☆ Controllers & Compensators:-

* Purpose:-

⇒ If the system is unstable then controllers and compensators are required to make it stable & to achieve the required performance.

⇒ If the system is stable then also required a compensator (or) controller to get the desired performance.

⇒ The Type-2 & Higher order sys. are usually un-stable. In this case it is essential to used lead compensator (or) PD controller to make the sys. stable & to get the desired performance.

⇒ In Type-0 & Type-1 sys., the stable operation is achieved by adjusting the sys. gain.

⇒ In this case we can use any compensator (or) controller to get the required specification.

<u>Type-2:</u>

$$G(s)\Big|_{w/c} = \frac{K}{s^2(s+2)(s+4)} \quad ; \quad H(s) = 1.$$

$$\xrightarrow{CE} \quad s^4 + 6s^3 + \underbrace{8s^2 + \cancel{K}}_{s^{+1} \text{ missing}} = 0 \quad \text{— (US)}$$

0 to ∞

With P-D controller $= (K_P + K_D s)$.

$$G(s)\Big|_{wc} = \frac{K(K_P + K_D s)}{s^2(s+2)(s+4)} \quad ; H(s) = 1.$$

$$\xrightarrow{CE} \quad s^4 + 6s^3 + 8s^2 + KK_D s + KK_P = 0$$

$\longrightarrow$ (S).

<u>Type-1:</u>

$$G(s)\Big|_{wo/c} = \frac{K}{s(s+2)(s+4)} \quad ; \quad H(s) = 1.$$

$$\xrightarrow{CE} \quad s^3 + 6s^2 + 8s + \cancel{K} = 0 \quad \longrightarrow \text{(S)}.$$

0 to 48.

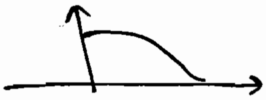
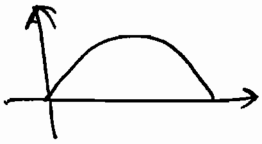
48

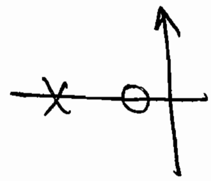
K

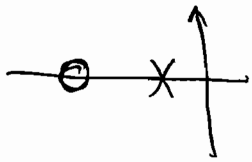
* <u>Compensators:</u>

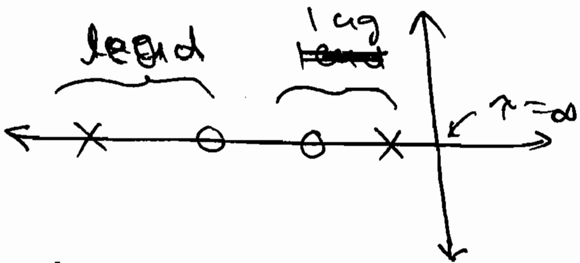
⇒ A Compensator is a electrical N/W which adds finite Poles & finite zeros to the System, so that the sys. performance is changed as per the requirement.

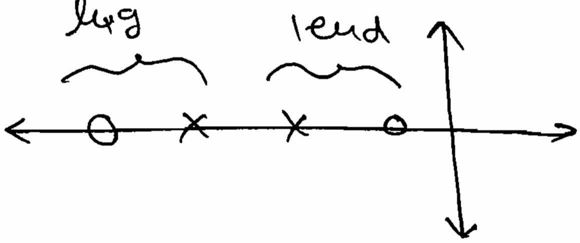
⇒ There are three types of Compensators

1. Lead Compensator  → High pass filter.
2. Lag Compensator  → Low pass filter.
3. Lag-Lead compensator  → Band Stop filter.
4. Lead-Lag Compensator  → Band pass filter.

* HPF → Lead Com. → +ve angle ⇒ zeros 

* LPF → Lag Comp. → −ve angle ⇒ Poles 

* BSF → $T_{lag} > T_{lead}$ 

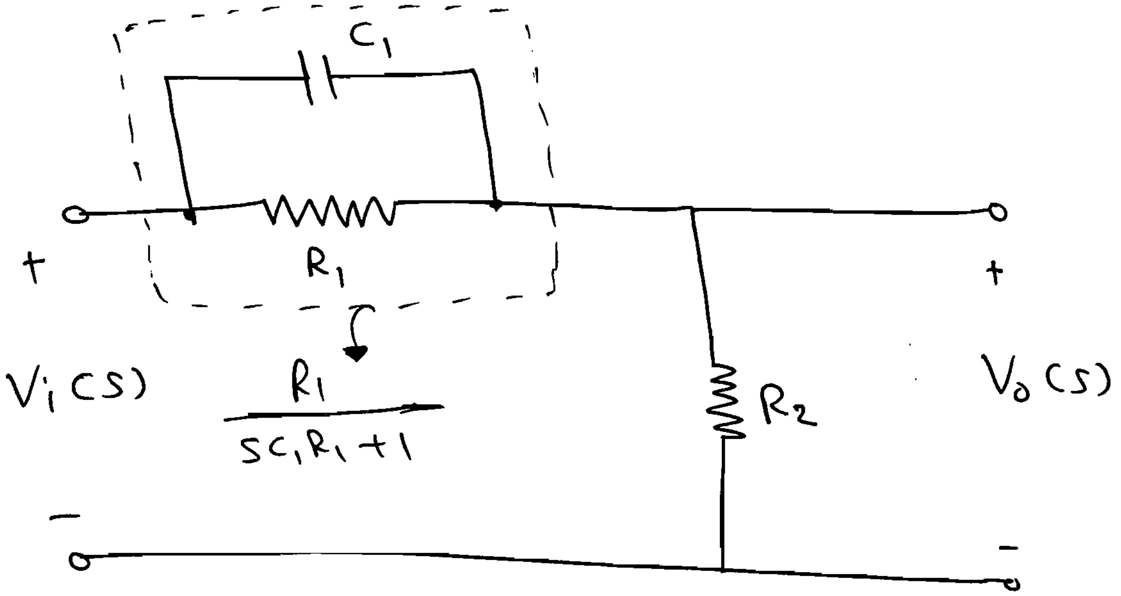
* BPF → $T_{lead} > T_{lag}$ 

1. Lead Compensators :-

⇒ When sinusoidal I/P is applied to a n/w it produce a sinusoidal steady state o/p, having a phase lead with respect to I/P, then the n/w is called lead compensator.

⇒ The lead compensator improves the transient performance & also margin for the sys. stability.

⇒



$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{R_2}{\frac{R_1}{sC_1R_1+1} + R_2}.$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2(sC_1R_1+1)}{R_1 + R_2(sC_1R_1+1)}.$$

⇒ S_1: T. F.

S_2: τ- const.

S_3: Poles & zeros → s-plane.

S_4: Bode plot.

S_5: Identify filters.

S_6: ω_m, ϕ_m, $M|\omega_m$.

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{R_2(1+sC_1R_1)}{R_1+R_2+sC_1R_1R_2}.$$

$$\therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{\frac{R_2}{R_1+R_2}\left(1+sC_1R_1\right)}{\left[1+\frac{R_2}{R_1+R_2}sC_1R_1\right]}.$$

$\Rightarrow$ let, α is called lead Const. $= \frac{R_2}{R_1+R_2} < 1$

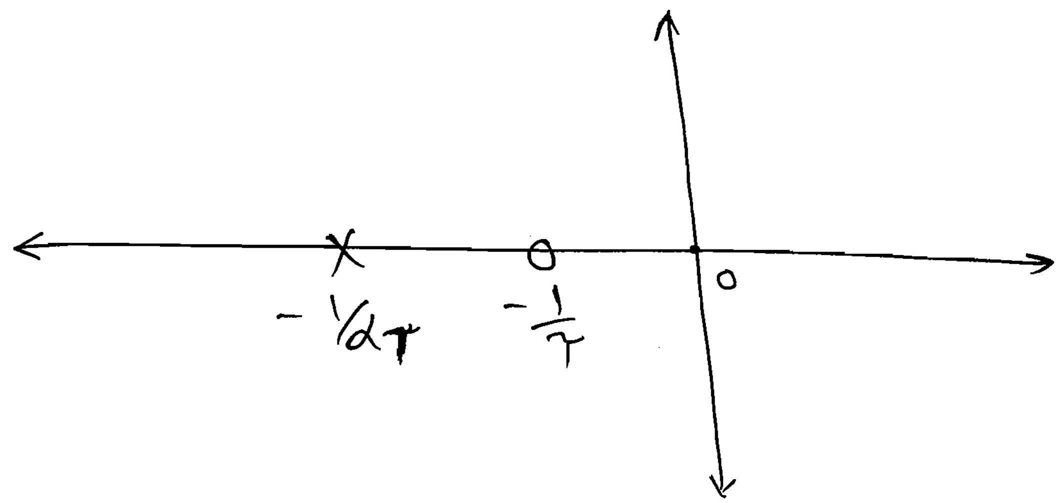
($\alpha \not< 0.07$).

