Some Mechanical Properties of Matter

Exercise Solutions

Solution 1:

(a) Stress = Force/Area = mg/A

= $(10x10)/(4x10^{-6})$

= 2.5×10^{-7} N/m^2

(b) Strain = Stress/Y

= $[2.5 \times 10^{-7}]/[2x10^{11}]$

= 1.25×10^{-4} m

(c) Let elongation be I

I = L x strain = 3.75×10^{-4} m

Solution 2:

Area of cross-section of the cylinder = $A = \pi r^2$

= $\pi \times 0.02^2$

= $4\pi \times 10^{-4}$ m^2

(a) stress = Force/Area

= $[10x10]/[4\pi \times 10^{-4}]$

= 7.96×10^5 N/m^2

(b) Strain = Stress/Y

= $[7.96 \times 10^5]/[2x10^{11}]$

= 3.98×10^{-6}

(c) Let compression be I

I = 2 x (3.98×10^{-6}) = 8×10^{-6} m (approx)

Solution 3:

The elastic limit of steel is 8×10^8 N m^{-2} and its Young modulus 2×10^{11} N m^{-2}.

Strain = Stress/Y and Stress = I/L

=> Stress/Y = I/L

=> I = (stress L)/Y

= $[8 x 10^8 x 0.5]/[2 x 10^{11}]$

= $2 x 10^{-3}$ m

Solution 4:

(a) Let the stress in steel wire be σ_s and in copper σ_c

Now, $\sigma_s = F/A$ and $\sigma_c = F/A$

=> $\sigma_s/\sigma_c = 1$

(b) Strain, $\varepsilon = \sigma/Y$

Strain in the steel wire = $\varepsilon_s = [F/A]/[2x10^{11}]$

and strain in the copper wire = $\varepsilon_c = [F/A]/[1.3x10^{11}]$

Hence, $\varepsilon_c/\varepsilon_s = 20/13$

Solution 5:

[Strain in the steel wire]/[strain in the copper wire] = $[2x10^{11}]/[1.3x10^{11}]$

= 20/13

= 1.54 (approx)

Solution 6:

(a) Stress on the lower wire = σ = [(total weight of lower wire)g]/[Area of cross section of upper wire]

= $[m+m_1]g/A_1$

Let say, A_1 = Area of cross section of upper wire
m = weight of the load and m_1 = mass of the block = 10 kg

For maximum load, we take in the maximum stress that the wire can withstand.

=> $8 \times 10^8 = [(m+10) \times 10]/[3 \times 10^{-7}]$

=> m = 14 kg

The upper wire has a total weight of $m+m_1+m_2$

Hence, stress on the lower wire = $[(m+m_1+m_2)g]/A_2$

For maximum load,

=> $8 \times 10^8 = [(m+10+20) \times 10]/[3 \times 10^{-7}]$

Given, m_2 = 20 kg

=> m = 18 kg

(b) For m_2 = 36 kg.

=> $8 \times 10^8 = [(m+10+36) \times 10]/[6 \times 10^{-7}]$

=> m = 2 kg

The maximum load that can be put is 2 kg, Upper wire will break first load is increased.

Solution 7:
We know, stress = F/A and strain = I/L

Young's modulus = Y = FL/AI

= $[100x2]/[2x10^{-4}x0.01]$

= 1×10^8 N/m^2

Solution 8:
We know, stress = T/A and strain = I/L

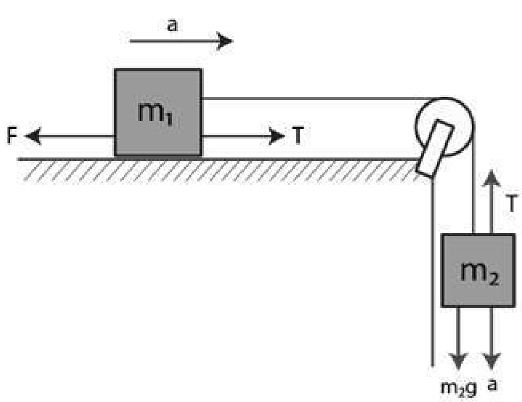
Young's modulus = Y = TL/AI

=> T = YAI/L

= $[1.9x10^{11}x4x10^{-4}x0.001]/[2]$

=> T = 3.8×10^4 N

Solution 9:



Force = F = m_2 (g/2) ......(given)

From free body diagram:

T - F = m_1 a [for mass m_1]

=> T - $m_2g/2 = m_1a$ ...(1)

m_2g - T = m_2a ....(2) [for mass m_2]

