Chapter 5 Quadratic Functions

Ex 5.7

Answer 1e.

The equation $(x-1)^2(x+2)=0$ has two distinct solutions, 1 and 2. Since the factor $x-1$ appears twice, we can count 1 twice. The solution 1 is said to be a repeated solution.

Thus, the statement can be completed as "For the equation $(x-1)^2(x+2)=0$, a **repeated** solution is 1 because the factor $x-1$ appears twice.

Answer 1gp.

The fundamental theorem of algebra states that a polynomial function $f(x)$ of degree n, where $n > 0$, will have at least n solutions. This is based on the fact that each solution repeated twice is counted as two solutions, each solution repeated three times is counted as three solutions, and so on.

The degree of the given polynomial equation is 4. Thus, the given equation will have at least four solutions.

Answer 2e.

We need to explain the difference between complex conjugates and irrational conjugates.

Multiplication of two irrational conjugates results in rational number and multiplication of two complex conjugates results in real number.
The number inside the square root, in case of irrational conjugates is not a perfect square and the number inside the square root, in case of complex conjugates is a negative number.

Answer 2gp.

We need to find the number of zeros of the function

$f(x)=x^3+7x^2+8x-16$

An nth degree polynomial has exactly n zeros.

Because

$f(x)=x^3+7x^2+8x-16$

is a polynomial of degree 3 it has $\boxed{\text{three zeros}}$ by the above mentioned corollary of the fundamental theorem of algebra.

Answer 3e.

The fundamental theorem of algebra states that a polynomial function $f(x)$ of degree n, where $n > 0$, will have at least n solutions. This is based on the fact that each solution repeated twice is counted as two solutions, each solution repeated three times is counted as three solutions, and so on.

The degree of the given polynomial equation is 4. Thus, the given equation will have at least four solutions or zeros.

Answer 3gp.

STEP 1 **Find** the rational zeros of f.

Since the degree of f is 3, the function will have 3 zeros. The possible rational zeros are ± 1, ± 3, and ± 9.

Use synthetic division to divide $f(x)$ by -1.

$$\begin{array}{r|rrrr} -1 & 1 & 7 & 15 & 9 \\ & & -1 & -6 & -9 \\ \hline & 1 & 6 & 9 & 0 \end{array}$$

The remainder is 0, which implies that -1 is a zero.

STEP 2 **Write** $f(x)$ in factored form.

The quotient obtained by dividing $f(x)$ by -1 is $x^2 + 6x + 9$.

The function can be thus written as $f(x) = (x + 1)(x^2 + 6x + 9)$.

STEP 2 **Find** the complex zeros of f.

Factor the trinomial $x^2 + 6x + 9$.
$f(x) = (x + 1)(x + 3)^2$

From the factor $(x + 3)^2$, we get -3 as a zero that is repeated twice.

Therefore, the zeros of f are -1, -3, and -3.

Answer 4e.

We need to find the number of zeros of the function

$$5y^3 - 3y^2 + 8y = 0$$

An nth degree polynomial has exactly n zeros.

Consider

$f(y)=5y^3-3y^2+8y$

Because

$f(y)=5y^3-3y^2+8y$

is a polynomial of degree 3 it has $\boxed{\text{three zeros}}$ by the above mentioned corollary of the fundamental theorem of algebra.

Answer 4gp.

We need to find all zeros of the polynomial function

$f(x)=x^5-2x^4+8x^2-13x+6$

An nth degree polynomial has exactly n zeros.

Because

$f(x)=x^5-2x^4+8x^2-13x+6$

is a polynomial of degree 5 it has five zeros by the above mentioned corollary of the fundamental theorem of algebra.

The possible rational zeros are $\pm1, \pm2, \pm3, \pm6$

Let us test these zeros using synthetic division.

Let $x=1$

$$\begin{array}{c|cccccc} 1 & 1 & -2 & 0 & 8 & -13 & 6 \\ & & 1 & -1 & -1 & 7 & -6 \\ \hline & 1 & -1 & -1 & 7 & -6 & 0 \end{array}$$

Since the remainder is zero, $x=1$ is a zero of the given function $f(x)$

Thus,

$f(x)=(x-1)(x^4-x^3-x^2+7x-6)$

Assume that

$g(x)=x^4-x^3-x^2+7x-6$

The possible rational zeros of $g(x)$ are also the zeros of $f(x)$

Again test for $x=1$:

$$\begin{array}{c|ccccc} 1 & 1 & -1 & -1 & 7 & -6 \\ & & 1 & 0 & -1 & 6 \\ \hline & 1 & 0 & -1 & 6 & 0 \end{array}$$

Synthetic division shows that $x=1$ is a zero of the function $g(x)$

Thus,

$$f(x) = (x-1)\left(x^4 - x^3 - x^2 + 7x - 6\right)$$
$$= (x-1)^2\left(x^3 - x + 6\right)$$

Assume that

$$h(x) = x^3 - x + 6$$

The possible rational zeros of $h(x)$ are also the zeros of $f(x)$

Now test for $x = -2$:

−2	1	0	−1	6
		−2	4	−6
	1	−2	3	0

Synthetic division shows that $x = -2$ is a zero of the function $h(x)$

Thus,

$$f(x) = (x-1)\left(x^4 - x^3 - x^2 + 7x - 6\right)$$
$$= (x-1)^2\left(x^3 - x + 6\right)$$
$$= (x-1)^2(x+2)\left(x^2 - 2x + 3\right)$$

Compute the discriminant of the quadratic equation

$x^2 - 2x + 3$

Thus,

$$d = b^2 - 4ac$$
$$= (-2)^2 - 4(1)(3)$$
$$= 4 - 12$$
$$= -8$$

Since the discriminant, $d < 0$, the zeros of the quadratic equation are imaginary.

Use quadratic formula to find the imaginary zeros of the function

$x^2 - 2x + 3$

Thus,

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{4-12}}{2}$$
$$= \frac{2 \pm \sqrt{-8}}{2}$$
$$= \frac{2 \pm 2i\sqrt{2}}{2}$$
$$= 1 \pm i\sqrt{2}$$

Thus, the zeros of f are $\boxed{1, 1, -2, 1+i\sqrt{2}, 1-i\sqrt{2}}$

Answer 5e.

The fundamental theorem of algebra states that a polynomial function $f(x)$ of degree n, where $n > 0$, will have at least n solutions. This is based on the fact that each solution repeated twice is counted as two solutions, each solution repeated three times is counted as three solutions, and so on.

The degree of the polynomial equation is 6. Thus, the given equation will have at least six solutions or zeros.

Answer 5gp.

Write $f(x)$ as a product of three factors using the three zeros and the factor theorem.

$f(x) = (x + 1)(x - 2)(x - 4)$

Multiply $x - 2$ and $x - 4$ first.
$f(x) = (x + 1)[x^2 - 4x - 2x + 8]$

Combine the like terms within the brackets.
$f(x) = (x + 1)[x^2 - 6x + 8]$

Multiply again and combine the like terms.

$$\begin{aligned} f(x) &= x^3 - 6x^2 + 8x + x^2 - 6x + 8 \\ &= x^3 - 5x^2 + 2x + 8 \end{aligned}$$

Thus, the required polynomial function is $f(x) = x^3 - 5x^2 + 2x + 8$.

Check the result by evaluating $f(x)$ at each of its three zeros. The function should evaluate to 0 each time.

$$f(-1) = (-1)^3 - 5(-1)^2 + 2(-1) + 8 = -1 - 5 - 2 + 8 = 0$$

$$f(2) = 2^3 - 5(2)^2 + 2(2) + 8 = 8 - 20 + 4 + 8 = 0$$

$$f(4) = 4^3 - 5(4)^2 + 2(4) + 8 = 64 - 80 + 8 + 8 = 0$$

The result checks.

Answer 6e.

We need to find the number of zeros of the function

$$f(z) = -7z^4 + z^2 - 25$$

An nth degree polynomial has exactly n zeros.

Consider

$f(z) = -7z^4 + z^2 - 25$

Because

$f(z) = -7z^4 + z^2 - 25$

is a polynomial of degree 4 it has $\boxed{\text{four zeros}}$ by the above mentioned corollary of the fundamental theorem of algebra.

Answer 6gp.

We need to write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1 and the given zeros, $4, 1+\sqrt{5}$

Irrational Conjugates Theorem:

Suppose f is a polynomial function with rational coefficients, and a and b are rational numbers such that $\sqrt{b}$ is irrational. If $a+\sqrt{b}$ is a zero of f, then $a-\sqrt{b}$ is also a zero of f.

Because the coefficients are rational and $1+\sqrt{5}$ is a zero, $1-\sqrt{5}$ must also be a zero by the irrational conjugates theorem. Use the three zeros and the factor theorem to write $f(x)$ as a product of three factors.

$$
\begin{aligned}
f(x) &= (x-4)\left[x-\left(1+\sqrt{5}\right)\right]\left[x-\left(1-\sqrt{5}\right)\right] && \text{[Write } f(x) \text{ in factored form]} \\
&= (x-4)\left[(x-1)-\sqrt{5}\right]\left[(x-1)+\sqrt{5}\right] && \text{[Regroup terms]} \\
&= (x-4)\left[(x-1)^2-5\right] && \text{[Multiply]} \\
&= (x-4)\left[\left(x^2-2x+1\right)-5\right] && \text{[Expand binomial]} \\
&= (x-4)\left(x^2-2x-4\right) && \text{[Simplify]} \\
&= x^3-2x^2-4x-4x^2+8x+16 && \text{[Multiply]} \\
&= x^3-6x^2+4x+16 && \text{[Combine like terms]}
\end{aligned}
$$

Thus, the required polynomial is $\boxed{f(x)=x^3-6x^2+4x+16}$

Check:

Let us check this result by evaluating $f(x)$ at each of its three zeros.

Consider

$$f(x)=x^3-6x^2+4x+16$$

Thus,

$$\begin{aligned} f(4)&=(4)^3-6(4)^2+4(4)+16 \\ &=64-96+16+16 \\ &=0 \end{aligned}$$

$$\begin{aligned} f\left(1+\sqrt{5}\right)&=\left(1+\sqrt{5}\right)^3-6\left(1+\sqrt{5}\right)^2+4\left(1+\sqrt{5}\right)+16 \\ &=1+5\sqrt{5}+3\sqrt{5}+15-6\left(1+5+2\sqrt{5}\right)+4+4\sqrt{5}+16 \\ &=8\sqrt{5}+16-6\left(6+2\sqrt{5}\right)+4+4\sqrt{5}+16 \\ &=8\sqrt{5}+16-36-12\sqrt{5}+4+4\sqrt{5}+16 \\ &=12\sqrt{5}-12\sqrt{5}+36-36 \\ &=0 \end{aligned}$$

Since $f\left(1+\sqrt{5}\right)=0$, by the irrational conjugates theorem, $f\left(1-\sqrt{5}\right)=0$

Answer 7e.

The fundamental theorem of algebra states that a polynomial function $f(x)$ of degree n, where $n > 0$, will have at least n solutions. This is based on the fact that each solution repeated twice is counted as two solutions, each solution repeated three times is counted as three solutions, and so on.

The degree of the given polynomial function is 7. Thus, the function will have at least seven solutions or zeros.

Answer 7gp.

Since the coefficients are rational and $2i$ is a zero of f, $-2i$ must also be a zero of f by complex conjugates theorem. Similarly, using irrational conjugates theorem, $4+\sqrt{6}$ is also a zero.

Write $f(x)$ as a product of five factors using the five zeros and the factor theorem.

$$f(x) = (x-2)(x-2i)(x+2i)\left[x-\left(4-\sqrt{6}\right)\right]\left[x-\left(4+\sqrt{6}\right)\right]$$

Expand the binomial.

$$f(x) = (x-2)(x^2+4)[x^2-8x+16-6]$$

Simplify.

$$f(x) = (x-2)(x^2+4)[x^2-8x+10]$$

Multiply again and combine the like terms.

$$\begin{aligned} f(x) &= (x-2)\left(x^4-8x^3+10x^2+4x^2-32x+40\right) \\ &= (x-2)\left(x^4-8x^3+14x^2-32x+40\right) \\ &= x^5-8x^4+14x^3-32x^2+40x-2x^4+16x^3-28x^2+64x-80 \\ &= x^5-10x^4+30x^3-60x^2+104x-80 \end{aligned}$$

Thus, the required polynomial function is $f(x) = x^5 - 10x^4 + 30x^3 - 60x^2 + 104x - 80$.

Check the result by evaluating $f(x)$ at the zeros 2, $2i$, and $4-\sqrt{6}$. The function should evaluate to 0 each time.

$$f(2) = 2^5 - 10(2)^4 + 30(2)^3 - 60(2)^2 + 104(2) - 80 = 0$$

$$f(2i) = (2i)^5 - 10(2i)^4 + 30(2i)^3 - 60(2i)^2 + 104(2i) - 80 = 0$$

$$f\left(4-\sqrt{6}\right) = \left(4-\sqrt{6}\right)^5 - 10\left(4-\sqrt{6}\right)^4 + 30\left(4-\sqrt{6}\right)^3 - 60\left(4-\sqrt{6}\right)^2 + 104\left(4-\sqrt{6}\right) - 80 = 0$$

As $2i$ and $4-\sqrt{6}$ checks, their conjugates $-2i$ and $4+\sqrt{6}$ also checks.

Answer 8e.

We need to find the number of zeros of the function

$$h(x) = -x^{12} + 7x^8 + 5x^4 - 8x + 6$$

An nth degree polynomial has exactly n zeros.

Consider

$$h(x) = -x^{12} + 7x^8 + 5x^4 - 8x + 6$$

Because

$$h(x) = -x^{12} + 7x^8 + 5x^4 - 8x + 6$$

is a polynomial of degree 12 it has $\boxed{\text{12 zeros}}$ by the above mentioned corollary of the fundamental theorem of algebra.

Answer 8gp.

We need to write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1 and the given zeros, $3, 3-i$

Complex Conjugates Theorem:

Suppose f is a polynomial function with real coefficients,and $a+ib$ is an imaginary zero of f, then $a-ib$ is also a zero of f.

Because the coefficients are rational and $3+i$ is a zero, $3-i$ must also be a zero by the complex conjugates theorem. Use the three zeros and the factor theorem to write $f(x)$ as a product of three factors.

$$\begin{aligned} f(x) &= (x-3)\left[x-(3+i)\right]\left[x-(3-i)\right] && \text{[Write } f(x) \text{ in factored form]} \\ &= (x-3)\left[(x-3)-i\right]\left[(x-3)+i\right] && \text{[Regroup terms]} \\ &= (x-3)\left[(x-3)^2-(-1)\right] && \text{[Multiply]} \\ &= (x-3)\left[(x^2-6x+9)+1\right] && \text{[Expand binomial]} \\ &= (x-3)(x^2-6x+10) && \text{[Simplify]} \\ &= x^3-6x^2+10x-3x^2+18x-30 && \text{[Multiply]} \\ &= x^3-9x^2+28x-30 && \text{[Combine like terms]} \end{aligned}$$

Thus, the required polynomial is $\boxed{f(x)=x^3-9x^2+28x-30}$

Check:

Let us check this result by evaluating $f(x)$ at each of its three zeros.

Consider

$f(x)=x^3-9x^2+28x-30$

Thus,

$$\begin{aligned} f(3) &= (3)^3-9(3)^2+28(3)-30 \\ &= 27-81+84-30 \\ &= 0 \end{aligned}$$

$$\begin{aligned} f(3-i) &= (3-i)^3-9(3-i)^2+28(3-i)-30 \\ &= (27+i+9-27i)-9(9+1-6i)+84-28i-30 \\ &= 27+i+9-27i-90+54i+84-28i-30 \\ &= (27+9-90+84-30)+(i-27i+54i-28i) \\ &= 0+0i \\ &= 0 \end{aligned}$$

Since $f(3-i)=0$, by the complex conjugates theorem, $f(3+i)=0$

Answer 9e.

The fundamental theorem of algebra states that a polynomial function $f(x)$ of degree n, where $n > 0$, will have at least n solutions. This is based on the fact that each solution repeated twice is counted as two solutions, each solution repeated three times is counted as three solutions, and so on.

Let us first rewrite the given function in standard form.
$f(x) = 6x^6 + 19x^5 - 22x^3 + 16x - 3$

Since the degree of $f(x)$ is 6, the function will have at least six zeros.
Thus, the correct answer is choice **D**.

Answer 9gp.

The number of positive and negative real zeros can be determined using the Descartes' rule of signs.

By this rule, if $f(x)$ is a polynomial function with real coefficients, then the number of positive real zeros is equivalent to the number of changes in sign of the coefficients of the function or less than this by an even number.

The number of negative real zeros is equivalent to the number of changes in sign of the coefficients of $f(-x)$ or less than this by an even number.

Let us first find the number of sign changes in the function.

$$f(x) = x^3 + 2x - 11$$

Since there is only one sign change in $f(x)$, there is only one positive real zero.

Now, find $f(-x)$ and the number of sign changes in it.

$$f(-x) = (-x)^3 + 2(-x) - 11$$
$$= -x^3 - 2x - 11$$

There is no sign change in $f(-x)$. This implies that there is no negative real zero(s).

The total number of zeros possible is equivalent to the degree of the function, which is 3. The number of imaginary zeros will be the difference between the total number of zeros and the sum of the numbers of positive and negative real zeros.

Summarize the possible zeros of f in a table.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
1	0	2	3

Answer 10e.

We need to find all zeros of the polynomial function

$f(x)=x^4-6x^3+7x^2+6x-8$

An nth degree polynomial has exactly n zeros.

Because

$f(x)=x^4-6x^3+7x^2+6x-8$

is a polynomial of degree 4 it has four zeros by the above mentioned corollary of the fundamental theorem of algebra.

The possible rational zeros are $\pm1, \pm2, \pm4, \pm8$

Let us test these zeros using synthetic division.

Let $x=1$

$$\begin{array}{r|rrrrr} 1 & 1 & -6 & 7 & 6 & -8 \\ & & 1 & -5 & 2 & 8 \\ \hline & 1 & -5 & 2 & 8 & 0 \end{array}$$

Since the remainder is zero, $x=1$ is a zero of the given function $f(x)$

Thus,

$f(x)=(x-1)(x^3-5x^2+2x+8)$

Assume that

$g(x)=x^3-5x^2+2x+8$

The possible rational zeros of $g(x)$ are also the zeros of $f(x)$

Again test for $x=1$:

$$\begin{array}{r|rrrr} -1 & 1 & -5 & 2 & 8 \\ & & -1 & 6 & -8 \\ \hline & 1 & -6 & 8 & 0 \end{array}$$

Synthetic division shows that $x=-1$ is a zero of the function $g(x)$

Thus,

$$\begin{aligned} f(x) &= (x-1)(x^3-5x^2+2x+8) \\ &= (x-1)(x+1)(x^2-6x+8) \end{aligned}$$

Assume that

$h(x)=x^2-6x+8$

The possible rational zeros of $h(x)$ are also the zeros of $f(x)$

Compute the discriminant of the quadratic equation

x^2-6x+8

Thus,

$$\begin{aligned} d &= b^2-4ac \\ &= (-6)^2-4(1)(8) \\ &= 36-32 \\ &= 4 \end{aligned}$$

Since the discriminant, $d>0$, the zeros of the quadratic equation are unique and real..

Use quadratic formula to find the imaginary zeros of the function

x^2-6x+8

Thus,

$$\begin{aligned} x &= \frac{-(-6)\pm\sqrt{(-6)^2-4(1)(8)}}{2(1)} \\ &= \frac{6\pm\sqrt{36-32}}{2} \\ &= \frac{6\pm 2}{2} \end{aligned}$$

$$x=\frac{6+2}{2} \text{ or } x=\frac{6-2}{2}$$

$$x=4 \quad \text{or } x=2$$

Thus, the zeros of f are $\boxed{-1, 1, 4 \text{ and } 2}$

Answer 10gp.

