Linear Programming

EXERCISE 6.1 [PAGES 97 - 99]

Exercise 6.1 | Q 1 | Page 97

A manufacturing firm produces two types of gadgets A and B, which are first processed in the foundry and then sent to machine shop for finishing. The number of man hours of labour required in each shop for production of A and B and the number of man hours available for the firm are as follows:

Gadgets	Foundry	Machine Shop
A	10	5
B	6	4
Time available (hours)	60	35

Profit on the sale of A is ₹ 30 and B is ₹ 20 per unit. Formulate the L.P.P. to have maximum profit.

Solution: Let the manufacturing firm produces x units of gadget A and y units of gadget B.

The profit on 1 unit of A is ₹ 30 and on 1 unit of B is ₹ 20.

∴ Total profit on selling x units of A and y units of B is ₹ 30x + 20y.

Thus the profit function Z = 30x + 20y

A and B are the products while the time required in the foundry and machine shop are constraints, we construct the given table with the products written column wise and the constraints row-wise.

Constraints/Gadgets	A (x)	B (x)	Time available in hours
Foundry	10	6	60
Machine shop	5	4	35

1 unit of A requires 10 hours in the foundry and 1 unit of B requires 6 hours.

∴ x units of A requires 10x hours and y units of B requires 6y hours in the foundry. But the maximum time available in the foundry is 60 hours.

∴ The 1^{st} constraint is $10x + 6y \le 60$.

The constraint for the machine shop is $5x + 4y \le 35$.

Since number of gadgets cannot be negative, we have $x \geq 0$, $y \geq 0$.

∴ Given problem can be formulated as follows:

Maximize $Z = 30x + 20y$

Subject to $10x + 6y \leq 60$, $5x + 4y \leq 35$, $x \geq 0$, $y \geq 0$.

Exercise 6.1 | Q 2 | Page 98

In a cattle breeding firm, it is prescribed that the food ration for one animal must contain 14, 22 and 1 unit of nutrients A, B and C respectively. Two different kinds of fodder are available. Each unit weight of these two contains the following amounts of these three nutrients:

Nutrient\Fodder	**Fodder 1**	**Fodder2**
Nutrient A	2	1
Nutrient B	2	3
Nutrient C	1	1

The cost of fodder 1 is ₹ 3 per unit and that of fodder ₹ 2, Formulate the L.P.P. to minimize the cost.

Solution: Let x units of fodder 1 and y units of fodder 2 be included in the food ration of an animal.
The cost of fodder 1 is ₹ 3 per unit and that of fodder 2 is ₹ 2 per unit.
∴ Total cost = ₹ $(3x + 2y)$
The minimum requirement of nutrients A, B, C for an animal are 14,22 and 1 unit respectively.
We construct the given table with the minimum requirement column as follows:

Nutrient\Fodder	**Fodder 1 (x)**	**Fodder 2 (y)**	**Minimum requirement**
Nutrient A	2	1	14
Nutrient B	2	3	22
Nutrient C	1	1	1

From the table, the food ration of an animal must contain $(2x + y)$ units of nutrient A, $(2x + 3y)$ units of B and $(x + y)$ units of C.
∴ The constraints are :
$2x + y \geq 14$,
$2x + 3y \geq 22$,
$x + y \geq 1$

Since x and y cannot be negative, we have $x \geq 0, y \geq 0$
∴ Given problem can be formulated as follows:
Minimize $Z = 3x + 2y$
Subject to $2x + y \geq 14$, $2x + 3y \geq 22$, $x + y \geq 1$, $x \geq 0$, $y \geq 0$

Exercise 6.1 | Q 3 | Page 98

A company manufactures two types of chemicals A and B. Each chemical requires two types of raw material P and Q. The table below shows number of units of P and Q required to manufacture one unit of A and one unit of B.

Raw Material \Chemical	**A**	**B**	**Availability**
p	3	2	120
Q	2	5	160

The company gets profits of ₹ 350 and ₹ 400 by selling one unit of A and one unit of B respectively. Formulate the problem as L.P.P. to maximize the profit.

Solution: Let x units of chemical A and y units of chemical B are manufactured by the company.

Here, $(3x + 2y)$ units of material P and $(2x + 5y)$ units of material Q is required and 120 units of material P and 160 units of material Q are available.

∴ The constraints are :

$3x + 2y \leq 120$,

$2x + 5y \leq 160$

Since x and y cannot be negative, we have $x \geq 0, y \geq 0$

Now, Profit on one unit of chemical A is ₹ 350.

∴ Profit on x units of chemical A is 350x.

Profit on one unit of chemical B is ₹ 400.

∴ Profit on y units of chemical B is 400y.

∴ Total Profit, $Z = 350x + 400y$

This is the objective function to be maximized.

∴ Given problem can be formulated as,

Maximize $Z = 350x + 400y$

Subject to $3x + 2y \leq 120$, $2x + 5y \leq 160$, $x \geq 0$, $y \geq 0$.

Exercise 6.1 | Q 4 | Page 98

A printing company prints two types of magazines A and B. The company earns ₹ 10 and ₹ 15 on magazines A and B per copy. These are processed on three machines I, II, III. Magazine A requires 2 hours on Machine I, 5 hours on Machine II and 2 hours on Machine III. Magazine B requires 3 hours on Machine I, 2 hours on Machine II and 6 hours on Machine III. Machines I, II, III are available for 36, 50, 60 hours per week respectively. Formulate the Linear programming problem to maximize the profit.

Solution: Let the company print x magazines of type A and y magazines of type B.
The profit on each copy of A and B is ₹ 10 and ₹ 15 respectively.
∴ Total profit = ₹ $(10x + 15y)$
We construct a table with the constraints of machines I, II, III as follows.

Machine\Magazine	**A (x)**	**B (y)**	**Available Time per week**
I	2	3	36
II	5	2	50
III	2	6	60

From the table, total time required for machines I, II, III are $(2x + 3y)$ hours, $(5x + 2y)$ hours and $(2x + 6y)$ hours respectively.
∴ The constraints are:
$2x + 3y \leq 36$,
$5x + 2y \leq 50$,
$2x + 6y \leq 60$
Since x, y cannot be negative, we have $x \geq 0, y \geq 0$
∴ Given problem can be formulated as,
Maximize $Z = 10x + 15y$
Subject to $2x + 3y \leq 36$, $5x + 2y \leq 50$, $2x + 6y \leq 60$, $x \geq 0$, $y \geq 0$.

Exercise 6.1 | Q 5 | Page 98

A manufacturer produces bulbs and tubes. Each of these must be processed through two machines M_1 and M_2. A package of bulbs requires 1 hour of work on Machine M_1 and 3 hours of work on M_2. A package of tubes requires 2 hours on Machine M_1 and 4 hours on Machine M_2. He earns a profit of ₹ 13.5 per package of bulbs and ₹ 55 per package of tubes. If maximum availability of Machine M_1 is 10 hours and that of Machine M_2 is 12 hours, then formulate the L.P.P. to maximize the profit.

Solution: Let the manufacturer produce ‘x’ packages of bulbs and ‘y’ packages of tubes.

The profit on a package of bulbs is ₹ 13.5 and that of tubes is ₹ 55.

∴ Total profit = ₹ (13.5 x + 55y)

We construct a table with the constraints of machines M_1 and M_2 as follows:

Machine\Product	**Bulbs x**	**Tubes y**	**Maximum Availability in hours**
M1	1	2	10
M2	3	4	12

From the table, the total time required on M_1 is (x + 2y) hours and on M_2 is (3x + 4y) hours.

∴ The constraints are:

$x + 2y \leq 10$, $3x + 4y \leq 12$

Since x and y cannot be negative, we have $x \geq 0$, $y \geq 0$

∴ Given problem can be formulated as follows:

Maximize Z = 13.5x + 55y

Subject to $x + 2y \leq 10$, $3x + 4y \leq 12$, $x \geq 0$, $y \geq 0$.

Exercise 6.1 | Q 6 | Page 98

A company manufactures two types of fertilizers F_1 and F_2. Each type of fertilizer requires two raw materials A and B. The number of units of A and B required to manufacture one unit of fertilizer F_1 and F_2 and availability of the raw materials A and B per day are given in the table below:

Raw Material\Fertilizers	**F_1**	**F_2**	**Availability**
A	2	3	40
B	1	4	70

By selling one unit of F_1 and one unit of F_2, company gets a profit of ₹ 500 and ₹ 750 respectively. Formulate the problem as L.P.P. to maximize the profit.

Solution: Let the company manufacture ‘x’ units of fertilizer F_1 and ‘y’ units of F_2.

The profit on one unit of F_1 is ₹ 500 and on unit of F_2 is ₹ 750.

∴ Total profit = ₹ (500x + 750y)

From the given table,

The raw material A required for x units of F_1 and y units of F_2 is (2x + 3y). The raw material B required is (x + 4y).

But maximum availability of raw materials A and B are 40 and 70 units respectively.

∴ The constraints are:

$2x + 3y \le 40$, $x + 4y \le 70$

Since x and y cannot be negative, we have $x \ge 0$, $y \ge 0$.

∴ Given problem can be formulated as follows:

Maximize $Z = 500x + 750y$

Subject to $2x + 3y \le 40$, $x + 4y \le 70$, $x \ge 0$, $y \ge 0$.

Exercise 6.1 | Q 7 | Page 98

A doctor has prescribed two different units of foods A and B to form a weekly diet for a sick person. The minimum requirements of fats, carbohydrates and proteins are 18, 28, 14 units respectively. One unit of food A has 4 units of fat, 14 units of carbohydrates and 8 units of protein. One unit of food B has 6 units of fat, 12 units of carbohydrates and 8 units of protein. The price of food A is ₹ 4.5 per unit and that of food B is ₹ 3.5 per unit. Form the LPP, so that the sick person's diet meets the requirements at a minimum cost.

Solution: Let x units of food A and y units of food B be prescribed in the weekly diet of a sick person.
The price for food A is ₹ 4.5 per unit and for food B is ₹ 3.5 per unit.

