Sample Question Paper - 34
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours ***Maximum Marks : 40***

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Find the roots of the equation $2x - \frac{3}{x} = 1$.

OR

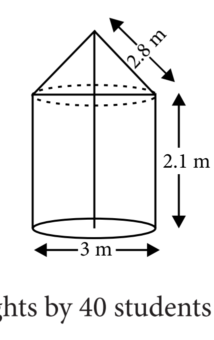
A school decided to award prizes to the most punctual and the most obedient student. The sum of the two prizes is ₹ 150 and their product is ₹ 5600. Find the prize money for punctuality and obedience.

2. If the n^{th} term of the A.P. 9, 7, 5, ... is same as the n^{th} term of the A.P. 15, 12, 9, ..., then find n.

3. A cone of height 20 cm and radius of base 5 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the diameter of the sphere.

OR

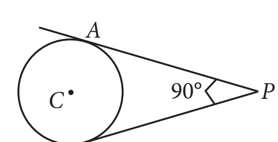
In the given figure, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of ₹500/sq. metre. $\left(\text{Use } \pi = \frac{22}{7}\right)$



4. A school keeps medical record of all the students. Following table shows the gain in weights by 40 students of class X in a year. Find the median gain in weight.

Gain in weight (in kg)	1	2	3	4	5	6	7
Number of students	3	6	8	10	3	8	2

5. In the given figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If $PA \perp PB$, then find the length of each tangent.



6. If $d_i = x_i - a$, $\sum_{i=1}^{n} f_i = 25$, $a = 250$, $\bar{x} = 250$, then find the value of $\sum_{i=1}^{n} f_i d_i$.

SECTION - B

7. If the sum of n terms of an A.P. is $(pn + qn^2)$, where p and q are constants, find the common difference.

OR

The first and last term of an A.P. is $2a$ and $4l$ respectively. If s is the sum of all terms of the A.P. then common difference is given by $\dfrac{16l^2 - ka^2}{2s-(2a+4l)}$. Find the value of k.

8. The length of a hall is 5 m more than its breadth. If the area of the floor of the hall is 84 m^2, what are the length and the breadth of the hall?

9. Draw a pair of tangents to a circle of radius 5 cm, which are inclined to each other at an angle of 60°.

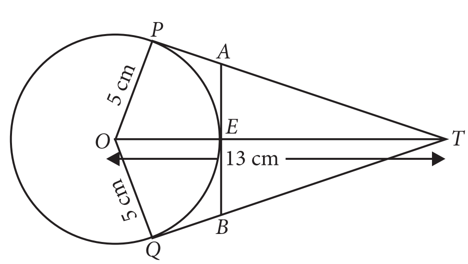
10. Two ships are anchored on opposite sides of a lighthouse. Their angles of depression as observed from the top of the lighthouse are 30° and 60°. The line joining the ships passes through the foot of the lighthouse. If the height of the lighthouse is 100 m, find the distance between the ships. (Use $\sqrt{3} = 1.732$)

SECTION - C

11. In given figure, a right triangle ABC, circumscribes a circle of radius r. If AB and BC are of length 8 cm and 6 cm respectively, find the value of r.

OR

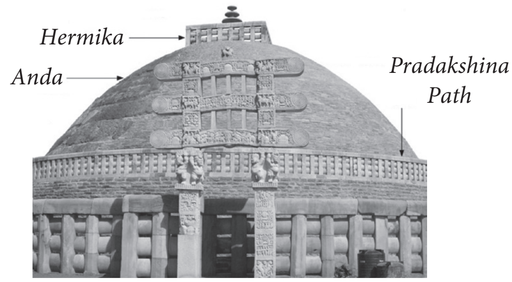
In the given figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



12. From the top of a vertical tower, the angles of depression of two cars, in the same straight line with the base of the tower, at an instant are found to be 45° and 60°. If the cars are 100 m apart and are on the same side of the tower, find the height of the tower. $[\text{Use } \sqrt{3} = 1.73]$

Case Study- 1

13. Ajay is a Class X student. His class teacher Mrs Kiran arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as *Anda*. It also contains a cubical shape part called *Hermika* at the top. Path around *Anda* is known as *Pradakshina Path*.



