Manipal – 2021

Solved Paper

Physics

1. The displacement of an oscillating particle is given by $y = A \sin[Bx + Ct + Dt]$. The dimensional formula for [ABCD] is :

A. $[M^0 L^{-1} T^0]$

B. $[M^0 L^0 T^{-1}]$

C. $[M^{\rho} L^{-1} T^{-1}]$

D. $[M^0 L^0 T^0]$

2. A cyclist moving on a circular track of radius 40 m completes half a revolution in 40 s . Its average velocity is :

A. zero

B. 4π m/s

C. 2 m/s

D. 8π m/s

3. A point initially at rest moves along x-axis. Its acceleration varies with time as $a = (6t + 5)$ in m/s^2. If it starts from origin, the distance covered in 2 s is :

A. 20 m

B. 18 m

C. 16 m

D. 25 m

4. A particle of mass m is moving in a horizontal circle of radius R under the centripetal force $= -k/R^2$ where k is a constant. What is the total energy of the particle?

A. $k/2R$

B. $-k/2R$

C. k/R

D.−k/R

5. A particle is projected from the ground with an initial speed of u at an angle θ with horizontal. The average velocity of the particle between its point of projection and highest point of trajectory is :

A. u cos θ

B. $\frac{u}{2}\sqrt{1+\cos^2\theta}$

C. $\frac{u}{2}\sqrt{1+2\cos^2\theta}$

D. $\frac{u}{2}\sqrt{1+3\cos^2\theta}$

6. A particle p is moving in a circle of radius r with a uniform speed v, C is the centre of the circle and AB is the diameter. The angular velocity of p about A and C is in the ratio :

A. 1:1

B. 1:2

C. 2:1

D. 4:1

7. A 4 kg block A is placed on the top of a block B of mass 8 kg , which rests on a smooth table. A just slips on B when a force of 12 N is applied on A.

Then, the maximum horizontal force required to make both A and B move together is :

A. 12 N

B. 24 N

C. 36 N

D. 48 N

8. A fireman weighing 80 kg slides down a pole. If the resisting force of friction is 720 N , his acceleration would be : (take g = 10 m/s^2)

A. 0.11 m/s^2

B. 0.9 m/s^2

C. 1 m/s^2

D. zero

9. A simple spring has length L and force constant k. It is cut into two springs of lengths l_1 and L_2 such that $l_1 = nL_2$ (n= an integer). The force constant of spring of length l_1 is :

A. $k(1 + n)$

B. $k/n(1 + n)$

C. k

D. $k/(n+1)$

10. A force of 5 N gives a mass M_1 an acceleration equal to 8 m/s^2 and M_2 an acceleration equal to 24 m/s^2. What is the acceleration, if both masses are tied together ?

A. 16 m/s^2

B. 6 m/s^2

C. 12 m/s^2

D. 4 m/s^2

11. An object A moving horizontally with kinetic energy of 800 J experiences a constant opposing force of 100 N, while moving from a place x to a place y, where xy is 2 m . What is the energy of A at y ?

A. 700 J

B. 400 J

C. 600 J

D. 300 J

12. The displacement x and time t for a particle are related to each other as $t = \sqrt{x} + 3$. What is work done in first 6 s of its motion ?

A. 6 J

B. Zero

C. 4 J

D. 2 J

13. A particle is moving in a circular path with a constant speed vv. If θ is the angular displacement. Then starting from $\theta = 0$, the maximum and minimum changes in the momentum will occur, when value of θ is respectively:

A. 45^0 and 90^0

B. 90^0 and 180^0

C. 180^0 and 360^0

D. 90^0 and 270^0

14. A bullet is fired by a light rifle and the other with a heavy rifle by the same force. Which rifle will cause more injury to the shoulder?

A. Light rifle

B. Heavy rifle

C. Both will cause the same injury

D. The information is insufficient

15. A ball falls vertically on to a floor, with momentum p, and then bounces repeatedly, the coefficient of restitution is e. The total momentum imparted by the ball to the floor is

A. $p(1 + e)$

B. $p/(1-e)$

C. $p\left(\frac{1+e}{1-e}\right)$

D. $p\left(1 - \frac{1}{e}\right)$

16. A particle of mass m is projected with a velocity v at an angle of 45^0 with horizontal. When the particle is at its maximum height, the magnitude of its angular momentum about the point of projection is :

A. zero

B. $\frac{mv^3}{4\sqrt{2}g}$

C. $\frac{mv^3}{\sqrt{2}g}$

D. $\frac{mv^3}{\sqrt{2gh^3}}$

17. A solid sphere of radius R has moment of inertia / about its diameter. What will be moment of inertia of a shell of same mass and same radius about its diameter ?

A. 3/5 I

B. 5/3 I

C. 2/3 I

D. 2/5 I

18. A body is moving with constant velocity parallel to x-axis. Its angular momentum w.r.t., the origin will :

A. be zero

B. decrease

C. increase

D. remain constant

19. A uniform thin bar of mass 6 m and length 12L is bent to make a regular hexagon. Its moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is :

A. 20 mL^2

B. 30 mL^2

C. 12/5mL^2

D. 6 mL^2

20. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/R}$, then T^2 is proportional to :

A. R^3

B. $R^{7/2}$

C. $R^{3/2}$

D. $R^{3.75}$

21. A satellite goes along an elliptical path arounds earth. The rate of change of area swept by the same line joining earth and the satellite is proportional to :

A. $r^{1/2}$

B. r

C. $r^{3/2}$

D. r^2

22. The ratio of KE required to be given to the satellite to escape earth's gravitational field to the KE required to be given, so that the satellite moves in a circular orbit just above earth's atmosphere is :

A. 1

B. 1/2

C. 2

D. infinity

23. A steel ring of radius r and cross-sectional area A is fitted on to a wooden disc of radius R(R > r). If Young's modulus be Y, then the force with which the steel ring is expanded, is

A. $AY\frac{R}{r}$

B. $AY\left(\frac{R-r}{r}\right)$

C. $\frac{Y}{A}\left[\frac{R-r}{r}\right]$

D. Yr/AR

24. To what height should a rectangular cylinder having square base of length 10 cm be filled, so that the total force on the bottom is equal to that on the sides ?

A.5 cm

B. 10 cm

C. 20 cm

D. 6.67 cm

25. A rectangulat vessel when full of water takes 10 min to be emptied through an orifice in its bottom. How much time will it take to be emptied, when half filled with water?

A. 4 min

B. 6 min

C. 7 min

D. 8 min

26. A thin copper wire of length L increases in length by 1%, when heated from T_1 to T_2. What is the percentage change in area when a thin copper plate having dimensions (10L × 2L) is heated from T_1 to T_2 ?

A. 2%

B. 20%

C. 10%

D. 40%

27. The rays of sun are focussed on a piece of ice through a lens of diameter 5 cm , as a result of which 10 g ice melts in 10 min . The amount of heat received from sun, per unit area per minute is :

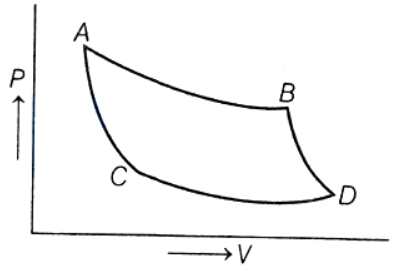
A. 4cal/cm^2 min

B. 40cal/cm^2 min

C. 4 J/m^2 min

D. 400cal/cm^2 min

28. In the indicator diagram, AB is an isotherm and BC is an adiabat, because :



A. AB and BC meet at B

B. BC is shorter than AB

C. slope of AB is less than slope of BC

D. none of the above

29. 8 g of O_2,14 g of N_2, and 2 g of CO_2 is mixed in a container of 10 L capacity at 27°C. The pressure exerted by the mixture in terms of atmospheric pressure is ($R = 0.082$ L atmk^{-1} mol^{-1}) :

A. 1.4 atm

B. 2.5 atm

C. 3.7 atm

D. 8.7 atm

30. During an adiabatic process, the pressure P of a fixed mass of an ideal gas changes by ΔP and its volume V changes by ΔV. If $\gamma = CP/CV$, then ΔV/V is given by:

A. $-\Delta p/p$

B. $-\gamma\Delta P/P$

C. $-\Delta P/\gamma P$

D. $\Delta P/\gamma^2 P$

31. An engine has an efficiency of 1/3. The amount of work this engine can perform per kcal of heat input is :

A. 1400 cal

B. 700 cal

C. 700 J

D. 1400 J

32. Two identical cylinders contain helium at 2.5 atm and argon at 1 atm respectively. If both the gases are filled in one of the cylinders, the pressure would be:

A. 3.5 atm

B. 1.50 atm

C. 1.75 atm

D. 1 atm

33. The displacement y in centimetre is given in terms of time t second by the equation

$y = 3\sin 314t + 4\cos 314t$

The amplitude of SHM is :

A. 7 cm

B. 3 cm

C. 4 cm

D. 5 cm

34. A wave travelling in a stretched string is described by the equation $y = A\sin(kx - \omega t)$. The maximum particle velocity is :

A. $A\omega$

B. ω/k

C. $d\omega/dk$

D. x/t-

35. A mass M is attached to a string, oscillates with a period of 2 s . If the mass is increased by 4 kg . The time period increases by 1 s . Assuming, that Hooke's law is obeyed, the initial mass M was :

A. 3.2 kg

B. 1 kg

C. 2 kg

D. 8 kg

36. A stretched sonometer wire is in unison with a tuning fork. When length of wire is increase by 1%, the number of beats heard per second is 5. Then the frequency of the fork is :

A. 500 Hz

B. 505 Hz

C. 255 Hz

D. 250 Hz

37. A heavy uniform rope hangs vertically from the ceiling, with its lower end free. A disturbance on the rope travelling upwards from the lower end has a velocity v at a distance x from the lower end such that :

A. $v \propto x$

B. $v \propto \sqrt{x}$

C. $v \propto 1/x$

D. $v \propto 1/\sqrt{x}$

38. If an electron has an initial velocity in a direction different from that of an electric field, the path of the electron is :

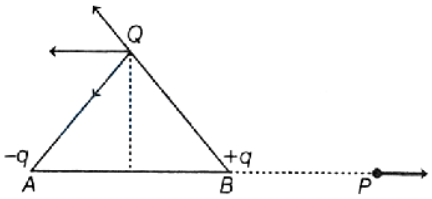
A. a straight line

B. a circle

C. an ellipse

D. a parabola

39. Due to the dipole shown in figure, the electric intensity will be parallel to the dipole axis at the point :



A. Q

B. P

C. both P and Q

D. neither P nor Q

40. A conducting sphere of radius R is charged to a potential of V volt. Then the electric field at a distance r(>R) from the centre of sphere would be :

A. RV/r^2

B. V/r

C. rV/R^2

D. R^2V/r^3

41. Given that $q_1 + q_2 = q$. For what ratio q_1/q_2 will the force between q_1 and q_2 be maximum ?

A. 0.25

B. 0.5

C. 1

D. 2

42. Two capacitors of capacitance 2μ F and 6μ F are connected in series. A potential difference of 800 V is applied to the outer plates of the two capacitors system. The charge on each capacitor will be :

A. 1200 C

B. 6000 C

C. 6000μC

D. 1200μC

43. A charge of 2×10^{-2}C moves at 30rev/s in a circle of diameter 80 cm . The current linked with the circuit is :

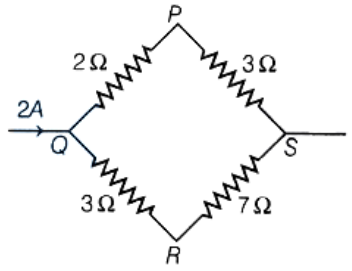
A. 0.02 A

B. 20 A

C. 0.60 A

D. 60 A

44. A current of 2 A flows in the system of conductors as shown in the figure. The potential difference $V_P - V_R$ will be :



A. -2V

B. -1V

C. $+1$V

D. $+2$V

45. A coil of wire of resistance 50Ω is embedded in a black of ice and a potential difference of 210 V is applied across it. The amount of ice which melts in 1 s is :

A. 0.262 g

B. 2.62 g

C. 26.2 g

D. 0.0262 g

46. When 1 g hydrogen

(ECE $= 1.044 \times 10^{-8}$ kgC^{-1}) forms water, 34 kcal heat is liberated. The minimum voltage required to decompose water is :

A. 0.75 V

B. 3 V

C. 1.5 V

D. 4.5 V

47. A copper wire of diameter 1.6 mm carries a current I. The maximum magnetic field due to this wire is 5×10^{-4} T. The value of l is :

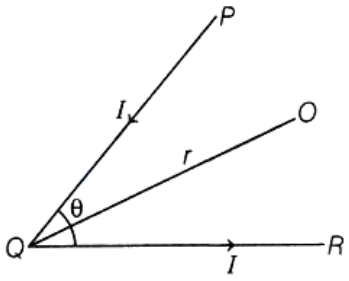
A. 0.2 A

B. 0.5 A

C. 2 A

D. 4 A

48. Two wires PQ and QR, carry equal currents I as shown in figure. One end of both the wires extends to infinity $\angle PQR = \theta$. The magnitude of the magnetic field at O on the bisector of these two wires at a distance rr from point O is



A. $\frac{\mu_0 I}{4\pi r}\sin\frac{\theta}{2}$

B. $\frac{\mu_0}{4\pi}\frac{1}{r}\cot\frac{\theta}{2}$

C. $\frac{\mu_0}{4\pi}\frac{1}{r}\tan\frac{\theta}{2}$

D. $\frac{\mu_0 I}{2\pi r}\frac{[1+\cos(\theta/2)]}{\sin(\theta/2)}$

49. A bar magnet has a magnetic moment equal to 5×10^{-5} Wb $-$ m. It is suspended in a magnetising field equal to $8\pi \times 10^{-4}$ A $-$ m^{-1}.