We need to determine the possible numbers of positive real zeros, negative real zeros and imaginary zeros for $g(x)=2x^4-8x^3+6x^2-3x+1$

Consider the given function:

$g(x)=2x^4-8x^3+6x^2-3x+1$

Thus, the coefficients in $g(x)$ have 4 sign changes, so $g(x)$ has 4 or 2 positive real zeros.

Consider

$$\begin{aligned} g(-x) &= 2(-x)^4-8(-x)^3+6(-x)^2-3(-x)+1 \\ &= 2x^4+8x^3+6x^2+3x+1 \end{aligned}$$

Thus, the coefficients in $g(-x)$ have 0 sign changes, so $g(x)$ has no negative real zeros.

The possible numbers of zeros for $g(x)$ are summarized in the table below:

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
4	0	0	4
2	0	2	4
0	0	4	4

Answer 11e.

STEP 1 **Find** the rational zeros of f.

Since the degree of f is 4, the function will have 4 zeros. The possible rational zeros are $\pm1, \pm2, \pm3, \pm5, \pm6, \pm10$, and ±30.

Use synthetic division to divide $f(x)$ by 1.

$$\begin{array}{r|rrrrr} 1 & 1 & 5 & -7 & -29 & 30 \\ & & 1 & 6 & -1 & -30 \\ \hline & 1 & 6 & -1 & -30 & 0 \end{array}$$

The remainder is 0, which implies that 1 is a zero.

Again, divide the quotient $x^3 + 6x^2 - x - 30$ by 2 using synthetic division.

$$\begin{array}{r|rrrr} 2 & 1 & 6 & -1 & -30 \\ & & 2 & 16 & 30 \\ \hline & 1 & 8 & 15 & 0 \end{array}$$

We get 2 as another zero.

STEP 2 **Write** $f(x)$ in factored form.

The quotient obtained by dividing $f(x)$ by 2 is $x^2 + 8x + 15$.

The function can be thus written as $f(x) = (x - 1)(x - 2)(x^2 + 8x + 15)$.

STEP 2 **Find** the complex zeros of f.

Factor the trinomial $x^2 + 8x + 15$.
$f(x) = (x - 1)(x - 2)(x + 3)(x + 5)$

Therefore, the zeros of f are $-5, -3, 1$, and 2.

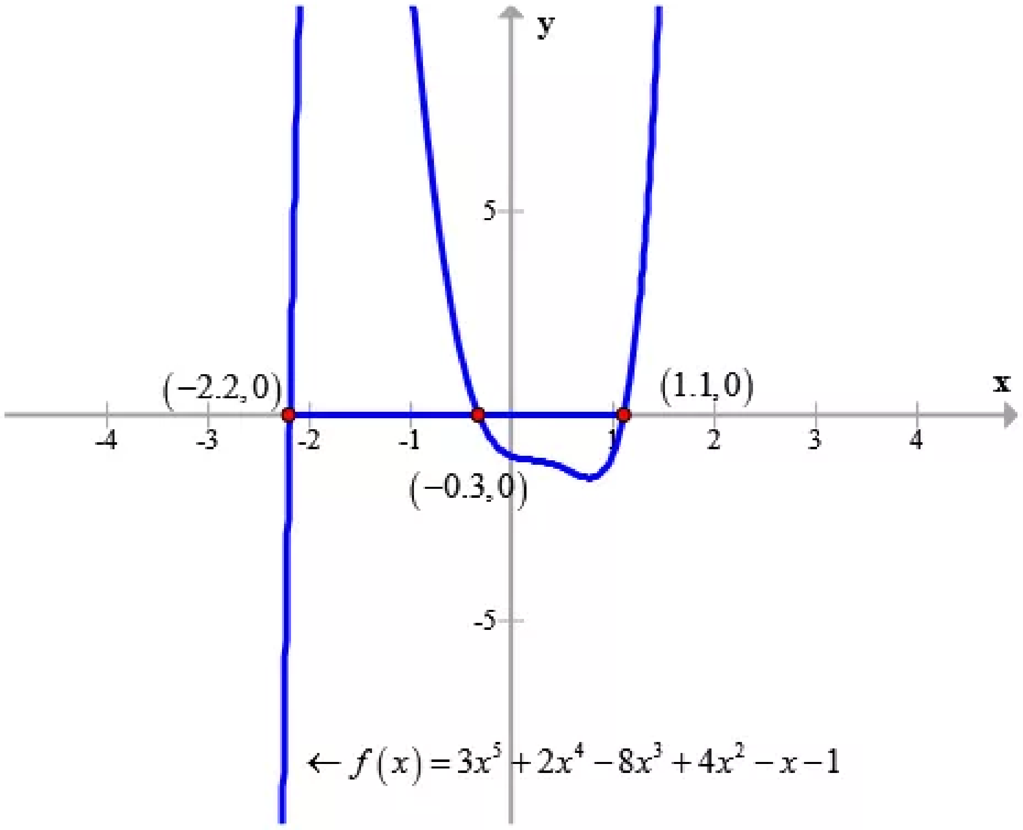
Answer 11gp.

We need to approximate the real zeros of
$f(x) = 3x^5 + 2x^4 - 8x^3 + 4x^2 - x - 1$

Consider the given function:

$f(x)=3x^5+2x^4-8x^3+4x^2-x-1$

Let us observe the following graph:



Thus, the real zeros of the given function are

$\boxed{x=-0.3, x=-2.2, x=1.1}$

Answer 12e.

We need to find all zeros of the polynomial function

$g(x)=x^4-9x^2-4x+12$

An nth degree polynomial has exactly n zeros.

Because

$g(x)=x^4-9x^2-4x+12$

is a polynomial of degree 4 it has four zeros by the above mentioned corollary of the fundamental theorem of algebra.

The possible rational zeros are $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

Let us test these zeros using synthetic division.

Let $x=1$

1	1	0	−9	−4	12
		1	1	−8	−12
	1	1	−8	−12	0

Since the remainder is zero, $x=1$ is a zero of the given function $g(x)$

Thus,

$g(x)=(x-1)(x^3+x^2-8x-12)$

Assume that

$h(x)=x^3+x^2-8x-12$

The possible rational zeros of $h(x)$ are also the zeros of $g(x)$

Now test for $x=-2$:

−2	1	1	−8	−12
		−2	2	12
	1	−1	−6	0

Synthetic division shows that $x=-2$ is a zero of the function $h(x)$

Thus,

$$\begin{aligned} g(x)&=(x-1)(x^3+x^2-8x-12) \\ &=(x-1)(x+2)(x^2-x-6) \end{aligned}$$

Assume that

$f(x)=x^2-x-6$

The possible rational zeros of $h(x)$ are also the zeros of $g(x)$

Compute the discriminant of the quadratic equation

$f(x)=x^2-x-6$

Thus,

$$\begin{aligned} d&=b^2-4ac \\ &=(-1)^2-4(1)(-6) \\ &=1+24 \\ &=25 \end{aligned}$$

Since the discriminant, $d>0$, the zeros of the quadratic equation are unique and real..

Use quadratic formula to find the imaginary zeros of the function

$x^2 - x - 6$

Thus,

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1-(-24)}}{2}$$

$$= \frac{1 \pm 5}{2}$$

$$x = \frac{1+5}{2} \text{ or } x = \frac{1-5}{2}$$

$$x = 3 \quad \text{ or } x = -2$$

Thus, the zeros of g are $\boxed{-2, -2, 1 \text{ and } 3}$

Answer 12gp.

For a certain boat, the speed x of the engine shaft (in 100 s of RPMs) and the speed s of the boat (in miles per hour) are modeled by

$s(x) = 0.00547x^3 - 0.225x^2 + 3.62x - 11.0$

We need to find the tachometer reading when the boat travels 20 miles per hour.

Consider the given function:

$s(x) = 0.00547x^3 - 0.225x^2 + 3.62x - 11.0$

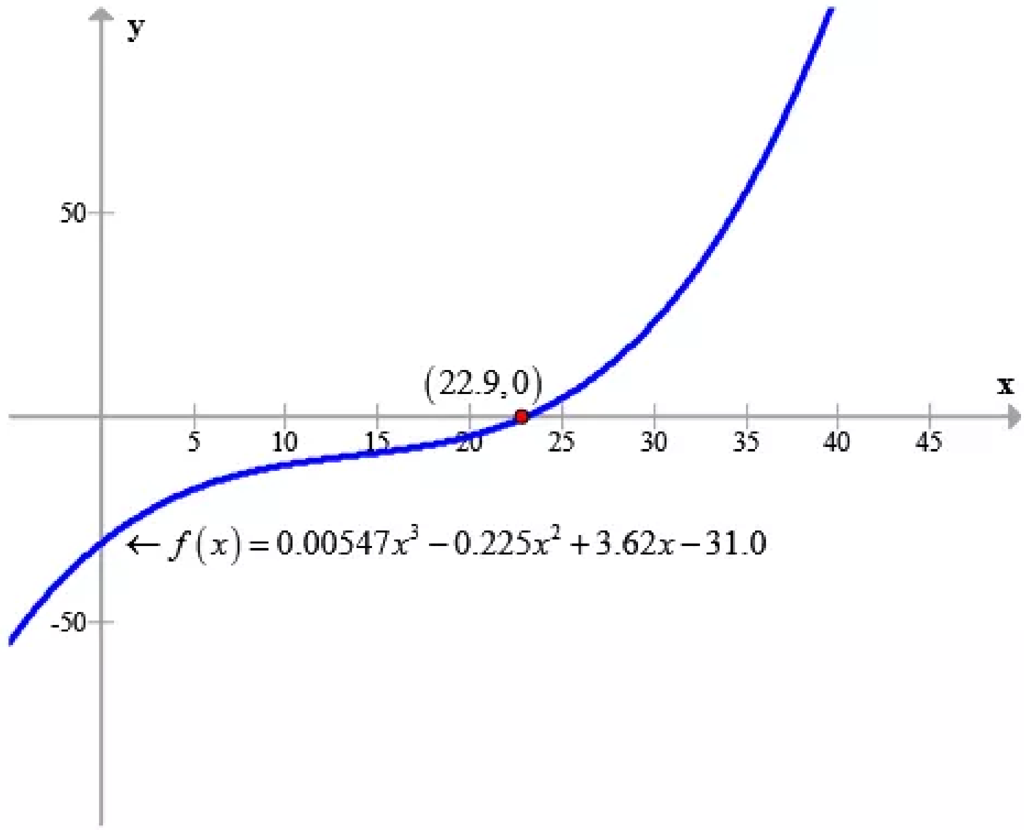
Substitute $s(x) = 20$ in the above function and rewrite it.

Thus, we have,

$0.00547x^3 - 0.225x^2 + 3.62x - 11.0 - 20 = 0$

$0.00547x^3 - 0.225x^2 + 3.62x - 31.0 = 0$

Let us observe the following graph:



Thus the real zero from the graph is
$x = 22.9$

Thus, the tachometer reading is about
$\boxed{2290 \text{ RPMs}}$

Answer 13e.

STEP 1 **Find** the rational zeros of h.

Since the degree of h is 4, the function will have 4 zeros. The possible rational zeros are ±1, ±2, ±4, ±5, ±10, and ±20.

Use synthetic division to divide $h(x)$ by 2.

$$\begin{array}{r|rrrr} 2 & 1 & 5 & -4 & -20 \\ & & 2 & 14 & 20 \\ \hline & 1 & 7 & 10 & 0 \end{array}$$

The remainder is 0, which implies that 2 is a zero.

STEP 2 **Write** $h(x)$ in factored form.

The quotient obtained by dividing $h(x)$ by 2 is $x^2 + 7x + 10$.

The function can be thus written as $h(x) = (x - 2)(x^2 + 7x + 10)$.

STEP 2 **Find** the complex zeros of h.

Factor the trinomial $x^2 + 7x + 10$.
$h(x) = (x - 2)(x + 2)(x + 5)$

Therefore, the zeros of h are -2, -5, and 2.

Answer 14e.

We need to find all zeros of the polynomial function
$f(x) = x^4 + 15x^2 - 16$

An nth degree polynomial has exactly n zeros.

Because
$f(x) = x^4 + 15x^2 - 16$
is a polynomial of degree 4 it has four zeros by the above mentioned corollary of the fundamental theorem of algebra.
The possible rational zeros are $\pm1, \pm2, \pm4, \pm8, \pm16$

Let us test these zeros using synthetic division.
Let $x = 1$

1	1	0	15	0	−16
		1	1	16	16
	1	1	16	16	0

Since the remainder is zero, $x = 1$ is a zero of the given function $f(x)$

Thus,
$f(x) = (x-1)(x^3 + x^2 + 16x + 16)$
Assume that
$h(x) = x^3 + x^2 + 16x + 16$
The possible rational zeros of $h(x)$ are also the zeros of $f(x)$

Now test for $x=-1$:

-1	1	1	16	16
		-1	0	-16
	1	0	16	0

Synthetic division shows that $x=-1$ is a zero of the function $h(x)$

Thus,

$$f(x)=(x-1)(x^3+x^2+16x+16)$$
$$=(x-1)(x+1)(x^2+16)$$

Assume that

$$g(x)=x^2+16$$

The possible rational zeros of $g(x)$ are also the zeros of $f(x)$

Compute the discriminant of the quadratic equation

$$g(x)=x^2+16$$

Thus,

$$d=b^2-4ac$$
$$=(0)^2-4(1)(16)$$
$$=0-64$$
$$=-64$$

Since the discriminant, $d<0$, the zeros of the quadratic equation are imaginary.

Use quadratic formula to find the imaginary zeros of the function

$$x^2+16$$

Thus,

$$x=\frac{\pm\sqrt{(0)^2-4(1)(16)}}{2(1)}$$
$$=\frac{\pm\sqrt{0-64}}{2}$$
$$=\frac{\pm 8i}{2}$$

$x=4i$ or $x=-4i$

Thus, the zeros of g are $\boxed{-1, 1, 4i \text{ and } -4i}$

Answer 15e.

STEP 1 **Find** the rational zeros of f.

Since the degree of f is 4, the function will have 4 zeros. The possible rational zeros are $\pm 1, \pm 2, \pm 4$, and ± 8.

Use synthetic division to divide $f(x)$ by 1.

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & 2 & 4 & -8 \\ & & 1 & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 8 & 0 \end{array}$$

The remainder is 0, which implies that 1 is a zero.

Again, divide the quotient $x^3 + 2x^2 + 4x + 8$ by -2 using synthetic division.

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 4 & 8 \\ & & -2 & 0 & -8 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

We get -2 as another zero.

STEP 2 **Write** $f(x)$ in factored form.

The quotient obtained by dividing $f(x)$ by -2 is $x^2 + 4$.

The function can be thus written as $f(x) = (x - 1)(x + 2)(x^2 + 4)$.

STEP 2 **Find** the complex zeros of f.

Factor the binomial $x^2 + 4$.

$f(x) = (x - 1)(x + 2)(x + 2i)(x - 2i)$

Therefore, the zeros of f are -2, 1, $-2i$, and $2i$.

Answer 16e.

We need to find all zeros of the polynomial function

$h(x) = x^4 + 4x^3 + 7x^2 + 16x + 12$

An nth degree polynomial has exactly n zeros.

Because

$h(x)=x^4+4x^3+7x^2+16x+12$

is a polynomial of degree 4 it has four zeros by the above mentioned corollary of the fundamental theorem of algebra.

The possible rational zeros are $\pm1,\pm2,\pm3,\pm4,\pm6,\pm12$

Let us test these zeros using synthetic division.

Let $x=-1$

-1	1	4	7	16	12
		-1	-3	-4	-12
	1	3	4	12	0

Since the remainder is zero, $x=-1$ is a zero of the given function $h(x)$

Thus,

$h(x)=(x+1)(x^3+3x^2+4x+12)$

Assume that

$g(x)=x^3+3x^2+4x+12$

The possible rational zeros of $g(x)$ are also the zeros of $h(x)$

Now test for $x=-3$:

-3	1	3	4	12
		-3	0	-12
	1	0	4	0

Synthetic division shows that $x=-3$ is a zero of the function $g(x)$

Thus,

$$\begin{aligned} h(x)&=(x+1)(x^3+3x^2+4x+12) \\ &=(x+3)(x+1)(x^2+4) \end{aligned}$$

Assume that

$f(x)=x^2+4$

The possible rational zeros of $f(x)$ are also the zeros of $h(x)$

Compute the discriminant of the quadratic equation

$f(x) = x^2 + 4$

Thus,

$$\begin{aligned} d &= b^2 - 4ac \\ &= (0)^2 - 4(1)(4) \\ &= 0 - 16 \\ &= -16 \end{aligned}$$

Since the discriminant, $d < 0$, the zeros of the quadratic equation are imaginary.

Use quadratic formula to find the imaginary zeros of the function

$f(x) = x^2 + 4$

Thus,

$$\begin{aligned} x &= \frac{\pm\sqrt{(0)^2 - 4(1)(4)}}{2(1)} \\ &= \frac{\pm\sqrt{0-16}}{2} \\ &= \frac{\pm 4i}{2} \end{aligned}$$

$x = 2i$ or $x = -2i$

Thus, the zeros of h are $\boxed{-1, -3, 2i \text{ and } -2i}$

Answer 17e.

STEP 1 **Find** the rational zeros of g.

Since the degree of g is 4, the function will have 4 zeros. The possible rational zeros are ± 1 and ± 2.

Use synthetic division to divide $g(x)$ by 1.

$$\begin{array}{r|rrrrr} 1 & 1 & -2 & -3 & 2 & 2 \\ & & 1 & -1 & -4 & -2 \\ \hline & 1 & -1 & -4 & -2 & 0 \end{array}$$

The remainder is 0, which implies that 1 is a zero.

Again, divide the quotient $x^3 - x^2 - 4x - 2$ by -1 using synthetic division.

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -4 & -2 \\ & & -1 & 2 & 2 \\ \hline & 1 & -2 & -2 & 0 \end{array}$$

We get -1 as another zero.

STEP 2 **Write** $g(x)$ in factored form.

The quotient obtained by dividing $g(x)$ by -1 is $x^2 - 2x - 2$.

The function can be thus written as $g(x) = (x - 1)(x + 1)(x^2 - 2x - 2)$.

STEP 2 **Find** the complex zeros of g.

Since the trinomial $x^2 - 2x - 2$ cannot be factored, use the quadratic formula to find its linear factors.

$$\begin{aligned} x &= \frac{2 \pm \sqrt{12}}{2} \\ &= \frac{2 \pm 2\sqrt{3}}{2} \\ &= 1 \pm \sqrt{3} \end{aligned}$$

The function can be factored as

$$g(x) = (x-1)(x+1)\left(x - 1 - \sqrt{3}\right)\left(x - 1 + \sqrt{3}\right).$$

Therefore, the zeros of g are $1, -1, 1+\sqrt{3}$, and $1-\sqrt{3}$.

Answer 18e.

We need to find all zeros of the polynomial function

$$g(x) = 4x^4 + 4x^3 - 11x^2 - 12x - 3$$

An nth degree polynomial has exactly n zeros.

Because

$g(x) = 4x^4 + 4x^3 - 11x^2 - 12x - 3$

is a polynomial of degree 4 it has four zeros by the above mentioned corollary of the fundamental theorem of algebra.

The possible rational zeros are $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

Let us test these zeros using synthetic division.

