STRAIGHT LINES [JEE ADVANCED PREVIOUS YEAR SOLVED PAPER]

JEE ADVANCED

Single Correct Answers Type

1. The straight lines $x + y = 0$, $3x + y - 4 = 0$, and $x + 3y - 4 = 0$ form a triangle which is

   a. isosceles **b.** equilateral
   c. right-angled **d.** none of these

   (IIT-JEE 1983)

2. If $P \equiv (1, 0)$, $Q \equiv (-1, 0)$, and $R \equiv (2, 0)$ are three given points, then the locus of the point S satisfying the relation $SQ^2 + SR^2 = 2SP^2$ is
   a. a straight line parallel to the x-axis
   b. a circle passing through the origin
   c. a circle with center at the origin
   d. a straight line parallel to the y-axis **(IIT-JEE 1988)**
3. Line L has intercepts a and b on the coordinate axes. When the axes are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q. Then,
   a. $a^2 + b^2 = p^2 + q^2$ b. $1/a^2 + 1/b^2 = 1/p^2 + 1/q^2$
   c. $a^2 + p^2 = b^2 + q^2$ d. $1/a^2 + 1/p^2 = 1/b^2 + 1/q^2$
   (IIT-JEE 1990)
4. If the sum of the distances of a point from two perpendicular lines in a plane is 1, then its locus is
   a. square b. circle
   c. straight line d. two intersecting lines
   (IIT-JEE 1992)
5. The equations to a pair of opposite sides of a parallelogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5 = 0$. The equations to its diagonals are
   a. $x + 4y = 13, y = 4x - 7$
   b. $4x + y = 13, 4y = x - 7$
   c. $4x + y = 13, y = 4x - 7$
   d. $y - 4x = 13, y + 4x = 7$ **(IIT-JEE 1994)**
6. The orthocenter of the triangle formed by the lines $xy = 0$ and $x + y = 1$ is
   a. (1/2, 1/2) b. (1/3, 1/3)
   c. (0, 0) d. (1/4, 1/4)
   (IIT-JEE 1995)
7. Let PQR be a right-angled isosceles triangle, right angled at $P(2, 1)$. If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is
   a. $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$
   b. $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$
   c. $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$
   d. $3x^2 - 3y^2 - 8xy - 15y - 20 = 0$ **(IIT-JEE 1999)**
8. Let PS be the median of the triangle with vertices $P(2, 2)$, $Q(6, -1)$, and $R(7, 3)$. The equation of the line passing through $(1, -1)$ and parallel to PS is
   a. $2x - 9y - 7 = 0$ b. $2x - 9y - 11 = 0$
   c. $2x + 9y - 11 = 0$ d. $2x + 9y + 7 = 0$
   (IIT-JEE 2000)
9. The number of integral values of m, for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is
   a. 2 b. 0 c. 4 d. 1
   (IIT-JEE 2001)
10. The area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$, and $y = nx + 1$ equals
    a. $|m + n|/(m - n)^2$ b. $2/|m + n|$
    c. $1/(|m + n|)$ d. $1/(|m - n|)$
    (IIT-JEE 2001)
11. Let $P = (-1, 0)$, $Q = (0, 0)$, and $R = (3, 3\sqrt{3})$ be three points. Then the equation of the bisector of $\angle PQR$ is
    a. $(\sqrt{3}/2)x + y = 0$ b. $x + \sqrt{3}y = 0$
    c. $\sqrt{3}x + y = 0$ d. $x + (\sqrt{3}/2)y = 0$
    (IIT-JEE 2002)
12. A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at points P and Q, respectively. Then the point O divides the segment PQ in the ratio
    a. 1 : 2 b. 3 : 4
    c. 2 : 1 d. 4 : 3 **(IIT-JEE 2002)**
13. Area of the triangle formed by the line $x + y = 3$ and the angle bisectors of the pairs of straight lines $x^2 - y^2 + 2y = 1$ is
    a. 2 sq. units b. 4 sq. units
    c. 6 sq. units d. 8 sq. units
    (IIT-JEE 2004)
14. The locus of the orthocenter of the triangle formed by the lines $(1 + p)x - py + p(1 + p) = 0$, $(1 + q)x - qy + q(1 + q) = 0$ and $y = 0$, where $p \neq q$, is
    a. a hyperbola b. a parabola
    c. an ellipse d. a straight line
    (IIT-JEE 2009)
15. A straight line L through the point $(3, -2)$ is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is
    a. $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$
    b. $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$
    c. $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$
    d. $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$ **(IIT-JEE 2011)**
16. For $a > b > c > 0$, the distance between (1, 1) and the point of intersection of the lines $ax + by + c = 0$ and $bx + ay + c = 0$ is less than $2\sqrt{2}$. Then
    a. $a + b - c > 0$ b. $a - b + c < 0$
    c. $a - b + c > 0$ d. $a + b - c < 0$
    (JEE Advanced 2013)

Multiple Correct Answers Type

1. Three lines $px + qy + r = 0$, $qx + ry + p = 0$, and $rx + py + q = 0$ are concurrent if
   a. $p + q + r = 0$ b. $p^2 + q^2 + r^2 = pr + rp + pq$
   c. $p^3 + q^3 + r^3 = 3pqr$ d. none of these
   (IIT-JEE 1985)
2. All points lying inside the triangle formed by the points (1, 3), (5, 0), and (−1, 2) satisfy
   a. $3x + 2y \geq 0$ b. $2x + y - 13 \geq 0$
   c. $2x - 3y \leq 12$ d. $-2x + y \geq 0$
   (IIT-JEE 1986)
3. The diagonals of a parallelogram $PQRS$ are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then $PQRS$ must be a
   a. rectangle b. square
   c. cyclic quadrilateral d. rhombus
   (IIT-JEE 1998)

Matching Column Type

1. Consider the lines given by

$L_1: x + 3y - 5 = 0$

$L_2: 3x - ky - 1 = 0$

$L_3: 5x + 2y - 12 = 0$

Column I	Column II
(a) L_1, L_2, L_3 are concurrent if	**(p)** $k = -9$
(b) One of L_1, L_2, L_3 is parallel to at least one of the other two if	**(q)** $k = -6/5$
(c) L_1, L_2, L_3 form a triangle if	**(r)** $k = 5/6$
(d) L_1, L_2, L_3 do not form a triangle if	**(s)** $k = 5$

(IIT-JEE 2008)

Assertion-Reasoning Type

1. Lines L_1, L_2 given by $y - x = 0$ and $2x + y = 0$ intersect the line L_3 given by $y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement 1: The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$.

Statement 2: In any triangle, the bisector of an angle divides the triangle into two similar triangles.

a. Both the statements are true and statement 2 is the correct explanation of statement 1.

b. Both the statements are true but statement 2 is not the correct explanation of statement 1.

c. Statement 1 is true and statement 2 is false.

d. Statement 1 is false and statement 2 is true.

(IIT-JEE 2007)

Fill in the Blanks Type

1. The area enclosed within the curve $|x| + |y| = 1$ is_____. **(IIT-JEE 1981)**

2. The set of lines $ax + by + c = 0$, where $3a + 2b + 4c = 0$, is concurrent at the point__________. **(IIT-JEE 1982)**

3. If a, b, and c are in AP, then the straight line $ax + by + c = 0$ will always pass through a fixed point whose coordinates are______________. **(IIT-JEE 1984)**

4. The orthocenter of the triangle formed by the lines $x + y = 1$, $2x + 3y = 6$, and $4x - y + 4 = 0$ lies in quadrant number__________________. **(IIT-JEE 1985)**

5. Let the algebraic sum of the perpendicular distance from the points (2, 0), (0, 2), and (1, 1) to a variable straight line be zero. Then the line passes through a fixed point whose coordinates are_____________. **(IIT-JEE 1991)**

6. The vertices of a triangle are $A(-1, -7)$, $B(5, 1)$, and $C(1, 4)$. The equation of the bisector of $\angle ABC$ is_________________. **(IIT-JEE 1993)**

True/False Type

1. The straight line $5x + 4y = 0$ passes through the point of intersection of the straight lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$. **(IIT-JEE 1983)**

2. The lines $2x + 3y + 19 = 0$ and $9x + 6y - 17 = 0$ cut the coordinate axes at concyclic points. **(IIT-JEE 1988)**

Subjective Type

1. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are $(-3, 1)$ and $(1, 1)$. Find the equations of the other three sides. **(IIT-JEE 1978)**

2. Find the equation of the line which bisects the obtuse angle between the lines $x - 2y + 4 = 0$ and $4x - 3y + 2 = 0$. **(IIT-JEE 1979)**

3. A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by line L and the coordinate axes is 5. Find the equation of line L. **(IIT-JEE 1980)**

4. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$. Find c and the remaining vertices. **(IIT-JEE 1981)**

5. The ends A, B of a straight line segment of constant length c slide upon the fixed rectangular axes OX, OY, respectively. If the rectangle $OAPB$ is completed, then show that the locus of the foot of the perpendicular drawn from P to AB is $x^{2/3} + y^{2/3} = c^{2/3}$. **(IIT-JEE 1983)**

6. Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point $(1, -10)$. Determine the equation of the third side. **(IIT-JEE 1984)**

7. One of the diameters of the circle circumscribing the rectangle $ABCD$ is $4y = x + 7$. If A and B are the points $(-3, 4)$ and $(5, 4)$, respectively, then find the area of the rectangle. **(IIT-JEE 1985)**

8. Two sides of a rhombus $ABCD$ are parallel to the lines $y = x + 2$ and $y = 7x + 3$. If the diagonals of the rhombus intersect at the point (1, 2) and the vertex A is on the y-axis, then find the possible coordinates of A. **(IIT-JEE 1985)**

9. The equations of the perpendicular bisectors of the sides AB and AC of triangle ABC are $x - y + 5 = 0$ and $x + 2y = 0$, respectively. If the point A is $(1, -2)$, then find the equation of the line BC. **(IIT-JEE 1986)**

10. Lines $L_1 \equiv ax + by + c = 0$ and $L_2 \equiv lx + my + n = 0$ intersect at the point P and make an angle θ with each other. Find the equation of a line different from L_2 which passes through P and makes the same angle θ with L_1. **(IIT-JEE 1988)**

11. Let ABC be a triangle with $AB = AC$. If D is the midpoint of BC, E is the foot of the perpendicular drawn from D to AC, and F is the midpoint of DE, then prove that AF is perpendicular to BE. **(IIT-JEE 1989)**

12. Straight lines $3x + 4y = 5$ and $4x - 3y = 15$ intersect at the point A. Points B and C are chosen on these two lines such that $AB = AC$. Determine the possible equations of the line BC passing through the point (1, 2). **(IIT-JEE 1990)**

13. Find the equation of the line passing through the point (2, 3) and making an intercept of length 2 units between the lines $y + 2x = 3$ and $y + 2x = 5$. **(IIT-JEE 1991)**

14. Show that all chords of the curve $3x^2 - y^2 - 2x + 4y = 0$, which subtend a right angle at the origin, pass through a fixed point. Find the coordinates of the point. **(IIT-JEE 1991)**

15. Determine all the values of α for which the point (α, α^2) lies inside the triangle formed by the lines
$2x + 3y - 1 = 0$
$x + 2y - 3 = 0$
$5x - 6y - 1 = 0$ **(IIT-JEE 1992)**

16. A line through $A(-5, -4)$ meets the lines $x + 3y + 2 = 0$, $2x + y + 4 = 0$, and $x - y - 5 = 0$ at the points B, C, and D, respectively. If $(15/AB)^2 + (10/AC)^2 = (6/AD)^2$, find the equation of the line. **(IIT-JEE 1993)**

17. A rectangle $PQRS$ has its side PQ parallel to the line $y = mx$ and vertices P, Q, and S on the lines $y = a$, $x = b$, and $x = -b$, respectively. Find the locus of the vertex R. **(IIT-JEE 1996)**

18. A straight line L through the origin meets the lines $x + y = 1$ and $x + y = 3$ at P and Q, respectively. Through P and Q, two straight lines L_1 and L_2 are drawn, parallel to $2x - y = 5$ and $3x + y = 5$, respectively. Lines L_1 and L_2 intersect at R. Show that the locus of R, as L varies, is a straight line. **(IIT-JEE 2002)**

19. A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q. Find the absolute minimum value of $OP + OQ$ as L varies, where O is the origin. **(IIT-JEE 2002)**

20. The area of the triangle formed by the intersection of a line parallel to the x-axis and passing through $P(h, k)$ with the lines $y = x$ and $x + y = 2$ is $4h^2$. Find the locus of the point P. **(IIT-JEE 2005)**

Answer Key

JEE Advanced

Single Correct Answer Type

1. a.	**2.** d.	**3.** b.	**4.** a.	**5.** c.
6. c.	**7.** b.	**8.** d.	**9.** a.	**10.** d.
11. c.	**12.** b.	**13.** a.	**14.** d.	**15.** b.
16. a.				