Using (1) and (2)

$$=> T - \frac{m_2 g}{2} = m_1\left(g - \frac{T}{m_2}\right)$$

$$=> T = \frac{m_2 g(2m_1 + m_2)}{2(m_1 + m_2)}$$

We know, stress = T/A
and Young's modulus = Y = stress/strain

Therefore, strain = T/AY

= $[m_2g(2m_1+m_2)]/[2AY(m_1+m_2)]$

Solution 10:
At the point of release, the tension = T = mg + mv^2/r

Force which brings elongation is centrifugal force = F = mv^2/r ...(1)

From work energy theorem,
$(1/2)mv^2 = mgr(1 - \cos\theta)$

or $v^2 = 2gr(1-\cos\theta)$

(1)=> $F = (m/r)\, 2gr(1-\cos\theta) = 2mg(1-\cos\theta)$

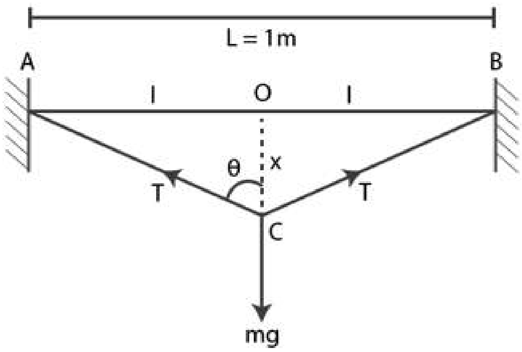
Again, Young's modulus = Y = FL/Al

=> $Y = [2mg(1-\cos\theta)L]/Al$

$$\cos\theta = 1 - \frac{2 \times 10^{11} \times \pi \times 0.25 \times 10^{-6} \times 0.002}{2 \times 20 \times 10 \times 4} = 0.803$$

=> $\theta = \cos^{-1}(0.803)$ or 36.4^0 (approx)

Solution 11:



From the figure,

$\cos\theta = x/\sqrt{x^2+l^2}$

$= (x/l)(1 - x^2/2l^2 + ...)$

[Using binomial theorem]

If $x << l$, $x^2/l^2 = 0$

=> $\cos\theta = (x/l)$

Again, change in length,

$$\Delta L = (AC + CB) - AB = 2(l^2 + x^2)^{1/2} - 2l$$

$$\Delta L = 2l\left(1 + \frac{x^2}{l^2}\right)^{1/2} - 2l$$

$$= \frac{x^2}{l}$$

Now, Using Young's modulus,

$T = (YA\, \Delta l)/L$

=> $T = 8 \times 10^5 \times (x^2/l)$

Forces are balance, $2T\cos\theta = mg$

=> $2 \times 8 \times 10^5 \times (x^2/l) \times (x/l) = 2.16 \times 10$

=> $x = 1.5$ cm

Solution 12:

The longitudinal strain= T/AY

Where T = tension, A = cross-sectional area and Y = Young's modulus

=> longitudinal strain = $20/[0.01 \times 10^{-4} \times 1.1 \times 10^{11}] = 1.82 \times 10^{-4}$

The area of circular section be of the diameter D, $A = \pi D^2/4$

=> $dA = 2\pi D\ dD/4$

=> $dA = 2 \times 0.01 \times 5.82 \times 10^{-5}$

$= 1.164 \times 10^{-6}\ cm^2$

Solution 13:

Let v be initial volume and v' be the final volume.

Change in volume = $\Delta V = -(v - v')$

and pressure, $P = -B(\Delta v/v)$

$= B(1 - v'/v)$

Therefore, $dP = B\ (dv'/v)$

We are given that, $dv'/v' = (0.01/100)$

=> $dP = B \times dv'/v' \times v'/v$

[Assuming, $v'/v = 1$]

=> $dP = B\ (dv'/v')$

$= 2.1 \times 10^9 \times (0.01/100$

$= 2.1 \times 10^5\ N/m^2$

Solution 14:

The bulk strain at depth h:

$\Delta v/v = P/B = \rho gh/B$

Now, the density at depth is $\rho' = \rho\ (v/v')$

V' is compressed volume at depth h and $v' = v - \Delta v$

$\Rightarrow \rho' = \rho \times [v/(v-\Delta v)]$

$= \rho B/(B-\rho gh)$

$= [1030 \times 2 \times 10^9]/[2 \times 10^9-(1030 \times 10 \times 400)]$

$= 1032\ kg/m^3$

Therefore, change in density = 1032 - 1030 = 2 kg/m^3

Solution 15:

Shearing strain = $\Delta x/d = \Delta x/0.0005 = 200\ \Delta x$

Shearing Stress = $F/A = 10/(4 \times 10^{-4}) = 25000\ N/m^2$

Rigidity modulus = (shearing stress)/(shearing strain)