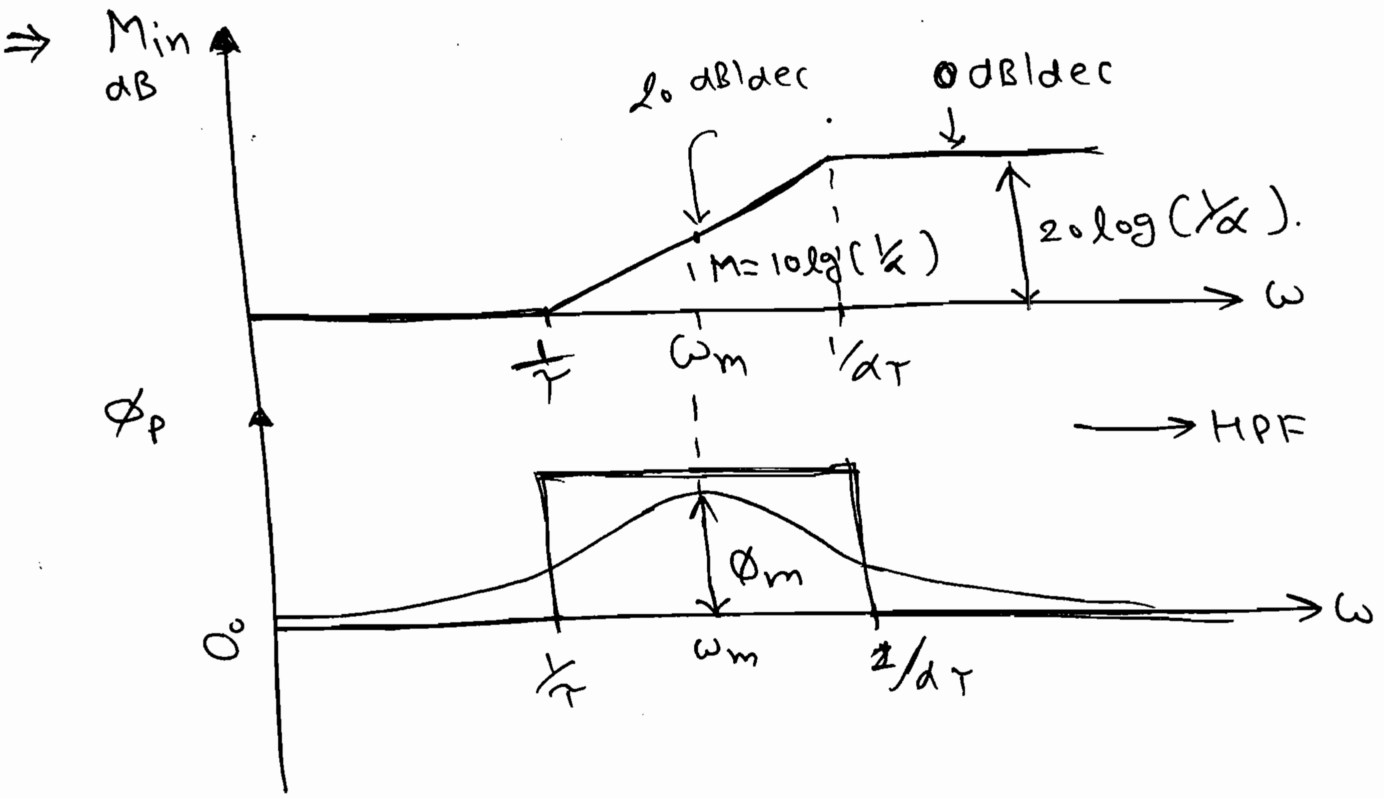
$\tau \rightarrow$ lead Time-Const. $= R_1C_1$.

$\alpha_{optimum} = 0.1$.

$$\Rightarrow \boxed{\frac{V_o(s)}{V_i(s)} = \frac{\alpha(1+\tau s)}{(1+\alpha\tau s)}}$$

$\Rightarrow$ The disadvantage of lead Componsator ~~it~~ is it creates the attenuation in the sys. To eliminate attenuation we require to add amp. with the gain of $1/\alpha$, which add the cost & space to the system.





$$\Rightarrow \quad \omega_m = \sqrt{\omega_{c1} \times \omega_{c2}} \quad , \quad \omega_m = \sqrt{\frac{1}{\tau} \times \frac{1}{\alpha\tau}}$$

$$\therefore \quad \omega_m = \frac{1}{\tau\sqrt{\alpha}} \text{ rad/sec.}$$

$$\therefore \quad \boxed{\phi_{max} = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right).} \quad ***$$

* Advantages:

⇒ A lead compensator improves the transient performance.

⇒ The lead compensator is a high pass filter hence the B.W. of the sys. improves.

⇒ As B.W. increases, the rise-time decreases the sys. gives very quick response.

⇒ The lead compensator improves the

damping of the system. ($\xi \omega_n$). Hence, setteling time (t_s) decreases (↓).

⇒ The lead componsator improves the Gain Margin & Phase margin of the sys. Hence, relative stability improves.

⇒ The Lead Compensator similar to P-D Contoller.

* **Disadvantages:**

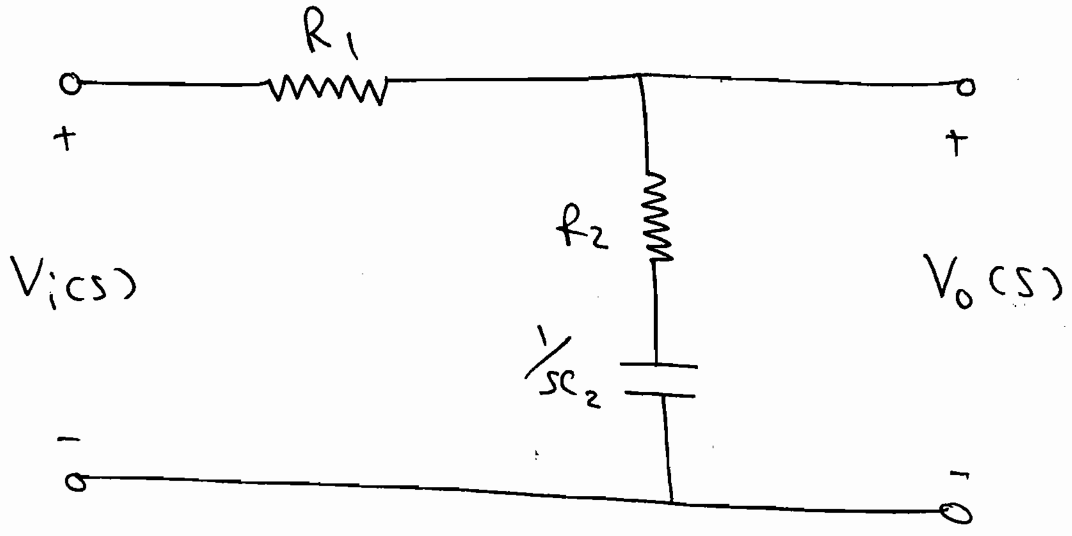
⇒ The lead Compensator creates the attenuation in the sys. to eliminate the attenuation we required to add an amplifier with a gain of $1/\alpha$.

⇒ The lead Comp. is a HPF. Hence Noise Power enter into the system. So, the SNR at output is poorer.

⇒ The max lead given by lead comp is 60°, if required more than 60° we required to use multi stage Compensator.

② Lag Compensator :-

⇒



⇒ $$\frac{V_o(s)}{V_i(s)} = \frac{R_2 + \frac{1}{sC_2}}{R_1 + R_2 + \frac{1}{sC_2}}.$$

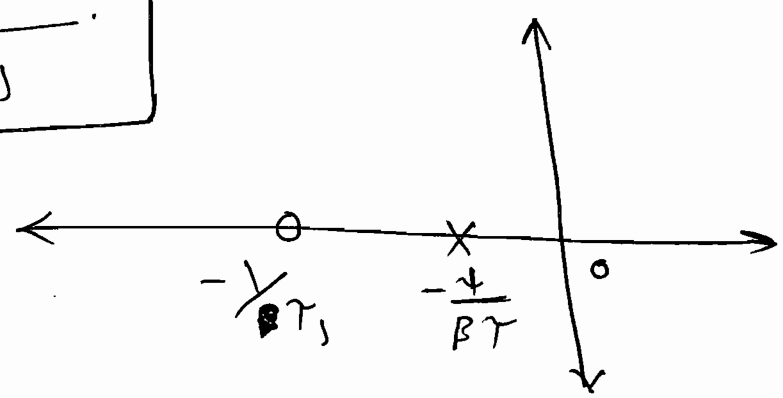
$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{sC_2R_2 + 1}{1 + sC_2(R_1+R_2)}.$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{sC_2R_2 + 1}{1 + sC_2R_2\left(\frac{R_1+R_2}{R_2}\right)}.$$

