DAY NINE

Gravitation

Learning & Revision for the Day

- Universal Law of Gravitation
- Acceleration due to Gravity
- Gravitational Field
- Gravitational Potential
- Gravitational Potential Energy
- Escape Velocity
- Kepler's Laws of Planetary Motion
- Satellite
- Geostationary Satellite

Universal Law of Gravitation

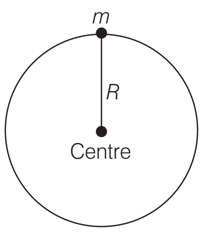
In this universe, each body attracts other body with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Let m_1 and m_2 be the masses of two bodies and r be the separation between them.

$$F = G\frac{m_1 m_2}{r^2}.$$

The proportionality constant G is called **universal gravitational constant**. In SI system, value of gravitational constant G is $6.67\times 10^{-11}\,\text{Nm}^2\,\text{kg}^{-2}$. Dimensional formula of G is $[\text{M}^{-1}\text{L}^3\text{T}^{-2}]$.

Acceleration due to Gravity



The acceleration of an object during its free fall towards the earth is called acceleration due to gravity.

If M is the mass of earth and R is the radius, the earth attracts a mass m on its surface with a force F given by

$$F = \frac{GMm}{R^2}$$

This force imparts an acceleration to the mass m which is known as acceleration due to gravity (g).

By Newton's law, we have

$$\text{Acceleration }(g) = \frac{F}{m} = \frac{\frac{GMm}{R^2}}{m} = \frac{GM}{R^2}$$

On the surface of earth, $g = \dfrac{GM}{R^2}$

Substituting the values of $G, M, R,$ we get $g = 9.81\ \text{ms}^{-2}$.

Mass of the earth $m = 6\times10^{24}$ kg and radius of the earth $R = 6.4\times10^{6}$ m.

Variation in g with Altitude and Depth with respect to Earth

The value of g is variable and can vary in same cases as mentioned below

1. **Value of acceleration due to gravity (g) at a height (h)** from the surface of the earth is given by $g' = \frac{gR^2}{(R+h)^2}$

   If $h << R$, then $g' = g\left[1 - \frac{2h}{R}\right]$

2. **Value of acceleration due to gravity (g) at a depth (d)** from the surface of the earth is given by

   $$g' = g\left(1 - \frac{d}{R}\right)$$

   At the centre of the earth $d = R$ and hence, $g' = 0$.

Variation in the Value of g Due to Rotation of the Earth

Due to rotation of the earth, the value of g decreases as the speed of rotation of the earth increases. The value of acceleration due to gravity at a latitude is

$$g'_\lambda = g - R\omega^2 \cos^2 \lambda$$

Following conclusions can be drawn from the above discussion

(a) The effect of centrifugal force due to rotation of the earth is to reduce the effective value of g.

(b) The effective value of g is not truely in vertical direction.

(c) At the equators, $\lambda = 0°$

Therefore, $g' = g - R\omega^2$ (minimum value)

(d) At the poles, $\lambda = 90°$

Therefore, $g' = g$ (maximum value)

Gravitational Field

The space surrounding a material body in which its gravitational force of attraction can be experienced is called its gravitational field.

Gravitational Field Intensity (I)

Gravitational field intensity at any point is defined as the force experienced by any test mass devided by the magnitude of test mass when placed at the desired point.

Mathematically,

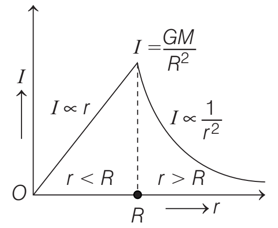
Gravitational field intensity, $\mathbf{I} = \frac{\mathbf{F}}{m_0}$

where, m_0 is a small test mass. The SI unit of gravitational intensity is N kg^{-1}.

- Gravitational intensity at a point situated at a distance r from a point mass M is given by $\mathbf{I} = \frac{GM}{r^2}$
- Gravitational field intensity due to a solid sphere (e.g. earth) of mass M and radius R at a point distant r from its centre $(r > R)$ is $\mathbf{I} = \frac{GM}{r^2}$

  and at the surface of solid sphere, $\mathbf{I} = \frac{GM}{R^2}$.

  However, for a point $r < R$, we find that $\mathbf{I} = \frac{GMr}{R^3}$



Variation of gravitational field intensity in solid sphere

- Due to a body in the form of uniform shell gravitational field intensity at a point outside the shell $(r > R)$ is given by $\mathbf{I} = \frac{GM}{r^2}$

  But at any point inside the shell, gravitational intensity is zero.
- Gravitational intensity at a point due to the combined effect of different point masses is given by the vector sum of individual intensities.

  Thus, $\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \dots$

Gravitational Potential

Gravitational potential at any point in a gravitational field is defined as the work done in bringing a unit mass from infinity to that point.

Gravitational potential, $V = \lim_{m_0 \to 0} \frac{W}{m_0}$.

Gravitational potential due to a point mass is $V = -\frac{GM}{r}$.

Gravitational potential is always negative. It is a scalar term and its SI unit is J kg^{-1}.

For Solid Sphere

- At a point outside the solid sphere, (e.g. earth), i.e.

  $$r > R, \ V = -\frac{GM}{r}.$$
- At a point on the surface, $V = -\frac{GM}{R}$.
- At a point inside the sphere, $(r < R)$.

  $$V = -\frac{GM}{2R^3}(3R^2 - r^2) = -\frac{GM}{2R}\left[3 - \frac{r^2}{R^2}\right]$$
- At the centre of solid sphere,

  $$V = -\frac{3GM}{2R} = \frac{3}{2}V_{\text{surface}}$$

For Spherical Shell

- At a point outside the shell,

$$V = -\frac{GM}{r} \text{ where, } r > R.$$

- At a point on the surface of spherical shell,

$$V = -\frac{GM}{R}$$

- At any point inside the surface of spherical shell

$$V = -\frac{GM}{R} = V_{\text{surface}}$$

Relation between Gravitational Field and Gravitational Potential

If $\mathbf{r}_1$ and $\mathbf{r}_2$ are position of two points in the gravitation field with intensity ($\mathbf{I}$), then change in gravitational potential

$$V(\mathbf{r}_2) - V(\mathbf{r}_1) = -\int_{r_1}^{r_2} \mathbf{I} \cdot d\mathbf{r}$$

$$\Rightarrow \quad dV = -\mathbf{I} \cdot d\mathbf{r}$$

$$dr = dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}}$$

and $\quad I = I_x\,\hat{\mathbf{i}} + I_y\,\hat{\mathbf{j}} + I_z\,\hat{\mathbf{k}}$, then

$$dV = -\mathbf{I}.d\mathbf{r} = -I_x dx - I_y dy - I_z dz$$

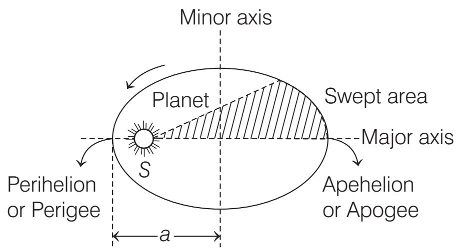
Thus, $\quad \mathbf{I} = -\frac{\partial V}{\partial x}\,\hat{\mathbf{i}} - \frac{\partial V}{\partial y}\,\hat{\mathbf{j}} - \frac{\partial V}{\partial z}\,\hat{\mathbf{k}}$

Remember that partial differentiation indicates that variation of gravitational potential in counter along the variation of x-coordinate, then other coordinates (i.e. y and z) are assumed to be constant.