The magnet vibrates with a period 15 s . The moment of inertia of the magnet is:

A. 11.14 kg $-$ m^2

B. 0.57 kg $-$ m^2

C. 22.28 kg $-$ m^2

D. 0.057 kg $-$ m^2

50. If $\vec{A}, \vec{B}$ and $\vec{C}$ are the unit vectors along the incident ray, reflected ray and outward normal to the reflecting surface, then:

A. $\vec{B} = \vec{A} - \vec{C}$

B. $\vec{B} = \vec{A} + (\vec{A}\cdot\vec{C})\vec{C}$

C. $\vec{B} = \vec{A} + \vec{C}$

D. $\vec{B} = \vec{A} - 2(\vec{A}\cdot\vec{C})\vec{C}$

Chemistry

1. In the reaction $N_2 + 3H_2 \longrightarrow 2NH_3$, ratio by volume of N_2, H_2 and NH_3 is 1 : 3 : 2. This illustrates law of :

A. definite proportion

B. multiple proportion

C. reciprocal proportion

D. gaseous volumes

2. 100 mL of 1.0 M HCl are mixed with 75 mL of $1.0M Na_2CO_3$. The resulting solution will be :

A. acidic

B. basic

C. neutral

D. amphoteric

3. The order of increasing energies of the orbitals follows:

A. $5p < 4f < 6s < 5d$

B. $5p < 6s < 4f < 5d$

C. $4f < 5p < 5d < 6s$

D. $5p < 5d < 4f < 6s$

4. Which of the following is not possible ?

A. $n = 3, l = 0, m = 0$

B. $n = 3, l = 1, m = -1$

C. $n = 2, l = 0, m = -1$

D. $n = 2, l = 1, m = 0$

5. What is the wavelength associated with an electron moving with a velocity of 10^6 m/s ?

$(h = 6.63 \times 10^{-34} Js)$

A. 72.7 nm

B. 7.27 nm

C. 0.727 nm

D. 0.0727 nm

6. Which of the following set of quantum number is not applicable for an electron in an atom?

A. $n = 1, l = 1, m = 1, s = +1/2$

B. $n = 1, l = 0, m = 0, s = +1/2$

C. $n = 2, l = 0, m = 0, s = +1/2$

D. $n = 2, l = 0, m = 0, s = -1/2$

7. For which of the following species Bohr's theory is not applicable?

A. Be^{3+}

B. Li^{2+}

C. He^{2+}

D. H

8. Whose name is not associated with the development of periodic table?

A. Prout's

B. Newlands

C. Rutherford

D. Lother Meyer

9. $C(s) + O_2(g) \longrightarrow CO_2(g)\ \Delta H = -94kcal$

$2CO + O_2(g) \longrightarrow 2CO_2(g) \Delta H = -135.2kcal$

The heat of formation of $CO(g)$ is :

A. -26.4 kcal

B. 41.2 kcal

C. 26.4 kcal

D. 229.2 kcal

10. Calculate the temperature at which $\Delta G = -5.2$ kJ mol^{-1} , $\Delta H =$ 145.6 kJ mol^{-1} and $\Delta S = 216 Jk^{-1}$ mol^{-1} for a chemical reaction :

A. $698.1^{o}C$

B. 698.1K

C. 130K

D. $130^{o}C$

11. Solution of 0.1 NNH_4OH and 0.1NH_4Cl has pH 9.25. Then pKK_b of NH_4OH is :

A. 9.25

B. 4.75

C. 3.75

D. 8.25

12. Which of the following is not correct for D_2O ?

A. BVP is higher than H_2O

B. D_2O reacts slowly than H_2O

C. Viscosity is higher than H_2O at 25^{o}

D. Solubility of NaCl in it is more than H_2O

13. The low density of ice compared to water is due to :

A. hydrogen bonding interactions

B. dibble-dipole interactions

C. dipole induced dipole interactions

D. induced dipole induced dipole interactions.

14. Ice floats on water because:

A. its density is less than that of water

B. crystal structure of ice has empty space

C. both of the above

D. none of the above

15. Sodium carbonate is manufactured by :

A. Lowing process

B. Leblanc process

C. Solvay process

D. Haber's process

16. When sodium hydroxide react with sand, it form :

A. sodium silicate

B. silicon oxide

C. silicon hydroxide

D. none of the above

17. Diborane on reaction with olefins forms :

A. mono acid derivatives

B. diethyl borane

C. triethyl borane

D. ethyl borane

18. Which is the decreasing order of stability?

(i) $CH_3 - \overset{+}{C}H - CH_3$

(ii) $CH_3 - \overset{+}{C}H - O - CH_3$

(iii) $CH_3\ \overset{+}{C}H \cdot CO \cdot CH_3$

A. (i) < (ii) < (iii)

B. (i) >> (ii) >> (iii)

C. (iii) >> (i) >> (ii)

D. (ii) >> (iii) >> (i)

19. Successive alkanes differ by :

A. CH_2

B. CH

C. CH_3

D. C_2H_4

20. The order of activity of the various o and p-director is :

A. $O^- > -OH > -OCOCH_3 > -COCH_3$

B. $OH > -O^- > -OCOCH_3 > -COCH_3$

C. $OH > -O^- > -COCH_3 > -OCOCH_3$

D. $-O^- > -COCH_3 > -OCOCH_3 > -OH$

21. Number of π electrons present in naphthalene is

A. 4

B. 6

C. 10

D. 14

22. Which of the following represents the given mode of hybridisation $sp^2 - sp^2 - sp - sp$ from left to right?

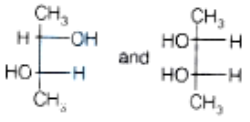
A. $H_2C = CH - C \equiv N$

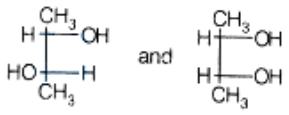
B. $CH \equiv C - C \equiv CH$

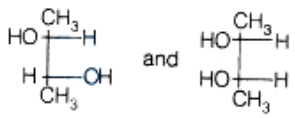
C. $H_2C = C = C = CH_2$

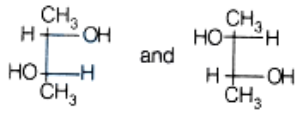
D. $CH_2 \approx CH_2$

23. Which of the following pairs of compounds are enantiomers?

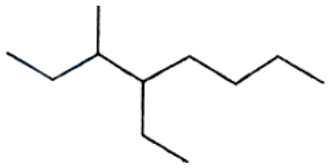
A. 

B. 

C. 

D.  and

24. Name of the compound given below is :



A. 3-methyl-4-ethyloctane

B. 2, 3-diethylheptane

C. 5-ethyl-6-methyloct1ane

D. 4-ethyl-3-methyloctane

25. Which isomeric form of benzene hexachloride is used as insecticide?

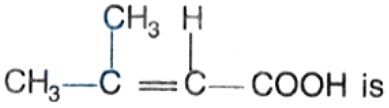
A. α-form

B. β-form

C. γ-form

D. δ-form

26. The IUPAC name for the formula



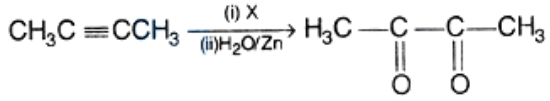
A. 2-methyl 2-butanoic acid

B. 3-methyl-3-butanoic acid

C. 3-methyl-2-butanoic-acid

D. 2-methyl-3-butanoic acid

27.



in the above reaction X is :

A. HNO_3

B. O_2

C. O_3

D. $KMnO_4$

28. Semiconductors are derived from compounds of

A. p-block elements

B. intrinsic semiconductor

C. mixed conductor

D. extrinsic semiconductor

29. Addition of arsenic to germanium makes the latter:

A. metallic conductor

B. intrinsic semiconductor

C. mixed conductor

D. extrinsic semiconductor

30. The standard reduction potential for Fe^{2+}/Fe and Sn^{2+}/Sn electrodes are -0.44 and -0.14 volt respectively. For the cell reaction $Fe^{2+} + Sn \longrightarrow F_e + Sn^{2+}$, the standard emf is

A. +0.30 V

B. -0.58 V

C. +0.58 V

D. -0.30 V

31. The rate of a gaseous reaction is given by the expression k[A][B]. If the volume of the reaction vessel is suddenly reduced to 1/4 th of the initial volume, the reaction rate relating to original rate will be :

A. 1/10

B. 1/8

C. 8

D. 16

32. Which of the following is neutral refractorie material?

A. SiO_2

B. MgO

C. CaO

D. SiC

33. When the more electropositive metal displace less electropositive metals form their salt solution this process is called:

A. auto reduction

B. electro reduction

C. hydrometallurgy

D. none of these

34. Pure cold conc. HNO_3 makes iron passive, as the surface is covered with protective layer of

A. Fe_2O_3

B. FeO

C. Fe_3O_4

D. $Fe(NO_3)_3$

35. Ozone when react with potassium iodide solution liberates certain product, which turns starch paper blue. The liberated substance is :

A. oxygen

B. iodine

C. hydrogen iodide

D. potassium hydroxide

36. Red hot iron absorbs SO_2 giving the product.

A. Fes + O_2

B. FeO + FeS

C. Fe_2O_3 + FeS

D. $FeO + S$

37. In group 15 elements which element show a fractional oxidation state?

A. N

B. As

C. Sb

D. Bi

38. What is the characteristic valence shell configuration of coinage metals ?

A. ns^2np^6

B. $(n-1)d^2ns^2$

C. nd^9ns^1

D. $(n-1)d^{10}ns^1$

39. 3.92 g of ferrous ammonium sulphate crystals are dissolved in 100 mL of water, 20 mL of this solution requires 18 mL of $KMnO_4$ during titrating for complete oxidation. the weight of $KMnO_4$ present in one liter of solution is :

A. 3.476 g

B. 12.38 g

C. 34.76 g

D. 1.238 g

40. Which one of the following is most reactive towards nucleophilic substitution reaction?

A. $CH_2 = CH - Cl$

B. C_6H_5Cl

C. $CH_3CH = CH - Cl$

D. $ClCH_2 - CH = CH_2$

41. A set of compound in which the reactivity of halogen atom in the ascending order is:

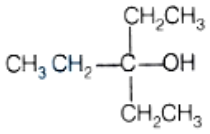
A. chlorobenzene, vinyl chloride, chloroethane

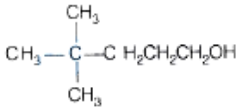
B. chloroethane, chlorobenzene, vinyl chloride

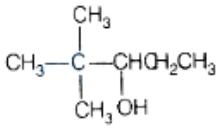
C. vinylchloride, chlorobenzene, chloroethane

D. vinylchloride; chloroethane, chlorobenzene.

42. The structure for neo-heptyl alcohol is :

A. 

B. 

C. 

D. $n-C_7H_{15}OH$

43. Cyclohexanol is dehydrated to cyclohexane on heating with conc- H_2SO_4. If the yield of this reaction is 75% how much cyclohexene will be obtained from 100 g of cyclohexanol?

A. 61.5 g

B. 75.0 g

C. 20.0 g

D. 41.0 g

44. A sample of 4.12 mg of unknown alcohol is added to CH_3MgBr then 1.56 mL of methane at STP was liberated. The alcohol is :

A. C_2H_5OH

B. CH_3OH

C. C_3H_7OH

D. C_4H_9OH

45. An ester (A) with molecular formula $C_9H_{10}O_2$ was treated with excess of CH_3MgBr and the compound so formed was treated with conc. H_2SO_4 to form olefin (B). Ozonolysis of B gav ketone with formula C_8H_8O. Which shows positive iodoform test the structure of A is :

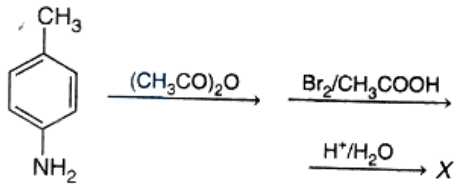
A. $CH_3CH_2COC_6CH_5$

B. $C_6H_5COOC_2H_5$

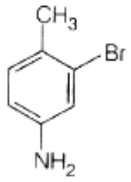
C. $C_6H_5COOC_6H_5$

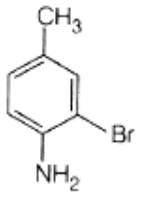
D. $CH_3COC_6H_4COCH_3$

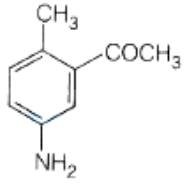
46.

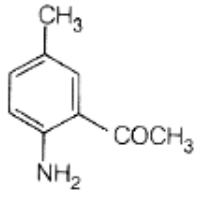


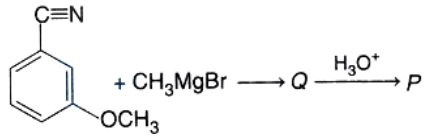
The product X is :

A. 

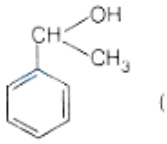
B. 

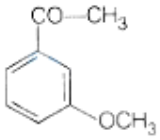
C. 

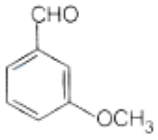
D. 

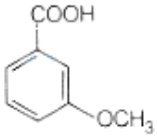
47. 

The product ' P ' in the above reaction is:

A. 

B. 

C. 

D. 

48. To become a carbohydrate a compound must contain at least :

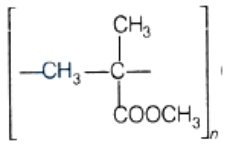
A. 2 carbons

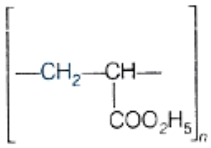
B. 3-carbons

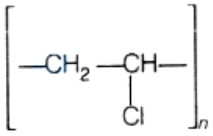
C. 4 carbons

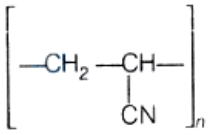
D. 6 carbons

49. Acrilan is a hard, horny and a high melting material. Its structure is :

A. 

B. 

C. 

D. 

