Linear Equations in Two Variables

Basic Concepts of Linear Equations In Two Variables

Recalling Linear Equations in One Variable

We know that **algebraic expressions** are those that have a few numbers, letters and operators. For example, $2x$, $3y + 4$ and $9z - \frac{1}{2}$ are all algebraic expressions and the letters x, y and z are the variables in the expressions.

If an algebraic expression is used for equating two different values or expressions, then it becomes an equation. For example, $2x = 4$, $3y + 4 = 2y$ and $9z - \frac{1}{2} = 4$ are all equations.

Now, consider the equation $2x = 4$. It has only one variable term, i.e., $2x$. The **exponent** of variable x is 1 and this is the highest exponent in the equation. We know that an equation having the highest exponent as 1 is known as a linear equation; so, $2x = 4$ is a linear equation. Also, since the equation has only one variable x, it is a **linear equation in one variable**. Similarly, $3y + 4 = 2y$ and $9z - \frac{1}{2} = 4$ are also linear equations in one variable.

There are also equations having more than one variable. In this lesson, we will learn about linear equations in two variables.

Introduction to Linear Equations in Two Variables

A linear equation comprising two different variables is called a **linear equation in two variables**. Let us consider the equation $C = \frac{5}{9}(F - 32)$. This equation is used to compare the temperatures on the Celsius (C) and Fahrenheit (F) scales.

In the equation, C and F are both variables; thus, it is an equation in two variables. Also, the degree of the equation is 1, so it is a linear equation in two variables.

Other examples of linear equations in two variables: $3x - 4y = 4$, $\frac{1}{2}y + 2z = 10$ and $p - q = -\frac{3}{4}$

The general form of a linear equation in two variables is **a*x* + b*y* + c = 0**. Here, *x* and *y* are variables while a, b and c are constants.

Concept Builder

The highest exponent of a variable involved in an equation is the degree of that equation.

For example, in the equation 3*y* + 4 = 2*y*, the highest exponent of variable *y* is 1; so, the degree of the equation is 1, or we can say that it is a **first-degree equation**.

Did You Know?
−40° is the only point at which the Celsius and Fahrenheit scales coincide. So, −40°C = −40°F

Solved Examples

Easy

Example: Identify the linear equations in two variables among the following equations.

i) $2x+5=0$

ii) $3D\sqrt{t^2}+D^2=89D$

iii) $4x^2+9y=54$

iv) $\frac{x}{y}=\frac{6}{34}$

v) $\frac{x}{2t}=\frac{z}{3}$

Solution:

i) Since the equation $2x+5=0$ consists of only one variable *x*, it is not a linear equation in two variables.

ii) The equation $3D\sqrt{t^2}+D^2=89D$ can be reduced to the general form of a linear equation in two variables, i.e., a*x* + b*y* + c = 0.

$$3D\sqrt{t^2} + D^2 = 89D$$
$$\Rightarrow 3Dt + D^2 = 89D \qquad (\because \sqrt{t^2} = t)$$
$$\Rightarrow D(3t + D) = 89D$$
$$\Rightarrow 3t + D = 89$$

The equation $3t + D = 89$ is a first-degree equation and consists of two variables *t* and *D*. Thus, it is a linear equation in two variables.

iii) The equation $4x^2 + 9y = 54$ consists of two variables *x* and *y*, but its degree is 2. Hence, it is not a linear equation in two variables.

iv) The equation $\frac{x}{y} = \frac{6}{34}$ can be reduced to the general form of a linear equation in two variables, i.e., ax + by + c = 0.

$$\frac{x}{y} = \frac{6}{34}$$

$\Rightarrow 34x = 6y$

$\Rightarrow 34x - 6y = 0$

The equation $34x - 6y = 0$ is a first-degree equation and consists of two variables *x* and *y*. Thus, it is a linear equation in two variables.

v) The equation $\frac{x}{2t} = \frac{z}{3}$ consists of three variables *x*, *t* and *z*; so, it is not a linear equation in two variables.

Expression of Situations In Linear Equations In Two Variables

Linear Equations in Real Life

Let us see what Arya and Madhuri are talking about.

Let us try to find Madhuri's age from what she has just told Arya.

We know that Madhuri's age is 12 less than twice Arya's age. So, we have two values to equate: 'Madhuri's age' and '12 less than twice Arya's age'.

Madhuri's age = 12 less than twice Arya's age

⇒ Madhuri's age = Twice Arya's age − 12

Here, Madhuri's age and Arya's age are two quantities that are related to each other and can change in value; so, we can represent them by using variables.

Thus, you can see how we need to form equations to solve such problems from our daily life. In this lesson, we will learn to represent real-life situations as linear equations.

Expressing Situations as Linear Equations in Two Variables

Sunita and Priyanka go shopping. On their way, Priyanka tells Sunita, 'I have Rs 150 more than two-fifth the amount you have.' Let us represent this situation mathematically.

As per what Priyanka says, if Sunita has Rs 200, then Priyanka has Rs 150 more than two-fifth of Rs 200, i.e., Rs $\left[\left(\frac{2}{5}\times 200\right)+150\right]$ = Rs 230. Similarly, if Sunita has Rs 500 then Priyanka has Rs $\left[\left(\frac{2}{5}\times 500\right)+150\right]$ = Rs 350.

So, the amount that Priyanka has depends upon the amount that Sunita has. Here, we have two unknown values, i.e., 'the amount with Priyanka' and 'the amount with Sunita'. Let us suppose that Priyanka has Rs x and Sunita has Rs y.

Now, two-fifth the amount with Sunita = Rs $\left(\frac{2}{5}y\right)$

Rs 150 more than the two-fifth the amount with Sunita = $\text{Rs}\left(\frac{2}{5}y+150\right)=\text{Rs}\left(\frac{2y+750}{5}\right)$

According to the given condition, we have:

The amount with Priyanka = Rs 150 more than two-fifth the amount with Sunita

$\Rightarrow x=\frac{2y+750}{5}\ \Rightarrow 5x-2y=750$

This linear equation in the two variables *x* and *y* is the mathematical representation of the given situation. Similarly, other real-life situations can be expressed as linear equations.

Solved Examples

Easy

Example 1: **Express the following situations as linear equations in two variables.**

i) The cost of three DVDs and five CDs is Rs 400.

ii) The perimeter of a rectangular garden is 230 m.

iii) The opposite angles of a parallelogram are $(2x + y)°$ and $(x − 3y + 100)°$.

Solution:

i) Here, we have two unknown quantities: 'the cost of a DVD' and 'the cost of a CD'.

Let the cost of each DVD be Rs *x* and that of each CD be Rs *y*.

∴ Cost of 3 DVDs = Rs $3x$

And, cost of 5 CDs = Rs $5y$

It is given that the cost of 3 DVDs and 5 CDs is Rs 400.

∴ $3x + 5y = 400$

This is the linear equation in two variables representing the given situation.

ii) We know that the perimeter of a rectangle is related to the length and breadth of the rectangle. In the given situation, the length and breadth are unknown values.

Let the length and breadth of the rectangular garden be *x* m and *y* m respectively.

Therefore, perimeter of the garden = 2 ($x + y$) m

It is given that the perimeter of the garden is 230 m.

$$\therefore 2(x+y)=230$$
$$\Rightarrow x+y=115$$

This is the linear equation in two variables representing the given situation.

iii) It is given that the opposite angles of the parallelogram are $(2x + y)°$ and $(x - 3y + 100)°$.

We know that the opposite angles of a parallelogram are equal.

$$\therefore 2x+y=x-3y+100$$
$$\Rightarrow x+4y=100$$

This is the linear equation in two variables representing the given situation.

Medium

Example 1: **Express the following situations as linear equations in two variables.**

i) The sum of a two-digit number and the number obtained by interchanging the digits is 121.

ii) Five years ago, a man's age was two years more than seven times his son's age at that time.

iii) Seven rupees are available in twenty-five paise and fifty paise coins.

Solution:

i) A two-digit number contains a digit in the ones place and a digit in the tens place. Here, both the digits are unknown values.

Let the ones-place digit be *x* and the tens-place digit be *y*.

Therefore, two-digit number = $10y + x$

On interchanging the digits, the ones-place digit and the tens-place digit of the two-digit number are y and x respectively.

Therefore, new two-digit number = $10x + y$

It is given that the sum of the two-digit number and the number obtained by interchanging the digits is 121.

$$\therefore (10y + x) + (10x + y) = 121$$
$$\Rightarrow 11x + 11y = 121$$
$$\Rightarrow 11(x + y) = 121$$
$$\Rightarrow x + y = 11$$

This is the linear equation in two variables representing the given situation.

ii) Let x represent the current age of the man and y represent the current age of his son.

Age of the man 5 years ago = $(x - 5)$ years

Age of the son 5 years ago = $(y - 5)$ years

7 times the age of the son 5 years ago = $7(y - 5)$ years

Two years more than 7 times the age of the son 5 years ago = $[7(y - 5) + 2]$ years

According to the question, we have:

$$x - 5 = 7(y - 5) + 2$$
$$\Rightarrow x - 5 = 7y - 35 + 2$$
$$\Rightarrow x - 5 = 7y - 33$$
$$\Rightarrow x - 7y = -28$$

This is the linear equation in two variables representing the given situation.

iii) Here, we have two unknown quantities: the number of 25 paise coins and the number of 50 paise coins.

Let the number of 25 paise coins be x and that of 50 paise coins be y.

Therefore, total amount = $(25x + 50y)$ paise

It is given that the total amount is Rs 7 or 700 paise.

$$\therefore 25x+50y=700$$
$$\Rightarrow 25(x+2y)=25\times 28$$
$$\Rightarrow x+2y=28$$

This is the linear equation in two variables representing the given situation.

Hard

Example 1: **The sum of the digits of a two-digit number is 14. If we add 18 to the original number, the digits get interchanged. Write two equations for these two statements.**

Solution:

Let the digit in the tens place be x and the digit in the ones place be y. Thus, the number can be expressed as $10x + y$.