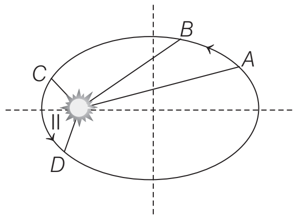
Kepler's Laws of Planetary Motion

Kepler discovered three empirical laws which accurately describe the motion of planets.
These laws are

1. **Law of Orbits** All the planets move around the sun in an elliptical orbit with sun at one of the focus of ellipse.



2. **Law of Areas** The line joining the sun to the planet sweeps out equal areas in equal intervals of time, i.e. areal velocity of the planet w.r.t. sun is constant. This is called the law of area, which indicates that a planet moves faster near the sun and slowly when away from the sun.



3. **Law of Periods** The square of the planet's time period of revolution is directly proportional to the cube of semi-major axis of its orbit.

$$T^2 \propto a^3$$

where a is the semi-major axis.

Gravitational Potential Energy

Gravitational potential energy of a body or system is negative of work done by the conservative gravitational forces F in bringing it from infinity to the present position.
Mathematically, gravitational potential energy

$$U = -W = -\int_{\infty}^{\mathbf{r}} \mathbf{F} \cdot d\mathbf{r}$$

- The gravitational potential energy of two particles of masses m_1 and m_2 separated by a distance r is given by

$$U = -\frac{Gm_1m_2}{r}$$

- The gravitational potential energy of mass m at the surface of the earth is

$$U = -\frac{GMm}{R}$$

- Difference in potential energy of mass m at a height h from the earth's surface and at the earth's surface is

$$U_{(R+h)} - U_R = \frac{mgh}{1 + \frac{h}{R}} \simeq mgh, \text{ if } h << R$$

- For three particles system,

$$U = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right]$$

- For n-particles system, $\frac{n(n-1)}{2}$ pairs form and total potential energy of the system is sum of potential energies of all such pairs.

Escape Velocity

It is the minimum velocity with which a body must be projected from the surface of the earth so that it escapes the gravitational field of the earth. We can also say that a body, projected with escape velocity, will be able to go to a point which is at infinite distance from the earth.

The value of escape velocity from the surface of a planet of mass M, radius R and acceleration due to gravity g is

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

Escape velocity does not depend upon the mass or shape or size of the body as well as the direction of projection of the body. For earth value of escape velocity is 11.2 kms^{-1}.

Satellites

Anybody that revolves around earth or any planet is called satellite. These can be natural (e.g. Moon) or artificial. The artificial satellites are man made satellites launched from the earth. The path of these satellites are elliptical with the centre of earth at a foci of the ellipse. However, as a first approximation we may consider the orbit of satellite as circular.

Orbital Velocity of Satellite

Orbital velocity of a satellite is the velocity required to put the satellite into its orbit around the earth. The orbital velocity of satellite is given by

$$v_o = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{r}} = \sqrt{\frac{gR^2}{(R+h)}}$$

If $h << R$ or $r \simeq R$, then

$$v_o = \sqrt{\frac{GM}{R}} = \sqrt{gR} = 7.9 \text{ kms}^{-1}$$

$$v_{\text{escape}} = \sqrt{2} v_{\text{orbital}}$$

Period of Revolution

It is the time taken by a satellite to complete one revolution around the earth.

Revolution period, $T = \dfrac{2\pi r}{v_o} = 2\pi\sqrt{\dfrac{r^3}{GM}} = 2\pi\sqrt{\dfrac{r^3}{gR^2}} = \sqrt{\dfrac{3\pi}{G.e}}$

If $r \simeq R$, then $T = 2\pi\sqrt{\dfrac{R}{g}} = 84.6$ min.

Height of Satellite in Terms of Period

The height of the satellite (from the earth planet) can be determined by its time period and *vice-versa*.

As the height of the satellite in terms of time period,

$$h = r - R = \left[\frac{gR^2T^2}{4\pi^2}\right]^{1/3} - R.$$

Energy of Satellite

Kinetic energy of satellite, $K = \dfrac{1}{2} mv_0^2 = \dfrac{GMm}{2r}$.

Potential energy of satellite, $U = -\dfrac{GMm}{r}$

and total energy of satellite $E = K + U = -\dfrac{GMm}{2r} = -K$.

Binding Energy of Satellite

It is the energy required to remove the satellite from its orbit and take it to infinity.

Binding energy $= -E = +\dfrac{GMm}{2r}$

Angular Momentum of Satellite

Angular momentum of a satellite, $L = mv_0 r = \sqrt{m^2 GMr}$

Geostationary Satellite

If an artificial satellite revolves around the earth in an equatorial plane with a time period of 24 h in the same sense as that of the earth, then it will appear stationary to the observer on the earth. Such a satellite is known as a **geostationary satellite** or **parking satellite**.

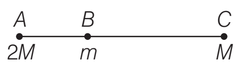
DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 A mass M is split into two parts m and $(M - m)$, which are separated by a certain distance. The ratio m/M which maximizes the gravitational force between the parts is

(a) 1 : 4 (b) 1 : 3
(c) 1 : 2 (d) 1 : 1

2 Particles of masses $2M$, m and M are respectively at points A, B and C with $AB = \frac{1}{2}(BC)$, m is much-much smaller than M and at time $t = 0$, they are all at rest as given in figure. At subsequent times before any collision takes place.



(a) M will remain at rest
(b) M will move towards M
(c) M will move towards $2M$
(d) M will have oscillatory motion

3 If one moves from the surface of the earth to the moon, what will be the effect on its weight?

(a) Weight of a person decreases continuously with height from the surface of the earth
(b) Weight of a person increases with height from the surface of the earth
(c) Weight of a person first decreases with height and then increases with height from the surface of the earth
(d) Weight of a person first increases with height and then decreases with height from the surface of the earth

4 At the surface of a certain planet acceleration due to gravity is one-quarter of that on the earth. If a brass ball is transported on this planet, then which one of the following statements is not correct?