Based on the above information, answer the following questions.

(i) The diameter and height of the cylindrical base part are respectively 42 m and 12 m. If the volume of each brick used is 0.01 m^3, then find the number of bricks used to make the cylindrical base.

(ii) The radius of the *Pradakshina path* is 25 m. If Buddhist priest walks 14 rounds on this *path*, then find the distance covered by the priest.

Case Study- 2

14. Transport department of a city wants to buy some Electric buses for the city. For which they wants to analyse the distance travelled by existing public transport buses in a day.

The following data shows the distance travelled by 60 existing public transport buses in a day.

Daily distance travelled (in km)	199.5-209.5	209.5-219.5	219.5-229.5	229.5-239.5	239.5-249.5
Number of buses	4	14	26	10	6

Based on the above information, answer the following questions.

(i) Find the median of the distance travelled.

(ii) If the mode of the distance travelled is 223.78 km, then find the mean of the distance travelled by the bus.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. Given, $2x - \frac{3}{x} = 1 \Rightarrow \frac{2x^2 - 3}{x} = 1 \Rightarrow 2x^2 - 3 = x$

$\Rightarrow 2x^2 - x - 3 = 0 \Rightarrow 2x^2 - 3x + 2x - 3 = 0$

$\Rightarrow x(2x - 3) + 1(2x - 3) = 0$

$\Rightarrow (2x - 3)(x + 1) = 0 \Rightarrow x = -1, 3/2$

OR

Let the prize money given for punctuality be ₹ x.

$\therefore$ Prize money given for obedience = ₹ $(150 - x)$

According to question, $x(150 - x) = 5600$

$\Rightarrow 150x - x^2 = 5600$

$\Rightarrow x^2 - 150x + 5600 = 0$

$\Rightarrow (x - 70)(x - 80) = 0$

$\Rightarrow x - 70 = 0$ or $x - 80 = 0$

$\therefore x = 70$ or $x = 80$

$\Rightarrow 150 - x = 80$ or $150 - x = 70$

Hence, prize money for punctuality and obedience are ₹ 70 and ₹ 80.

2. Given A.P.s are 9, 7, 5 .... and 15, 12, 9 ....

Let a_1, d_1 and a_2, d_2 be the first terms and common difference of two A.P.s respectively. So $a_1 = 9$, $d_1 = -2$, $a_2 = 15$, $d_2 = -3$.

According to question n^{th} term of two A.P.s are same.

$\Rightarrow a_1 + (n - 1)d_1 = a_2 + (n - 1)d_2$

$\Rightarrow 9 + (n - 1)(-2) = 15 + (n - 1)(-3)$

$\Rightarrow 3(n - 1) - 2(n - 1) = 15 - 9$

$\Rightarrow (n - 1) = 6 \Rightarrow n = 7.$

3. Let the radius of the sphere be r and R, h are the radius, height of cone respectively.

Radius of the cone = 5 cm

Height of the cone = 20 cm

Now, Volume of sphere = Volume of cone

$\Rightarrow \frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^2 h \Rightarrow \frac{4}{3}\pi r^3 = \frac{1}{3}\pi(5)^2 \times (20)$

$\Rightarrow r^3 = 125 \Rightarrow r = 5$ cm

$\therefore$ Diameter of the sphere = $2r$ = 10 cm

OR

Let r and h be radius and height of cylinder respectively and l be the slant height of cone.