∴ Total cost is z = ₹ (4.5x + 3.5y)
We construct a table with constraints of fats, carbohydrates and proteins as follows:

Nutrients\Foods	**A (x)**	**B (y)**	**Minimum requirement**
Fats	**4**	**6**	**18**
Carbohydrates	**14**	**12**	**28**
Protein	**8**	**8**	**14**

From the table, diet of sick person must include (4x + 6y) units of fats, (14x + 12y) units of carbohydrates and (8x + 8y) units of proteins
∴ The constraints are
$4x + 6y \ge 18$,

$14x + 12y \geq 28$,
$8x + 8y \geq 14$.
Since x and y cannot be negative, we have $x \geq 0, y \geq 0$
∴ Given problem can be formulated as follows:
Minimize $z = 4.5x + 3.5y$
Subject to $4x + 6y \geq 18$, $14x + 12y \geq 28$, $8x + 8y \geq 14$, $x \geq 0$, $y \geq 0$.

Exercise 6.1 | Q 8 | Page 99

If John drives a car at a speed of 60 km/hour, he has to spend ₹ 5 per km on petrol. If he drives at a faster speed of 90 km/hour, the cost of petrol increases ₹ 8 per km. He has ₹ 600 to spend on petrol and wishes to travel the maximum distance within an hour. Formulate the above problem as L.P.P.

Solution: Let John travel x_1 km at speed of 60 km/hr and x_2 km at a speed of 90 km/hr.
∴ Total distance = $(x_1 + x_2)$ km

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Time to travel x1 km = $\left(\frac{x_1}{60}\right)$ hours and time to travel x_2 km = $\left(\frac{x_2}{90}\right)$ hours.

∴ Total time = $\left(\frac{x_1}{60} + \frac{x_2}{90}\right)$hours

But John wishes to travel maximum distance within an hour.

$$\therefore \frac{x_1}{60} + \frac{x_2}{90} \leq 1$$

John has to spend ₹ 5 per km at 60 km/hr and ₹ 8 per km at 90 km/hr.

∴ Total cost = ₹ $(5x_1 + 8x_2)$

But John has ₹ 600 to spend on petrol

∴ $5x_1 + 8x_2 \leq 600$

Since x_1 and x_2 cannot be negative, we have $x_1 \geq 0$, $x_2 \geq 0$

∴ Given problem can be formulated as follows:

Maximize Z = x1 + x2,

Subject to $\frac{x_1}{60} + \frac{x_2}{90} \leq 1, 5x_1 + 8x_2 \leq 600$, $x_1 \geq 0$, $x_2 \geq 0$.

Exercise 6.1 | Q 9 | Page 99

The company makes concrete bricks made up of cement and sand. The weight of a concrete brick has to be at least 5 kg. Cement costs ₹ 20 per kg and sand costs of ₹ 6 per kg. Strength consideration dictates that a concrete brick should contain minimum 4 kg of cement and not more than 2 kg of sand. Form the L.P.P. for the cost to be minimum.

Solution 1:

Let the company use x_1 kg of cement and x_2 kg of sand to make concrete bricks.

Cement costs ₹ 20 per kg and sand costs ₹ 6 per kg.

∴ the total cost c = ₹ $(20x_1 + 6x_2)$

This is a linear function which is to be minimized.

Hence, it is an objective function.

Total weight of brick = $(x_1 + x_2)$ kg

Since the weight of concrete brick has to be at least 5 kg,

∴ $x_1 + x_2 \geq 5$

Since concrete brick should contain minimum 4 kg of cement and not more than 2 kg of sand,

$x_1 \geq 4$ and $0 \leq x_2 \leq 2$

Hence, the given LPP can be formulated as:

Minimize $c = 20x_1 + 6x_2$, subject to

$x_1 + x_2 \geq 5$, $x_1 \geq 4$, $0 \leq x_2 \leq 2$.

Solution 2:

Let the concrete brick contain x_1 kg of cement and x_2 kg of sand Cement costs ₹ 20 per kg and sand costs ₹ 6 per kg.

∴ Total cost = ₹ $(20x_1 + 6x_2)$

Weight of a concrete brick has to be at least 5 kg.

∴ $x_1 + x_2 \geq 5$

The brick should contain minimum 4 kg of cement.

∴ $x_1 \geq 4$

The brick should contain not more than 2 kg of sand.

∴ $x_2 \leq 2$

Since x_1 and x_2 cannot be negative, we have $x_1 \geq 0$, $x_2 \geq 0$

∴ Given problem can be formulated as follows:

Minimize $Z = 20x_1 + 6x_2$

Subject to $x_1 + x_2 \geq 5$, $x1 \geq 4$, $x_2 \leq 2$, $x_1 \geq 0$, $x_2 \geq 0$.

EXERCISE 6.2 [PAGE 101]

Exercise 6.2 | Q 1 | Page 101

Solve the following L.P.P. by graphical method :

Maximize : $Z = 11x + 8y$ subject to $x \leq 4$, $y \leq 6$, $x + y \leq 6$, $x \geq 0$, $y \geq 0$.

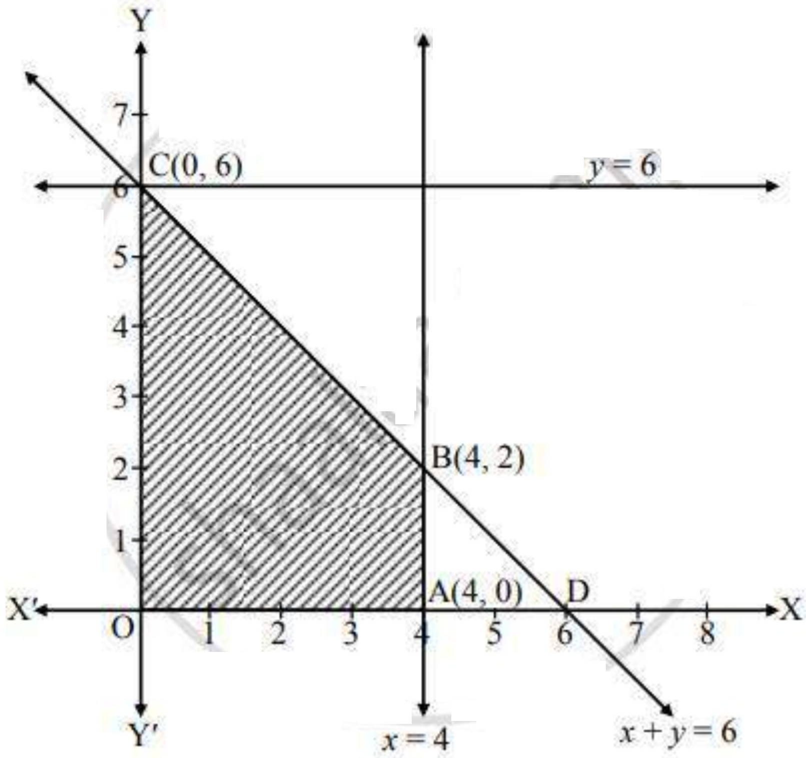
Solution: To draw the feasible region, construct table as follows:

Inequality	$x \leq 4$	$y \leq 6$	$x + y \leq 6$
Corresponding equation (of line)	$x = 4$	$y = 6$	$x + y = 6$
Intersection of line with X-axis	(4, 0)	–	(6, 0)
Intersection of line with Y-axis	–	(0, 6)	(0, 6)
Region	Origin side	Origin side	Origin side

Shaded portion OABC is the feasible region,

whose vertices are O(0, 0), A(4, 0), B and C(0, 6).

B is the point of intersection of the lines $x = 4$ and $x + y = 6$



Substituting x = 4 in x + y = 6, we get

y = 2

∴ B ≡ (4, 2)

Here, the objective function is Z = 11x + 8y

∴ Z at O(0, 0) = 11(0) + 8(0) = 0

Z at A(4, 0) = 11(4) + 8(0) = 44

Z at B(4, 2) = 11(4) + 8(2) = 44 + 16 = 60

Z at C(0, 6) = 11(0) + 8(6) = 48

∴ Z has maximum value 60 at B(4, 2)

∴ Z is maximum, when x = 4 and y = 2.

Exercise 6.2 | Q 2 | Page 101

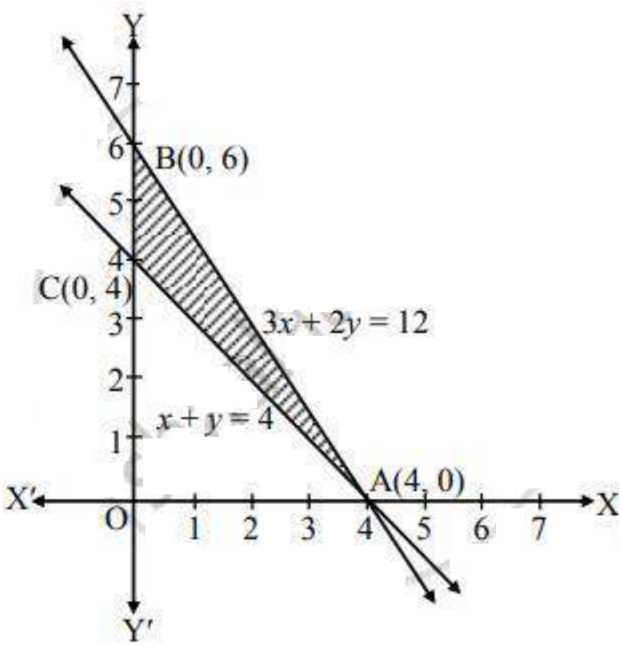
Solve the following L.P.P. by graphical method :

Maximize : Z = 4x + 6y subject to 3x + 2y ≤ 12, x + y ≥ 4, x, y ≥ 0.