Let $x = -\frac{1}{2}$

$$\begin{array}{c|ccccc} -\frac{1}{2} & 4 & 4 & -11 & -12 & -3 \\ & & -2 & -1 & 6 & 3 \\ \hline & 4 & 2 & -12 & -6 & 0 \end{array}$$

Since the remainder is zero, $x = -\frac{1}{2}$ is a zero of the given function $g(x)$

Thus,

$g(x) = (2x+1)(4x^3 + 2x^2 - 12x - 6)$

Assume that

$h(x) = 4x^3 + 2x^2 - 12x - 6$

The possible rational zeros of $h(x)$ are also the zeros of $g(x)$

Now test for $x = -\frac{1}{2}$:

$$\begin{array}{c|cccc} -\frac{1}{2} & 4 & 2 & -12 & -6 \\ & & -2 & 0 & 6 \\ \hline & 4 & 0 & -12 & 0 \end{array}$$

Synthetic division shows that $x = -\frac{1}{2}$ is a zero of the function $h(x)$

Thus,

$$g(x) = (2x+1)(4x^3+2x^2-12x-6)$$
$$= (2x+1)^2(4x^2-12)$$

Assume that

$$f(x) = 4x^2 - 12$$

The possible rational zeros of $f(x)$ are also the zeros of $g(x)$

Compute the discriminant of the quadratic equation

$$f(x) = 4x^2 - 12$$

Thus,

$$d = b^2 - 4ac$$
$$= (0)^2 - 4(4)(-12)$$
$$= 0 + 192$$
$$= 192$$

Since the discriminant, $d < 0$, the zeros of the quadratic equation are imaginary.

Use quadratic formula to find the imaginary zeros of the function

$$f(x) = x^2 + 4$$

Thus,

$$x = \frac{\pm\sqrt{(0)^2 - 4(4)(-12)}}{2(4)}$$
$$= \frac{\pm\sqrt{0+192}}{8}$$
$$= \frac{\pm 8\sqrt{3}}{8}$$
$$x = \sqrt{3} \text{ or } x = -\sqrt{3}$$

Thus, the zeros of g are $\boxed{-\frac{1}{2}, -\frac{1}{2}, \sqrt{3} \text{ and } -\sqrt{3}}$

Answer 19e.

STEP 1 **Find** the rational zeros of h.

Since the degree of h is 4, the function will have 4 zeros. The possible rational zeros are $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24, \pm\frac{1}{2}$, and $\pm\frac{3}{2}$.

Use synthetic division to divide $h(x)$ by -2.

$$\begin{array}{r|rrrrr} -2 & 2 & 13 & 19 & -10 & -24 \\ & & -4 & -18 & -2 & 24 \\ \hline & 2 & 9 & 1 & -12 & 0 \end{array}$$

The remainder is 0, which implies that -2 is a zero.

Again, divide the quotient $2x^3 + 9x^2 + x - 12$ by -41 using synthetic division.

$$\begin{array}{r|rrrr} -4 & 2 & 9 & 1 & -12 \\ & & -8 & -4 & 12 \\ \hline & 2 & 1 & -3 & 0 \end{array}$$

We get -4 as another zero.

STEP 2 **Write** $h(x)$ in factored form.

The quotient obtained by dividing $h(x)$ by -4 is $2x^2 + x - 3$.

The function can be thus written as $h(x) = (x + 2)(x + 4)(2x^2 + x - 3)$.

STEP 2 **Find** the complex zeros of h.

Factor the trinomial $2x^2 + x - 3$.
$h(x) = (x + 2)(x + 4)(x - 1)(2x + 3)$

Therefore, the zeros of h are $-4, -2, -\frac{3}{2}$, and 1.

Answer 20e.

We need to write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1 and the given zeros, 1, 2, 3

Use the three zeros and the factor theorem to write

$f(x)$ as a product of three factors.

$$
\begin{aligned}
f(x) &= (x-1)(x-2)(x-3) && \text{[Write } f(x) \text{ in factored form]}\\
&= (x-1)\left[x^2-3x-2x+6\right] && \text{[Multiply]}\\
&= (x-1)\left[x^2-5x+6\right] && \text{[Multiply]}\\
&= x^3-5x^2+6x-x^2+5x-6\\
&= x^3-6x^2+11x-6
\end{aligned}
$$

Thus, the required polynomial is $\boxed{f(x)=x^3-6x^2+11x-6}$

Check:

Let us check this result by evaluating $f(x)$ at each of its three zeros.

Consider

$f(x)=x^3-6x^2+11x-6$

Thus,

$$
\begin{aligned}
f(1) &= (1)^3-6(1)^2+11(1)-6\\
&= 1-6+11-6\\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
f(2) &= (2)^3-6(2)^2+11(2)-6\\
&= 8-24+22-6\\
&= -16+22-6\\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
f(3) &= (3)^3-6(3)^2+11(3)-6\\
&= 27-54+33-6\\
&= -27+33-6\\
&= 0
\end{aligned}
$$

Answer 21e.

Write $f(x)$ as a product of three factors using the three zeros and the factor theorem.

$f(x) = (x + 2)(x - 1)(x - 3)$

Multiply $x-1$ and $x-3$ first.
$f(x)=(x+2)[x^2-3x-x+3]$

Combine the like terms within the brackets.
$f(x)=(x+2)[x^2-4x+3]$

Multiply again and combine the like terms.

$$\begin{aligned} f(x) &= x^3-4x^2+3x+2x^2-8x+6 \\ &= x^3-2x^2-5x+6 \end{aligned}$$

Thus, the required polynomial function is $f(x)=x^3-2x^2-5x+6$.

Check the result by evaluating $f(x)$ at each of its three zeros. The function should evaluate to 0 each time.

$$f(-2) = (-2)^3-2(-2)^2-5(-2)+6 = -8-8+10+6 = 0$$

$$f(1) = (1)^3-2(1)^2-5(1)+6 = 1-2-5+6 = 0$$

$$f(3) = (3)^3-2(3)^2-5(3)+6 = 27-18-15+6 = 0$$

The result checks.

Answer 22e.

We need to write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1 and the given zeros, $-5,-1,2$

Use the three zeros and the factor theorem to write $f(x)$ as a product of three factors.

$$\begin{aligned} f(x) &= (x+5)(x+1)(x-2) && \text{[Write } f(x) \text{ in factored form]} \\ &= (x+5)\left[x^2-2x+x-2\right] && \text{[Multiply]} \\ &= (x+5)\left[x^2-x-2\right] && \text{[Multiply]} \\ &= x^3-x^2-2x+5x^2-5x-10 \\ &= x^3+4x^2-7x-10 \end{aligned}$$

Thus, the required polynomial is $\boxed{f(x)=x^3+4x^2-7x-10}$

Check:

Let us check this result by evaluating $f(x)$ at each of its three zeros.

Consider

$$f(x)=x^3+4x^2-7x-10$$

Thus,

$$\begin{aligned}f(-5)&=(-5)^3+4(-5)^2-7(-5)-10\\&=-125+100+35-10\\&=0\end{aligned}$$

$$\begin{aligned}f(-1)&=(-1)^3+4(-1)^2-7(-1)-10\\&=-1+4+7-10\\&=0\end{aligned}$$

$$\begin{aligned}f(2)&=(2)^3+4(2)^2-7(2)-10\\&=8+16-14-10\\&=24-24\\&=0\end{aligned}$$

Answer 23e.

Write $f(x)$ as a product of three factors using the three zeros and the factor theorem.

$$f(x) = (x + 3)(x - 1)(x - 6)$$

Multiply $x - 1$ and $x - 6$ first.

$$f(x) = (x + 3)[x^2 - 6x - x + 6]$$

Combine the like terms within the brackets.

$$f(x) = (x + 3)[x^2 - 7x + 6]$$

Multiply again and combine the like terms.

$$\begin{aligned}f(x) &= x^3 - 7x^2 + 6x + 3x^2 - 21x + 18\\&= x^3 - 4x^2 - 15x + 18\end{aligned}$$

Thus, the required polynomial function is $f(x) = x^3 - 4x^2 - 15x + 18$.

Check the result by evaluating $f(x)$ at each of its three zeros. The function should evaluate to 0 each time.

$$f(-3) = (-3)^3 - 4(-3)^2 - 15(-3) + 18 = -27 - 36 + 45 + 18 = 0$$

$$f(1) = (1)^3 - 4(1)^2 - 15(1) + 18 = 1 - 4 - 15 + 18 = 0$$

$$f(6) = (6)^3 - 4(6)^2 - 15(6) + 18 = 216 - 144 - 90 + 18 = 0$$

The result checks.

Answer 24e.

We need to write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1 and the given zeros, $2, -i, i$

Use the three zeros and the factor theorem to write $f(x)$ as a product of three factors.

$$
\begin{aligned}
f(x) &= (x-2)[x-(-i)][x-(i)] && \text{[Write } f(x) \text{ in factored form]}\\
&= (x-2)[x+i][x-i]\\
&= (x-2)[x^2-(-1)] && \text{[Multiply]}\\
&= (x-2)[x^2+1]\\
&= x^3+x-2x^2-2 && \text{[Multiply]}\\
&= x^3-2x^2+x-2
\end{aligned}
$$

Thus, the required polynomial is $\boxed{f(x)=x^3-2x^2+x-2}$

Check:

Let us check this result by evaluating $f(x)$ at each of its three zeros.

Consider

$f(x)=x^3-2x^2+x-2$

Thus,

$$
\begin{aligned}
f(2) &= (2)^3-2(2)^2+(2)-2\\
&= 8-8+2-2\\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
f(i) &= (i)^3-2(i)^2+(i)-2\\
&= -i+2+i-2\\
&= 0
\end{aligned}
$$

Since $f(i)=0$, by the complex conjugates theorem, $f(-i)=0$

Answer 25e.

Since the coefficients are rational and $3i$ and $2-i$ are zeros of f, $-3i$ and $2+i$ must also be zeros of f by complex conjugates theorem.

Write $f(x)$ as a product of four factors using the four zeros and the factor theorem.

$f(x) = (x-3i)(x+3i)[x-(2-i)][x-(2+i)]$

Regroup the terms within the brackets.

$f(x) = (x-3i)(x+3i)[(x-2)-i][(x-2)+i]$

Multiply the expressions of the form $(a+b)(a-b)$.

$f(x) = (x^2+9)[(x-2)^2+1]$

Expand the binomial.
$f(x) = (x^2 + 9)[x^2 - 4x + 4 + 1]$

Simplify.
$f(x) = (x^2 + 9)[x^2 - 4x + 5]$

Multiply again and combine the like terms.

$$f(x) = x^4 - 4x^3 + 5x^2 + 9x^2 - 36x + 45$$
$$= x^4 - 4x^3 + 14x^2 - 36x + 45$$

Thus, the required polynomial function is $f(x) = x^4 - 4x^3 + 14x^2 - 36x + 45$.

Check the result by evaluating $f(x)$ at the zeros $3i$ and $2 - i$. The function should evaluate to 0 each time.

$$f(3i) = (3i)^4 - 4(3i)^3 + 14(3i)^2 - 36(3i) + 45 = 0$$
$$f(2-i) = (2-i)^4 - 4(2-i)^3 + 14(2-i)^2 - 36(2-i) + 45 = 0$$

As $3i$ and $2 - i$ checks, their conjugates $-3i$ and $2 + i$ also checks.

Answer 26e.

We need to write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1 and the given zeros, $-1, 2, -3i$

Complex Conjugates Theorem:
Suppose f is a polynomial function with real coefficients, and $a+ib$ is an imaginary zero of f, then $a-ib$ is also a zero of f.

Because the coefficients are rational and $-3i$ is a zero, $3i$ must also be a zero by the complex conjugates theorem. Use the three zeros and the factor theorem to write $f(x)$ as a product of three factors.

$$f(x) = (x+1)(x-2)[x+3i][x-3i] \quad \text{[Write } f(x) \text{ in factored form]}$$
$$= (x+1)(x-2)\left[x^2-(-9)\right] \quad \text{[Multiply]}$$
$$= \left(x^2-2x+x-2\right)\left(x^2+9\right) \quad \text{[Multiply]}$$
$$= \left(x^2-x-2\right)\left(x^2+9\right)$$
$$= x^4+9x^2-x^3-9x-2x^2-18 \quad \text{[Multiply]}$$
$$= x^4-x^3+7x^2-9x-18$$

Thus, the required polynomial is $\boxed{f(x)=x^4-x^3+7x^2-9x-18}$

Check:

Let us check this result by evaluating $f(x)$ at each of its three zeros.

Consider

$$f(x)=x^4-x^3+7x^2-9x-18$$

Thus,

$$\begin{aligned} f(-1)&=(-1)^4-(-1)^3+7(-1)^2-9(-1)-18 \\ &=1+1+7+9-18 \\ &=0 \end{aligned}$$

$$\begin{aligned} f(2)&=(2)^4-(2)^3+7(2)^2-9(2)-18 \\ &=16-8+28-18-18 \\ &=0 \end{aligned}$$

$$\begin{aligned} f(-3i)&=(-3i)^4-(-3i)^3+7(-3i)^2-9(-3i)-18 \\ &=81-27i-63+27i-18 \\ &=0 \end{aligned}$$

Since $f(-3i)=0$, by the complex conjugates theorem, $f(3i)=0$

Answer 27e.

Since the coefficients are rational and $4+i$ is a zero of f, $4-i$ must also be zeros of f by complex conjugates theorem.

Write $f(x)$ as a product of four factors using the four zeros and the factor theorem.

$f(x) = (x-5)(x-5)[x-(4+i)][x-(4-i)]$

Regroup the terms within the brackets.

$f(x) = (x-5)^2[(x-4)+i][(x-4)-i]$

Multiply the expressions of the form $(a+b)(a-b)$.

$f(x) = (x-5)^2[(x-4)^2+1]$

Expand the binomials.

$f(x) = (x^2-10x+25)[x^2-8x+16+1]$

Simplify.

$f(x) = (x^2-10x+25)[x^2-8x+17]$

Multiply again and combine the like terms.

$$\begin{aligned} f(x) &= x^4-8x^3+17x^2-10x^3+80x^2-170x+25x^2-200x+425 \\ &= x^4-18x^3+122x^2-370x+425 \end{aligned}$$

Thus, the required polynomial function is $f(x)=x^4-18x^3+122x^2-370x+425$.

Check the result by evaluating $f(x)$ at the zeros 5 and $4+i$. Since 5 is a repeated zero, we need to evaluate the function at 5 only once. The function should evaluate to 0 each time.

$$f(5) = (5)^4 - 18(5)^3 + 122(5)^2 - 370(5) + 425 = 0$$

$$f(4+i) = (4+i)^4 - 18(4+i)^3 + 122(4+i)^2 - 370(4+i) + 425 = 0$$

As $4+i$ checks, its conjugate $4-i$ also checks.

Answer 28e.

We need to write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1 and the given zeros, $4, -\sqrt{5}, \sqrt{5}$

Use the three zeros and the factor theorem to write $f(x)$ as a product of three factors.

$$f(x) = (x-4)\left[x-\sqrt{5}\right]\left[x-\left(-\sqrt{5}\right)\right] \quad \text{[Write } f(x) \text{ in factored form]}$$
$$= (x-4)\left[x-\sqrt{5}\right]\left[x+\sqrt{5}\right]$$
$$= (x-4)\left[x^2-5\right] \quad \text{[Multiply]}$$
$$= x^3 - 5x - 4x^2 + 20$$
$$= x^3 - 4x^2 - 5x + 20$$

Thus, the required polynomial is $\boxed{f(x) = x^3 - 4x^2 - 5x + 20}$

Check:

Let us check this result by evaluating $f(x)$ at each of its three zeros.

Consider

$$f(x) = x^3 - 4x^2 - 5x + 20$$

Thus,

$$f(4) = (4)^3 - 4(4)^2 - 5(4) + 20$$
$$= 64 - 64 - 20 + 20$$
$$= 0$$

$$f\left(\sqrt{5}\right) = \left(\sqrt{5}\right)^3 - 4\left(\sqrt{5}\right)^2 - 5\left(\sqrt{5}\right) + 20$$
$$= 5\sqrt{5} - 20 - 5\sqrt{5} + 20$$
$$= 0$$

Since $f\left(\sqrt{5}\right) = 0$, by the irrational conjugates theorem, $f\left(-\sqrt{5}\right) = 0$

Answer 29e.

Since the coefficients are rational and $2 - \sqrt{6}$ is a zero of f, $2 + \sqrt{6}$ must also be a zero of f by irrational conjugates theorem.

Write $f(x)$ as a product of four factors using the four zeros and the factor theorem.

$$f(x) = (x+4)(x-1)\left[x-\left(2-\sqrt{6}\right)\right]\left[x-\left(2+\sqrt{6}\right)\right]$$

Regroup the terms within the brackets.

$$f(x) = (x+4)(x-1)\left[(x-2)-\sqrt{6}\right]\left[(x-2)+\sqrt{6}\right]$$

Multiply the expressions of the form $(a+b)(a-b)$.

$f(x) = (x+4)(x-1)[(x-2)^2-6]$

Expand the binomial.

$f(x) = (x+4)(x-1)[x^2-4x+4-6]$

Simplify.

$f(x) = (x+4)(x-1)[x^2-4x-2]$

Multiply again and combine the like terms.

$$\begin{aligned} f(x) &= (x+4)\left(x^3-4x^2-2x-x^2+4x+2\right) \\ &= (x+4)\left(x^3-5x^2+2x+2\right) \\ &= x^4-5x^3+2x^2+2x+4x^3-20x^2+8x+8 \\ &= x^4-x^3-18x^2+10x+8 \end{aligned}$$

Thus, the required polynomial function is $f(x) = x^4-x^3-18x^2+10x+8$.

Check the result by evaluating $f(x)$ at the zeros -4, 1, and $2-\sqrt{6}$. The function should evaluate to 0 each time.

$$f(-4) = (-4)^4-(-4)^3-18(-4)^2+10(-4)+8 = 0$$

$$f(-1) = (1)^4-(1)^3-18(1)^2+10(1)+8 = 0$$

$$f\left(2-\sqrt{6}\right) = \left(2-\sqrt{6}\right)^4-\left(2-\sqrt{6}\right)^3-18\left(2-\sqrt{6}\right)^2+10\left(2-\sqrt{6}\right)+8 = 0$$

As $2-\sqrt{6}$ checks, its conjugate $2+\sqrt{6}$ also checks.

Answer 30e.

We need to write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1 and the given zeros, $-2,-1,2,3,\sqrt{11}$

Irrational Conjugates Theorem:

Suppose f is a polynomial function with rational coefficients, and a and b are rational numbers such that $\sqrt{b}$ is irrational. If $a+\sqrt{b}$ is a zero of f, then $a-\sqrt{b}$ is also a zero of f.

Because the coefficients are rational and $\sqrt{11}$ is a zero, $-\sqrt{11}$ must also be a zero by the irrational conjugates theorem. Use the three zeros and the factor theorem to write $f(x)$ as a product of three factors.

$$f(x) = (x+2)(x-2)(x+1)(x+3)\left(x-\sqrt{11}\right)\left(x+\sqrt{11}\right) \quad \text{[Write } f(x) \text{ in factored form]}$$

$$= \left(x^2-4\right)\left(x^2+3x+x+3\right)\left(x^2-11\right) \quad \text{[Multiply]}$$

$$= \left(x^4-11x^2-4x^2+44\right)\left(x^2+3x+x+3\right)$$

$$= \left(x^4-15x^2+44\right)\left(x^2+4x+3\right)$$

$$= x^6+4x^5+3x^4-15x^4-60x^3-45x^2+44x^2+176x+132$$

$$= x^6+4x^5-12x^4-60x^3-x^2+176x+132$$

Thus, the required polynomial is $\boxed{f(x) = x^6+4x^5-12x^4-60x^3-x^2+176x+132}$

Check:

Let us check this result by evaluating $f(x)$ at each of its three zeros.