Total surface area of tent = $2\pi rh + \pi rl$

$= \frac{22}{7}\left[\left(2 \times \frac{3}{2} \times 2.1\right) + \left(\frac{3}{2} \times 2.8\right)\right] = \frac{22}{7} \times 10.5 = 33 \text{ m}^2$

$\therefore$ Total cost of canvas needed to make the tent = ₹(500 × 33) = ₹16500

4. Cumulative frequency table for the given data is as follows:

Gain in weight (in kg)	Number of students (f_i)	Cumulative frequency (*c.f.*)
1	3	3
2	6	3 + 6 = 9
3	8	9 + 8 = 17
4	10	17 + 10 = 27
5	3	27 + 3 = 30
6	8	30 + 8 = 38
7	2	38 + 2 = 40
Total	$\sum f_i = 40$	

Here, $n = 40 \Rightarrow n/2 = 20$

The cumulative frequency just greater than $n/2$ *i.e.*, 20 is 27 and the value of gain in weight corresponding to 27 is 4. Therefore, median gain in weight is 4 kg.

5. We know that, tangent to a circle is $\perp$ to radius at the point of contact. So, $CA \perp AP$ and $CB \perp BP$

Also, $\angle APB = 90°$

Now, in quadrilateral, $ACBP$,

$\angle ACB = 360° - (90° + 90° + 90°) = 90°$

(By angle sum property)

Now, $\angle CAP = \angle CBP = \angle ACB = \angle APB = 90°$ ...(i)

Also, $PA = PB$ ...(ii)

(Tangents drawn from an external point are equal)

So, from (i) and (ii), $ACBP$ is a square.

Hence, $PA = PB = CA = CB = 4$ cm

6. We know, mean $(\overline{x}) = a + \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$

$\Rightarrow 250 = 250 + \frac{\sum_{i=1}^{n} f_i d_i}{25} \Rightarrow \frac{\sum_{i=1}^{n} f_i d_i}{25} = 0 \Rightarrow \sum_{i=1}^{n} f_i d_i = 0$

7. Let $a_1, a_2, a_3, \ldots, a_n$ be the given A.P., then

Sum = $a_1 + a_2 + a_3 + \ldots + a_n = pn + qn^2$ ...(1)

Clearly, S_1 is the sum of single term.

S_2 is the sum of the 1st two terms
S_3 is the sum of the 1st three terms
Putting $n = 1, 2$ in (1), we get

$S_1 = a_1 = p \times (1) + q \times (1)^2 = p + q$ ...(2)

$S_2 = a_1 + a_2 = p \times (2) + q \times (2)^2 = 2p + 4q$

$\Rightarrow\ a_2 = 2p + 4q - a_1$ ...(3)

Putting $a_1 = p + q$ in (3), we get

$a_2 = 2p + 4q - (p + q) = p + 3q$

Now, the common difference,

$d = a_2 - a_1 = p + 3q - (p + q) = 2q$

OR

Let d be the common difference. First term = $2a$ and last term = $4l$

Sum of all terms = s

Now, $A + (n - 1)d = L \Rightarrow (n - 1)d = L - A$

$$\Rightarrow\ d = \frac{L-A}{n-1} \Rightarrow d = \frac{4l-2a}{n-1} \qquad \text{...(i)}$$

$$\text{Also } s = \frac{n}{2}(A+L) \Rightarrow n(A+L) = 2s$$

$$\Rightarrow\ n = \frac{2s}{L+A} \Rightarrow n = \frac{2s}{4l+2a} \qquad \text{...(ii)}$$

Putting value of n from (ii) in (i), we get

$$d = \frac{4l-2a}{\frac{2s}{4l+2a}-1} \Rightarrow d = \frac{(4l-2a)(4l+2a)}{2s-(4l+2a)}$$

$$= \frac{16l^2-4a^2}{2s-(2a+4l)} = \frac{16l^2-ka^2}{2s-(2a+4l)} \quad \therefore \quad k = 4$$

8. Let breadth of the hall = x m

Then, the length of the hall = $(x + 5)$ m

Area = $x(x + 5) = 84 \Rightarrow x^2 + 5x - 84 = 0$ ...(1)