(a) The brass ball has same mass on the other planet as on the earth
(b) The mass of the brass ball on this planet is a quarter of its mass as measured on the earth
(c) The weight of the brass ball on this planet is a quarter of the weight as measured on the earth
(d) The brass ball has the same volume on the other planet as on the earth

5 If both the mass and radius of the earth, each decreases by 50%, the acceleration due to gravity would

(a) remain same (b) decrease by 50%
(c) decrease by 100% (d) increase by 100%

6 A research satellite of mass 200 kg circles the earth in an orbit of average radius $\frac{3R}{2}$,where R is the radius of the earth. Assuming the gravitational pull on a mass of 1kg on the earth's surface to be 10 N, the pull on the satellite will be

(a) 880 N (b) 889 N
(c) 885 N (d) 892 N

7 The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where, g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth is

(a) $2h$ (b) $\frac{h}{\sqrt{3}}$ (c) $\frac{h}{2}$ (d) $\sqrt{2}\,h$

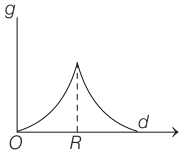
8 The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the surface of the earth. When both d and h are much smaller than the radius of the earth, then which one of the following is correct?

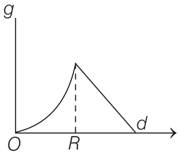
(a) $d = \frac{h}{2}$ (b) $d = \frac{3h}{2}$ (c) $d = 2h$ (d) $d = h$

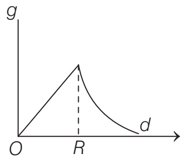
9 Average density of the earth

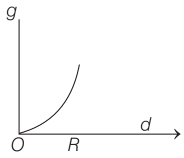
(a) does not depend on g
(b) is a complex function of g
(c) is directly proportional to g
(d) is inversely proportional to g

10 The variation of acceleration due to gravity g with distance d from centre of the Earth is best represented by (R = Earth's radius) **→ JEE Main 2017 (Offline)**

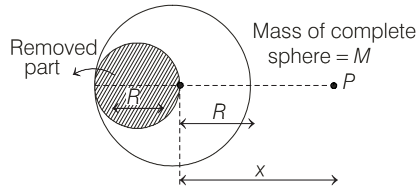
(a)


(b)


(c)


(d)


11 The gravitational field, due to the 'left over part' of a uniform sphere (from which a part as shown has been 'removed out') at a very far off point, P located as shown, would be (nearly) **→ JEE Main 2015**



(a) $\frac{5}{6}\frac{GM}{R^2}$ (b) $\frac{8}{9}\frac{GM}{R^2}$ (c) $\frac{7}{8}\frac{GM}{R^2}$ (d) $\frac{6}{7}\frac{GM}{R^2}$

12 A solid sphere is of density ρ and radius R. The gravitational field at a distance r from the centre of the sphere, when $r < R$, is

(a) $\frac{\rho\pi GR^3}{r}$ (b) $\frac{4\pi G\rho r^2}{3}$ (c) $\frac{4\pi G\rho R^3}{3r^2}$ (d) $\frac{4\pi G\rho r}{3}$

13 The maximum vertical distance through which a full dressed astronaut can jump on the earth is 0.5 m. Estimate the maximum vertical distance through which he can jump on the moon, which has a mean density 2/3rd that of the earth and radius one quarter that of the earth.

(a) 1.5 m (b) 3 m (c) 6 m (d) 7.5 m

14 The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be

(a) 6.4×10^{11} J (b) 6.4×10^{8} J
(c) 6.4×10^{9} J (d) 6.4×10^{10} J

15 If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth, is

(a) $2mgR$ (b) $\frac{1}{2}mgR$ (c) $\frac{1}{4}mgR$ (d) mgR

16 Two bodies of masses m and $4\,m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

(a) $-\frac{4Gm}{r}$ (b) $-\frac{6Gm}{r}$ (c) $-\frac{9Gm}{r}$ (d) zero

17 A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 kms^{-1}, the escape velocity from the surface of the planet would be

(a) 1.1 kms^{-1} (b) 11 kms^{-1}
(c) 110 kms^{-1} (d) 0.11 kms^{-1}

18 A projectile is fired vertically upwards from the surface of the earth with a velocity kv_e, where v_e is the escape velocity and $k < 1$. If R is the radius of the earth, the maximum height to which it will rise measured from the centre of the earth will be

(a) $\dfrac{1-k^2}{R}$ (b) $\dfrac{R}{1-k^2}$

(c) $R(1-k^2)$ (d) $\dfrac{R}{1+k^2}$

19 Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is

(a) $m_1r_1 : m_2r_2$ (b) $m_1 : m_2$ (c) $r_1 : r_2$ (d) 1 : 1

20 What is the direction of areal velocity of the earth around the sun?

(a) Perpendicular to positon of the earth w.r.t. the sun at the focus
(b) Perpendicular to velocity of the earth revolving in the elliptical path
(c) Parallel to angular displacement
(d) Both (a) and (b)

21 The time period of a satellite of the earth is 5 h. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become

(a) 10 h (b) 80 h (c) 40 h (d) 20 h

22 A satellite goes along an elliptical path around earth. The rate of change of area swept by the line joining earth and the satellite is proportional to

(a) $r^{1/2}$ (b) r (c) $r^{3/2}$ (d) r^2

23 A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is

(a) gx (b) $\dfrac{gR}{R-x}$ (c) $\dfrac{gR^2}{R+x}$ (d) $\left(\dfrac{gR^2}{R+x}\right)^{1/2}$

24 Two particles of equal mass m go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is **→ AIEEE 2011**

(a) $\sqrt{\dfrac{Gm}{R}}$ (b) $\sqrt{\dfrac{Gm}{4R}}$ (c) $\sqrt{\dfrac{Gm}{3R}}$ (d) $\sqrt{\dfrac{Gm}{2R}}$

25 Suppose the gravitational force varies inversely as the nth power of distance. Then the time period of a planet in circular orbit of radius R around the sun will be proportional to

(a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$ (c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$

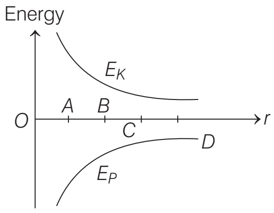
26 The gravitational force exerted by the sun on the moon is about twice as great as the gravitational force exerted by the earth on the moon, but still the moon is not escaping from gravitational influence of the earth. Mark the option which correctly explains the above system.