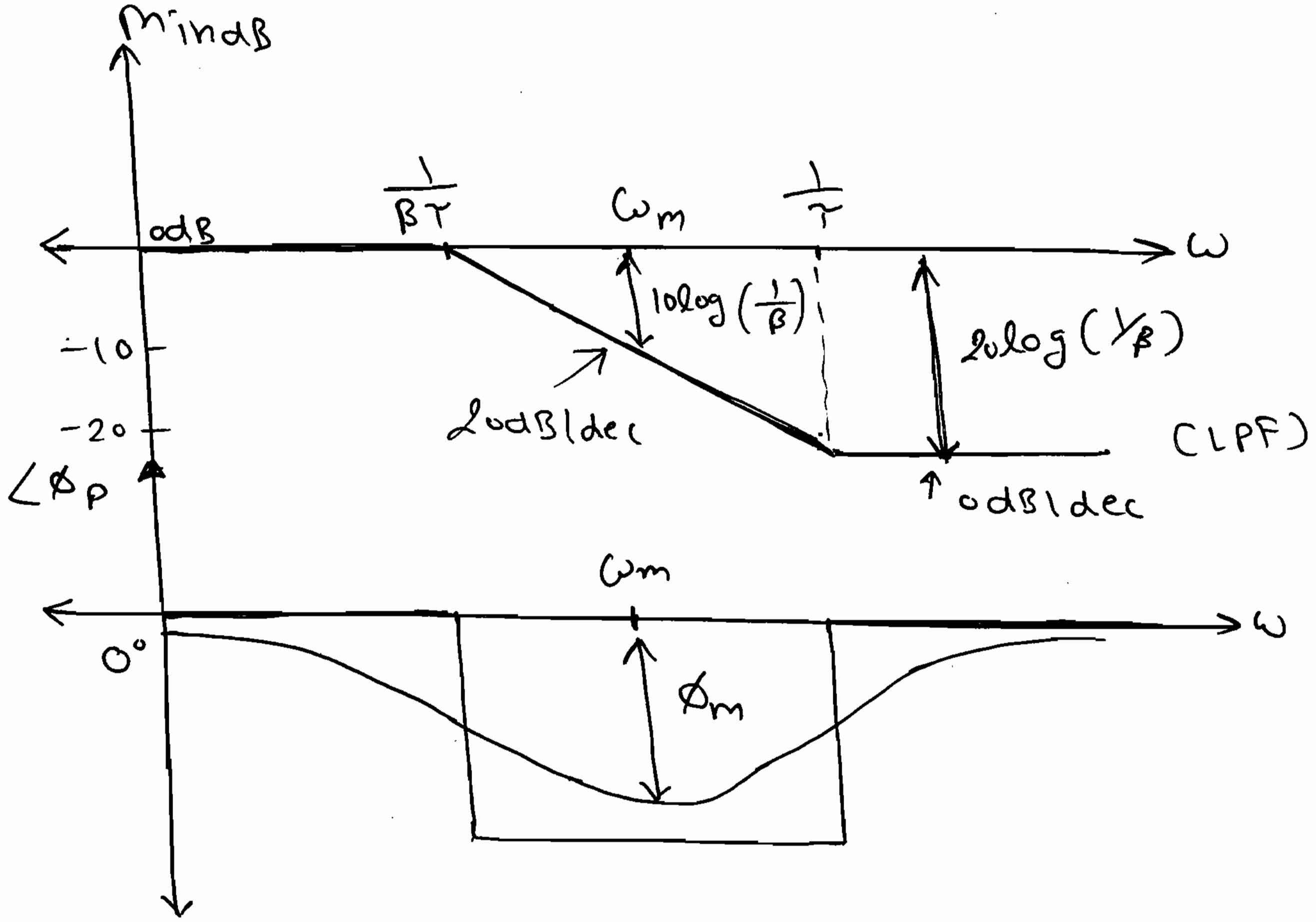
$\beta \Rightarrow$ lag Constant $= \dfrac{R_1+R_2}{R_2} > 1$ ($\beta_{opt} = 10$).

τ = lag time const. = R_2C_2.

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{1+\tau s}{1+\beta\tau s}}.$$



$\Rightarrow$



$\Rightarrow$ $\omega_m = \sqrt{\omega_{c1} \times \omega_{c2}} = \sqrt{\frac{1}{\beta\tau} \times \frac{1}{\tau}}$.

$\omega_m = \frac{1}{\sqrt{\beta}\cdot\tau}$. rad/sec.

$\therefore$ $\boxed{\phi_m = \sin^{-1}\left(\frac{\beta-1}{\beta+1}\right).}$ ***

* Advantages:

$\Rightarrow$ The lag Compensator is a LPF, it improves the steady state performance (steady state error ↓es, accurate o/p).

$\Rightarrow$ The lag Compensator is a LPF, it eliminate the noise in the system, hence SNR at the o/p is improved.

⇒ The main ~~advantage~~ purpose of lag compensator is to provide the sufficient phase margin to the system.

* **Disadvantages:-**

⇒ The lag compensator & decreases the BW; hence the rise time increases hence the system gives the slow response.

⇒ The lag compensator is similar to the PI controller. With lag comp. system becomes very sensitive with parameter variation.

③ **Lag-Lead Compensators:-**

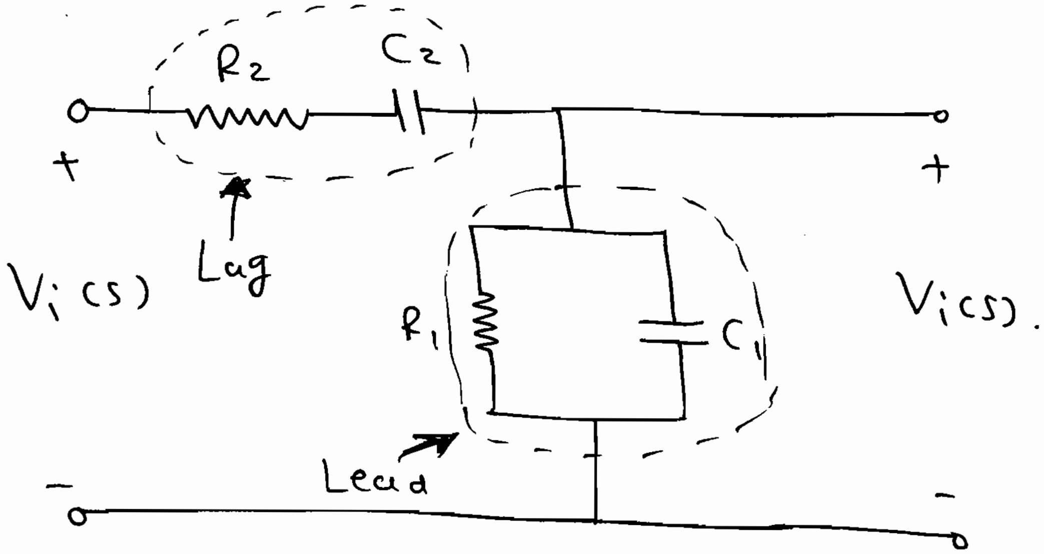
$(\tau_{lag} > T_{lead})$.

⇒ The Lag-Lead compensator is used to get the very quick response and good static accuracy.

(Rise time ↓ & e_{ss} ↓).

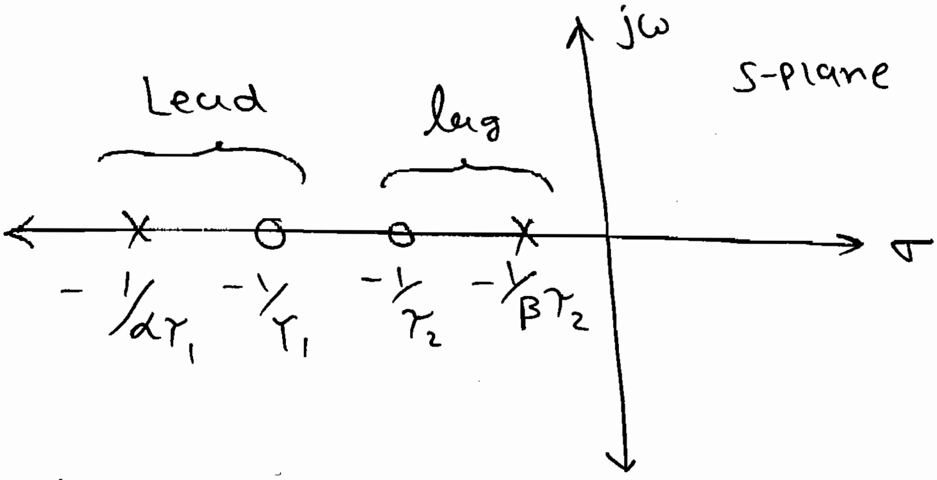
⇒ The ckt of lag-lead compensator is shown in fig.

⇒



$$\frac{V_o(s)}{V_i(s)} = \underbrace{\left(\frac{1+\tau_1 s}{1+\alpha\tau_1 s}\right)}_{\text{Lead}} \underbrace{\left(\frac{1+T_2 s}{1+\beta T_2 s}\right)}_{\text{Lag.}}$$