Here $a = 1, b = 5, c = -84$

$\therefore\ D = b^2 - 4ac = (5)^2 - 4 \times 1 \times (-84) = 361$

Then, $\sqrt{D} = \sqrt{361} = 19$

Roots of the quadratic (1) are given by

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm 19}{2 \times 1} = \frac{-5+19}{2} \text{ or } \frac{-5-19}{2}$$

$\Rightarrow\ x = 7$ or -12 (We reject $x = -12$)

$\therefore\ x = 7$. Hence, breadth of the hall = 7 m

and length of the hall = 7 + 5 = 12 m.

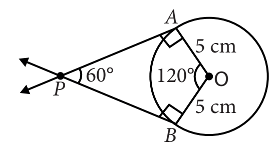
9. Steps of construction :

Step-I : Draw a circle of radius 5 cm with centre O.

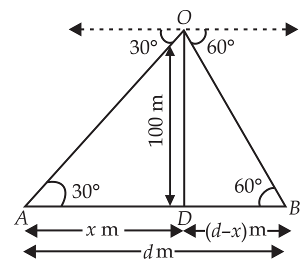
Step-II : At O construct radii OA and OB such that $\angle AOB = 120°\ (180° - 60°)$.

Step-III : Draw perpendiculars at A and B such that these perpendiculars intersect at P.

Hence, PA and PB are required tangents.



10. Let OD be the light house and A and B be two ships such that $AB = d$ m. Suppose the distance of one of the ships from the light house is x m, then the distance of the other ship from the light house is $(d - x)$ m.



In right ΔADO, we have

$$\tan 30° = \frac{OD}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{x} \Rightarrow x = 100\sqrt{3} \qquad \text{...(i)}$$

In right ΔBDO, we have

$$\tan 60° = \frac{OD}{BD} \Rightarrow \sqrt{3} = \frac{100}{d-x}$$

$$\Rightarrow\ (d-x)\sqrt{3} = 100 \Rightarrow \sqrt{3}d - x\sqrt{3} = 100$$

$$\Rightarrow\ \sqrt{3}d - 100\sqrt{3}\cdot\sqrt{3} = 100 \qquad \text{[(Using (i)]}$$

$$\Rightarrow\ \sqrt{3}d = 400 \Rightarrow d = \frac{400}{\sqrt{3}} = 230.94$$

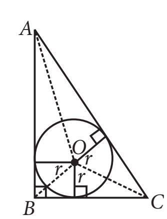
Thus, the distance between two ships is approximately 230.94 m.

11. We have, $AB = 8$ cm, $BC = 6$ cm

In right ΔABC, $AC^2 = AB^2 + BC^2$

$\Rightarrow\ AC^2 = 8^2 + 6^2 = 64 + 36 = 100$

$\Rightarrow\ AC = 10$ cm



$$\text{Area of } \Delta ABC = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$$

Also, area of ΔABC = ar (ΔAOB) + ar (ΔBOC) + ar (ΔAOC)

$$\Rightarrow\ 24 = \frac{1}{2} \times AB \times r + \frac{1}{2} \times BC \times r + \frac{1}{2} \times AC \times r$$

$$\Rightarrow\ 24 = \frac{1}{2} r[8+6+10] \Rightarrow r = \frac{48}{24} = 2 \text{ cm}$$

OR

We know, tangent to a circle is perpendicular to its radius at the point of contact.