50. The reagent NH_4Cl and aqueous NH_3 will precipitate :

A. Ca^{2+}

B. Al^{3+}

C. Mg^{2+}

D. Zn^{2+}

Mathematics

1. $\frac{5^{\frac{3}{2}}-2^{\frac{3}{2}}}{\sqrt{5}-\sqrt{2}}+\frac{5^{\frac{3}{2}}+2^{\frac{3}{2}}}{\sqrt{5}+\sqrt{2}}$

A. 7

B. 14

C. 12

D. 8

2. If $9^{-x}=\frac{1}{27\sqrt{27}}=(81)^{-y}$, then (x,y) is equal to

A. (9/4, 9/8)

B. (3/2, 3/4)

C. (3,6)

D. (6,3)

3. If $x^{4/3}+x^{-1/3}=1$, x^5+3x^2-x is equal to

A. 0

B. 1

C. -1

D. 2

4. If $x=\frac{2\sqrt{2}-\sqrt{7}}{2\sqrt{2}+\sqrt{7}}$, then $x+x^{-1}$ is equal to

A. 28

B. 32

C. 30

D. 24

5. $\left|\frac{x}{2}-1\right|<3$ implies that x lies in the interval

A. $(-4,8)$

B. $(-3,6)$

C. $(-4,6)$

D. $(-3,8)$

6. y is the sum of three numbers, one of which is a constant, the 2nd varies as x and the 3rd varies inversely as x. The values of y at $x = 1, -1$ and 3 are respectively 6, -4 and 8. Then, y is equal to

A. $1 + x - 1/x$

B. $1 + 2x + 3/x$

C. $2 + x + 1/x$

D. $2 - x + 1/x$

7. $\log x + \log x^3 + \log x^5 + ... + \log x^{2n-1}$ is equal to

A. $2n \log x$

B. $(2n - 1)\log x$

C. $n^2 \log x$

D. $(n^2 + 1)\log x$

8. $\frac{1}{\log_{25} 10} + \frac{1}{\log_4 10} + \frac{1}{\log_{\sqrt{2}} 10} + \frac{1}{\log_{\sqrt{5}} 10}$ is equal to 11

A. $3/2$

B. 2

C. 3

D. $5/2$

9. If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in the interval

A. $[1/2,2]$

B. $[-1, 1/2]$

C. $[-1/2, 1]$

D. $[-1, 1]$

10. If $a < b$, then $a < \frac{a+b}{2} < \ldots$.

A. 2a

B. 2b

C. b

D. None of these

11. If the roots $x^2 + ax + 9 = 0$ are complex, then

A. $a < 6$

B. $a < -6$

C. $|a| < 6$

D. $|a| > 6$

12. The equation $(\cos p - 1)x^2 + \cos px + \sin p = 0$ in the variable x, has real roots. Then, p can take any value in the interval

A. $(0, \pi)$

B. $(-\pi, 0)$

C. $(-\pi/2, \pi/2)$

D. $(-\pi, \pi)$

13. If α and β are the roots of $4x^2 + 3x + 7 = 0$, then the value of $1/\alpha + 1/\beta$ is

A. $-3/4$

B. $3/7$

C. $-3/7$

D. $4/7$

14. If ω is a cube root of unity, then $(1 + \omega - \omega^2)(1 - \omega + \omega^2)$ is

A. 1

B. 0

C. 2

D. 4

15. The complex number z which satisfy the equation $\left|\frac{i+z}{i-z}\right| = 1$ lies on

A. a circle $x^2 + y^2 = 1$

B. the x-axis

C. the y-axis

D. the line $x + y = 1$

16. The number of three digit numbers which have at least two identical digit, is

A. 648

B. 729

C. 252

D. 452

17. $1 + \frac{3}{1!} + \frac{5}{2!} + \frac{7}{3!} + \ldots \infty$ is

A. 5

B. 2e

C. 3e

D. None of these

18. The sum of the even multiples of 9 between 300 and 500 is

A. 4356

B. 5336

C. 5346

D. 3456

19. The sum of the first n terms of two AP's are in the ratio $(2n + 3) : (3n - 1)$. The ratio of the 5th terms is

A. 11:6

B. 21:26

C. 13:16

D. 8:5

20. The value of $\sum_{r=1}^{\infty} \left[3 \cdot 2^{-r} - 2 \cdot 3^{1-r}\right]$ is

A. 2

B. $1/2$

C. 1

D. 0

21. The sum of the series $0.2 + 0.004 + 0.00006 + 0.0000008 + \ldots \infty$ is

A. $200/891$

B. $2000/9801$

C. $1000/9801$

D. None of these

22. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$ is equal to

A. e^{-1}

B. $\log 2 - 1$

C. 1

D. 0

23. If $\log_3 2$, $\log_3(2^x - 5)$ and $\log_3(2^x - 7/2)$ are in AP, the value of x is

A. 2

B. 3

C. 0

D. 13

24. If $C(2n, 3) : C(n,2) = 44 : 3$, then n is equal to

A. 6

B. 7

C. 5

D. 8

25. The middle term in the expansion of $(x - 1/2y)^{10}$ is

A. $-63\frac{x^5}{y^5}$

B. $-\frac{63x^5y^5}{8}$

C. $\frac{63x^5}{y^5}$

D. $\frac{63x^5}{8y^5}$

26. The term independent of x in the expansion of $\left(2x^4 - \frac{1}{x^2}\right)^{12}$ is

A. 6920

B. 7920

C. 7900

D. 3960

27. In the usual notation, $\frac{{}^nC_1}{2} + \frac{{}^nC_2}{3} + \ldots + \frac{{}^nC_n}{n+1}$ is equal to

A. $\frac{2^{n+1}-1}{n+1}$

B. $\frac{2^{n+1}-n-1}{n+1}$

C. $\frac{2^{n+1}-n}{n+1}$

D. None of these

28. The middle term of $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^6$ is

A. -20

B. -1

C. 1

D. None of these

29. The coefficient of x in the expansion of $(1 + x + x^2 + x^3)^{-3}$ is

A. 6

B. 9

C. 5

D. -3

30. The value of $\frac{C(n,2)}{(n+1)!}$ is

A. $\frac{1}{2}e + 1$

B. e + 1

C. $\frac{1}{2}e - 1$

D. e

31. If $\sin\theta = -24/25$ and θ is in the 4 th quadrant, $7\tan\theta + 25\cos\theta$ is equal to

A. 17

B. -17

C. 14

D. -14

32. $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta}$ is equal to

A. sec θ + cosec θ

B. sin θ + cos θ

C. tan θ + cot θ

D. sin θ – cos θ

33. $\cos\pi/12 + \cos 17\pi/12 + \cos 11\pi/12$ is equal to

A. 1

B. -1

C. 0

D. $\frac{1-\sqrt{3}}{2\sqrt{2}}$

34. $\cot^2(\pi/4 + \theta/2)$ is equal to

A. $\frac{1-\sin\theta}{1+\sin\theta}$

B. $\frac{1-\cos\theta}{1+\cos\theta}$

C. $\frac{1+\sin\theta}{1-\sin\theta}$

D. $\frac{2-\sin\theta}{2+\sin\theta}$

35. If A, B, C are the angles of a triangle, then cos B + cos C − cos A + 1 is equal to

A. 4sin A/2 cos B/2 cos C/2

B. −4 sin A/2 cos B/2 cos C/2

C. 4 cos A/2 sin B/2 sin C/2

D. 4 sin A/2 sin B/2 sin C/2

36. In a simple regular graph, total degree is 28. If the graph has more than one cycle in it, then the degree of each vertex is

A. 2

B. 4

C. 7

D. 14

37. If sec A – tan A + a = 0, sin A is equal to

A. $\frac{a^2-1}{a^2+1}$

B. $\frac{1-a^2}{1+a^2}$

C. $\frac{a^2+1}{a^2-1}$

D. $\frac{1+a^2}{1-a^2}$

38. The point on the line y = x equidistant from (4,0) and (5,1) is

A. (2,2)

B. (3,3)

C. (5/2, 5/2)

D. $(1/2, 1/2)$

39. If $A(-1, 2)$, $B(5, 1)$, $C(6, 5)$ are the vertices of a parallelogram ABCD. The equation to the diagonal through B is

A. $x + y + 6 = 0$

B. $x + y - 6 = 0$

C. $x - y - 4 = 0$

D. $x - 2y - 1 = 0$

40. A line cuts off on the coordinate axes positive intercepts whose sum is 4. If it passes through $(9/2, -5)$, its equation is

A. $10x + 6y = 15$

B. $2x - y = 14$

C. $4x + y = 13$

D. None of these

41. The ratio in which the segment joining $(2, 1)$ and $(0, -2)$ is divided by the line $2x - 3y + 4 = 0$ is

A. 1:2

B. 2:1

C. -1:2

D. -2:1

42. The equation to the line through the point of intersection of $x - y + 1 = 0, 3x + 2y + 4 = 0$ and perpendicular to $x - 4y = 0$ is

A. $4x + y + 5 = 0$

B. $4x + y + 3 = 0$

C. $4x + y - 5 = 0$

D. $4x + y - 3 = 0$

43. The eccentricity of the conic $9x^2 - 16y^2 = 144$ is

A. $4/5$

B. $4/3$

C. 5/4

D. $\sqrt{7}$

44. The value of $\lim_{\theta \to 0} \frac{\tan \theta}{\theta}$ is

A. 0

B. 1

C. ∞

D. None of these

45. $\lim_{x \to 0} \frac{\sin 3x - \sin x}{\sin x}$ is

A. -2

B. 2

C. 0

D. None of these

46. Pick out the wrong statement. If A and B are square matrices of the same order, then

A. $A + B = B + A$

B. $(AB)' = B'A'$

C. $A - B = I$

D. $|AB| = |A||B|$

47. If $\begin{bmatrix} a & 2 & 3 \\ b & 5 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 13 \\ 12 & 11 \end{bmatrix}$, (a, b) is

A. $(1, -2)$

B. $(-1, -4)$

C. (1,3)

D. $(1, -4)$

48. If $a \neq \pm b$ and $\begin{bmatrix} a & a^2 & 1 + a^3 \\ b & b^2 & 1 + b^3 \\ 1 & 1 & 2 \end{bmatrix} = 0$, then ab is equal to

A. -1

B. 1

C. 2

D. -2

49. . If $A+B+C=\pi$, then $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$ is equal to

A. sin A

B. sin A cos B

C. 0

D. None of these

50. If $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then A^2 is equal to

A. unit matrix

B. null matrix

C. A

D. −A

51. In the determinant $\begin{vmatrix} 3 & x & -1 \\ 2 & -1 & 4 \\ 1 & y & -3 \end{vmatrix}$, the sum of the cofactors of xx and y is

A. -24

B. 24

C. -4

D. 4

52. The value of ' a ' for which the system of equations

ax + y + z = 0, x + ay + z = 0, x + y + z = 0

possesses non-zero solutions are given by

A. 1,2

B. 1,−1

C. 1

D. None of these

53. The system of linear equations $x + y + z = 0$, $2x + y - z = 0$, $3x + 2y = 0$ has

A. no solution

B. a unique solution

C. infinitely many solutions

D. None of these

54. $\sin^{-1}\frac{1}{\sqrt{5}} + \cos^{-1}\frac{3}{\sqrt{10}}$ is equal to

A. $\pi/6$

B. $\pi/4$

C. $\pi/3$

D. $2\pi/3$

55. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, then the value of $x + y + z - xyz$ is

A. 1

B. 0

C. -1

D. 1/2

56. The function and its derivative are same for

A. sin x

B. cos x

C. log x

D. e^x

57. If $y = \sqrt{\sin x + \sqrt{\sin x + \ldots\ldots}}$, then $\frac{dy}{dx}$ is

A. $\frac{\sin x}{2y-1}$

B. $\frac{\cos x}{1-2y}$

C. $\frac{\cos x}{2y-1}$

D. 0

58. The function sin x(1 + cos x), $0 \leq x \leq \pi/2$ has maximum value, when x is equal to

A. 0

B. $\pi/2$

C. $\pi/6$

D. None of these

59. $\int_0^1 x(1-x)^{12}dx$ is equal to

A. 1/132

B. 1/156

C. 1/182

D. None of these

60. $\cdot \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x|dx$ is

A. 2

B. 0

C. $\pi/2$

D. 1

61. $\int_1^e \frac{\log x}{x}dx$ is

A. 1/2

B. 1

C. e

D. None of these

62. $\int_0^1 \sin^{-1} x dx$ is

A. $\pi/2 - 1$

B. 1

C. π

D. 0

63. The area included between the curves $y^2 = 2x$ and $x^2 = 2y$ is

A. 3/4 sq unit

B. 4/3 sq unit

C. 1/2 squnit

D. 4/3 sq unit

64. Differential equations of the family of curves $y = a \cos \mu x + b \sin \mu x$, where a and b are arbitrary constants is given by

A. $\frac{d^2y}{dx^2} + \mu y = 0$

B. $\frac{d^2y}{dx^2} - \mu^2 y = 0$

C. $\frac{d^2y}{dx^2} + \mu^2 y = 0$

D. None of these

65. The value of $|\vec{a} \times \vec{b} + \vec{b} \times \vec{a}|$ is

A. 1

B. $2|\vec{a} \times \vec{b}|$

C. 0

D. None of these

66. $\vec{\mathbf{a}} = \frac{1}{7}(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}), \vec{\mathbf{b}} = \frac{1}{7}(3\hat{\mathbf{i}} - \lambda\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$. If $\vec{\mathbf{a}}$ and $\vec{b}$ are mutually perpendicular, then value of λ is

A. 2

B. -1

C. 6

D. -6

67. The value of $\left\{(\vec{\mathbf{a}} \times \vec{\mathbf{b}})^2 + (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})^2\right\} \div a^2b^2$ is

A. 0

B. 1

C. 2

D. None of these

68. $[\vec{a} + \vec{b}\vec{b} + \vec{c}\vec{c} + \vec{a}]$ is equal to

A. $[\vec{a}\vec{b}\vec{c}]$

B. $\Sigma(\vec{a} \cdot \vec{b})\vec{c}$

C. $2[\vec{a}\vec{b}\vec{c}]$

D. $|\vec{a}||\vec{b}||\vec{c}|$

69. A parallelogram is constructed on the vectors $\vec{\mathbf{a}} = 3\vec{\alpha} - \vec{\beta}, \vec{b} = \vec{\alpha} + 3\vec{\beta}$. If $|\vec{\alpha}| = |\vec{\beta}| = 2$ and the angle between $\vec{\alpha}$ and $\vec{\beta}$ is $\frac{\pi}{3}$, then length of a diagonal of the parallelogram is

A. $4\sqrt{3}$

B. $4\sqrt{5}$

C. $4\sqrt{7}$

D. None of these

70. If α, β, γ are the angles which a half ray makes with the positive direction of the axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$ is equal to

A. 1

B. 2

C. 0

D. -1

Solution

Physics

Ans 1

Correct Option. B

Solution:

$y = A\sin[Bx + Ct + D]$

As each term inside the bracket is dimension less,

$\therefore \quad [A] = [y] = [L], \quad [B] = \frac{1}{[x]} = \left[L^{-1}\right]$

$[C] = \frac{1}{[t]} = \left[1^{-1}\right]$ and D is dimensionless.

