7. VISCOUS FLOW OF INCOMPRESSIBLE FLUID

In viscous flow fluid particles move along straight parallel paths in *layers*. It occurs at low velocity and ***viscous force*** predominates inertial force.

REYNOLD'S NUMBER (R_e)

$$R_e = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho v d}{\mu}$$

Here, ρ = Density
d = characteristics length
V = average velocity

NATURE OF FLOW ACCORDING TO REYNOLDS NUMBER FOR PIPE AND OPEN CHANNEL FLOW

The limiting values of Reynold's Number corresponding to which flow is Laminar is given by:

Flow Condition	Pipe flow	Open channel flow
Laminar Flow	$Re \leq 2000$	$Re \leq 500$
Transitional Flow	$2000 < Re < 4000$	$500 < Re < 1000$
Turbulent Flow	$Re > 4000$	$Re > 1000$

ENTRANCE LENGTH (L_e)

The length of pipe from its entrance upto the point where flow attains fully developed velocity profile and which remains unaltered beyond that known as entrance length.

The entrance length required to establish fully developed ***laminar flow*** is given by

$$\frac{L_e}{D} = 0.07\,R_e$$

The entrance length for fully developed ***turbulent flow*** is given by

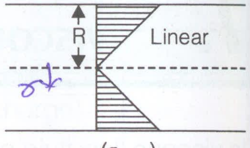
$$\frac{L_e}{D} = 50$$

LAMINAR FLOW THROUGH CIRCULAR PIPE (HAGEN-POISEULLE FLOW)

- Shear stress (τ) distribution

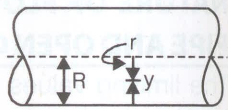
$$\tau = \left(-\frac{\partial P}{\partial x}\right).\frac{r}{2}$$

The negative sign on $\frac{\partial p}{\partial x}$ indicates decrease in pressure in the direction of flow. The pressure must decrease because pressure force is the only means available to compensate for resistance to the flow, the potential and kinetic energy remain constant.



> **Remember**
> - Above equation is valid for steady and uniform flow.
> - The maximum value of stress occur at r = R.
> - In laminar flow shear stress is entirely due to *viscous* action.

- **Velocity distribution**

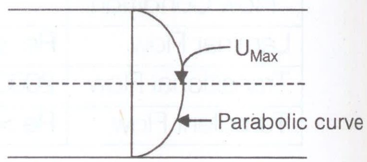


r is the distance measured from axis

$$U = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)(R^2 - r^2)$$

At center, r = 0 thus

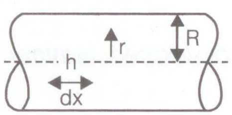
$$U_{max} = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)\cdot R^2$$



Here, U_{max} = Maximum velocity

$$U = U_{max}\left(1 - \frac{r^2}{R^2}\right)$$

- **Discharge (Q) through a pipe**



Q ≠ AV, because 'V' is changing

- **Heigen-Poiseulle equation, for *viscous* flow**

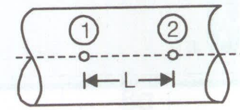
$$Q = \frac{\pi}{128\mu}\cdot\left(\frac{-\partial P}{\partial x}\right)\cdot D^4$$

$$\text{Mean velocity} = \frac{U_{max}}{2}$$

> **Remember**
> - The maximum velocity occur at axis.
> - The point where local velocity is equal to mean velocity is given by
>
> $$r = \frac{R}{\sqrt{2}} = 0.707\,R$$

- **Pressure drop in pipe**

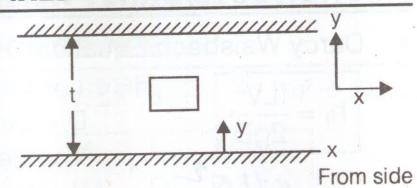
$$P_1 - P_2 = \frac{32\mu V_{avg.} L}{D^2}$$



LAMINAR FLOW BETWEEN TWO FIXED PARALLEL PLATE

- **Velocity**

$$u = \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)(ty - y^2)$$



> **Remember**
> - The maximum velocity occurs at $y = \frac{t}{2}$ and it is given by,
>
> $$u_m = \frac{t^2}{8\mu}\left(-\frac{\partial p}{\partial x}\right)$$
>
> - $V_{avg} = 0.66\, u_{max}$

- **Discharge per unit width**

$$Q = \frac{1}{12\mu}\left(-\frac{\partial P}{\partial x}\right)t^3$$

- **Pressure drop in given length**

$$P_1 - P_2 = \frac{12\mu VL}{t^2}$$

MOMENTUM CORRECTION FACTOR (β)

It is defined as the ratio of momentum/sec based on *actual* velocity to the momentum/sec based on *average* velocity.

$$\beta = \frac{1}{AV^2}\int u^2 \cdot dA$$

V = Average velocity

u = Local velocity at distance r

- For laminar flow, β = 1.33
- For turbulent flow, β = 1.2

KINETIC ENERGY CORRECTION FACTOR (α)

It is defined as the ratio of kinetic energy/second based on *actual* velocity to the kinetic energy/second based on *average* velocity.

$$\alpha = \frac{1}{AV^3}\int u^3 \cdot dA$$

- For laminar flow, α = 2
- For turbulent flow, α = 1.33

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