(a) At some point of time the moon will escape from the earth
(b) Separation between the moon and sun is larger than the separation between the earth and moon
(c) The moon-earth system is bounded one and a minimum amount of energy is required to escape the moon from the earth
(d) None of the above

27 What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$? **→ JEE Main 2013**

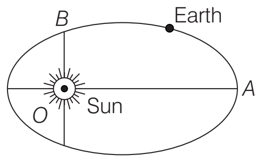
(a) $\dfrac{5\,GmM}{6R}$ (b) $\dfrac{2GmM}{3R}$ (c) $\dfrac{GmM}{2R}$ (d) $\dfrac{GmM}{3R}$

28 The curves for potential energy (E_P) and kinetic energy (E_K) of two particles system are as shown in the figure. At what points the system will be bound



(a) Only at point A
(b) Only at point D
(c) At points A and D
(d) At points A, B and C

29 The earth moves around the sun in an elliptical orbit as shown in the figure. The ratio $\dfrac{OA}{OB} = x$. The ratio of the speeds of the earth at B and at A is



(a) $\sqrt{x}$ (b) x (c) x^2 (d) $x\sqrt{x}$

30 A satellite of mass m_s revolving in a circular orbit of radius r_s around the earth of mass M, has a total energy E. Then its angular momentum will be

(a) $(2Em_sr_s)^{1/2}$ (b) $(2Em_sr_s)$

(c) $(2Em_sr_s^2)^{1/2}$ (d) $(2Em_sr_s^2)$

31 Match the term related to gravitation given in Column I with their formula given in Column II and select the correct option from the choices given below :

	Column I		Column II
A.	Gravitational potential at a point outside the solid sphere	1.	$\dfrac{3GM}{2R}$
B.	Gravitational potential at a point on the surface of sphere	2.	$\dfrac{GM}{r}$
C.	Gravitational potential at the centre of solid sphere	3.	$\dfrac{GM}{R}$

	A	B	C
(a)	1	3	2
(b)	2	3	1
(c)	1	2	3
(d)	3	2	1

Direction (Q. Nos. 32-33) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

32 **Statement I** An astronaut in an orbiting space station above the earth experiences weightlessness.

Statement II An object moving around the earth under the influence of the earth's gravitational force is in a state of 'free-fall'.

33 **Statement I** Kepler's laws for planetary motion are consequence of Newton's laws.

Statement II Kepler's laws can be derived by using Newton's laws.

34 **Assertion** (A) If the earth were a hollow sphere, gravitational field intensity at any point inside the earth would be zero.

Reason (R) Net force on a body inside the sphere is zero.

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion
(c) If Assertion is true but Reason is false
(d) If Assertion is false but Reason is true

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 Satellites orbitting the earth have finite life and sometimes debris of satellites fall to the earth. This is because

(a) the solar cells and batteries in satellites run out
(b) the laws of gravitation predict a trajectory spiralling inwards
(c) of viscous forces causing the speed of satellite and hence height to gradually decrease
(d) of collisions with other satellites

2 A body is released from a point distance r from the centre of earth. If R is the radius of the earth and $r > R$, then the velocity of the body at the time of striking the earth will be

(a) $\sqrt{gR}$ (b) $\sqrt{2gR}$
(c) $\sqrt{\frac{2gRr}{r-R}}$ (d) $\sqrt{\frac{2gR(r-R)}{r}}$

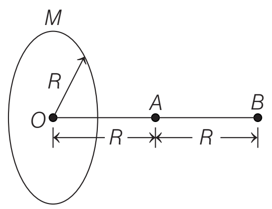
3 Two small satellites move in circular orbits around the earth, at distances r and $r + \Delta r$ from the centre of the earth. Their time periods of rotation are T and $T + \Delta T$, $(\Delta r << r, \Delta T << T)$. Then, ΔT is equal to

(a) $\frac{3}{2}T\frac{\Delta r}{r}$ (b) $\frac{2}{3}T\frac{\Delta r}{r}$ (c) $\frac{-3}{2}T\frac{\Delta r}{r}$ (d) $T\frac{\Delta r}{r}$

4 A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force, it exerts on a point mass m at $x = 0$, if the mass per unit length is $A + Bx^2$, is

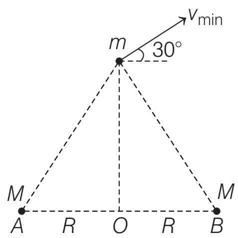
(a) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)+BL\right]$ (b) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)-BL\right]$
(c) $Gm\left[A\left(\frac{1}{(a+L)}-\frac{1}{a}\right)-BL\right]$ (d) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$

5 A ring having non-uniform distribution of mass having mass M and radius R is being considered. A point mass m_0 is taken slowly from A to B along the axis of the ring. In doing so, work done by the external force against the gravitational force exerted by ring is



(a) $\frac{GMm_0}{\sqrt{2}R}$
(b) $\frac{GMm_0}{R}\left[\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{5}}\right]$
(c) $\frac{GMm_0}{R}\left[\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{2}}\right]$
(d) It is not possible to find the required work as the nature of distribution of mass is not known

6 With what minimum speed should m be projected from point C in the presence of two fixed masses M each at A and B as shown in the figure, such that mass m should escape the gravitational attraction of A and B?



(a) $\sqrt{\frac{2GM}{R}}$ (b) $\sqrt{\frac{2\sqrt{2}GM}{R}}$

(c) $2\sqrt{\frac{GM}{R}}$ (d) $2\sqrt{2}\sqrt{\frac{GM}{R}}$

7 A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them, to take the particle far away from the sphere, (you may take $G = 6.67\times 10^{-11}\ \text{Nm}^2/\text{kg}^2$)

(a) 13.34×10^{-10} J (b) 3.33×10^{-10} J
(c) 6.67×10^{-9} J (d) 6.67×10^{-10} J

8 Two metallic spheres each of mass M are suspended by two strings each of length L. The distance between the upper ends of strings is L. The angle which the strings will make with the vertical due to mutual attraction of the spheres is

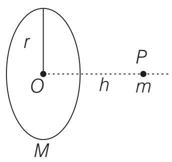
(a) $\tan^{-1}\left[\frac{GM}{gL}\right]$ (b) $\tan^{-1}\left[\frac{GM}{2gL}\right]$

(c) $\tan^{-1}\left[\frac{GM}{gL^2}\right]$ (d) $\tan^{-1}\left[\frac{2GM}{gL^2}\right]$

9 A body of mass 2 kg is moving under the influence of a central force whose potential energy is given by $U = 2r^3$ joule. If the body is moving in a circular orbit of 5 m, its energy will be

(a) 250 J (b) 125 J
(c) 625 J (d) 650 J

10 A mass m is placed at P at a distance h along the normal through the centre O of a thin circular ring of mass M and radius r as shown in figure. If the mass is moved further away such that OP becomes $2h$, by what factor, the force of gravitation will decrease, if $h = r$?