$\therefore [ABCD] = [\mathrm{L}]\left[\mathrm{L}^{-1}\right]\left[\mathrm{T}^{-1}\right][1] = \left[\mathrm{N}^{0}\,\mathrm{L}^{0}\,\mathrm{T}^{-1}\right]$

Ans 2

Correct Option. C

Solution:

$\text{Average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{2r}{t} = \frac{2\times 40}{40}$

= 2 m/s

Ans 3

Correct Option. B

Solution:

Given, $a = \frac{dv}{dt} = 6t + 5$ Integrating it, $\int_0^v dv = \int_0^t (6t+5)dt$

$v = \frac{6t^2}{2} + 5t$

As $v = \frac{ds}{dt}$, so $ds = \left(\frac{6t^2}{2} + 5t\right)dt$ Integrating it, $\int_0^s ds = \int_0^t \left(\frac{6t^2}{2} + 5t\right)dt$

$s = \frac{3t^3}{3} + \frac{5t^2}{2}$

When $t = 2\ \mathrm{s}, s = 3\frac{(2)^3}{3} + \frac{5(2)^2}{2}$

$= 8 + 10 = 18\ \mathrm{m}$

Ans 4

Correct Option. B

Solution:

Centripetal force $= \frac{mv^2}{R} = \frac{k}{R^2}$ (in magnitude)

$\therefore \quad \mathrm{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\frac{k}{R}$

$PE = -\int F dR = -\int \frac{-k}{R^2} dR = -\frac{k}{R}$

$\therefore$ Total energy $= \mathrm{KE} + \mathrm{PE}$

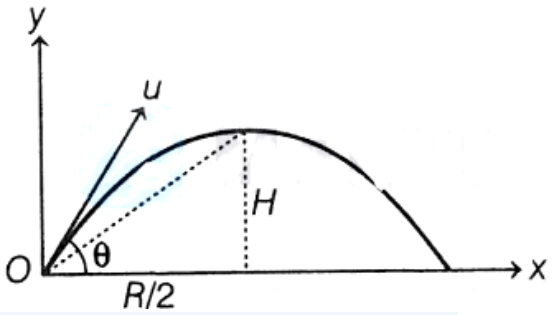
$= \frac{k}{2R} - \frac{k}{R} = -\frac{k}{2R}$

Ans 5

Correct Option. D

Solution:

Refer figure as shown, the average velocity



$= \frac{\text{displacement}}{\text{time taken}}$

$v_{av} = \frac{\sqrt{H^2 + R^2/4}}{T/2}$ ...(i)

Here, $H = \frac{u^2 \sin^2\theta}{2g}$

$R = \frac{u^2 \sin 2\theta}{g}$

Putting these values in Eq. (i), we get

$v_{av} = \frac{u}{2}\sqrt{1 + 3\cos^2\theta}$

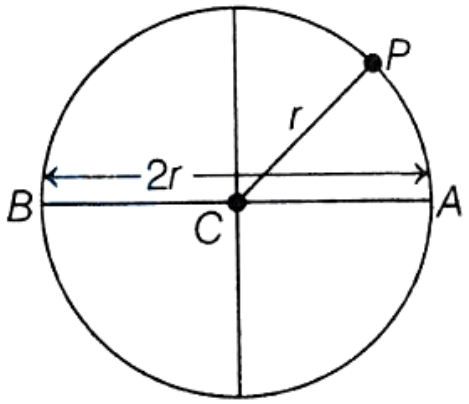
Ans 6

Correct Option. B

Solution:

Angular velocity about A,

$\omega_1 = \frac{v}{2r}$



Angular velocity about C,

$\omega_2 = \frac{v}{r}$

$\therefore \frac{\omega_1}{\omega_2}$

$= \frac{v/2r}{v/r} = 1:2$

Ans 7

Correct Option. C

Solution:

Solution:

The mass of block A = 4 kg

Force of friction between A and B = 12 N Mass of block A and B = 4 + 8 = 12 kg ie, mass is increased 3 times.

The force of friction also increases 3 times

= 12 × 3 = 36N

Hence, maximum horizontal force required = 36 N

Ans 8

Correct Option. C

Solution:

Force of friction $=720$ N

If aa is the acceleration of fireman, sliding down the pole, then

$$\begin{aligned} mg - F &= ma \\ a &= \frac{mg - F}{m} \\ &= \frac{(80 \times 10 - 720)}{80} \\ &= 1\ \mathrm{m/s^2} \end{aligned}$$

Ans 9

Correct Option. B

Solution:

Since $L_1 + nL_2$ where n is an integer, so the spring is made of $(n+1)$ equal parts in length each of length I_2

$\therefore \quad \| \quad \frac{1}{k} = \frac{(n+1)}{k'}$

or $k' = (n+1)k$

The spring of length $l_1\ (= nl_2)$ will be equivalent to n springs connected in series where spring constant

$k'' = \frac{k'}{n} = \frac{(n+1)k}{n}$

Ans 10

Correct Option. B

Solution:

Mass $M_1 = \frac{5}{8}$ kg, Mass $M_2 = \frac{5}{24}$ kg

Total maks $= M_1 + M_2 = \frac{5}{8} + \frac{5}{24} = \frac{20}{24}$ kg

Therefore, total acceleration in two masses

$$\begin{aligned} &= \frac{F}{M_1 + M_2} = \frac{5}{20/24} \\ &= \frac{5 \times 24}{20} = 6\ \mathrm{ms^{-2}} \end{aligned}$$

Ans 11

Correct Option. C

Solution:

Energy at y = total KE - work done from x to y

$= 800 - 100 \times 2 = 600$ J

Ans 12

Correct Option. B

Solution:

$t = \sqrt{x} + 3$

or $x = (t-3)^2$

Now, $v = \frac{dx}{dt} = 2(t-3)$

At $t = 0, v_1 = 2(-3) = -6$

At $t = 6, v_2 = 2(6-3) = 6$

Work done = change in KE

$= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = 0$

Ans 13

Correct Option. C

Solution:

When $\theta = 180^0$, the particle will be at diametrically opposite point. Where its velocity is opposite to the direction of motion.

The change in momentum = mv − (−mv) + 2mv (maximum).

When $\theta = 360^0$, the particle is at the initial position with momentum mv change in momentum = mv – mv = 0 (minimum).

Ans 14

Correct Option. A

Solution:

While firing a bullet from a rifle, there will be equal back thrust on light rifle as well as heavy rifle. Due to this, the light rifle will move backwards with greater speed than heavy rifle. That is why light rifle will cause more injury.

Ans 15

Correct Option. C

Solution:

When a particle undergoes normal collision with a floor or a wall, with coefficient of restitution e, the speed after collision is e times the speed before collision. Therefore, change in momentum after Ist impact is

$\mathrm{ep} - (-\mathrm{p}) = \mathrm{p}(1 + \mathrm{e})$

After the second impact, change in momentum would be $\mathrm{e}(\mathrm{ep}) - (-\mathrm{ep}) = \mathrm{ep}(1 + \mathrm{e})$ and soon.

Therefore, total change in momentum of ball

$$= p(1+e)\left[1+e+e^2+\ldots\right]$$
$$= p\left(\frac{1+e}{1-e}\right)$$

Ans 16

Correct Option. B

Solution:

Maximum height

$$H = \frac{v^2 \sin^2 45^\circ}{2g} = \frac{v^2}{2g} \times \frac{1}{2} = \frac{v^2}{4g}$$

Momentum of particle at the highest point

$$p = mv\cos 45^\circ = mv/\sqrt{2}$$

Angular momentum $= \mathrm{pH}$

$$= \frac{mv}{\sqrt{2}} \times \frac{v^2}{4g} = \frac{mv^3}{4\sqrt{2}g}$$

Ans 17

Correct Option. B

Solution:

For a solid sphere,

$I = \frac{2}{5}MR^2$

For a spherical shell,

$$I' = \frac{2}{3}MR^2$$
$$= \frac{5}{3} \times \frac{2}{5}MR^2 = \frac{5}{3}I$$

Ans 18

Correct Option. D

Solution:

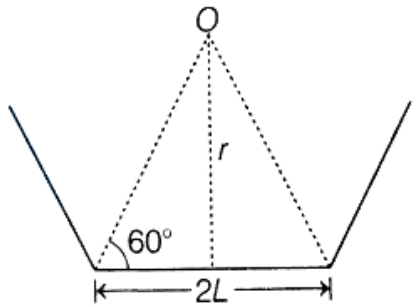
Angular momentum w.r.t. origin (L = mvr) will remain constant.

Ans 19

Correct Option. A

Solution:

Length of each side of hexagon =2L and mass of each side = m



Let O be the centre of mass of hexagon. Therefore, perpendicular distance of O from each side, r = L tan $60^0 = L\sqrt{3}$

The desired moment of inertia of hexagon about O is

$$I = 6\,[I_{\text{one side}}] = 6\left[\frac{m(2L)^2}{12} + mr^2\right]$$
$$= 6\left[\frac{mL^2}{3} + m(L\sqrt{3})^2\right]$$
$$= 20mL^2$$

Ans 20

Correct Option. B

Solution:

Gravitational force $\left(=\frac{GMm}{R^{6/2}}\right)$ provides the necessary centripetal force $\left(ie, mR\omega^2\right)$. So,

$$\frac{GMm}{R^{6/2}} = mR\omega^2 = mR\left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2 mR}{T^2}$$

or

$$T^2 = \frac{4\pi^2 R^{7/2}}{GM}$$

i.e,

$$T^2 \propto R^{7/2}$$

Ans 21

Correct Option. A

Solution:

Areal velocity $= \frac{dA}{dt} = \frac{L}{2m}$

$$= \frac{mvr}{2m} = \frac{vr}{2} = \frac{r}{2}\sqrt{\frac{GM}{r}} = \frac{1}{2}\sqrt{GMr}$$

So, $\frac{dA}{dt} \propto \sqrt{r}$

Ans 22

Correct Option. C

Solution:

$v_e = \sqrt{2gR}$ and $v_o = \sqrt{gR}$

$$\therefore \quad \frac{\mathrm{KE}_1}{\mathrm{KE}_2} = \frac{\frac{1}{2}mv_e^2}{\frac{1}{2}mv_o^2} = \frac{v_\theta^2}{v_o^2} = 2$$

Ans 23

Correct Option. B

Solution:

$$\text{Strain} = \frac{\Delta l}{l} = \frac{2\pi R - 2\pi r}{2\pi r} = \left(\frac{R-r}{r}\right)$$

$$\text{Stress} = Y \times \text{strain} = \frac{F}{A}$$

$$\therefore \quad F = Y \times A \times \text{strain} = AY \times \left(\frac{R-r}{r}\right)$$

Ans 24

Correct Option. A

Solution:

If h is the height of liquid column in cylinder and I is the length of each side of base. Then

$$F_B = (h\rho g)(I \times l) = h\rho g l^2$$
$$F_S = \left(\frac{h\rho g}{2}\right)(4I \times h) = 2h^2 \rho g l$$

$$\therefore \quad h\rho g l^2 = 2h^2 \rho g l$$

$$\text{or } h = \frac{1}{2} = \frac{10}{2} = 5 \text{ cm}$$

Ans 25

Correct Option. C

Solution:

Velocity of liquid flowing out of orifice,

$V = \sqrt{2gh}$

Average velocity of liquid flowing out of orifice till the whole vessel is emptied

$$v_{av} = \frac{v+0}{2} = \frac{1}{2}\sqrt{2gh}$$

If A is area of cross-section of the orifice, then the time taken by liquid of volume V to flow out of tank through orifice is

$$t = \frac{V}{A \times \frac{1}{2}\sqrt{2gh}}$$

In the second case, volume is V/2 and height of liquid in vessel is $h/2$. Therefore, time t is :

$$t' = \frac{V/2}{A \times \frac{1}{2}\sqrt{2gh/2}} = \frac{t}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7 \text{ min}$$

Ans 26

Correct Option. A

Solution:

$$A = 10L \times 2L = 20L^2$$
$$\frac{\Delta A}{A} \times 100 = 2\frac{\Delta L}{L} \times 100 = 2 \times 1 = 2\%$$

Ans 27

Correct Option. A

Solution:

Mass of ice melted per minute $= \frac{10}{10} = 1$ g

Quantity of heat used

Area of lens $= \pi r^2$

$= 19.625 \text{ cm}^2$

$\therefore$ Amount of heat received from sun

$= \frac{80}{19.625} = 4\text{cal/cm}^2 \text{ min}$

Latent heat of fusion $= 80$

Ans 28

Correct Option. C

Solution:

Slope of isothermal curve (AB) is smaller than slope of adiabatic curve (BC).

Ans 29

Correct Option. C

Solution:

$$P = \frac{n_1RT + n_2RT + n_3RT}{V}$$

$$= (n_1 + n_2 + n_3)\frac{RT}{V}$$

$$= \left(\frac{8}{16} + \frac{14}{28} + \frac{22}{44}\right) \times \frac{0.082 \times 300}{10}$$

$$= \left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right) \times 2.46$$

$$= \frac{3}{2} \times 2.46 = 3.69 \text{ atm} \approx 3.7 \text{ atm}$$

Ans 30

Correct Option. C

Solution:

$$K_{\text{adia}} = \gamma P = -\frac{\Delta P}{\Delta V/V}$$

$$\therefore \quad \frac{\Delta V}{V} = -\frac{\Delta P}{\gamma P}$$

Ans 31

Correct Option. D

Solution:

$$\eta = \frac{W}{Q_1}$$

$$\Rightarrow \quad w = \eta Q_1 = \frac{1}{3} \times 1000\text{cal}$$

$$= \frac{1000}{3} \times 4.2 \text{ J} = 1400 \text{ J}$$

Ans 32

Correct Option. A

Solution:

Total pressure, when the two gases are filled in one cylinder is equal to sum of the pressures of the individual gases.

Thus, total pressure = 2.5 + 1 = 3.5 atm

Ans 33

Correct Option. D

Solution:

$$y = 3\sin 314t + 4\cos 314t$$
$$= r\cos\theta \sin 314t + r\sin\theta \cos 314t$$

$$= r\sin(314t + \theta) \text{...(i)}$$

$$\therefore \quad r^2\left(\cos^2\theta + \sin^2\theta\right) = 3^2 + 4^2 = 25$$

or

$$r = 5 \text{ cm}$$

Ans 34

Correct Option. A

Solution:

$$y = A\sin(kx - \omega t)$$
$$\frac{dy}{dt} = A\cos(kx - \omega t) \times (\omega)$$
$$\left(\frac{dy}{dt}\right)_{\max} = A(1)(\omega) = A\omega$$

Ans 35

Correct Option. A

Solution:

$$T = 2 = 2\pi\sqrt{\frac{M}{k}}$$

and $2 + 1 = 2\pi\sqrt{\frac{M+4}{k}}$

or $3 = 2\pi\sqrt{\frac{M+4}{k}}$

$$\therefore \quad \frac{4}{9} = \frac{M}{M+4}$$

or $4M + 16 = 9M$

or $M = \frac{16}{5} = 3.2 \text{ kg}$

Ans 36

Correct Option. B

Solution:

Let frequency of tuning fork be n_1

$\frac{n_2}{n_1} = \frac{100}{101}$

As $n_1 - n_2 = 5$

So, $n_1 = 505$ Hz

Ans 37

Correct Option. B

Solution:

Let m = mass per unit length of rope

T = tension in the rope at a distance x from the lower end

$\therefore T = (\mathrm{mg})\mathrm{x}$ = weight of x metre of rope

As $v = \sqrt{\frac{T}{m}}$

$\therefore \quad v = \sqrt{\frac{mgx}{m}} = \sqrt{gx}$

i.e., $\quad v \propto \sqrt{x}$

Ans 38

Correct Option. D

Solution:

The path is a parabola, because initial velocity can be resolved into two rectangular components, one along $\vec{E}$ and other perpendicular to $\vec{E}$. The former decreases at a constant rate and later is unaffected. The resultant path is, therefore a parabola.

Ans 39

Correct Option. C

Solution:

$\vec{E}$ is parallel to dipole axis, at both the points P and Q, i.e. on axial line as well as equatorial line.

Ans 40

Correct Option. A

Solution:

$$As V = \frac{q}{4\pi\varepsilon_0 R}$$

$$\therefore \quad \frac{q}{4\pi\varepsilon_0} = VR \text{ and } E = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$\therefore \quad E = \frac{VR}{r^2}$$

Ans 41

Correct Option. B

Solution:

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{q_1(q-q_1)}{4\pi\varepsilon_0 r^2}$$

For maximum force between q_1 and

$$\frac{dF}{dq_1} = 0$$

$$\therefore \quad q - 2q_1 = 0$$

$$\text{or} \quad q_1 = \frac{q}{2} \quad \text{or } \frac{q_1}{q} = 0.5$$

Ans 42

Correct Option. D

Solution:

$$\frac{1}{C_s} = \frac{1}{2} + \frac{1}{6} = \frac{3+1}{6} = \frac{4}{6} = \frac{2}{3}$$

$$\therefore \quad C_S = \frac{3}{2}\mu\,\text{F}$$

In combination charge on each capacitor in same.

