\Rightarrow (x^2+a^2)\frac{d^2y}{dx^2} = \frac{-x}{\sqrt{x^2+a^2}}$$

$$\Rightarrow (x^2+a^2)\frac{d^2y}{dx^2} = -x\frac{dy}{dx}$$

$$\Rightarrow (x^2+a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$$

~~Hence Proved~~

~~**Question: 29**~~

~~If <~~

~~**Solution:**~~

~~Differentiating with respect to θ~~

$$\frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)\ \frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta)$$

$$\Rightarrow \frac{dx}{d\theta} = a\theta\cos\theta \Rightarrow \frac{dy}{d\theta} = a\theta\sin\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \frac{dy}{dx} = \tan\theta$$

~~Differentiating with respect to x~~

$$\frac{d^2y}{dx^2} = \sec^2\theta \times \frac{d\theta}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \sec^2\theta \times \frac{1}{a\theta\cos\theta}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \sec^2\theta \times \frac{\sec\theta}{a\theta}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{\sec^3\theta}{a\theta}$$

~~Hence Proved~~

~~**Question: 30**~~

~~If <~~

~~**Solution:**~~

$$\frac{dx}{d\theta} = -a\sin\theta + b\cos\theta\ \frac{dy}{d\theta} = a\cos\theta + b\sin\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{a\cos\theta + b\sin\theta}{-a\sin\theta + b\cos\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{y}$$

~~Differentiating with respect to x~~

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{y - x\frac{dy}{dx}}{y^2}$$

$$\Rightarrow y^2\frac{d^2y}{dx^2} = y - x\frac{dy}{dx}$$

$$\Rightarrow y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$$

~~Hence Proved~~