CBSE Class 11 Mathematics
Sample Papers 02 (2020-21)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
ii. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
iii. Both Part A and Part B have choices.

Part – A:

i. It consists of two sections- I and II.
ii. Section I comprises of 16 very short answer type questions.
iii. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

i. It consists of three sections- III, IV and V.
ii. Section III comprises of 10 questions of 2 marks each.
iii. Section IV comprises of 7 questions of 3 marks each.
iv. Section V comprises of 3 questions of 5 marks each.
v. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part - A Section - I

1. Write the interval in set builder form [-23, 5)

OR

List all the elements of set {x: x is a vowel in the word EQUATION}.

2. Show that if $x^2 + y^2 = 1$, then the point $(x, y, \sqrt{1 - x^2 - y^2})$ is at a distance 1 unit from the origin.

3. Evaluate: cos 47° cos 13° - sin 47° sin 13°.

OR

Determine whether the function is even or odd or neither: $h(x) = x^2 + x + 4$.

4. Express the complex numbers $(5i)\left(-\frac{3}{5}i\right)$ in the form a + ib.

5. A, B and C are three cities. There are 5 routes from A to B and 3 routes from B to C. How many different routes are there from A to C via B?

OR

Is 3! + 4! = 7!?

6. If the mth term of an A.P. be $\frac{1}{n}$, and nth term be $\frac{1}{m}$ then show that its (mn)th term is 1.

7. Find the slope of line, whose inclination is 60°.

OR

Find all points on x + y = 4 that lie at a unit distance from the line 4x + 3y - 10 = 0.

8. Find the equation of parabola when the vertex is at (0, 0) and focus is at (0, 4).

9. Write the interval in the set-builder form: E = [-10, 0)

OR

State whether A = B or not if set A = {2, 4, 6, 8, 10} and set B = {x: x is a positive even integer and $x \leqslant 10$}

10. Three coins are tossed once. Find the probability of getting: 3 tails

11. Name the octant in (-7, 2, -5) point lies.

12. Evaluate: ${}^{14}C_3$.

13. Prove that: $\frac{\tan\left(\frac{\pi}{2}-x\right)\sec(\pi-x)\sin(-x)}{\sin(\pi+x)\cot(2\pi-x)\operatorname{cosec}\left(\frac{\pi}{2}-x\right)} = 1.$

14. Find the value of cos 135°.

15. Solve for x, the inequalities: $\frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}, (x > 0)$

16. If A = {a, b, c, d, e, f}, B = {c, e, g, h} and C = {a, e, m, n}, find $A \cup B$

Section - II

17. **Read the Case study given below and attempt any 4 subparts:**

A farmer, Ramgarh, took a bank loan from SBI for repairing his house. But he could not pay the amount on time.

This resulted in the accumulation of interest and the amount to pay reached Rs.1,00,000. After a few months, the farmer opened a shop that resulted in enough income and the income increased on a regular basis. So he decided to pay the bank loan in a different manner.

The farmer visited the bank. He made an agreement with the bank that he will start paying the amount of Rs.1,00,000 without interest from Jan 2020. In January he will pay Rs.5000 and will increase the payment by Rs. 100 in each month, as shown in the figure.

Now answer the following questions:

i. In how many months will the farmer clear the loan amount?
   a. 16
   b. 15
   c. 18
   d. 20

ii. How much amount he has to pay in last month in rupees?
   a. 1400
   b. 1500
   c. 1800
   d. 2000

iii. In which month he will pay Rs.6000?
   a. 14th
   b. 10th
   c. 12th
   d. 11th

iv. How much amount he will pay in 10th month in Rs.?

a. 6000
b. 6400
c. 7500
d. 7000

v. How much amount in rupees till 10th month he will have paid?

a. 54500
b. 50000
c. 55000
d. 60000

18. **Read the Case study given below and attempt any 4 subparts:**

A state cricket authority has to choose a team of 11 members, to do it so the authority asks 2 coaches of a government academy to select the team members that have experience as well as the best performers in last 15 matches. They can make up a team of 11 cricketers amongst 15 possible candidates. In how many ways can the final eleven be selected from 15 cricket players if:

i. there is no restriction

a. 1365
b. 2365
c. 1465
d. 1375

ii. one of then must be included

a. 1002
b. 1003
c. 1001
d. 1004

iii. one of them, who is in bad form, must always be excluded

a. 480
b. 364

c. 1365

d. 640

iv. Two of them being leg spinners, one and only one leg spinner must be included?

a. ${}^{2}C_{1} \times {}^{13}C_{10}$

b. ${}^{2}C_{1} \times {}^{10}C_{13}$

c. ${}^{1}C_{2} \times {}^{13}C_{10}$

d. ${}^{2}C_{10} \times {}^{13}C_{10}$

v. If there are 6 bowlers, 3 wicket-keepers, and 11 batsmen in all. The number of ways in which a

A team of 4 bowlers, 2 wicket-keepers, and 5 batsmen can be chosen.

a. ${}^{6}C_{2} \times {}^{3}C_{4} \times {}^{11}C_{5}$

b. ${}^{6}C_{2} \times {}^{3}C_{4} \times {}^{11}C_{5}$

c. ${}^{6}C_{2} \times {}^{3}C_{5} \times {}^{11}C_{4}$

d. ${}^{6}C_{2} \times {}^{3}C_{1} \times {}^{11}C_{5}$

Part - B Section - III

19. If A = {1, 3, 5, 7, 9}, B = [2, 4, 6, 8} and C = {2, 3, 5, 7, 11}, find (A ∩ B) and (A ∩ C). What do you conclude?

20. Let A and B be two sets such that n (A) = 5 and n (B) = 2. If a, b, c, d, e are distinct and (a, 2), (b 3), (c, 2), (d, 3), (e, 2) are in A × B, find A and B.