Consider

$f(x) = x^6+4x^5-12x^4-60x^3-x^2+176x+132$

Thus,

$$f(-1) = (-1)^6+4(-1)^5-12(-1)^4-60(-1)^3-(-1)^2+176(-1)+132$$
$$= 1-4-12+60-1-176+132$$
$$= 193-193$$
$$= 0$$

$$f(-2) = (-2)^6+4(-2)^5-12(-2)^4-60(-2)^3-x^2+176(-2)+132$$
$$= 64-128-192+480-4-352+132$$
$$= 676-676$$
$$= 0$$

$$f(2) = (2)^6+4(2)^5-12(2)^4-60(2)^3-(2)^2+176(2)+132$$
$$= 64+128-192-480-4+352+132$$
$$= 676-676$$
$$= 0$$

$$f(-3) = (-3)^6+4(-3)^5-12(-3)^4-60(-3)^3-(-3)^2+176(-3)+132$$
$$= 729-972-972+1620-9-528+132$$
$$= 2481-2481$$
$$= 0$$

$$f\left(\sqrt{11}\right) = \left(\sqrt{11}\right)^6+4\left(\sqrt{11}\right)^5-12\left(\sqrt{11}\right)^4-60\left(\sqrt{11}\right)^3-\left(\sqrt{11}\right)^2+176\left(\sqrt{11}\right)+132$$
$$= 1331+484\sqrt{11}-1452-660\sqrt{11}-11+176\sqrt{11}+132$$
$$= 660\sqrt{11}-660\sqrt{11}+1463-1463$$
$$= 0$$

Since $f\left(\sqrt{11}\right) = 0$, by the irrational conjugates theorem, $f\left(-\sqrt{11}\right) = 0$

Answer 31e.

Since the coefficients are rational and $4 + 2i$ is a zero of f, $4 - 2i$ must also be a zero of f by complex conjugates theorem. Similarly, using irrational conjugates theorem, $1 - \sqrt{7}$ is also a zero.

Write $f(x)$ as a product of five factors using the five zeros and the factor theorem.

$$f(x) = (x-3)[x-(4+2i)][x-(4-2i)]\left[x-\left(1+\sqrt{7}\right)\right]\left[x-\left(1-\sqrt{7}\right)\right]$$

Regroup the terms within the brackets.

$$f(x) = (x-3)[(x-4)+2i][(x-4)-2i]\left[(x-1)+\sqrt{7}\right]\left[(x-1)-\sqrt{7}\right]$$

Multiply the expressions of the form $(a+b)(a-b)$.

$$f(x) = (x-3)[(x-4)^2+4][(x-1)^2-7]$$

Expand the binomials.

$$f(x) = (x-3)[x^2-8x+16+4][x^2-2x+1-7]$$

Simplify.

$$f(x) = (x-3)[x^2-8x+20][x^2-2x-6]$$

Multiply again and combine the like terms.

$$\begin{aligned} f(x) &= (x-3)\left(x^4-2x^3-6x^2-8x^3+16x^2+48x+20x^2-40x-120\right) \\ &= (x-3)\left(x^4-10x^3+30x^2+8x-120\right) \\ &= x^5-10x^4+30x^3+8x^2-120x-3x^4+30x^3-90x^2-24x+360 \\ &= x^5-13x^4+60x^3-82x^2-144x+360 \end{aligned}$$

Thus, the required polynomial function is $f(x) = x^5 - 13x^4 + 60x^3 - 82x^2 - 144x + 360$.

Check the result by evaluating $f(x)$ at the zeros 3, $4 + 2i$, and $1 + \sqrt{7}$. The function should evaluate to 0 each time.

$$f(3) = (3)^5 - 13(3)^4 + 60(3)^3 - 82(3)^2 - 144(3) + 360 = 0$$

$$f(4+2i) = (4+2i)^5 - 13(4+2i)^4 + 60(4+2i)^3 - 82(4+2i)^2 - 144(4+2i) + 360 = 0$$

$$f\left(1+\sqrt{7}\right) = \left(1+\sqrt{7}\right)^5 - 13\left(1+\sqrt{7}\right)^4 + 60\left(1+\sqrt{7}\right)^3 - 82\left(1+\sqrt{7}\right)^2 - 144\left(1+\sqrt{7}\right) + 360 = 0$$

As $4 + 2i$ and $1 + \sqrt{7}$ checks, their conjugates $4 - 2i$ and $1 - \sqrt{7}$ also checks.

Answer 32e.

We need to find the error in writing a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1 and the given zeros, 2 and $1+i$

$$
\begin{aligned}
f(x) &= (x-2)[x-(1+i)] \\
&= x(x-1-i)-2(x-1-i) \\
&= x^2 - x - ix - 2x + 2 + 2i \\
&= x^2 - (3+i)x + (2+2i)
\end{aligned}
$$

Let us consider the following Complex Conjugates Theorem to solve this problem

Complex Conjugates Theorem:

Suppose f is a polynomial function with real coefficients, and $a+ib$ is an imaginary zero of f, then $a-ib$ is also a zero of f.

Thus, by the above Complex Conjugates Theorem, if $1+i$ is a zero, then $1-i$ is also a zero.

Therefore, we need to write $f(x)$ in factored form of three zeros, $2, 1+i$ and $1-i$

That is use the three zeros $2, 1+i$ and $1-i$ and the factor theorem to write $f(x)$ as a product of three factors.

$$
\begin{aligned}
f(x) &= (x-2)[x-(1+i)][x-(1-i)] \\
&= (x-2)[(x-1)-i][(x-1)+i] \\
&= (x-2)\left[(x-1)^2 - i^2\right] \\
&= (x-2)\left[(x-1)^2 + 1\right] \\
&= (x-2)\left[x^2 - 2x + 1 + 1\right] \\
&= (x-2)\left[x^2 - 2x + 2\right] \\
&= x^3 - 2x^2 + 2x - 2x^2 + 4x - 4 \\
&= x^3 - 4x^2 + 6x - 4
\end{aligned}
$$

Thus, the required polynomial should be $\boxed{f(x)=x^3-4x^2+6x-4}$

Check:

Let us check this result by evaluating $f(x)$ at each of its three zeros.

Consider

$f(x)=x^3-4x^2+6x-4$

Thus,

$$\begin{aligned} f(2) &= (2)^3-4(2)^2+6(2)-4 \\ &= 8-16+12-4 \\ &= 20-20 \\ &= 0 \end{aligned}$$

$$\begin{aligned} f(1+i) &= (1+i)^3-4(1+i)^2+6(1+i)-4 \\ &= 1-i+3i-3-4(1-1+2i)+6+6i-4 \\ &= -2+2i-8i+6+6i-4 \\ &= 6-6+8i-8i \\ &= 0 \end{aligned}$$

Since $f(1+i)=0$, by the complex conjugates theorem, $f(1-i)=0$

Answer 33e.

Let the function be $f(x)$.

Since $-i$ is a zero, i is also a zero by complex conjugates theorem. Now, we have 4 zeros. As the required function must have degree 5, there must be 5 zeros. This implies that one of the given zeros has to a repeated zero.

Assume 1 as the repeated zero. The function can be written in factored form as
$f(x) = (x-1)^2(x-2)(x+i)(x-i)$

Multiply.

$$\begin{aligned} f(x) &= (x^2-2x+1)(x-2)(x^2+1) \\ &= (x^2-2x+1)(x^3+x-2x^2-2) \\ &= x^5+x^3-2x^4-2x^2-2x^4-2x^2+4x^3+4x+x^3+x-2x^2-2 \\ &= x^5-4x^4+6x^3-6x^2+5x-2 \end{aligned}$$

Thus, the function can be $f(x)=x^5-4x^4+6x^3-6x^2+5x-2$.
Other answers are also possible.

Answer 34e.

We need to determine the possible numbers of positive real zeros, negative real zeros and imaginary zeros for $f(x)=x^4-x^2-6$

Consider the given function:

$$f(x) = x^4 - x^2 - 6$$

Thus, the coefficients in $f(x)$ have 1 sign changes, so $f(x)$ has 1 positive real zeros.

Consider

$$f(-x) = (-x)^4 - (-x)^2 - 6$$
$$= x^4 - x^2 - 6$$

Thus, the coefficients in $f(-x)$ have 1 sign changes, so $f(x)$ has 1 negative real zeros.

The possible numbers of zeros for $f(x)$ are summarized in the table below:

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
1	1	2	4

Answer 35e.

The number of positive and negative real zeros can be determined using the Descartes' rule of signs.

By this rule, if $g(x)$ is a polynomial function with real coefficients, then the number of positive real zeros is equivalent to the number of changes in sign of the coefficients of the function or less than this by an even number.

The number of negative real zeros is equivalent to the number of changes in sign of the coefficients of $g(-x)$ or less than this by an even number.

Let us first find the number of sign changes in the given $g(x)$.

$$g(x) = -x^3 + 5x^2 + 12$$

There is only one sign change in $g(x)$, and so there is only 1 positive real zero.

Now, find $g(-x)$ and the number of sign changes in it.

$$g(-x) = -(-x)^3 + 5(-x)^2 + 12$$
$$= x^3 + 5x^2 + 12$$

There is no sign change in $g(-x)$. This implies that there is no negative real zero(s).

The total number of zeros possible is equivalent to the degree of the function, which is 3. The number of imaginary zeros will be the difference between the total number of zeros and the sum of the number of positive and negative real zeros.

Summarize the possible zeros of g in a table.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
1	0	2	3

Answer 36e.

We need to determine the possible numbers of positive real zeros, negative real zeros and imaginary zeros for $g(x)=x^3-4x^2+8x+7$

Consider the given function:

$$g(x)=x^3-4x^2+8x+7$$

Thus, the coefficients in $g(x)$ have 2 sign changes, so $g(x)$ has 2 or 0 positive real zeros.

Consider

$$\begin{aligned} g(-x)&=(-x)^3-4(-x)^2+8(-x)+7 \\ &=-x^3-4x^2-8x+7 \end{aligned}$$

Thus, the coefficients in $g(-x)$ have 1 sign changes, so $g(x)$ has 1 negative real zeros.

The possible numbers of zeros for $g(x)$ are summarized in the table below:

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
2	1	0	3
0	1	2	3

Answer 37e.

The number of positive and negative real zeros can be determined using the Descartes' rule of signs.

By this rule, if $h(x)$ is a polynomial function with real coefficients, then the number of positive real zeros is equivalent to the number of changes in sign of the coefficients of the function or less than this by an even number.

The number of negative real zeros is equivalent to the number of changes in sign of the coefficients of $h(-x)$ or less than this by an even number.

Let us first find the number of sign changes in the given function.

$$h(x) = x^5 - 2x^3 - x^2 + 6x + 5$$

Since there are two sign changes in $h(x)$, there are 2 or 0 positive real zero(s).

Now, find $h(-x)$ and the number of sign changes in it.

$$h(-x) = (-x)^5 - 2(-x)^3 - (-x)^2 + 6(-x) + 5$$
$$= -x^5 + 2x - x^2 - 6x + 5$$

There are three sign changes in $h(-x)$. This implies that there are 3 or 1 negative real zero(s).

The total number of zeros possible is equivalent to the degree of the function, which is 5. The number of imaginary zeros will be the difference between the total number of zeros and the sum of the numbers of positive and negative real zeros.

Summarize the possible zeros of h in a table.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
2	3	0	5
2	1	2	5
0	3	2	5
0	1	4	5

Answer 38e.

We need to determine the possible numbers of positive real zeros, negative real zeros and imaginary zeros for $h(x) = x^5 - 3x^3 + 8x - 10$

Consider the given function:

$h(x) = x^5 - 3x^3 + 8x - 10$

Thus, the coefficients in $h(x)$ have 3 sign changes, so $h(x)$ has 3 or 1 positive real zeros.

Consider

$$h(-x) = (-x)^5 - 3(-x)^3 + 8(-x) - 10$$
$$= -x^5 + 3x^3 - 8x - 10$$

Thus, the coefficients in $h(-x)$ have 2 sign changes, so $h(x)$ has 2 or 0 negative real zeros.

The possible numbers of zeros for $h(x)$ are summarized in the table below:

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
3	2	0	5
3	0	2	5
1	2	2	5
1	0	4	5

Answer 39e.

The number of positive and negative real zeros can be determined using the Descartes' rule of signs.

By this rule, if $f(x)$ is a polynomial function with real coefficients, then the number of positive real zeros is equivalent to the number of changes in sign of the coefficients of the function or less than this by an even number.

The number of negative real zeros is equivalent to the number of changes in sign of the coefficients of $f(-x)$ or less than this by an even number.

Let us first find the number of sign changes in the given function.

$$f(x) = x^5 + 7x^4 - 4x^3 - 3x^2 + 9x - 15$$

Since there are three sign changes in $f(x)$, there are 3 or 1 positive real zero(s).

Now, find $f(-x)$ and the number of sign changes in it.

$$f(x) = (-x)^5 + 7(-x)^4 - 4(-x)^3 - 3(-x)^2 + 9(-x) - 15$$
$$= -x^5 + 7x^4 + 4x^3 - 3x^2 - 9x - 15$$

There are two sign changes in $f(-x)$. This implies that there are 2 or 0 negative real zero(s).

The total number of zeros possible is equivalent to the degree of the function, which is 5. The number of imaginary zeros will be the difference between the total number of zeros and the sum of the numbers of positive and negative real zeros.

Summarize the possible zeros of f in a table.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
3	2	0	5
3	0	2	5
1	2	2	5
1	0	4	5

Answer 40e.

We need to determine the possible numbers of positive real zeros, negative real zeros and imaginary zeros for $g(x)=x^6+x^5-3x^4+x^3+5x^2+9x-18$

Consider the given function:

$g(x)=x^6+x^5-3x^4+x^3+5x^2+9x-18$

Thus, the coefficients in $g(x)$ have 3 sign changes, so $g(x)$ has 3 or 1 positive real zeros.

Consider

$$\begin{aligned} g(-x)&=(-x)^6+(-x)^5-3(-x)^4+(-x)^3+5(-x)^2+9(-x)-18 \\ &=x^6-x^5-3x^4-x^3+5x^2-9x-18 \end{aligned}$$

Thus, the coefficients in $g(-x)$ have 3 sign changes, so $g(x)$ has 3 or 1 negative real zeros.

The possible numbers of zeros for $g(x)$ are summarized in the table below:

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
3	3	0	6
3	1	2	6
1	3	2	6
1	1	4	6

Answer 41e.

The number of positive and negative real zeros can be determined using the Descartes' rule of signs.

By this rule, if $f(x)$ is a polynomial function with real coefficients, then the number of positive real zeros is equivalent to the number of changes in sign of the coefficients of the function or less than this by an even number.

The number of negative real zeros is equivalent to the number of changes in sign of the coefficients of $f(-x)$ or less than this by an even number.

Let us first find the number of sign changes in the given function.

$$f(x) = x^7 + 4x^4 - 10x + 25$$

Since there are two sign changes in $f(x)$, there are 2 or 0 positive real zero(s).

Now, find $f(-x)$ and the number of sign changes in it.

$$f(x) = (-x)^7 + 4(-x)^4 - 10(-x) + 25$$
$$= -x^7 + 4x^4 + 10x + 25$$

There is only one sign change in $f(-x)$. This implies that there is 1 negative real zero.

The total number of zeros possible is equivalent to the degree of the function, which is 7. The number of imaginary zeros will be the difference between the total number of zeros and the sum of the numbers of positive and negative real zeros.

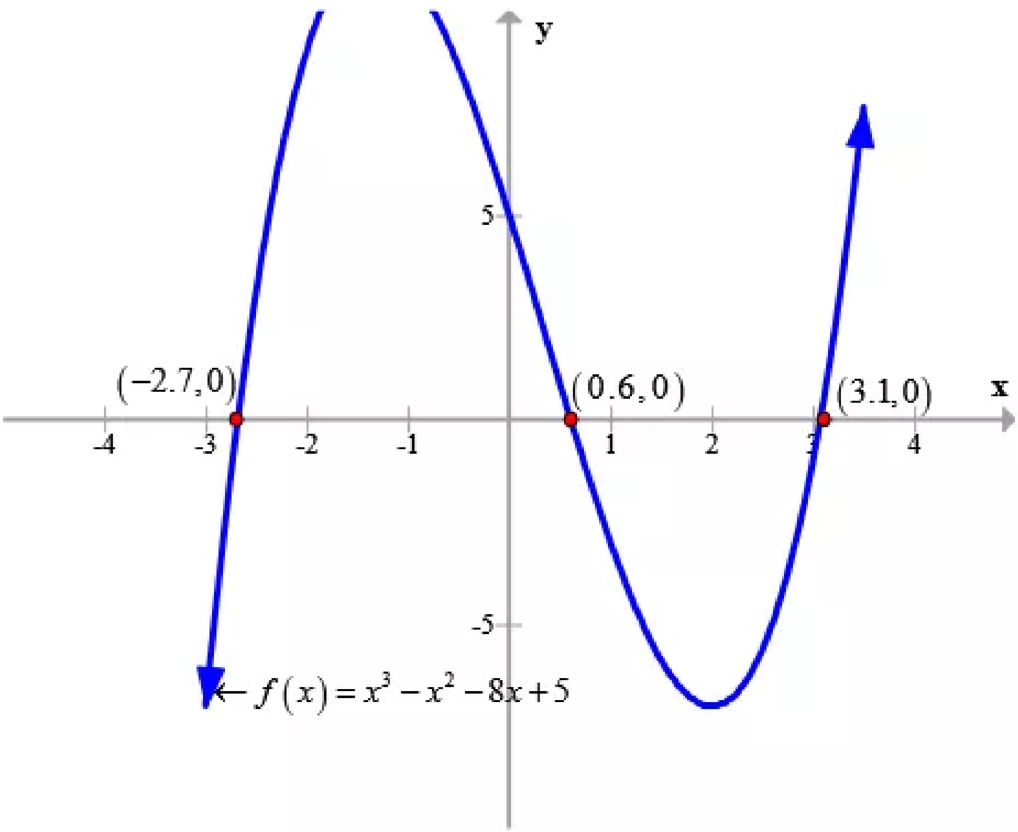
Summarize the possible zeros of f in a table.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
2	1	4	7
0	1	6	7

Answer 42e.

We need to approximate the real zeros of
$f(x) = x^3 - x^2 - 8x + 5$

Consider the given function:
$f(x) = x^3 - x^2 - 8x + 5$
Let us observe the following graph:



Thus, the real zeros of the given function are

$\boxed{x=-2.7, x=0.6, x=3.1}$

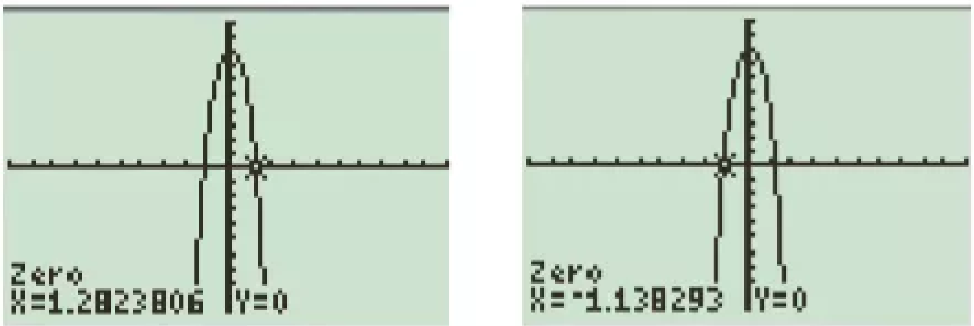
Answer 43e.

To approximate the real zeros of the following function.

$$f(x)=-x^4-4x^2+x+8$$

First, we graph the function $f(x)=-x^4-4x^2+x+8$

The screen shots of the graph of the function is shown below:



Therefore the real zeros of the functions are $\boxed{-1.2, 1.2}$

Answer 44e.

