Chapter 1

Simple Equations

CHAPTER HIGHLIGHTS

- *One Equation in One Unknown*
- *Two Equations in Two Unknowns*
- *Three Equations in Three Unknowns*
- *Additional Cases in Linear Equations*

INTRODUCTION

There will be linear equations of one or two unknowns invariably in every problem. A linear equation is one where each variable occurs only in its first power and not in any higher powers. Sometimes, we get three equations in three unknowns. In general, we need as many equations as the variables we will have to solve for. So, for solving for the values of two unknowns, we need two equations (or two conditions given in the problem) and for solving for the values of three unknowns, we need three equations (and hence the problem should give three conditions from which we can frame three equations). Solving the equations by itself is not a difficult task. The most important part of the problem is framing the equation/equations. Once the equations are framed, solving them is very easy. In this chapter, we will deal with problems involving as many equations (of first degree) as the number of unknowns. Later on, we will look at equations of second degree (quadratic equations) and linear equations where the number of equations will be less than that of the number of variables (under the chapter special equations).

ONE EQUATION IN ONE UNKNOWN

An equation like $2x + 4 = 26$ is an equation in one unknown. We have only one variable x whose value we have to find out. The steps in solving this are:

Step 1: Take all quantities added to (or subtracted from) the x term (term with the unknown) to the right side with a change of sign.
i.e., $2x = 26 - 4 = 22$.

Step 2: Take the co-efficient of x from left-hand side and divide right-hand side with this term to get the value of x:
i.e. $x = 22/2 = 11$. Therefore, $x = 11$.

TWO EQUATIONS IN TWO UNKNOWNS

A set of equations like

$$2x + 3y = 8 \quad (1)$$

$$5x + 4y = 13 \quad (2)$$

is called a system of simultaneous equations in two unknowns. Here, we have two variables (or unknowns) x and y whose values we have to find out. This can be done using the two given equations. The steps for this are as follows:

Step 1: Using both the equations, we first eliminate one variable (so that we can then have one equation in one unknown).
For this purpose, we multiply equation (1) with 5 (the co-efficient of x in the second equation) and multiply equation (2) with 2 (the co-efficient of x in the first equation) to eliminate x. Thus, we have

$$(1) \times 5 \Rightarrow 10x + 15y = 40 \quad (3)$$

$$(2) \times 2 \Rightarrow 10x + 8y = 26 \quad (4)$$

Now, subtracting equation (4) from equation (3) we have

$$7y = 14 \quad (5)$$

This is one equation in one unknown.

Step 2: Solve for the value of one variable from the equation (in one unknown) obtained from Step I above. Therefore, $y = 2$.

Step 3: Substitute this value of the variable in one of the two equations to get the value of the second variable.

Substituting the value of y in equation (1) or equation (2), we get $x = 1$. Therefore the values of x and y that satisfy the given set of equations are $x = 1$ and $y = 2$.

Three Equations in Three Unknowns

A set of equations like

$$x + 2y + 3z = 14 \quad (6)$$

$$2x + y + 2z = 10 \quad (7)$$

$$3x + 3y + 4z = 21 \quad (8)$$

is a system of three equations in three unknowns.

Here we have three unknowns x, y and z which we have to solve for from the three given equations. The procedure for the same is as follows:

Step 1: Take two out of the three equations [say, eqn. (6) and (7)] and eliminate one variable (say x) so that we get an equation in two unknowns (y and z in this case).

For this purpose, take equations (6) and (7). Multiply equation (6) by 2 and subtract equation (7) from it.

$$\text{Equation (6)} \times 2 \Rightarrow 2x + 4y + 6z = 28$$

$$2x + y + 2z = 10$$

$$3y + 4z = 18 \quad (9)$$

Step 2: Repeat Step 1 for two other equations [say equations (7) and (8)] and eliminate the same variable (x in this case) so that we get one more equation in two unknowns (y and z).