⇒



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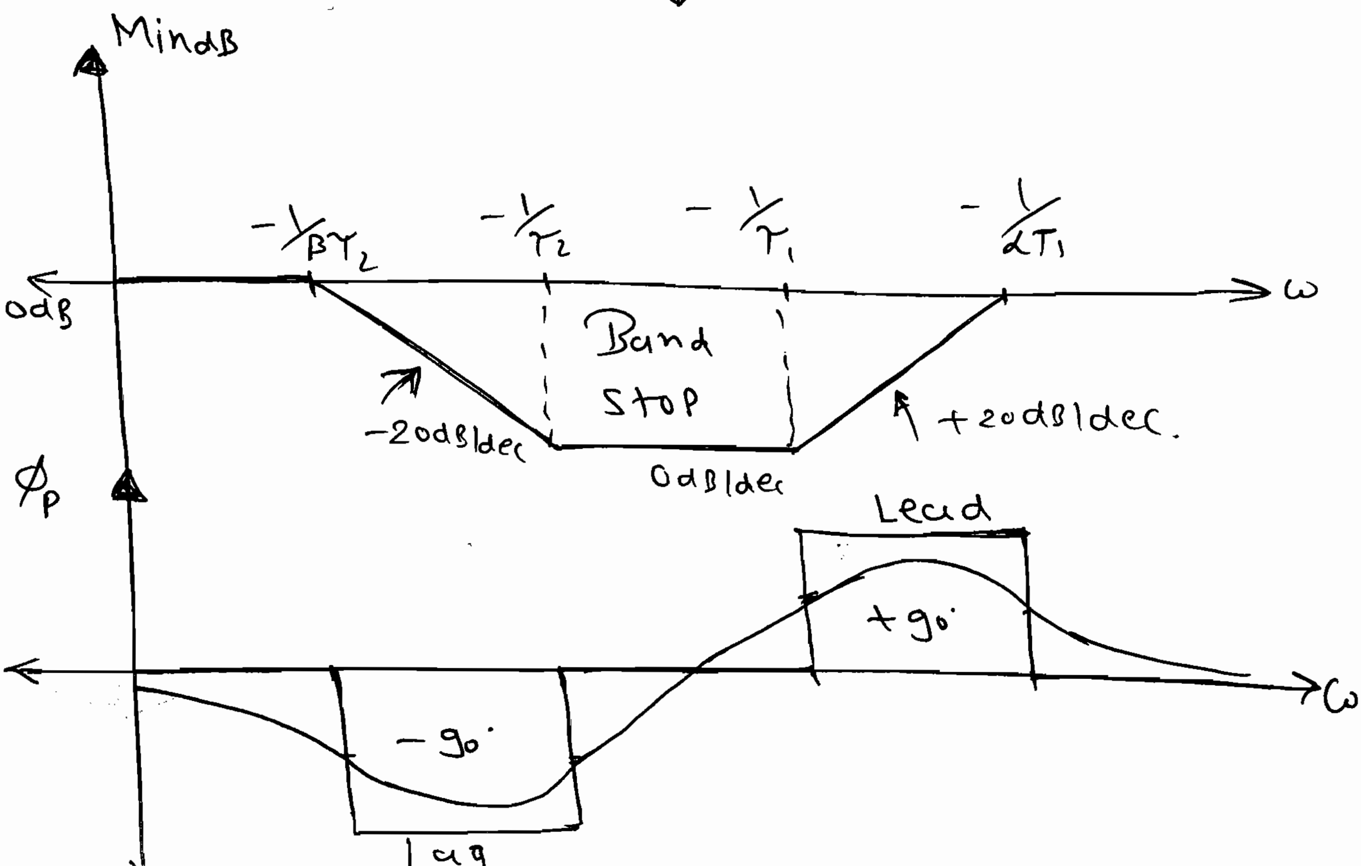
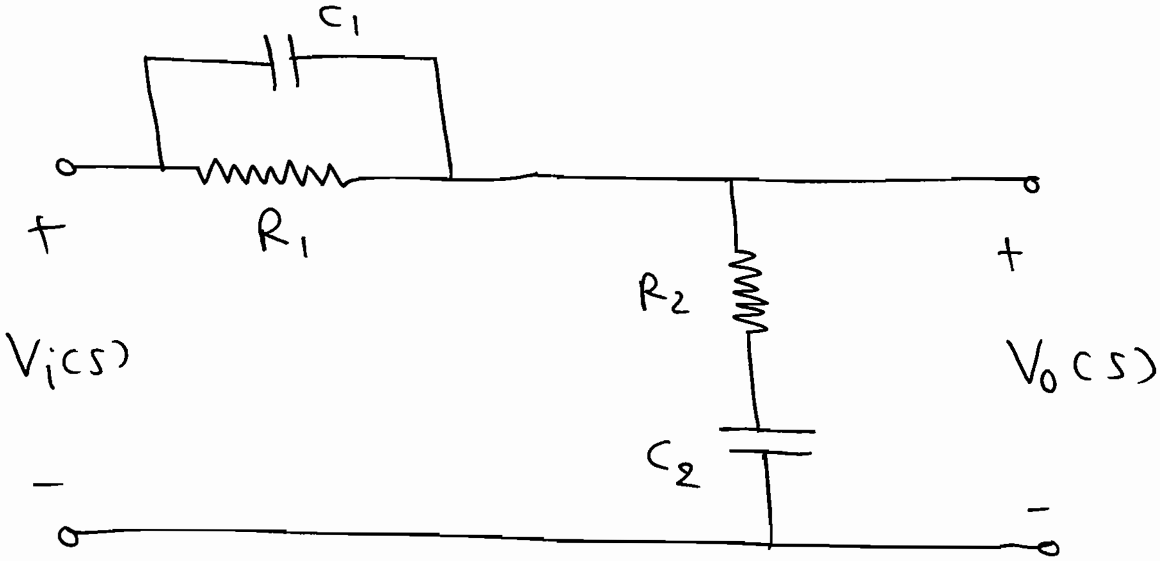


④ Lead-lag Compensators:

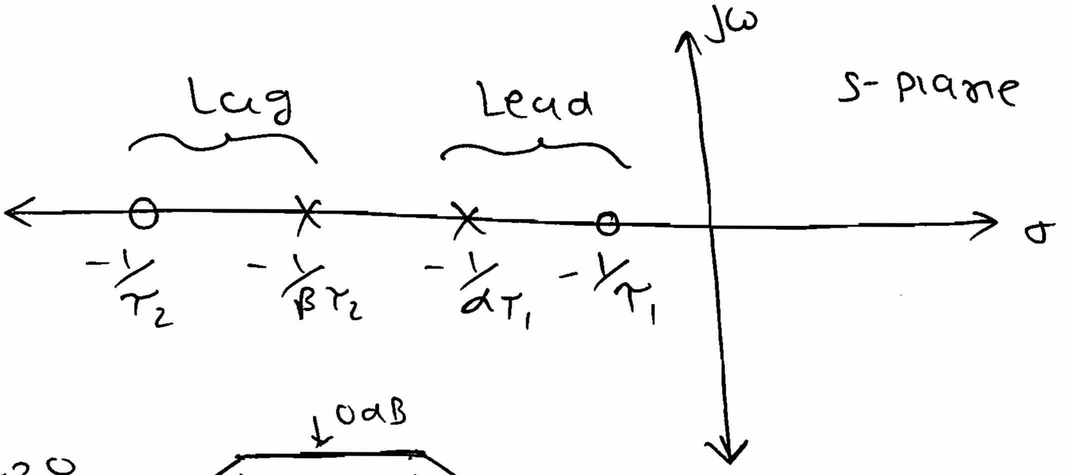
⇒



T.F. $$\frac{V_o(s)}{V_i(s)} = \left(\frac{1+\tau_1 s}{1+\alpha\tau_1 s}\right) \times \left(\frac{1+\tau_2 s}{1+\beta\tau_2 s}\right).$$

$$\boxed{\tau_{lead} > \tau_{lag}}$$

⇒



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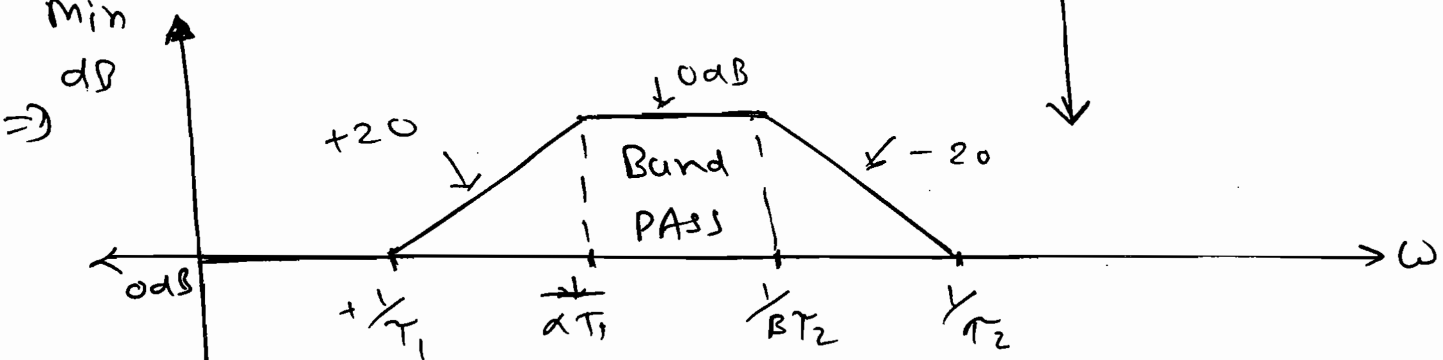
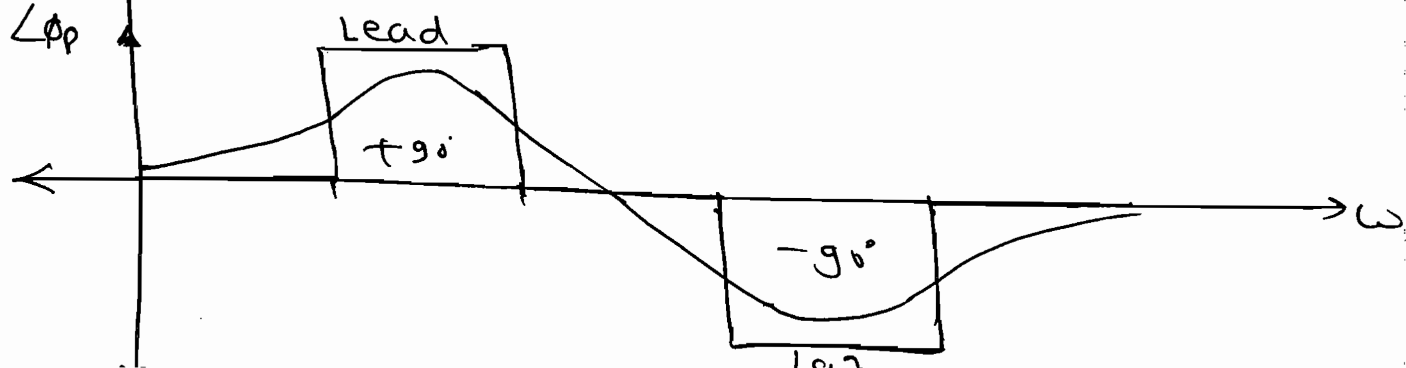