According to the question, we have:

$$10x+y=14 \qquad ...(1)$$

When 18 is added to the number $10x + y$, the digits get interchanged.

$$\therefore (10x+y)+18=10y+x$$
$$\Rightarrow 9y-9x=18$$
$$\Rightarrow 9(y-x)=18$$
$$\Rightarrow y-x=2 \qquad ...(2)$$

Equations (1) and (2) are the required equations.

Example 2: **The area of a rectangle decreases by 15 cm^2 when the length is increased by 3 cm and the breadth is decreased by 1 cm. Express this scenario as a linear equation in two variables.**

Solution:

We know that the area of a rectangle is related to the length and breadth of the rectangle. In this case, the length and breadth of the rectangle are unknown values.

Let the length and breadth of the rectangle be x cm and y cm respectively.

Therefore, area of the rectangle = xy cm^2

When the length is increased by 3 cm and the breadth is decreased by 1 cm, the new rectangle thus formed will have:

Length = (x + 3) cm

And, breadth = (y − 1) cm

∴ Area of the new rectangle = (x + 3) (y − 1) cm^2

It is given that the area of the new rectangle is 15 cm^2 less than that of the old rectangle.

$$\therefore (x+3)(y-1) = xy-15$$
$$\Rightarrow x(y-1)+3(y-1) = xy-15$$
$$\Rightarrow xy-x+3y-3 = xy-15$$
$$\Rightarrow -x+3y-3 = -15$$
$$\Rightarrow x-3y = 12$$

This is the linear equation in two variables expressing the given scenario.

Solutions of Linear Equations In Two Variables

The Need to Solve Linear Equations

A car moving with **uniform speed** travels for two hours. How can we find the distance travelled by the car?

Speed of a moving object is given by the formula

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

This formula can be reduced to

Distance = Speed × Time

Let us represent 'Distance' by D, 'Speed' by S and 'Time' by T.

Now, the formula becomes $D = ST$

In the given situation, $T = 2$ hours

So, the formula becomes $D = 2S$.

This is a linear equation in the two variables D and S. If any of these variables is known, then the other can be calculated easily. The numeric values of D and S are known as the solution of the equation $D = 2S$.

In this lesson, we will learn to:

• Check the solutions of linear equations

• Find the solutions of linear equations

Solution of a Linear Equation

Solved Examples

Easy

Example 1: Which of the following ordered pairs are solutions of the equation $2x - y = 6$?

i)(0, 6)

ii)(0, −6)

iii)(−3, 0)

iv)(1, −4)

v)(2, 2)

Solution:

We know that the solution of a linear equation in two variables satisfies the equation, i.e., after putting the values of x and y in the equation, we obtain LHS = RHS. For each of the given ordered pairs, let us check whether it is a solution of the equation $\mathbf{2x - y = 6}$.

i)On substituting $x = 0$ and $y = 6$ on the left hand side of the given equation, we obtain:

$2 \times 0 - 6 = -6$

The right hand side of the given equation is 6.

So, LHS ≠ RHS

Therefore, (0, 6) is not a solution of $2x - y = 6$.

ii)On substituting $x = 0$ and $y = -6$ on the left hand side of the given equation, we obtain:

$2 \times 0 - (-6) = 6$

The right hand side of the given equation is 6.

So, LHS = RHS

Therefore, (0, −6) is a solution of $2x - y = 6$.

iii)On substituting $x = -3$ and $y = 0$ on the left hand side of the given equation, we obtain:

$2 \times (-3) - 0 = -6$

The right hand side of the given equation is 6.

So, LHS ≠ RHS

Therefore, (−3, 0) is not a solution of $2x - y = 6$.

iv)On substituting $x = 1$ and $y = -4$ on the left hand side of the given equation, we obtain:

$2 \times 1 - (-4) = 2 + 4 = 6$

The right hand side of the given equation is 6.

So, LHS = RHS

Therefore, $(1, -4)$ is a solution of $2x - y = 6$.

v)On substituting $x = 2$ and $y = 2$ on the left hand side of the given equation, we obtain:

$2 \times 2 - 2 = 4 - 2 = 2$

The right hand side of the given equation is 6.

So, LHS ≠ RHS

Therefore, $(2, 2)$ is not a solution of $2x - y = 6$.

Hence, we have $(0, -6)$ and $(1, -4)$ as solutions of the equation $2x - y = 6$.

Medium

Example 1: Find the value of m for which $(1, -1)$ is a solution of the equation $mx + 5y = 1$.

Solution:

It is given that $(1, -1)$ is a solution of the equation $mx + 5y = 1$. We know that the solution of a linear equation in two variables satisfies the equation, i.e., after putting the values of x and y in the equation, we obtain LHS = RHS.

So,

$m(1) + 5(-1) = 1$

$\Rightarrow m - 5 = 1$

$\Rightarrow m - 5 + 5 = 1 + 5$

$\Rightarrow m = 6$

Solved Examples

Easy

Example 1: Find four different solutions of the equation $4x + 5y = 20$.

Solution:

The given linear equation in two variables is $4x + 5y = 20$. We can obtain the solution of the equation by substituting the value of one variable and obtaining the value of the other variable.

If we take $x = 0$, we obtain:

$4 \times 0 + 5y = 20$

$\Rightarrow 5y = 20$

$\Rightarrow \therefore y = 4$

So, (0, 4) is a solution of the given equation.

If we take $y = 0$, we obtain:

$4x + 5 \times 0 = 20$

$\Rightarrow 4x = 20$

$\Rightarrow \therefore x = 5$

So, (5, 0) is a solution of the given equation.

If we take $x = 1$, we obtain:

$4 \times 1 + 5y = 20$

$\Rightarrow 4 + 5y = 20$

$\Rightarrow 5y = 16$

$\Rightarrow \therefore y = \frac{16}{5}$

So, $\left(1, \frac{16}{5}\right)$ is a solution of the given equation.

If we take $x = 3$, we obtain:

$4 \times 3 + 5y = 20$

$\Rightarrow 12 + 5y = 20$

$\Rightarrow 5y = 20 - 12$

$\Rightarrow 5y = 8$

$\Rightarrow \therefore y = \frac{8}{5}$

So, $\left(3, \frac{8}{5}\right)$ is a solution of the given equation.

Thus, we have (0, 4), (5, 0), $\left(1, \frac{16}{5}\right)$ and $\left(3, \frac{8}{5}\right)$ as four different solutions of the equation $4x + 5y = 20$.

Example 2: Find three integer solutions of the linear equation $4x - 3y = 4$.

Solution:

The given linear equation in two variables is $4x - 3y = 4$. To get integral solutions, we just need to consider the integer values of the variables *x* and *y*.

On substituting $x = 1$, we obtain:

$4 \times 1 - 3y = 4$,

$\Rightarrow 3y = 0$

$\Rightarrow \therefore y = 0$

So, (1, 0) is a solution of the given equation.

On substituting $y = -4$, we obtain:

$4x - 3 \times (-4) = 4$

$\Rightarrow 4x = -8$

$\Rightarrow \therefore x = -2$

So, (−2, −4) is a solution of the given equation.

On substituting $x = 4$, we obtain:

$4 \times 4 - 3y = 4$

$\Rightarrow 16 - 3y = 4$

$\Rightarrow 3y = 12$

$\Rightarrow \therefore y = 4$

So, (4, 4) is a solution of the given equation.

Thus, we have (1, 0), (−2, −4) and (4, 4) as three integral solutions of the equation $4x - 3y = 4$.

Medium

Example 1: **The length of a rectangular field is 3 m more than twice the width of the field. Express this situation as a linear equation in two variables and find three possible dimensions of the given field.**

Solution:

Let x and y denote the length and breadth of the rectangular field respectively.

According to the question, we have:

$x = 2y + 3$

This is the required linear equation in two variables.

On substituting $x = 6$ m, we get:

$6 = 2y + 3$

$\Rightarrow 2y = 3$

$\Rightarrow y = 1.5$ m

On substituting $y = 3$ m, we get:

$x = 2 \times 3 + 3$

$\Rightarrow x = 9$ m

On substituting $y = 7$ m, we get:

$x = 2 \times 7 + 3$

$\Rightarrow x = 17$ m

Thus, we have three possible dimensions of the field as follows:

i)Length = 6 m

Breadth = 1.5 m

ii)Length = 9 m

Breadth = 3 m

iii)Length = 17 m

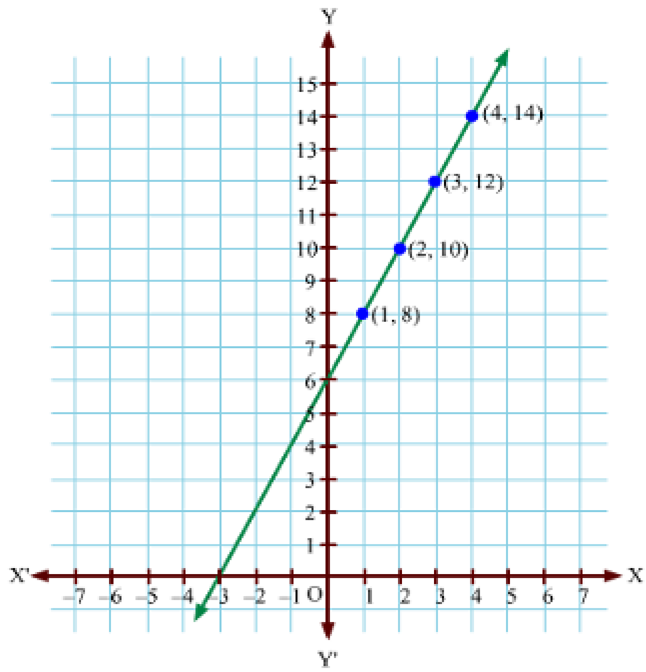
Breadth = 7 m

Graphs of Linear Equations in Two Variables

Graph of a Linear Equation in Two Variables

Algebraically, we know that a linear equation in two variables has infinitely many solutions. These solutions are in the form of ordered pairs showing the values of x and y. Each ordered pair represents a point on the Cartesian plane.