So, $Q = C_S V = \frac{3}{2} \times 800 = 1200\mu\text{C}$

Ans 43

Correct Option. C

Solution:

$$I = \frac{q}{T} = qv$$

$$= 2 \times 10^{-2} \times 30 = 0.60\ \text{A}$$

Ans 44

Correct Option. B

Solution:

Resistance between Q and S,

$R' = \frac{5\times10}{5+10} = \frac{50}{15} = \frac{10}{3}\Omega$

Potential difference across Q and S,

$V_Q - V_S = 2 \times \frac{10}{3} = \frac{20}{3}$ V

Current through arm QPS,

$$I_1 = \frac{20}{3 \times 5} = \frac{4}{3} \text{ A}$$

$$V_Q - V_P = \frac{4}{3} \times 2 = \frac{8}{3} \text{ V}$$

Current through arm QRS,

$$I_2 = \frac{20}{3 \times 10} = \frac{2}{3} \text{ A}$$

$$V_Q - V_R = \frac{2}{3} \times 3 = 2 \text{ V}$$

Hence,

$$V_P - V_R = (V_Q - V_R) - (V_Q - V_P)$$
$$= 2 - \frac{8}{3} = -\frac{2}{3} \approx -1 \text{ V}$$

Ans 45

Correct Option. B

Solution:

$$\text{Heat produced } = \frac{V^2 t}{4.2R} = mL$$

$$\text{or } m = \frac{V^2 t}{4.2RL} = \frac{(210)^2 \times 1)}{4.2 \times 50 \times 80} = 2.62 \text{ g}$$

Ans 46

Correct Option. C

Solution:

$q = \frac{m}{z} = \frac{1\times10^{-3}}{1.044\times10^{-8}} = \frac{10^5}{1.044}C$

Given, $\mathrm{H} = 34\mathrm{k} - \mathrm{cal} = 34 \times 10^3 \times 4.2\ \mathrm{J}$

Heat produced, $H = Vq$

$$\therefore \quad 34 \times 10^3 \times 4.2 = \frac{V \times 10^5}{1.044}$$

$$\therefore \quad V = \frac{34 \times 10^3 \times 4.2 \times 1.044}{10^5} = 1.5\ \mathrm{V}$$

Ans 47

Correct Option. C

Solution:

Maximum magnetic field due to wire carrying current is at surface and is given by

$B = \frac{\mu_0}{4\pi}\frac{2I}{R}$

or $\quad I = \frac{BR}{2\times(\mu_0/4\pi)}$

$$\therefore \quad I = \frac{5\times10^{-4}\times\left(\frac{1.0\times10^{-3}}{2}\right)}{2\times10^{-7}} = 2\ \mathrm{A}$$

Ans 48

Correct Option. D

Solution:

Perpendicular of O from PQ or QR, $a = r\sin\theta/2$ magnetic field induction at O due to current through PQ and QR is

$$B = \frac{\mu_0}{4\pi}\cdot\frac{1}{a}[\sin(90^\circ - \theta/2) + \sin 90^\circ] \times 2$$
$$= \frac{\mu_0}{2\pi}\cdot\frac{1}{r\sin\theta/2}(\cos\theta/2 + 1)$$
$$= \frac{\mu_0}{2\pi}\cdot\frac{l}{r}\frac{(1+\cos\theta/2)}{\sin\theta/2}$$

Ans 49

Correct Option. B

Solution:

The SI unit of magnetic moment is $\mathrm{A}-\mathrm{m}^2$, but the magnetic moment is given in $\mathrm{Wb}-\mathrm{m}^2$, which is the unit of $\mu_0 M$.

$$\therefore \quad \mu_0 M = 5 \times 10^{-5}$$

$$M = \frac{5 \times 10^{-5}}{\mu_0}\ \mathrm{Wb}-\mathrm{m}$$

$$= \frac{5 \times 10^{-5}}{4\pi \times 10^{-7}} = \frac{125}{\pi}$$

As

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

$$\therefore \quad I = \frac{T^2 MB}{4\pi^2} = \frac{(15)^2\left(\frac{125}{\pi}\right) \times 8\pi \times 10^{-4}}{4\pi^2}$$

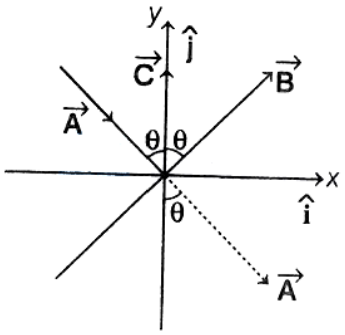
$$= 0.57\ \mathrm{kg\ m^2}$$

Ans 50

Correct Option. D

Solution:

Let $\vec{A}, \vec{B}$ and $\vec{C}$ be as shown in figure. Let θ be the angle of incidence, which is equal to the angle of reflection. Resolving these vectors in rectangular components, we have



$$\vec{\mathbf{A}} = \sin\theta\hat{\mathbf{i}} - \cos\theta\hat{\mathbf{j}}$$

$$\vec{\mathbf{B}} = \sin\theta\hat{\mathbf{i}} + \cos\theta\hat{\mathbf{j}}$$

$$\vec{\mathbf{B}} - \vec{\mathbf{A}} = 2\cos\theta\hat{\mathbf{j}}$$

or $\vec{\mathbf{B}} = \vec{\mathbf{A}} + 2\cos\theta\hat{\mathbf{j}}$

Now, $\vec{\mathbf{A}} \cdot \vec{\mathbf{C}} = (1)(1)\cos(180^\circ - \theta) = -\cos\theta$

$$\therefore \quad \vec{B} = \vec{A} - 2(\vec{A}\cdot\vec{C})\hat{j}$$

or $\vec{B} = \vec{\mathbf{A}} - 2(\vec{\mathbf{A}} \cdot \vec{\mathbf{C}})\vec{\mathbf{C}}$ (as $\hat{j} = \vec{\mathbf{C}}$)

Chemistry

Ans 1

Correct Option. D

Solution:

The volumes of the reacting gases and those of gaseous products bear the simple ratio. This is called the law of gaseous volumes or Gay-Lussac's law.

Ans 2

Correct Option. B

Solution:

$Na_2CO_3 + 2HCl - 2NaCl + H_2O + CO_2$
Meq of $HCl = 1.0 \times 100$

Meq of $Na_2CO_3 = 2 \times 1.0 \times 75 = 150$

Thus, Na_2CO_3 is in excess, and being the salt of weak acid (H_2CO_3) and strong base $(NaOH)$, undergoes hydrolysis to produce basic character.

$Na_2CO_3 + 2H_2O \longrightarrow 2Na^+ + 2OH^- + H_2CO_3$

Ans 3

Correct Option. B

Solution:

According to aufbau principle. The order of increasing energies may be summed up as below. 1s, 2s, 2p, 3s, 3p, 4s, 3d, 4p.5s, 4d, 5p, 6s, 4f, 5d, 6p, 7s, 5f, 6d and 7p

So, the correct order is 5p < 6s < 4f < 5d

Ans 4

Correct Option. C

Solution:

When $n = 3, l = 0, 1, 2$ and

For

$l = 0, m = 0$
$l = 1, m = -1, 0, +1$
$l = 2, m = -2, -1, 0, +1, +2$

So, for the value $l = 0, m$ cannot be -1

Ans 5

Correct Option. C

Solution:

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 10^{6}} = 0.727 \times 10^{-9} = 0.727 \text{ nm}$$

Ans 6

Correct Option. A

Solution:

For a given n value possible l values are 0 to $(n-1)$ possible m values for a given l value is

$-l, 0, +I$ and value of s for an electron $= +\frac{1}{2}$ or $-\frac{1}{2}$. So for $n = 1, I = 0$ not 1 and $m = 0$ not 1 .

Ans 7

Correct Option. C

Solution:

Bohr's atom model is applicable only to one electron splcies, He^{2+} does not contain any electron all other species have one electron each.

Ans 8

Correct Option. C

Solution:

Prout's gave unitary theory for the development of periodic table. He suggested that the atoms of all the element are made up of hydrogen atom.

Newlands gave law of octaves: According to him, "If elefents are arranged in the increasing order of their atomic weights, the eighth element starting from the given

one is the repetition of the first like the eighth node in an octave of music. Lother Meyer's plotted the atomic volumes (i.e., at. wt/density) of various elements against their atomic weight and pointed out that the similar element occupy similar position on the curve.

Rutherford is associated with structure of atom. He bombarded high speed α-particles on nitrogen and proved that nuclei of all atoms have protons.

Ans 9

Correct Option. A

Solution:

The required reaction is

$$\mathrm{C(s)} + \frac{1}{2}\mathrm{O_2}(g) \longrightarrow \mathrm{CO}(g)$$

$$\mathrm{CO_2}(g) \longrightarrow \mathrm{CO}(g) + \frac{1}{2}\mathrm{O_2}(g)$$

$\Delta H = \frac{+135.2}{2}\mathrm{kcal}$...(i)

$$\mathrm{C(s)} + \mathrm{O_2(\,g)} \longrightarrow \mathrm{CO_2(\,g)}$$

$\Delta H = -94\mathrm{kcal}$...(ii)

The required equation is come out by adding Eq. (i) and (ii)

So, $\Delta H_{f(\mathrm{CO})} = -94 + \frac{135.2}{2} = -26.4\mathrm{kcal}$

Ans 10

Correct Option. B

Solution:

$$\Delta G_1 = \Delta H - T\Delta S \text{ or } T = \frac{\Delta H - \Delta G}{\Delta S}$$

$$= \frac{[145.6 - (-5.2)] \times 10^3}{216} = 698.1\ \mathrm{K}$$

Ans 11

Correct Option. B

Solution:

$$\mathrm{pOH} = \mathrm{p}K_b + \log\frac{[\,\mathrm{salt}\,]}{[\,\mathrm{base}\,]} = pK_b + 0$$

$$\mathrm{pK_b} = \mathrm{pOH} = 14 - \mathrm{pH} = 14 - 9.25 = 4.75$$

Ans 12

Correct Option. D

Solution:

The b.p. and viscosity of D_2O is higher than water. Solubility of NaCl in it is less than H_2O because reaction velocity is slightly less in D_2O due to the that deuterium bond is strong than protium bond.

Ans 13

Correct Option. A

Solution:

In ice each O-atom is bonded with 4 H -atoms in a tetrahedral fashion. Two bonds are normal covalent bonds, while two are H-bonds (weaker and larger in length) and hence give rise to loose cage structure.

Ans 14

Correct Option. C

Solution:

Ice floats on water because its density is less than that of water which in turn is due to empty space in its crystal structure. Crystal structure of ice is regular hexagon with empty space at the centre.

Ans 15

Correct Option. C

Solution:

Na_2CO_3 is manufactured by Solvay process.
Lowing process is for the manufacture of NaOH Leblanc process is for the manufacture of K_2CO_3 Haber's process is for the manufacture of NH_3

Ans 16

Correct Option. A

Solution:

NaOH when react with sand SiO_2, it forms sod. silicate which is generally called glass.

$$2NaOH + SiO_2 \longrightarrow \underset{\text{Sod. silicate (glass)}}{Na_2SiO_3} + H_2O$$

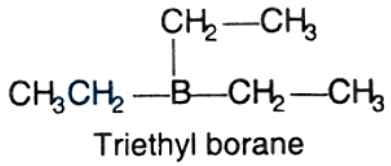
Ans 17

Correct Option. C

Solution:

Diborane readily adds at 0^0C to 25^0C to the olefin or acetylenic compound to form trialkyl boranes. This reaction is known as hydro-boration.

$$\underset{\text{Ethylene}}{6CH_2 = CH_2} + B_2H_6 \xrightarrow{\text{ether}}$$



Triethyl borane

Ans 18

Correct Option. B

Solution:

Presence of -/ group in a carbocation decreases delocalisation of the positive charge and hence decreases stability of the carbocation and vice versa. Thus the carbocation (iii) having $-COCH_3$ grouping is the least stable, followed by (ii) having $-OCH_3$ grouping, followed by (i) having only alkyl group.

Ans 19

Correct Option. A

Solution:

Alkanes have a general formula C_nH_{2n+2}

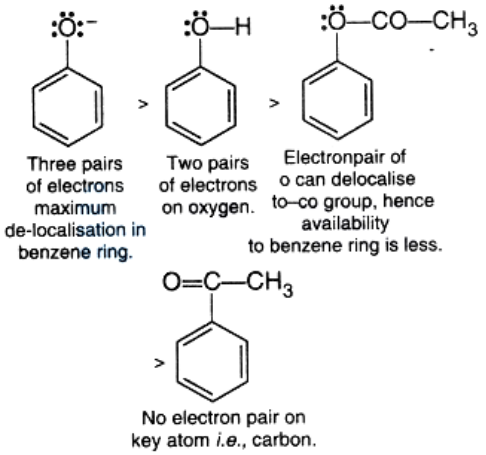
So, when

$n = 1, C_1H_{2\times1+2} = CH_4$
$n = 2, C_2H_{2\times2} + 2 = C_2H_6$
$n = 3, C_3H_{2\times3+2} = C_3H_8$
So, the all have a difference of CH_2.

Ans 20

Correct Option. A

Solution:

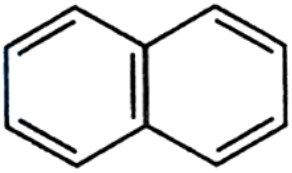


Ans 21

Correct Option. C

Solution:

Naphthalene has a structure,

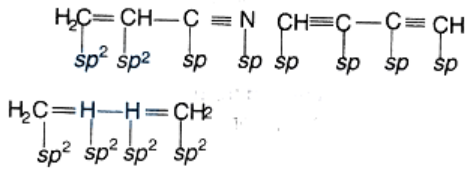


It has 5π bonds and thus the number of ππ electrons is $5 \times 2 = 10$

Ans 22

Correct Option. A

Solution:

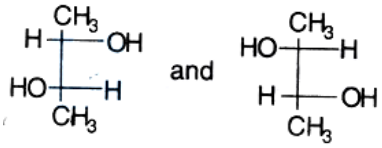


Ans 23

Correct Option. D

Solution:

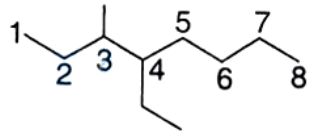
A pair of molecule which cannot be super imposed on its mirror image (i.e., asymmetrical) related to each other as an object to its mirror image are knowns as enantiomers, So,



Ans 24

Correct Option. D

Solution:



4-ethyl 3-methyloctane

Ans 25

Correct Option. C

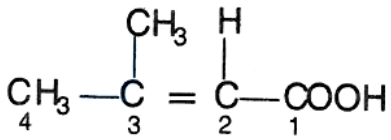
Solution:

Benzene-hexachloride exists in eight isomeric forms namely α,β,γ and δ etc. Out of which γ-isomer is most active and used as insecticide under the name of gammexene.