OR

If a function f: R → R be defined by

$$f(x) = \begin{Bmatrix} 3x-2, & x<0 \\ 1 & , x=0 \\ 4x+1, & x>0 \end{Bmatrix}$$

Find: f(1), f(-1), f(0), f(2)

21. If z_1 = (l + i) and z_2 = (-2 + 4i), prove that $\text{Im}\left(\frac{z_1 z_2}{\bar{z}_1}\right) = 2$.

22. If $\left|\frac{z-5i}{z+5i}\right| = 1$, show that z is a real number.

23. Solve: $21x^2 + 9x + 1 = 0$.

OR

Express $\frac{3+2i}{-2+i}$ the complex numbers in the standard form a + ib.

24. Three unbiased coins are tossed once. What is the probability of getting at least 1 head?

25. Differentiate $e^{ax}\cos(bx + c)$

26. List all events associated with the random experiment of tossing of two coins. How many of them are elementary events?

27. Find the mean deviation from the mean for the data:

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

28. Prove that: $\sin A + \sin 2A + \sin 4A + \sin 5A = 4\cos\frac{A}{2}\cos\frac{3A}{2}\sin 3A$.

OR

Prove the identities: $1 - \frac{\sin^2 x}{1+\cot x} - \frac{\cos^2 x}{1+\tan x} = \sin x \cos x$.

Section - IV

29. Find the mean deviation from the mean for the following data:

Classes:	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequencies	2	3	8	14	8	3	2

30. Find the domain and range of the following relations:

    i. $R = \{(x,y): x, y \in N, y = x^2 + 3 \text{ and } 0 < x < 5\}$

    ii. $R = \{(x, y); x, y \in N, y = \frac{1}{1+x}$ and x is odd natural number$\}$

31. A man saved ₹16500 in ten years. In each year after the first he saved ₹ 100 more than he did in the receding year. How much did he save in the first year?

OR

There are n A.M.s between 3 and 17. The ratio of the last mean to the first mean is 3 : 1. Find the value of n.

32. Prove that the centres of the three circles $x^2 + y^2 - 4x - 6y - 12 = 0$, $x^2 + y^2 + 2x + 4y - 5 = 0$ and $x^2 + y^2 - 10x - 16y + 7 = 0$ are collinear.

33. Write the number of numbers that can be formed using all for digits 1, 2, 3, 4.

34. Find a point on the x-axis, which is equidistant from the points (7, 6) and (3, 4).

OR

Find the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ when :

(i) PQ is parallel to the y-axis

(ii) PQ is parallel to the x-axis.

35. A school awarded 42 medals in hockey, 18 in basketball and 23 in cricket. If these medals were bagged by a total of 65 students and only 4 students got medals in all the three sports, how many students received medals in exactly two of the three sports?

Section - V

36. Find the derivative of $\sqrt[3]{\sin x}$ from first principles.

OR

Evaluate: $\lim\limits_{x \to 0} \frac{e^{\sin x} - 1}{x}$

37. Write the domain and the range of the function, $f(x) = \sqrt{x - [x]}$

OR

For any three sets, A, B and C, prove that

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

38. Solve the system of inequalities graphically.

$x + y \leq 5$

$4x + y \geq 4$

$x + 5y \geq 5$

$x \leq 4$

$y \leq 3$

OR

Solve the system of inequality graphically: $2x - y > 1$, $x - 2y < -1$

CBSE Class 11 Mathematics

Sample Papers 02 (2020-21)

Solution

Part - A Section - I

1. The interval [-23, 5) can be written in set builder form as $(x : x \in R, -23 \leqslant x < 5)$

OR

The vowels in the word EQUATION are E, U, A, I, O

Since, the order in which the elements of a set are written doesn't matter, Hence the set is {A, E, I, O, U}

2. Given points is $(x, y, \sqrt{1-x^2-y^2})$

$\therefore$ Distance between the origin and the point is

$= \sqrt{(x-0)^2+(y-0)^2+(\sqrt{1-x^2-y^2-0})^2}$ [using distance formula]

$= \sqrt{1}$ = 1 Hence, proved.

3. cos 47^o cos 13^o - sin 47^o sin 13^o [cos(A + B) = cos A cos B - sin A sin B]

= cos (47^o + 13^o)

= cos 60^o

$= \frac{1}{2}$

$\therefore$ Cos 47^o cos 13^o - sin 47^o sin 13^o $= \frac{1}{2}$

OR

The given function is h(x) = x^2 + x + 4

Therefore, h(-x) = ($-x^2$) + (-x) + 4 = x^2 - x + 4

Clearly, h(-x) is neither equal to h(x) nor to -h(x). So, h(x) is neither even nor odd function.

4. $(5i)\left(-\frac{3}{5}i\right) = -3i^2 = -3 \times -1 (\because i^2 = -1)$

= 3 = 3+0i

5. Given: 5 routes from A to B and 3 routes from B to C.

To find: number of different routes from A to C via B.

Let E_1 be the event : 5 routes from A to B

Let E_2 be the event : 3 routes from B to C

Since going from A to C via B is only possible if both the events E_1 and E_2 occur simultaneously.