For this purpose, take equations (7) and (8). Multiply equation (7) by 3 and from that subtract equation (8) multiplied by 2.

$$\text{Equation (7)} \times 3 \Rightarrow 6x + 3y + 6z = 30$$

$$\text{Equation (8)} \times 2 \Rightarrow 6x + 6y + 8z = 42$$

$$-3y - 2z = -12 \quad (10)$$

Step 3: Now the equations in two unknowns that have been obtained from the above two steps have to be solved as discussed previously (in TWO EQUATIONS IN TWO UNKNOWNS) to get the values of two of the three variables (y and z in this case).

In this case, solving equations (9) and (10), we get $y = 2$ and $z = 3$.

Step 4: Substitute these values of the two variables in one of the three equations to get the value of the third variable.

Substitute the value of y and z in equation (6) to get the value of $x = 1$.

Thus, the values of the three variables x, y and z that satisfy the three given equations are $x = 1$; $y = 2$ and $z = 3$

Solved Examples

Example 1

The cost of 3 tables and 4 chairs is ₹2500. The cost of 4 tables and 3 chairs is ₹2400. Find the costs of each table and each chair.

Solution

Let the cost of each table be ₹x.
Let the cost of each chair be ₹y.

$$3x + 4y = 2500 \quad (1)$$

$$4x + 3y = 2400 \quad (2)$$

Method 1:
Multiplying (1) by 3 and subtracting it from (2) multiplied by 4, we get

$$7x = 2100$$

$$x = 300$$

Substituting $x = 300$ in (1),

$$y = 400$$

Method 2:
Adding both the equations (1) and (3), we get $7(x + y) = 4900$

$$x + y = 700 \quad (3)$$

subtracting (2) from (1),

$$-x + y = 100 \quad (4)$$

Adding (3) and (4), $2y = 800$

$$y = 400$$

Substituting $y = 400$ in either (3) or (4), $x = 300$

Example 2

Raju bought 6 pens, 5 erasers, and 4 sharpeners for ₹32. Had he bought 4 pens, 3 erasers, and 5 sharpeners, his total expenditure would have been ₹23. Had he bought 7 pens, 2 erasers, and 6 sharpeners, his total expenditure would have been ₹31. Find the cost of 1 pen, 1 eraser, and 2 sharpeners.

Solution

Let the prices of each pen, each eraser, and each sharpener be ₹p, ₹e, and ₹s respectively.

$$6p + 5e + 4s = 32 \quad (1)$$

$$4p + 3e + 5s = 23 \quad (2)$$

$$7p + 2e + 6s = 31 \quad (3)$$

Multiplying (1) by 2 and subtracting from (2) multiplied by 3,

$$-e + 7s = 5 \quad (4)$$

Multiplying (3) by 4 and subtracting it from (2) multiplied by 7,

$$13e + 11s = 37 \quad (5)$$

Multiplying (4) by 13 and adding it to (5), $102s = 102$

$$s = 1$$

Substituting $s = 1$ in (4),

$$e = 2$$

Substituting values of e and s in (1), $p = 3$.

Example 3

In a two digit number, the digits differ by 2. 10 times the number exceeds 5 times the sum of the number formed by reversing its digits and the sum of its digits by 90. Find the number.

Solution

Let the number be xy. Hence, the value of the number is $10x + y$.

$$x - y = 2 \quad \text{or} \quad y - x = 2 \quad (6)$$

$$10\,(10x + y) - 5\,(10y + x + x + y) = 90$$

$$90x - 45y = 90$$

$$2x - y = 2;\ 2x - (x \pm 2) = 2$$

$$x = 4 \quad \text{or} \quad 0$$

As x cannot be 0, $x = 4$

$\therefore \quad y = 6$

$\therefore$ the number is 46.

Example 4

The age of a man 15 years ago was 5 times his son's age. His age 10 years ago was thrice his son's age. After how may years will their combined age become 80 years?

Solution

Let the present age of the man and his son be f years and s years, respectively.

$$f - 15 = 5\,(s - 15) \quad \Rightarrow \quad f = 5s - 60$$

$$f - 10 = 3\,(s - 10) \quad \Rightarrow \quad f = 3s - 20$$

$$f = 5s - 60 = 3s - 20$$

$$s = 20,\ f = 40$$

Their combined present age is 60 years. For the combined present age to become 80 years, the age of each of them must increase by 10 years.