Multiple Correct Answers Type

1. a., b., c. **2.** a., c. **3.** d.

Matching Column Type

1. (a)–(s); (b)–(p), (q); (c)–(r); (d)–(p), (q), (s)

Assertion–Reasoning Type

1. c.

Fill in the Blanks Type

1. 2 sq. units
2. (3/4, 1/2)
3. (1, –2)
4. first quadrant
5. (1, 1)
6. $x - 7y + 2 = 0$

True/False Type

1. True **2.** True

Subjective Type

1. $4x + 7y - 11 = 0$, $7x - 4y - 3 = 0$, $7x - 4y + 25 = 0$
2. $(4 - \sqrt{5})x + (2\sqrt{5} - 3)y - (4\sqrt{5} - 2) = 0$
3. $x + 5y - 5\sqrt{2} = 0$ or $x + 5y + 5\sqrt{2} = 0$
4. $c = -4$, (4, 4), (2, 0)
6. $x - 3y - 31 = 0$, $3x + y + 7 = 0$
7. 32 sq. units
8. (0, 0) or (0, 5/2)
9. $14x + 23y - 40 = 0$
10. $2(al + bm)(ax + by + c) - (a^2 + b^2)(lx + my + n) = 0$
12. $x - 7y + 13 = 0$ or $7x + y - 9 = 0$
13. $3x + 4y - 18 = 0$, $x = 2$
15. $x \in (-3/2, -1) \cup (1/2, 1)$
16. $2x + 3y + 22 = 0$
17. $(m^2 - 1)x - my + b(m^2 + 1) + am = 0$
19. 18
20. $y = 2x + 1$ or $y = -2x + 1$

Hints and Solutions

JEE Advanced

Single Correct Answer Type

1. **a.** Solving the given equations of lines pairwise, we get the vertices of triangle as $A(-2, 2)$, $B(2, -2)$, $C(1, 1)$. Then,

   $AB = \sqrt{16+16} = 4\sqrt{2}$

   $BC = \sqrt{1+9} = \sqrt{10}$

   $CA = \sqrt{9+1} = \sqrt{10}$

   Hence, the triangle is isosceles.

2. **d.** We have $P \equiv (1, 0)$, $Q \equiv (-1, 0)$, $R \equiv (2, 0)$. Let $S \equiv (x, y)$.

   Now, given that $SQ^2 + SR^2 = 2SP^2$. Hence,

   $(x+1)^2 + y^2 + (x-2)^2 + y^2 = 2[(x-1)^2 + y^2]$

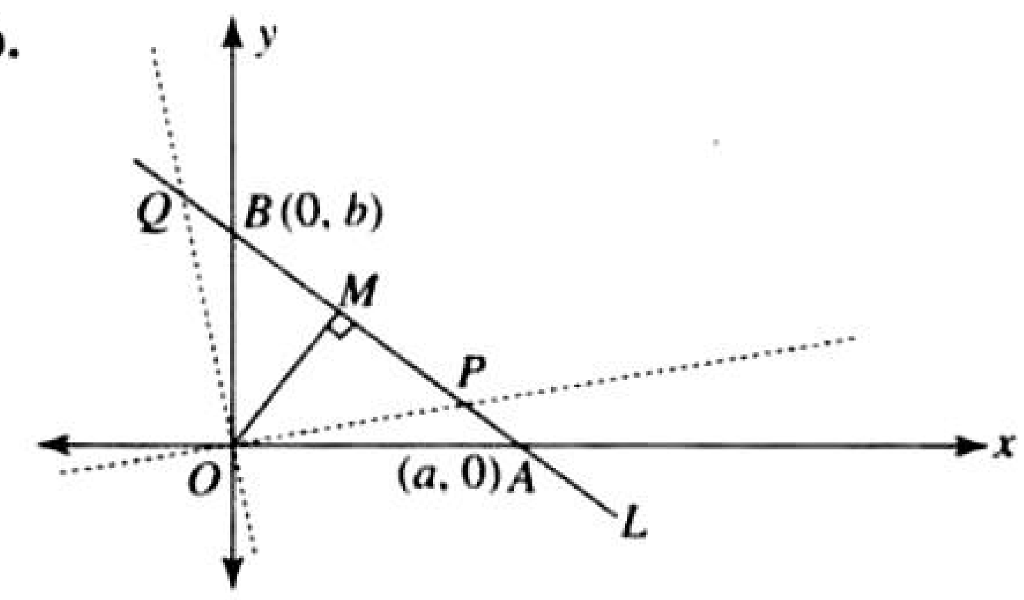
   or $2x^2 + 2y^2 - 2x + 5 = 2x^2 + 2y^2 - 4x + 2$

   or $2x + 3 = 0$

   or $x = -\dfrac{3}{2}$

   which is a straight line parallel to the y-axis.

3. **b.**



   As L has intercepts a and b on the axes, the equation of L is

   $$\frac{x}{a} + \frac{y}{b} = 1 \qquad \text{(i)}$$

   Let the x-axis and the y-axis be rotated through an angle θ in the anticlockwise direction. In the new system, the intercepts are p and q ($OP = p$, $OQ = q$). Therefore, the equation of L w.r.t. new coordinate system becomes

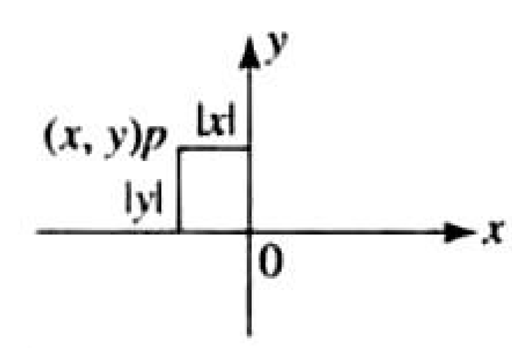
   $$\frac{x}{p} + \frac{y}{q} = 1 \qquad \text{(ii)}$$

   As the origin is fixed in rotation, the distance of line from the origin in both the cases should be same. Hence, we get

   $$d = \left|\frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}\right| = \left|\frac{1}{\sqrt{\frac{1}{p^2} + \frac{1}{q^2}}}\right|$$

   or $\dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{1}{p^2} + \dfrac{1}{q^2}$

4. **a.** Let the two perpendicular lines be the coordinate axes. Let (x, y) be the point, sum of whose distances from two axes is 1. Then we must have



   $|x| + |y| = 1$

   or $\pm x \pm y = 1$

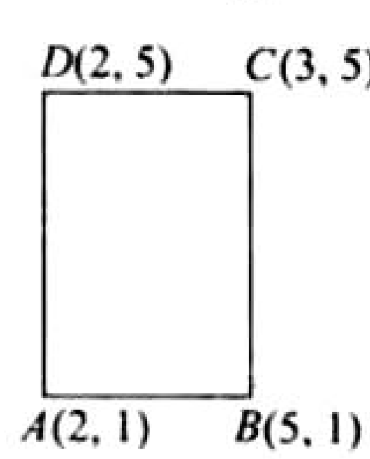
   The four lines are

   $x + y = 1$, $x - y = 1$, $-x + y = 1$, $-x - y = 1$

   Any two adjacent sides are perpendicular to each other. Also, each line is equidistant from the origin. Therefore, the figure formed is a square.

5. **c.** The sides of parallelogram are $x = 2$, $x = 3$, $y = 1$, $y = 5$.

   $\therefore$ Vertices of parallelograms are as shown in the figure



   The equation of diagonal AC is

   $$\frac{y-1}{5-1} = \frac{x-2}{3-2}$$

   or $y = 4x - 7$

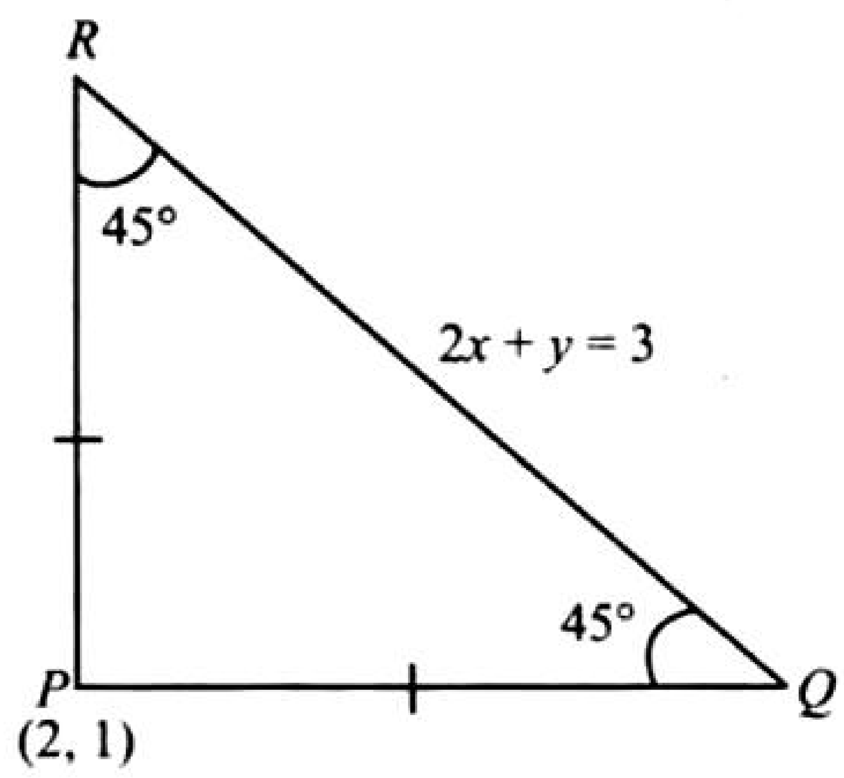
   and the equation of diagonal BD is

   $$\frac{x-2}{3-2} = \frac{y-5}{1-5}$$

   or $4x + y = 13$

6. **c.** The lines by which triangle is formed are $x = 0$, $y = 0$, and $x + y = 1$. Clearly, it is a right triangle. We know that in a right-angled triangle, the orthocenter coincides with the vertex at which right angle is formed. Therefore, the orthocenter is $(0, 0)$.

7. b.