* Controllers :-

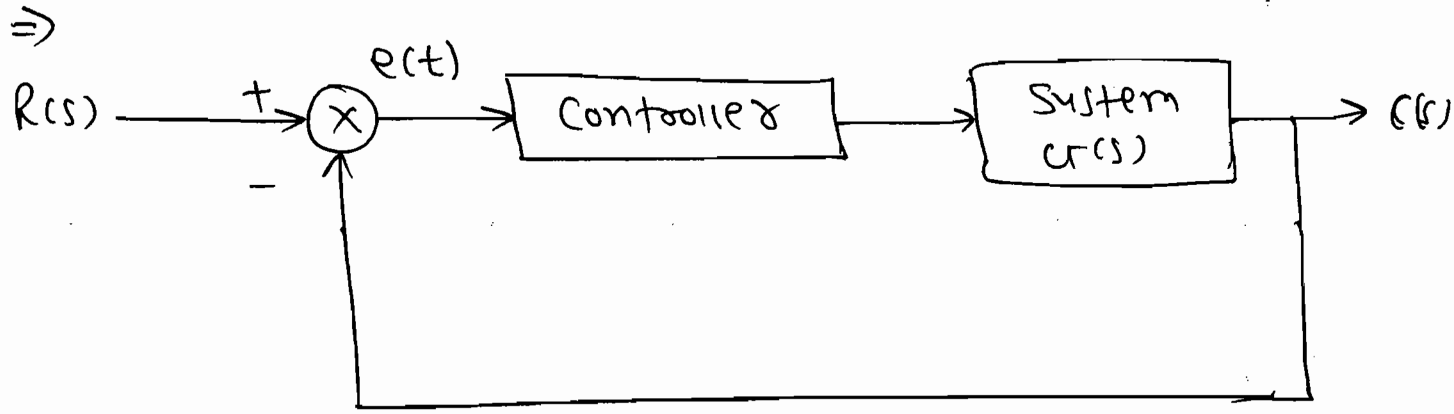
⇒ The Controller is a device which is used to Controll the transient and Steady State response as per requirement.

⇒ The best system demands Smallest t_r, Smallest t_s, Smallest e_{ss}, Smallest M_p.

⇒ To get above requirements we require to add a Controller to the System.

⇒ The block diagram with the Controller is shown in fig.

⇒



⇒ ① P Controller
② D Controller
③ I controller
④ PD controller.
⑤ PI Controller.
⑥ PID Controller.

① Proportional Controller :-

*** Purpose :-**

⇒ To change the transient response as per the requirement.

⇒ The T.F. of Proportional Controller is $\boxed{K_p}$

P controller $= K_p$.

for (e.g) → $G(s)\Big|_{\substack{\text{without}\\ \text{Controller}}} = \dfrac{1}{s(s+10)}$.

⇒ $CLTF = \dfrac{1}{s^2+10s+1}$ ⇒ $\omega_n = 1$ rad/sec

$2\xi\omega_n = 10$

$\xi = 5 > 1$

⇒ overdamped system.

⇒ $G(s)\Big|_{\substack{\text{with}\\ \text{Controller}}} = \dfrac{K_p}{s(s+10)}$.

→ $CLTF = \dfrac{K_p}{s^2+10s+K_p}$.

→ let, $K_p = 100$ ⇒ $\omega_n = 10$ r/sec.

$2\xi\omega_n = 10$

$\boxed{\xi = 0.5}$ ⇒ Under damped sys.

→ let, $K_p = 25$, $\omega_n = 5$

& $\boxed{\xi = 1}$ ⇒ Critical damped sys.

⇒ If selecting the Proper Value of K_p we can get the required Transient response.

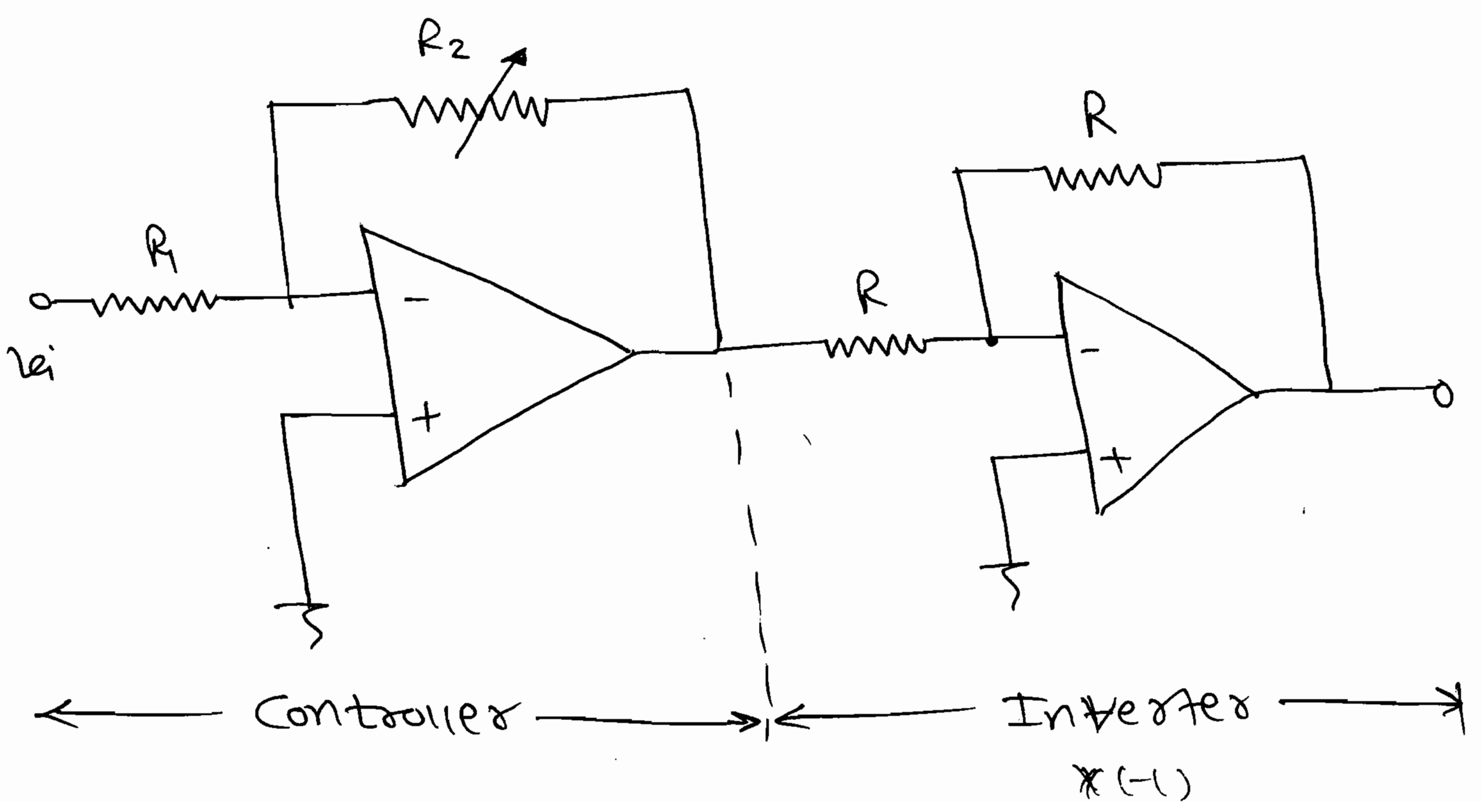
** ⇒

$$\boxed{K_p \uparrow \rightarrow \omega_n \uparrow \rightarrow \xi \downarrow \rightarrow \% M_p \uparrow \rightarrow \text{less RS \& more osc.}}$$

⇒ Proportional Controller can not eliminate error in the system.

⇒ If the $K_p \uparrow$, to get the better transient response $\xi \downarrow$. Hence the $\% M_p$ increases, the sys. become more oscillatory & ~~xxx~~ less Relative Stable.

⇒ <u>Practical Proportional Controller:-</u>



$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1} = K_p.}$$

② Integrated Controller (or) RESET Controller :-

* Purpose:-

⇒ To decrease the steady state error (e_{ss}).

⇒ The T.F. of Integral Controller is $\frac{K_I}{s}$.

⇒ The integral Controller added the one pole at origin Hence, Type is increases.

⇒ As the Type increases, the e_{ss} ↓ but the system stability is affected.

e.g.:

→ $G(s)\Big|_{\text{without Controller}} = \frac{1}{s(s+10)}$, Type-1.

$\xrightarrow{CE}$ $s^2 + 10s + 1 = 0$ → Stable.