So there are 5 × 3 = 15 different routes from A to C via B

OR

Here $3! + 4! = 3 \times 2 \times 1 + 4 \times 3 \times 2 \times 1 = 6 + 24 = 30$

$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

$3! + 4! \neq 7!$

6. Let a and d be the first term and common difference respectively of the given A.P.

Then,we can write ,

$\frac{1}{n}$ = mth term ⇒ $\frac{1}{n}$ = a + (m - 1)d ..(i)

$\frac{1}{m}$ = nth term ⇒ $\frac{1}{m}$ = a + (n - 1)d ...(ii)

Subtract (ii) from (i), we get

$\frac{1}{n}\frac{1}{m}$ = (m -n)d = ⇒ $\frac{m-n}{mn}$ = (m-n)d ⇒ $d = \frac{1}{mn}$

Put $d = \frac{1}{mn}$ in (i), we get ,

$\frac{1}{n} = a + \frac{(m-1)}{mn} \Rightarrow \frac{1}{n} = a + \frac{1}{n} - \frac{1}{mn} \Rightarrow a = \frac{1}{mn}$

∴ (mn)th term = a + (mn-1) $d = \frac{1}{mn}$ + (mn -1)$\frac{1}{mn}$ = 1

7. Let θ be the inclination of a line with X-axis, then its slope = tan θ.

At θ = 60°, slope of a line = tan 60°= $\sqrt{3}$

OR

Coordinates of an arbitrary point on x + y = 4 can be obtained by putting x = t (or y = t) and then obtaining y (or x) from the equation of the line, where t is a parameter.

Substituting x = t in the equation x + y = 4 of the given line, we get y = 4 - t.

Therefore, coordinates of an arbitrary point on the given line are P (t, 4 - t).

Let P (t, 4 - t) be the required point.

Then, distance of P from the line 4x + 3y - 10 = 0 is unity.

$= \left|\frac{4+3(4-1)-10}{\sqrt{4^2+3^2}}\right| = 1 \Rightarrow |t+2| = 5$

$\Rightarrow t + 2 = \pm 5$

$\Rightarrow$ t = -7 or t = 3

Therefore, the required points are (-7, 11) and (3, 1).

8. Since, the vertex is at (0, 0) and focus is at (0, 4) which lies on Y-axis. The Y-axis is the axis of the parabola.

   $\therefore$ Equation of parabola is of the form

   $x^2 = -4ay \Rightarrow x^2 = -4(4)y$ [$\because$ a = 4]

   $\Rightarrow x^2 = -16y$

9. The answer is E = {x : x $\in$ R, -10 $\leq$ x < 0}

OR

A= {2, 4, 6, 8, 10} and B = {x : x is a positive even integer and $x \leqslant 10$} which can be written in roster form as B = {2, 4, 6, 8, 10} are equal sets.

$\therefore$ A = B = {2, 4, 6, 8, 10}

10. When three coins are tossed then total outcomes, S = {HHH, HHT, HTH, THH, TTH, HTT, TTT, THT}

    Where s is sample space and here n(S) = 8

    Let A be the event of getting 3 tails

    n(A) = 1

    P(getting 3 tails) = P(A) = $\frac{n(A)}{n(S)} = \frac{1}{8}$

11. x coordinate is -ve

    y coordinate is +ve

    z coordinate is -ve

    Therefore, this point lies in X'OYZ' octant.

12. We have,

    $^{14}C_3 = \frac{14}{3} \times \frac{13}{2} \times \frac{12}{1} \times {}^{11}C_0$ [$\because {}^nC_r = \frac{n}{r} {}^{n-1}C_{r-1}$]

    $\Rightarrow {}^{14}C_3 = 364$ [$\because {}^nC_0 = 1$]

13. To prove $\frac{\tan\left(\frac{\pi}{2}-x\right)\sec(\pi-x)\sin(-x)}{\sin(\pi+x)\cot(2\pi-x)\operatorname{cosec}\left(\frac{\pi}{2}-x\right)} = 1$

    Take L.H.S

$$\frac{\tan\left(\frac{\pi}{2}-x\right)\sec(\pi-x)\sin(-x)}{\sin(\pi+x)\cot(2\pi-x)\operatorname{cosec}\left(\frac{\pi}{2}-x\right)}$$

$$=\frac{(\cot x)(-\sec x)(-\sin x)}{(-\sin x)(-\cot x)(\sec x)}$$

= 1

= R.H.S

Hence proved.

14. $cos135^o = cos(90^o + 45^o)$

$= -sin45^o$ [$\because \cos(90^0 + \theta) = -\sin\theta$]

$= -\frac{1}{\sqrt{2}}$

15. Given that $\frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}, (x > 0)$

$\Rightarrow 4 \leq 3(x+1) \leq 6$

$\Rightarrow 4 \leq 3x+3 \leq 6$

$\Rightarrow 4-3 \leq 3x \leq 6 - 3$

$\Rightarrow 1 \leq 3x \leq 3$

$\Rightarrow \frac{1}{3} \leq x \leq 1$

Therefore, solution set = $[\frac{1}{3}, 1]$

16. Given; A = {a, b, c, d, e, f}, B = {c, e, g, h} and C = {a, e, m, n}

Therefore $A \cup B$ = {a, b, c, d, e, f, g, h}

Section - II

17. i. (c) 18
 ii. (a) 1400
 iii. (d) 11th
 iv. (b) 6400
 v. (a) 54,500

18. i. (a) 1365
 ii. (c) 1001
 iii. (b) 364
 iv. (a) ${}^{2}C_{1} \times {}^{13}C_{10}$
 v. (d) ${}^{6}C_{2} \times {}^{3}C_{1} \times {}^{11}C_{5}$

Part - B Section - III

19. We have,

$A \cap B = \{1, 3, 5, 7, 9\} \cap \{2, 4, 6, 8\} = \phi$

and A ∩ C = {1, 3, 5, 7, 9} ∩ {2, 3, 5, 7, 11} = {3, 5, 7} $\neq \phi$.