Solution: The draw the feasible region, construct table as follows:

Inequality	3x + 2y ≤ 12	x + y ≥ 4

Corresponding equation (of line)	3x + 2y = 12	x + y = 4
Intersection of line with X-axis	(4, 0)	(4, 0)
Intersection of line with Y-axis	(0, 6)	(0, 4)
Region	Origin side	Non-origin side



Shaded portion ABC is the feasible region,
whose vertices are A(4, 0), B(0, 6), C(0, 4).
Here, the objective function is Z = 4x + 6y
∴ Z at A(4, 0) = 4(4) + 6(0) = 16
Z at B(0, 6) = 4(0) + 6(6) = 36
Z at C(0, 4) = 4(0) + 6(4) = 24
∴ Z has maximum value 36 at B(0, 6)
∴ Z is maximum, when x = 0 and y = 6.

Exercise 6.2 | Q 3 | Page 101

Solve the following L.P.P. by graphical method :

Maximize : Z = 7x + 11y subject to 3x + 5y ≤ 26, 5x + 3y ≤ 30, x ≥ 0, y ≥ 0.

Solution: The draw the feasible region, construct table as follows:

Inequality	$3x + 5y \leq 26$	$5x + 3y \leq 26$
Corresponding equation (of line)	$3x + 5y \ \ 26$	$5x + 3y = 30$
Intersection of line with X-axis	$\left(\frac{26}{3}, 0\right)$	(6, 0)
Intersection of line withY-axis	$\left(0,\ \frac{26}{5}\right)$	(0, 10)
Region	Origin side	Origin side

Shaded portion OABC is the feasible region,

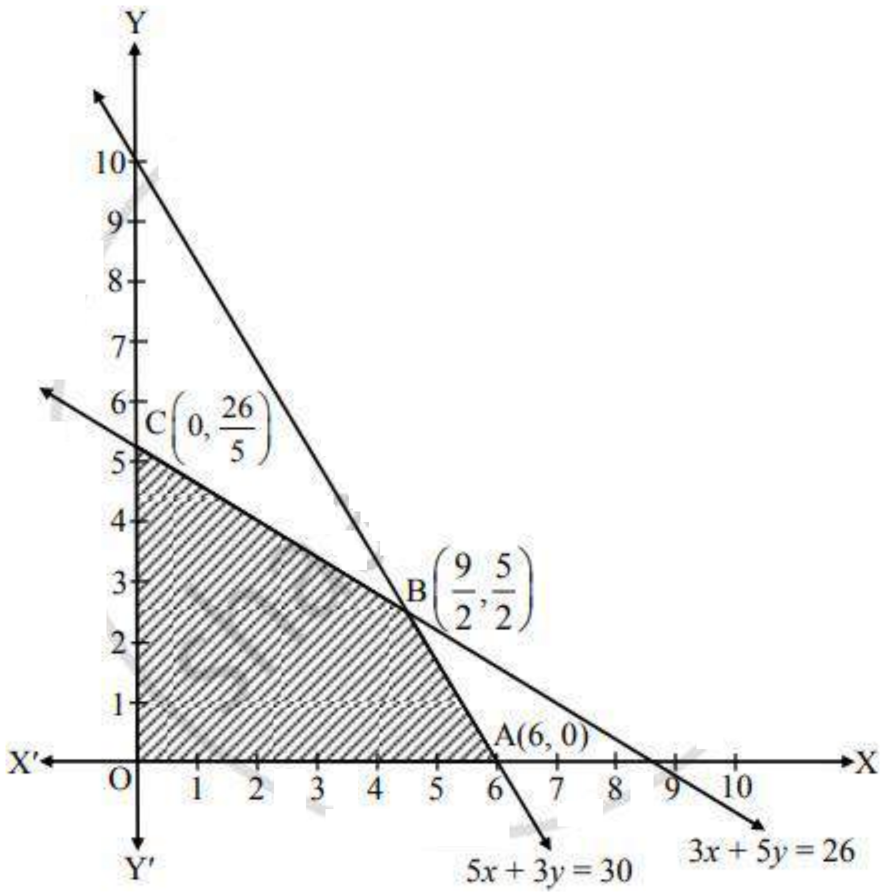
whose vertices are O(0, 0), A(6, 0), B and C $\left(0,\ \frac{26}{5}\right)$.

B is the point of intersection of the lines $5x + 3y = 30$ and $3x + 5y = 26$.

Solving the above equations, we get

$x = \frac{9}{2}, y = \frac{5}{2}$

$\therefore$ B = $\left(\frac{9}{2}, \frac{5}{2}\right) \equiv (4.5, 2.5)$



Here, the objective function is Z = 7x + 11y

∴ Z at O(0, 0) = 7(0) + 11(0) = 0

Z at A(6, 0) = 7(6) + 11(0) = 42

Z at B $\left(\frac{9}{2}, \frac{5}{2}\right) = 7\left(\frac{9}{2}\right) + 11\left(\frac{5}{2}\right) = \frac{63 + 55}{2}$ = 59

Z at C $\left(0, \frac{26}{5}\right) = 7(0) + 11\left(\frac{26}{5}\right) \; \frac{286}{5}$ = 57.2

∴ Z has maximum value 59 at B $\left(\frac{9}{2}, \frac{5}{2}\right)$.

i.e. at B(4.5, 2.5)

∴ Z is maximum, when $x = \frac{9}{2}$ and $y = \frac{5}{2}$

i.e. when x = 4.5 and y = 2.5

Exercise 6.2 | Q 4 | Page 101

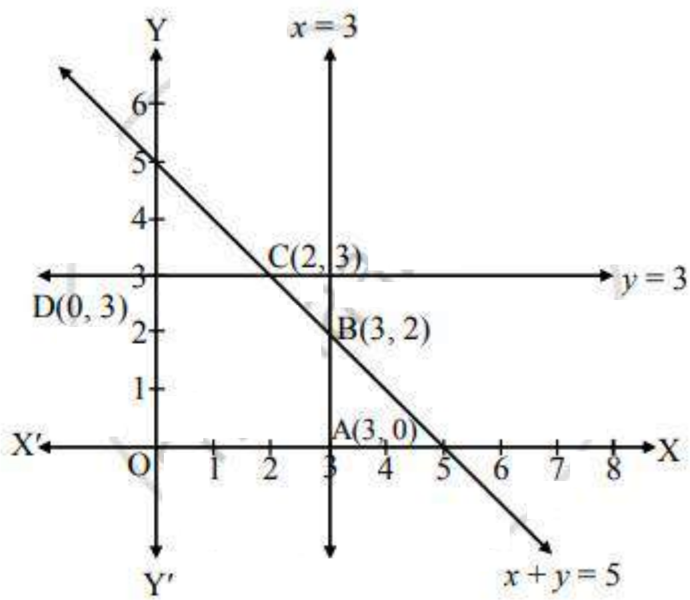
Solve the following L.P.P. by graphical method :

Maximize : Z = 10x + 25y subject to $0 \le x \le 3$, $0 \le y \le 3$, $x + y \le 5$ also find maximum value of z.

Solution: To draw the feasible region, construct table as follows:

Inequality	$x \le 3$	$y \le 3$	$x + y \le 5$
Corresponding equation (of line)	x = 3	y = 3	x + y = 5
Intersection of line with X-axis	(3, 0)	–	(5, 0)
Intersection of line with Y-axis	–	(0, 3)	(0, 5)
Region	Origin side	Origin side	Origin side

Shaded portion OABCD is the feasible region,
whose vertices are O(0, 0), A(3, 0), B, C and D(0, 3)
B is the point of intersection of the lines x = 3 and x + y = 5.
Substituting x = 3 in x + y = 5, we get y = 2
∴ B ≡ (3, 2)
C is the point of intersection of the lines y = 3 and x + y = 5.
Substituting y = 3 in x + y = 5, we get
x = 2
∴ C ≡ (2, 3)



Here, the objective function is Z = 10x + 25y
∴ Z at O(0, 0) = 10(0) + 25(0) = 0
Z at A(3, 0) = 10(3) + 25(0) = 30
Z at B(3, 2) = 10(3) + 25(2) = 30 + 50 = 80

Z at C(2, 3) = 10(2) + 25(3) = 20 + 75 = 95
Z at D(0, 3) = 10(0) + 25(3) = 75
∴ Z has maximum value 95 at C(2, 3).
∴ Z is maximum, when x = 2 and y = 3.

Exercise 6.2 | Q 5 | Page 101

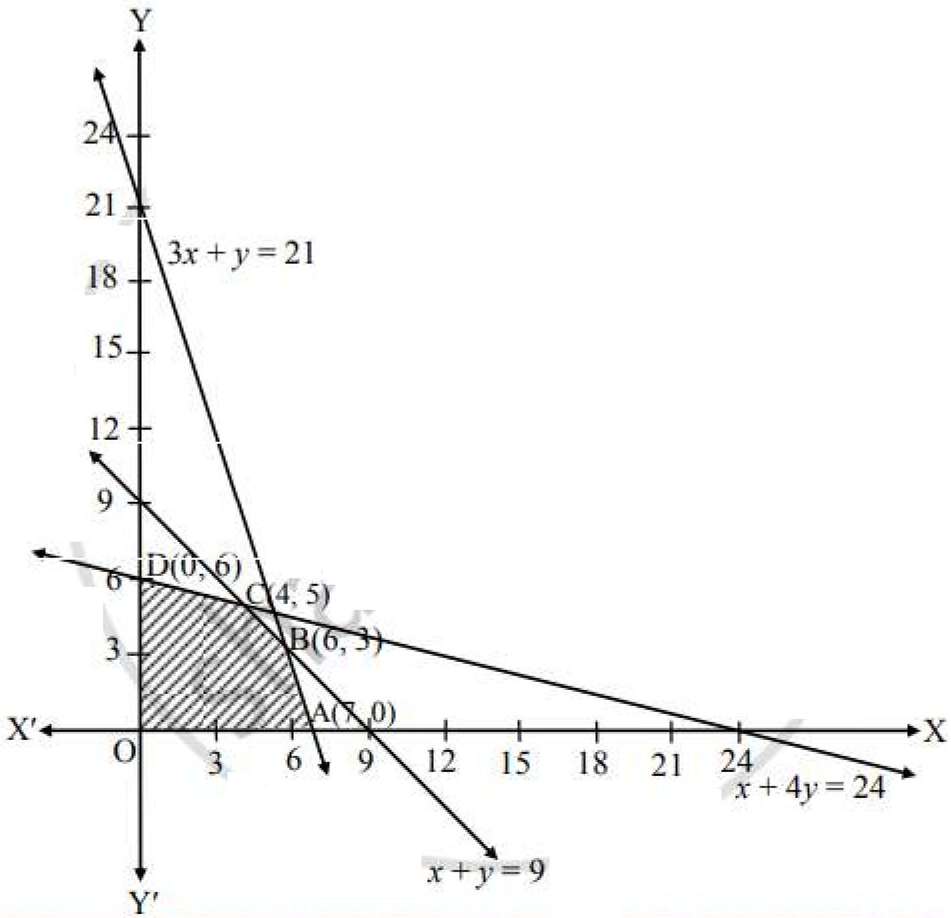
Solve the following L.P.P. by graphical method :

Maximize: Z = 3x + 5y subject to x + 4y ≤ 24, 3x + y ≤ 21, x + y ≤ 9, x ≥ 0, y ≥ 0 also find maximum value of Z.