→ $G(s)\Big|_{\text{with Controller}} = \frac{K_I}{s^2(s+10)}$, Type-2, ↑ e_{ss} ↓ (more accurate)

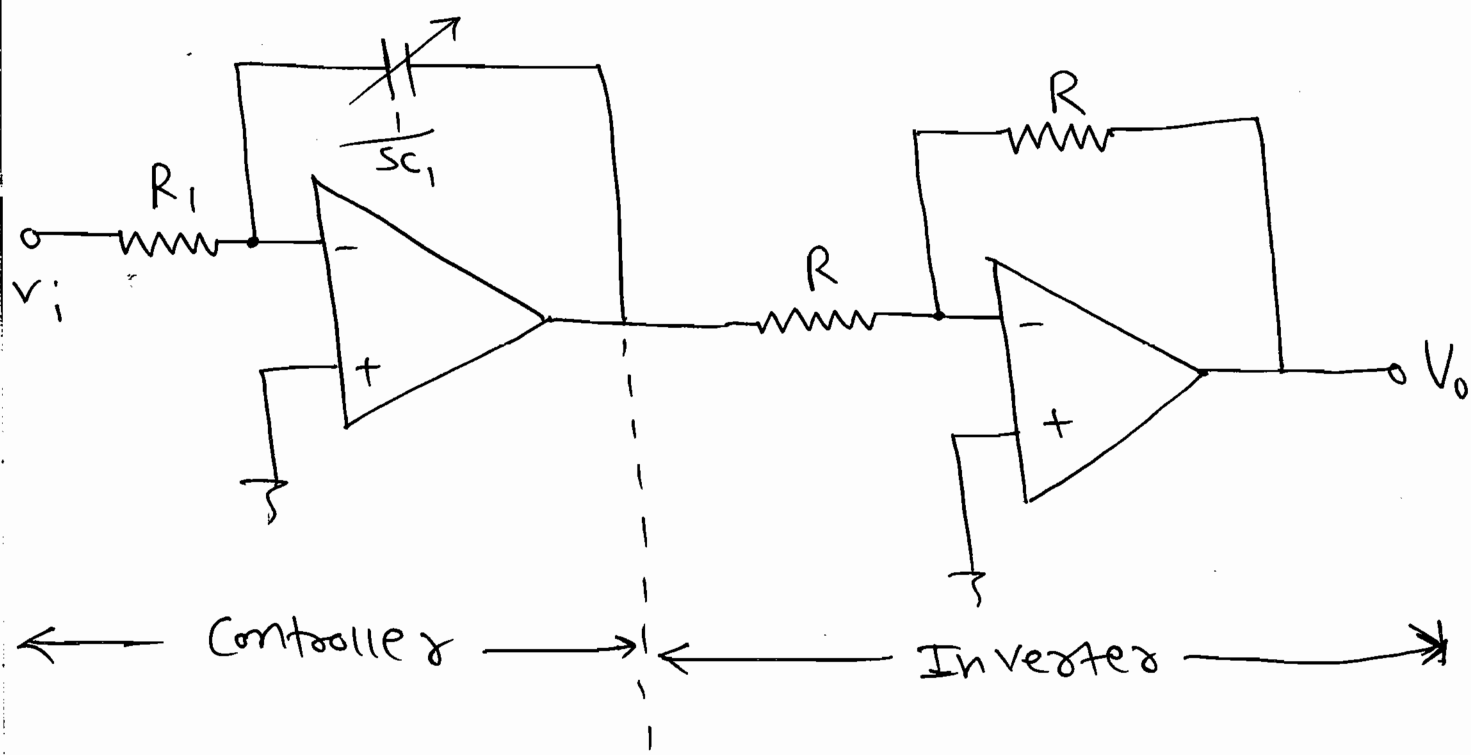
CE $\rightarrow$ $s^3 + 10s^2 + K_I = 0$ $\longrightarrow$ Un-stable.

$\Rightarrow$ The integral controller effect the sys. stability. Hence, before using the integral controller we required to verify the sys. stability.

$\Rightarrow$ If the system stability is affected the integral controllers are not used.

* **Practical ckt of Integral controller:-**

$\Rightarrow$



$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{1/sC}{R} = \frac{1}{sCR} = \frac{1}{\tau_I s} = \frac{K_I}{s}$$

Where, $K_I = \frac{1}{\tau_I} = \frac{1}{RC}$

(3) Derivative Controller :- (RATE Controller)

* **Purpose:-**

⇒ To improve the Stability.

⇒ T.F. of Derivative controller is $K_D s$.

⇒ The Derivative Controller adds 1 zero at origin.

$$\text{T.F. of D Controller} = K_D s.$$

⇒ The best example of derivative Controller is Techo-meter.

→ With D Controller added one zero at origin. Hence the type is ↓.

→ As type ↓, the sys. Stability improved but sys. become less accurate. (e_{ss} ↑).

E.g. → $G(s)\Big|_{\substack{\text{without}\\ \text{Controller}}} = \dfrac{1}{s^2(s+10)}$ Type-2.

$\xrightarrow{CE}$ $s^3 + 10s^2 + 1 = 0 \longrightarrow$ Unstable.

→ $G(s)\Big|_{\substack{\text{with}\\ \text{Controller}}} = \dfrac{K_D \cancel{s}}{s^{\cancel{2}}(s+10)} = \dfrac{K_D}{s(s+10)}$

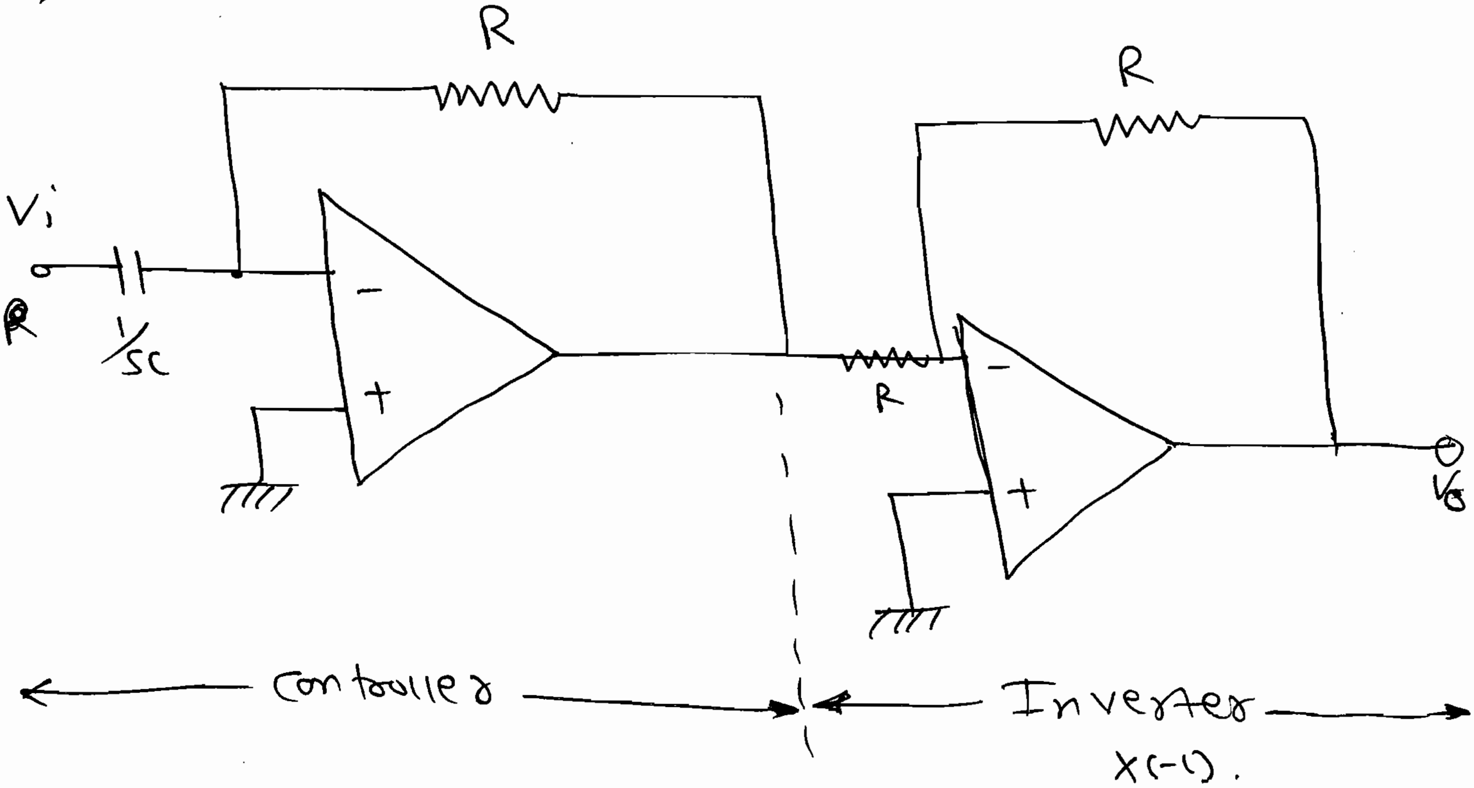
Type-1 ↓ e_{ss} ↑

less accurate

$CE \rightarrow \quad s^2 + 10s + K_D = 0 \longrightarrow$ Stable.

* Practical ckt for Derivative Controller :-

⇒



$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{R}{\frac{1}{sC}} = sCR = T_d s = K_D s.$$

where, $K_D = T_D = RC.$

(4) PI Controller :-

* Purpose:-

⇒ To decrease the steady-state error without affecting the stability.

⇒ The ~~PI Controller~~ T.F. of the PI Controller is

$$\boxed{T.F = K_P + \frac{K_I}{s}.}$$

$$\Rightarrow \quad T.F. = \left(\frac{sK_p + K_I}{s}\right).$$

⇒ The P-I Controller added one Pole at origin which increases the Type of the system.

⇒ As type ↑, the e_{ss} ↓.