Therefore,A and B are disjoint sets while A and C are intersecting sets.

20. Since (a, 2), (b, 3), (c, 2), (d, 3), (e, 2) are elements of A × B. Therefore, a, b, c, d, e ∈ A and 2, 3 ∈ B.

It is given that n (A) = 5 and n (B) = 2

∴ a, b, c, d, e ∈ A and n(A) = 5 ⇒ A = {a, b, c, d, e}

2, 3 ∈ B and n (B) = 2 ⇒ B = {2, 3}

OR

Given,

For x < 0, f(x) = 3x - 2,

For x = 0, f(x) = 1,

For x > 0, f(x) = 4x + 1

Thus,

f(1) = 4 × 1 + 1 = 5,

f(-1) = 3 × (-1) - 2 = – 3 – 2 = – 5,

f(0) = 1,

and f(2) = 4 × 2 + 1 = 9

21. We have, z_1 = (1 + i) and z_2 = (-2 + 4i)

now, $\frac{z_1 z_2}{z_1} = \frac{(1+i)(-2+4i)}{(1+1)}$

$= \frac{-2+4i-2i+4i^2}{(1-i)} = \frac{-2+4i-2i-4}{(1-i)}$

$= \frac{-6+2i}{(1-i)}$

$= \frac{-6+2i}{(1-i)} \times \frac{(1+i)}{(1+i)}$ [(a + b)(a - b) = a^2 - b^2]

$= \frac{-6-6i+2i+2i^2}{1+1}$

$= \frac{-6-4i-2}{2} = \frac{-8-4i}{2}$

= -4 - 2i

Hence , $\mathbf{Im}\left(\frac{z_1 z_2}{z_2}\right) = -2$

22. Let z = (x + iy). Then,

$\left|\frac{z-5i}{z+5i}\right| = 1 \Rightarrow \frac{|z-5i|}{|z+5i|} = 1 \left[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$

⇒ |z - 5i| = |z + 5i| ⇒ $|z - 5i|^2 = |z + 5i|^2$ {squaring both the sides}

$\Rightarrow |(x + iy) - 5i|^2 = |(x+ iy)+ 5i|^2$ [$\because z = (x + iy)$]

$\Rightarrow |x + i(y - 5)|^2 = |x + z(y + 5)|^2$

$\Rightarrow x^2 + (y - 5)^2 = x^2 + (y + 5)^2$ [$\because |x + iy|^2 = (x^2 + y^2)$]

$\Rightarrow (y + 5)^2 - (y - 5)^2 = 0 \Rightarrow 4 \times y \times 5 = 0 \Rightarrow y = 0.$

$\therefore z = x + i0$

=> z = x {where x is real}

Hence, z is a real number.

23. Given: $21x^2 + 9x + 1 = 0$

Comparing $21x^2 + 9x + 1 = 0$ with the general form of the quadratic equation $ax^2 + bx + c = 0$, we get a = 21, b = 9 and c = 1.

Substituting these values in $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$, we get

$\alpha = \frac{-9+\sqrt{81-4\times21\times1}}{2\times21}$ and $\beta = \frac{-9+\sqrt{81-4\times21\times1}}{2\times21}$

$\Rightarrow \alpha = \frac{-9+\sqrt{3i}}{42}$ and $\beta = \frac{-9-\sqrt{3i}}{42}$

$\Rightarrow \alpha = -\frac{9}{42} + \frac{\sqrt{3i}}{42}$ and $\beta = -\frac{9}{42} - \frac{\sqrt{3i}}{42}$

$\Rightarrow \alpha = -\frac{3}{14} + \frac{\sqrt{3i}}{42}$ and $\beta = -\frac{3}{14} - \frac{\sqrt{3i}}{42}$

Hence, the roots of the equation are $-\frac{3}{14} \pm \frac{\sqrt{3i}}{42}$

OR

$\frac{3+2i}{-2+i}$

$= \frac{3+2i}{-2+i} \times \frac{-2-i}{-2-i}$ [multiply and divide by -2 -i]

$= \frac{-6-3i-4i-2i^2}{4-i^2}$ ($\because i^2 = -1$)

$= \frac{-6-7i+2}{4+1}$

$= \frac{-4-7i}{5}$

$= \frac{-4}{5} - \frac{7}{5}i$

24. In tossing three coins, then the sample space of event is given by

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}. And, therefore, n(S) = 8.

Let E_4 = event of getting at least 1 head. Then,

E_4 = {HTT, THT, TTH, HHT, HTH, THH, HHH). And, therefore, $n(E_4) = 7$.

Therefore, P(getting at least 1 head) = $P(E_4) = \frac{n(E_4)}{n(S)} = \frac{7}{8}$.

25. Using product rule, we have

$\frac{d}{dx}$ (e^{ax} cos (bx + c)

$= e^{ax} \cdot \frac{d}{dx}$ cos (bx + c) + cos (cos (bx + c) $\frac{d}{dx}(e^{ax})$

= e^{ax} (- sin (bx +c) $\frac{d}{dx}$ (bx + c) + cos (bx + c) e^{ax} $\frac{d}{dx}(ax)$ [using chain rule]

= - be^{ax} sin (bx + c)+a e^{ax} cos (bx + c)

= e^{ax}{ (a cos (bx + c) - b sin (bx + c)}.