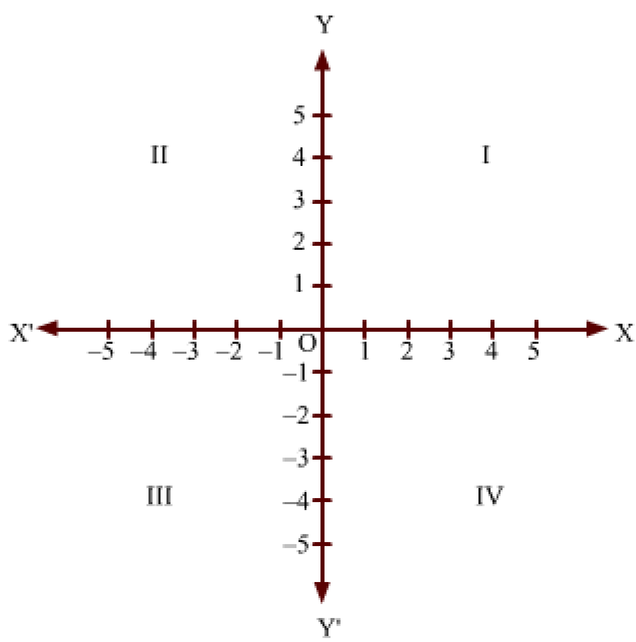
Look at the following figure.
A few points are plotted and joined in the figure to obtain a straight line. When the solutions of a linear equation in two variables are plotted and joined, a similar straight line is obtained. This is why we call such an equation a 'linear' equation.



In this lesson, we will learn to represent linear equations geometrically on graph paper by plotting the **coordinates** and then joining them to obtain straight lines.

Concept Builder

Cartesian plane:The plane divided by two perpendicular lines to represent or define the location of different points as well as two-dimensional figures is called the Cartesian or coordinate plane. The perpendicular lines are called the coordinate axes.



The given figure represents the Cartesian or coordinate plane. The perpendicular lines are represented as X-axis and Y-axis. The regions represented by the Roman numerals I, II, III and IV are the quadrants of the plane.

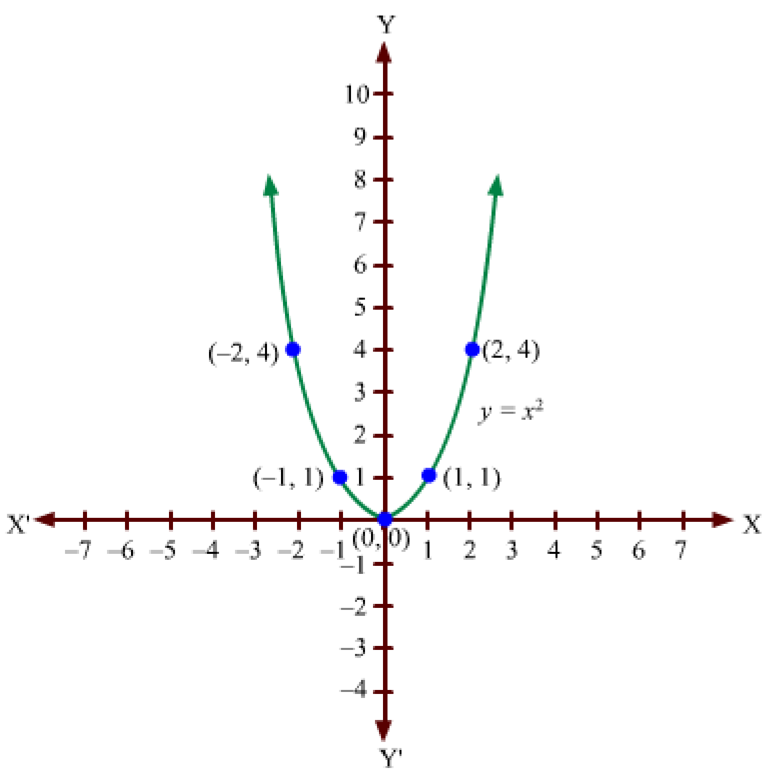
Whiz Kid

Since there are infinitely many solutions of a linear equation, the line corresponding to the equation can go to any length in both directions.

Know More

The graph of a quadratic equation in two variables is always a curve.

For example, the graph of $y = x^2$ is shown.



This symmetrical bell-shaped curve is called a **parabola.**

Solved Examples

Easy

Example 1: **Draw the graph of the linear equation 2*x* – 3*y* = 7.**

Solution:

The given equation is $2x - 3y = 7$. We have to find three solutions of the equation to draw its graph.

We can rewrite the given equation as $x = \frac{3y+7}{2}$.

On substituting:

$y = 1$, we obtain $x = 5$

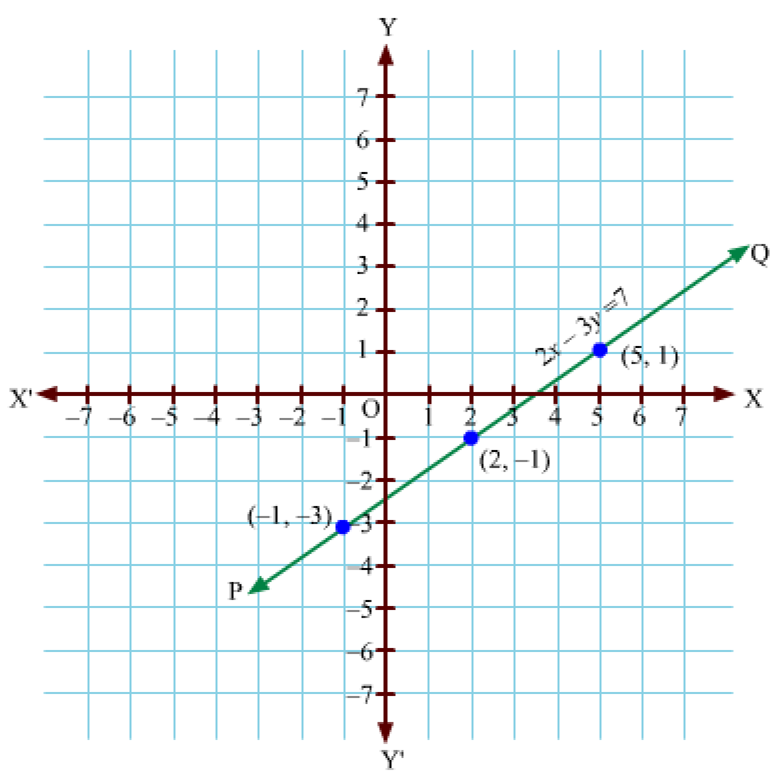
$y = -1$, we obtain $x = 2$

$y = -3$, we obtain $x = -1$

Now, we have three solutions of the given equation. These solutions are shown in a tabular form.

x	5	2	−1
y	1	−1	−3

By plotting and joining the points (5, 1), (2, −1), and (−1, −3), we obtain the following graph.



The line PQ represents the graph of the equation $2x - 3y = 7$.

Example 2: Draw the graph of the linear equation $2x + 3y = 6$.

Solution:

The given equation is $2x + 3y = 6$. We have to find three solutions of the equation to draw its graph.

We can rewrite the given equation as $x = \frac{6-3y}{2}$.

On substituting:

$y = 0$, we obtain $x = 3$

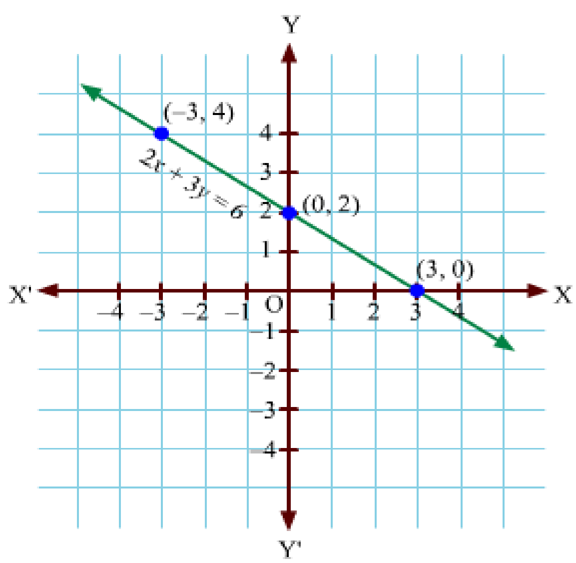
$y = 4$, we obtain $x = -3$

$y = 2$, we obtain $x = 0$

Now, we have three solutions of the given equation. These solutions are shown in a tabular form.

x	3	−3	0
y	0	4	2

By plotting and joining the points (3, 0), (−3, 4) and (0, 2), we obtain the following graph.



The drawn line represents the graph of the equation $2x + 3y = 6$.

Example 1:**Draw the graph of the linear equation** $\frac{1}{2}x+\frac{1}{3}y=2$.

Solution:

We can rewrite the given equation as follows:

$$\frac{1}{2}x+\frac{1}{3}y=2$$
$$\Rightarrow \frac{3x+2y}{6}=2$$
$$\Rightarrow 3x+2y=12$$
$$\Rightarrow x=\frac{12-2y}{3}$$
.

On substituting:

$y = 6$, we obtain $x = 0$

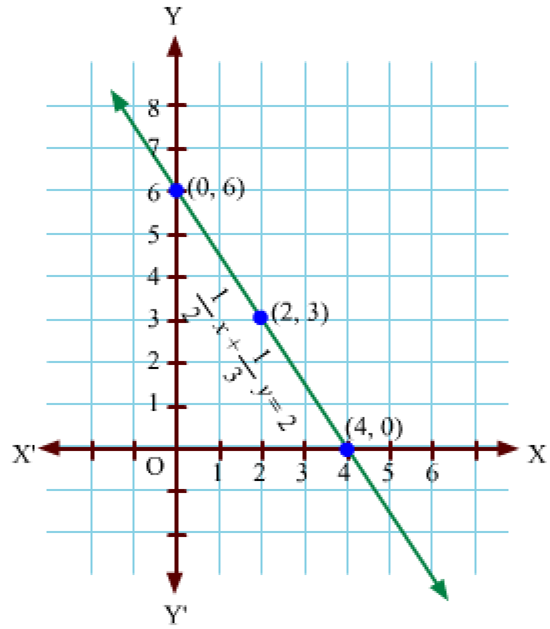
$y = 3$, we obtain $x = 2$

$y = 0$, we obtain $x = 4$

Now, we have three solutions of the given equation. These solutions are shown in a tabular form.

x	0	2	4
y	6	3	0

By plotting and joining the points (0, 6), (2, 3) and (4, 0), we obtain the following graph.