$\Rightarrow 8.4 \times 10^{10} = 25000/200\Delta x$

$\Rightarrow \Delta x = 1.5 \times 10^{-9}\ m$

Solution 16:

Length of thread = L = 5 cm or 0.05 m
Surface tension of water = S = 0.076 N/m

Therefore, force due to surface tension: $F = SL = 0.076 \times 0.05$

$= 3.8 \times 10^{-3}\ N$

Solution 17:

(a) excess pressure inside a drop of mercury of radius 2 mm:

$P = 2S/r = [2x0.465]/0.002 = 465\ N/m^2$

(b) excess pressure inside a snap bubble of radius 4 mm

$P = 4S/r = [4x0.03]/0.004 = 30\ N/m^2$

(c) excess pressure inside an air bubble of radius 4 mm formed inside a tank of water.

$P = 2S/r = [2x0.076]/0.004 = 38\ N/m^2$

Solution 18:

(a) Force exerted by air above:

$F = PA = 10^5 \times 10^{-6} = 0.1\ N$

(b) Force exerted by the mercury below the surface area:
Let P' be the required pressure,

=> $P' = P_o + 2T/r$

Now, $F = P'A = (P_o + 2T/r)r$

$= (0.1 + (2x0.465)/(4x10^{-3})) \times 10^{-6}$

$= 0.10023\ N$

(c) Force exerted by the mercury surface in contact with it:

$F = PA = (2T/r)\ A$

$= (2x0.465)/(4x10^{-3})$

$= 0.00023\ N$

Solution 19:

Surface tension of water = T = 7.5×10^{-2} N/m
Considering cos θ = 1
Radius of capillary A = r_A = 0.5 mm = 0.5×10^{-3} m

Height of water level in capillary A:

$h_A = [2T \cos\theta]/[r_A \rho g]$

$= [2 \times 7.5 \times 10^{-2}]/[0.5 \times 10^{-3} \times 1000 \times 10]$

$= 3 \times 10^{-2}$ m or 3 cm

Radius of capillary B = r_B= 1 mm = 1×10^{-3} m

Height of water level in capillary B:

$h_B = [2T \cos\theta]/[r_B \rho g]$

$= [2 \times 7.5 \times 10^{-2}]/[1 \times 10^{-3} \times 1000 \times 10]$

$= 15 \times 10^{-3}$ m or 1.5 cm

Radius of capillary C = r_C = 1.5 mm = 1.5×10^{-3} m

Height of water level in capillary C:

$h_C = [2T \cos\theta]/[r_C \rho g]$

$= [2 \times 7.5 \times 10^{-2}][1.5 \times 10^{-3} \times 1000 \times 10]$

$= 15/1.5 \times 10^{-3}$ m or 1 cm

Solution 20:
We know, The rise or depression in capillary = h= [2T cosθ]/[rρg]

Let the angle of mercury, θ = 0°

depression in mercury = -2 cm = -0.02 cm (Given)

For mercury = h_{hg} = -0.02 = [2 T_{hg}]/[rρ_{hg} g] ...(1)

Let h_w be the rise in water.

h_w = [2T_w]/[r_ww g] ....(2)

Dividing (2) by (1), we get

h_w/h_{hg} = (0.075/0.465) x 13.6

= 2.19

Now, height of the water level = h_w = 2 x 2.19 = 4.38 cm

Solution 21:
Rise of mercury column = h = [2 T cosθ]/[rρg]
Substituting given values,

h = [2x0.466xcos130°]/[0.001x13600x10]

= -0.005 m or -0.5 cm
[The negative sign is used as mercury goes down in a capillary tube.]

Height of mercury in the tube = h+H = -0.5 + 76 = 75.5 cm

Solution 22:
Pressure = p = 2T/r
Where T = surface tension = 0.075 N/m (given)
and r = Radius of the capillary tube = 0.5 mm or 0.0005 m (given)

=> p = 300 N/m^2

Pressure at a depth of 5 cm = p' = ρgh = 1000 x 9 x 0.05 490 N/m^2

Let P be the pressure of water just below surface, then
Difference in pressure : ΔP = (P + p') - (P + p)

= (P + 490) - (P + 300)

= 190 N/m^2

Solution 23:
Area of surface = A = πr^2
where r = 6 cm = 0.06 m (given)

=> A = 0.0113 m^2

Now, surface energy = E = SA = 0.075 x 0.0113 = 8.4 x 10^4 J

Solution 24:
Volume of drop = v = $(4/3)\pi r^3$
Volume of each drop after division = v' = v/8 = $(1/6)\pi r^3$

Now the new radius of each drop be r':

$(4/3)\pi r'^3 = (1/6)\pi r^3$

or r' = r/2

The total area of all droplets= A' = 8 x $4\pi r^{2'}$ = $8\pi r^2$

Therefore, increase in area = $8\pi r^2 - 4\pi r^2 = 4\pi r^2$

Therefore, increase in energy = E = S x (increase in area) = 2.34 x 10^{-5} J

[S = 0.465 J m^{-2} and r = 0.002 cm]

Solution 25:
Let h be the height of water raised in the capillary.