Let m be the slope of PQ. Then,

$$\tan 45° = \left|\frac{m-(-2)}{1+m(-2)}\right| = \left|\frac{m+2}{1-2m}\right|$$

$\therefore \quad m + 2 = 1 - 2m$ or $-1 + 2m = m + 2$

$\therefore \quad m = -\frac{1}{3}$ or $m = 3$

Hence, the equation of PQ is

$$y - 1 = -\frac{1}{3}(x - 2)$$

or $\quad x + 3y - 5 = 0$

and the equation of PR is

$$y - 1 = 3(x - 2)$$

or $\quad 3x - y - 5 = 0$

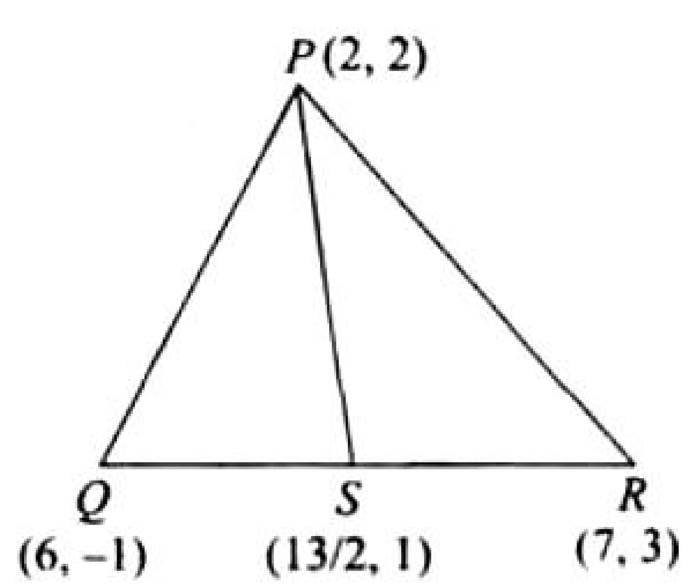
Hence, the combined equation of PQ and PR is

$$(x + 3y - 5)(3x - y - 5) = 0$$

or $\quad 3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

8. d. S is the midpoint of Q and R. Therefore,

$$S \equiv \left(\frac{7+6}{2}, \frac{3-1}{2}\right) \equiv \left(\frac{13}{2}, 1\right)$$



Now, the slope of PS is $m = \frac{2-1}{2-13/2} = -\frac{2}{9}$

Then the equation of the line passing through $(1, -1)$ and parallel to PS is

$$y + 1 = -\frac{2}{9}(x - 1)$$

or $\quad 2x + 9y + 7 = 0$

9. a. Solving given lines for the x-coordinate of the point of intersection

$$3x + 4(mx + 1) = 9$$

or $\quad (3 + 4m)x = 5$ or $x = \frac{5}{3+4m}$

For x to be an integer, $3 + 4m$ should be a divisor of 5, i.e., of 1, -1, 5, or -5. Hence,

$3 + 4m = 1$ or $m = -\frac{1}{2}$ (not integer)

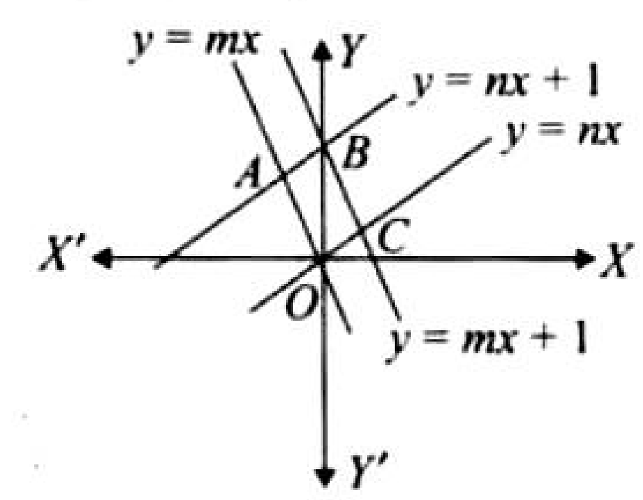
$3 + 4m = -1$ or $m = -1$ (integer)

$3 + 4m = 5$ or $m = \frac{1}{2}$ (not an integer)

$3 + 4m = -5$ or $m = -2$ (integer)

Hence, there are two integral values of m.

10. d. $y = mx$ and $y = nx$ are lines through $(0, 0)$, $y = mx + 1$ and $y = nx + 1$ are lines parallel to $y = mx$ and $y = nx$ respectively with y-intercept 1.



The vertices are $O(0, 0)$, $A(1/(m-n), m/(m-n))$. The area of parallelogram is given by

$$2 \times \text{Ar}(\Delta OAB)$$

$$= 2 \times \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ \frac{1}{m-n} & \frac{m}{m-n} & 1 \end{vmatrix}$$

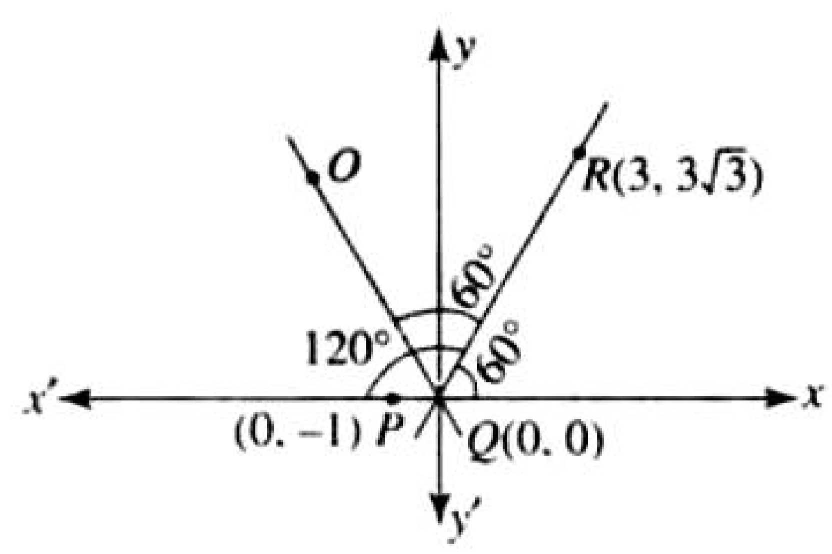
$$= \frac{1}{|m-n|}$$

11. c. The slope of QR is

$(3\sqrt{3} - 0)/(3 - 0) = \sqrt{3}$

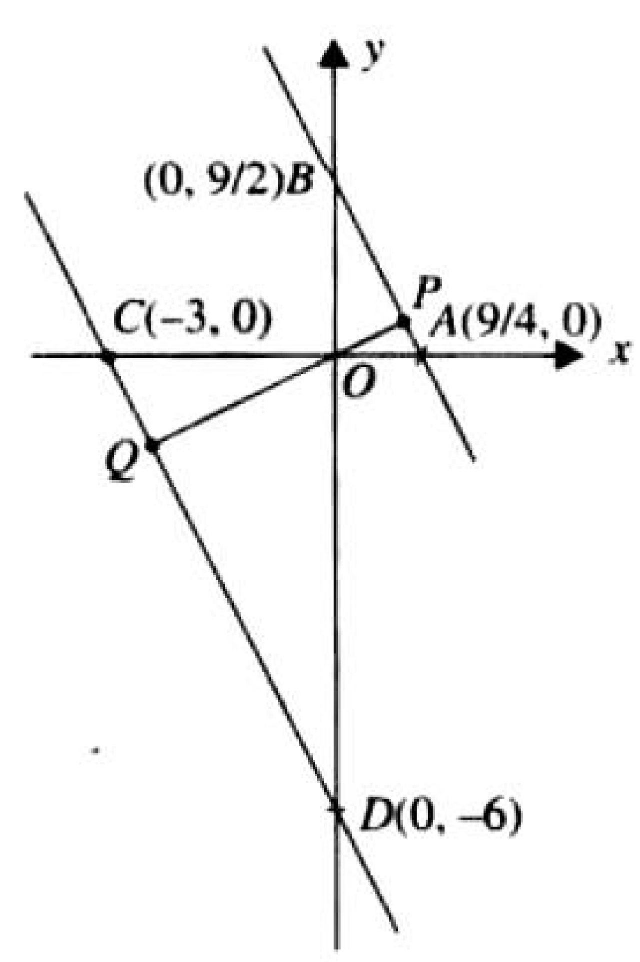
i.e., $\theta = 60°$

Clearly, $\angle PQR = 120°$. OQ is the angle bisector of the angle. So, line OQ makes $120°$ with the positive direction of the x-axis.



Therefore, the equation of the bisector of $\angle PQR$ is $y = x \tan 120°$ or $y = -\sqrt{3}x$, i.e., $\sqrt{3}x + y = 0$.

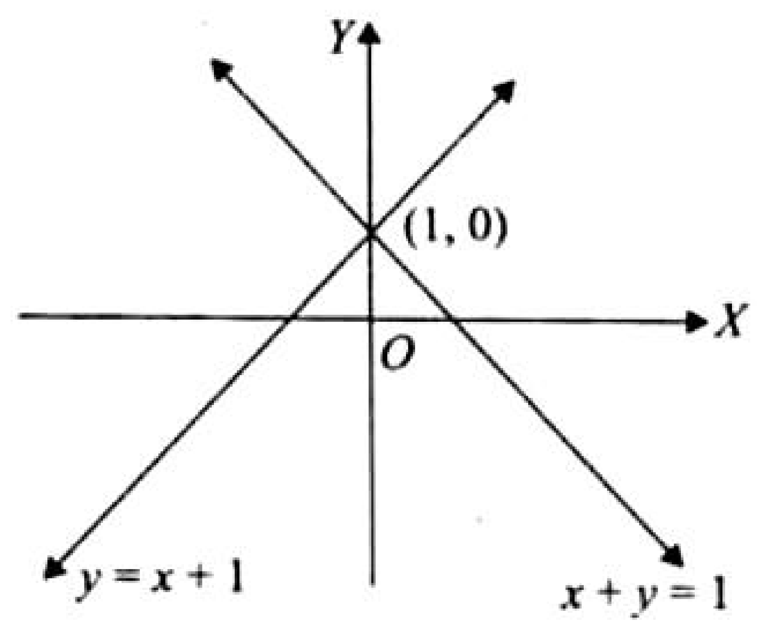
12. b. Let any line through the origin meets the given lines at P and Q as shown in figure.



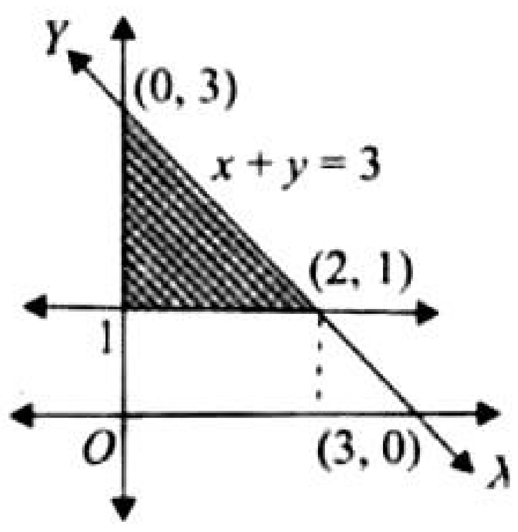
Now, from the figure, triangles OAP and OCQ are similar. Therefore,

$$\frac{OP}{OQ} = \frac{OA}{OC} = \frac{9/4}{3} = \frac{3}{4}$$

13. a. $x^2 - y^2 + 2y = 1$ or $x = \pm(y - 1)$



The bisectors of the above lines are $x = 0$ and $y = 1$.



So, the area between $x = 0$, $y = 1$, and $x + y = 3$ is the shaded region shown in the figure. The area is given by $(1/2) \times 2 \times 2 = 2$ sq. units.

14. d. The intersection point of $y = 0$ with the first line is $B(-p, 0)$.

The intersection point of $y = 0$ with the second line is $A(-q, 0)$.

The intersection point of the two lines is

$C(pq, (p + 1)(q + 1))$

The altitude from C to AB is $x = pq$.

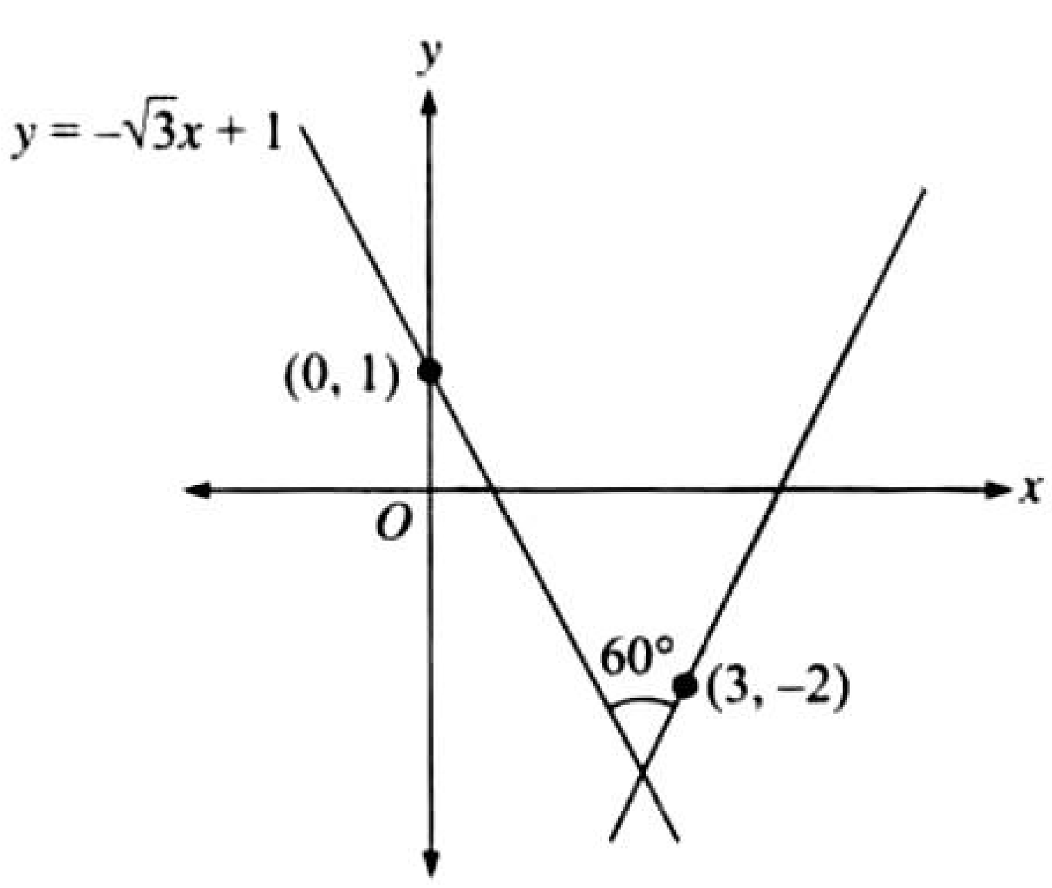
The altitude from B to AC is

$$y = -\frac{q}{1+q}(x + p)$$

Solving these two, we get $x = pq$ and $y = -pq$.