The drawn line represents the graph of the equation $\frac{1}{2}x+\frac{1}{3}y=2$.

Example 2: **The number of eggs needed to make a cake varies directly as the number of cups of flour used. To make a cake, a chef used 2 cups of flour and half a dozen eggs. Write an equation and draw the graph of the egg–flour relation.**

Solution:

Let the number of eggs be y and the number of cups of flour be x.

According to the question, y is directly related to x.

$\therefore y \propto x$

$\Rightarrow y = kx$ (Here, k is a constant)

According to the question, when 2 cups of flour are used, the number of eggs used is 6.

$\therefore 6 = k(2)$

$\Rightarrow k = 3$

Thus, the linear equation representing the egg–flour relation becomes:

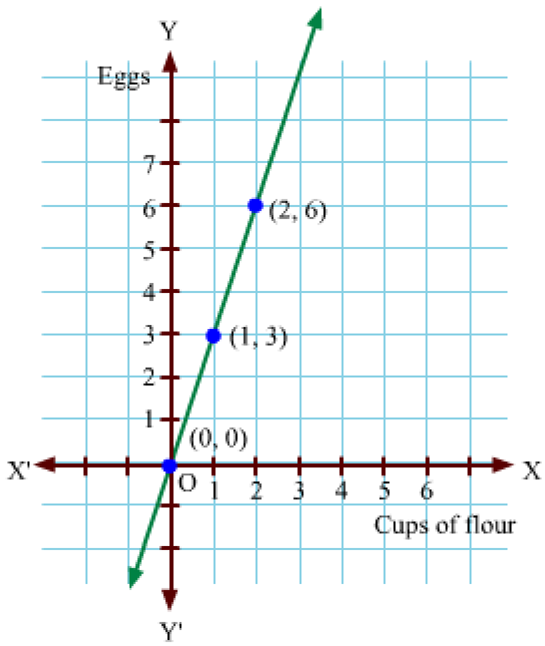
$y = 3x$

$\Rightarrow y - 3x = 0$

Three solutions of this equation are shown in the following table.

x	0	1	2
y	0	3	6

By plotting and joining the points (0, 0), (1, 3) and (2, 6), we obtain the following graph.



The drawn line represents the relation between the number of eggs and the number of cups of flour.

Hard

Example 1: **The graph of a linear equation passes through the points (−1, 4), (*p*, 3) and (2, 1). Find the value of *p* and draw the graph with the line passing through all the three points.**

Solution: Let the equation of the line be $ax + by + c = 0$.

The line passes through (−1, 4), which means it is a solution of the equation.

$\therefore a(-1) + b(4) + c = 0$

$\Rightarrow -a + 4b + c = 0$

$\Rightarrow c = a - 4b$ … (1)

The line also passes through (2, 1).

$\therefore a\,(2) + b\,(1) + c = 0$

$\Rightarrow 2a + b + c = 0$

$\Rightarrow 2a + b + a - 4b = 0$ (Using equation 1)

$\Rightarrow 3a - 3b = 0$

$\Rightarrow a = b$

On substituting this value in equation 1, we get:

$c = a - 4b$

$= a - 4a$

$= -3a$

Therefore, the equation of the line becomes:

$ax + ay - 3a = 0$

$\Rightarrow x + y - 3 = 0$

Since the line passes through $(p, 3)$, it is a solution of the equation $x + y - 3 = 0$.

$\therefore p + 3 - 3 = 0$

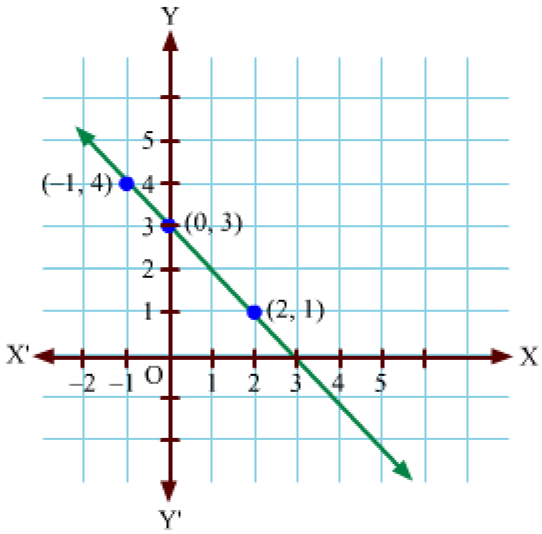
$\Rightarrow p = 0$

Therefore, the third point is (0, 3).

Three solutions of the equation $x + y - 3 = 0$ are shown in the following table.

x	−1	2	0
y	4	1	3

By plotting and joining the points (−1, 4), (2, 1) and (0, 3), we obtain the following graph.



The drawn line represents the equation $x + y - 3 = 0$.

Example 2: **Draw the graphs of $x + y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis. Also, calculate the area of the triangle.**

Solution:

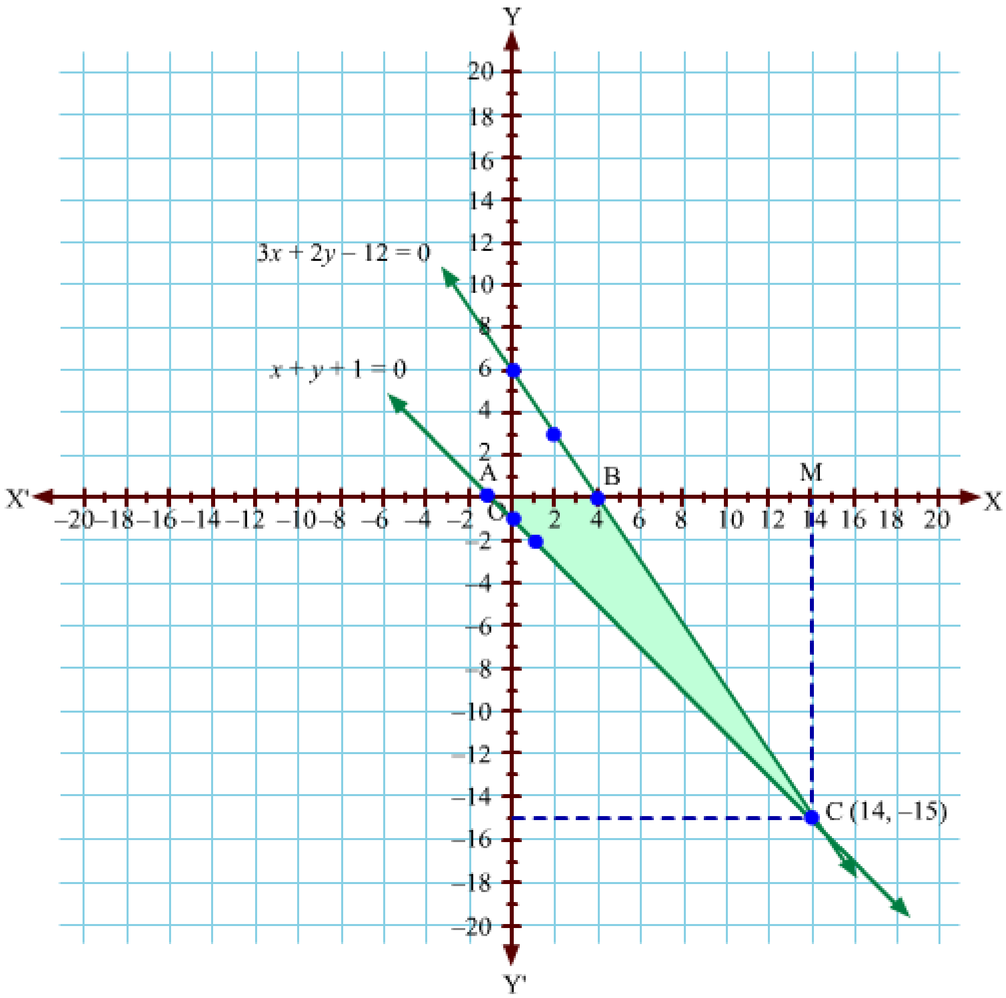
Three solutions of the equation $x + y + 1 = 0$ are shown in the following table.

x	0	1	−1
y	−1	−2	0

Three solutions of the equation is $3x + 2y - 12 = 0$ are shown in the following table.

x	0	2	4
y	6	3	0

The graphs of the two equations can be drawn as shown.