Ans 26

Correct Option. C

Solution:



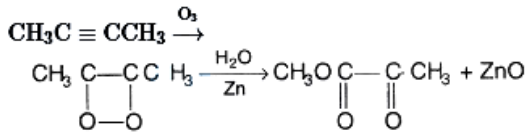
3-methyl-2-butanoic acid.

Ans 27

Correct Option. C

Solution:

This reaction is ozonolysis. O_3 is added to triple bond and then on hydrolysis form aldehyde or ketones



Ans 28

Correct Option. A

Solution:

Semiconductor are derived from compounds of p-block elements.

Ans 29

Correct Option. D

Solution:

Since, As is added from outside, it is an extrinsic semiconductor.

Ans 30

Correct Option. D

Solution:

For the cell reaction, Fe acts as cathode and Sn as anode

Hence,

$$\begin{aligned} E^{\circ}_{\text{cell}} &= E^{\circ}_{\text{cathode}} - E^{\circ}_{\text{anode}} \\ &= -0.44 - (-0.14) = -0.30\ \text{V} \end{aligned}$$

The negative emf suggests that the reaction goes spontaneously in reversed direction.

Ans 31

Correct Option. D

Solution:

On reducing the volume 1/4 th of the initial volume, each concentration will be increased by 4 times.

$(\text{Rate})_1 = k[A][B]$
$(\text{Rate})_2 = k[4[\text{A}][4[\text{B}]]$

Hence, $\frac{[\text{Rate}]_2}{[\text{Rate}]} = 4 \times 4 = 16$

Ans 32

Correct Option. D

Solution:

Substances which resist high temperatures and do not become soft are called refractories material. The neutral refractories are graphite, chromite and carboren, dum (SiC). Acidic refractories are SiO_2, basic refractories are CaO and MgO.

Ans 33

Correct Option. C

Solution:

When more electropositive metal displace less electropositive metals from their salt solution, this is reduction by precipitation and the process is called hydrometallurgy.

Ans 34

Correct Option. C

Solution:

Cold conc. HNO_3 makes it passive in which state it fails to exhibit the properties of normal iron. Passivity is said to be due to the deposit of a thin layer of iron oxide (Fe_3O_4) on the surface.

Ans 35

Correct Option. B

Solution:

When potassium iodide react with ozone it oxidise Kl and liberate I_2 it turns starch paper blue.

Ans 36

Correct Option. B

Solution:

Red hot iron absorbs SO_2 and give FeO and FeS.

$Fe + SO_2 \longrightarrow FeO + FeS$

Ans 37

Correct Option. A

Solution:

In group number 15 elements shows -III and III oxidation state (by gain or loss of three p electrons). These shows V oxidation state due to loss of $np^3 + ns^2$ electrons. But nitrogen is unique in having a large number of oxidation states including a fractional one (i.e., $-1/3$ in N_3H)

Ans 38

Correct Option. D

Solution:

Coinage metals are Cu,Ag and Au . They are transition metal and present in group II or IB. The valence shell configuration is $(n-1)d^{10}ns^1$.

Ans 39

Correct Option. A

Solution:

Normality of salt solution $= \frac{3.92}{392} \times \frac{1}{100} \times 1000$

$= 0.1\ \mathrm{N}$

20 mL of 0.1 N salt solution $\equiv 18\ \mathrm{mL}$ of $\mathrm{KMnO_4}$ solution

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$\therefore$ Normality of $\mathrm{KMnO_4}$ solution $= \frac{20 \times 0.1}{18}$

$\therefore$ Normality of $\mathrm{KMnO_4}$ solution $= \frac{20 \times 0.1}{18}$

$= \frac{1}{9}\ \mathrm{N}$

$\therefore$ Strength of $\mathrm{KMnO_4}$ solution $= \frac{1}{9} \times 31.6$

$= 3.5\ \mathrm{g\ L^{-1}}$

Ans 40

Correct Option. D

Solution:

Stability of corresponding carbocation

$$CH_2 = CH - \overset{+}{C}H_2 > CH_2 = \overset{+}{C}H > C_6H_5^+$$

So, $ClCH_2 - CH = CH_2$ is most reactive towards nucleophilic substitution reaction.

Ans 41

Correct Option. A

Solution:

Stability of corresponding carbocation

$$CH_3CH_2 > CH_2 = \overset{+}{C}H > C_6H_5^+.$$

So, the correct order of reactivity of halogen atom is chlorobenzene, vinyl chloride, chloroethane.

Ans 42

Correct Option. B

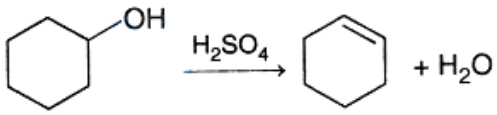
Solution:

The prefix neo means a carbon atom having four alkyl groups, position of alcohol group should be at C_1.

Ans 43

Correct Option. A

Solution:



The weight of cyclohexene formed

$$= \frac{82 \times 100}{100} \times \frac{75}{100} = 61.5 \text{ g}$$

Ans 44

Correct Option. C

Solution:

$1.56\ \text{mL}^{-1}\text{CH}_4$ at STP is liberated by 4.12 mg of alcohol. 22400 mL of CH_4 at STP is liberated by

$$= \frac{4.12}{1.56} \times 22400\text{mg of } ROH$$
$$= 61800\text{mg of } ROH = 61.8\text{ g of ROH}$$

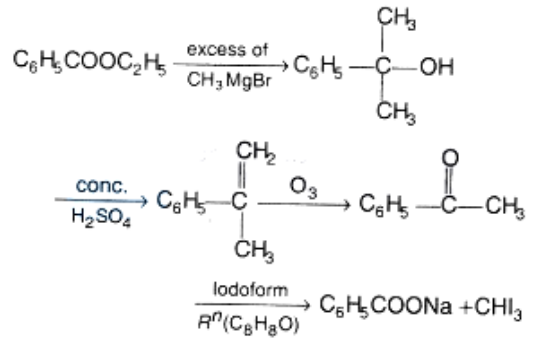
Thus, the molecular wt. of $ROH = 61.8$. This coincides with the molecular weight of $\text{C}_3\text{H}_7\text{OH} = 60$.

Ans 45

Correct Option. B

Solution:

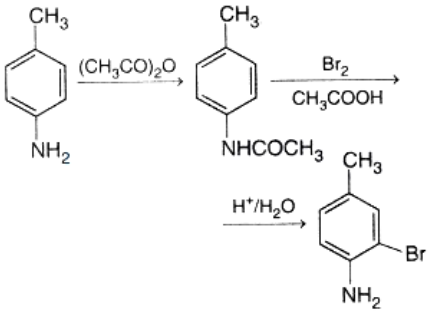
Only $C_6H_5COOC_2H_5$ corresponds with the molecular formula of the ester A. Further it explains all the given reactions.



Ans 46

Correct Option. B

Solution:

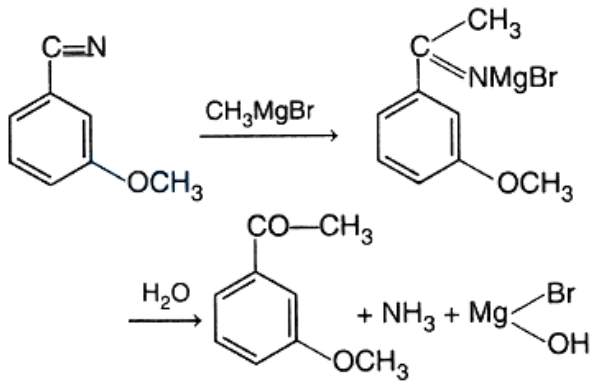


Ans 47

Correct Option. B

Solution:

Cyanide with Grignard reagent forms ketone.



Ans 48

Correct Option. B

Solution:

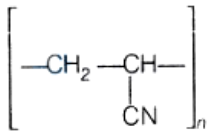
Carbohydrates are optically active polyhydroxy aldehyde ketones it must contain at least three carbon atom. Hydroxyaldehyde with, 2 carbon atom i.e., $CH_2OH \cdot CHO$ is not carbohydrate as it is optically inactive.

Ans 49

Correct Option. D

Solution:

Acrilan is a cyanide compound. It has a structure



Ans 50

Correct Option. B

Solution:

NH_4Cl and aqueous $NH_3(NH_4OH)$ precipitates basic radicals of 3rd group of qualitative analysis hydroxides of this group have very low value of solubility

product. So, this reagent will precipitate Al^{3+} because the radicals present in 3rd group are Al^{3+}, Cr^{3+}, and Fe^{3+}.

Mathematics

Ans 1

Correct Option. B

Solution:

Now, $$\begin{aligned}&\frac{5^{\frac{3}{2}}-2^{\frac{3}{2}}}{\sqrt{5}-\sqrt{2}}+\frac{5^{\frac{3}{2}}+2^{\frac{3}{2}}}{\sqrt{5}+\sqrt{2}}\\ =&\frac{\sqrt{5}^3-\sqrt{2}^3}{\sqrt{5}-\sqrt{2}}+\frac{\sqrt{5}^3+\sqrt{2}^3}{\sqrt{5}+\sqrt{2}}\\ =&(5+2+\sqrt{10})+(5+2-\sqrt{10})\\ =&7+7=14\end{aligned}$$

Ans 2

Correct Option. A

Solution:

Given, $9^{-x}=\frac{1}{27\sqrt{27}}=(81)^{-y}$

$$\begin{aligned}\Rightarrow\quad &(3)^{-2x}=(3)^{-3}(3)^{-\frac{3}{2}}=(3)^{-4y}\\ \Rightarrow\quad &-2x=-\frac{9}{2}\text{ and }-4y=-\frac{9}{2}\\ \Rightarrow\quad &x=\frac{9}{4}\text{ and }y=\frac{9}{8}\end{aligned}$$

Ans 3

Correct Option. C

Solution:

$x^{4/3}+x^{-1/3}=1\Rightarrow x^{4/3}\cdot x^{1/3}+1=x^{1/3}$

$$\Rightarrow\quad x^{5/3}-x^{1/3}=-1$$

On cubing both sides, we get

$$\begin{aligned}&x^5-x-3x^2\left(x^{5/3}-x^{1/3}\right)=-1\\ \Rightarrow\quad &x^5-x+3x^2=-1\end{aligned}$$

Ans 4

Correct Option. C

Solution:

Given, $x = \frac{2\sqrt{2}-\sqrt{7}}{2\sqrt{2}+\sqrt{7}}$

$$\therefore \quad x + \frac{1}{x} = \frac{2\sqrt{2}-\sqrt{7}}{2\sqrt{2}+\sqrt{7}} + \frac{2\sqrt{2}+\sqrt{7}}{2\sqrt{2}-\sqrt{7}}$$
$$= \frac{8+7-4\sqrt{14}+8+7+4\sqrt{14}}{8-7} = 30$$

Ans 5

Correct Option. A

Solution:

$$\left|\frac{x}{2}-1\right| < 3 \Rightarrow -3 < \frac{x}{2} - 1 < 3$$
$$-3+1 < \frac{x}{2} < 3+1$$

$$\Rightarrow \quad -2 < \frac{x}{2} < 4$$
$$\Rightarrow \quad -4 < x < 8$$
$$\therefore \quad x \in (-4, 8)$$

Ans 6

Correct Option. B

Solution:

$y = a + kx + p/x$

$$\therefore 6 = a + k + p$$
$$-4 = a - k - p$$
$$\text{and } 8 = a + 3k + p/3$$
$$\Rightarrow 6 - 1 = k + p \Rightarrow 5 = k + p$$

On solving these equations, we get

$a = 1, k = 2, p = 3$

$\therefore$ Required equation is $y = 1 + 2x + \frac{3}{x}$

Ans 7

Correct Option. C

Solution:

$$\log x + \log x^3 + \log x^5 + \ldots + \log x^{2n-1}$$

$$= \log\left(x \cdot x^3 \cdot x^5 \ldots x^{2n-1}\right)$$
$$= \log\left(x^{1+3+5+\ldots+2n-1}\right)$$
$$= \log x^{\frac{n}{2}[2+(n-1)2]} = \log x^{n^2} = n^2 \log x$$

Ans 8

Correct Option. D

Solution:

$$\frac{1}{\log_{25} 10}+\frac{1}{\log_{4} 10}+\frac{1}{\log_{\sqrt{2}} 10}+\frac{1}{\log_{\sqrt{5}} 10}$$
$$=\frac{\log 25}{\log 10}+\frac{\log 4}{\log 10}+\frac{\log \sqrt{2}}{\log 10}+\frac{\log \sqrt{5}}{\log 10}$$
$$=\frac{2\log 5+\frac{1}{2}\log 5}{\log 10}+\frac{2\log 2+\frac{1}{2}\log 2}{\log 10}$$
$$=\frac{5}{2}\left(\frac{\log 5+\log 2}{\log 10}\right)=\frac{5}{2}\frac{\log 10}{\log 10}=\frac{5}{2}$$

Ans 9

Correct Option. C

Solution:

We know, $(a+b+c)^2 \geq 0$

$\Rightarrow a^2+b^2+c^2+2ab+2bc+2ca \geq 0$
$\Rightarrow 1+2(ab+bc+ca) \geq 0$

$\Rightarrow \quad (ab+bc+ca) \geq \frac{-1}{2}$...(i)

Also, $\quad a^2+b^2+c^2=1$

$\Rightarrow a^2+b^2+c^2-(ab+bc+ca)$
$=\frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right] \geq 0$
$\Rightarrow \quad 1-(ab+bc+ca) \geq 0$

$\Rightarrow \quad ab+bc+ca \leq 1$...(ii)

Hence, from Eqs. (i) and (ii),

$-\frac{1}{2} \leq ab+bc+ca \leq 1$

Ans 10

Correct Option. C

Solution:

$a<b \Rightarrow \frac{a+b}{2}<\frac{b+b}{2}$

$\Rightarrow \quad \frac{a+b}{2}<b$

Ans 11

Correct Option. C

Solution:

Since, roots are complex.

$$\begin{aligned} &\therefore \quad a^2 - 36 < 0 \quad (\because \text{ discriminant } < 0) \\ &\Rightarrow \qquad a^2 < 36 \\ &\Rightarrow \qquad |a| < 6 \end{aligned}$$

Ans 12

Correct Option. A

Solution:

Since, the given equation has real roots.