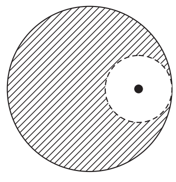


(a) $\frac{3\sqrt{2}}{4\sqrt{3}}$ (b) $\frac{5\sqrt{2}}{\sqrt{3}}$ (c) $\frac{4\sqrt{3}}{5}$ (d) $\frac{4\sqrt{2}}{5\sqrt{5}}$

11 An artificial satellite of the earth is launched in circular orbit in equatorial plane of the earth and satellite is moving from West to East. With respect to a person on the equator, the satellite is completing one round trip in 24 h. Mass of the earth is, $M = 6\times 10^{24}$kg. For this situation, orbital radius of the satellite is

(a) 2.66×10^4km (b) 6400 km
(c) 36,000 km (d) 29,600 km

12 From a solid sphere of mass M and radius R, a spherical portion of radius $\left(\frac{R}{2}\right)$ is removed as shown in the figure.



Taking gravitational potential $V = 0$ at $I = \infty$, the potential at the centre of the cavity thus formed is (**I** = gravitational constant). → **JEE Main 2015**

(a) $\frac{-GM}{2R}$ (b) $\frac{-GM}{R}$ (c) $\frac{-2GM}{3R}$ (d) $\frac{-2GM}{R}$

13 A satellite is revolving in a circular orbit at a height h from the Earth's surface (radius of Earth $R, h << R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the Earth's gravitational field, is close to (Neglect the effect of atmosphere) → **JEE Main 2017 (Offline)**

(a) $\sqrt{2gR}$ (b) $\sqrt{gR}$ (c) $\sqrt{gR/2}$ (d) $\sqrt{gR}\,(\sqrt{2}-1)$

14 Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction, the speed of each particle is → **JEE Main 2015**

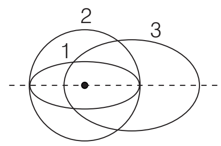
(a) $\sqrt{\frac{GM}{R}}$ (b) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

(c) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (d) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

Direction (Q. Nos. 15-16) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

15 **Statement I** Three orbits are marked as 1, 2 and 3. These three orbits have same semi-major axis although their shapes (eccentricities) are different. The three identical satellites are orbiting in these three orbits, respectively. These three satellites have the same binding energy.



Statement II Total energy of a satellite depends on the semi-major axis of orbit according to the expression,

$$E = \frac{-GMm}{2r}$$

16 **Statement I** Two satellites are following one another in the same circular orbit. If one satellite tries to catch another (leading one) satellite, then it can be done by increasing its speed without changing the orbit.

Statement II The energy of the earth satellites system in circular orbits is given by $E = \frac{-GMm}{2r}$, where, r is the radius of the circular orbit.

ANSWERS

SESSION 1	**1** (c)	**2** (c)	**3** (c)	**4** (b)	**5** (d)	**6** (b)	**7** (c)	**8** (c)	**9** (c)	**10** (c)
	11 (c)	**12** (d)	**13** (b)	**14** (d)	**15** (b)	**16** (c)	**17** (c)	**18** (b)	**19** (c)	**20** (d)
	21 (c)	**22** (a)	**23** (d)	**24** (b)	**25** (a)	**26** (c)	**27** (a)	**28** (d)	**29** (b)	**30** (c)
	31 (b)	**32** (a)	**33** (d)	**34** (a)						
SESSION 2	**1** (c)	**2** (d)	**3** (a)	**4** (d)	**5** (b)	**6** (b)	**7** (d)	**8** (c)	**9** (c)	**10** (d)
	11 (a)	**12** (b)	**13** (d)	**14** (d)	**15** (a)	**16** (d)				

Hints and Explanations

SESSION 1

1 As, $F = \dfrac{Gm(M-m)}{x^2}$

For maximum gravitational force

$\dfrac{dF}{dm} = \dfrac{G}{x^2}(M-2m) = 0$ or $\dfrac{m}{M} = \dfrac{1}{2}$

2 The particle B will move towards the greater force between forces by A and B.

Force on B due to $A = F_{BA} = \dfrac{G(2Mm)}{(AB)^2}$ towards BA

Force on B due to $C = F_{BC} = \dfrac{GMm}{(BC)^2}$ towards BC

As, $(BC) = 2AB$

$\Rightarrow \quad F_{BC} = \dfrac{GMm}{(2AB)^2} = \dfrac{GMm}{4(AB)^2} < F_{BA}$

Hence, m will move towards BA (i.e. $2M$).

3 The gravitational attraction on a body due to the earth decreases with height and increases due to the moon at a certain height.

At one point it becomes zero and with further increase in height the gravitational attraction of the moon becomes more than that of the earth.

4 The mass of a body is always constant and does not change with position.

5 Here, $g = GM/R^2$

and $g' = \dfrac{G(M/2)}{(R/2)^2} = \dfrac{2GM}{R^2} = 2g$

$\therefore$ % increase in $g = \left(\dfrac{g'-g}{g}\right) \times 100$

$= \left(\dfrac{2g-g}{g}\right) \times 100$

$= 100\%$

6 Here, $mg = 10$ or $1 \times g = 10$

$\Rightarrow \quad g = 10\,\text{ms}^{-2}$

Now, $g' = g\dfrac{R^2}{r^2} = 10 \times \dfrac{R^2}{(3R/2)^2} = \dfrac{40}{9}$

Pull on satellite $= m'g'$

$= 200 \times \dfrac{40}{9}$

$\approx 889\,\text{N}$

7 Acceleration due to gravity at height h is, $g' = \dfrac{GM}{(R+h)^2}$

$\Rightarrow \dfrac{g}{9} = \dfrac{GM}{R^2} \cdot \dfrac{R^2}{(R+h)^2} = g\left(\dfrac{R}{R+h}\right)^2$

$\Rightarrow \dfrac{1}{9} = \left(\dfrac{R}{R+h}\right)^2$

$\Rightarrow \quad \dfrac{R}{R+h} = \dfrac{1}{3}$

$\Rightarrow \quad 3R = R + h$

$\Rightarrow \quad R = \dfrac{h}{2}$

8 $g_h = g\left(1 - \dfrac{2h}{R}\right)$ ...(i)

and $g_d = g\left(1 - \dfrac{d}{R}\right)$ ...(ii)

As per statement of the problem,

$g_h = g_d$

i.e. $g\left(1 - \dfrac{2h}{R}\right) = g\left(1 - \dfrac{d}{R}\right)$

$\Rightarrow \quad 2h = d$

9 As, $g = \dfrac{GM}{R^2}$;

$M = \left(\dfrac{4}{3}\pi R^3\right)\rho$

$\therefore \quad g = \dfrac{4G}{3}\dfrac{\pi R^3}{R^2}\rho$

$\Rightarrow \quad \rho = \left(\dfrac{3}{4\pi GR}\right)g$;

$\therefore \quad \rho \propto g$

(where, ρ = average density of the earth)

10 Inside the earth's surface,

$g = \dfrac{GM}{R^3}d$ i.e. $g \propto d$

Outside the earth's surface,

$g = \dfrac{Gm}{d^2}$ i.e. $g \propto \dfrac{1}{d^2}$

So, till earth's surface g increases linearly with distance r, shown only in graph (c).