The obtained triangle is ΔABC. The coordinates of its vertices are as follows:

A = (−1, 0), B = (4, 0), C = (14, −15)

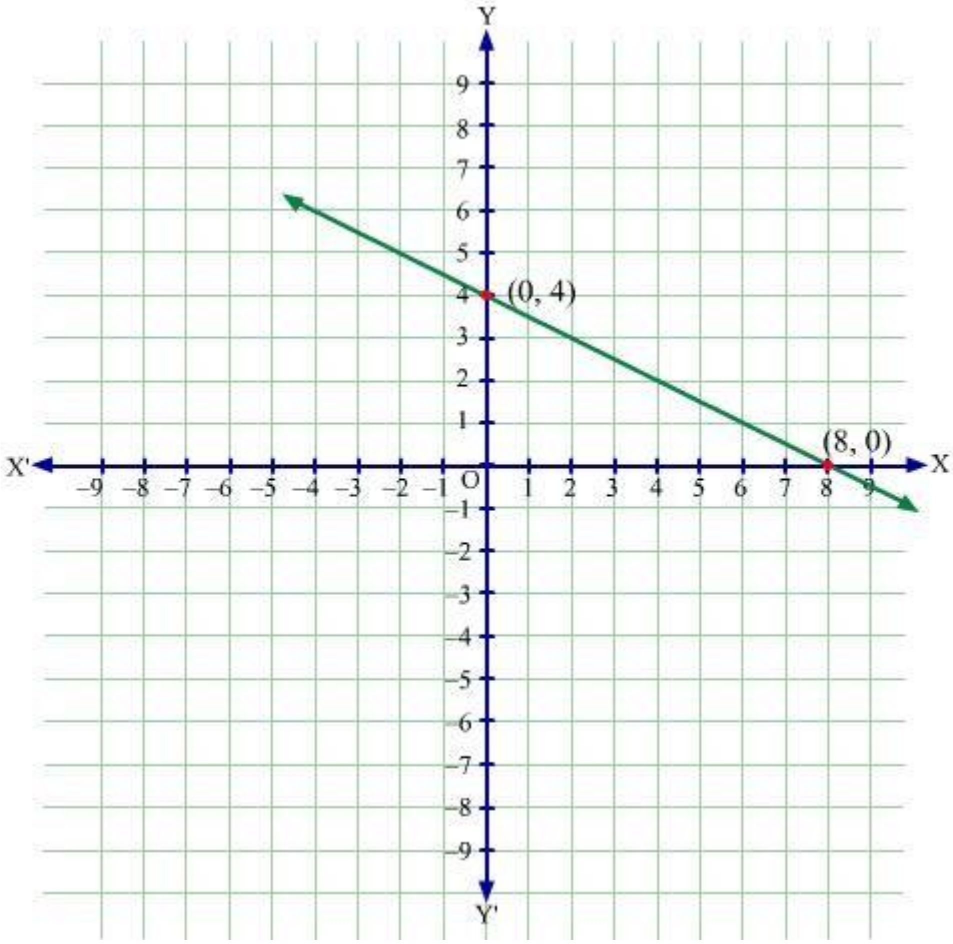
Area of ΔABC = $\frac{1}{2}$ × Base × Height

= $\frac{1}{2}$ × AB × CM

= $\frac{1}{2}$ × 5 × 15

= 37.5 sq units

Example 3: Find the equation of the line shown in the graph given below.



Solution:

Let the equation of straight line be $y = ax + b$.

From the graph, we observe that the line passes through the points (0, 4) and (8, 0). Therefore,

$$4 = (a \times 0) + b$$
$$b = 4$$

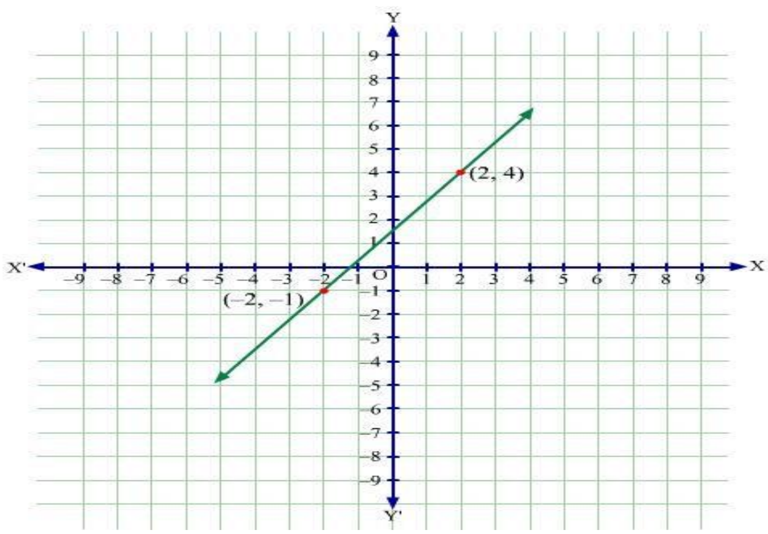
Also,

$$0=(a\times 8)+4$$
$$8a=-4$$
$$a=-\frac{4}{8}$$
$$a=-\frac{1}{2}$$

Thus, the equation of the given straight line is

$$y=-\frac{1}{2}x+4$$
$$y=\frac{-x+8}{2}$$
$$2y=-x+8$$
$$2y+x=8$$

Example 4: Determine the equation of the line as shown in the graph given below.



Solution:

Let the equation of straight line be $y=ax+b$.

From the graph, it can be observed that the line passes through the points (−2, −1) and (2, 4). Thus, we have

$$-1=a\times(-2)+b$$
$$-2a+b=-1 \qquad \ldots(1)$$

Also,

$$4=(a\times 2)+b$$
$$2a+b=4 \qquad \ldots(2)$$

Solving (1) and (2), we have

$$-2a+b+2a+b=-1+4$$
$$2b=3$$
$$b=\frac{3}{2}$$

Using (2), we have

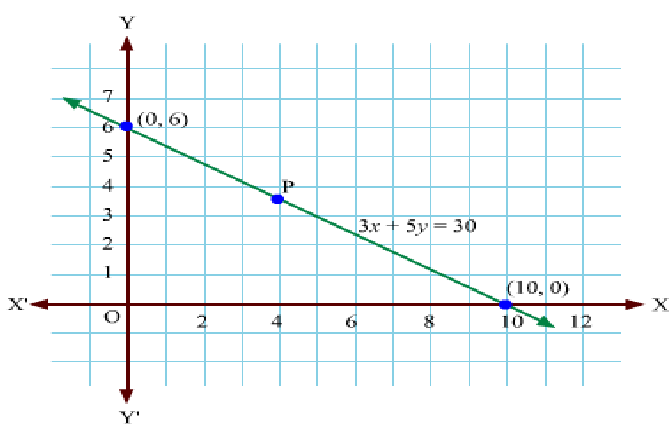
$$2a+\frac{3}{2}=4$$
$$2a=4-\frac{3}{2}$$
$$2a=\frac{5}{2}$$
$$a=\frac{5}{4}$$

Thus, the equation of the straight line is $y=\frac{5}{4}x+\frac{3}{2}$.

Graphical Solution of a Linear Equation In Two Variables

Finding the Solution of a Linear Equation by Using Its Graph

We have learned to draw graphs for linear equations in two variables. Take, for example, the linear equation 3*x* + 5*y* = 30. Its graph is drawn as is shown.



The drawn line represents the equation $3x + 5y = 30$. Every point lying on this line is a solution of the equation. Point P is one such solution of the equation.

In this lesson, we will learn how to find solutions of linear equations using the graphs of those equations.

Whiz Kid

Graphical method is also used to find the solutions of a system of linear equations. In this case, we draw the lines for the linear equations and mark the points where the lines intersect. The points of intersection are solutions of the system of linear equations.

Solved Examples

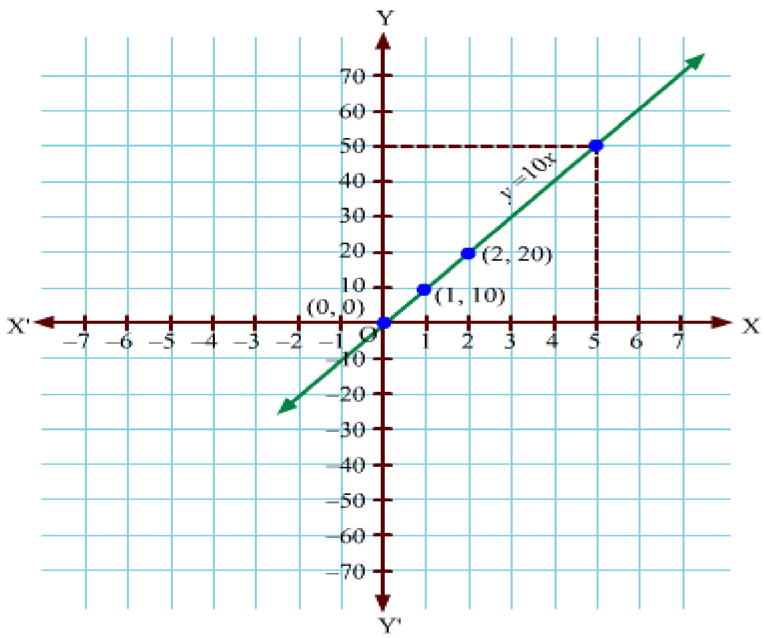
Easy

Example 1: **Using the graph of $y = 10x$, find the solution of the equation when $y = 50$.**

Solution: The given equation is $y = 10x$. Three solutions of this equation are shown in the following table.

x	0	1	2
y	0	10	20

By plotting and joining the points (0, 0), (1, 10) and (2, 20), we obtain the following graph.



The drawn line represents the graph of the equation $y = 10x$. On this line, the value of x corresponding to $y = 50$ is 5.