Solution: To draw the feasible region, construct table as follows:

Inequality	x + 4y ≤ 24	3x + y ≤ 21	x + y ≤ 9
Corresponding equation (of line)	x + 4y = 24	3x + y = 21	x + y = 9
Intersection of line with X-axis	(24, 0)	(7, 0)	(9, 0)
Intersection of line with Y-axis	(0, 6)	(0, 21)	(0, 9)
Region	Origin side	Origin side	Origin side

Shaded portion OABCD is the feasible region,
whose vertices are O (0, 0), A (7, 0), B, C and D (0, 6)
B is the point of intersection of the lines 3x + y = 21 and x + y = 9.
Solving the above equations, we get
x = 6, y = 3
∴ B ≡ (6, 3)
C is the point of intersection of the lines x + 4y = 24
and x + y = 9.
Solving the above equations, we get
x = 4y = 5
∴ C ≡ (4, 5)



Here, the objective function is
Z = 3x + 5y
∴ Z at O(0, 0) = 3(0) + 5(0) = 0
Z at A(7, 0) = 3(7) + 5(0) = 21
Z at B(6, 3) = 3(6) + 5(3)
= 18 + 15 = 33
Z at C(4, 5) = 3(4) + 5(5)
= 12 + 25 = 37
Z at D(0, 6) = 3(0) + 5(6) = 30
∴ Z has maximum value 37 at C(4, 5).
∴ Z is maximum, when x = 4, y = 5.

Exercise 6.2 | Q 6 | Page 101

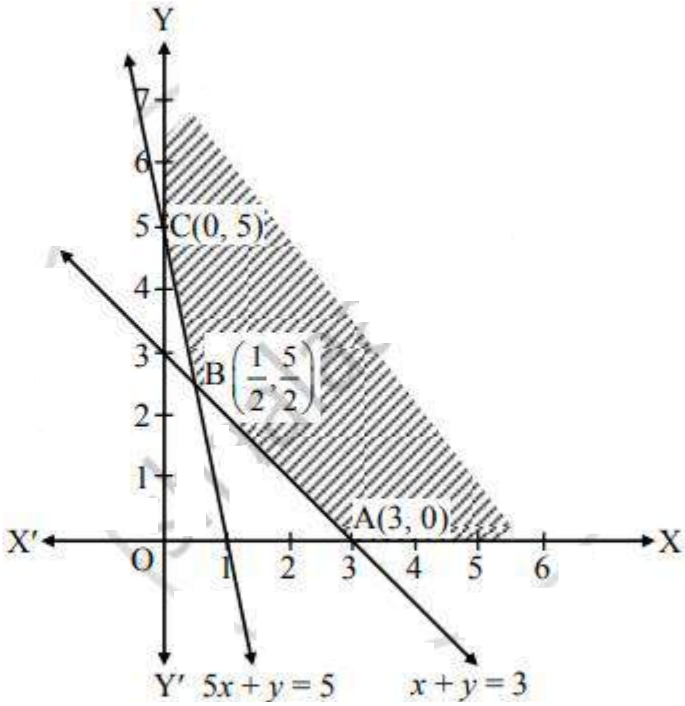
Solve the following L.P.P. by graphical method :

Minimize : Z = 7x + y subject to 5x + y ≥ 5, x + y ≥ 3, x ≥ 0, y ≥ 0.

Solution: To draw the feasible region, construct table as follows:

Inequality	5x + y ≥ 5	x + y ≥ 3
Corresponding equation (of line)	5x + y = 5	x + y = 3

Intersection of line with X-axis	(1, 0)	(3, 0)
Intersection of line with Y-axis	(0, 5)	(0, 3)
Region	Non-origin side	Non-origin side



Shaded portion XABCY is the feasible region,
whose vertices are A(3, 0), B and C (0, 5).
B is the point of intersection of the lines x + y = 3 and 5x + y = 5
Solving the above equations, we get

x = $\frac{1}{2}, y = \frac{5}{2}$

$\therefore$ B ≡ $\left(\frac{1}{2}, \frac{5}{2}\right)$

Here, the objective function is Z = 7x + y

Z at A(3, 0) = 7(3) + 0 = 21

Z at B $\left(\frac{1}{2}, \frac{5}{2}\right) = 7\left(\frac{1}{2}\right) + \frac{5}{2}$

= $\frac{7}{2} + \frac{5}{2}$ = 6

Z at C (0, 5) = 7(0) + 5 = 5

$\therefore$ Z has minimum value 5 at C(0, 5).

$\therefore$ Z is minimum, when x = 0 and y = 5.

Exercise 6.2 | Q 7 | Page 101

Solve the following L.P.P. by graphical method :

Minimize: Z = 8x + 10y subject to 2x + y ≥ 7, 2x + 3y ≥ 15, y ≥ 2, x ≥ 0, y ≥ 0.

Solution: To draw the feasible region, construct table as follows:

Inequality	2x + y ≥ 7	2x + 3y ≥ 15	y ≥ 2
Corresponding equation (of line)	2x + y = 7	2x + 3y = 15	y = 2
Intersection of line with X-axis	$\left(\frac{7}{2}, 0\right)$	$\left(\frac{15}{2}, 0\right)$	–
Intersection of line with Y-axis	(0, 7)	(0, 5)	(0, 2)
Region	Non-origin side	Non-origin side	Non-origin side

Shaded portion EABCY is the feasible region,
whose vertices are A, B and C(0, 7)
A is the point of intersection of the lines y = 2 and 2x + 3y = 15
Subtituting y = 2 in 2x + 3y = 15, we get
2x + 6 = 15

$\therefore$ x = $\frac{9}{2}$

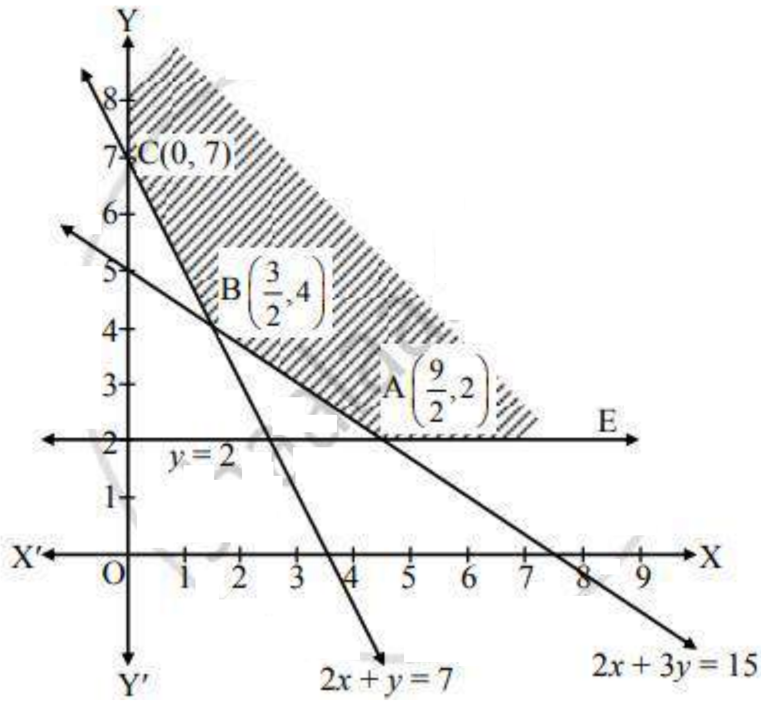
$\therefore$ A = $\left(\frac{9}{2}, 2\right)$

B is the point of intersection of the lines 2x + y = 7 and 2x + 3y = 15.

Solving the above equations, we get

x = $\frac{3}{2}$, y = 4

$\therefore$ B ≡ $\left(\frac{3}{2}, 4\right)$ ≡ (1 5, 4)



Here, the objective function is Z = 8x + 10y

Z at A = $\left(\frac{9}{2}, 2\right) = 8\left(\frac{9}{2}\right) + 10(2)$ = 36 + 20 = 56

Z at B = $\left(\frac{3}{2}, 4\right) = 8\left(\frac{3}{2}\right) + 10(4)$ = 12 + 40 = 52

Z at C(0, 7) = 8(0) + 10(7) = 70

∴ Z has minimum value 52 at B $\left(\frac{3}{2}, 4\right)$ i.e. at (1, 5, 4)

∴ Z is minimum, when x = $\frac{3}{2}$ i.e. 1.5 and y = 4.

Exercise 6.2 | Q 8 | Page 101

Solve the following L.P.P. by graphical method :

Minimize: Z = 6x + 2y subject to x + 2y ≥ 3, x + 4y ≥ 4, 3x + y ≥ 3, x ≥ 0, y ≥ 0.