11 We have,

$M' = \dfrac{M}{\frac{4}{3}\pi R^3} \times \dfrac{4}{3}\pi\left(\dfrac{R}{2}\right)^3 = \dfrac{M}{8}$

$\therefore$ Gravitational field at

$P = \dfrac{GM}{R^2} - \dfrac{GM}{8R^2}$

$= G \times \dfrac{M}{R^2}\left(1 - \dfrac{1}{8}\right) = \dfrac{7}{8}\dfrac{GM}{R^2}$

12 Intensity,

$I = \dfrac{GM}{R^3}r = \dfrac{Gr}{R^3}\left(\dfrac{4}{3}\pi R^3\rho\right) = \dfrac{4\pi G\rho r}{3}$

13 On the moon, $g_m = \dfrac{4}{3}\pi G(R/4)(2\rho/3)$

$= \dfrac{1}{6}\left(\dfrac{4}{3}\pi GR\rho\right) = \dfrac{1}{6}g$

Work done in jumping

$= m \times g \times 0.5 = m \times (g/6)h_1$

$h_1 = 0.5 \times 6 = 3.0$ m

14 Potential energy on the earth surface is $-mgR$ while in free space it is zero. So, to free the spaceship, minimum required energy is

$K = mgR = 10^3 \times 10 \times 6400 \times 10^3\,\text{J}$

$= 6.4 \times 10^{10}\,\text{J}$

15 Gravitational potential energy of body on the earth's surface is

$$U = -\frac{GM_e m}{R}$$

At a height h from earth's surface, its value is

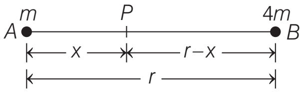
$$U_h = -\frac{GM_e m}{(R+h)} = -\frac{GM_e m}{2R} \quad [\text{as } h = R]$$

where, M_e = mass of earth,
m = mass of body
and R = radius of earth.

$\therefore$ Gain in potential energy

$$= U_h - U$$
$$= -\frac{GM_e m}{2R} - \left(-\frac{GM_e m}{R}\right)$$
$$= -\frac{GM_e m}{2R} + \frac{GM_e m}{R}$$
$$= \frac{GM_e m}{2R} = \frac{gR^2 m}{2R} \quad \left[\text{as } g = \frac{GM_e}{R^2}\right]$$
$$= \frac{1}{2} mgR$$

16 Let gravitational field is zero at P as shown in figure.



$$\therefore \quad \frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$$
$$\Rightarrow \quad 4x^2 = (r-x)^2 \Rightarrow 2x = r - x$$
$$\Rightarrow \quad x = \frac{r}{3}$$
$$\therefore \quad V_P = -\frac{Gm}{x} - \frac{G(4m)}{r-x} = -\frac{9Gm}{r}$$

$(\because x = r/3)$

17 Mass of planet, $M_p = 10M_e$,

where, M_e is mass of earth.

Radius of planet, $R_p = \frac{R_e}{10}$,

where R_e is radius of earth.

Escape velocity is given by,

$$v_e = \sqrt{\frac{2GM}{R}}$$

For planet, $v_p = \sqrt{\frac{2G \times M_p}{R_p}}$

$$= \sqrt{\frac{100 \times 2GM_e}{R_e}}$$
$$= 10 \times v_e$$
$$= 10 \times 11$$
$$= 110 \text{ kms}^{-1}$$

18 From law of conservation of energy,

$$\frac{1}{2}mv^2 = \frac{mgh}{1 + \frac{h}{R}}$$

Here, $v = kv_e = k\sqrt{2gR}$

and $h = r - R$

$$\frac{1}{2}m\, k^2 \cdot 2gR = \frac{mg\,(r-R)}{1 + \frac{r-R}{R}},$$
$$k^2 R\left[1 + \frac{r-R}{R}\right] = r - R$$
$$k^2 r = r - R$$
$$\Rightarrow \quad r = \frac{R}{1-k^2}$$

19 As their period of revolution is same, so their angular speed is also same. Centripetal acceleration is circular path, $a = \omega^2 r$.

Thus, $\frac{a_1}{a_2} = \frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2}$

20 Areal velocity of the earth around the sun is given by

$$\frac{d\mathbf{A}}{dt} = \frac{\mathbf{L}}{2m}$$

where, $\mathbf{L}$ is the angular momentum and m is the mass of the earth. But angular momentum

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times m\mathbf{v}$$

$\therefore$ Areal velocity

$$\left(\frac{d\mathbf{A}}{dt}\right) = \frac{1}{2m}(\mathbf{r} \times m\mathbf{v})$$
$$= \frac{1}{2}(\mathbf{r} \times \mathbf{v})$$

Therefore, the direction of areal velocity $\left(\frac{d\mathbf{A}}{dt}\right)$ is the direction of $(\mathbf{r} \times \mathbf{v})$, i.e. perpendicular to the plane containing $\mathbf{r}$ and $\mathbf{v}$.

21 According to Kepler's law, $T^2 \propto r^3$

$$\therefore \quad \left(\frac{T}{T^1}\right)^2 = \left(\frac{r}{r^1}\right)^3 \Rightarrow \frac{25}{(T')^2} = \frac{r^3}{64r^3}$$

or $T' = \sqrt{1600}$ or $T' = 40\,\text{h}$

22 Areal velocity $= \frac{dA}{dt} = \frac{L}{2m} = \frac{mvr}{2m} = \frac{vr}{2}$

$$= \frac{r}{2}\sqrt{\frac{GM}{r}} = \frac{1}{2}\sqrt{GMr}$$

So, $\frac{dA}{dt} \propto \sqrt{r}$

23 The gravitational force exerted on satellite at a height x is

$$F_G = \frac{GM_e m}{(R+x)^2}$$

where, M_e = mass of the earth.