Example 2: **Draw the graph of $4x - 3y + 12 = 0$. Using the graph, find the solution of the equation when:**

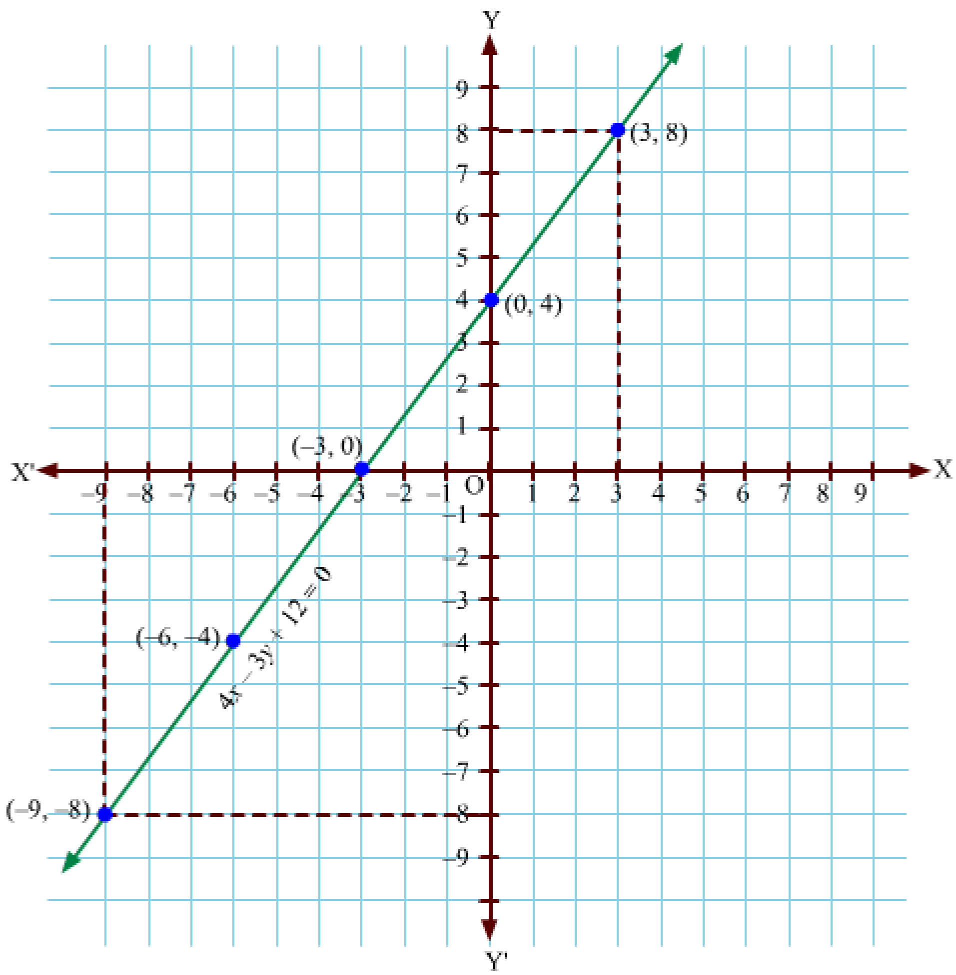
i) $x = 3$

ii) $y = -8$

Solution: The given equation is $4x - 3y + 12 = 0$. Three solutions of this equation are shown in the following table.

x	0	−3	−6
y	4	0	−4

By plotting and joining the points (0, 4), (−3, 0) and (−6, −4), we obtain the following graph.



The drawn line represents the graph of the equation $4x - 3y + 12 = 0$. On this line:

i) The value of y corresponding to $x = 3$ is 8.

ii) The value of x corresponding to $y = -8$ is -9.

Medium

Example 1: **A boy has some twenty-five paise coins and some fifty paise coins which add up to Rs 2.25. Form a linear equation in two variables for this information. Using the graph of the equation, find the number of fifty paise coins if there are 3 coins of twenty-five paise.**

Solution: Let x be the number of twenty-five paise coins and y be the number fifty paise coins.

We know that Re 1 = 100 paise

∴ Rs 2.25 = 225 paise

Amount obtained from twenty-five paise coins = $25x$

Amount obtained from fifty paise coins = $50y$

According to the given information, we have:

$25x + 50y = 225$

$x + 2y = 9$

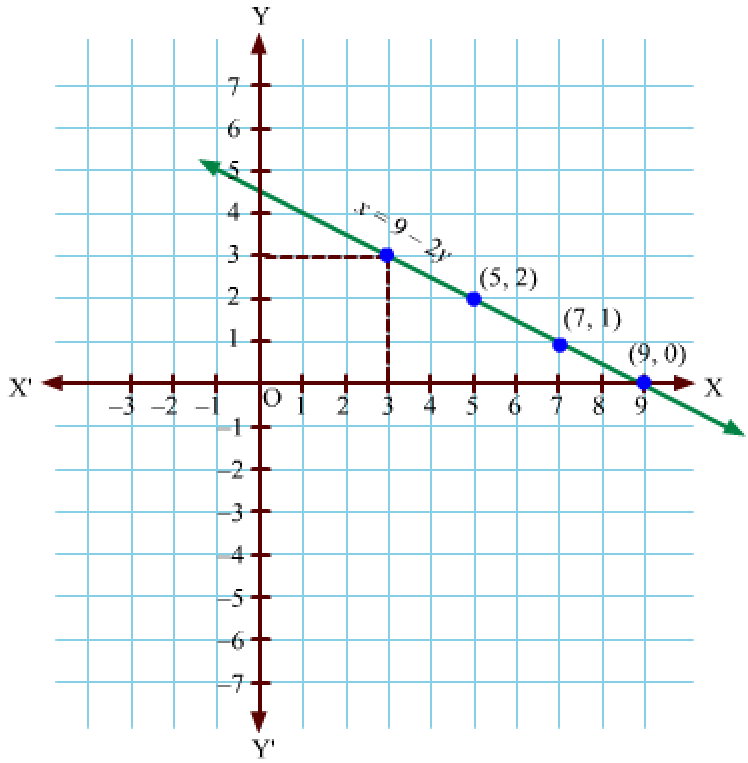
$x = 9 - 2y$

This is the required linear equation in two variables for the given information.

Three solutions of this equation are shown in the following table.

x	9	7	5
y	0	1	2

By plotting and joining the points (9, 0), (7, 1) and (5, 2), we obtain the following graph.



The drawn line represents the graph of the equation $x = 9 - 2y$. On this line, the value of y corresponding to $x = 3$ is 3. Therefore, if there are 3 coins of twenty-five paise, then there will be 3 coins of fifty paise.

Example 2: **Aarushi is driving a car with a uniform speed of 60 km/hour. Draw the distance−time graph. Using the graph, find the distance travelled by Aarushi in:**

i) Two and half hours

ii) Half an hour

Solution: Uniform speed of the car = 60 km/hr(Given)

Let D be the distance travelled by Aarushi in t hours.

We know that:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\Rightarrow 60 = \frac{D}{t}$$

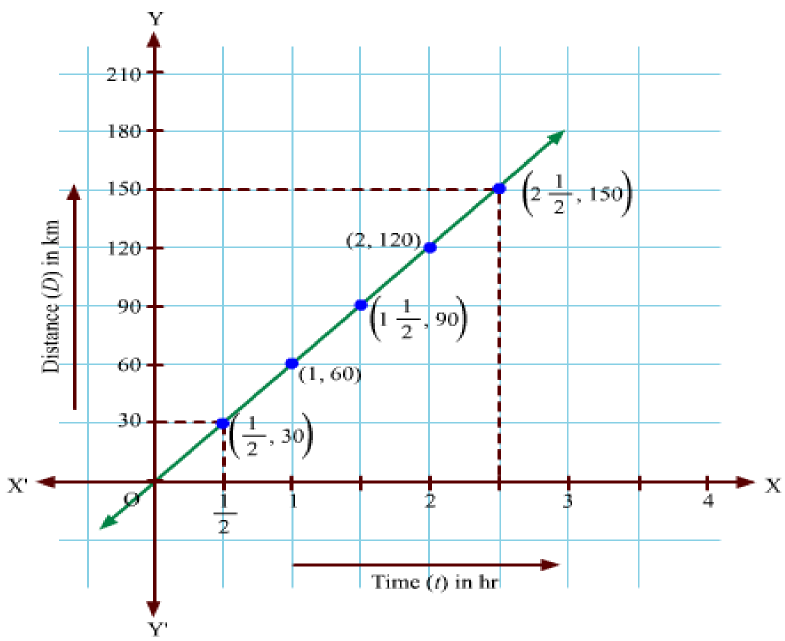
$$\Rightarrow D = 60t$$

Three solutions of this equation are shown in the following table.

t	1	$1\frac{1}{2}$	2
D	60	90	120

Let us keep time (t) along the x-axis and distance (D) along the y-axis on the Cartesian plane.

By plotting and joining the points (1, 60), $\left(1\frac{1}{2}, 90\right)$ and (2, 120), we obtain the following graph.



The drawn line represents the distance–time equation $D = 60t$. On this line:

i) The value of D corresponding to $t = 2\frac{1}{2}$ is 150. So, the distance travelled by Aarushi in two and a half hours is 150 km.

ii) The value of D corresponding to $t = \frac{1}{2}$ is 30. So, the distance travelled by Aarushi in half an hour is 30 km.

Example 1: The linear equation $C = \frac{5F - 160}{9}$ is used to compare the temperatures on the Celsius (C) and Fahrenheit (F) scales. Draw the graph for the equation and find the following information from the graph.

i) If the temperature is 0°F, then what is the approximate temperature in degree Celsius?

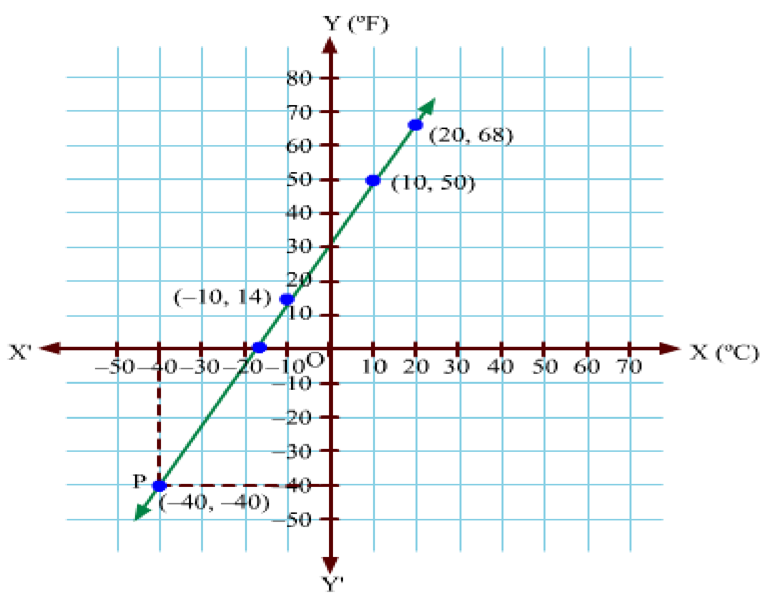
ii) Find and locate the point where the numerical value of temperature is the same on both the scales?

Solution: The given equation is $C = \frac{5F - 160}{9}$. Three solutions of this equation are shown in the following table.