Solution: To draw the feasible region, construct table as follows:

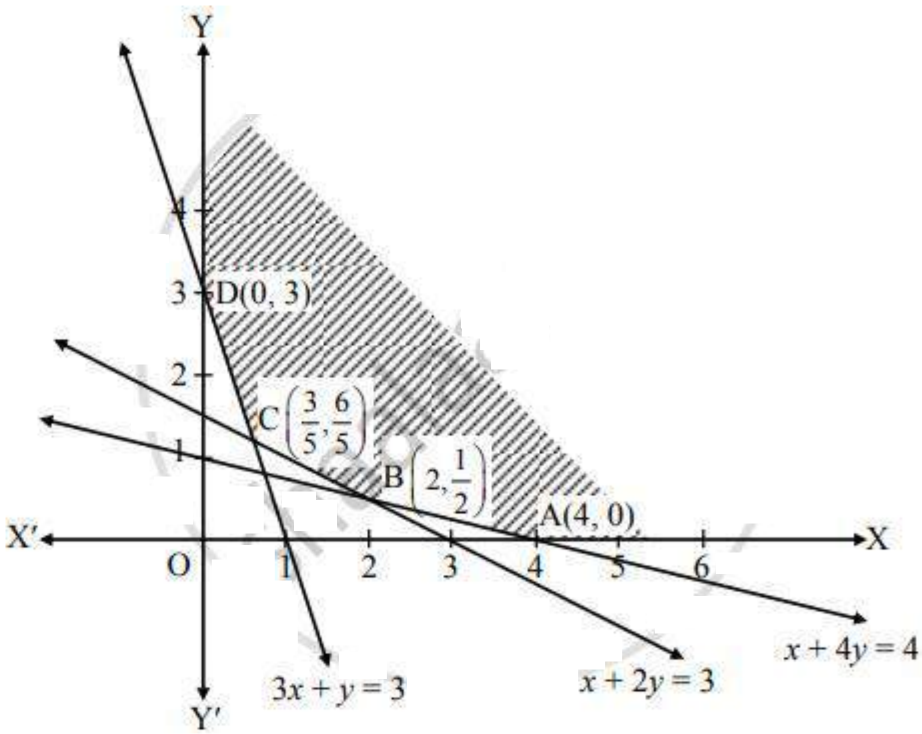
Inequality	x + 2y ≥ 3	x + 4y ≥ 4	3x + y ≥ 3
Corresponding equation (of line)	x + 2y = 3	x + 4y = 4	3x + y = 3
Intersection of line with X-axis	(3, 0)	(4, 0)	(1, 0)
Intersection of line with Y-axis	$\left(0, \frac{3}{2}\right)$	(0, 1)	(0, 3)
Region	Non-origin side	Non-origin side	Non-origin side

Shaded portion XABCDY is the feasible region,
whose vertices are A(4, 0), B, C and D(0, 3).
B is the point of intersection of the lines x + 4y = 4 and x + 2y = 3
Solving the above equations, we get

$x = 2, y = \frac{1}{2} \quad \therefore B \equiv \left(2, \frac{1}{2}\right)$

C is the point of intersection of the lines x + 2y = 3 and 3x + y = 3.
Solving the above equations, we get

$x = \frac{3}{5}, y = \frac{6}{5} \quad \therefore C \equiv \left(\frac{3}{5}, \frac{6}{5}\right)$



Here, the objective function is Z = 6x + 2y

Z at A(4, 0) = 6(4) + 2(0) = 24

Z at B $\left(2, \frac{1}{2}\right) = 6(2) + 2\left(\frac{1}{2}\right)$ = 12 + 1 = 13

Z at C $\left(\frac{3}{5}, \frac{6}{5}\right) = 6\left(\frac{3}{5}\right) + 2\left(\frac{6}{5}\right) = \frac{18}{5}\Big) + \frac{12}{5}$ = 6

∴ Z at D(0, 3) = 6(0) + 2(3) = 6

∴ Z has minimum value 6 at C $\left(\frac{3}{5}, \frac{6}{5}\right)$ and D(0, 3).

∴ Z is minimum when, x = $\left(\frac{3}{5}\right), y = \left(\frac{6}{5}\right)$, Z = 6 and x = 0, y = 3, Z = 6.

MISCELLANEOUS EXERCISE 6 [PAGES 102 - 105]

Miscellaneous Exercise 6 | Q 1.01 | Page 102

Choose the correct alternative :

The value of objective function is maximize under linear constraints.

1. **at the centre of feasible region**
2. at (0, 0)
3. at any vertex of feasible region.
4. The vertex which is at maximum distance from (0, 0).

Solution: The value of objective function is maximize under linear constraints **at the centre of feasible region**.

Miscellaneous Exercise 6 | Q 1.02 | Page 102

Choose the correct alternative :

Which of the following is correct?

1. Every LPP has on optional solution
2. Every LPP has unique optional solution.
3. **If LPP has two optional solution then it has infinitely many solutions.**
4. The set of all feasible solutions of LPP may not be a convex set.

Solution: If LPP has two optional solution then it has infinitely many solutions.

Miscellaneous Exercise 6 | Q 1.03 | Page 102

Choose the correct alternative :

Objective function of LPP is

1. a constraint
2. **a function to be maximized or minimized**
3. a relation between the decision variables
4. a feasible region.

Solution: Objective function of LPP is **a function to be maximized or minimized**.

Miscellaneous Exercise 6 | Q 1.04 | Page 102

Choose the correct alternative :

The maximum value of z = 5x + 3y. subject to the constraints

1. 235
2. 235/9

3. **235/19**
4. 235/3

Solution: Z = 5x + 3y

The inequalities are 3x + 5y ≤ 15, 5x + 2y ≤ 10

Consider lines L_1 and L_2 where

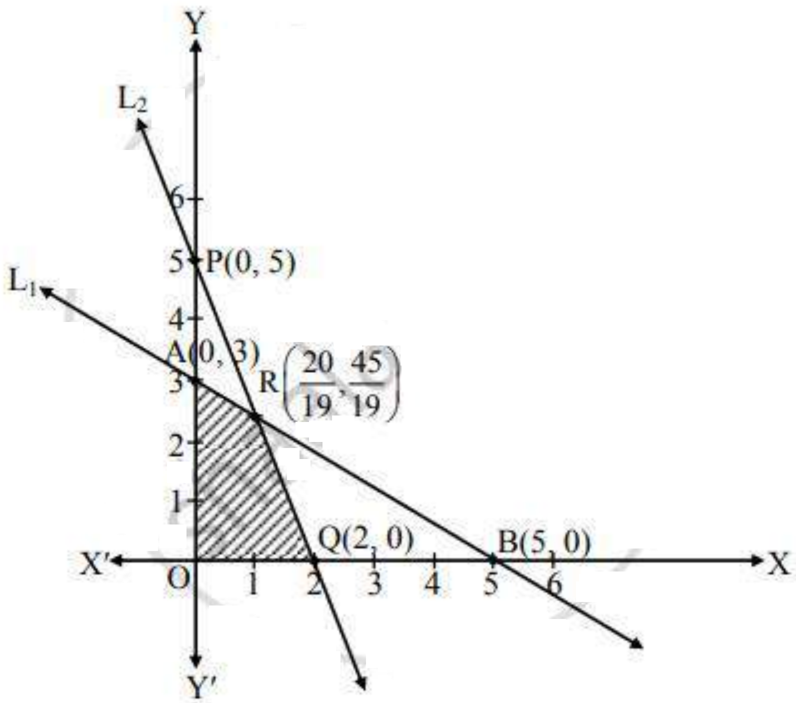
L_1 : 3x + 5y = 15, 5x + 2y = 10

For line L_1, Plot A (0, 3) and B (5, 0)

For line L_2, plot P(0, 5) and Q(2, 0)

Solving both line, we get x = $\frac{20}{19}, y = \frac{45}{19}$

The coordinates of the origin O (0, 0) satisfies both the inequalities.



∴ The required region is on the origin side of both the lines L_1 and L_2.

As x ≥ 0, y ≥ 0; the feasible region is in the 1^{st} quadrant.

OQRAO is the required feasible region.

At O (0, 0), Z = 0

At Q (2, 0), Z = 5(2) + 0 = 10

At R $\left(\frac{20}{19}, \frac{45}{19}\right), z = 5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right) = \frac{235}{19}$.

At A (0, 3), Z = 0 + 3(3) = 9

The maximum value of Z is $\frac{235}{19}$ and it occurs at point R $\left(\frac{20}{19}, \frac{45}{19}\right)$

Miscellaneous Exercise 6 | Q 1.05 | Page 102

Choose the correct alternative :

The maximum value of z = 10x + 6y, subjected to the constraints 3x + y ≤ 12, 2x + 5y ≤ 34, x ≥ 0, y ≥ 0 is.

1. **56**
2. `65
3. 55
4. 66

Solution: Z = 10x + 6y

The given inequalities are 3x + y ≤ 12 and 2x + 5y ≤ 34.

Consider line L_1 and L_2 where

L_1 : 3x + y = 12, L_2 : 2x + 5y = 34

For line L_1, plot A (0, 12) and B (4, 0)

For line L_2, plot P (0, 6.8) and Q (17, 0)

Solving both lines, we get x = 2, y = 6.

The coordinates of origin O (0, 0) satisfies both the inequalities.

∴ The required region is on the origin side of both the lines L_1 and L_2.

As x ≥ 0, y ≥ 0, the feasible region is in the 1st quadrant.

OBRPO is the required feasible region.