Since, gravitational force provides the necessary centripetal force, so

$$\frac{GM_e m}{(R+x)^2} = \frac{mv_o^2}{(R+x)}$$

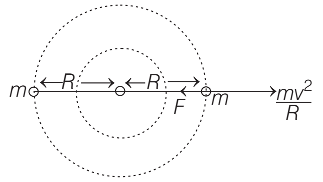
where, v_o is orbital speed of satellite

$$\Rightarrow \quad \frac{GM_e m}{(R+x)} = mv_o^2$$

or $\frac{gR^2 m}{(R+x)} = mv_o^2 \quad \left(\because g = \frac{GM_e}{R^2}\right)$

or $v_o = \sqrt{\left[\frac{gR^2}{(R+x)}\right]} = \left[\frac{gR^2}{(R+x)}\right]^{1/2}$

24 As gravitational force provides necessary centripetal force.



i.e. $F = \frac{Gm^2}{(2R)^2} = \frac{mv^2}{R}$

$$\Rightarrow \quad v = \sqrt{\frac{Gm}{4R}}$$

25 The necessary centripetal force required for a planet to move around the sun = gravitational force exerted on it

i.e. $\frac{mv^2}{R} = \frac{GM_e m}{R^n}$ or $v = \left(\frac{GM_e}{R^{n-1}}\right)^{1/2}$

Now, $T = \frac{2\pi R}{v} = 2\pi R \times \left(\frac{R^{n-1}}{GM_e}\right)^{1/2}$

$$= 2\pi\left(\frac{R^2 \times R^{n-1}}{Gm_e}\right)^{1/2}$$
$$= 2\pi\left(\frac{R^{(n+1)/2}}{(Gm_e)^{1/2}}\right)$$

or $T \propto R^{(n+1)/2}$

26 Option (c) is correct, a minimum amount of energy equal to $|\text{TE}|$ of the moon-earth system has to be given to break (unbound) the system, the sun is exerting force on the moon but not providing any energy.

27 From conservation of energy

Total energy at the planet = Total energy at the altitude

$$\frac{-GMm}{R} + (\text{KE})_{\text{surface}}$$
$$= \frac{-GMm}{3R} + \frac{1}{2}mv_A^2 \qquad \ldots\text{(i)}$$

In its orbit, the necessary centripetal force is provided by gravitational force.

$$\therefore \quad \frac{mv_A^2}{(R+2R)} = \frac{GMm}{(R+2R)^2}$$
$$\Rightarrow \quad v_A^2 = \frac{GM}{3R} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$(\text{KE})_{\text{surface}} = \frac{5}{6}\frac{GMm}{R}$$

28 The system will be bound at all these points where the total energy $= (E_P + E_K)$ is negative.

In the given curve, at points A, B and C, the $E_P > E_K$.

29 According to law of conservation of angular momentum;

$$mv_A \times OA = mv_B \times OB,$$
$$\frac{v_B}{v_A} = \frac{OA}{OB} = x$$

30 Total energy of satellite,

$$E = -\frac{GMm_s}{2r_s} \quad \ldots(i)$$

Orbital velocity of satellite,

$$v_s = \sqrt{\frac{GM}{r_s}}$$

Angular momentum of satellite is given by

$$L = m_s v_s r_s = m_s\left(\frac{GM}{r_s}\right)^{1/2} r_s = (GM\, m_s^2 r_s)^{1/2}$$
$$= (2Em_s r_s^2)^{1/2} \quad \text{[from Eq. (i)]}$$

31 Gravitational potential,

At outside point (solid sphere) $= \frac{GM}{r}$

At the surface (solid sphere) $= \frac{GM}{R}$

At the centre (solid sphere) $= \frac{3GM}{2R}$

Hence, (b) is correct.

32 Force acting on astronaut is utilised in providing necessary centripetal force, thus he feels weightlessness, as he is in the state of free fall.

33 Kepler's laws are based on observations, hit and trial method and already recorded data but later on Newton proved their correctness using his laws.

34 It is clear that the net force on the body inside the hollow sphere is zero hence, then net gravitational field intensity $\left(E = \frac{F}{m}\right)$ at any point inside the earth must also be zero.

SESSION 2

1 As the total energy of the earth satellite bounded system is negative $\left(\frac{-GM}{2a}\right)$.

Where, a is radius of the satellite and M is mass of the earth. Due to the viscous force acting on satellite, energy decreases continuously and radius of the orbit or height decreases gradually.

2 Using law of conservation of energy,

$$-\frac{GMm}{r} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$\Rightarrow \frac{v^2}{2} = \frac{GM}{R} - \frac{GM}{r} = GM\left(\frac{r-R}{rR}\right)$$
$$= gR\left(\frac{r-R}{r}\right) \quad \left(\because \frac{GM}{R^2} = g\right)$$
$$\therefore \quad v = \sqrt{2gR(r-R)/r}$$

3 According to Kepler's law, $T^2 \propto r^3$

$$T^2 = kr^3 \quad \ldots(i)$$

Differentiating it, we have

$$2T\,\Delta T = 3kr^2\Delta r$$

Dividing it by Eq. (i), we get

$$\frac{2T\,\Delta T}{T^2} = \frac{3kr^2\Delta r}{kr^3} \Rightarrow \Delta T = \frac{3}{2}T\frac{\Delta r}{r}$$

4 Mass of the element of length dx at a distance x from the origin

$$= (A + Bx^2)\,dx$$
$$dF = \frac{Gm(A + Bx^2)dx}{x^2}$$

On integrating,

$$F = Gm\int_a^{a+L}\frac{(A + Bx^2)dx}{x^2}$$
$$= Gm\int_a^{a+L}\left(\frac{A}{x^2} + B\right)dx$$
$$= Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$$

5 Even though the distribution of mass is unknown we can find the potential due to ring on any axial point because from any axial point the entire mass is at the same distance (whatever would be the nature of distribution).

Potential at A due to ring is,

$$V_A = -\frac{GM}{\sqrt{2}R}$$

Potential at B due to ring is,

$$V_B = -\frac{GM}{\sqrt{5}R}$$

$$dU = U_f - U_i$$
$$= U_B - U_A = m_0(V_B - V_A)$$
$$= \frac{GMm_0}{R}\left[-\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{2}}\right]$$

$$W_{gr} = -W_{ext}$$
$$\Rightarrow \quad W_{gr} = -dU = -W_{ext}$$
$$\therefore W_{ext} = dU = \frac{GMm_0}{R}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{5}}\right]$$

6 Here, $K_i + U_i = K_f + U_f$

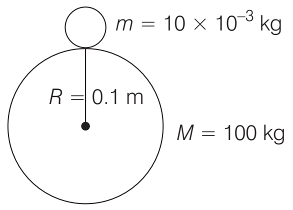
$$\therefore \quad \frac{1}{2}mv^2 - \frac{2GMm}{\sqrt{2}R} = 0 + 0$$

$$\text{or} \quad \frac{1}{2}mv^2 = \frac{2GMm}{\sqrt{2}R} \quad \text{or} \quad v = \sqrt{\frac{2\sqrt{2}GM}{R}}$$

Hence, (b) is the correct option.