Therefore, the locus of the orthocenter is $x + y = 0$.

15. b.



Let the slope of the required line be m. Then

$$\left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}$$

$\therefore \quad m + \sqrt{3} = \pm(\sqrt{3} - 3m)$

$\therefore \quad m = 0$ or $m = \sqrt{3}$

Therefore, the equation is

$y + 2 = \sqrt{3}(x - 3)$ ($m \neq 0$ as given that line cuts the x-axis)

or $\quad \sqrt{3}x - y - (2 + 3\sqrt{3}) = 0$

16. a. Solving given lines for their point of intersection, we get the point of intersection as $(-c/(a + b), -c/(a + b))$.

Its distance from (1, 1) is

$$\sqrt{\left(1+\frac{c}{a+b}\right)^2 + \left(1+\frac{c}{a+b}\right)^2} < 2\sqrt{2} \quad \text{(given)}$$

or $\quad (a + b + c)^2 < 4(a + b)^2$ or $(a + b + c)^2 - (2a + 2b)^2 < 0$

or $\quad (c - a - b)(c + 3a + 3b) < 0$

Since $a > b > c > 0$, $(c - a - b) < 0$ or $a + b - c > 0$.

Multiple Correct Answers Type

1. a., b., c. For the concurrency of three lines $px + qy + r = 0$, $qx + ry + p = 0$, $rx + py + q = 0$, we must have

$$\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

or $\quad 3pqr - p^3 - q^3 - r^3 = 0$

or $\quad (p + q + r)(p^2 + q^2 + r^2 - pq - pr - rq) = 0$

2. a., c. Substituting the coordinates of the points (1, 3) (5, 0), and (−1, 2) in $3x + 2y$, we obtain the values 9, 15, and 1, respectively which are all positive. Therefore, all the points lying inside the triangle formed by the given points satisfy $3x + 2y \geq 0$.

Substituting the coordinates of the given points in $2x + 3y - 13$, we find the values −2, −3, and −9, respectively, which are all negative. So, (b) is not correct.

Again, substituting the given points in $2x - 3y - 12$, we get −19, −2, and −20, respectively, which are all negative. It follows that all points lying inside the triangle formed by the given point satisfy $2x - 3y - 12 \leq 0$. So, (c) is the correct answer.

Finally, substituting the coordinates of the given point in $-2x + y$, we get 1, −10, and 4, respectively, which are not all positive. So, (d) is not correct.

3. d. The slope of $x + 3y = 4$ is $-1/3$ and the slope of $6x - 2y = 7$ is 3. Therefore, these two lines are perpendicular, which shows that both diagonals are perpendicular. Hence, $PQRS$ must be a rhombus.

Matching Column Type

1. (a)–(s); (b)– (p), (q); (c)–(r); (d)–(p), (q), (s)

Given lines are

$L_1: x + 3y - 5 = 0$

$L_2: 3x - ky - 1 = 0$

$L_3: 5x + 2y - 12 = 0$

L_1 and L_3 intersect at (2, 1)

$\therefore L_1, L_2, L_3$ are concurrent if

$6 - k - 1 = 0$ or $k = 5$

For L_1, L_2 to be parallel

$$\frac{1}{3} = \frac{3}{-k} \Rightarrow k = -9$$

For L_2, L_3 to be parallel

$$\frac{3}{5} = \frac{-k}{2} \Rightarrow k = \frac{-6}{5}$$

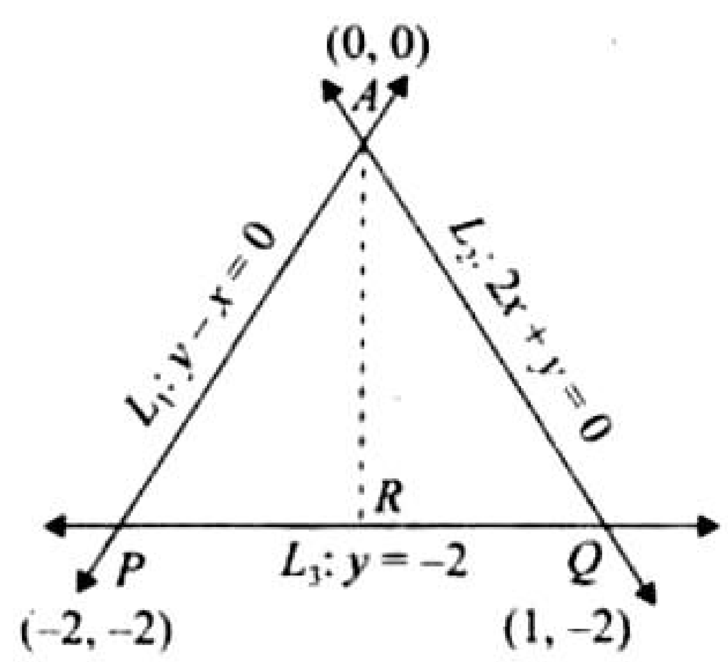
Thus, for $k = 5$, lines are concurrent and for $k = -9, \frac{-6}{5}$, at least two lines are parallel. So, for these values of k, lines will not form triangle.

Obviously, for $k \neq 5, -9, \frac{-6}{5}$, lines form triangle.

Assertion-Reasoning Type

1. c. The point of intersection of L_1 and L_2 is $A(0, 0)$.

Also, $P \equiv (-2, -2)$, $Q \equiv (1, -2)$.

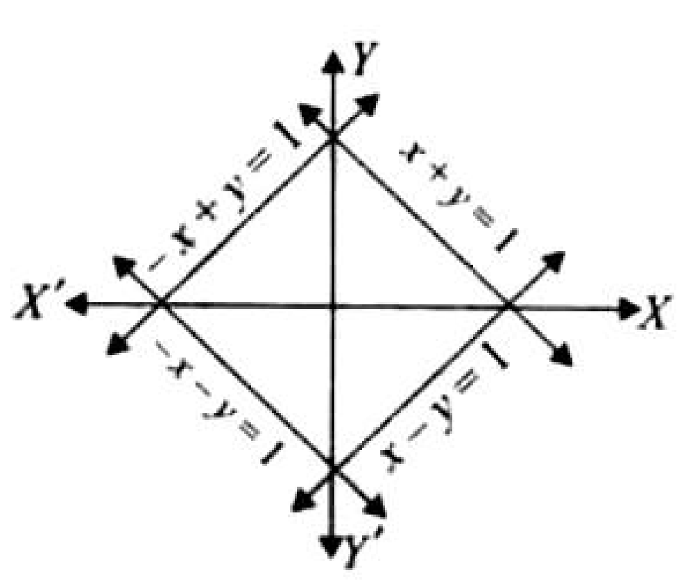


Since AR is the bisector of $\angle PAQ$, R divides PQ in the same ratio as $AP : AQ$. Thus $PR : RQ = AP : AQ = 2\sqrt{2} : \sqrt{5}$. Hence, statement 1 is true. Statement 2 is clearly false.

Fill in the Blanks Type

1. $|x| + |y| = 1$

The curve represents four lines: $x + y = 1$, $x - y = 1$, $-x + y = 1$, $-x - y = 1$.



These lines enclose a square of side equal to the distance between opposite sides $x + y = 1$ and $x + y = -1$, which is given by $(1 + 1)/\sqrt{1+1} = \sqrt{2}$. Therefore, the required area is 2 sq. units.

2. Given that $3a + 2b + 4c = 0$.

Hence, $\frac{3}{4}a + \frac{1}{2}b + c = 0$

Hence, the set of lines $ax + by + c = 0$ pass through the point (3/4, 1/2). Therefore, the given lines are concurrent at the point (3/4, 1/2).

3. If a, b, c are in AP, then

$a + c = 2b$

or $a - 2b + c = 0$

Hence, $ax + by + c = 0$ passes through $(1, -2)$.

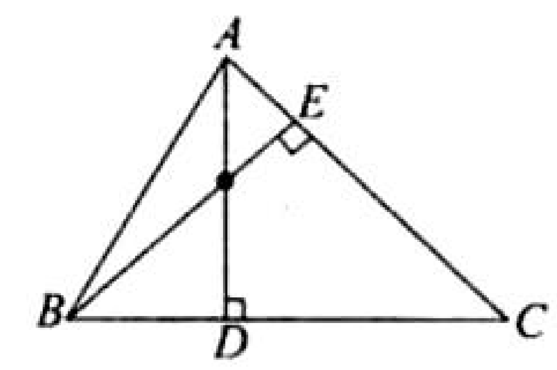
4. The equations of sides of triangle ABC are as follows:

AB: $x + y = 1$

BC: $2x + 3y = 6$

CA: $4x - y = -4$

Solving these pairwise, we get the vertices of the triangle as $A(-3/5, 8/5)$, $B(-3, 4)$, $C(-3/7, 16/7)$.



Now, AD is perpendicular to BC and passes through A. Any line perpendicular to BC is $3x - 2y + \lambda = 0$. As it passes through $(-3/5, 8/5)$, we have

$$\frac{-9}{5} - \frac{16}{5} + \lambda = 0$$

or $\lambda = 5$

Hence, the equation of altitude AD is

$$3x - 2y + 5 = 0 \quad \text{(i)}$$

Any line perpendicular to side AC is $x + 4x + \mu = 0$. As it passes through point $B(-3, 4)$, we have

$-3 + 16 + \mu = 0$ or $\mu = -13$

Therefore, the equation of altitude BE is

$$x + 4x - 13 = 0 \quad \text{(ii)}$$

Now, the orthocenter is the intersection point of (i) and (ii) (AD and BE).

Solving (i) and (ii), we get $x = 3/7$, $y = 22/7$. Hence, the orthocenter lies in the first quadrant.

5. Let the variable line be

$$ax + by + c = 0 \quad \text{(i)}$$

Then the perpendicular distance of the line from (2, 0) is

$$p_1 = \frac{2a + c}{\sqrt{a^2 + b^2}}$$

The perpendicular distance of the line from (0, 2) is

$$p_2 = \frac{2b + c}{\sqrt{a^2 + b^2}}$$

The perpendicular distance of the line from (1, 1) is

$$p_3 = \frac{a + b + c}{\sqrt{a^2 + b^2}}$$

According to question,

$$p_1 + p_2 + p_3 = 0$$

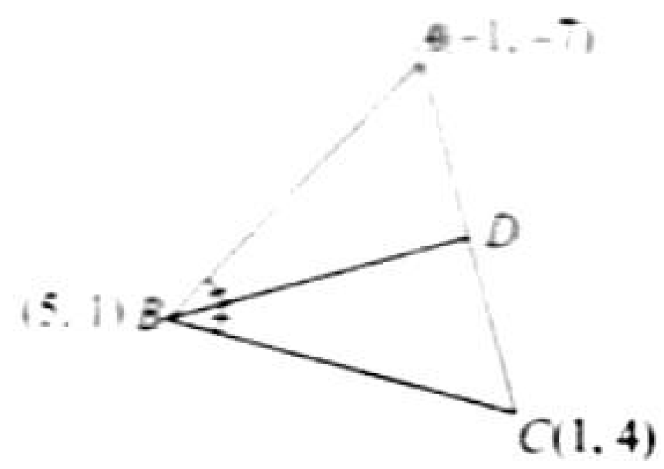
or $$\frac{2a + c + 2b + c + a + b + c}{\sqrt{a^2 + b^2}} = 0$$

or $3a + 3b + 3c = 0$

or $a + b + c = 0 \quad \text{(ii)}$

From (i) and (ii), we can say that variable line (i) passes through the fixed point (1, 1).