$h = [2T \cos\theta]/r\rho g$ ....(1)

Here T = Surface tension of water = 0.076 N/m, $\theta = 0$^o, r = 0.001, $\rho = 1000$ and g = 10]

=> h = 15 cm

(b) length of capillary, h' = h/2 ...(A), then

$h' = [2T \cos\theta]/r\rho g$ ...(2)

Dividing (2) by (1)

$h'/h = \cos\theta$
using (A)

or $\theta = 60$ degrees

Solution 26:
(a) depression of mercury column in the capillary

$h = [2T \cos\theta]/r\rho g$

$= [2 \text{x} 0.465 \text{x} \cos 135^o]/[0.001 \text{x} 13600 \text{x} 9.8]$

= 4.9 cm

(b) Let ϕ be the angle made by mercury.

new height is h' = h/2

Now, $h' = [2T \cos\phi]/r\rho g$

From above equations, $h'/h = \cos\phi/\cos(135^o)$

=> $\cos\phi = -0.3535$

or $\phi = 111$ degrees (approx)

Solution 27:

The weight of water in unit volume = W = ρ h dg x 1

and given force = F = 2Tl = 2 x 0.0075 x 1 = 0.150 N

Now, W = ρ h dg = 1000h x 0.001 x 10 = 10h N

or 10h = 0,150

or h = 0.15/10 = 1.5 cm

Solution 28:

Volume of ice cube of edge = V = 1 cm^3 = 10^{-6} m^3
keeping in mind that, volumes do not change when state changes.

As gravity has no effect here, the water after melting will form a sphere,

$(4/3)\pi r^3 = 10^{-6}$

or $r = [[10^{-6} x3]/4\pi]^{1/3}$ m

Now, The surface area of the sphere

$A = 4\pi r^2$

Using value of r, and solving we get

$A = (36\pi)^{1/3}$ cm^2

Solution 29:

The loop will tale circular shape after pricking. Radius can be calculated by using relation,
$l = 2\pi R$

=> $R = (6.28)/[2 \times 3.14]$

= 1 cm or 10^{-2} m

$2T \sin(d\theta)$ force inward direction is balanced by surface tension force in outward direction.

Therefore, $2T \sin(d\theta)$ = length of arc x surface tension

for small angle, $\sin(d\theta) = d\theta$

$2T\, d\theta = 2R\, d\theta \times S$
Where S is the surface tension.

Now, $T = SR$

$= 0.030 \times 10^{-2}$

$= 3 \times 10^{-4}$ N

Solution 30:

(a) viscous force = F = $6\pi\eta rv$

= 6x0.8x3.14x0.001x0.01

= 1.5×10^{-4} N

(b) The hydrostatic force exerted by the glycerin on the sphere:

Force = $F_B = (4/3)\, \pi r^3 \times \rho g$

= $(4/3) \times 3.14 \times 0.001^3 \times 1260 \times 9.8$

= 5.2×10^{-5} N (approx)

(c) Terminal velocity with which the sphere will move down without acceleration

viscous force= $F = 6\pi\eta rv = 6\pi \times 0.8 \times 0.001 \times v = 0.0048\, \pi v$

The upwards force is due to the buoyant force in glycerin.

$F_B = 5.2 \times 10^{-5}$ N

and the weight downwards, $F_m = 50 \times 10^{-6} \times 9.8$

= 4.9×10^{-4} N

As per situation, there should be no acceleration downwards:

$F + F_B = F_m$

=> $0.0048\pi v + 5.2 \times 10^{-5} = 4.9 \times 10^{-4}$

=> v = 0.029 m/s

=> v = 2.9 cm/s

Solution 31:

Viscous force is balanced by the weight:

$6\pi\eta rv = \rho\ vg$

=> $v = [1000 \times (4/3)\ \pi r^3 \times 9.9]/[6\pi\eta r]$

=> $v = [2000 \times 0.002^2 \times 1.1]/[1.8 \times 10^{-5}] = 4.9$ m/s

Solution 32:

Reynolds number = $N = \rho v(2r)/\eta$

$= [1000 \times 0.06 \times 2 \times 0.01]/[0.001]$

$N = 1200$ (approx)