7 The initial potential energy of the particle = Work done.

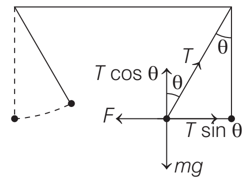
$$U_i = -\frac{GMm}{r}$$
$$U_i = -\frac{6.67 \times 10^{-11} \times 100 \times 10^{-2}}{0.1}$$
$$U_i = -\frac{6.67 \times 10^{-11}}{0.1}$$
$$= -6.67 \times 10^{-10}\ \text{J}$$



We know that, work done = Difference in potential energy

$$\therefore \quad W = \Delta U = U_f - U_i$$
$$\Rightarrow \quad W = -U_i \quad [\because U_f = 0]$$
$$= 6.67 \times 10^{-10}\ \text{J}$$

8 The metallic spheres will be at positions as shown in the figure.



From the figure,

$$T\sin\theta = F = \frac{GM \times M}{L^2} = \frac{GM^2}{L^2}$$

and $\quad T\cos\theta = Mg$

$$\Rightarrow \quad \tan\theta = \frac{GM}{gL^2}$$
$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{GM}{gL^2}\right)$$

9 Given, $\quad U = 2r^3$

$$\therefore \quad F = \frac{-dU}{dr} = -6r^2$$

Now, $\frac{mv^2}{r} = |F| = 6r^2$

or $\quad \frac{1}{2}mv^2 = \frac{1}{2}(6r^3) = 3r^3$

As, $\quad \text{TE} = K + U$

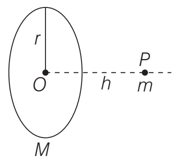
$$= 3r^3 + 2r^3$$
$$= 5r^3$$

$\therefore$ At $\quad r = 5\,\text{m}$,

$$\text{TE} = 625\,\text{J}$$

Hence, (c) is the correct option.

10 Gravitational force acting on an object of mass m, placed at point P at a distance h along the normal through the centre of a circular ring of mass M and radius r is given by



$$F = \frac{GMmh}{(r^2 + h^2)^{3/2}}$$

When mass is displaced upto distance $2h$, then

$$F' = \frac{GMm \times 2h}{(r^2 + (2h)^2)^{3/2}} = \frac{2GMmh}{(r^2 + 4h^2)^{3/2}}$$

When $h = r$, then

$$F = \frac{GMm \times r}{(r^2 \times r^2)^{3/2}} = \frac{GMm}{2\sqrt{2}r^2}$$

and $F' = \dfrac{2GMmr}{(r^2 + 4r^2)^{3/2}} = \dfrac{2GMm}{5\sqrt{5}\,r^2}$

$\therefore \quad \dfrac{F'}{F} = \dfrac{4\sqrt{2}}{5\sqrt{5}}$ or $F' = \dfrac{4\sqrt{2}}{5\sqrt{5}} F$

11 Here, time period of satellite w.r.t. observer on equator is 24 h and the satellite is moving from West to East, so angular velocity of satellite w.r.t. the earth's axis of rotation (considered as fixed) is,

$$\omega = \frac{2\pi}{T_s} + \frac{2\pi}{T_e},$$

where, T_s and T_e are time periods of satellite and the earth, respectively

$$\omega = \frac{\pi}{6} \text{ rad/h}$$
$$= 1.45 \times 10^{-4} \text{ rad s}^{-1}$$

From $\quad v = \sqrt{\dfrac{GM}{r}}$

$\Rightarrow \quad r\omega = \sqrt{\dfrac{GM}{r}}$

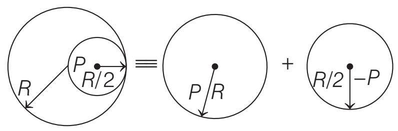
$\Rightarrow \quad r^{3/2} = \dfrac{\sqrt{GM}}{\omega}$

$$= \frac{\sqrt{6.67 \times 10^{-11} \times 6 \times 10^{24}}}{1.45 \times 10^{-4}}$$

$\Rightarrow \quad r = 2.66 \times 10^7$ m

$= 2.66 \times 10^4$ km

12 Consider cavity as negative mass and apply superposition of gravitational potential. Consider the cavity formed in a solid sphere as shown in figure.



According to the equation, potential at an internal point P due to complete sphere.

$$V_3 = -\frac{GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$

$$\frac{-GM}{2R^3}\left[3R^2 - \frac{R^2}{4}\right] = \frac{-GM}{2R^3}\left[\frac{11R^2}{4}\right]$$
$$= \frac{-11\,GM}{8R}$$

Mass of removed part

$$= \frac{M}{\frac{4}{3} \times \pi R^3} \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{M}{8}$$

Potential at point P due to removed part

$$V_2 = \frac{-3}{2} \times \frac{GM/8}{R/2} = \frac{-3GM}{8R}$$

Thus, potential due to remaining part at point P.

$$V_P = V_3 - V_2 = \frac{-11GM}{8R} - \left(-\frac{3GM}{8R}\right)$$
$$= \frac{(-11+3)\,GM}{8R} = \frac{-GM}{R}$$

13 Given, a satellite is revolving in a circular orbit at a height h from the Earth's surface having radius of Earth R, i.e. $h << R$.

Orbital velocity of a satellite,

$$v = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{GM}{R}} \quad (\text{as } h << R)$$

Velocity required to escape,

$$\frac{1}{2}mv'^2 = \frac{GMm}{R+h}$$

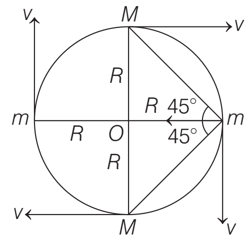
$$v' = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2GM}{R}}$$

$(h << R)$

$\therefore$ Minimum increase in its orbital velocity required to escape from the Earth's gravitational field.

$$v' - v = \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}}$$
$$= \sqrt{2gR} - \sqrt{gR}$$
$$= \sqrt{gR}\,(\sqrt{2} - 1) \qquad \left(\because g = \frac{GM}{R^2}\right)$$

14 Net force acting on any one particle M.



$$= \frac{GM^2}{(2R)^2} + \frac{GM^2}{(R\sqrt{2})^2}\cos 45^\circ + \frac{GM^2}{(R\sqrt{2})^2}\cos 45^\circ$$

$$= \frac{GM^2}{R^2}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right)$$

This force will equal to centripetal force.

So, $\quad \dfrac{Mv^2}{R} = \dfrac{GM^2}{R^2}\left(\dfrac{1}{4} + \dfrac{1}{\sqrt{2}}\right)$

$$v = \sqrt{\frac{GM}{4R}(1 + 2\sqrt{2})}$$
$$= \frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2} + 1)}$$

Hence, speed of each particle in a circular motion is

$$\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2} + 1)}$$

15 Total energy of earth (planet)-satellite system is independent of eccentricity of orbit and it depends on semi-major axis and masses of the planet and satellite.

16 Here, Statement I is wrong because as speed of one satellite increases, its kinetic energy and hence total energy increases, i.e. total energy becomes less negative and hence r increases, i.e. orbit changes.