6. Let BD be the bisector of $\angle ABC$. Then



$$AD : DC = AB : BC$$

and $AB = \sqrt{(5+1)^2 + (1+7)^2} = 10$

$BC = \sqrt{(5-1)^2 + (1-4)^2} = 5$

$\therefore \quad AD : DC = 2 : 1$

Therefore, by section formula, $D \equiv (1/3, 1/3)$. Therefore, the equation of BD is

$$y - 1 = \frac{1/3 - 1}{1/3 - 5}(x - 5)$$

or $\quad y - 1 = \frac{-2/3}{-14/3}(x - 5)$

or $\quad 7y - 7 = x - 5$

or $\quad x - 7y + 2 = 0$

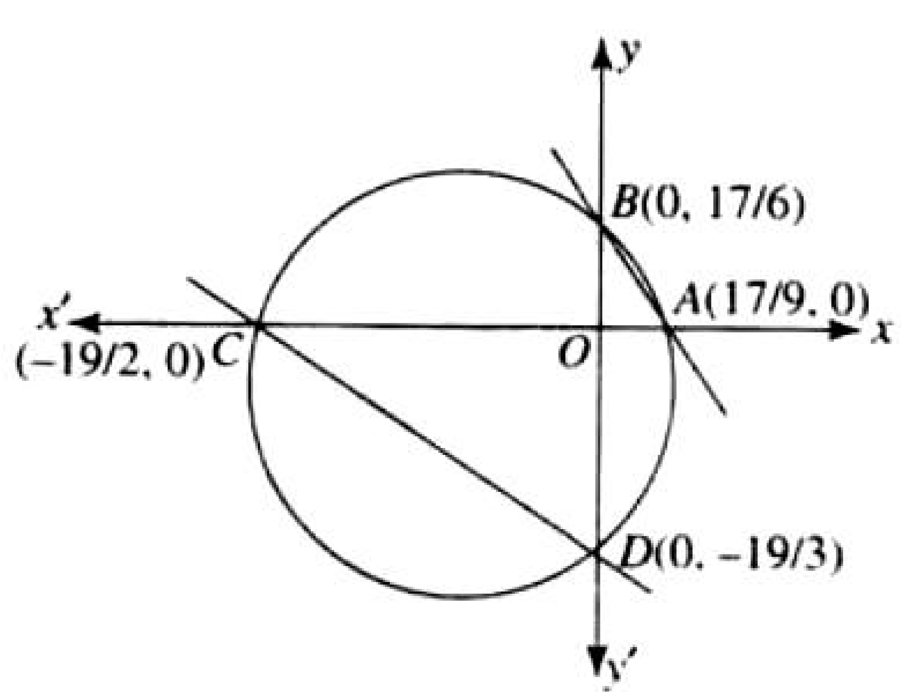
True/False Type

1. For the given lines, we have

$$\begin{vmatrix} 5 & 4 & 0 \\ 1 & 2 & -10 \\ 2 & 1 & 5 \end{vmatrix} = 5(10 + 10) - 4(5 + 20) = 100 - 100 = 0$$

Therefore lines are concurrent
Hence, the statement is true.

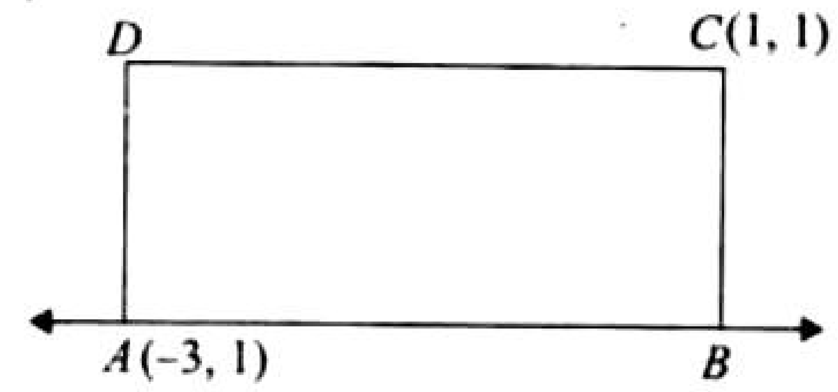
2. Given lines cut the x-axis at $A(17/9, 0)$ and $C(-19/2, 0)$.



The given lines cut the y-axis at $B(0, 17/6)$ and $D(0, -19/3)$. Now, A, B, C, and D are concyclic if $AO \times OC = BO \times OD$, which holds. Hence, the given statement is true.

Subjective Type

1. Let the side AB of rectangle $ABCD$ lies along $4x + 7y + 5 = 0$. As $(-3, 1)$ lies on this line, let it be vertex A. Now, $(1, 1)$ is either vertex C or D.



If $(1, 1)$ is vertex D, then the slope of AD is 0. Hence, AD is not perpendicular to AB. But it is a contradiction as $ABCD$ is a rectangle. Therefore, $(1, 1)$ are the coordinates of vertex C.
CD is a line parallel to AB and passing through C. Therefore, the equation of CD is

$$y - 1 = -\frac{4}{7}(x - 1)$$

or $\quad 4x + 7y - 11 = 0$

Also, BC is a line perpendicular to AB and passing through C. Therefore, the equation of BC is

$$y - 1 = \frac{7}{4}(x - 1)$$

or $\quad 7x - 4y - 3 = 0$

Similarly, AD is a line perpendicular to AB and passing through $A(-3, 1)$. Therefore, the equation of line AD is

$$y - 1 = \frac{7}{4}(x + 3)$$

or $\quad 7x - 4y + 25 = 0$

2. The given lines are

$$x - 2y + 4 = 0 \qquad \text{(i)}$$

and $4x - 3y + 2 = 0 \qquad$ (ii)

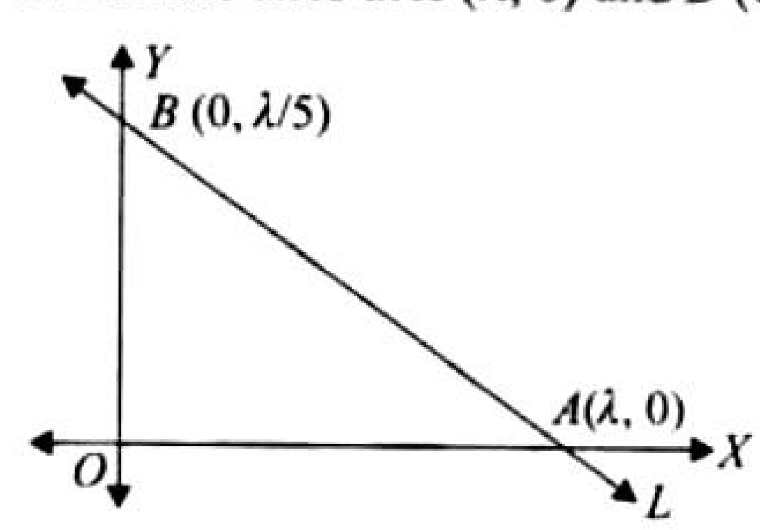
Both the lines have constant terms of same sign. Therefore, the equations of bisectors of the angles between the given lines are

$$\frac{x - 2y + 4}{\sqrt{1 + 4}} = \pm \frac{4x - 3y + 2}{\sqrt{16 + 9}}$$

Here, $a_1a_2 + b_1b_2 > 0$. Therefore, taking positive sign on the RHS, we get the obtuse angle bisector as

$$(4 - \sqrt{5})x + (2\sqrt{5} - 3)y - (4\sqrt{5} - 2) = 0 \qquad \text{(iii)}$$

3. The given line is $5x - y = 1$. Therefore, the equation of line L which is perpendicular to the given line is $x + 5y = \lambda$. This line meets the coordinate axes at $A(\lambda, 0)$ and $B(0, \lambda/5)$.



$\therefore \quad$ Area of $\Delta OAB = \frac{1}{2} \times OA \times OB$

or $\quad 5 = \frac{1}{2} \times \lambda \times \frac{\lambda}{5}$

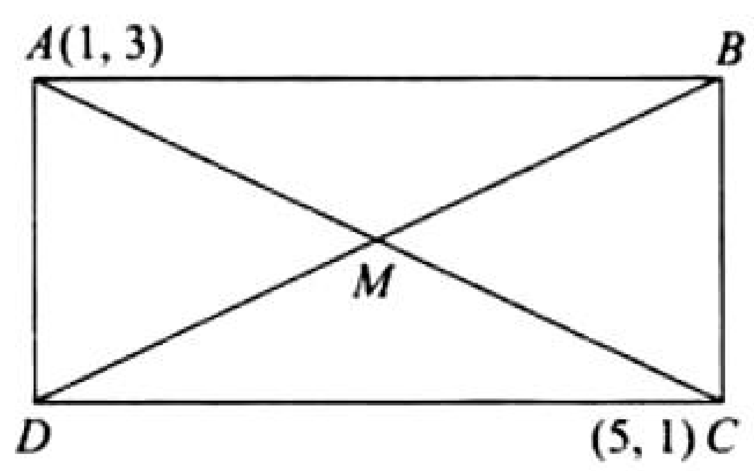
or $\quad \lambda^2 = 5^2 \times 2$

or $\quad \lambda = \pm 5\sqrt{2}$

Hence, the equation of line L is

$$x + 5y - 5\sqrt{2} = 0 \text{ or } x + 5y + 5\sqrt{2} = 0$$

4. Let $ABCD$ be a rectangle and the coordinates of its opposite vertices A and C be $(1, 3)$ and $(5, 1)$, respectively.



Given that other two vertices B and D lie on $y = 2x + c$. Hence, the equation of diagonal BD is $2x - y + c = 0$.

Now, we know that the diagonals of a rectangle bisect each other. Therefore, the midpoint of AC will lie on BD, which is given by

$$M \equiv \left(\frac{1+5}{2}, \frac{3+1}{2}\right) \equiv (3, 2)$$

As (3, 2) lies on $2x - y + c = 0$, we have

$$2 \times 3 - 2 + c = 0$$

or $\quad c = -4$

Hence, the equation of BD is $2x - y - 4 = 0$. Now, the slope of BD is $m = 2$ or $\tan\theta = 2$. Therefore,

$$\cos\theta = \frac{1}{\sqrt{5}};\ \sin\theta = \frac{2}{\sqrt{5}}$$

Therefore, the equation of BD in symmetric form is

$$\frac{x-3}{1/\sqrt{5}} = \frac{y-2}{2/\sqrt{5}} = r \qquad \text{(i)}$$

Now, the length of diagonal AC is $\sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$. As the diagonals of a rectangle are equal in length, $BD = 2\sqrt{5}$.