$\therefore$ Their combined age will become 80 years after 10 years.

Example 5

If the numerator and the denominator of a fraction are both increased by 1, the fraction becomes $\frac{3}{5}$. If both are decreased by 1, it becomes $\frac{5}{9}$. Find the fraction.

Solution

Let the fraction be $\frac{x}{y}$.

$$\frac{x+1}{y+1} = \frac{3}{5}$$

$$\Rightarrow \quad 5x + 5 = 3y + 3$$

$$5x + 2 = 3y \quad (1)$$

$$\frac{x-1}{y-1} = \frac{5}{9}$$

$$\Rightarrow \quad 9x - 9 = 5y - 5$$

$$9x - 4 = 5y \quad (2)$$

Multiplying (1) by 5 and subtract it from (2) after multiplying by 3,

$$5\,(5x + 2) = 3\,(9x - 4)$$

$$\Rightarrow \quad x = 11$$

substituting $x = 11$ in (1), $y = 19$

$\therefore$ The fraction $= \frac{11}{19}$

Additional Cases in Linear Equations

1. If the number of equations is less than the number of unknowns, then we say the variables are 'indeterminate' or we have an 'indeterminate' system of equations. Here, we cannot uniquely determine the values of all the variables. There will be infinite sets of solutions that satisfy the equations.
   For example, if we take the following two equations in three unknowns,

   $$x + y + 2z = 8$$

   $$2x - y + 3z = 13$$

   this system of equations have infinite number of solutions and no unique solution is possible. For any value we take for x, we can find a corresponding set of values for y and z.
2. However, even in case of indeterminate equations, say, of three variables, it is possible that the value of one of the variables may be uniquely determined, i.e. if we have two equations and three unknowns, we may be still able to determine the value of one variable uniquely but the other two variables will

have infinite number of values. This will happen if the ratio of the co-efficients of two variables in one equation is the same as the ratio of the co-efficients of the same two variables in the second equation.
This depends on the equations given. Example 8 will clarify this aspect.

Example 6

Tarun bought 2 shirts, 4 trousers, and 5 pairs of shoes for ₹3600. Had he bought 6 shirts, 5 trousers, and 15 pairs of shoes, his total expenditure would have been ₹8700. Find the price of each trouser.

Solution

Let the prices of each shirt, each trouser and each pair of shoe be ₹x, ₹y, and ₹z, respectively.

$$2x + 4y + 5z = 3600 \quad (1)$$

$$6x + 5y + 15z = 8700 \quad (2)$$

Multiplying (1) by 3 and subtracting (2) from it, $7y = 2100$

$\therefore \quad y = 300$

3. Even in case of indeterminate equations, when some additional conditions are either implicitly built into the problem or explicitly imposed by specifying some constraints on the values of the variables, we may some times be able to determine the values of the variables uniquely or find out a finite set of values that the variables may take. Such problems are separately considered under the chapter 'SPECIAL EQUATIONS.'
4. Sometimes, even if we have equations less in number than the number of variables (i.e., indeterminate equations), while we cannot find out the values of ALL the variables uniquely, it may be possible to find out the value of some specific combination of the variables.

Example 7

The cost of 3 dosas, 5 idlis, and 7 vadas is ₹154. The cost of 5 dosas, 8 idlis, and 11 vadas is ₹246. Find the total cost of one idli, one dosa, and one vada.

Solution

Let the cost of each dosa, each idli, and each vada be ₹d, ₹i, and ₹v, respectively.

$$3d + 5i + 7v = 154 \quad (1)$$

$$5d + 8i + 11v = 246 \quad (2)$$

Multiplying (1) by 3 and subtracting it, from twice (2), $d + i + v = 30$

5. Sometimes, even if we have three equations in three unknown, we may not be able to uniquely determine the values of the variables if the equations are not 'INDEPENDENT,' i.e. one of the given equations can be written as a 'linear combination' of the other two equations.