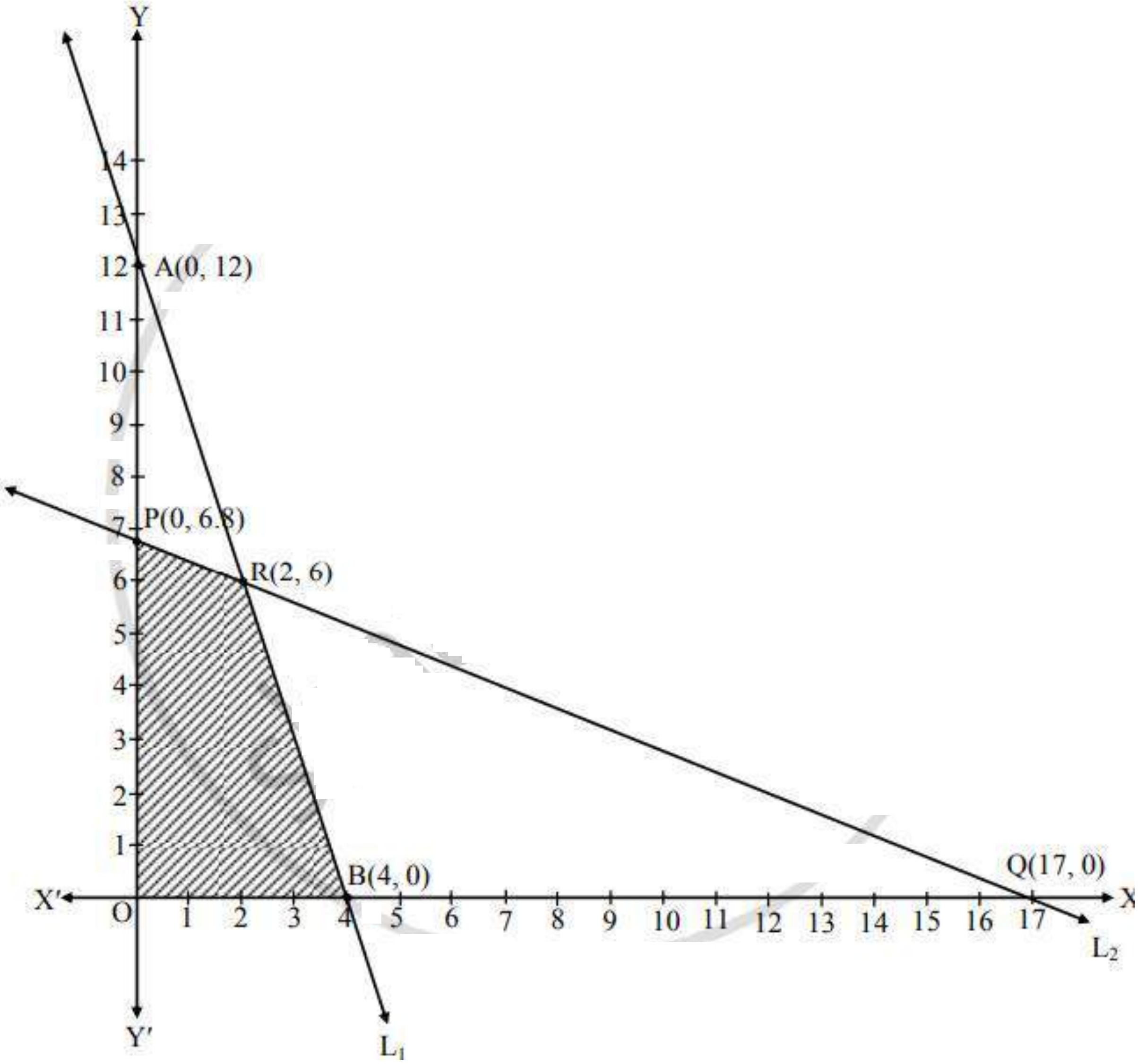
At O (0, 0), Z = 0

At B (4, 0), Z = 10 (4) + 0 = 40

At R (2, 6), Z = 10 (2) + 6 (6) = 56

At P (0, 6.8), Z = 0 + 6 (6.8) = 40.8

The maximum value of Z is **56** and it occurs at R (2, 6).



Miscellaneous Exercise 6 | Q 1.06 | Page 103

Choose the correct alternative :

The point at which the maximum value of z = x + y subject to the constraints x + 2y ≤ 70, 2x + y ≤ 95, x ≥ 0, y ≥ 0 is

1. (36, 25)
2. (20, 35)
3. (35, 20)
4. **(40, 15)**

Solution: Z = x + y

The given inequalities are x + 2y ≤ 70, 2x + y ≤ 95.

Consider lines L_1 and L_2 where L_1 : x + 2y = 70 and L2 : 2x + y = 95.

For line L_1, plot A (0, 35) and B (70, 0)

For line L_2, plot P (0, 95) and Q (47.5, 0).

Solving both lines we get x = 40, y = 15

The coordinates of origin O (0, 0) satisfies both the inequalities.

∴ The required region is on the origin side of both the lines L_1 and L_2.

As x ≥ 0, y ≥ 0, the feasible region is in the first quadrant.

OQRAO is the required feasible region.

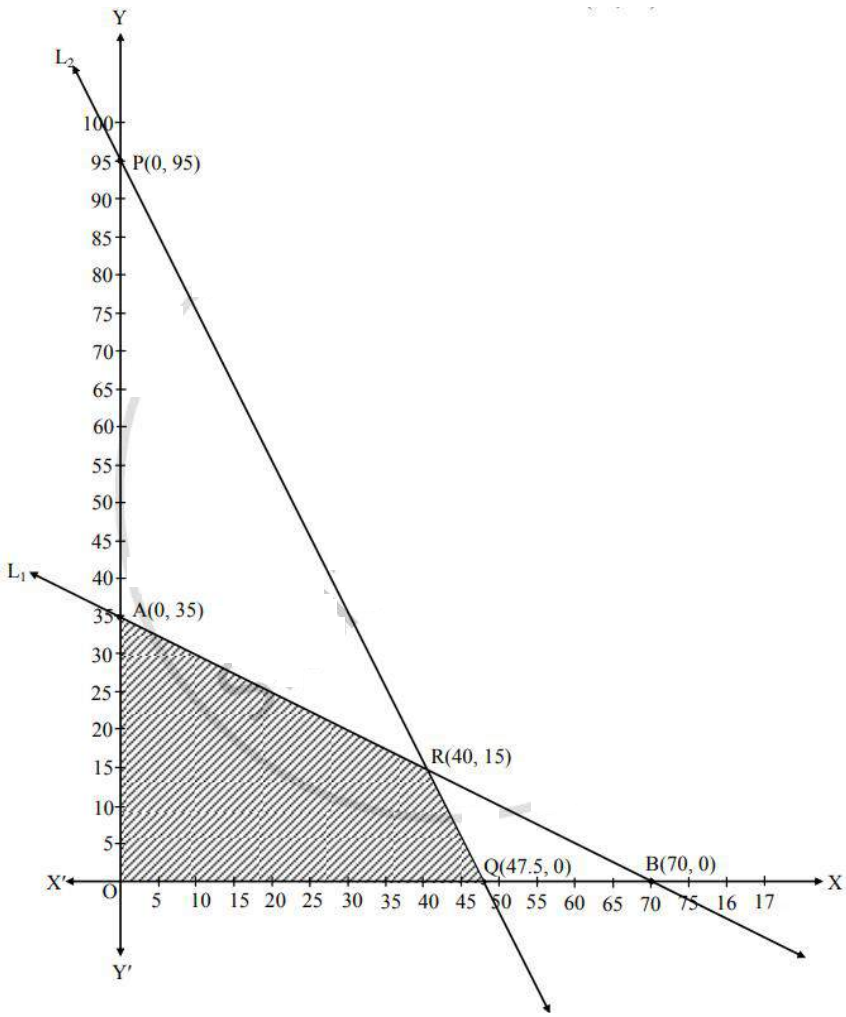
At O (0, 0), z = 0

At Q (47.5, 0), Z = 47.5 + 0 = 47.5

At R (40, 15), Z = 40 + 15 = 55

At A (0, 35), Z = 0 + 35 = 35.

The maximum value of Z is 55 and it occurs at R **(40, 15)**.



Miscellaneous Exercise 6 | Q 1.07 | Page 103

Choose the correct alternative :

Of all the points of the feasible region the optimal value of z is obtained at a point

1. inside the feasible region.
2. at the boundary of the feasible region.
3. **at vertex of feasible region.**
4. on x - axis.

Solution: Of all the points of the feasible region the optimal value of z is obtained at a point **at vertex of feasible region**.

Miscellaneous Exercise 6 | Q 1.08 | Page 103

Choose the correct alternative :

Feasible region; the set of points which satify.

1. The objective function.
2. **All of the given constraints.**
3. Some of the given constraints
4. Only non-negative constrains

Solution: All of the given constraints.

Miscellaneous Exercise 6 | Q 1.09 | Page 103

Choose the correct alternative :

Solution of LPP to minimize $z = 2x + 3y$ st. $x \geq 0$, $y \geq 0$, $1 \leq x + 2y \leq 10$ is

1. **$x = 0$, $y = 1/2$**
2. $x = 1/2$, $y = 0$
3. $x = 1$, $y = -2$
4. $x = y = 1/2$

Solution: $Z = 2x + 3y$

The given inequalities are $1 \leq x + 2y \leq 10$

i.e. $x + 2y \geq 1$ and $x + 2y \leq 10$

consider lines L_1 and L_2 where $L_1 : x + 2y = 1$, $L_2 : x + 2y = 10$.

For line L_1 plot A (0, 1/2), B(1, 0)

For line L_2 plot P (0, 5), Q (10, 0).

The coordinates of origin O (0, 0) do not satisfy $x + 2y \geq 1$.

Required region lies on non – origin side of L_1.

The coordinates of origin O(0, 0) satisfies the inequalities $x + 2y \leq 10$.

Required region lies on the origin side of L_2.

Lines L_1 and L_2 are parallel.