Also, M is the midpoint of BD. Therefore, $BM = MD = \sqrt{5}$. In order to find B, we use the symmetrical form of line BD with $r = \sqrt{5}$. Therefore,

$$\frac{x-3}{1/\sqrt{5}} = \frac{y-2}{2/\sqrt{5}} = \sqrt{5}$$

or $\quad x = 1 + 3, y = 2 + 2$

or $\quad B \equiv (4, 4)$

Similarly, for point D (being in opposite direction of BD we use $r = -\sqrt{5}$), we get

$$\frac{x-3}{1/\sqrt{5}} = \frac{y-2}{2/\sqrt{5}} = -\sqrt{5}$$

or $\quad x = -1 + 3, y = -2 + 2$

or $\quad x = 2, y = 0$

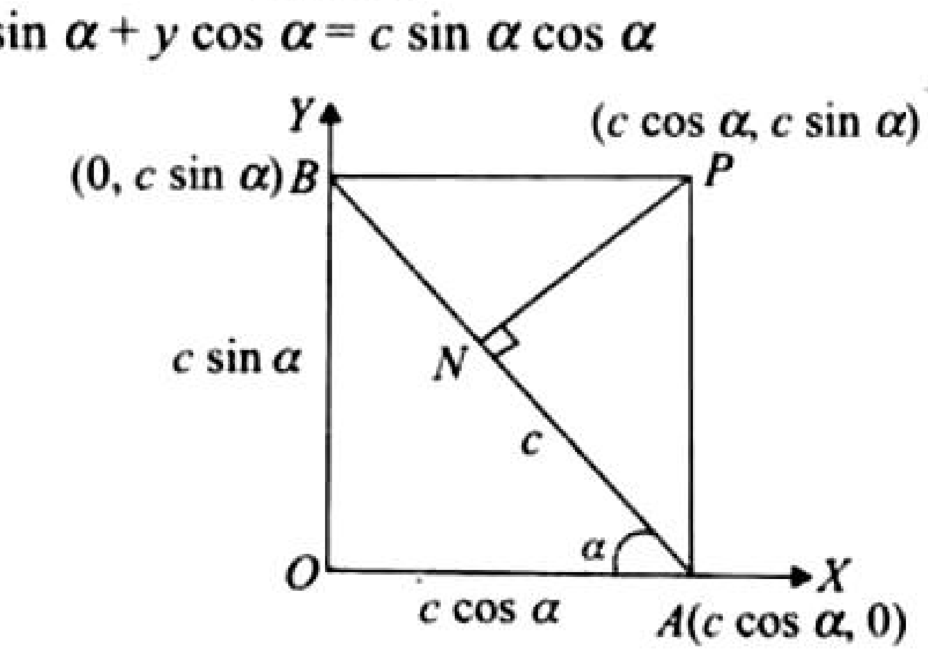
$\therefore \quad D \equiv (2, 0)$

Hence, $c = -4$ and the remaining vertices are (4, 4) and (2, 0).

5. Here, the equation of AB is

$$\frac{x}{c\cos\alpha} + \frac{y}{c\sin\alpha} = 1$$

or $x\sin\alpha + y\cos\alpha = c\sin\alpha\cos\alpha \qquad$ (i)



The equation of PN (perpendicular to AB and through P) is

$$y - c\sin\alpha = \cot\alpha\,(x - c\cos\alpha)$$

or $\quad x\cos\alpha - y\sin\alpha = c(\cos^2\alpha - \sin^2\alpha) \qquad$ (ii)

N is the intersection point of (i) and (ii). Solving (i) and (ii), we get

$$x = c\cos^3\alpha,\ y = c\sin^3\alpha$$

$$\therefore \quad \cos\alpha = \left(\frac{x}{c}\right)^{1/3},\ \sin\alpha = \left(\frac{y}{c}\right)^{1/3}$$

Therefore, the locus of (x, y) is

$$\left(\frac{x}{c}\right)^{2/3} + \left(\frac{y}{c}\right)^{2/3} = 1$$

or $\quad x^{2/3} + y^{2/3} = c^{2/3}$

6. The given equations of equal sides are

$$7x - y + 3 = 0 \qquad \text{(i)}$$

$$x + y - 3 = 0 \qquad \text{(ii)}$$

Let m be the slope of line which passes through the point $(1, -10)$.

Thus, the equation of line will be

$$y + 10 = m(x - 1) \qquad \text{(iii)}$$

Now, the given lines make same angle with the line in (iii). Therefore,

$$\left|\frac{m-7}{1+7m}\right| = \left|\frac{m+1}{1-m}\right|$$

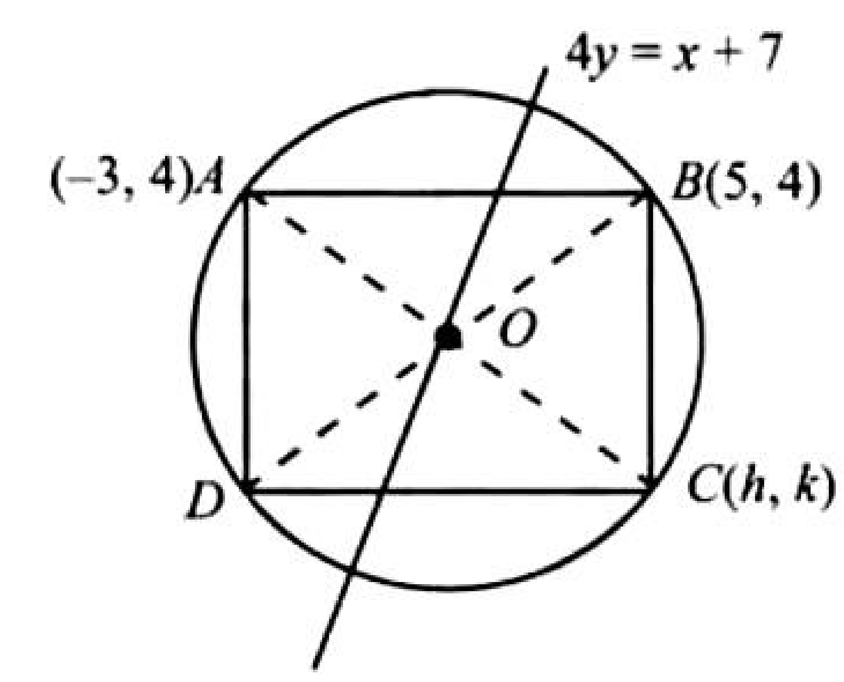
i.e., $m = -3$ or $\frac{1}{3}$

Hence, the required line is

$$y + 10 = -3(x - 1) \text{ or } y + 10 = \frac{1}{3}(x - 1)$$

i.e., $3x + y + 7 = 0$ or $x - 3y - 31 = 0$

7.



From the diagram, $BC \perp AB$.

The slope of AB is 0. Then,

From BC, $h - 5 = 0 \qquad$ (i)

Also, the midpoint of AC lies on the given diameter. Therefore,

$$4\left(\frac{k+4}{2}\right) = \frac{h-3}{2} + 7 \qquad \text{(ii)}$$

From (i) and (ii),

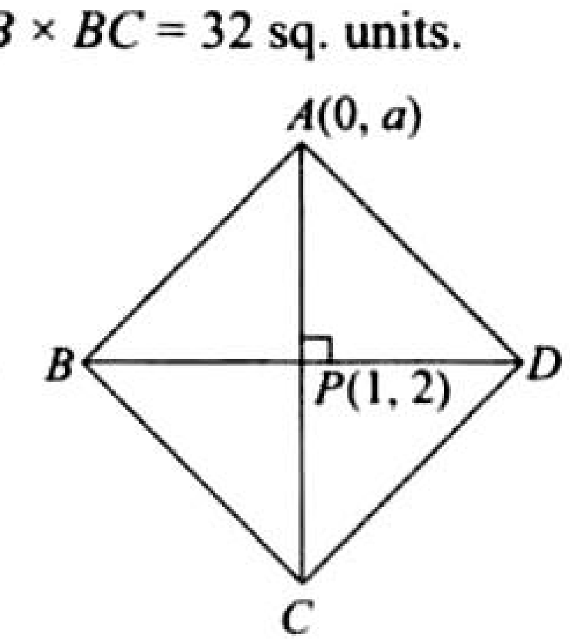
$$h = 5 \text{ and } k = 0$$

Hence, C is (5, 0). Therefore,

$$BC = 4 \text{ and } AB = 8$$

Therefore, the area of rectangle is $AB \times BC = 32$ sq. units.

8. A being on the y-axis may be chosen as $(0, a)$. The diagonals intersect at $P(1, 2)$.



Again, we know that the diagonals will be parallel to the bisectors of the two given lines $y = x + 2$ and $y = 7x + 3$, i.e.,

$$\frac{x-y+2}{\sqrt{2}} = \pm\frac{7x-y+3}{5\sqrt{2}}$$

or $\quad 5x - 5y + 10 = \pm(7x - y + 3)$

or $\quad 2x + 4y - 7 = 0$ and $12x - 6y + 13 = 0$

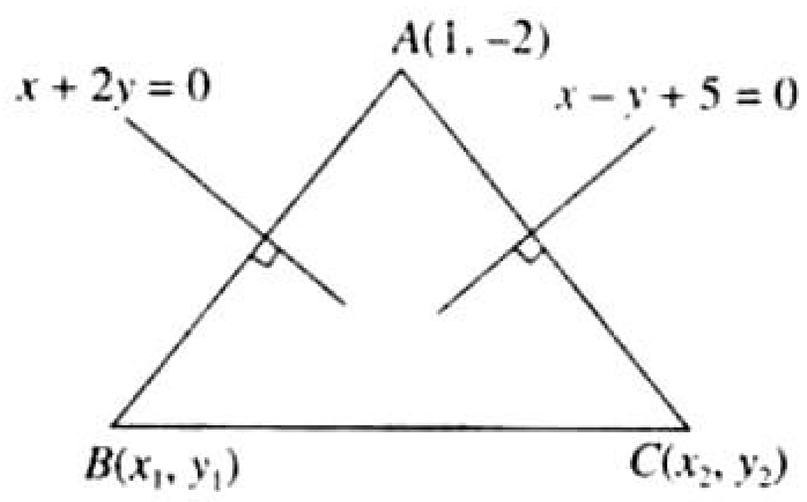
$\therefore$ Slope of diagonals are $m_1 = -1/2$ and $m_2 = 2$. Let diagonal d_1 be parallel to $2x + 4y - 7 = 0$ and diagonal d_2 be parallel to $12x - 6y + 13 = 0$. The vertex A could be on any of the two diagonals. Hence, the slope of AP is either $-1/2$ or 2. Therefore,

$$\frac{2-a}{1-0} = 2 \text{ or } -\frac{1}{2}$$

i.e., $a = 0$ or $\frac{5}{2}$

Hence, A is (0, 0) or (0, 5/2).

9.



Since the given lines are perpendicular bisectors of the sides as shown in the figure, points B and C are the images of the point A in these lines. Therefore,

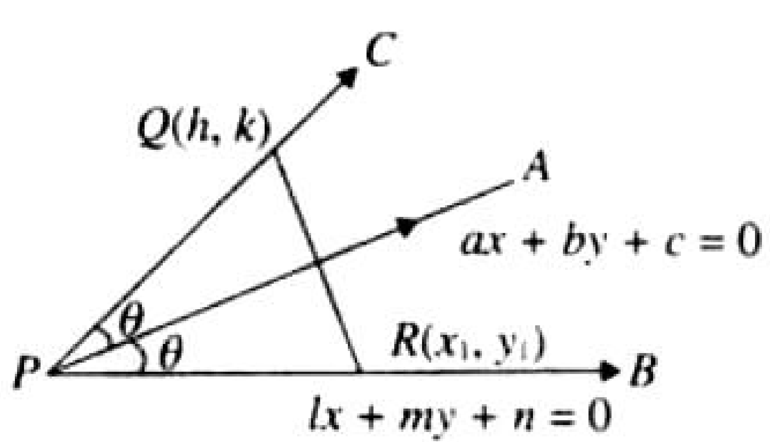
$$\frac{x_1-1}{1} = \frac{y_1+2}{-1} = -\frac{2(1-4)}{1+4}$$

and
$$\frac{x_2-1}{1} = \frac{y_2+2}{2} = -\frac{2(1+2+5)}{1+1}$$

$\therefore \quad B(x_1, y_1) \equiv (11/5, 2/5)$ and $C(x_2, y_2) \equiv (-7, 6)$

Hence, the line passing through the points B and C is $14x + 23y - 40 = 0$.

10.



Clearly, AP is the bisector of $\angle CPB$.