   For example, let us take the following system of three equations in three unknowns.

$$3x + 5y + 7z = 12 \quad (3)$$

$$x - 3y + 9z = 16 \quad (4)$$

$$9x + 8y + 31z = 54 \quad (5)$$

   If we try to solve these equations, we will find that we cannot get a unique solution. That is because these equations are not independent. In this case, equation (5) can be obtained by multiplying equation (3) by 2.5 and equation (4) by 1.5 and adding them.

   If there are three equations l_1, l_2, and l_3 in three unknowns, we say that they are linearly dependent if one of the three equations can be written as a linear combination of the other two, i.e. $l_3 = l_1 + kl_2$ where k is any constant.

   In such a case, the system of equations will have infinite number of solutions.

   If it is not possible to write the three equations in the form above, then they are linearly independent and the system of equations will have a unique solution.
6. Sometimes, we can have 'inconsistent' equations. For example, if we know that $x + 2y = 4$, then the value of $2x + 4y$ has to be 8. The expression $(2x + 4y)$ cannot take any other value. If it is given any other value, there will be inconsistency in the data because then we will effectively be saying that $x + 2y = 4$ and at the same time $x + 2y \neq 4$.

   So, if we have the system of equations $x + 2y = 4$ and $2x + 4y = k$, this system of equations will be consistent ONLY If the value of $k = 8$. For any other value of k, the system of equations will be inconsistent.

   In the above system of equations, when $k = 8$, there will be infinite number of solutions (and not a unique solution).

Example 8

Find the value of k for which the following system of equations will be consistent.

$$2x - 5y = 10 \quad \text{and} \quad 6x - 15y = k$$

Solution

In the given system of equations, the ratio of the coefficients of x equals the ratio of the coefficients of y.

$\therefore$ They would be consistent only if this ratio equals the ratio of the constant terms.

$\therefore$ If $\dfrac{10}{k} = \dfrac{2}{6} = \dfrac{-5}{-15}$ i.e.

if $k = 30$, the given system of equations would be consistent.

Exercises

Direction for questions 1 to 25: Select the correct alternative from the given choices.

1. P, Q, and R are successive even natural numbers in ascending order. Five times R is eight more than seven times P. Find Q.
(A) 6 (B) 8 (C) 12 (D) 14

2. Divide 1 kg weight into two parts such that the sum of the parts is $5/4^{th}$ the difference.
(A) 550 g, 450 g (B) 200 g, 800 g
(C) 900 g, 100 g (D) 400 g, 600 g

3. A is greater than B by $1/3^{rd}$ the sum of A and B. If B is increased by 40, it becomes greater than twice A by 10. Find A, B.
(A) 30, 20 (B) 60, 30
(C) 20, 10 (D) 20, 40

4. Ajay was asked to find $(2/9)^{th}$ of a number. He instead multiplied the number by (9/2) and obtained an answer which was 4235 more than the correct answer. Find the number.
(A) 900 (B) 945
(C) 990 (D) 810

5. An amount of ₹5,600 is divided among A, B, and C. The sum of the shares of B and C is equal to thrice the share of A. The sum of the shares of A and C is equal to nine-fifths the share of B. What is the share of C?
(A) ₹1,400 (B) ₹2,400
(C) ₹2,200 (D) ₹2,000

6. Four times the sum of the digits of a two-digit number is 18 less than the number and is also 9 less than the number formed by reversing its digits. Find the product of its digits.
(A) 12 (B) 20 (C) 30 (D) 42

7. Six years ago, Ram's age was four times Shyam's age. Six years hence, Ram's age will be thrice Shyam's age. After how many years from now will their combined age be 150 years?
(A) 21 (B) 9 (C) 36 (D) 18

8. The sum of the ages of Bharat and Sharat is twice the sum of their ages seven years ago. What is the product of their present ages, if the sum of the squares of their ages is 400?
(A) 192 (B) 180 (C) 200 (D) 164

9. Ashok has a total of 30 notes in denominations of ₹20 and ₹5. The total value of the notes with him is ₹300. Find the number of ₹20 notes with him.
(A) 5 (B) 10 (C) 8 (D) 6

10. A fraction is such that the numerator is five less than the denominator. Also four times, the numerator is one more than the denominator. Find the fraction.
(A) 4/9 (B) 3/8 (C) 2/7 (D) 7/12