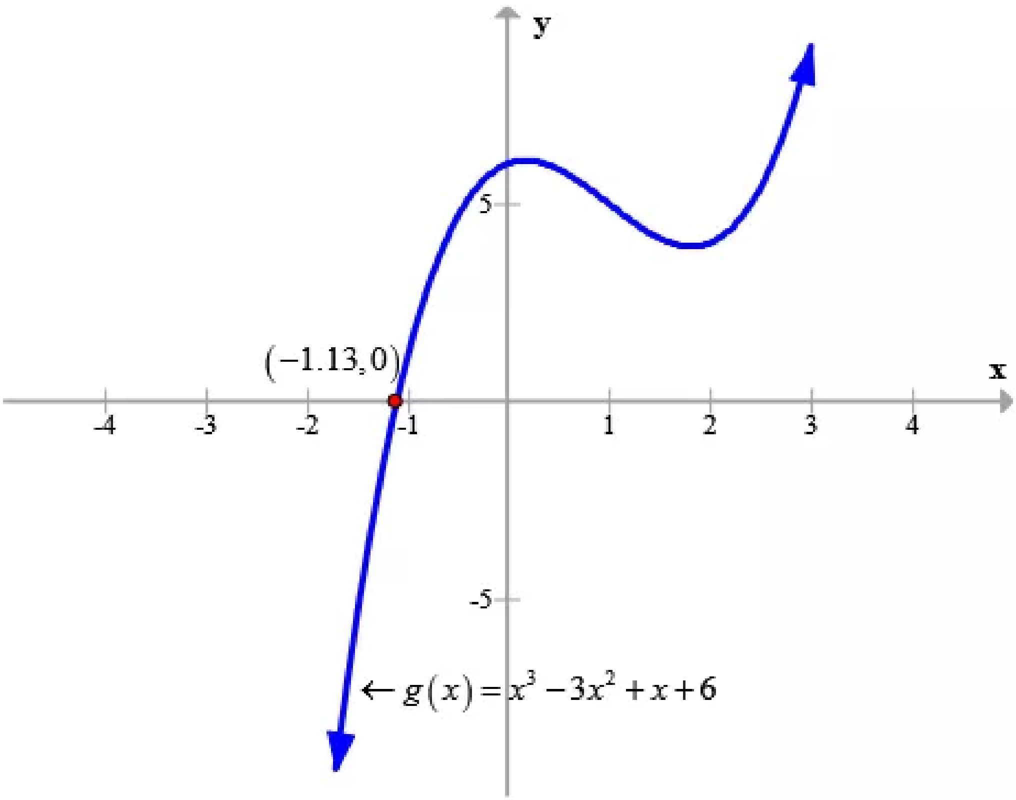
We need to approximate the real zeros of

$g(x)=x^3-3x^2+x+6$

Consider the given function:

$g(x)=x^3-3x^2+x+6$

Let us observe the following graph:



Thus, the real zero of the given function is

$\boxed{x = -1.13}$

Answer 45e.

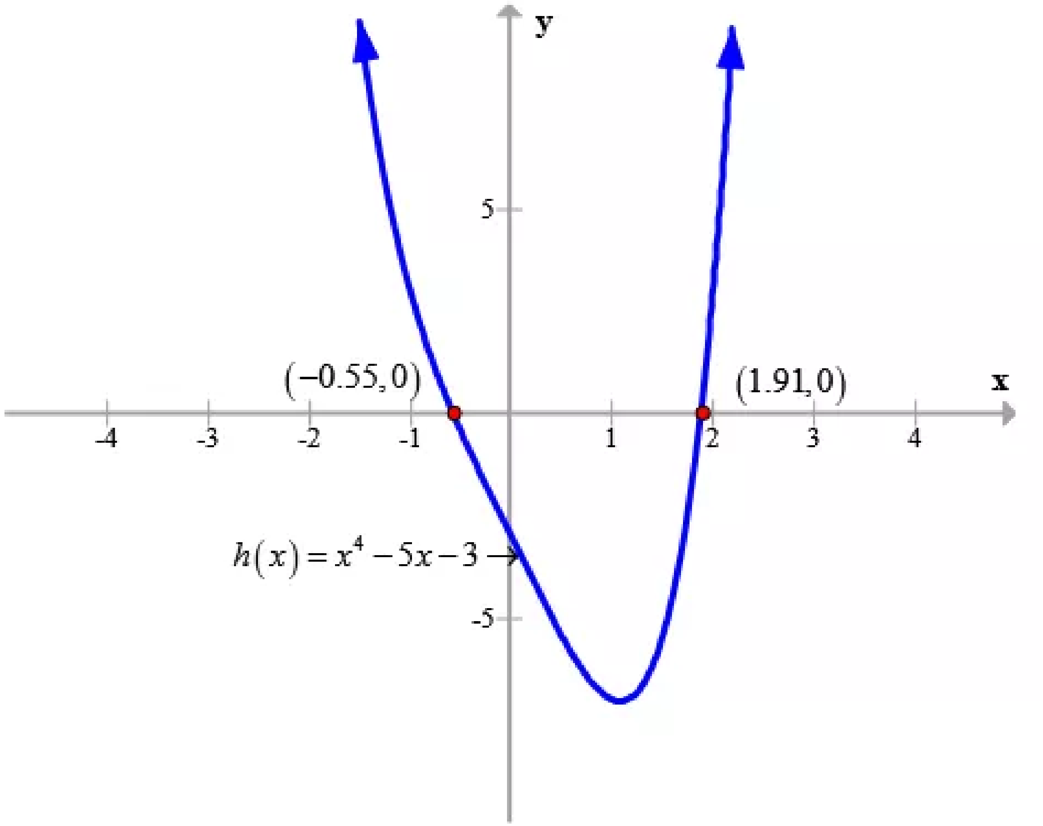
We need to approximate the real zeros of

$h(x) = x^4 - 5x - 3$

Consider the given function:

$h(x) = x^4 - 5x - 3$

Let us observe the following graph:



Thus, the real zeros of the given function are

$\boxed{x = -0.55 \text{ and } x = 1.91}$

Answer 46e.

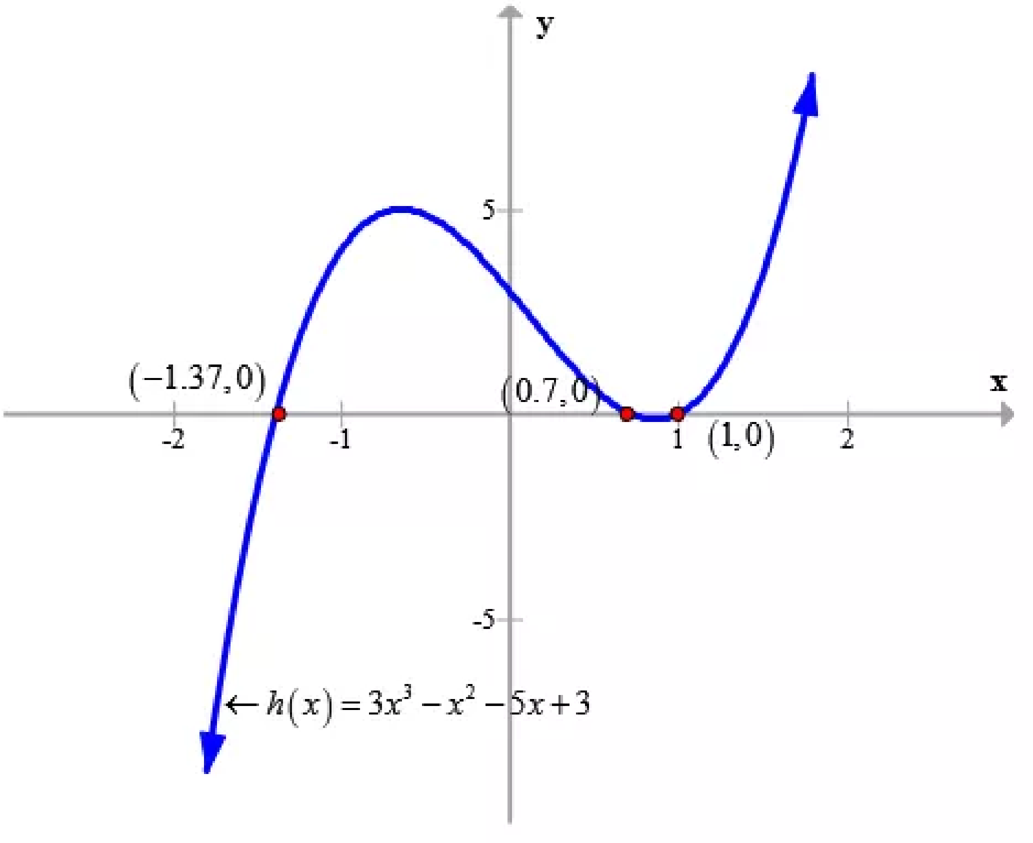
We need to approximate the real zeros of

$h(x) = 3x^3 - x^2 - 5x + 3$

Consider the given function:

$h(x) = 3x^3 - x^2 - 5x + 3$

Let us observe the following graph:



Thus, the real zeros of the given function are

$\boxed{x = -1.37, x = 0.7 \text{ and } x = 1}$

Answer 47e.

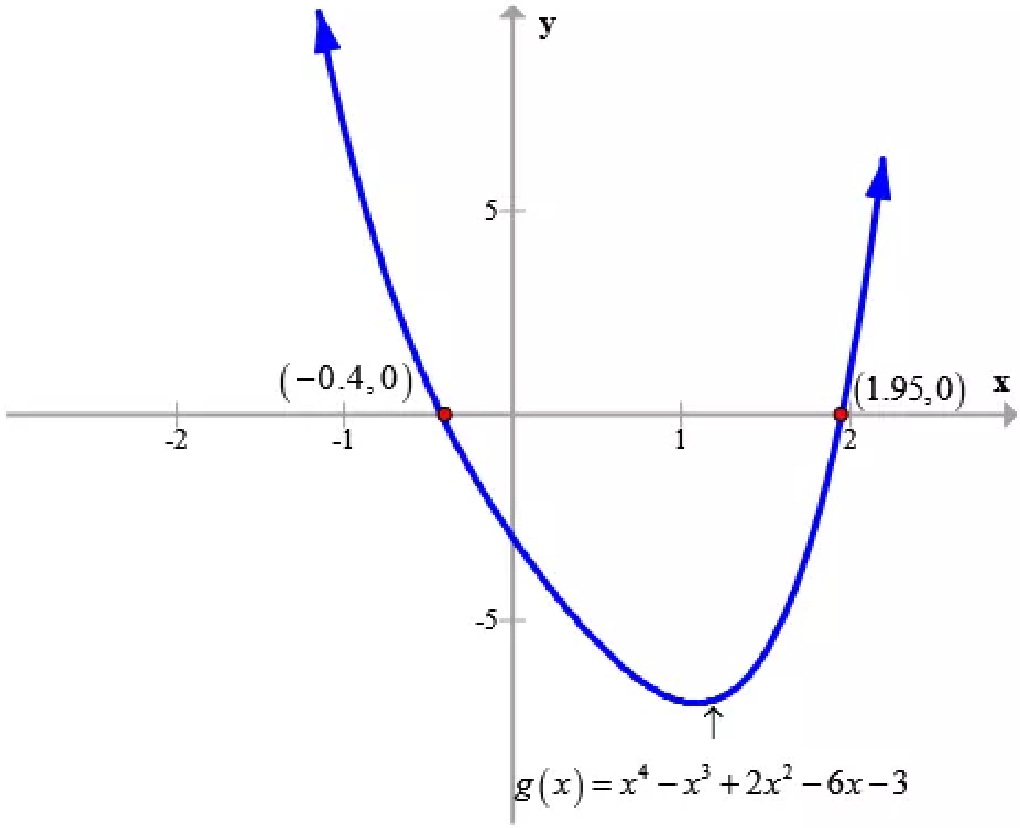
We need to approximate the real zeros of

$g(x) = x^4 - x^3 + 2x^2 - 6x - 3$

Consider the given function:

$g(x) = x^4 - x^3 + 2x^2 - 6x - 3$

Let us observe the following graph:



Thus, the real zeros of the given function are

$\boxed{x = -0.4 \text{ and } x = 1.95}$

Answer 48e.

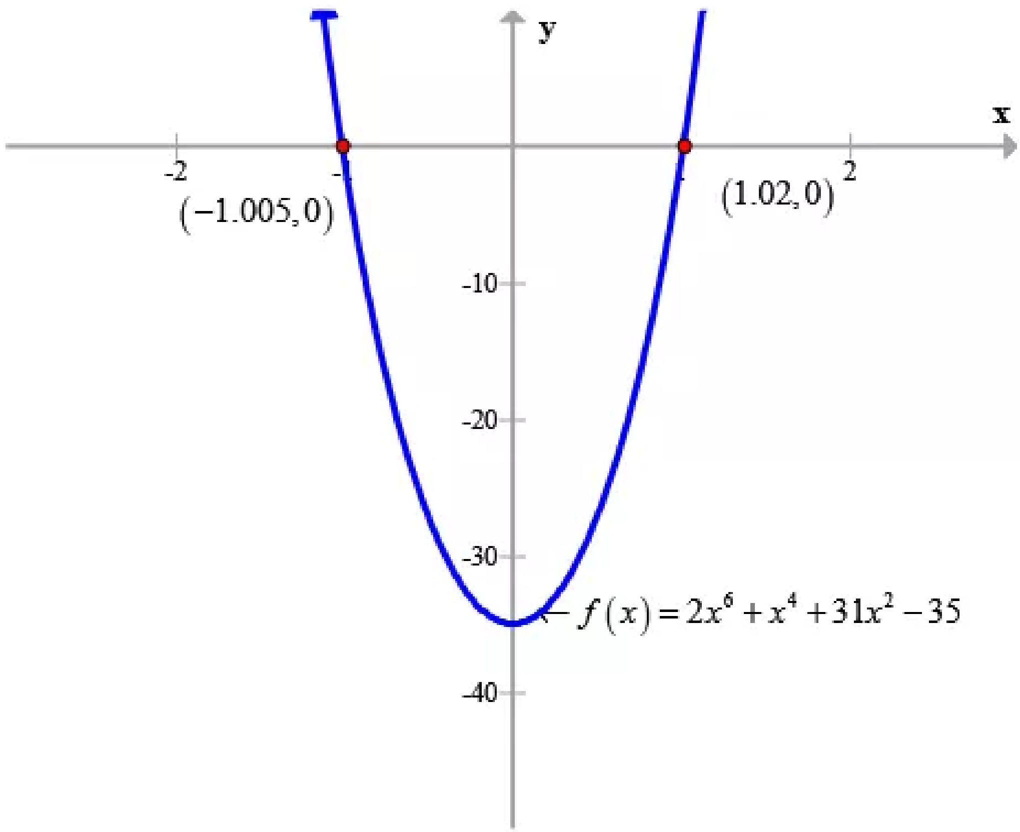
We need to approximate the real zeros of

$f(x) = 2x^6 + x^4 + 31x^2 - 35$

Consider the given function:

$f(x)=2x^6+x^4+31x^2-35$

Let us observe the following graph:



Thus, the real zeros of the given function are

$\boxed{x=-1.005 \text{ and } x=1.02}$

Answer 49e.

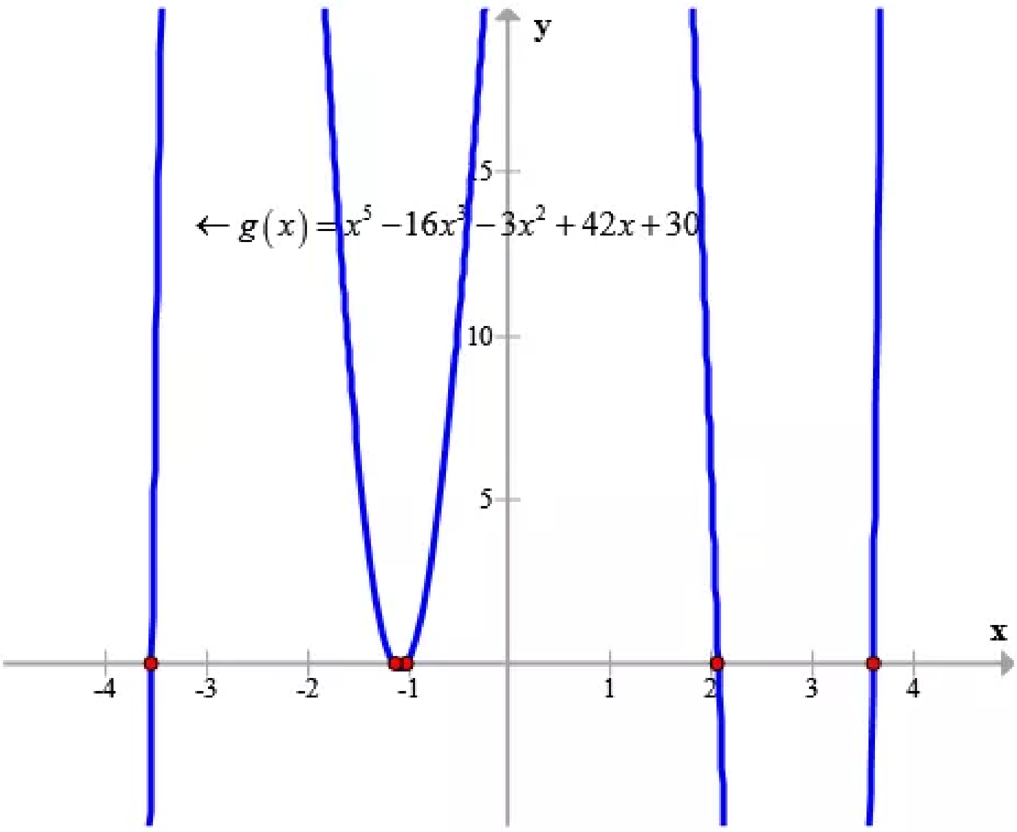
We need to approximate the real zeros of

$g(x)=x^5-16x^3-3x^2+42x+30$

Consider the given function:

$g(x)=x^5-16x^3-3x^2+42x+30$

Let us observe the following graph:



Thus, the real zeros of the given function are

$\boxed{x=-1.0056, x=-1.1265, x=-3.53, x=2.068 \text{ and } x=3.605}$

Answer 50e.

Two zeros of

$f(x)=x^3-6x^2-16x+96$

are 4 and -4.

We need to explain why the third zero must also be a real number

Consider the given function:

$f(x) = x^3 - 6x^2 - 16x + 96$

Since it is a cubic equation, there are three zeros of the function.

The coefficients in $f(x)$ have 2 sign changes, so $f(x)$ has 2 or 0 positive real zeros.

Now consider

$$f(-x) = (-x)^3 - 6(-x)^2 - 16(-x) + 96$$
$$= -x^3 - 6x^2 + 16x + 96$$

Thus, the coefficients in $f(-x)$ have 1 sign changes, so $f(x)$ has 1 negative real zeros.

The possible numbers of zeros for $f(x)$ are summarized in the table below:

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
2	1	0	3
0	1	2	3

One negative real zero, that is -4, given.

By the Complex Conjugates Theorem, imaginary zeros occur in pairs.