26. It is given that: Two coins are tossed once.

We have to find: How many events are elementary events

Explanation: We know, when Two coins are tossed then the no. of possible outcomes are $2^2 = 4$

So, The Sample spaces are {HH, HT, TT, TH}

Hence, there is a total of 4 events associated with the given experiment.

27. To find mean deviation about mean for following data we need to make the following table,

x_i	f_i	$f_i x_i$	$\|d_i\| = \|x_i - 9\|$	$f_i \|d_i\|$
5	8	40	4	32
7	6	42	2	12
9	2	18	0	0
10	2	20	1	2
12	2	24	3	6
15	6	90	6	36
	N = 26	Total = 234		Total = 88

$$Mean = \frac{\sum f_i x_i}{N} = \frac{234}{26} = 9$$
$$M.D. = \frac{\sum f_i |d_i|}{N} = \frac{88}{26} = 3.39$$

28. To prove:

sin A + sin 2A + sin 4A + sin 5A = 4 cos $\left(\frac{A}{2}\right)$ cos $\left(\frac{3}{2}A\right)$ cos 3A

Consider LHS:

sin A + sin 2A + sin 4A + sin 5A

= 2 sin $\left(\frac{A+2A}{2}\right)$ cos $\left(\frac{A-2A}{2}\right)$ + 2 sin $\left(\frac{4A+5A}{2}\right)$ cos $\left(\frac{4A-5A}{2}\right)$ {∵ sin A + sin B = 2 sin $\left(\frac{A+B}{2}\right)$ cos $\left(\frac{A-B}{2}\right)$}

= 2 sin $\left(\frac{3}{2}A\right)$ cos $\left(-\frac{A}{2}\right)$ + 2 sin $\left(\frac{9}{2}A\right)$ cos $\left(-\frac{A}{2}\right)$

= 2 sin $\left(\frac{3}{2}A\right)$ cos $\left(\frac{A}{2}\right)$ + 2 sin $\left(\frac{9}{2}A\right)$ cos $\left(\frac{A}{2}\right)$

= 2 cos $\left(\frac{A}{2}\right)$ {sin $\left(\frac{3}{2}A\right)$ + sin $\left(\frac{9}{2}A\right)$}

= 2 cos $\left(\frac{A}{2}\right)$ × 2 sin $\left(\frac{\frac{3}{2}A+\frac{9}{2}A}{2}\right)$ cos $\left(\frac{\frac{3}{2}A-\frac{9}{2}A}{2}\right)$

= 4 cos $\left(\frac{A}{2}\right)$ cos 3A cos $\left(-\frac{3}{2}A\right)$

= 4 cos $\left(\frac{A}{2}\right)$ cos $\left(\frac{3}{2}A\right)$ cos 3A

= RHS

Hence proved.

OR

$1-\frac{\sin^2 x}{1+\cot x}-\frac{\cos^2 x}{1+\tan x}$ = sin x cos x

$\text{LHS}=1-\frac{\sin^3 x}{\sin x+\cos x}-\frac{\cos^3 x}{\sin x+\cos x}$

$=\frac{\sin x+\cos x-\left(\sin^3 x+\cos^3 x\right)}{\sin x+\cos x}$

$=\frac{(\sin x+\cos x)\left(1-\sin^2 x-\cos^2 x+\sin x\cos x\right)}{\sin x+\cos x}$

= (1 - sin^2 x - cos^2 x + sin x cos x)

= (1 - 1 + sin x cos x)

= sin x cos x

= RHS

Hence proved.

Section - IV

29.

Classes	Mid-values x_i	frequencies f_i	$f_i x_i$	$\left\|x_i-\overline{X}\right\|=\left\|x_i-45\right\|$	$f_i\left\|x_i-\overline{X}\right\|$
10-20	15	2	30	30	60
20-30	25	3	75	20	60

30-40	35	8	280	10	80
40-50	45	14	630	0	0
50-60	55	8	440	10	80
60-70	65	3	195	20	60
70-80	75	2	150	30	60
		$N=\Sigma f_i$ $=40$	$\Sigma f_i x_i$ $=1800$		$\Sigma f_i \left\|x_i-\overline{X}\right\|$ $=400$

$N=40$ and $\Sigma f_i x_i=1800$

$\therefore \quad \overline{X}=\frac{\Sigma f_i x_i}{N}=\frac{1800}{40}=45$

$\Sigma f_i\left|x_i-\overline{X}\right|=400$ and $N=\Sigma f_i=40$

$\therefore \quad M.D.=\frac{1}{N}\Sigma f_i\left|x_i-\overline{X}\right|=\frac{400}{40}=10$

30. i. We may also write the given relation as,

R = $\{(x, x^2+3): 0<x<5, x \in N\}$

As per condition x takes the values $1,2,3,4$ and therefore, y takes the values $4,7,12,19$.

Thus, R = $\{(1,4),(2,7),(3,12),(4,19)\}$

Hence, domain of R = $\{1,2,3,4\}$ and range of R = $\{4,7,12,19\}$

ii. We may also write the given relation as,

$R=\left\{\left(x, \frac{1}{1+x}\right)\right.$: x is an odd natural number}

As per condition x takes the values $1,3,5,7$,.....