⇒ PI Controller added one finite zero in the left of the s-plane which avoid the effect on sys. Stability.

eg let, $G(s)\Big|_{\text{without Controller}} = \dfrac{1}{s(s+10)}$ Type-1

$\xrightarrow{CE}$ $s^2 + 10s + 1 = 0 \longrightarrow$ Stable

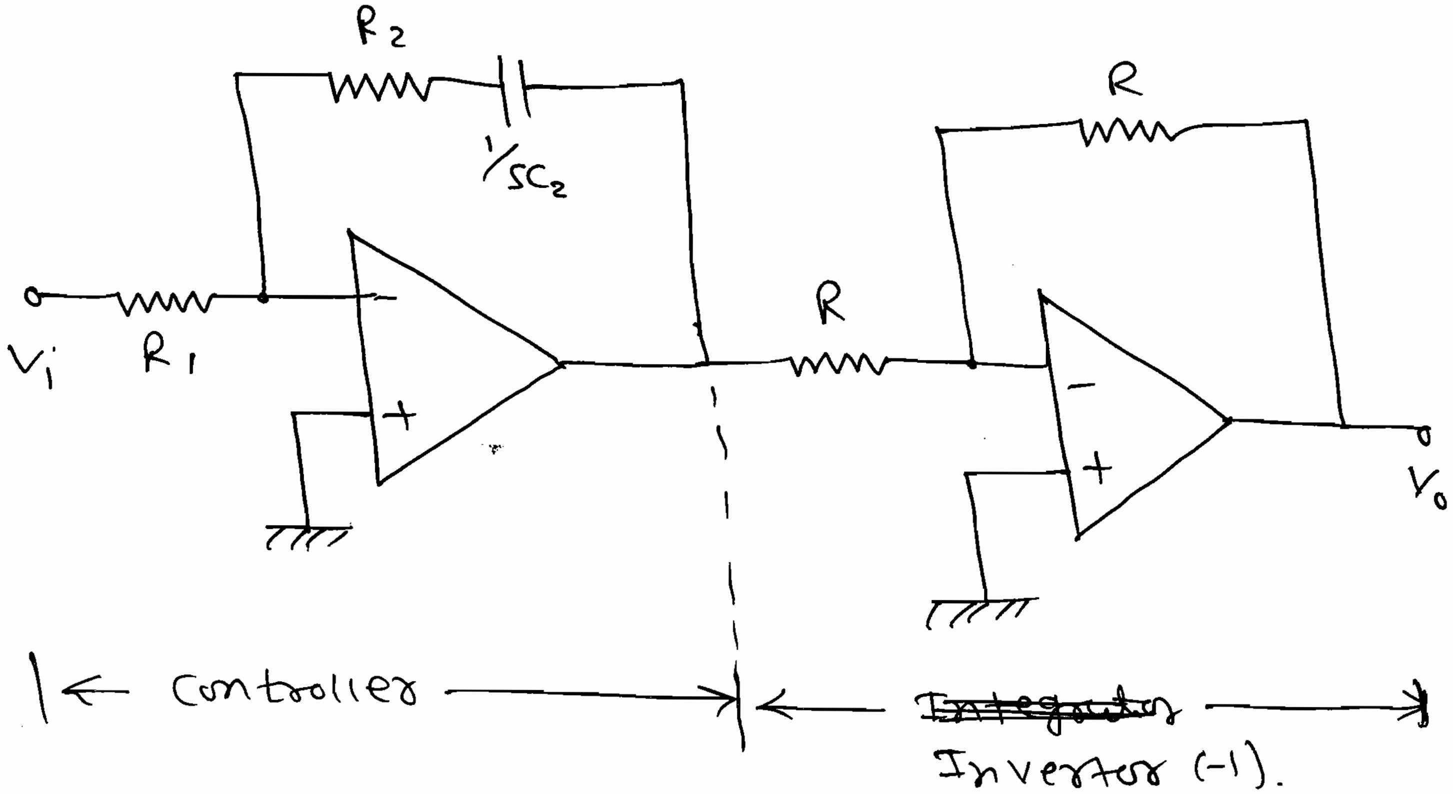
$\rightarrow G(s)\Big|_{\text{with Controller}} = \dfrac{(sK_p + K_I)}{s^2(s+10)}$, Type-2 ↑, e_{ss} ↓ more accurate

$\xrightarrow{CE}$ $s^3 + 10s^2 + sK_p + K_I = 0 \longrightarrow$ Stable

Stability is not affected.

* Practical ckt for P-I Controller:-

⇒ The Practical ckt is shown in fig.



$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{R_2 + \frac{1}{sC_2}}{R_1}.$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1} + \frac{1}{sC_2R_1}.$$

$$T.F. = K_P + \frac{K_I}{s}.$$

Where, $K_P = \frac{R_2}{R_1}$, $K_I = \frac{1}{R_1 C_2}$.

5 PD Controller :-

*** Purpose :-**

=> To improve the stability without affecting the Steady State error (ess).

⇒ The T.F. of PD Controller is $(K_P + K_D s)$.

⇒ The P.D. Controller added one finite zero in the left hand side, which improves the sys. stability.

⇒ PD Controller do not change the type, hence No effect on Steady state error.

⇒ The damping ratio with PD Controller is

$$\xi_{PD} = \left[\xi + \frac{\omega_n K_D}{2}\right].$$

⇒ $$G(s)\Big|_{\substack{\text{without}\\ \text{Controller}}} = \frac{1}{s^2(s+10)}\ ; \quad \text{Type-2}$$

$$\xrightarrow{CE} \quad s^3 + 10s^2 + 1 = 0 \longrightarrow \text{Unstable.}$$

No change in Type. Hence No change in e_{ss}

⇒ $$G(s)\Big|_{\substack{\text{with}\\ \text{Controller}}} = \frac{(K_P + K_D s)}{s^2(s+10)}\ ; \quad \text{Type-2.}$$

improved

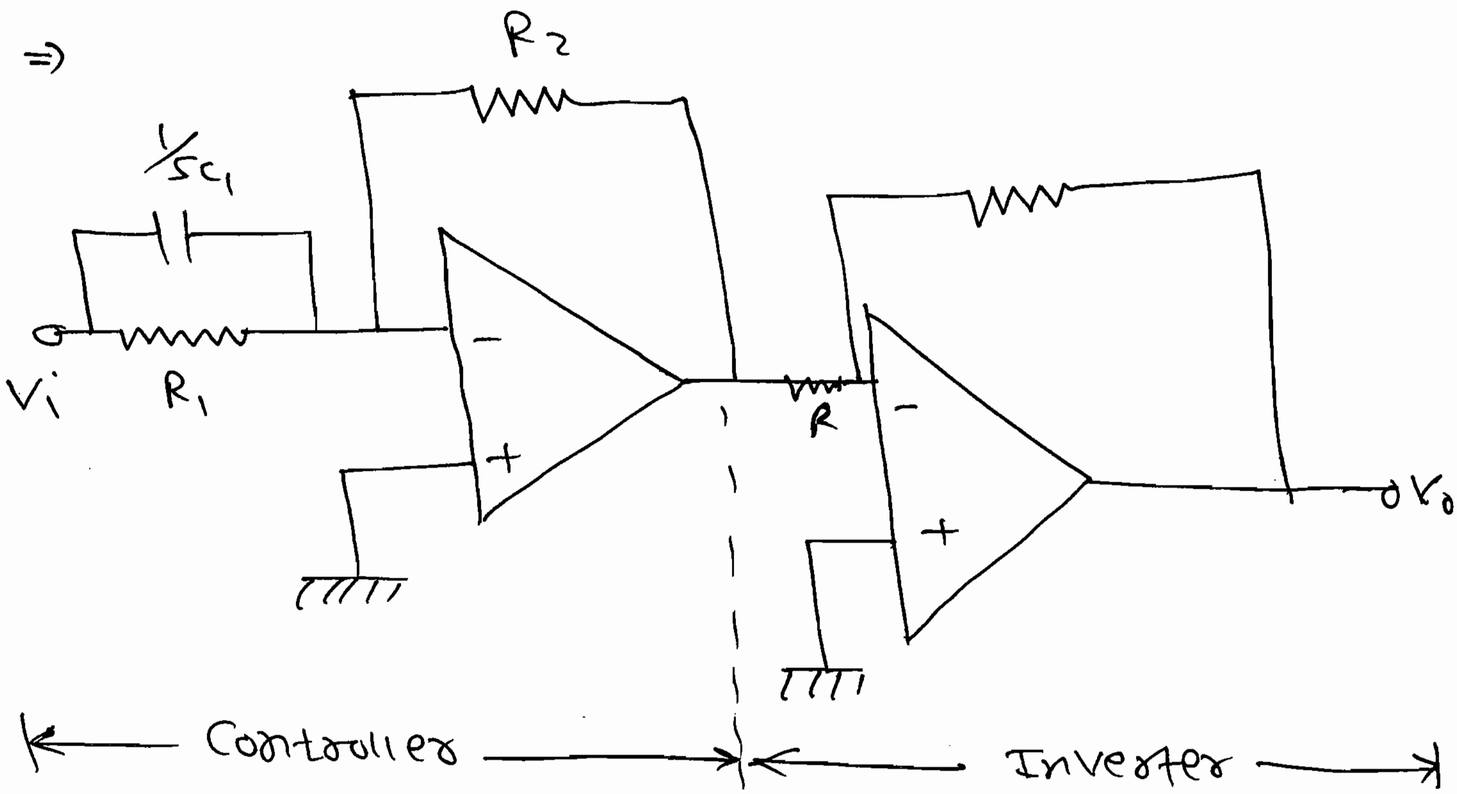
$$\xrightarrow{CE} \quad s^3 + 10s^2 + K_D s + K_P = 0 \longrightarrow \text{Stable.}$$

⇒ No Change in Type, Hence No Change in e_{ss}.