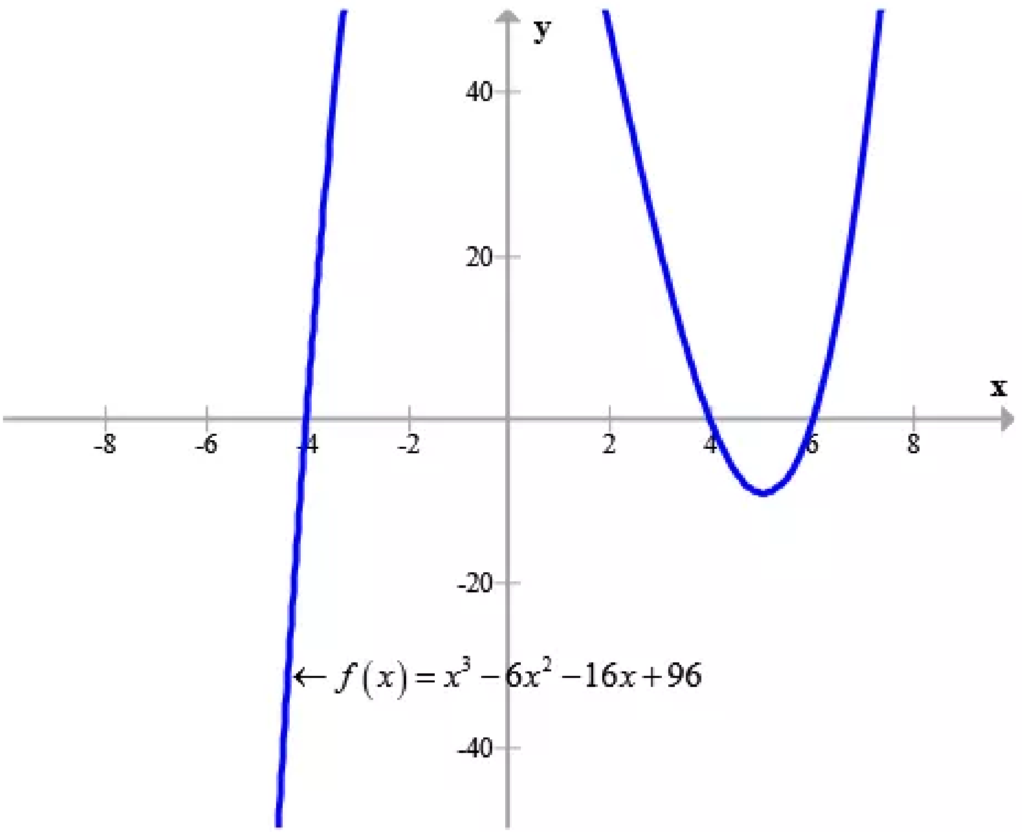
And another zero is positive real, that is 4.

Thus we have two real zeros.

The third one cannot be imaginary according to the Complex Conjugates Theorem and also due to the fact that there are 3 zeros in total.

Thus the third zero has to be real.

Let us observe the following graph:



The graph suggests result we obtained is correct. There are three real zeros of the given function.

Answer 51e.

The total number of possible zeros of a function is equivalent to the degree of the function. A cubic function has degree 3 and so it can have a maximum of 3 zeros.

The zeros can be positive or negative depending on the number of sign changes in the function. The number of imaginary zeros is the difference between the total number of zeros and the sum of the number of positive and negative real zeros.

We can write a cubic function in a variety of ways such that there could be 3, 2, 1, or 0 positive real zeros, 3, 2, 1, or 0 negative real zeros.
By the complex conjugates theorem, a function can have either even number of imaginary zeros or none. So, a cubic function can have either 2 imaginary zeros or none.

Summarize the possible zeros of the function in a table.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
3	0	0	3
2	1	0	3
1	2	0	3
1	0	2	3
0	1	2	3
0	3	0	3

Thus, there can be 3, 2, 1, or 0 positive real zeros, 3, 2, 1, or 0 negative real zeros, and 2 or 0 imaginary zeros.

Answer 52e.

We need to select the impossible classification of numbers of positive real, negative real and imaginary zeros for

$$f(x)=x^5-4x^3+6x^2+12x-6$$

from the following options:

(a) 3 positive real zeros, 2 negative real zeros and 0 imaginary zeros
(b) 3 positive real zeros, 0 negative real zeros and 2 imaginary zeros
(c) 1 positive real zeros, 4 negative real zeros and 0 imaginary zeros
(d) 1 positive real zero, 2 negative real zeros and 2 imaginary zeros

Consider the given function:

$$f(x) = x^5 - 4x^3 + 6x^2 + 12x - 6$$

Since it is a polynomial function of degree 5, there are five zeros of the function.

The coefficients in $f(x)$ have 3 sign changes, so $f(x)$ has 3 or 1 positive real zeros.

Now consider

$$f(-x) = (-x)^5 - 4(-x)^3 + 6(-x)^2 + 12(-x) - 6$$
$$= -x^5 + 4x^3 + 6x^2 - 12x - 6$$

Thus, the coefficients in $f(-x)$ have 2 sign changes, so $f(x)$ has 2 or 0 negative real zeros.

The possible numbers of zeros for $f(x)$ are summarized in the table below:

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
3	2	0	5
3	0	2	5
1	2	2	5
1	0	4	5

Compare the given classification with the data in the table.

The classification mentioned in the third option does not match with any of the other combinations and hence option (c) is the impossible classification according to Descarte's Rule of Signs.

Answer 53e.

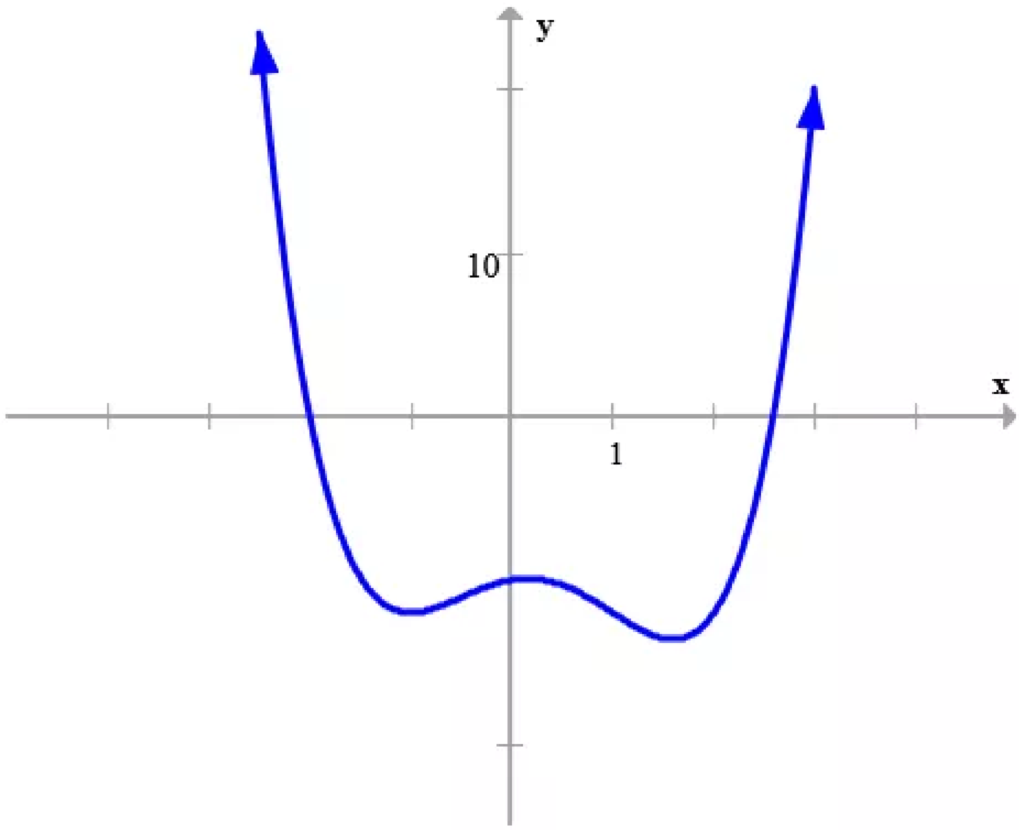
The real zeros of a function will appear as the x-intercepts on the graph of the function. The given graph has intercepts at -3, -1, and 2.

The graph crosses the negative x-axis at -3 and -1 and so they are the two negative real zeros. The graph crosses the positive x-axis at 2, which implies that there is one positive real zero. Since the degree of the function is only 3, there are no remaining zeros that will be imaginary.

Therefore, the given graph represents a function that has 1 positive real zero, 2 negative real zeros, and 0 imaginary zeros.

Answer 54e.

We need to determine the numbers of positive real zeros, negative real zeros and imaginary zeros for the following graph of the function with degree 4.



Let us observe the given graph.

A real zero is the zero where the curve crosses the x-axis.

The given graph crosses the x-axis at two points, one on the positive x-axis and the other on the negative x-axis.

Thus it has two real zeros, one positive real and the other negative real zero.

By Complex Conjugates Theorem, imaginary zeros occur in pairs.

Thus the remaining two zeros should be imaginary.

Let us summarize the classification of zeros:

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
1	1	2	4

Answer 55e.

The real zeros of a function will appear as the x-intercepts on the graph of the function. The given graph has intercepts at -2.

The graph crosses the negative x-axis at -2 and so it must be a negative real zero. The graph does not cross the positive x-axis, which implies that there are no positive real zeros. Since the degree of the function is 5, the remaining 4 zeros must imaginary.

Therefore, the given graph represents a function that has 0 positive real zero, 1 negative real zero, and 4 imaginary zeros.

Answer 56e.

Consider the function

$$f(x)=x^3-2x^2+2x+5i \qquad \ldots\ldots (1)$$

To show that $2-i$ is a zero and but the conjugate of the number, $2+i$ is not zero for the polynomial $x^3-2x^2+2x+5i$.

Putting $x=2-i$ in equation (1), we have

$$f(2-i)=(2-i)^3-2(2-i)^2+2(2-i)+5$$

$$=8-i^3-12i+6i^2-2\left(4+i^2-4i\right)+4-2i+5i$$

since $(a-b)^3=a^3-3a^2b+3ab^2-b^3$ and $(a-b)^2=a^2+b^2-2ab$

$$=8-i\left(i^2\right)-12i+6\left(i^2\right)-2\left(4+\left(i^2\right)-4i\right)+4-2i+5i$$

$$=8+i-12i-6-8-2i^2+8i+4+3i \qquad \text{since } i^2=-1$$

$$=8-11i-14+2+11i+4$$

$$=0$$

Therefore $2-i$ is a $\boxed{\text{zero}}$ of x.

Now putting the value of $x=2+i$ in equation (1), we have

$$f(2+i)=(2+i)^3-2(2+i)^2+2(2+i)+5i$$

$$=8+i^3+12i+6i^2-2\left(4+i^2+4i\right)+4+2i+5i$$

$$=8-i+12i-6-8-2i^2-8i+4+7i$$

$$=-2+2-9i$$

$$=-9i$$

Therefore, the conjugate of $2-i$ is $\boxed{\text{not a zero}}$ of $f(x)=x^3-2x^2+2x+5i$

Answer 57e.

If $-1+i$ is a zero of $g(x)$, then $g(-1+i)$ must evaluate to 0.

Replace x with $-1+i$ in $g(x)$.
$g(-1+i) = (-1+i)^3 + 2(-1+i)^2 + 2i - 2$

Evaluate.

$$\begin{aligned} g(-1+i) &= -1+3i+3-i+2(1-2i-1)+2i-2 \\ &= -1+3i+3-i+2-4i-2+2i-2 \\ &= 0 \end{aligned}$$

Thus, $-1+i$ is a zero of $g(x)$.

The conjugate of $-1+i$ is $-1-i$. Now, evaluate the function at $-1-i$.

$$\begin{aligned} g(-1-i) &= (-1-i)^3 + 2(-1-i)^2 + 2i - 2 \\ &= -1-3i+3-i+2(1+2i-1)+2i-2 \\ &= -1-3i+3-i+2+4i-2+2i-2 \\ &= 2i \end{aligned}$$

Since the function does not evaluate to 0, it is clear that the conjugate of $-1+i$ is not a zero.

Answer 58e.

To explain why the result of 56 and 57 do not contradict the complex conjugate theorem.
According to complex conjugate theorem if f is a polynomial function with real Coefficients and $a+bi$ is an imaginary zero of f, then $a-bi$ is also zero of f.
In the exercise 56, the polynomial $f(x)=x^3-2x^2+2x+5i$ is not a polynomial with real coefficients.
In the exercise 57, the polynomial $f(x)=x^3+2x^2+2i-2$ is not a polynomial with real coefficients.
Therefore the condition for the complex conjugate theorem does not satisfied.
Thus, the result of exercise 56 and 57 do not contradict the complex conjugate theorem.

Answer 59e.

Replace R with 1.5 in the given function.
$1.5 = 0.0001(-t^4 + 12t^3 - 77t^2 + 600t + 13{,}650)$

Divide both the sides by 0.0001.
$15{,}000 = -t^4 + 12t^3 - 77t^2 + 600t + 13{,}650$

Obtain a zero on one side.
$t^4 - 12t^3 + 77t^2 - 600t + 1350 = 0$

Solve the equation for t using synthetic division. Divide the polynomial using any of the possible rational zeros, say, 9.

$$\begin{array}{r|rrrrr} 9 & 1 & -12 & 77 & -600 & 1350 \\ & & 9 & -27 & 450 & -1350 \\ \hline & 1 & -3 & 50 & -150 & 0 \end{array}$$

The quotient is $x^3 - 3x^2 + 50x - 150$.

Divide the quotient again by 3.

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 50 & -150 \\ & & 3 & 0 & 150 \\ \hline & 1 & 0 & 50 & 0 \end{array}$$

The equation in t now becomes $(t-9)(t-3)(t^2+50) = 0$.

The solutions are 9, 3, and $\pm\sqrt{-50}$. Since the number of years cannot be imaginary, the solution $\pm\sqrt{-50}$ is not feasible.

Therefore, the revenue was \$1.5 million in the year 3 and year 9 since the store opened.

Answer 60e.

The number of lakes in Michigan infested with zebra mussels can be modeled by the function

$N(t) = -0.028t^4 + 0.59t^3 - 2.5t^2 + 8.3t - 2.5$ ……(1)

where t is the number years since 1990.
We need to find the year in which the number of infested inland lakes first reach 120.

That is we need to find t, when $N(t) = 120$

Substitute $N(t) = 120$ in equation (1).

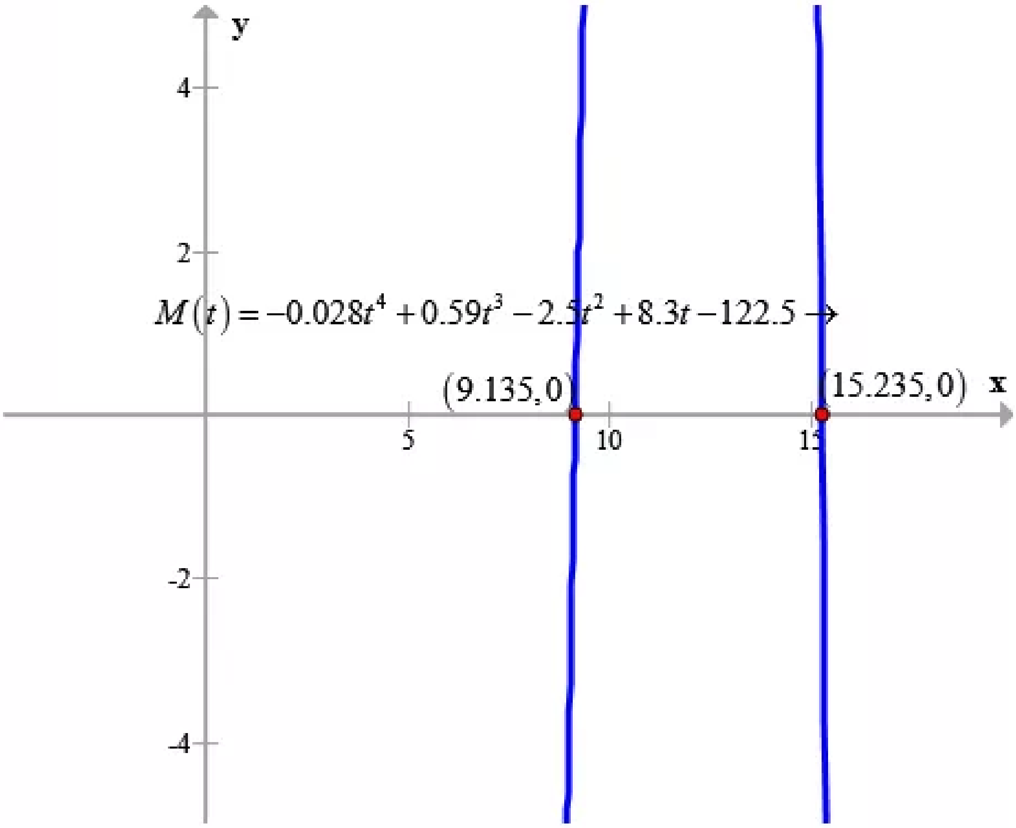
Thus, we have,

$$-0.028t^4 + 0.59t^3 - 2.5t^2 + 8.3t - 2.5 = 120$$
$$-0.028t^4 + 0.59t^3 - 2.5t^2 + 8.3t - 2.5 - 120 = 0$$
$$-0.028t^4 + 0.59t^3 - 2.5t^2 + 8.3t - 122.5 = 0$$

That is,

$-0.028t^4 + 0.59t^3 - 2.5t^2 + 8.3t - 122.5 = 0$ ……(2)

Let us sketch the graph of the above equation to estimate the real zeros of the equation (2).



Since the degree of the function

$-0.028t^4 + 0.59t^3 - 2.5t^2 + 8.3t - 122.5 = 0$

is 4, there are 4 zeros for the function.

Let us observe the given graph.

The curve crosses the x-axis at two points and hence there are two real zeros and they lie on the positive x-axis, thus both the zeros are positive.

By the Complex Conjugates Theorem, the remaining two zeros should be imaginary since imaginary zeros occur in pairs.

The graph suggests that $t = 9.135, t = 15.235$ are the two real zeros of the given function.

Thus in the year $\boxed{1999}$ and in the year $\boxed{2005}$ the number of infested inland lakes reach 120.

Answer 61e.

A person's score S on a step-climbing exercise test is modeled by the following function:

$$S(x) = -0.015x^3 + 0.6x^2 - 2.4x + 19 \quad \text{...... (1)}$$

where x is the amount of hemoglobin in grams per 100 milliliters of blood.

Given that the normal range of hemoglobin is 12-18 grams per 100 milliliters of blood.

We need to find the likely amount of hemoglobin for a person who scores 75 in the above mentioned test.

That is we need to find x, when $S(x)=75$

Substitute $S(x)=75$ in equation (1).

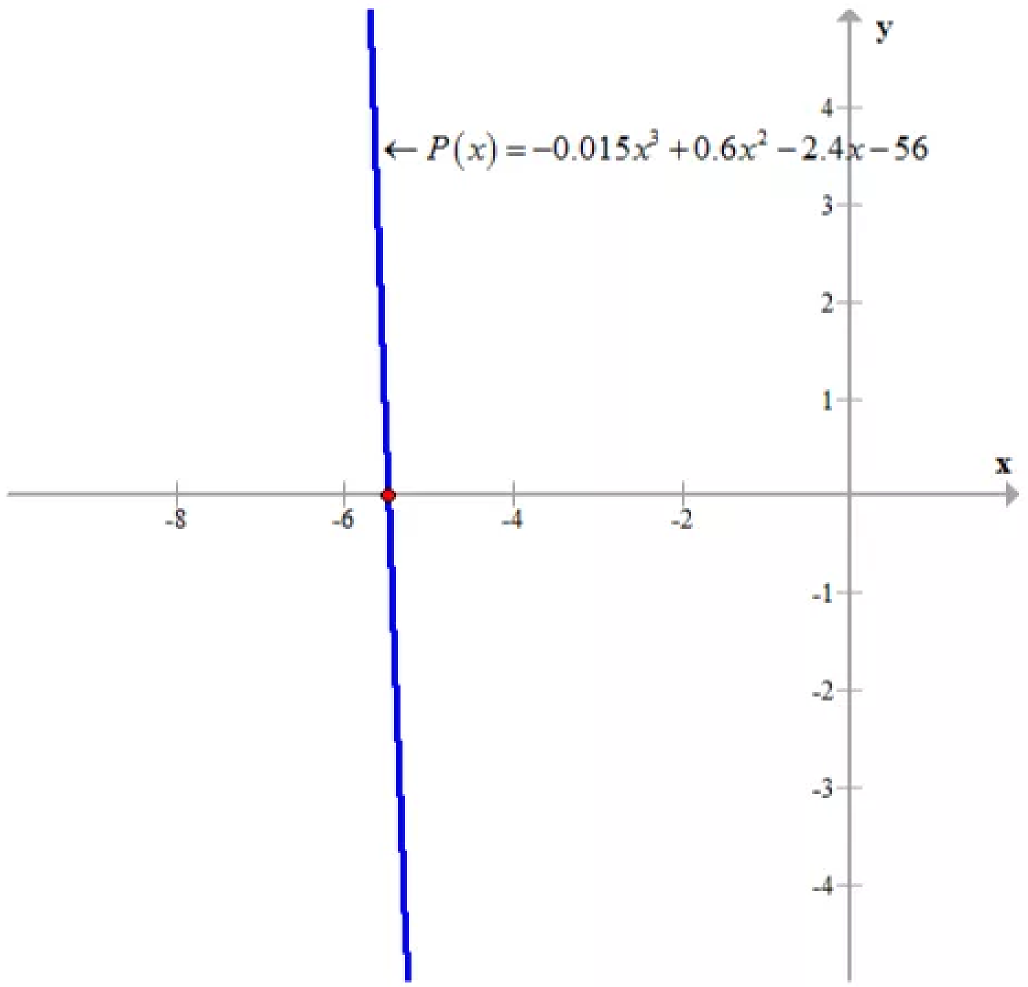
Thus, we have,

$$-0.015x^3+0.6x^2-2.4x+19=75$$
$$-0.015x^3+0.6x^2-2.4x+19-75=0$$
$$-0.015x^3+0.6x^2-2.4x-56=0$$

That is,

$$-0.015x^3+0.6x^2-2.4x-56=0 \qquad \text{...... (2)}$$

Let us sketch the graph of the above equation to estimate the real zeros of the equation (2).