$\therefore$

$R=\left\{\left(1,\frac{1}{2}\right),\left(3,\frac{1}{4}\right),\left(5,\frac{1}{6}\right),\left(7,\frac{1}{8}\right),\ldots\right\}$

Hence, domain of R = $\{1,3,5,\ldots\}$ and range of $R=\left\{\frac{1}{2},\frac{1}{4},\frac{1}{6},\ldots\right\}$

31. According to the question , we can write,

Let the amount saved by the man in the first year be ₹ A. Let d be the common difference.

Let S_{10} denote the amount he saves in ten years.

Here, n = 10, d = 100

We know:

$S_n = \frac{n}{2}\{2A + (n - 1)d\}$

$\therefore S_{10} = \frac{10}{2}\{2A + (10 - 1)100\}$

$\Rightarrow 16500 = 5\{2A + 900\}$

$\Rightarrow 3300 = 2A + 900$

$\Rightarrow A = 1200$

Therefore, the man saved ₹1200 in the first year.

OR

As per the question , we can write it as ,

Let the n A.M's between 3 and 17 be $A_1, A_2, A_3, \ldots, A_n$ Then,

ATQ

$\frac{A_n}{A_1} = \frac{3}{1}$ ...(i)

We know that 3, $A_1, A_2, A_3, \ldots, A_n$, 17$ are in A.P of n + 2 terms

So, 17 is the (n + 2)th terms. i.e. 17 = 3 + (n + 2 - 1) d [Using $a_n = a + (n - 1)d$]

or

$d = \frac{14}{(n+1)}$

$A_n = 3 + (n + 1 - 1)d$

$= 3 + \frac{14n}{n+1} = \frac{17n+3}{n+1}$

$A_1 = 3 + d = \frac{3n+17}{n+1}$

From (i), (iii) and iv

$\frac{A_n}{A_1} = \frac{17n+3}{3n+17} = \frac{3}{1}$

n = 6

There are 6 A.M.s between 3 and 17

32. Given, $x^2 + y^2 - 4x - 6y - 12 = 0$

centre $(-g_1, -f_1) = (2, 3)$

$x^2 + y^2 + 2x + 4y - 5 = 0$

centre $(-g_2, -f_2) = (-1, -2)$

$x^2 + y^2 - 10x - 16y + 7 = 0$

centre $(-g_3, -f_3) = (5, 8)$

to prove collinearity of the center of three circle

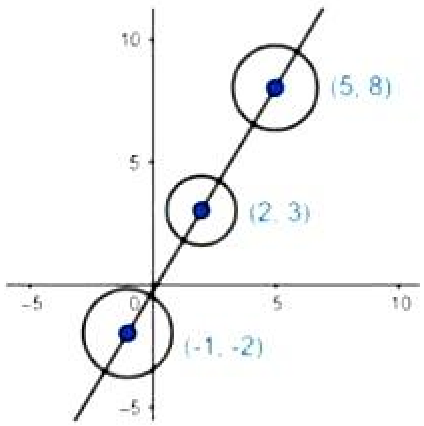
$$D = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Where x_i, y_i are the coordinates of ist centre and so on.

$$\Rightarrow D = \begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix}$$

= 2 (-2 - 8) - 3 (-1 - 5) + 1 (-8 + 10)

D= -20 + 18 + 2 = 0



The centers are collinear.

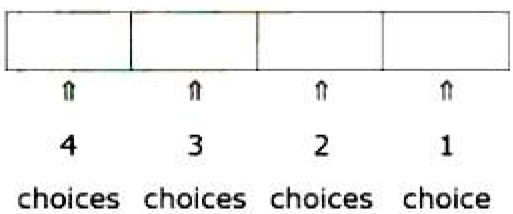
33. Thousands' place (10^3) can be filled with 1, 2, 3 and 4 i.e. thousands' place can be filled in 4 ways.

Hundreds' place (10^2) can be filled in 3 ways [i.e. with the remaining 3 digits].

Similarly, tens' place (10^1) can be filled in 2 ways and ones' place can be filled only in 1 way.

As, the operations are dependent, so, total number of ways = 24

The discussion can be shown pictorially as:



ALTERNATIVE APPROACH: The number of permutations of 4 objects taken 4 at a time is =

$^4P_4 = 4! = 24$

34. Let P(x, 0) be any point on the x-axis which is equidistant from Q(7, 6) and R (3, 4).

Then $PQ = \sqrt{(x-7)^2 + (0-6)^2} = \sqrt{x^2 - 14x + 49 + 36}$

$= \sqrt{x^2 - 14x + 85}$

$PR = \sqrt{(x-3)^2 + (0-4)^2} = \sqrt{x^2 - 6x + 9 + 16}$

$= \sqrt{x^2 - 6x + 25}$

Since PQ = PR

$\therefore \sqrt{x^2 - 14x + 85} = \sqrt{x^2 - 6x + 25}$

Squaring both sides, we have

$x^2 - 14x + 85 = x^2 - 6x + 25$

$\Rightarrow -14x + 6x = 25 - 85 \Rightarrow 8x = -60$

$\Rightarrow x = \frac{15}{2}$

Thus coordinates of point on the x-axis is $\left(\frac{15}{2}, 0\right)$.

OR

Here $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points.