11. The digits of a two digit number differ by 3. Find the difference of the number and the number formed by reversing its digits.
(A) 18 (B) 27 (C) 36 (D) 45

12. Two chocolates, three milk shakes and four cakes cost ₹190. Four chocolates and eight cakes cost ₹320. Find the cost of a milkshake (in ₹).
(A) 10 (B) 20
(C) 30 (D) Cannot be determined

13. Three consecutive even integers are such that one-third of the second number is equal to one-fourth of the third number. Find the three numbers.
(A) 4, 6, 8 (B) 8, 10, 12
(C) 12, 14, 16 (D) 2, 4, 6

14. Amar, Bhavan, Chetan, and Dinesh have a total of ₹150 with them. Amar has one-fourth of the total amount with the others. Find the amount with Amar (in ₹).
(A) 20 (B) 25 (C) 30 (D) 37.5

15. Ramesh is thrice as old as Suresh. Two years hence, Ramesh will be twice as old as Suresh. Find Ramesh's present age (in years).
(A) 2 (B) 3 (C) 4 (D) 6

16. Nalini has an amount of ₹20 in coins of denominations of 50 paise and ₹1. If she has a total of 30 coins with her, how many ₹1 coins does she have?
(A) 20 (B) 10 (C) 15 (D) 30

17. A two-digit number is one more than six times the sum of its digits and also five more than forty six times the difference of its digits. Find the number.
(A) 79 (B) 97 (C) 49 (D) 94

18. Find the value of k if the equations $3x + (k/3 + 2)y = 1$ and $kx + 2ky = 4$ have infinite solutions.
(A) 9 (B) 6 (C) 18 (D) 12

19. Cost of two pens, five pencils, and seven erasers is ₹37. Cost of seven pens, one eraser, and two pencils is ₹49. What is the cost of nine pencils and fortyseven pens?
(A) ₹184 (B) ₹276
(C) ₹284 (D) None of these

20. The sum of two numbers is 250. The difference of their squares is 12,500. Find the larger number.
(A) 130 (B) 140 (C) 150 (D) 160

21. Five three-digit numbers including N, were to be added. While adding, the reverse of N was added by mistake instead of N. Hence, the sum increased by 11 times the sum of the digits of N. Eight times the difference of N's units and hundreds digits is 6 more than twice its hundreds digit. Find its tens digit.
(A) 4 (B) 6 (C) 8 (D) 2

22. The cost of two pens, one eraser, and three sharpeners, is ₹23. The cost of six pens, three erasers, and one sharpener is ₹45. The cost of fourteen pens, seven erasers, and twenty one sharpeners is ₹161. Find the cost of each pen (in ₹).
(A) 3
(B) 4
(C) 5
(D) Cannot be determined

23. A child went to a shop to buy a pen, a pencil and a ruler where costs are integral values (in ₹) and are in decreasing order. Each item costs at least ₹4. The total cost is ₹15 and the cost of a pencil is ₹5. How many pencils can he purchase with the amount required to purchase ten rulers?
(A) 10 (B) 12 (C) 8 (D) 9

24. Nitya and Satya have some marbles with them. Nitya says to Satya, 'If you give one marble to me, we will have equal number of marbles'. Satya says to Nitya, 'If you give me one marble, I will have twice the number of marbles you have'. How many marbles do Nitya and Satya have respectively?
(A) 4, 6 (B) 5, 7
(C) 6, 4 (D) 7, 5

25. John covers 10 km per hour more than Peter while driving. On doubling his speed, Peter covers 15 km per hour more than John who is driving at his normal speed. What is John's speed?
(A) 40 km/hr
(B) 25 km/hr
(C) 45 km/hr
(D) 35 km/hr

Answer Keys

1. B	**2.** C	**3.** C	**4.** C	**5.** C	**6.** B	**7.** B	**8.** A	**9.** B	**10.** C
11. B	**12.** A	**13.** A	**14.** C	**15.** D	**16.** B	**17.** B	**18.** D	**19.** D	**20.** C
21. B	**22.** D	**23.** C	**24.** B	**25.** D					