Since the degree of the function

$$-0.015x^3+0.6x^2-2.4x-56=0$$

is 3, there are 3 zeros for the function.
Let us observe the given graph.
The curve crosses the x-axis at only one point and hence there is only one real zero and it lies on the negative x-axis, thus the zero is negative.
By the Complex Conjugates Theorem, the remaining two zeros should be imaginary since imaginary zeros occur in pairs.
The graph suggests that $x=-5.5$ is the only real zero of the given function.

Thus the amount of hemoglobin for a person who scores 75 in the above mentioned test is $\boxed{-5.5 \text{ grams per 100 milliliters}}$.

Answer 62e.

From 1890 to 2000, the population(in thousands) of American Indian, Eskimo and Aleut can be modeled by the function:

$$P(t) = 0.0035t^3 - 0.235t^2 + 4.87t + 243 \qquad \text{...... (1)}$$

where t is the number of years since 1890.

We need to find the year in which the population first reach 722,000

That is we need to find t, when $P(t) = 722{,}000$

Substitute $P(t) = 722{,}000$ in equation (1).

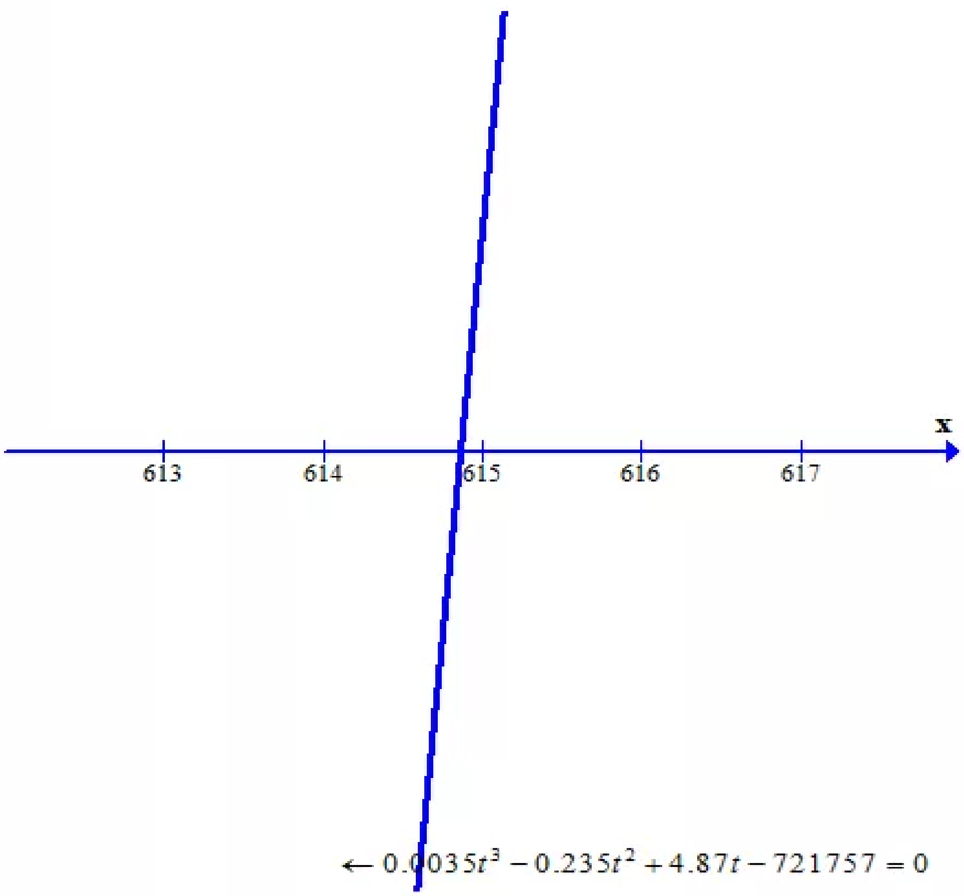
Thus, we have,

$$0.0035t^3 - 0.235t^2 + 4.87t + 243 = 722000$$
$$0.0035t^3 - 0.235t^2 + 4.87t + 243 - 722000 = 0$$
$$0.0035t^3 - 0.235t^2 + 4.87t - 721757 = 0$$

That is,

$$0.0035t^3 - 0.235t^2 + 4.87t - 721757 = 0 \qquad \text{......(2)}$$

Let us sketch the graph of the above equation to estimate the real zeros of the equation (2).



Since the degree of the function

$$0.0035t^3 - 0.235t^2 + 4.87t - 721757 = 0$$

is 3, there are 3 zeros for the function.

Let us observe the given graph.

The curve crosses the x-axis at only one point and hence there is only one real zero and it lies on the positive x-axis, thus the zero is positive.

By the Complex Conjugates Theorem, the remaining two zeros should be imaginary since imaginary zeros occur in pairs.

The graph suggests that $t = 614.857$ is the only real zero of the given function.

Therefore, after 614 years the population reaches 722000

Thus in the year $\boxed{2504}$ in which the population reach $722,000$.

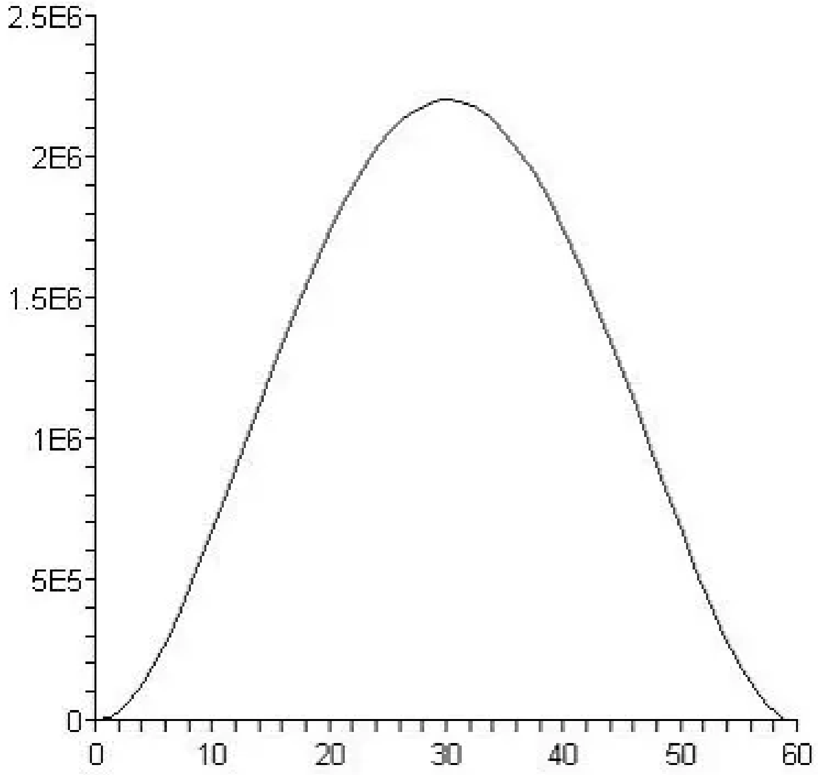
Answer 63e.

A 60-inch long bookshelf is wrapped under 180 pounds of books. The deflection *d* the bookshelf is given by

$$d = (2.724\times10^{-7})x^4 - (3.269\times10^{-5})x^3 + (9.806\times10^{-4})x^2 \quad \text{...... (1)}$$

Where *x* is the distance from the bookshelf left end. Approximate the real zeros of the function on the domain $0 \le x \le 60$.

We can draw the graph for equation (1), as below:



From the graph we can notice that the deflection $d = 0$ for $x = 0$ and $x = 60$.

When $x = 0$ and $x = 60$, books are placed at the end of the shelf. Therefore there is no deflection in this situation.

Therefore all the answers in this situation make sense.

Answer 64e.

Consider the table shows the value of deposits over the four years period. In this table, *g* is the growth factor , where *r* is the annual interest rate expressed as a decimal.

	Year 1	Year 2	Year 3	Year 4
Value of 1st deposit	1000			
Value of 2nd deposit		1000	?	?
Value of 3rd deposit			1000	?
Value of 4th deposit				1000

(a)

Now we need to complete the table.

Consider second row second column represents at end the summer of second year deposit 1000, so from growth factor it will become for next year is $1000g$ and for next year is $(1000g)g = 1000g^2$

	Year 1	Year 2	Year 3	Year 4
Value of 1st deposit	1000	$1000g$	$1000g^2$	$1000g^3$
Value of 2nd deposit	–	1000	$1000g$	$1000g^2$
Value of 3rd deposit	–	–	1000	$1000g$
Value of 4th deposit	–	–	–	1000

(a)

Now we need to write a polynomial function that gives the value *v* at the end of the forth summer in terms of *g*.

For finding the polynomial function at the end of forth summer we need to add all the deposits in forth year. Therefore, we have

$v = 1000g^3 + 1000g^2 + 1000g + 1000$ ...... (1)

Therefore the polynomial function that gives the value *v* at the end of forth summer is:

$\boxed{v = 1000g^3 + 1000g^2 + 1000g + 1000}$.

(b)

We want to buy a car that cost that cost about \$4300. Putting this value in equation (1), we have

$$4300 = 1000g^3 + 1000g^2 + 1000g + 1000$$

$$1000g^3 + 1000g^2 + 1000g - 3300 = 0 \qquad \begin{bmatrix}\text{Write in standard} \\ \text{form}\end{bmatrix}$$

$$\frac{1}{1000}\left(1000g^3 + 1000g^2 + 1000g - 3300\right) = 0 \qquad \begin{bmatrix}\text{Multipling each} \\ \text{side by } \frac{1}{1000}\end{bmatrix}$$

$$g^3 + g^2 + g - 3.3 = 0$$

To finding out the growth factor we need to solve this equation. Therefore, we can use synthetic division to solve this equation. For synthetic division we can write the equation $g^3 + g^2 + g - 3.3 = 0$ as below:

$$\begin{array}{r|rrrr} 1.05 & 1 & 1 & 1 & -3.3 \\ & & 1.05 & 2.15 & 3.3 \\ \hline & 1 & 2.05 & 3.15 & 0 \end{array}$$

So, $g = 1.05$ is one solution for the equation $g^3 + g^2 + g - 3.3 = 0$.

For the amount of \$4300 we need the growth factor $\boxed{g = 1.05}$.

Given that growth factor $g = 1 + r$, where *r* is the annual interest rate expressed as a decimal.

So we can put $g = 1.05$ in $g = 1 + r$. Then we have,

$$1.05 = 1 + r$$
$$r = 1.05 - 1$$
$$r = 0.05$$

So the annual interest rate $\boxed{r=0.5}$.

Answer 65e.

The volume of the monument must be equal to the volume of the marble.

From the figure given, we can say that the volume of the monument is the sum of the volumes of the rectangular solid base and the pyramid.

The volume of the rectangular solid base is the product of its length, breadth, and height, which we get as $(6+2x)(6+2x)x$ using the figure. The volume of the pyramid is $\frac{1}{3}Bh$ where B is the base area, which is $(2x)^2$, and h is the height of the base from the apex of the pyramid, which is x. Using this, we get the volume of the pyramid as $\frac{1}{3}(2x)^2 x$.

The volume of the monument is thus $x(6+2x)^2+\frac{1}{3}(2x)^2 x$. This value is equivalent to 1000.

$$x(6+2x)^2+\frac{1}{3}(2x)^2 x = 1000$$

Clear the fraction.

$$3x(6+2x)^2+x(2x)^2=3000$$

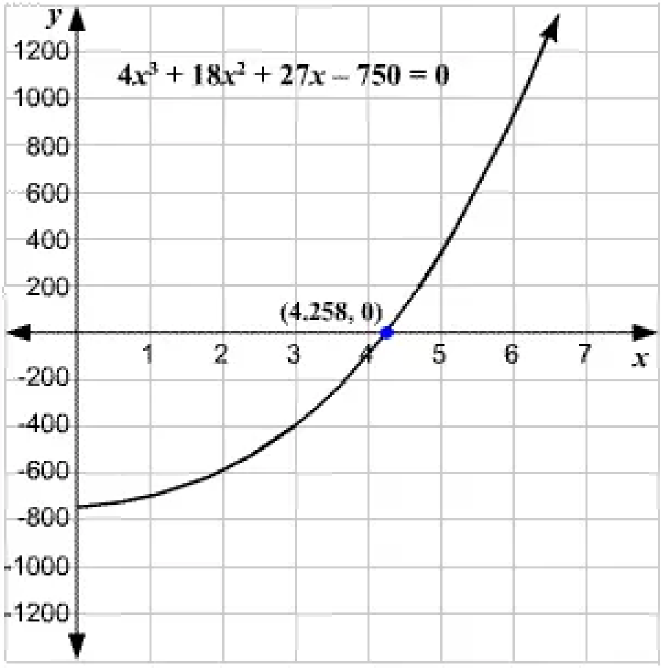
Remove the parentheses.

$$\begin{aligned} 3x\left(36+24x+4x^2\right)+4x^3 &= 3000 \\ 16x^3+72x^2+108x &= 3000 \\ 4x^3+18x^2+27x &= 750 \end{aligned}$$

Obtain a zero on one side of the equation.

$$4x^3+18x^2+27x-750=0$$

Let us draw a graph representing the equation.



We k now that the x-intercepts of the graph of an equation are the zeros or solutions of that equation. From the graph, we get the solution for our equation as about 4.258.

Therefore, the value of x is about 4.258 ft.

Answer 66e.

We need to evaluate the determinant of the following matrix:

$$\begin{bmatrix} 5 & -9 & 2 \\ 4 & 2 & 3 \\ 0 & 1 & -1 \end{bmatrix}$$

Consider the given matrix.

$$\begin{bmatrix} 5 & -9 & 2 \\ 4 & 2 & 3 \\ 0 & 1 & -1 \end{bmatrix}$$

Compute the determinant of the matrix by multiplying each element of the first row by their cofactors and then adding the results.

Thus,

$$\begin{vmatrix} 5 & -9 & 2 \\ 4 & 2 & 3 \\ 0 & 1 & -1 \end{vmatrix} = 5(-2-3)+9(-4-0)+2(4-0)$$

$$= -25-36+8$$

$$= -53$$

Thus, the determinant of the given matrix is $\boxed{-53}$

Answer 67e.

The determinant of a 3×3 matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ is given by

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

From the given matrix, we get a as 3, b as 12, c as -1, d as 5, e as 9, f as 0, g as -6, h as 4, and i as -2. Substitute for the variables in the equation.

$$\begin{vmatrix} 3 & 12 & -1 \\ 5 & 9 & 0 \\ -6 & 4 & -2 \end{vmatrix} = (-54 + 0 - 20) - (54 + 0 - 120)$$

Evaluate.

$$\begin{aligned} (-54 + 0 - 20) - (54 + 0 - 120) &= -74 - (-66) \\ &= -8 \end{aligned}$$

Therefore, the determinant of the given matrix evaluates to -8.

Answer 68e.

We need to evaluate the determinant of the following matrix:

$$\begin{bmatrix} 15 & 4 & -9 \\ 10 & 0 & 2 \\ -8 & 2 & -7 \end{bmatrix}$$

Consider the given matrix.

$$\begin{bmatrix} 15 & 4 & -9 \\ 10 & 0 & 2 \\ -8 & 2 & -7 \end{bmatrix}$$

Compute the determinant of the matrix by multiplying each element of the first row by their cofactors and then adding the results.

Thus,

$$\begin{aligned} \begin{vmatrix} 15 & 4 & -9 \\ 10 & 0 & 2 \\ -8 & 2 & -7 \end{vmatrix} &= 15(0-4) - 4(-70+16) - 9(20-0) \\ &= -60 + 216 - 180 \\ &= -24 \end{aligned}$$

Thus, the determinant of the given matrix is $\boxed{-24}$

Answer 69e.

The determinant of a 3×3 matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ is given by

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

From the given matrix, we get a as -2, b as 1, c as -3, d as 7, e as 0, f as 2, g as -6, h as 2, and i as -4. Substitute for the variables in the equation.

$$\begin{vmatrix} -2 & 1 & -3 \\ 7 & 0 & 2 \\ -6 & 2 & -4 \end{vmatrix} = (0 - 12 - 42) - (0 - 8 - 28)$$

Evaluate.

$$\begin{aligned}(0 - 12 - 42) - (0 - 8 - 28) &= -54 - (-36) \\ &= -18\end{aligned}$$

Therefore, the determinant of the given matrix evaluates to -18.

Answer 70e.

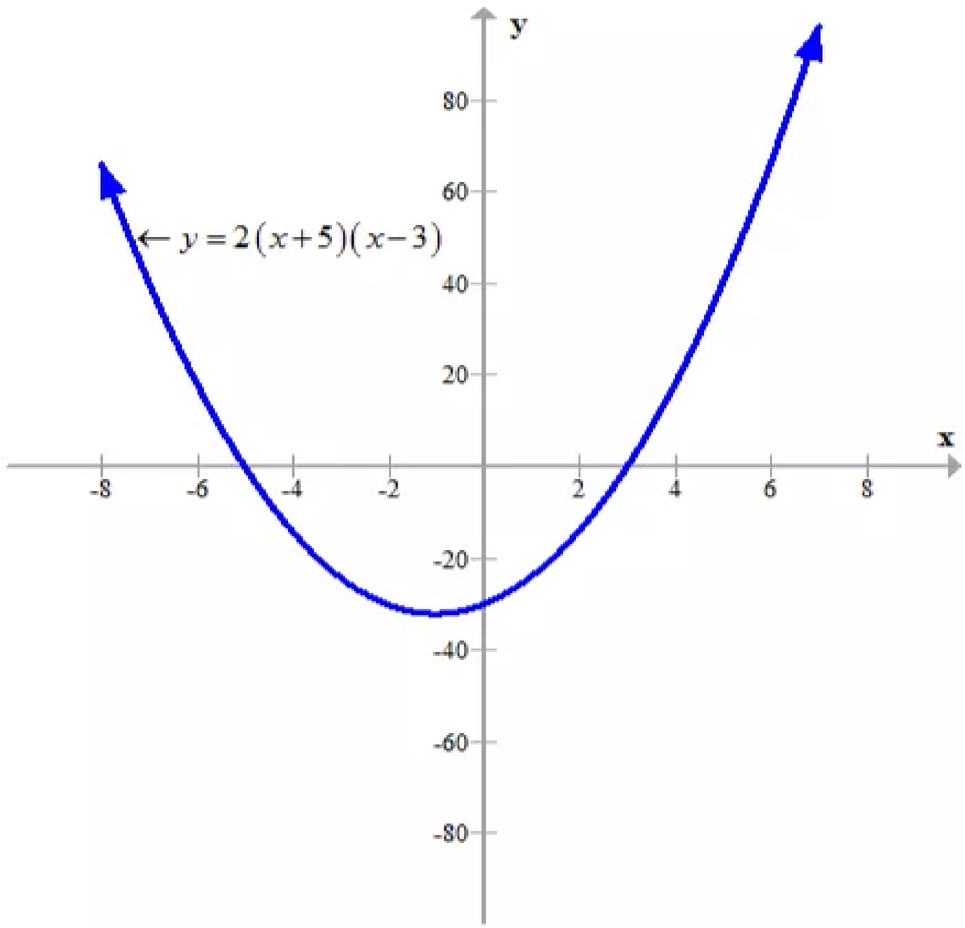
We need to graph the following function:

$$y = 2(x+5)(x-3)$$

Consider the given function.

$$y = 2(x+5)(x-3)$$

Let us sketch the given function:



Answer 71e.