So, $OP \perp PT$ and $OQ \perp QT$

In ΔOPT, $(OP)^2 + (PT)^2 = OT^2 \Rightarrow PT^2 = (OT)^2 - (OP)^2$

$\Rightarrow (PT)^2 = 169 - 25 = 144 \Rightarrow PT = 12$ cm

$\Rightarrow PT = QT = 12$ cm

($\because$ Tangents drawn from an external point are equal)

Let $PA = x$ cm $\Rightarrow AT = (12 - x)$cm

Hence, in right angled ΔAET

$(AE)^2 + (ET)^2 = (AT)^2$ ($\because OE \perp AB$)

$\Rightarrow (x)^2 + (8)^2 = (12 - x)^2$

($\because PA = AE$ and $ET = OT - OE$)

$\Rightarrow x^2 + 64 = 144 + x^2 - 24x$

$\Rightarrow x = \frac{80}{24} = 3.33$

In ΔAET and ΔBET

$\angle ETA = \angle ETB$

$ET = ET$ (Common)

$\angle AET = \angle BET$ (90° each, as $OE \perp AB$)

$\Rightarrow \Delta AET \cong \Delta BET$ (By *ASA* congruence)

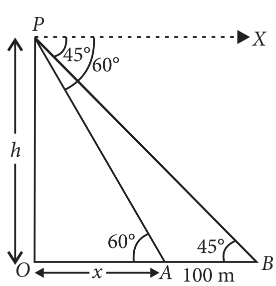
$\Rightarrow AE = EB$ (By *CPCT*)

Now, $AB = AE + EB$

$\Rightarrow AB = AE + AE$

$\Rightarrow AB = 2AE = 2 \times 3.33 = 6.66$ cm

12.



Let OP be the tower of height h m and A, B be two cars such that $AB = 100$ m.

Now in ΔOPA, $\tan 60° = \frac{OP}{OA}$

$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$ ...(i)

From ΔOPB, $\tan 45° = \frac{h}{OB} \Rightarrow 1 = \frac{h}{x + 100}$

$\Rightarrow x + 100 = h \Rightarrow h - x = 100$ ...(ii)

$\Rightarrow h - \frac{h}{\sqrt{3}} = 100$ [From (i) and (ii)]

$\Rightarrow \frac{(\sqrt{3} - 1)}{\sqrt{3}} h = 100$

$\Rightarrow h = \frac{100\sqrt{3}}{\sqrt{3} - 1} = \frac{100\sqrt{3}(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$

$= \frac{100(3 + \sqrt{3})}{2} = 50 \times (3 + 1.73) = 236.5$ m

Therefore, the height of the tower is 236.5 m.

13. (i) Diameter of cylindrical base = 42 m

$\therefore$ Radius of cylindrical base (r) = 21 m

Height of cylindrical base (h) = 12 m

$\therefore$ Number of bricks used $= \frac{\frac{22}{7} \times 21 \times 21 \times 12}{0.01}$

$= 1663200$

(ii) Given, radius of *Pradakshina Path* (r) = 25 m

$\therefore$ Perimeter of *path* = $2\pi r$

$= \left(2 \times \frac{22}{7} \times 25\right)$ m

$\therefore$ Distance covered by priest $= 14 \times 2 \times \frac{22}{7} \times 25$

$= 2200$ m

14.

Class interval	**Frequency (f_i)**	**Cumulative frequency (*c.f.*)**
199.5-209.5	4	4
209.5-219.5	14	18
219.5-229.5	26	44
229.5-239.5	10	54
239.5-249.5	6	60

(i) Here, $\sum f_i$ *i.e.*, $N = 60 \Rightarrow \frac{N}{2} = 30$

Now, the class interval whose cumulative frequency is just greater than 30 is 219.5 – 229.5.

So, median class is 219.5 – 229.5.

$\therefore$ Median $= l + \left[\frac{\frac{N}{2} - c.f.}{f}\right] \times h$

$= 219.5 + \left(\frac{30 - 18}{26}\right) \times 10$

$= 219.5 + \frac{12 \times 10}{26} = 219.5 + 4.62 = 224.12$

$\therefore$ Median of the distance travelled is 224.12 km

(ii) We know, Mode = 3 Median – 2 Mean

$\therefore$ Mean $= \frac{1}{2}$ (3 Median – Mode)

$= \frac{1}{2}$ (672.36 – 223.78) = 224.29 km