C	10	−10	20
F	50	14	68

Let us keep temperature in degree Celsius along the *x*-axis and temperature in degree Fahrenheit along the *y*-axis on the Cartesian plane.

By plotting and joining the points (10, 50), (−10, 14) and (20, 68), we obtain the following graph.



The drawn line represents the Celsius−Fahrenheit equation $C = \frac{5F - 160}{9}$. On this line:

i) The approximate value of C corresponding to $F = 0$ is -20. So, when the temperature is 0°F, the temperature on the Celsius scale is approximately −20°C.

ii) At point P, both C and F have the same value, i.e., -40. So, -40 is the point at which the Celsius and Fahrenheit scales coincide, i.e., −40°C = −40°F.

Example 2: The acceleration x of a small object having a mass of 6 kg is directly proportional to the force y applied on it. Express this statement as a linear equation in two variables and draw its graph. Using the graph, find the force required to produce an acceleration of:

i) 5 cm/sec^2

ii) 6 cm/sec^2

Solution: It is given that the force applied is y and the acceleration produced is x.

$\therefore y \propto x$

$\Rightarrow y = \mathrm{m}x$

The constant mass 'm' of the object is given as 6 kg.

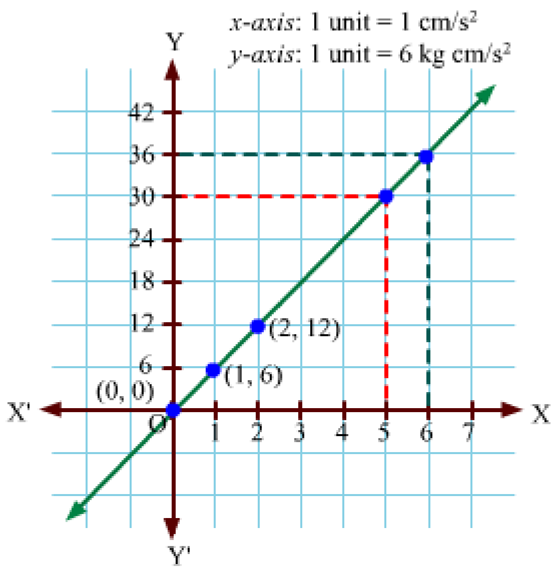
$\therefore y = 6x$

This is the required linear equation in two variables.

Three solutions of this equation are shown in the following table.

x	0	1	2
y	0	6	12

By plotting and joining the points (0, 0), (1, 6) and (2, 12), we obtain the following graph.



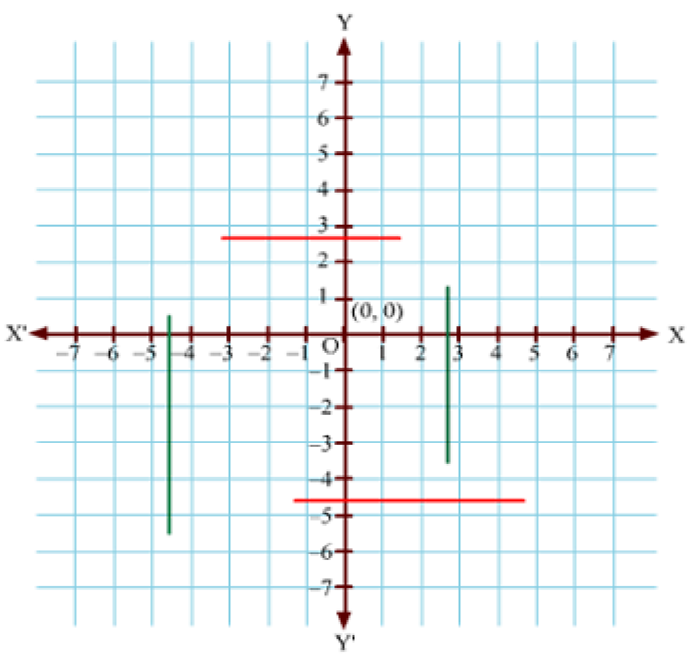
The drawn line represents the equation $y = 6x$. On this line:

i) The value of y corresponding to $x = 5$ is 30. So, a force of 30 kg cm/s^2 is required to produce an acceleration of 5 cm/s^2.

ii) The value of y corresponding to $x = 6$ is 36. So, a force of 36 kg cm/s^2 is required to produce an acceleration of 6 cm/s^2.

Graphs of Linear Equations Parallel to Coordinate Axes

Lines Parallel to the Coordinate Axes



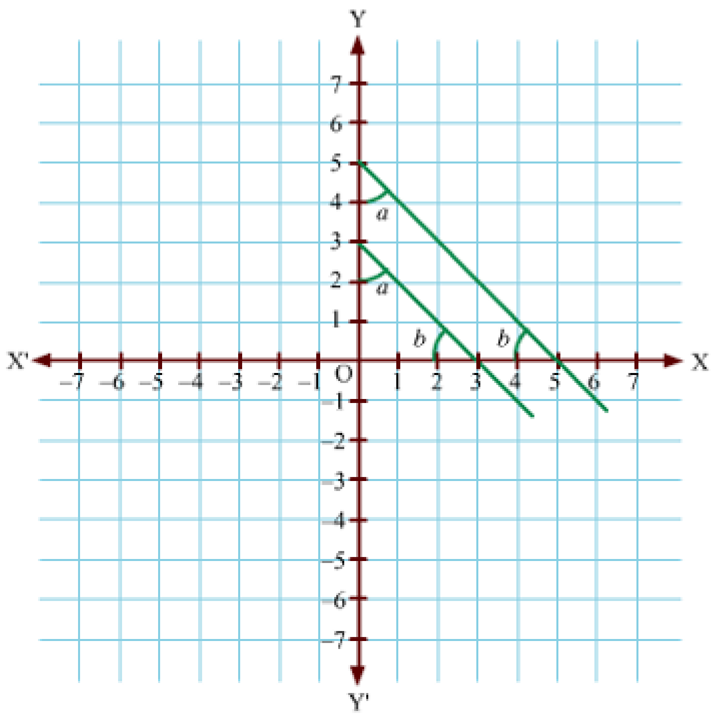
We have studied that the graphs of linear equations are always straight lines that can be easily drawn on the coordinate plane. Sometimes these lines are **parallel** to the coordinate axes. Look at the following figure which shows a few such parallel line graphs.

In the figure, the green graphs are parallel to the *y*-axis and the red graphs are parallel to the *x*-axis. Each of these graphs represents a linear equation.

In this lesson, we will study about linear equations whose graphs are lines parallel to the coordinate axes.

Whiz Kid

Parallel lines have the same slopes. This means that parallel lines make the same angle with a coordinate axis or a common line. For example, take a look at the parallel green lines in the following figure.

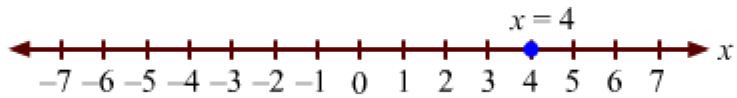


In the figure, the parallel green lines make the same angle *a* with the *y*-axis and the same angle *b* with the *x*-axis. So, both the lines have the same slopes with the coordinate axes.

Concept Builder

Consider the linear equation in one variable $4x - 16 = 0$. We know that linear equations in one variable have unique solutions. In this case, the unique solution of the equation is $x = 4$.

Now, we can represent $x = 4$ on a number line as follows:



We can represent other linear equations in one variable in the same way.

Did You Know?

- Since graphs of equations of the form '$x = a$' are always parallel to the y-axis, they are always perpendicular to the x-axis.

- Two lines '$x = a$'and'$x = b$' are always parallel to each other if 'a' and 'b' are non-zero real numbers and'$a \neq b$'.

Solved Examples

Easy

Example 1: Find the distance between the lines x = 3.5 and x = −1.5.

Solution: Distance between the y-axis and the line x = 3.5 is 3.5 units. The positive value of x shows that the line is on the right hand side of the y-axis.

Distance between the y-axis and the line x = −1.5 is 1.5 units. The negative value of x shows that the line is on the left hand side of the y-axis.

The lines are on different sides of the y-axis, so the distance between the lines will be the sum of their distances from the y-axis.

∴ Distance between the given lines = (3.5 + 1.5) units = 5 units

Example 1: Solve the equation $2x-1=\frac{x}{3}$ and represent the solution on:

i) The number line

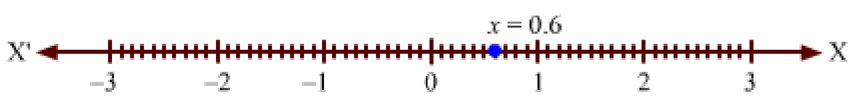
ii) The Cartesian plane

Solution: The given equation can be reduced as follows:

$$2x-1=\frac{x}{3}$$
$$\Rightarrow 2x-\frac{x}{3}=1$$
$$\Rightarrow \frac{6x-x}{3}=1$$
$$\Rightarrow 5x=3$$
$$\Rightarrow x=\frac{3}{5}$$
$$\Rightarrow x=0.6$$

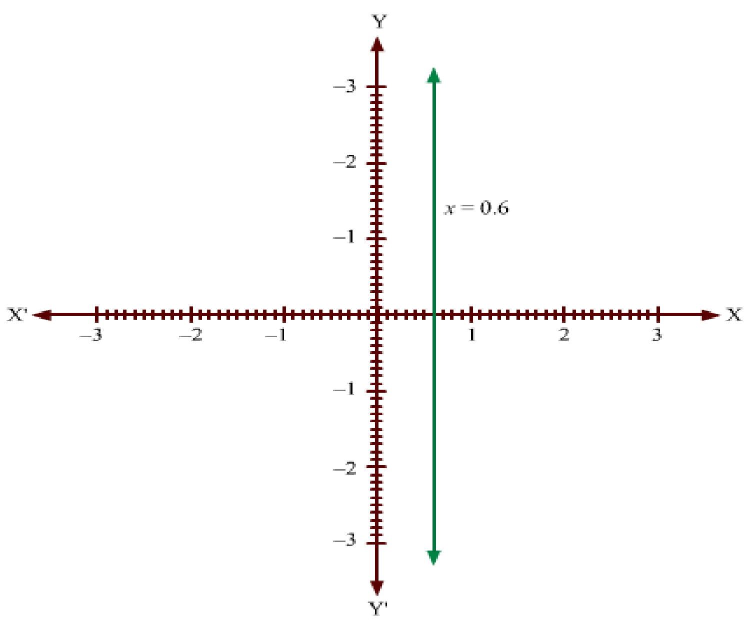
Thus, the equation $2x-1=\frac{x}{3}$ can be rewritten as *x* = 0.6, which is of the form '*x* = *a*'.

i) *x* = 0.6 can be represented on the number line as follows:



ii) We know that the graph of the equation'*x* = *a*' is a line parallel to the *y*-axis, situated at a distanceof '*a*' unitsfrom the *y*-axis. So, the graph of the equation *x* = 0.6 will be a line parallel to the *y*-axis, situated at a distanceof 0.6 unitfrom the *y*-axis.