∴ Discriminant,

$$\begin{aligned} &D = \cos^2 p - 4(\cos p - 1)\sin p \geq 0 \\ \Rightarrow \quad &D = (\cos p - 2\sin p)^2 + 4\sin p(1 - \sin p) \geq 0 \\ \Rightarrow \quad &0 < p < \pi \end{aligned}$$

Ans 13

Correct Option. C

Solution:

Since, α and β are the roots of the equation

$$4x^2 + 3x + 7 = 0$$

$$\therefore \quad \alpha + \beta = -3/4, \alpha\beta = 7/4$$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$$= -\frac{3 \cdot 4}{4 \cdot 7} = -\frac{3}{7}$$

Ans 14

Correct Option. D

Solution:

If ω is a cube root of unity, then

$1+\omega+\omega^2=0$ and $\omega^3=1$.

$$\therefore (1+\omega-\omega^2)(1-\omega+\omega^2)$$
$$=(-\omega^2-\omega^2)(-\omega-\omega)$$
$$=2\omega^2\cdot 2\omega=4$$

Ans 15

Correct Option. B

Solution:

Let $z=x+iy$

$$\therefore \quad i+z=x+i(1+y)$$

and $\quad i-z=-x+i(1-y)$

$$\because \quad \left|\frac{i+z}{i-z}\right|=1$$
$$\Rightarrow |i+z|=|i-z|$$
$$\Rightarrow x^2+(1+y)^2=x^2+(1-y)^2$$
$$\Rightarrow x^2+y^2+2y+1=x^2+y^2+1-2y$$
$$\Rightarrow 4y=0$$
$$\Rightarrow y=0$$

ie, x-axis.

Ans 16

Correct Option. C

Solution:

Number of three digits number when two 1's occur = 9 + 8 + 9 = 26.

Similarly, for two 2's ... 9's

We have, 26 × 9 numbers = 234 numbers

Number of numbers when two zero's occur = 9

Number of numbers when all three digits are same = 9

∴ Total three digit numbers having at least two identical digits =234 + 9 + 9 = 252

Ans 17

Correct Option. C

Solution:

Let T_n be the nth term of a given series:

$$\therefore \quad T_n = \frac{(2n-1)}{(n-1)!}; n = 1, 2, 3, \ldots, \infty$$

$$\Rightarrow \quad T_n = \frac{2n}{(n-1)!} - \frac{1}{(n-1)!}$$
$$= 2\left[\frac{(n-1)}{(n-1)!} + \frac{1}{(n-1)!}\right] - \frac{1}{(n-1)!}$$
$$= \frac{2}{(n-2)!} + \frac{1}{(n-1)!}$$
$$\therefore \sum_{n=1}^{\infty} T_n = \sum_{n=1}^{\infty} \frac{2}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$$
$$= 2e + e = 3e$$

Ans 18

Correct Option. A

Solution:

Even multiples of 9 are of the form $18n$.

$18n$ for $n = 1, \ldots, 27$ lies between 1 to 500

and $18n$ for $n = 1, \ldots, 16$ lies between 1 to 300 .

$$\therefore \text{ Required sum } = 18\left[\sum_{n=1}^{27} n - \sum_{n=1}^{16} n\right]$$
$$= 18\left[\frac{27 \times 28}{2} - \frac{16 \times 17}{2}\right] = 4356$$

Ans 19

Correct Option. B

Solution:

Let $\quad S_{m_1} = \frac{n}{2}[2a_1 + (n-1)d_1]$

and $\quad S_{n_2} = \frac{n}{2}[2a_2 + (n-1)d_2]$

$$\Rightarrow \quad \frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \frac{2n+3}{3n-1}$$

(given)

For 5th term, replace n by $2 \times 5 - 1 = 9$

$$\Rightarrow \quad \frac{2a_1 + 8d_1}{2a_2 + 8d_2} = \frac{18+3}{27-1}$$
$$\Rightarrow \quad \frac{a_1 + 4d_1}{a_2 + 4d_2} = \frac{21}{26}$$

Ans 20

Correct Option. D

Solution:

$$\sum_{r=1}^{\infty}\left[3\cdot 2^{-r}-2\cdot 3^{1-r}\right]=3\sum_{r=1}^{\infty}\frac{1}{2^r}-2\sum_{r=1}^{n}3^{1-r}$$

$$=3\left[\frac{1}{2}+\frac{1}{2^2}+\ldots\infty\right]-2\left[1+\frac{1}{3}+\frac{1}{3^2}+\ldots\infty\right]$$

$$=\frac{3}{2}\left[1+\frac{1}{2}+\frac{1}{2^2}+\ldots\right]-2\left[1+\frac{1}{3}+\frac{1}{3^2}+\ldots\infty\right]$$

$$=\frac{3}{2}\left(\frac{1}{1-\frac{1}{2}}\right)-2\left(\frac{1}{1-\frac{1}{3}}\right)=\frac{3}{2}\times 2-\frac{2\times 3}{2}=0$$

Ans 21

Correct Option. B

Solution:

$$\text{Sum}=\frac{2}{10}+\frac{4}{10^3}+\frac{6}{10^5}+\frac{8}{10^7}+\ldots\infty$$

It is an arithmetic-geometric series.

$$\therefore\quad S-\frac{S}{10^2}=\frac{2}{10}+\frac{2}{10^3}+\frac{2}{10^5}+\ldots\infty$$

$$\Rightarrow\quad \frac{99S}{100}=\frac{20}{99}\Rightarrow S=\frac{2000}{9801}$$

Ans 22

Correct Option. B

Solution:

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n+1}=-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots\infty$$

$$=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots\infty-1$$

$$=\log 2-1$$

Ans 23

Correct Option. B

Solution:

$\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in AP.

$\Rightarrow \log_3(2^x - 5) = \frac{\log_3(2^x - \frac{7}{2}) + \log_3 2}{2}$

$\Rightarrow \quad (2^x - 5)^2 = 2 \cdot \left(2^x - \frac{7}{2}\right)$

$\Rightarrow \quad (2^x)^2 + 25 - 10 \cdot 2^x = 2 \cdot 2^x - 7$

$\Rightarrow (2^x)^2 - 12 \cdot 2^x + 25 + 7 = 0$

$\Rightarrow \quad (2^x)^2 - 12 \cdot 2^x + 32 = 0$

$\Rightarrow \quad 2^x = 8$ or $2^x = 4$

$\Rightarrow x = 3$ or 2

$\Rightarrow x = 3 (\because x = 2$ does not satisfy the given series $)$

Ans 24

Correct Option. A

Solution:

Given, ${}^{2n}C_3 : {}^nC_2 = \frac{44}{3}$

$\Rightarrow \quad \frac{2n!}{(2n-3)!3!} \times \frac{2!(n-2)!}{n!} = \frac{44}{3}$

$\Rightarrow \frac{2n(2n-1)(2n-2)}{3} \times \frac{1}{n(n-1)} = \frac{44}{3}$

$\Rightarrow \quad \frac{2}{3} \times 2(2n-1) = \frac{44}{3}$

$\Rightarrow \quad 2n - 1 = 11 \Rightarrow 2n = 12 \Rightarrow n = 6$

Ans 25

Correct Option. B

Solution:

Middle term $= T_6 = T_{5+1}$

$= {}^{10}C_5 (x)^5 \left(-\frac{1}{2}y\right)^5 = \frac{10!}{5! \times 5!} \frac{(x)^5}{(-2)^5} y^5$

$= -\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 2 \times 3 \times 2^5} x^5 y^5$

$= -\frac{63}{8} x^5 y^5$

Ans 26

Correct Option. B

Solution:

The general term is

$$T_{r+1} = {}^{12}C_r (2x^4)^r \left(-\frac{1}{x^2}\right)^{12-r}$$
$$= {}^{12}C_r (2)^r x^{4r} (-1)^{12-r} (x)^{-2(12-r)}$$

For term independent of x, we put

$$4r - 24 + 2r = 0 \Rightarrow 6r = 24 \Rightarrow r = 4$$
$$T_5 = {}^{12}C_4 (2)^4 (-1)^{12-4}$$
$$= \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} \times (2)^4 = 7920$$

Ans 27

Correct Option. D

Solution:

$$\frac{{}^nC_1}{2} + \frac{{}^nC_2}{3} + \ldots + \frac{{}^nC_n}{n+1} = \sum_{r=0}^{n} \frac{{}^nC_r}{r+1} - {}^nC_0$$
$$= \sum_{r=0}^{n} \frac{1}{n+1} \frac{n+1}{r+1} {}^nC_r - {}^nC_0$$
$$= \frac{1}{n+1} \sum_{r=0}^{n} {}^{n+1}C_{r+1} - {}^nC_0$$

$$= \frac{1}{n+1} \left[\left({}^{n+1}C_0 + {}^{n+1}C_1 + \ldots + {}^{n+1}C_{n+1} \right) - {}^{n+1}C_0 \right] - C_0$$
$$= \frac{1}{n+1} \left(2^{n+1} - 1\right) - 1$$
$$= \frac{2^{n+1} - n - 1 - 1}{n+1} = \frac{2^{n+1} - n - 2}{n+1}$$

Ans 28

Correct Option. A

Solution:

Middle term is $\left(\frac{n}{2} + 1\right)$ th term for n even.

$\therefore$ Middle term $= \left(\frac{6}{2} + 1\right) = 4$ th term

$$\therefore \quad T_4 = {}^6C_3 (\sqrt{x})^3 \left(-\frac{1}{\sqrt{x}}\right)^{6-3}$$

$$= \frac{6!}{3!3!} (\sqrt{x})^3 \frac{(-1)^3}{(\sqrt{x})^3}$$
$$= \frac{-6 \times 5 \times 4}{6}$$
$$= -20$$

Ans 29

Correct Option. D

Solution:

$$\left(1+x+x^2+x^3\right)^{-3}=\left[\frac{\left(x^4-1\right)}{x-1}\right]^{-3}$$

$$=\left(x^4-1\right)^{-3}(x-1)^3$$
$$=(-1)^{-3}\left(1-x^4\right)^{-3}(x-1)^3$$
$$=(-1)\left(1+3x^4+\ldots\right)\left(x^3-3x^2+3x-1\right)$$

$\therefore$ Coefficient of x in $\left(1+x+x^2+x^3\right)^{-3}$ is -3 .

Ans 30

Correct Option. C

Solution:

Now, $\frac{{}^nC_2}{(n+1)!}=\frac{\frac{n!}{(n-2)!2!}}{(\theta+1)!}$

$$=\frac{n!}{2(n+1)!(n-2)!}=\frac{n(n-1)}{2(n+1)!}$$
$$=\frac{n^2-1+1-n}{2(n+1)!}$$
$$=\frac{1}{2}\left[\frac{(n-7)}{n!}-\frac{1}{n!}+\frac{2}{(n+1)!}\right]$$
$$=\frac{1}{2}\left[\frac{1}{(n-1)!}-\frac{2}{n!}+\frac{2}{(n+1)!}\right]$$

Put $n=0,1,2,3,\ldots$, we get

$$=\frac{1}{2}[e-2(e)+2(e-1)]$$
$$=\frac{1}{2}[e-2]=\frac{e}{2}-1$$

Ans 31

Correct Option. B

Solution:

$$\text{Since, } \sin\theta = -\frac{24}{25}$$
$$\therefore \text{ Base } AB = \sqrt{(25)^2 - (24)^2} = 7$$
$$\therefore \quad \cos\theta = \frac{7}{25}$$
$$\text{and } \tan\theta = -\frac{24}{7} \quad (\because \theta \in 4 \text{ th quadrant})$$
$$\therefore 7\tan\theta + 25\cos\theta = -24 + 7 = -17$$

Ans 32

Correct Option. B

Solution:

$$\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta}$$
$$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} + \frac{\sin^2\theta}{\sin\theta - \cos\theta}$$
$$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta} = \cos\theta + \sin\theta$$

Ans 33

Correct Option. D

Solution:

$$\cos\tfrac{\pi}{12} + \cos\tfrac{17\pi}{12} + \cos\tfrac{11\pi}{12}$$
$$= \cos\frac{\pi}{12} + \cos\left(\frac{3\pi}{2} - \frac{\pi}{12}\right) + \cos\left(\pi - \frac{\pi}{12}\right)$$
$$= \cos\frac{\pi}{12} - \cos\frac{\pi}{12} - \sin\frac{\pi}{12}$$
$$= -\sin\frac{\pi}{12} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

Ans 34

Correct Option. A

Solution:

$$\cot^2\left(\tfrac{\pi}{4} + \tfrac{\theta}{2}\right)$$
$$= \left[\cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right]^2 = \left[\frac{\cot\frac{\theta}{2}\cot\frac{\pi}{4} - 1}{\cot\frac{\theta}{2} + \cot\frac{\pi}{4}}\right]^2$$
$$= \left[\frac{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}\right]^2 = \frac{1-\sin\theta}{1+\sin\theta}$$

Ans 35

Correct Option. A

Solution:

Now, $\cos B + \cos C + (1 - \cos A)$

$$= 2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right) + 2\sin^2\frac{A}{2}$$

$$= 2\cos\left(\frac{\pi}{2} - \frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right) + 2\sin^2\frac{A}{2}$$

$$= 2\sin\frac{A}{2}\left[\cos\left(\frac{B-C}{2}\right) + \sin\frac{A}{2}\right]$$

$$= 2\sin\frac{A}{2}\left[\cos\left(\frac{B-C}{2}\right) + \cos\left(\frac{B+C}{2}\right)\right]$$

$$= 2\sin\frac{A}{2} \cdot 2\cos\frac{B}{2} \cdot \cos\frac{C}{2}$$

$$= 4\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

Ans 36

Correct Option. B

Solution:

Total degree = number of vertices × degree

$= 4 \times 7$ or 2×14

If there are 14 vertices each of degree 2, then graph is a cycle.

Hence, there are 7 vertices and each of degree 4.

Ans 37

Correct Option. B

Solution:

Given, $\sec A - \tan A = -a$

$$\Rightarrow \quad \frac{1-\sin A}{\cos A} = -a$$
$$\Rightarrow \quad \frac{1-\sin A}{\sqrt{1-\sin^2 A}} = -a$$
$$\Rightarrow \quad 1+\sin^2 A - 2\sin A = a^2\left(1-\sin^2 A\right)$$
$$\Rightarrow \left(1+a^2\right)\sin^2 A - 2\sin A + \left(1-a^2\right) = 0$$
$$\Rightarrow \quad (\sin A - 1)\left[\sin A - \frac{\left(1-a^2\right)}{\left(1+a^2\right)}\right] = 0$$
$$\Rightarrow \quad \sin A = 1 \text{ or } \sin A = \frac{1-a^2}{1+a^2}$$

Ans 38

Correct Option. C

Solution:

Let the required point is (a, a).

$$\therefore (4-a)^2 + a^2 = (5-a)^2 + (1-a)^2$$

$$\Rightarrow \quad 16 + a^2 - 8a + a^2 = 25 + a^2 - 10a + 1 + a^2 - 2a$$
$$\Rightarrow \quad 16 - 25 - 1 + a^2 + a^2 - a^2 - a^2 - 8a + 10a + 2a = 0$$
$$\Rightarrow \quad 4a = 10$$
$$\Rightarrow \quad a = \tfrac{10}{4} = \tfrac{5}{2}$$

$\therefore$ Required point is $\left(\tfrac{5}{2}, \tfrac{5}{2}\right)$.

Ans 39

Correct Option. B

Solution:

Diagonals of parallelogram bisects each other.

$\Rightarrow$ Diagonal BD passes through mid point of AC.