Then $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(i) PQ is parallel to the y-axis then $x_2 - x_1 = 0$

Then $PQ = \sqrt{(y_2 - y_1)^2} = |y_2 - y_1|$

(ii) PQ is parallel to the x-axis then $y_2 - y_1 = 0$

Then $PQ = \sqrt{(x_2 - x_1)} = |x_2 - x_1|$

35. Given: Total number of students = 65

Medals awarded in Hockey = 42

Medals awarded n Basketball = 18

Medals awarded in Cricket = 23

4 students got medals in all the three sports.

We have to find: Number of students who received medals in exactly two of the three sports.

Total number of medals = Medals awarded in Hockey + Medals awarded in Basketball + Medals awarded in Cricket

Total number of medals = 42 + 28 + 23 = 83

It is given that 4 students got medals in all the three sports.

Thus, the number of medals received by those 4 students = 4 × 3 = 12

Now, the number of medals received by the rest of 61 students = 83 - 12 = 71

Among these 61 students, everyone at least received 1 medal.

Thus, the number of extra medals = 71 - 1 × 61 = 10

Thus, we can say that 10 students received medals in exactly two of three sports.

Section - V

36. Let f (x) = $\sqrt[3]{\sin x}$. Then, f(x + h) = $\sqrt[3]{\sin(x+h)}$

$\therefore \frac{d}{dx}$ (f (x)) = $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim\limits_{h\to 0}\frac{\sqrt[3]{\sin(x+h)}-\sqrt[3]{\sin x}}{h}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim\limits_{h\to 0}\frac{\{\sqrt[3]{\sin(x+h)}\}^3-\{\sqrt[3]{\sin x}\}^3}{h\left[\sin^{2/3}(x+h)+\sin^{1/3}(x+h)\sin^{1/3}x+\sin^{2/3}x\right]}$ $\left[\because a-b=\frac{a^3-b^3}{a^2+ab+b^2}\right]$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim\limits_{h\to 0}\frac{\sin(x+h)-\sin x}{h}\times\frac{1}{\sin^{2/3}(x+h)+\sin^{1/3}(x+h)\sin^{1/3}x+\sin^{2/3}x}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim\limits_{h\to 0}\frac{2\sin\frac{h}{2}\cos\left(x+\frac{h}{2}\right)}{h}\times\frac{1}{\sin^{2/3}(x+h)+\sin^{1/3}(x+h)\sin^{1/3}x+\sin^{2/3}x}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim\limits_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)\cos\left(x+\frac{h}{2}\right)}{\frac{h}{2}}\times\lim\limits_{h\to 0}\frac{1}{\sin^{2/3}(x+h)+\sin^{1/3}(x+h)\sin^{1/3}x+\sin^{2/3}x}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\cos x\times\frac{1}{\sin^{2/3}x+\sin^{2/3}x+\sin^{1/3}x\sin^{1/3}x}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\frac{\cos x}{3\sin^{2/3}x}$

OR

$\lim\limits_{x\to 0}\frac{e^{\sin x}-1}{x}$

$=\lim\limits_{x\to 0}\frac{\left[\frac{e^{\sin x}-1}{\sin x}\right]}{\left[\frac{x}{\sin x}\right]}=\frac{1}{1}$

= 1

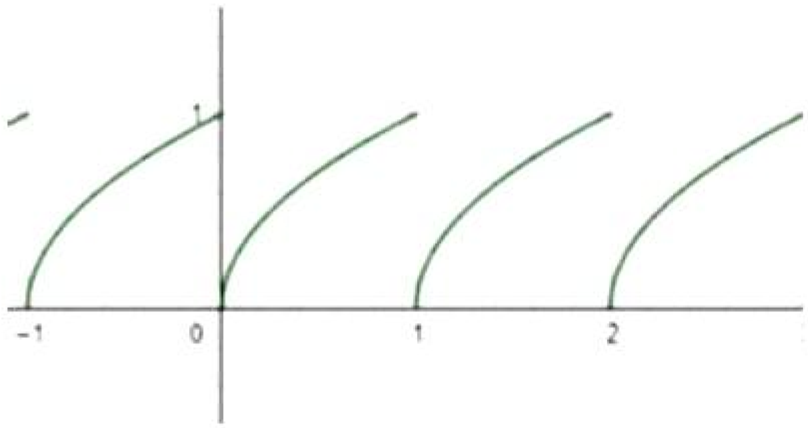
37. Here we are given that, f(x) = $\sqrt{x-[x]}$

Where [x] is the Greatest integer Function of x.

f(x) = $\sqrt{\{x\}}$

Where {x} is fractional part of x

The graph of f(x) is:



i. **Domain:**

Domain of {x} is R.

The value of the fractional part of x is always either positive or zero.

Hence domain of x is R

ii. **Range:**

Range of {x} is [0, 1)

As the root value [0, 1) between interval lies between [0, 1)

Hence range of f(x) is [0, 1).

OR

Suppose (a, b) be an arbitrary element of $A \times (B \cup C)$.

$\Rightarrow (a, b) \in A \times (B \cup C)$

$\Rightarrow a \in A$ and $b \in B \cup C$

$\Rightarrow a \in A$ and $(b \in B$ or $b \in C)$

$\Rightarrow (a \in A$ and $b \in B)$ or $(a \in A$ and $b \in C)$

$\Rightarrow (a, b) \in A \times B$ or $(a, b) \in A \times C$

$\Rightarrow (a, b) \in (A \times B) \cup (A \times C)$.....(i)

Suppose (x, y) be an arbitrary element of $(A \times B) \cup (A \times C)$.