⇒ Stability improved.

* Practical ckt for PD Controller :-

⇒



$$\Rightarrow T.F. = \frac{V_0(s)}{V_i(s)} = \frac{R_2}{R_1 \parallel \frac{1}{sC_1}} = \frac{R_2}{\frac{R_1}{1+sC_1R_1}}$$

$$= \frac{R_2(1+sC_1R_1)}{R_1}$$

$$= \frac{R_2}{R_1} + sC_1R_2$$

$$= K_P + K_D s$$

Where, $K_P = \frac{R_2}{R_1}$, $K_D = C_1R_2$.

$\boxed{6}$ PID Controller :-

* Purpose :-

⇒ To decrease the steady state error (e_{ss}) & improve the stability.

⇒ The T.F. of the PID Controller:

$$T.F. = \left(K_P + \frac{K_I}{S} + K_D S\right).$$

$$T.F. = \left(\frac{K_D S^2 + K_P S + K_I}{S}\right).$$

⇒ The PID Controller added one Pole at origin. Hence, the Type is ↑ Steady State error ↓.

⇒ The PID Controller added two finite zero in left-hand side.

⇒ one zero avoid the effect on system stability and other zero improves the system stability.

Eg: $G(s)\Big|_{\text{without Controller}} = \frac{1}{S^2(S+10)}$; Type-2.

$\xrightarrow{CE}$ $S^3 + 10S^2 + 1 = 0 \longrightarrow$ Unstable.

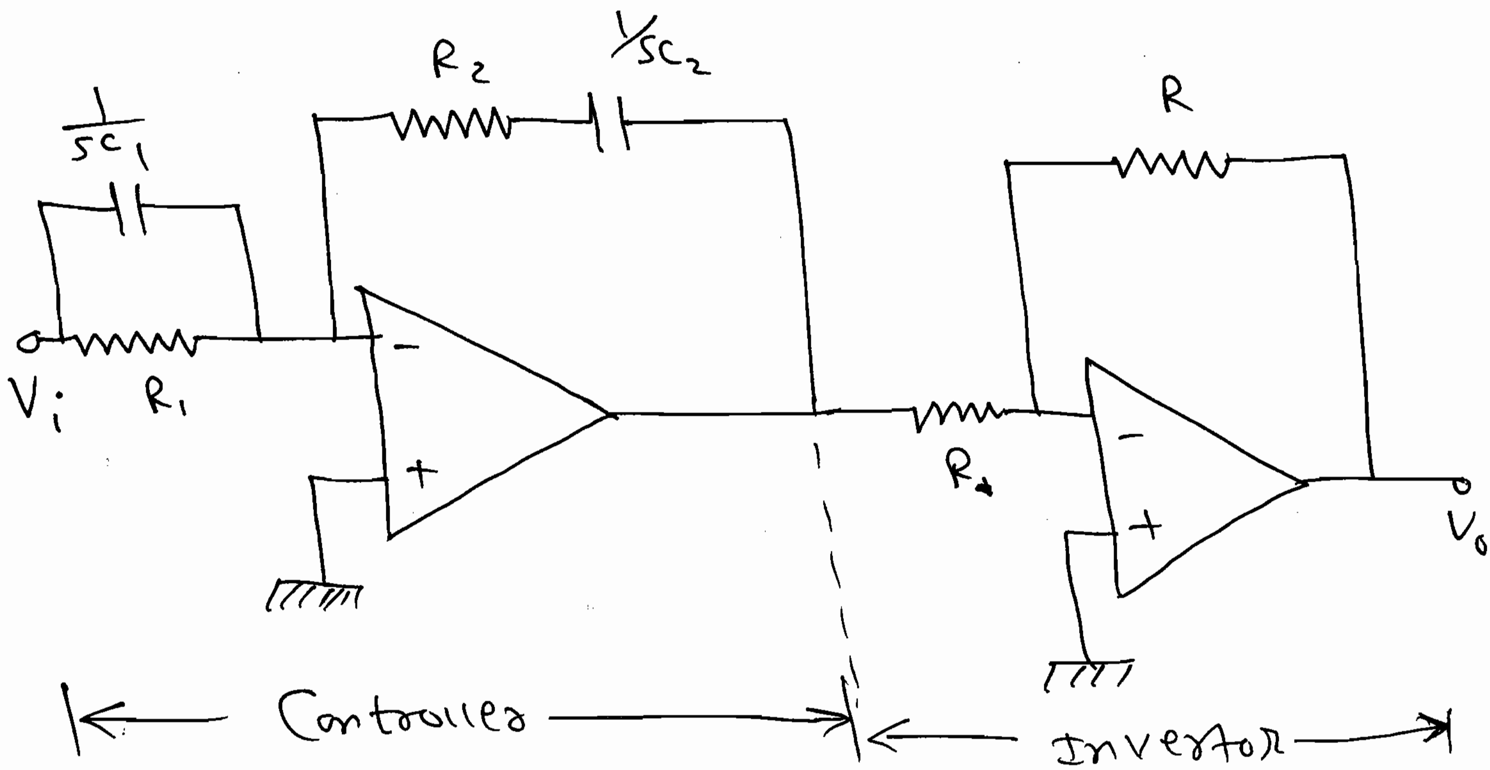
⇒ $G(s)\Big|_{\text{with Controller}} = \frac{K_D S^2 + K_P S + K_I}{S^3(S+10)}$; Type-3 ↑

e_{ss} ↓

improved

$\xrightarrow{CE}$ $S^4 + 10S^3 + K_D S^2 + K_P S + K_I = 0 \longrightarrow$ Stable (more accurate)

* Practical ckt for PID Controller :-



$$\Rightarrow \text{T.F.} = \frac{V_o(s)}{V_i(s)}.$$

$$= \frac{R_2 + \frac{1}{sC_2}}{\frac{R_1}{sC_1R_1+1}}$$

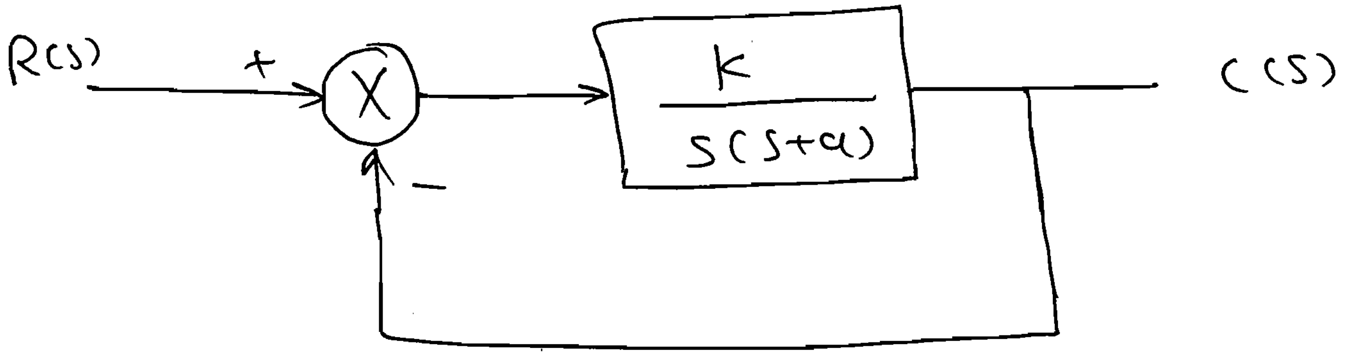
$$= \frac{(1+sC_2R_2)(1+sC_1R_1)}{sC_2R_1}$$

$$= \frac{1 + s(C_2R_2 + C_1R_1) + s^2 C_2R_2C_1R_1}{sC_2R_1}$$

$$= \underbrace{\left(\frac{R_2}{R_1} + \frac{C_1}{C_2}\right)}_{K_p} + \underbrace{\left(\frac{1}{sC_2R_1}\right)}_{K_I/s} + \underbrace{\left(sC_1R_2\right)}_{K_D \cdot s}.$$

$$\text{T.T.} = K_p + \frac{K_I}{s} + K_D \cdot s$$

Q Find the Steady State error of Sensitivity to Change in Parameters (i) K (ii) a to the unit-ramp I/P to the following system.



Sol^n:

$$G(s) = \frac{K}{s(s+a)}.$$

R(s) = unit ramp

$r(t) = t\,u(t)$. & Type-1

$$\therefore \quad e_{ss} = \frac{1}{K/a} = a/K.$$

(i) $$S_K^{e_{ss}} = \left(\frac{\partial e_{ss}}{\partial K}\right) \times \left(\frac{K}{e_{ss}}\right) = \frac{-\cancel{K}\,a}{K^{\cancel{2}}} \times \frac{\cancel{K}}{a/K}$$

$$\therefore \quad \boxed{S_K^{e_{ss}} = -1}$$

(ii) $$S_a^{e_{ss}} = \left(\frac{\partial e_{ss}}{\partial a}\right) \times \left(\frac{a}{e_{ss}}\right) = \frac{1}{\cancel{K}} \times \frac{\cancel{a}}{\cancel{a}/\cancel{K}}$$

$$\therefore \quad \boxed{S_K^{e_{ss}} = 1}$$