ABQPA is the required feasible region

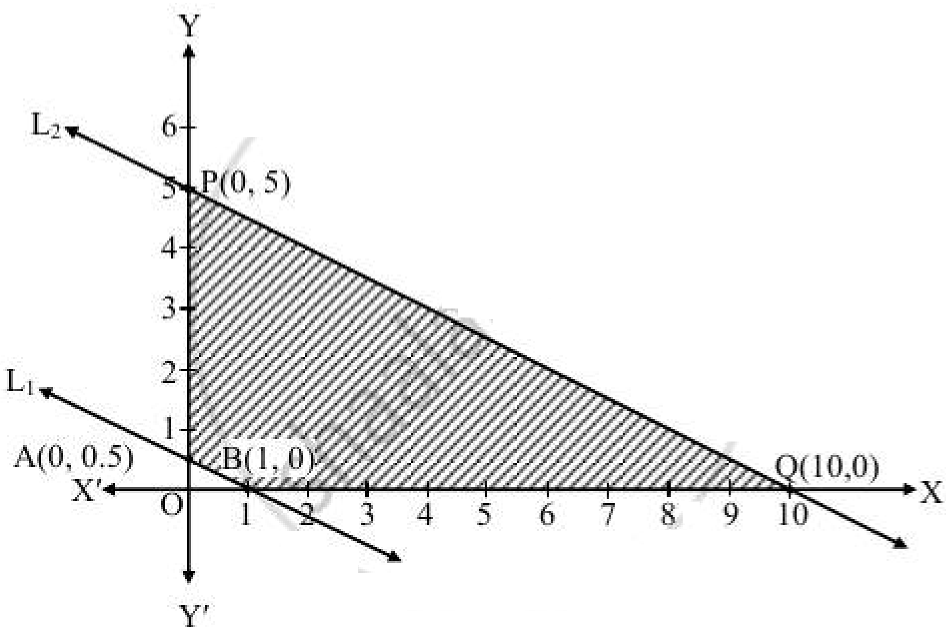
At $\mathbf{A}\left(0, \frac{1}{2}\right), \mathbf{Z} = 0 + 3\left(\frac{1}{2}\right)$ = 1.5

At B (1, 0), Z = 2 (1) + 0 = 2

At P (0, 5), Z = 0 + 3(5) = 15

At Q (10, 0), Z = 2 (10) + 0 = 20

The maximum value of Z is 1.5 and it occurs at $\mathbf{A}\left(0, \frac{1}{2}\right)$ i.e. x = 0, $y = \frac{1}{2}$



Miscellaneous Exercise 6 | Q 1.1 | Page 103

Choose the correct alternative :

The corner points of the feasible region given by the inequations $x + y \leq 4$, $2x + y \leq 7$, $x \geq 0$, $y \geq 0$, are

1. (0, 0), (4, 0), (3, 1), (0, 4).
2. **(0, 0), (7/2, 0), (3, 1), (0, 4).**
3. (0, 0), (7/2, 0),(3,1), (5, 7).
4. (6, 0), (4, 0), (3, 1), (0, 7).

Solution: Given inequalities are $x + y \leq 4$, $2x + y \leq 7$.

Consider line $L_1 : x + y = 4$ and $L_2 : 2x + y = 7$

For line L_1, A (0, 4) and B (4, 0)

For line L_2, P(0, 7) and Q(7/2, 0)

Solving both lines, we get x = 3, y = 1.

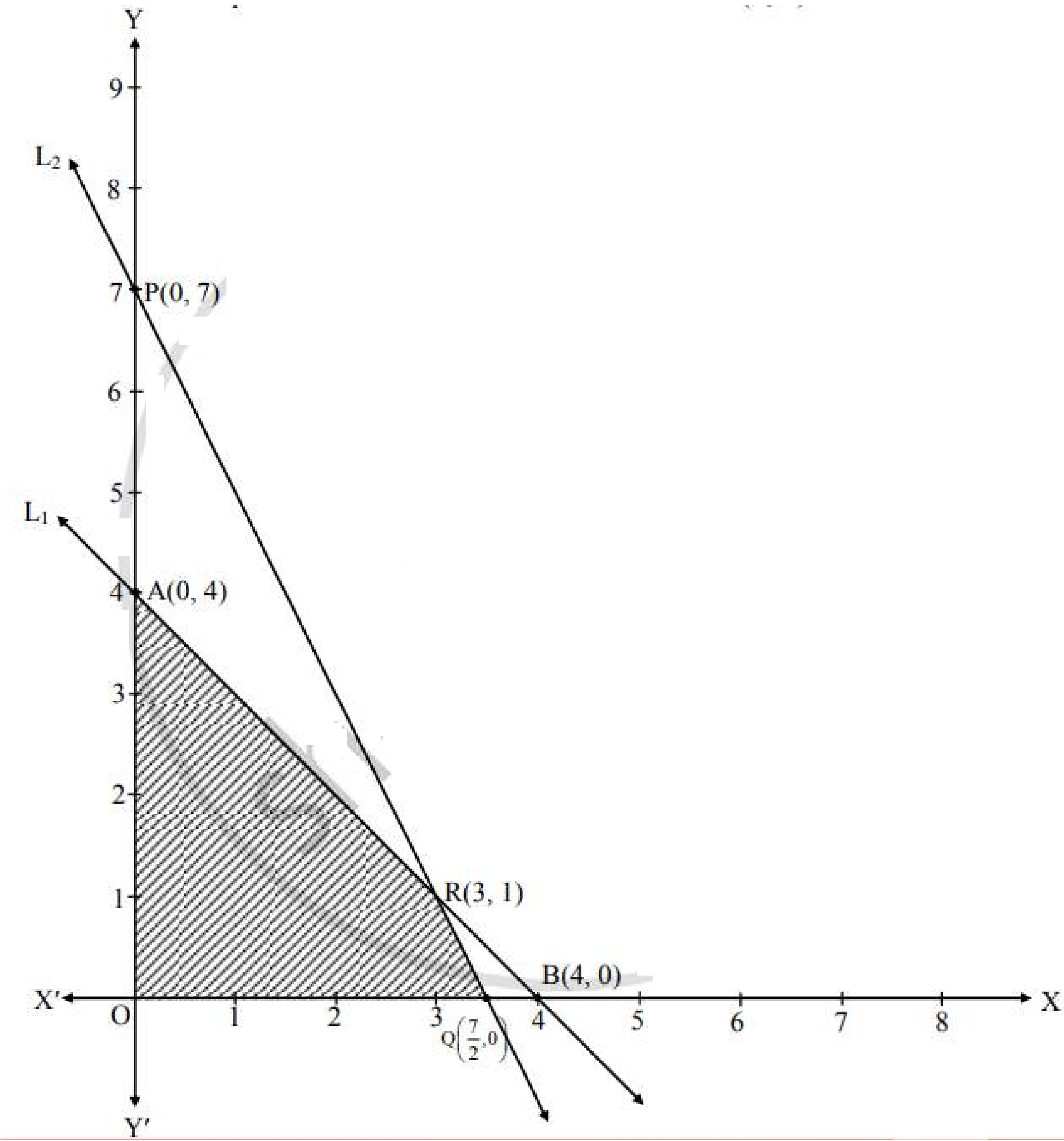
The coordinates of origin O (0, 0) satisfies both the inequalities.

∴ The required region is on the origin side of both the lines L_1 and L_2.

As $x \geq 0$, $y \geq 0$, the feasible region is in the first quadrant.

OQRAO is the required feasible region.

∴ **The corner points are O (0, 0), Q $\left(\frac{7}{2}, 0\right)$, R (3, 1), A (0, 4).**



Miscellaneous Exercise 6 | Q 1.11 | Page 103

Choose the correct alternative :

The corner points of the feasible region are (0, 0), (2, 0), (12/7, 3/7) and (0,1) then the point of maximum z = 7x + y

1. (0, 0)
2. **(2, 0)**
3. (12/7, 3/7)
4. (0, 1)

Solution:

Z = 7x + y

At (0, 0), Z = 0 + 0 = 0

At (2, 0), Z = 7 (2) + 0 = 14

At $\left(\frac{12}{7}, \frac{3}{7}\right), \mathrm{Z} = 7\left(\frac{12}{7}\right) + \frac{3}{7} = \frac{87}{7}$ = 12.428

At (0, 1), Z = 0 + 1 = 1.

The maximum value of Z is 14 and it occurs at **(2, 0)**.

Miscellaneous Exercise 6 | Q 1.12 | Page 103

Choose the correct alternative :

If the corner points of the feasible region are (0, 0), (3, 0), (2, 1) and (0, 7/3) the maximum value of z = 4x + 5y is .

1. 12
2. **13**
3. 35/2
4. 0

Solution: Z = 4x + 5y

At (0, 0), Z = 0 + 0 = 0

At (3, 0), Z = 4 (3) + 0 = 12

At (2, 1), Z = 4 (2) + 5 (1) = 13

At $\left(0, \frac{7}{3}\right), \mathrm{Z} = 0 + 5\left(\frac{7}{3}\right)$ = 11.67

The maximum value of Z is **13**.

Miscellaneous Exercise 6 | Q 1.13 | Page 103

Choose the correct alternative :

If the corner points of the feasible region are (0, 10), (2, 2), and (4, 0) then the point of minimum z = 3x + 2y is.

1. **(2, 2)**
2. (0, 10)
3. (4, 0)
4. (2, 4)

Solution: Z = 3x + 2y

At (0, 10) = Z = 0 + 2 (10) = 20

At (2, 2), = Z = 3(2) + 2 (2) = 10

At (4, 0), Z = 3 (4) + 0 = 12

The maximum value of Z is 10 and it occurs at **(2, 2)**.

Miscellaneous Exercise 6 | Q 1.14 | Page 103

Choose the correct alternative :

The half plane represented by 3x + 2y ≤ 0 constraints the point.

1. (1, 5/2)
2. (2, 1)
3. **(0, 0)**
4. (5, 1)

Solution: Only (0, 0) satisfies the given inequality.

Miscellaneous Exercise 6 | Q 1.15 | Page 103

Choose the correct alternative :

The half plane represented by 4x + 3y ≥ 14 contains the point

1. (0, 0)
2. (2, 2)
3. **(3, 4)**
4. (1, 1)

Solution: Only (3, 4) satisfies the given inequality**.**

Miscellaneous Exercise 6 | Q 2.1 | Page 103

Fill in the blank :

Graphical solution set of the in equations x ≥ 0, y ≥ 0 is in _______ quadrant

Solution: Graphical solution set of the in equations x ≥ 0, y ≥ 0 is in **I** quadrant.