$\Rightarrow (x, y) \in (A \times B) \cup (A \times C)$

$\Rightarrow (x, y) \in A \times B$ or $(x, y) \in A \times C$

$\Rightarrow (x \in A$ and $y \in B)$ or $(x \in A$ and $y \in C)$

$\Rightarrow x \in A$ and $(y \in B$ or $y \in C)$

$\Rightarrow x \in A$ and $y \in (B \cup C)$

$\Rightarrow (x, y) \in A \times (B \cup C)$

$\therefore \quad (A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$....(ii)

From equations (i) and (ii),

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

Hence proved.

38. We have,

x + y $\leq$ 5 ... (i)

4x + y $\geq$ 4 ... (ii)

x + 5y $\geq$ 5 ... (iii)

x $\leq$ 4 ... (iv)

y $\leq$ 3 ... (v)

Take inequality (i)

x + y $\leq$ 5

Convert it into linear equation i.e.,

x + y = 5

x	0	5
y	5	0

Thus, the line x + y = 5 passes through points (0,5) and (5.0)

Now, putting x = 0 and y = 0 in inequality (i), we get 0 $\leq$ 5, which is true.

$\therefore$ For inequality x + y $\leq$ 5 shade the region which contains the origin.

Take inequality (ii)

4x + y $\geq$ 4

Convert it into linear equation i.e.,

4x + y = 4

x	0	1
y	4	0

Thus, the line 4x + y = 4 passes through points (0, 4) and (1, 0)

Now, on putting x = 0 and y = 0 in inequality (ii), we get

4 (0) + 0 $\geq$ 4

$\Rightarrow$ 0 $\geq$ 4, which is false.

$\therefore$ For inequality 4x + y $\geq$ 4, shade the region which does not contain origin.

Take inequality (iii)

$x + 5y \geq 5$

Convert it into linear equation i.e.,

$x + 5y = 5$

x	0	5
y	1	0

Thus, the line $x + 5y = 5$ passes through points (0, 1) and (5,0).

Now, on putting $x = 0$ and $y = 0$ in inequality (iii), we get

$0 \geq 5$, which is false.

$\therefore$ For inequality $x + 5y \geq 5$, shade the region which does not contain the origin.

Take inequality (iv)

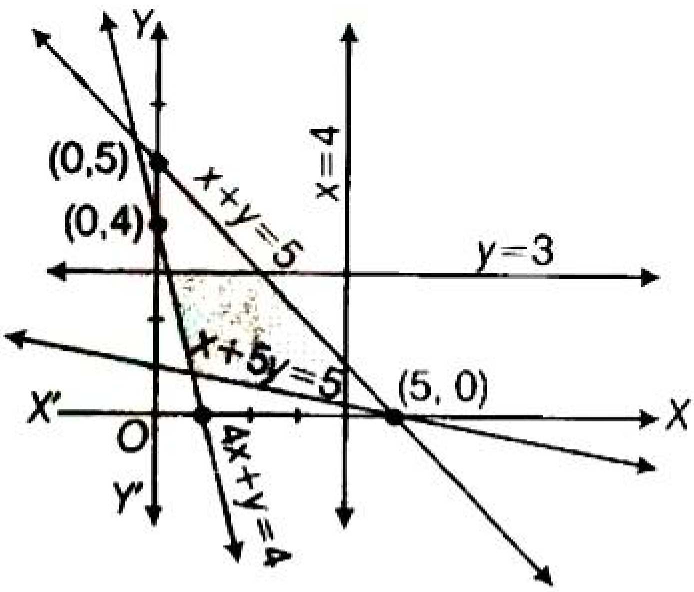
$x \leq 4$

Linear equation corresponding to inequality (iv) is $x = 4$. This is a line parallel to Y-axis at a distance 4 units to the right of Y-axis and for this inequality shaded region contains the origin.

Take inequality (v)

$y \leq 3$

Linear equation corresponding to inequality (v) is $y = 3$. This is a line parallel to X-axis at a distance of 3 units above the X-axis and for this inequality shaded region contains the origin.



Hence, the common region is shaded. Any point in this region represents a solution of given inequalities.

OR

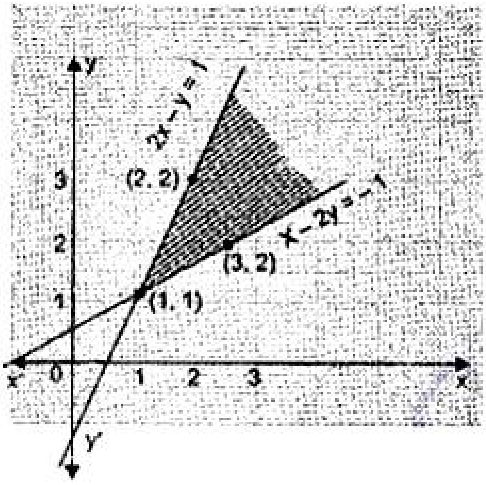
The given inequality is 2x - y > 1

Draw the graph of the line 2x - y = 1

Table of values satisfying the equation

2x - y = 1

X	1	2
Y	1	3



Putting (0, 0) in the given inequation, we have

$2 \times 0 - 0 > 1 \Rightarrow 0 > 1$, which is false.

∴ Half plane of 2x - y > 1 is away from origin.

Also the given inequality is x - 2y < -1

Draw the graph of the line x - 2y = -1

Table of values satisfying the equation x - 2y = -1

X	1	2
Y	1	2

Putting (0, 0) in the given inequation, we have

$0 - 2 \times 0 < -1 \Rightarrow 0 < -1$ which is false

∴ Half plane of x - 2y < - 1 is away from origin.