Hence, the image of any point Q on line AC in line PA must lie on line PB. Therefore,

$$\frac{x_1-h}{a} = \frac{y_1-h}{b} = -\frac{2(ah+bk+c)}{a^2+b^2}$$

or
$$x_1 = -\frac{2a(ah+bk+c)}{a^2+b^2} + h$$

and
$$y_1 = -\frac{2b(ah+bk+c)}{a^2+b^2} + k$$

Now, $R(x_1, y_1)$ lies on the line $lx + my + n = 0$. Therefore,

$$l\left[-\frac{2a(ah+bk+c)}{a^2+b^2} + h\right] + m\left[-\frac{2b(ah+bk+c)}{a^2+b^2} + k\right] + n = 0$$

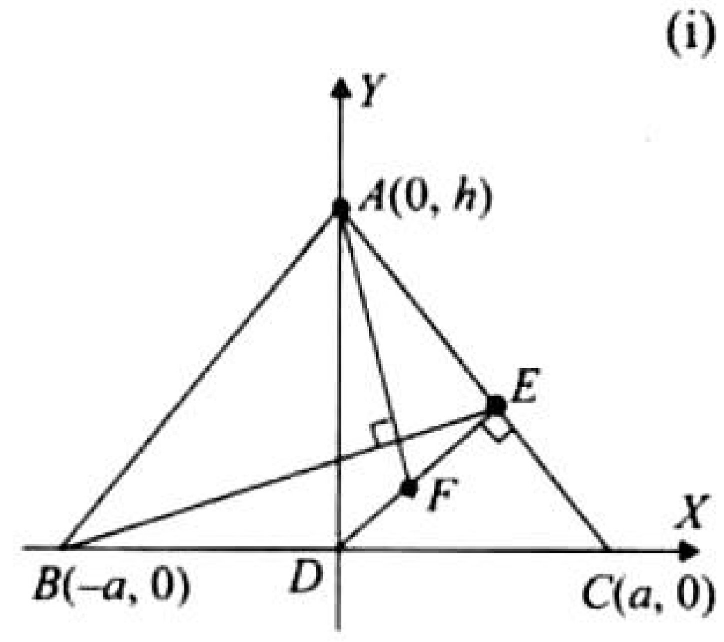
Hence, the locus of Q is

$$2(al + mb)(ax + by + c) - (a^2 + b^2)(lm + my + n) = 0$$

11. Let BC be taken as the x-axis with the origin at D. Let A be taken on y-axis. Let $BC = 2a$. Now $AB = AC$, then the coordinates of B and C are $(-a, 0)$ and $(a, 0)$, respectively. Let $DA = h$. Then the coordinates of A are $(0, h)$.

Hence, the equation of AC is

$$\frac{x}{a} + \frac{y}{h} = 1 \qquad \text{(i)}$$



and the equation of DE perpendicular to AC and passing through the origin is

$$\frac{x}{h} - \frac{y}{a} = 0$$

or
$$x = \frac{hy}{a} \qquad \text{(ii)}$$

Solving (i) and (ii), we get the coordinates of E as follows:

$$\frac{hy}{a^2} + \frac{y}{h} = 1$$

or
$$h^2y + a^2y = a^2h$$

or
$$y = \frac{a^2h}{a^2+h^2}$$

$$\therefore \quad x = \frac{ah^2}{a^2+h^2}$$

$$\therefore \quad E \equiv \left(\frac{ah^2}{a^2+h^2}, \frac{a^2h}{a^2+h^2}\right)$$

Since F is the midpoint of DE, its coordinates are

$$\left(\frac{ah^2}{2(a^2+h^2)}, \frac{a^2h}{2(a^2+h^2)}\right)$$

The slope of AF is

$$m_1 = \frac{h - \frac{a^2h}{2(a^2+h^2)}}{0 - \frac{ah^2}{2(a^2+h^2)}}$$

$$= \frac{2h(a^2+h^2) - a^2h}{-ah^2}$$

$$= -\frac{a^2+2h^2}{ah} \qquad \text{(i)}$$

and the slope of BE is

$$m_2 = \frac{\frac{a^2h}{a^2+h^2} - 0}{\frac{ah^2}{a^2+h^2} + a}$$

$$= \frac{a^2h}{ah^2 + a^3 + ah^2}$$

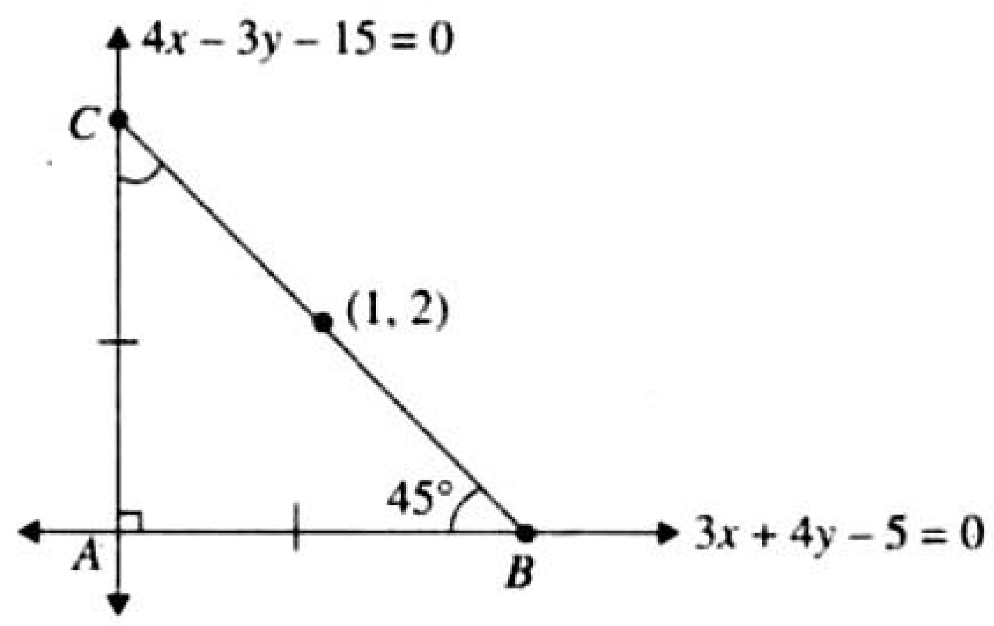
$$= \frac{ah}{a^2+2h^2} \qquad \text{(ii)}$$

From (i) and (ii), we have

$$m_1 m_2 = -1$$

or $AF \perp BE$

12. The given straight lines are $3x + 4y = 5$ and $4x - 3y = 15$. Clearly, these straight lines are perpendicular to each other $(m_1 m_2 = -1)$, and intersect at A. Now, B and C are the points on these lines such that $AB = AC$ and BC passes through (1, 2). From the figure, it is clear that $\angle B = \angle C = 45°$.



Let the slope of BC be m. Then,

$$\tan 45° = \left|\frac{m + 3/4}{1 - (3/4)m}\right|$$

or $4m + 3 = \pm(4 - 3m)$

i.e., $4m + 3 = 4 - 3m$ or $4m + 3 = -4 + 3m$

i.e., $m = \frac{1}{7}$ or $m = -7$

Hence, the equation of BC is

$$y - 2 = \frac{1}{7}(x - 1) \text{ or } y - 2 = -7(x - 1)$$

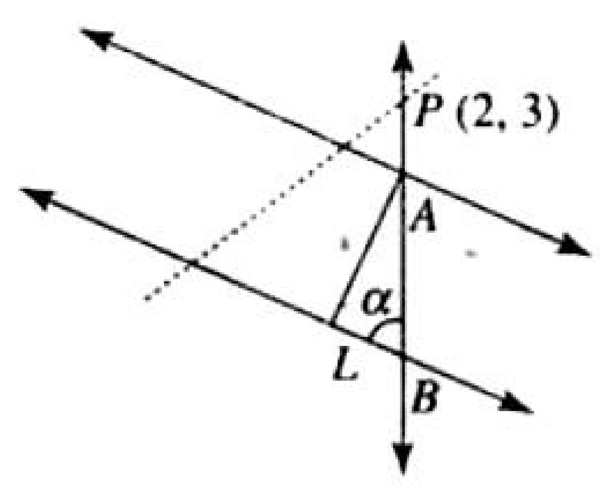
i.e., $7y - 14 = x - 1$ or $y - 2 = -7x + 7$

i.e., $x - 7y + 13 = 0$ or $7x + y - 9 = 0$

13. Given lines are

$$2x + y - 3 = 0 \quad \text{(i)}$$

and $\quad 2x + y - 5 = 0 \quad$ (ii)



Given $AB = 2$

AL = distance between parallel lines (i) and (ii) $= \dfrac{|-3 + 5|}{\sqrt{2^2 + 1^2}} = \dfrac{2}{\sqrt{5}}$

From ΔALB, $\sin \alpha = \dfrac{AL}{AB} = \dfrac{\frac{2}{\sqrt{5}}}{2} = \dfrac{1}{\sqrt{5}}$

$\therefore \quad \tan \alpha = \dfrac{1}{2}$

Now slope of given lines is – 2.

Let the slope of AB be m. Then

$$\tan \alpha = \frac{|m - (-2)|}{|1 + m(-2)|}$$

$\Rightarrow \quad \pm\dfrac{1}{2} = \dfrac{m + 2}{1 - 2m}$

$\Rightarrow \quad 1 - 2m = 2m + 4$ or $2m - 1 = 2m + 4$

$\Rightarrow \quad m = -3/4$ or $m = \infty$

Therefore, equation of lines can be $x - 2 = 0$ or $(y - 3) = (-3/4)(x - 2)$

or $\quad 3x + 4y - 18 = 0$

14. The given curve is

$$3x^2 - y^2 - 2x + 4y = 0 \quad \text{(i)}$$

Let $y = mx + c$ be the chord of curve (i) which subtends an angle of 90° at the origin. Then the combined equations of lines joining the points of intersection of curve (i) and chord $y = mx + c$ to the origin can be obtained by making the equation of curve homogeneous with the help of the equation of chord as

$$3x^2 - y^2 - 2x\left(\frac{y - mx}{c}\right) + 4y\left(\frac{y - mx}{c}\right) = 0$$

or $\quad 3cx^2 - cy^2 - 2xy + 2mx^2 + 4y^2 - 4mxy = 0$

or $\quad (3c + 2m)x^2 - 2(1 + 2m)xy + (4 - c)y^2 = 0$

As the lines represented by this pair are perpendicular to each other, we must have

Coefficient of x^2 + Coefficient of $y^2 = 0$

Hence, $3c + 2m + 4 - c = 0$

or $\quad -2 = m + c$

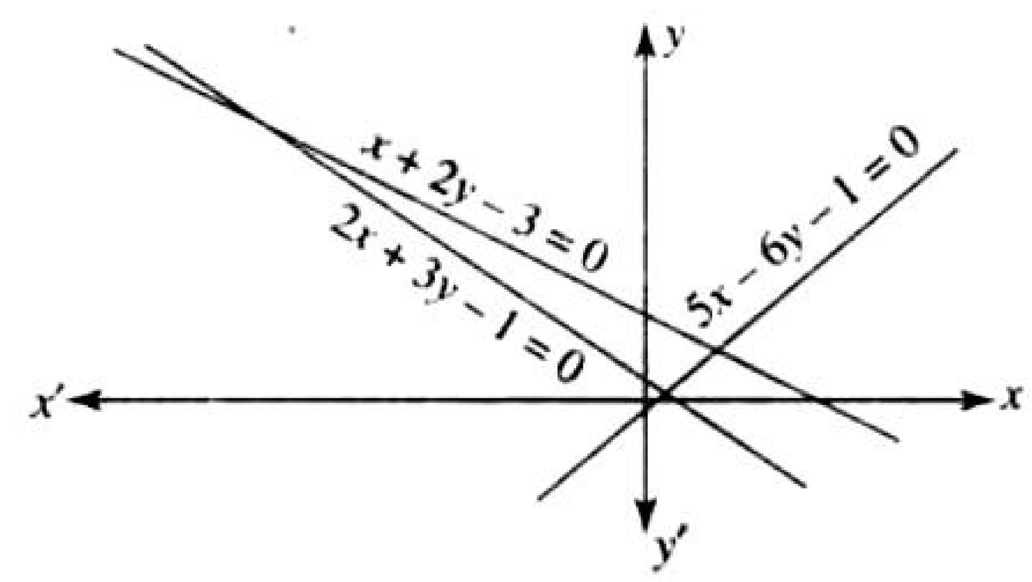
Comparing this result with $y = mx + c$, we can see that $y = mx + c$ passes though (1, –2).

15. Given lines are $L_1 : 2x + 3y - 1 = 0$

$L_2 : 5x - 6y - 1 = 0$

$L_3 : x + 2y - 3 = 0$

These lines can be drawn on coordinate axes as shown:



Clearly, from the figure, origin O and point $P(\alpha, \alpha^2)$ must lie on the opposite sides w.r.t $2x + 3y - 1 = 0$.

$L_1(0, 0) : -1 < 0$

$\Rightarrow \quad L_1(\alpha, \alpha^2) : 2\alpha + 3\alpha^2 - 1 > 0$

$\Rightarrow \quad (3\alpha - 1)(\alpha + 1) > 0$

$\Rightarrow \quad \alpha < -1$ or $\alpha > 1/3 \quad$ (i)

O and the point (α, α^2) must lie to the same side w.r.t. $x + 2y - 3 = 0$.