The graph of the equation *x* = 0.6 is drawn on the Cartesian plane as is shown.



Did You Know?

• Since graphs of equations of the form '$y = b$' are always parallel to the x-axis, they are always perpendicular to the y-axis.

• The line '$y = b$' is always perpendicular to the line '$x = a$' if drawn on the same Cartesian plane.

Solved Examples

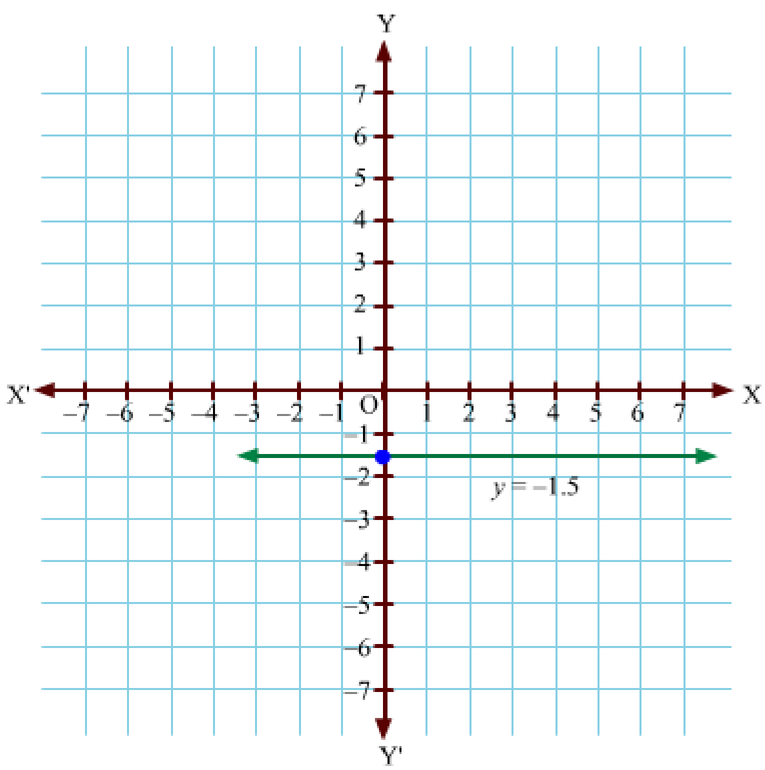
Easy

Example 1: Represent the equation 2y + 3 = 0 graphically on the Cartesian plane.

Solution: The equation $2y + 3 = 0$ can be rewritten as $y = -1.5$, which is of the form '$y = b$'.

We know that the graph of the equation'$y = b$'is a line parallel to the x-axis, situated at a distance of 'b' unitsfrom the x-axis. So, the graph of $y = -1.5$ will be a line parallel to the x-axis, situated at a distance of 1.5 unitsfrom the x-axis.

The graph of the equation $y = -1.5$ is drawn on the Cartesian plane as is shown.



Medium

Example 1: Consider the equation $\frac{2y-2}{3}=\frac{3y-3}{2}$ **.**

Represent this equation on:

i) The number line

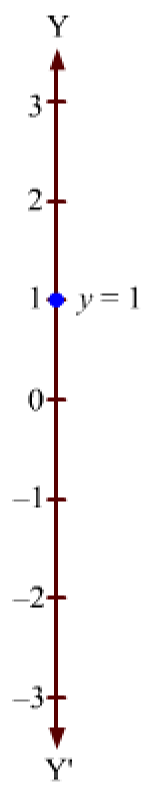
ii) The Cartesian plane

Solution: The given equation can be reduced as follows:

$$\frac{2y-2}{3}=\frac{3y-3}{2}$$
$$\Rightarrow 4y-4=9y-9$$
$$\Rightarrow -5y=-5$$
$$\Rightarrow y=1$$

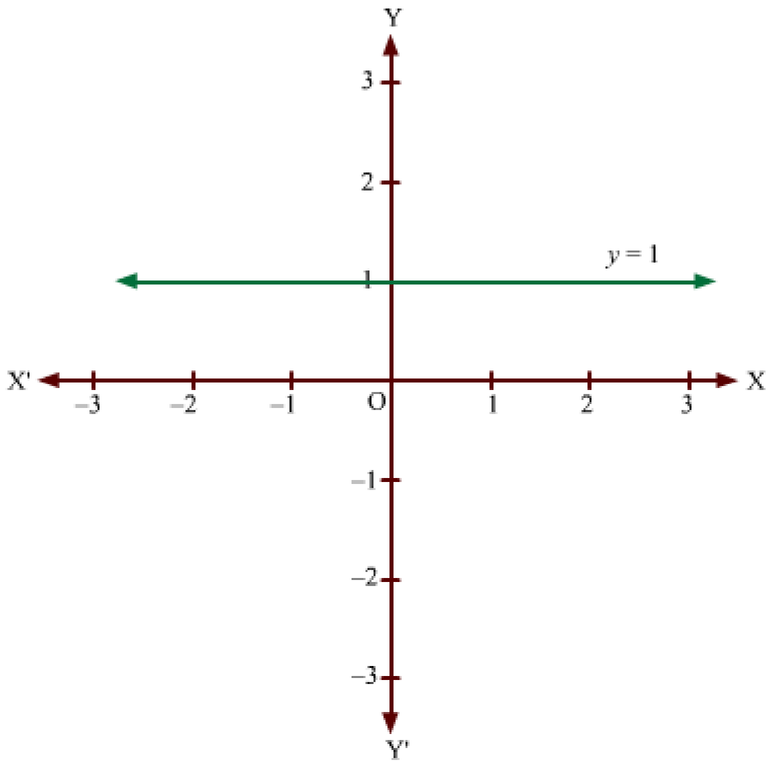
Thus, the equation $\frac{2y-2}{3}=\frac{3y-3}{2}$ can be rewritten as y = 1, which is of the form 'y = b'.

i) The equation y = 1 can be plotted on the number line as is shown.



ii) Since the equation $y = 1$ is of the form '$y = b$', its graph will be a line parallel to the x-axis, situated at a distance of 1 unit from the x-axis.

The graph of the equation $y = 1$ is drawn on the Cartesian plane as is shown.



Solved Examples

Medium

Example 1: **Find the area of the triangle formed by the lines $x = 3$, $y = 4$ and $y = x$.**

Solution: The given equations are $x = 3$, $y = 4$ and $y = x$.

The graph of the equation $x = 3$ will be a line parallel to the y-axis, situated at a distance of 3 units from the y-axis.

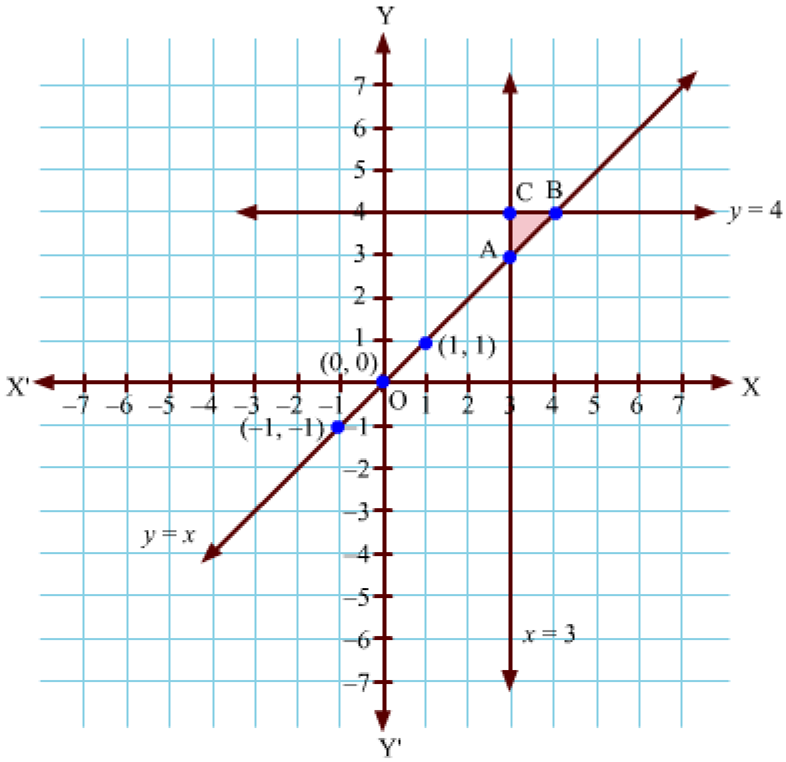
The graph of the equation $y = 4$ will be a line parallel to the x-axis, situated at a distance of 4 units from the x-axis.

Now, consider the equation $y = x$. Three solutions of this equation are shown in the following table.

x	0	1	−1

y	0	1	−1

The graphs of the given equations are drawn on the Cartesian plane as is shown.



$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times BC \times CA$$

$$= \frac{1}{2} \times 1 \times 1$$

$$= \frac{1}{2}$$

Thus, the area of the triangle formed by the given lines is 0.5 sq. units.