STEP 1 The intercept form of a quadratic function is $y = a(x - p)(x - q)$, where p and q are the x-intercepts and $x = \frac{p+q}{2}$ is the axis of symmetry.

In order to graph the given function, first we have to identify the x-intercepts.
On comparing the given equation with the intercept form, we find that $a = 1$, $p = 2$, and $q = 9$. Thus, the x-intercepts occur at (2, 0) and (9, 0).
Since $a > 0$, the parabola opens up.

STEP 2 Then, find the coordinates of the vertex. For this, first we have to find the axis of symmetry. Substitute for p and q in $x = \frac{p+q}{2}$ and evaluate.

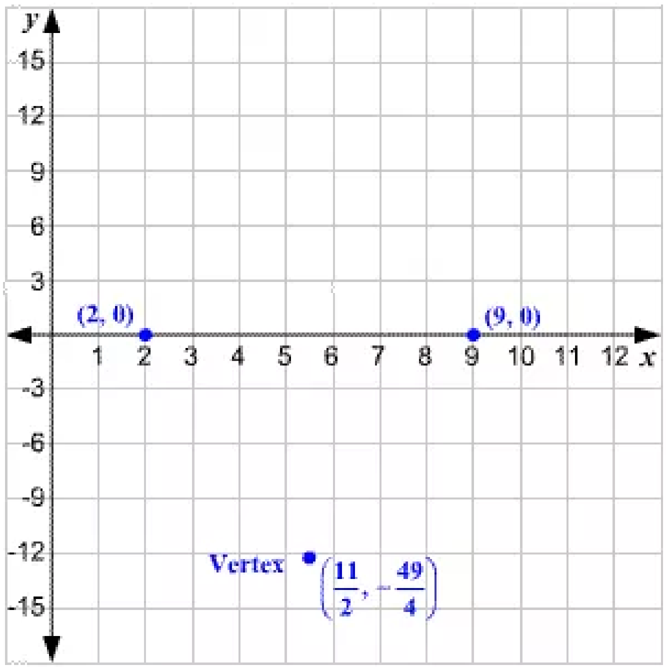
$$x = \frac{2+9}{2} = \frac{11}{2}$$

Replace x with $\frac{11}{2}$ in the given function and evaluate y.

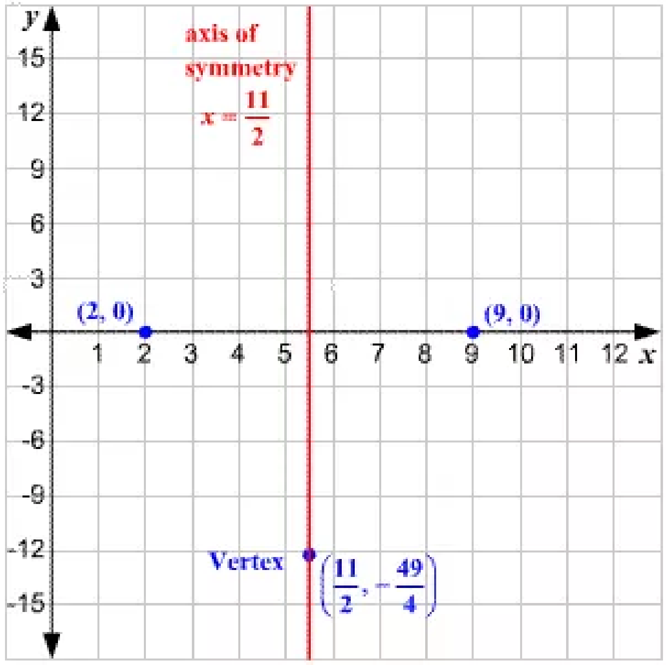
$$\begin{aligned} y &= \left(\frac{11}{2} - 2\right)\left(\frac{11}{2} - 9\right) \\ &= \left(\frac{7}{2}\right)\left(-\frac{7}{2}\right) \\ &= -\frac{49}{4} \end{aligned}$$

Thus, the vertex is $\left(\frac{11}{2}, -\frac{49}{4}\right)$.

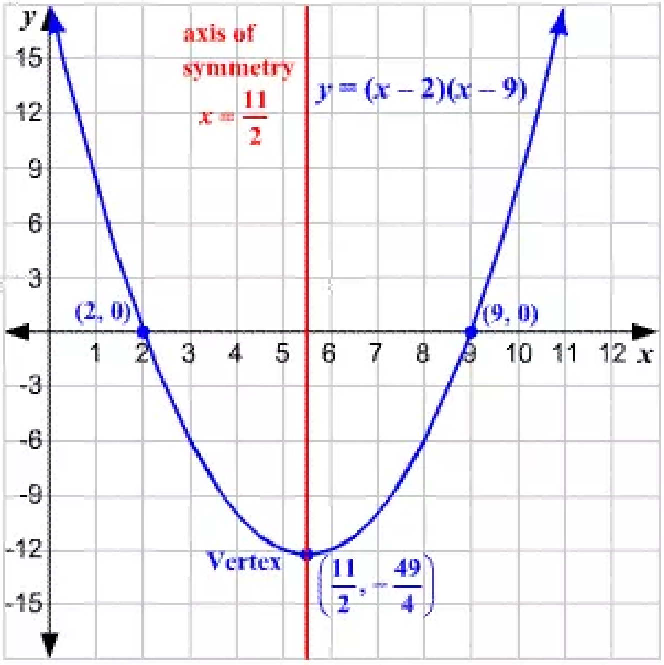
STEP 3 Now, plot the vertex and the x-intercepts on a coordinate plane.



Draw the axis of symmetry $x = \frac{11}{2}$ on the same coordinate plane.



Draw a parabola through the points plotted.



Answer 72e.

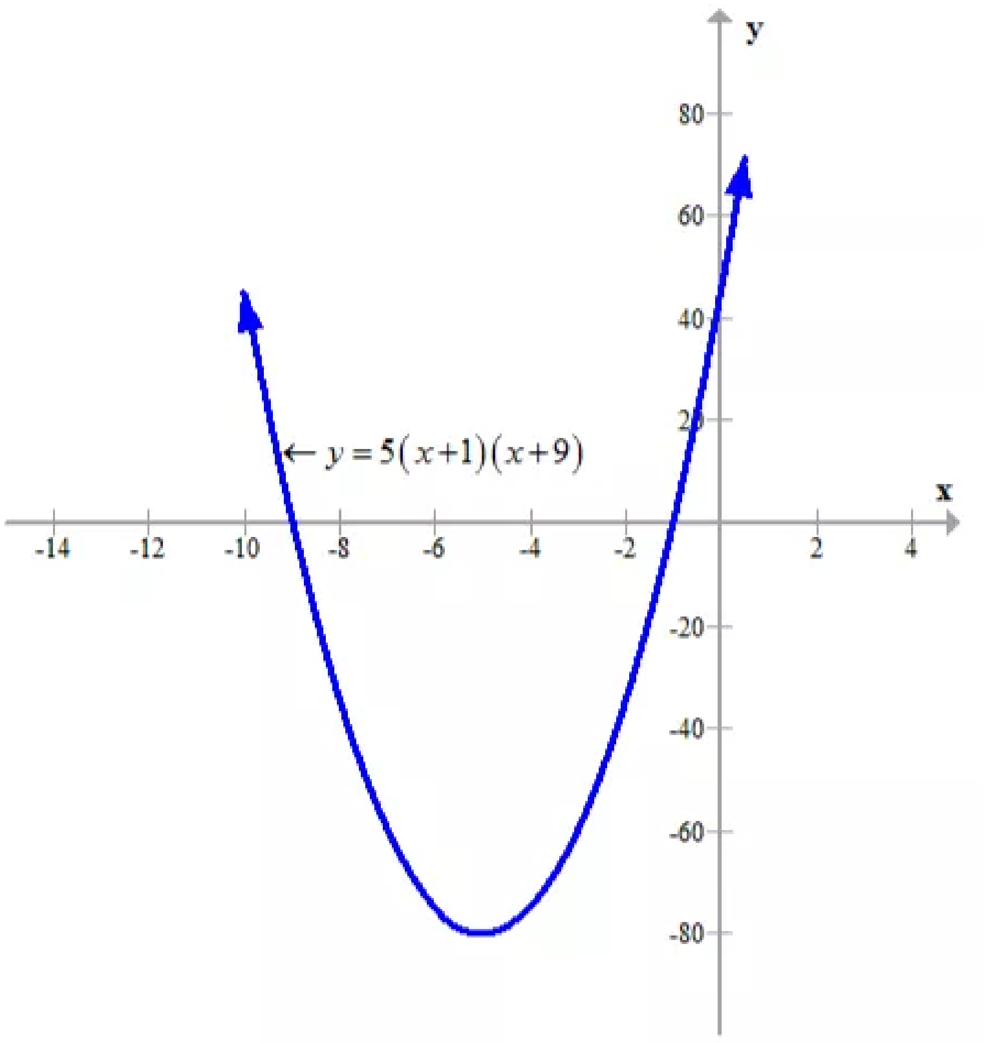
We need to graph the following function:

$$y = 5(x+1)(x+9)$$

Consider the given function.

$$y = 5(x+1)(x+9)$$

Let us sketch the given function:

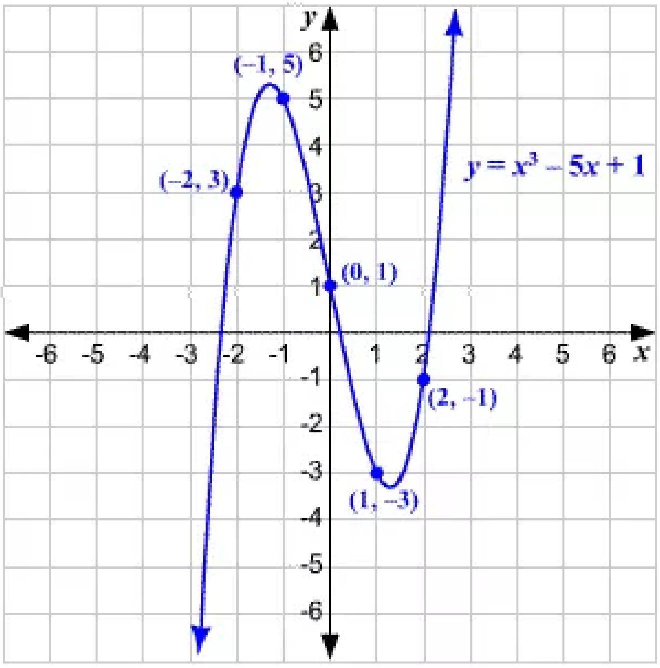


Answer 73e.

Make a table of values that satisfy the given function. For this, select some values for x and find the corresponding values of y.

x	-2	-1	0	1	2
y	3	5	1	-3	-1

Plot the points and connect them using a smooth curve.



The degree of the function is odd and the leading coefficient is positive. The end behavior of the graph is such that $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

Answer 74e.

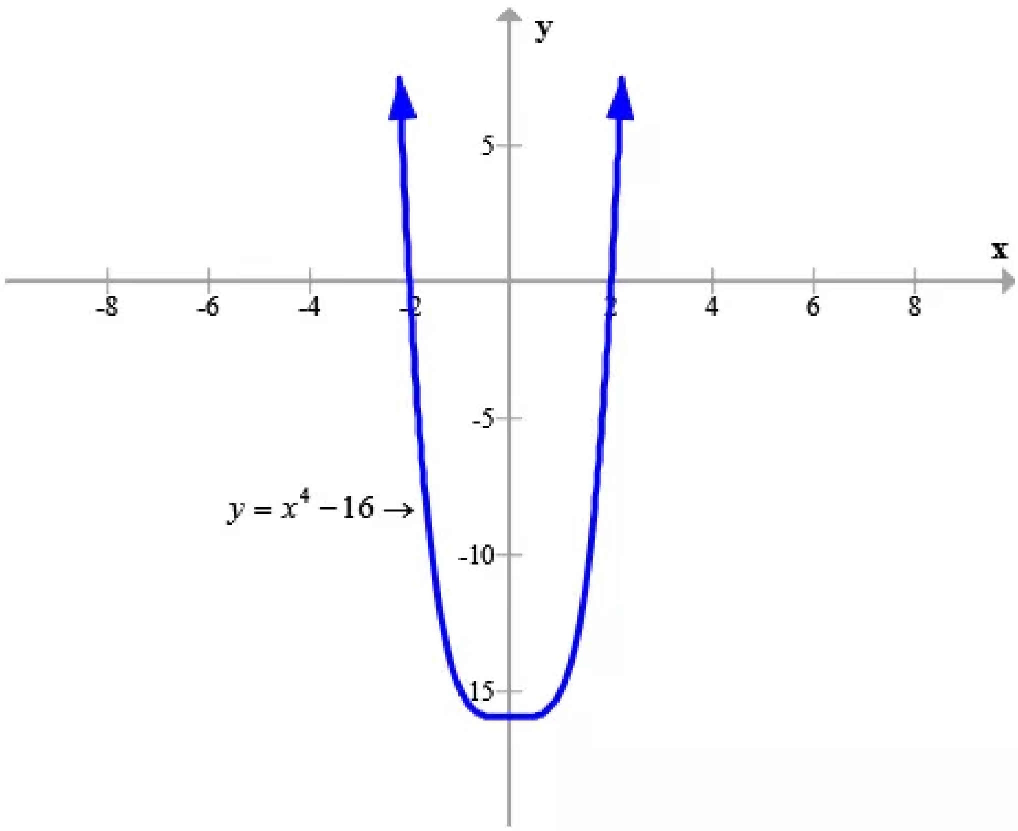
We need to graph the following function:

$$y = x^4 - 16$$

Consider the given function.

$$y = x^4 - 16$$

Let us sketch the given function:

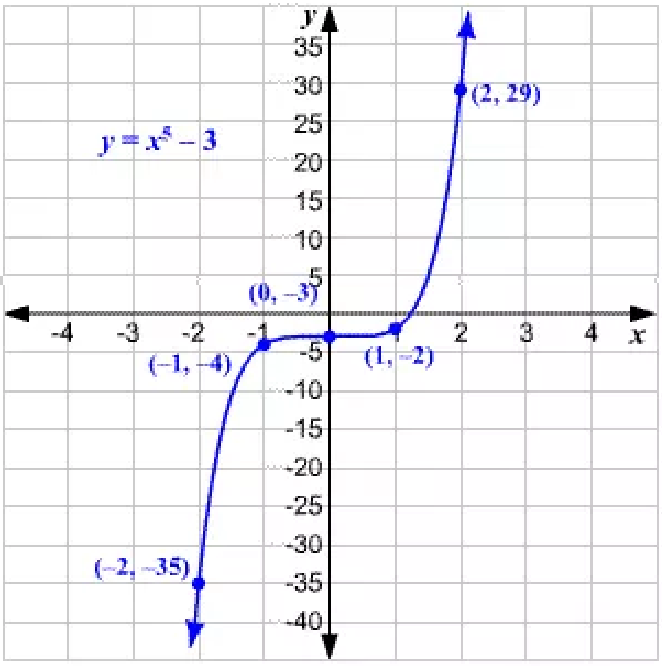


Answer 75e.

Make a table of values that satisfy the given function. For this, select some values for x and find the corresponding values of y.

x	-2	-1	0	1	2
y	-35	-4	-3	-2	29

Plot the points and connect them using a smooth curve.



The degree of the function is odd and the leading coefficient is positive. The end behavior of the graph is such that $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

Answer 76e.

We need to write a quadratic function in intercept form whose graph has the x-intercepts: $-2, 4$ and passes through the point : $(2, -4)$

The quadratic function in intercept form is

$$y = a(x-p)(x-q) \qquad \text{......(1)}$$

where $x = p$ and $x = q$ are the x-intercepts.

Substitute the values of the x-intercepts as $-2, 4$ in equation (1).

Thus, we have

$$y = a(x+2)(x-4) \qquad \text{......(2)}$$

Since the graph passes through the point $(2,-4)$, the point $(2,-4)$ satisfies the equation (2).

Therefore, we have

$$-4 = a(2+2)(2-4)$$
$$-4 = a(4)(-2)$$
$$8a = 4$$
$$a = \frac{4}{8}$$
$$a = \frac{1}{2}$$

Substitute the value $a = \frac{1}{2}$ in equation (2),

Therefore the quadratic function is

$$y = \frac{1}{2}(x+2)(x-4)$$
$$2y = (x+2)(x-4)$$

Thus the required quadratic function in intercept form is

$$\boxed{2y = (x+2)(x-4)}$$

Answer 77e.

We know that the intercept form of a quadratic function is $y = a(x-p)(x-q)$, where p and q are the intercepts.

It is given that the x-intercepts are -5 and -1. Substitute -5 for p, and -1 for q in the equation.

$$y = a[x-(-5)][x-(-1)]$$
$$y = a(x+5)(x+1)$$

The parabola passes through the point $(-2, 6)$. Substitute -2 for x, and 6 for y in the equation to find a.

$6 = a(-2+5)(-2+1)$

Solve for a.

$$6 = a(3)(-1)$$
$$6 = -3a$$
$$-2 = a$$

Substitute -2 for a in $y = a(x+5)(x+1)$.
$y = -2(x+5)(x+1)$

Therefore, the quadratic equation is $y = -2(x+5)(x+1)$.

Answer 78e.

We need to write a quadratic function in intercept form whose graph has the x-intercepts: $2, 7$ and passes through the point : $(4, -2)$

The quadratic function in intercept form is

$$y = a(x-p)(x-q) \qquad \ldots\ldots(1)$$

where $x = p$ and $x = q$ are the x-intercepts.
Substitute the values of the x-intercepts as $2, 7$ in equation (1).
Thus, we have

$$y = a(x-2)(x-7) \qquad \ldots\ldots(2)$$

Since the graph passes through the point $(4, -2)$, the point $(4, -2)$ satisfies the equation (2).
Therefore, we have

$$-2 = a(4-2)(4-7)$$
$$-2 = a(2)(-3)$$
$$6a = 2$$
$$a = \frac{2}{6}$$
$$a = \frac{1}{3}$$

Substitute the value $a = \frac{1}{3}$ in equation (2),
the required quadratic function is

$$y = \frac{1}{3}(x-2)(x-7)$$
$$3y = (x-2)(x-7)$$

Thus the quadratic function in intercept form is

$$\boxed{3y = (x-2)(x-7)}$$