$L_3(0, 0) : -3 < 0$

$\Rightarrow \quad L_3(\alpha, \alpha^2) : \alpha + 2\alpha^2 - 3 < 0$

$\Rightarrow \quad (2\alpha + 3)(\alpha - 1) < 0$

$\Rightarrow \quad -3/2 < \alpha < 1 \quad$ (ii)

Again O and $P(\alpha, \alpha^2)$ must lie to the same side w.r.t. $5x - 6y - 1 = 0$

$\Rightarrow \quad L_2(0, 0) : -1 < 0$

$\Rightarrow \quad L_2(\alpha, \alpha^2) : 5\alpha - 6\alpha^2 - 1 < 0$

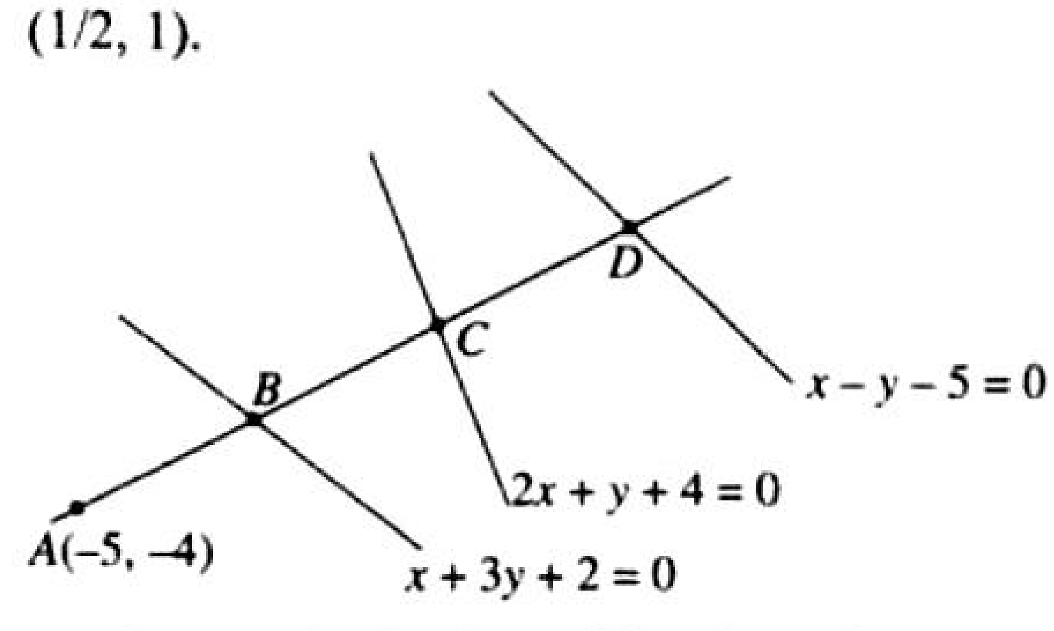
$\Rightarrow \quad 6\alpha^2 - 5\alpha + 1 > 0$

$\Rightarrow \quad (3\alpha - 1)(2\alpha - 1) > 0$

$\Rightarrow \quad \alpha < 1/3$ or $\alpha > 1/2 \quad$ (iii)

From (i), (ii) and (iii) common values of α are $(-3/2, -1) \cup (1/2, 1)$.

16.



Let θ be the inclination of line through $A(-5, -4)$.

Therefore, equation of this line is $\dfrac{x+5}{\cos\theta} = \dfrac{y+4}{\sin\theta} = r$ (i)

Any point on this line at distance r from point $A(-5, -4)$ is given by

$(-5 + r\cos\theta, -4 + r\sin\theta)$

If $AB = r_1$, $AC = r_2$ and $AD = r_3$, then

$B(r_1\cos\theta - 5, r_1\sin\theta - 4)$

$C(r_2\cos\theta - 5, r_2\sin\theta - 4)$

$D(r_3\cos\theta - 5, r_3\sin\theta - 4)$

But B lies on $x + 3y + 2 = 0$. Therefore,

$r_1\cos\theta - 5 + 3r_1\sin\theta - 12 + 2 = 0$

or $\dfrac{15}{\cos\theta + 3\sin\theta} = r_1$

or $\dfrac{15}{AB} = \cos\theta + 3\sin\theta$ (i)

As C lies on $2x + y + 4 = 0$, we have

$2(r_2\cos\theta - 5) + (r_2\sin\theta - 4) + 4 = 0$

or $r_2 = \dfrac{10}{2\cos\theta + \sin\theta} = AC$

or $\dfrac{10}{AC} = 2\cos\theta + \sin\theta$ (ii)

Similarly, D lies on $x - y - 5 = 0$. Therefore,

$r_3\cos\theta - 5 - r_3\sin\theta + 4 - 5 = 0$

or $r_3 = \dfrac{6}{\cos\theta - \sin\theta} = AD$

or $\dfrac{6}{AD} = \cos\theta - \sin\theta$ (iii)

Now, given that

$$\left(\frac{15}{AB}\right)^2 + \left(\frac{10}{AC}\right)^2 = \left(\frac{6}{AD}\right)^2$$

or $(\cos\theta + 3\sin\theta)^2 + (2\cos\theta + \sin\theta)^2 = (\cos\theta - \sin\theta)^2$ [Using (i), (ii), and (iii)]

or $4\cos^2\theta + 9\sin^2\theta + 12\sin\theta\cos\theta = 0$

or $2\cos\theta + 3\sin\theta = 0$

or $\tan\theta = -\dfrac{2}{3}$

Hence, the equation of the required line is

$y + 4 = -\dfrac{2}{3}(x+5)$

or $3y + 12 = -2x - 10$

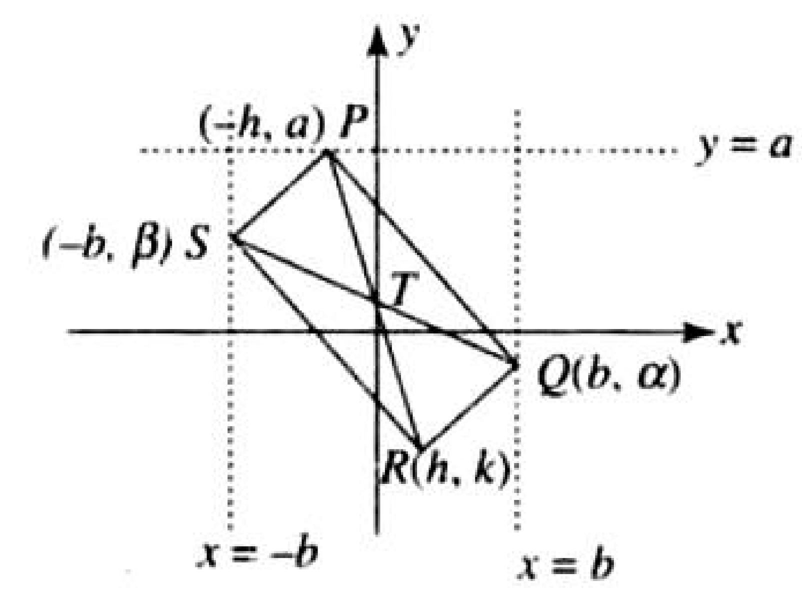
or $2x + 3y + 22 = 0$

17. Let the coordinate of Q be (b, α) and that of S be $(-b, \beta)$. Let PR and SQ intersect each other at T.

$\because$ T is the mid point of SQ

Since diagonals of a rectangle bisect each other, x co-ordinates of P is $-h$.

As P lies on $y = a$, therefore coordinates of P are $(-h, a)$.



Given that PQ is parallel to $y = mx$ and slope of $PQ = m$.

$\therefore$ $\dfrac{\alpha - a}{b + h} = m$

$\Rightarrow$ $\alpha = a + m(b+h)$ (1)

Also, $RQ \perp PQ \Rightarrow$ slope of $RQ = \dfrac{-1}{m}$

$\therefore$ $\dfrac{k-\alpha}{h-b} = \dfrac{-1}{m} \Rightarrow \alpha = k + \dfrac{1}{m}(h-b)$ (2)

From (1) and (2), we get

$a + m(b+h) = k + \dfrac{1}{m}(h-b)$

$\Rightarrow$ $am + m^2(b+h) = km + (h-b)$

$\Rightarrow$ $(m^2-1)h - mk + b(m^2+1) + am = 0$

Therefore, locus of (h, k) is

$(m^2-1)x - my + b(m^2+1) + am = 0$

18. The line $y = mx$ meets the given lines at

$P\left(\dfrac{1}{m+1}, \dfrac{m}{m+1}\right)$ and $Q\left(\dfrac{3}{m+1}, \dfrac{3m}{m+1}\right)$

Hence, the equation of L_1 is

$\left(y - \dfrac{m}{m+1}\right) = 2\left(x - \dfrac{1}{m+1}\right)$

or $y - 2x - 1 = -\dfrac{3}{m+1}$ (i)

and that of L_2 is

$\left(y - \dfrac{3m}{m+1}\right) = -3\left(x - \dfrac{3}{m+1}\right)$

or $y + 3x - 3 = \dfrac{6}{m+1}$ (ii)

From (i) and (ii), eliminating m, we get

$\dfrac{y - 2x - 1}{y + 3x - 3} = -\dfrac{1}{2}$

or $x - 3y + 5 = 0$

which is a straight line.

19. Let the equation of the line be

$(y - 2) = m(x - 8)$, where $m < 0$

$\therefore$ $P \equiv \left(8 - \frac{2}{m}, 0\right)$ and $Q \equiv (0, 2 - 8m)$

Now, $OP + OQ = \left|8 - \frac{2}{m}\right| + |2 - 8m|$

$= 10 + \dfrac{2}{-m} + (-8m)$

$\geq 10 + 2\sqrt{\dfrac{2}{-m} \times (-8m)} \geq 18$

(using AM $\geq$ GM)

20. A line passing through $P(h, k)$ and parallel to the x-axis is

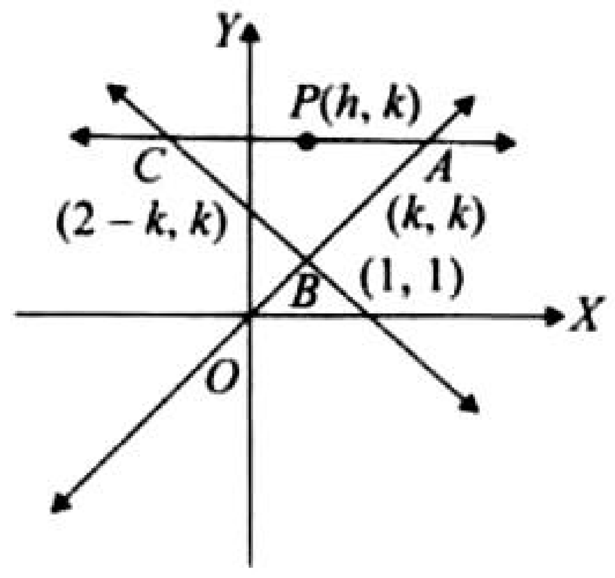
$y = k$ (i)

The other two lines given are

$y = x$ (ii)

and $x + y = 2$ (iii)

Let ABC be the triangle formed by the points of intersection of lines (i), (ii), and (iii), as shown in the figure.



Then, $A \equiv (k, k)$, $B \equiv (1, 1)$, and $C \equiv (2 - k, k)$. Therefore,

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} k & k & 1 \\ 1 & 1 & 1 \\ 2-k & k & 1 \end{vmatrix} = 4h^2$$

Operating $C_1 \to C_1 - C_2$, we get

$$\frac{1}{2}\begin{vmatrix} 0 & k & 1 \\ 0 & 1 & 1 \\ 2-2k & k & 1 \end{vmatrix} = 4h^2$$

$\therefore \quad \frac{1}{2}|(2 - 2k)(k - 1)| = 4h^2$

$\therefore \quad (k - 1)^2 = 4h^2$

$\therefore \quad k - 1 = 2h$ or $k - 1 = -2h$

$\therefore \quad k = 2h + 1$ or $k = -2h + 1$

Hence, the locus of (h, k) is

$y = 2x + 1$ or $y = -2x + 1$