$ie,$

$\left(\frac{5}{2}, \frac{7}{2}\right)$

$\therefore$ Equation of diagonal passing through

$(5,1)$ and $\left(\frac{5}{2}, \frac{7}{2}\right)$ is

$$y-1=\frac{\frac{7}{2}-1}{\frac{5}{2}-5}(x-5)$$

$$\Rightarrow \quad y-1=-1(x-5)$$

$$\Rightarrow \quad y+x-6=0$$

Ans 40

Correct Option. A

Solution:

Let the equation of line be $\frac{x}{a}+\frac{y}{b}=1$.

Then $a+b=4$

$$\Rightarrow \quad \begin{array}{l} \Rightarrow \\ \Rightarrow \\ \therefore \quad \frac{x}{a}+\frac{y}{4-a}=1 \\ \Rightarrow \quad x(4-a)+ya \end{array}$$

$$\Rightarrow a(4-a)$$

This passes through $\left(\frac{9}{2}, -5\right)$.

$$\therefore \quad \frac{9}{2}(4-a)-5a=a(4-a)$$

$$\Rightarrow \quad 36-9a-10a=8a-2a^2$$

$$\Rightarrow \quad 2a^2-27a+36=0$$

$$\Rightarrow \quad a=12 \text{ or } \frac{3}{2} \text{ but } a \neq 12$$

$$\therefore \quad a=\frac{3}{2} \Rightarrow b=\frac{5}{2}$$

$\therefore$ Required equation is

$$\begin{array}{l} \frac{2x}{3}+\frac{2y}{5}=1 \\ \Rightarrow \quad 10x+6y=15 \end{array}$$

Ans 41

Correct Option. C

Solution:

Suppose the segment $2x - 3y + 4 = 0$ divides line segment joining A(2,1) and B(0,−2) in the ratio k:1 at point C.

$\therefore \quad C$ is $\left(\frac{2}{k+1}, \frac{-2k+1}{k+1}\right)$

Since, C lies on $2x - 3y + 4 = 0$

$$\Rightarrow \quad \frac{4}{k+1} - 3\frac{(-2k+1)}{k+1} + 4 = 0$$
$$\Rightarrow \quad 4 + 6k - 3 + 4k + 4 = 0$$
$$\Rightarrow \quad 10k + 5 = 0$$
$$\Rightarrow \quad k = -\frac{1}{2}$$
$$\therefore \quad \text{Ratio is } -1:2.$$

Ans 42

Correct Option. A

Solution:

Point of intersection of $x - y + 1 = 0$ and $3x + 2y + 4 = 0$ is Slope of required line is -4 .

$\therefore$ Required line is $\left(y + \frac{1}{5}\right) = -4\left(x + \frac{6}{5}\right)$

$$\Rightarrow \quad 5y + 1 = -20x - 24$$
$$\Rightarrow \quad 5y + 20x = -25$$
$$\Rightarrow \quad y + 4x + 5 = 0$$

Ans 43

Correct Option. C

Solution:

Given conic is $9x^2 - 16y^2 = 144$

$$\Rightarrow \quad \frac{x^2}{16} - \frac{y^2}{9} = 1$$
$$\therefore \quad e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$$

Ans 44

Correct Option. B

Solution:

$\lim_{\theta \to 0} \frac{\tan\theta}{\theta}$. Applying L'Hospital's rule,

$$= \lim_{\theta \to 0} \frac{\sec^2\theta}{1} = 1$$

Ans 45

Correct Option. B

Solution:

$$\lim_{x \to 0} \frac{\sin 3x - \sin x}{\sin x}$$

$$= \lim_{x \to 0} \frac{2\cos 2x \sin x}{\sin x} = 2$$

Ans 46

Correct Option. C

Solution:

The only wrong statement is option(c).

∵ It is not necessary that A – B = I,∀A, B square matrix of same order.

Ans 47

Correct Option. D

Solution:

$$\begin{bmatrix} a & 2 & 3 \\ b & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} a+6-3 & 2a+8+3 \\ b+15+1 & 2b+20-1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+3 & 2a+11 \\ b+16 & 2b+19 \end{bmatrix} = \begin{bmatrix} 4 & 13 \\ 12 & 11 \end{bmatrix} \text{ (given)}$$

$$\Rightarrow \quad a+3=4 \Rightarrow a=1$$

and $b+16=12 \Rightarrow b=12-16=-4$

$$\therefore \quad (a,b)=(1,-4)$$

Ans 48

Correct Option. A

Solution:

Given that, $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ 1 & 1 & 1+1 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ 1 & 1 & 1 \end{vmatrix} + ab \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (ab+1) \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad (ab+1)\left[a\left(b^2-1\right)-a^2(b-1)+1\left(b-b^2\right)\right]=0$$

$$\Rightarrow \quad (ab+1)(b-1)[a(b+1)-a-b)]=0$$

$$\Rightarrow \quad (ab+1)(b-1)[ab+a-a-b]=0$$

$$\Rightarrow \quad b(ab+1)(b-1)(a-1)=0$$

$$\Rightarrow \quad b=0,1; a=1, ab=-1$$

Ans 49

Correct Option. C

Solution:

In a triangle ABC,

$A+B+C=\pi$,

$\Rightarrow \sin(A+B+C)=0, \cos(A+B)=-\cos C$

$$\therefore \quad \begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ -\cos C & -\tan A & 0 \end{vmatrix}$$

=0 ∴ It is the determinant of a skew symmetric matrix of odd order.

Ans 50

Correct Option. A

Solution:

Given, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a-a & b-b & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ans 51

Correct Option. C

Solution:

Given, determinant $\begin{vmatrix} 3 & x & -1 \\ 2 & -1 & 4 \\ 1 & y & -3 \end{vmatrix}$

Here, cofactor of $x = 10$ and cofactor of $y = -14$

$\therefore$ Their sum $= -4$

Ans 52

Correct Option. C

Solution:

Given, system of equations are

$ax + y + z = 0, x + ay + z = 0$
and $x + y + z = 0$ possesses non-zero solution.

$$\therefore \quad \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad a(a-1) - 1(1-1) + 1(1-a) = 0$$
$$\Rightarrow \quad a^2 - a + 0 + 1 - a = 0$$
$$\Rightarrow \quad a^2 - 2a + 1 = 0$$
$$\Rightarrow \quad (a-1)^2 = 0$$
$$\Rightarrow \quad a = 1$$

Ans 53

Correct Option. C

Solution:

Given system of equations are

$x+y+z=0, 2x+y-z=0, 3x+2y=0$

Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & 0 \end{vmatrix} = 1(0+2)-1(0+3)+1(4-3)$

$=+2-3+1=0$

Hence, it has infinitely many solutions.

Ans 54

Correct Option. B

Solution:

$\sin^{-1}\frac{1}{\sqrt{5}}+\cos^{-1}\frac{3}{\sqrt{10}}$

$$\text{Now, } \sin^{-1}\frac{1}{\sqrt{5}}=\tan^{-1}\left(\frac{1/\sqrt{5}}{\sqrt{1-\frac{1}{5}}}\right)=\tan^{-1}\frac{1}{2}$$

$$\text{and } \cos^{-1}\frac{3}{\sqrt{10}}=\tan^{-1}\left(\frac{\sqrt{1-\frac{9}{10}}}{\frac{3}{\sqrt{10}}}\right)=\tan^{-1}\frac{1}{3}$$

$$\therefore \tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}}\right)=\tan^{-1}(1)=\frac{\pi}{4}$$

Ans 55

Correct Option. B

Solution:

Given, $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$

$$\Rightarrow \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$$

$\Rightarrow x+y+z-xyz=0$

Ans 56

Correct Option. D

Solution:

Let $y=e^x$

$\frac{d}{dx}(y)=\frac{d}{dx}(e^x)\Rightarrow\frac{dy}{dx}=e^x$

Ans 57

Correct Option. C

Solution:

$$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots}}}$$

$$\Rightarrow \quad y = \sqrt{\sin x + y}$$
$$\Rightarrow \quad y^2 = \sin x + y$$

On differentiating both sides w.r.t. x, we get

$$2y\frac{dy}{dx} = \cos x + \frac{dy}{dx} \Rightarrow \frac{dy}{dx}(2y-1) = \cos x$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{\cos x}{2y-1}$$

Ans 58

Correct Option. D

Solution:

$$(\text{ Let } z = \sin x(1+\cos x)$$
$$\text{or } z = \sin x + \frac{1}{2}\sin 2x$$

On differentiating w.r.t. x, we get

$$\frac{dz}{dx} = \cos x + \cos 2x$$

For maximum or minimum, put $\frac{dz}{dx} = 0$

$$\Rightarrow \quad \cos x + \cos 2x = 0$$
$$\Rightarrow \quad 2\cos^2 x - 1 + \cos x = 0$$
$$\Rightarrow \quad (\cos x + 1)(2\cos x - 1) = 0$$
$$\Rightarrow \quad \cos x = -1, \frac{1}{2}$$
$$\Rightarrow \quad x = \frac{\pi}{3}, \pi$$

But $0 \le x \le \frac{\pi}{2}$, therefore we take only $x = \frac{\pi}{3}$.

$$\therefore \quad \frac{d^2z}{dx^2} = -\sin x - 2\sin 2x = -\text{ ve, for } x = \pi/3$$
$\therefore$ It is maximum at $x = \pi/3$.

Ans 59

Correct Option. C

Solution:

$$\int_0^1 x(1-x)^{12}dx = \left[\frac{x(1-x)^{13}}{-13} + \int_0^1 \frac{(1-x)^{13}}{13}dx\right]_0^1$$

$$= \left[\frac{x(1-x)^{13}}{-13}\right]_0^1 + \left[\frac{(1-x)^{14}}{-14\times 13}\right]_0^1$$
$$= [0-0] + \left[0 - \frac{1}{-14\times 13}\right] = \frac{1}{182}$$

Ans 60

Correct Option. A

Solution:

$$\int_{-\pi/2}^{\pi/2} |\sin x|dx = 2\int_0^{\pi/2} \sin x dx$$
$$= 2[-\cos x]_0^{\pi/2} = 2$$

Ans 61

Correct Option. A

Solution:

Let $I = \int_1^e \frac{\log x}{x}dx$

Put $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$\therefore \quad I = \int_0^1 tdt = \left[\frac{t^2}{2}\right]_0^1 = \frac{1}{2}$

Ans 62

Correct Option. A

Solution:

$$\text{Let } I = \int_0^1 \sin^{-1} x dx$$
$$= \left[x\sin^{-1} x\right]_0^1 - \int_0^1 \frac{x}{\sqrt{1-x^2}}dx$$

Put $1-x^2 = t^2 \Rightarrow -2xdx = 2tdt$

$\therefore \quad I = \left[\frac{\pi}{2} - 0\right] + \int_1^0 \frac{tdt}{t}$

$= \frac{\pi}{2} + \int_1^0 dt = \frac{\pi}{2} + [t]_1^0 = \frac{\pi}{2} - 1$

Ans 63

Correct Option. B

Solution:

The point of intersection of the curves $y^2 = 2x$ and $x^2 = 2y$ are $O(0,0), A(2,2)$.

$\therefore$ Required area $= \int_0^2 \left(\sqrt{2x} - \frac{x^2}{2}\right) dx$

$$= \left[\sqrt{2}\frac{x^{3/2}}{3/2} - \frac{x^3}{6}\right]_0^2$$
$$= \left[\frac{2}{3} \cdot 2^2 - \frac{2^3}{6}\right] = \left[\frac{8}{3} - \frac{4}{3}\right]$$
$$= \frac{4}{3} \text{ sq unit}$$

Ans 64

Correct Option. C

Solution:

Given, $y = a\cos\mu x + b\sin\mu x$...(i)

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = -a\mu\sin\mu x + b\mu\cos\mu x$$

Again, differentiating, we get

$$\frac{d^2y}{dx^2} = -a\mu^2\cos\mu x - b^2\mu\sin\mu x$$

$= -\mu^2 y$ [from Eq. (i)]

$$\Rightarrow \frac{d^2y}{dx^2} + \mu^2 y = 0$$

Ans 65

Correct Option. C

Solution:

$$|\vec{a} \times \vec{b} + \vec{b} \times \vec{a}| = |\vec{a} \times \vec{b} - \vec{a} \times \vec{b}| = 0$$

Ans 66

Correct Option. C

Solution:

Given, $\vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}), \vec{b} = \frac{1}{7}(3\hat{i} - \lambda\hat{j} + 2\hat{k})$

Since, the vectors $\vec{a}$ and $\vec{b}$ are perpendicular.

$\therefore \quad \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$

$\Rightarrow \frac{1}{7}(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}) \cdot \frac{1}{7}(3\hat{\mathbf{i}} - \lambda\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 0$

$\Rightarrow \quad \frac{1}{49}(6 - 3\lambda + 12) = 0$

$\Rightarrow \quad 3\lambda = 18 \Rightarrow \lambda = 6$

Ans 67

Correct Option. B

Solution:

$$\left\{(\vec{\mathbf{a}} \times \vec{\mathbf{b}})^2 + (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})^2\right\} \div a^2b^2$$

$$= \left\{(|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sin\theta\hat{\mathbf{n}})^2 + (|\vec{\mathbf{a}}\|\vec{\mathbf{b}}|\cos\theta)^2\right\} + a^2b^2$$

$$= a^2b^2\left(\sin^2\theta + \cos^2\theta\right) + a^2b^2 = 1$$

Ans 68

Correct Option. C

Solution:

$$[\vec{a} + \vec{b}\vec{b} = +\vec{a}$$

$$= (\vec{a} + \vec{b}) \cdot [(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})]$$

$$= (\vec{a} + \vec{b}) \cdot [(\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + 0 + \vec{c} \times \vec{a})]$$

$$= [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] + 0 + 0 + 0 + 0 + [\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,\vec{\mathbf{a}}]$$

$$= [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] + [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$$

$$= 2[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$$

Ans 69

Correct Option. C

Solution:

Given, $\vec{a} = 3\vec{\alpha} - \vec{\beta}, \vec{b} = \vec{\alpha} + 3\vec{\beta}$

and $|\vec{\alpha}| = |\vec{\beta}| = 2$

$$\begin{aligned}
\therefore |\vec{a} + \vec{b}|^2 &= |4\vec{\alpha} + 2\vec{\beta}|^2 \\
&= 16|\vec{\alpha}|^2 + 4|\vec{\beta}|^2 + 16|\vec{\alpha}||\vec{\beta}| \cos\frac{\pi}{3} \\
&= 16(4) + 4(4) + 16(2 \times 2) \times \frac{1}{2} \\
&= 64 + 16 + 32 = 112 \\
\Rightarrow |\vec{a} + \vec{b}| &= 4\sqrt{7}
\end{aligned}$$

Ans 70

Correct Option. B

Solution:

Since, a ray makes angles α, β, γ with the positive direction of the axes.

$$\begin{aligned}
\therefore \quad & \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \\
\Rightarrow \quad & 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1 \\
\Rightarrow \quad & \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2
\end{aligned}$$