Miscellaneous Exercise 6 | Q 2.2 | Page 103

Fill in the blank :

The region represented by the in equations x ≤ 0, y ≤ 0 lines in _______ quadrants

Solution: The region represented by the in equations x ≤ 0, y ≤ 0 lines in **II** quadrant.

Miscellaneous Exercise 6 | Q 2.3 | Page 103

Fill in the blank :

The optimal value of the objective function is attained at the _______ points of feasible region.

Solution: The optimal value of the objective function is attained at the **vertex** points of feasible region.

Miscellaneous Exercise 6 | Q 2.4 | Page 103

Fill in the blank :

The region represented by the inequality y ≤ 0 lies in _______ quadrants.

Solution: The region represented by the inequality y ≤ 0 lies in **III and IV** quadrants.

Miscellaneous Exercise 6 | Q 2.5 | Page 103

Fill in the blank :

The constraint that a factory has to employ more women (y) than men (x) is given by_______

Solution: The constraint that a factory has to employ more women (y) than men (x) is given by **y > x**.

Miscellaneous Exercise 6 | Q 2.6 | Page 103

Fill in the blank :

"A gorage employs eight men to work in its shownroom and repair shop. The constraints that there must be at least 3 men in showroom and at least 2 men in repair shop are ______ and _______ respectively.

Solution: "A gorage employs eight men to work in its shownroom and repair shop. The constraints that there must be at least 3 men in showroom and at least 2 men in repair shop are **x ≥ 3** and **x ≥ 2** respectively.

Miscellaneous Exercise 6 | Q 2.7 | Page 103

Fill in the blank :

A train carries at least twice as many first class passengers (y) as second class passengers (x) The constraint is given by_______

Solution: A train carries at least twice as many first class passengers (y) as second class passengers (x) The constraint is given by **$x \geq 2y$**.

Miscellaneous Exercise 6 | Q 2.8 | Page 103

Fill in the blank :

A dish washing machine holds up to 40 pieces of large crockery (x) This constraint is given by_______.

Solution: A dish washing machine holds up to 40 pieces of large crockery (x) This constraint is given by **$x \leq 40$**.

Miscellaneous Exercise 6 | Q 3.1 | Page 104

State whether the following is True or False :

The region represented by the inequalities $x \geq 0$, $y \geq 0$ lies in first quadrant.

1. **True**
2. False

Solution: The region represented by the inequalities $x \geq 0$, $y \geq 0$ lies in first quadrant **True**.

Miscellaneous Exercise 6 | Q 3.2 | Page 104

State whether the following is True or False :

The region represented by the inqualities $x \leq 0$, $y \leq 0$ lies in first quadrant.

1. True
2. **False**

Solution: The region represented by the inqualities $x \leq 0$, $y \leq 0$ lies in first quadrant **False**.

Miscellaneous Exercise 6 | Q 3.3 | Page 104

State whether the following is True or False :

The optimum value of the objective function of LPP occurs at the center of the feasible region.

1. True
2. **False**

Solution: The optimum value of the objective function of LPP occurs at the center of the feasible region **False**.

Miscellaneous Exercise 6 | Q 3.4 | Page 104

State whether the following is True or False :

Graphical solution set of $x \le 0$, $y \ge 0$ in xy system lies in second quadrant.

1. **True**
2. False

Solution: Graphical solution set of $x \le 0$, $y \ge 0$ in xy system lies in second quadrant **True**.

Miscellaneous Exercise 6 | Q 3.5 | Page 104

State whether the following is True or False :

Saina wants to invest at most ₹ 24000 in bonds and fixed deposits. Mathematically this constraints is written as $x + y \le 24000$ where x is investment in bond and y is in fixed deposits.

1. **True**
2. False

Solution: Saina wants to invest at most ₹ 24000 in bonds and fixed deposits. Mathematically this constraints is written as $x + y \le 24000$ where x is investment in bond and y is in fixed deposits **True**.

Miscellaneous Exercise 6 | Q 3.6 | Page 104

State whether the following is True or False :

The point (1, 2) is not a vertex of the feasible region bounded by $2x + 3y \le 6$, $5x + 3y \le 15$, $x \ge 0$, $y \ge 0$.

1. **True**
2. False

Solution: Since (1, 2) does not satisfy any of the equations $2x + 3y = 6$ and $5x + 3y = 15$, it is not a vertex of the feasible region **True**.

Miscellaneous Exercise 6 | Q 3.7 | Page 104

State whether the following is True or False :

The feasible solution of LPP belongs to only quadrant I.

1. **True**
2. False

Solution: The feasible solution of LPP belongs to only quadrant I **True**.

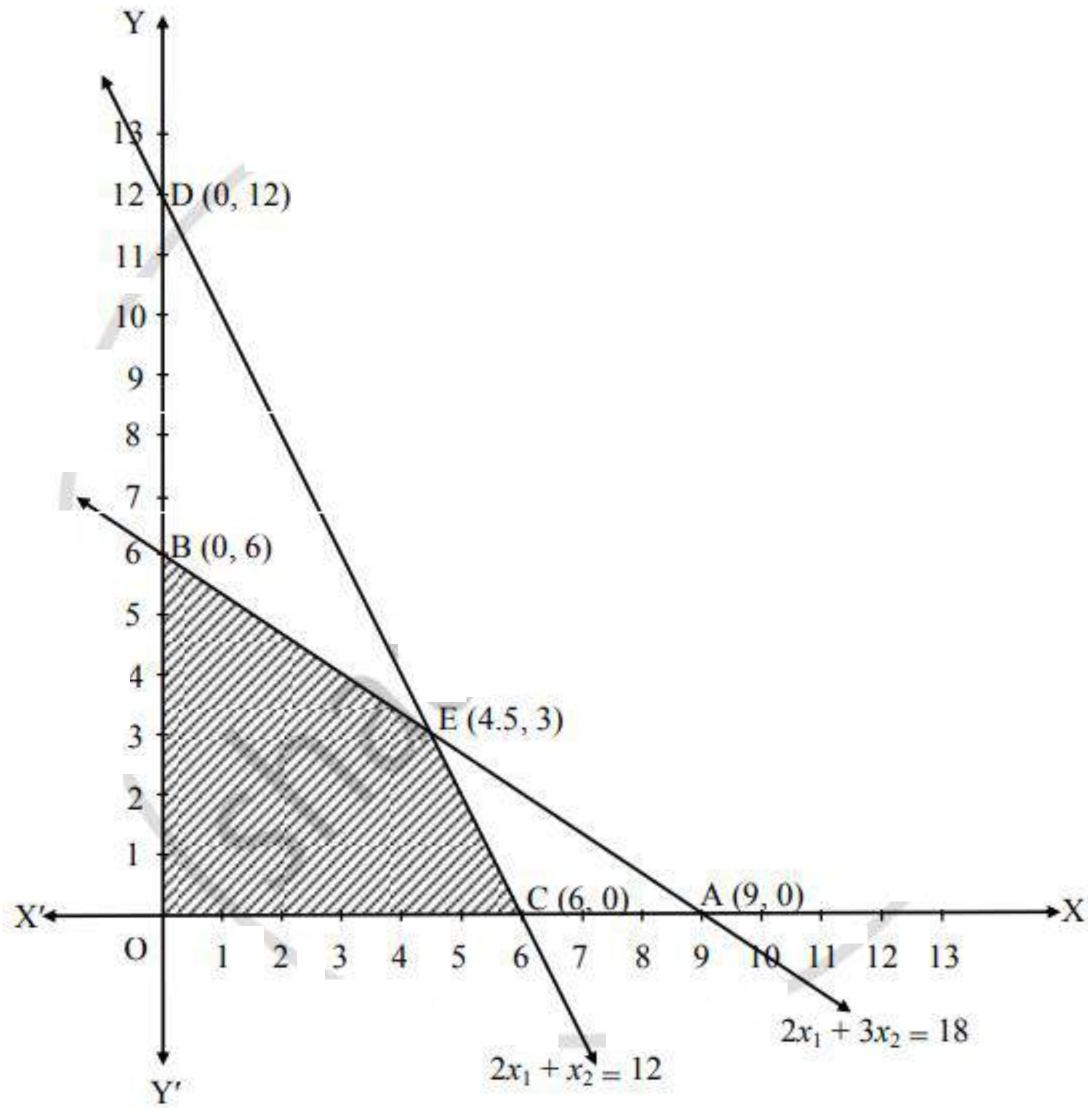
Miscellaneous Exercise 6 | Q 4.01 | Page 104

Solve the following problem :

Maximize Z = $5x_1 + 6x_2$ Subject to $2x_1 + 3x_2 \leq 18$, $2x_1 + x_2 \leq 12$, $x \geq 0$, $x_2 \geq 0$

Solution: To find the graphical solution, construct the table as follows:

Inequation	equation	Double intercept form	Points (x_1, x_2)	Region
$2x_1 + 3x_2 \leq 18$	$2x_1 + 3x_2 = 18$	$\frac{x_1}{9} + \frac{x_2}{6} = 1$	A (9, 0) B (0, 6)	$2(0) + 3(0) \leq 18$ $\therefore 0 \leq 18$ ∴ Origin-side
$2x_1 + x_2 \leq 12$	$2x_1 + x_2 = 12$	$\frac{x_1}{6} + \frac{x_2}{12} = 1$	C (6, 0) D (0, 12)	$2(0) + 1(0) \leq 12$ $\therefore 0 \leq 12$ ∴ Origin-side
$x_1 \geq 0$	$x_1 = 0$	–	–	R.H.S. of Y-axis
$x_2 \geq 0$	$x_2 \geq 0$	–	–	above X-axis



The shaded portion OBEC is the feasible region.
Whose vertices are O (0, 0), B (0, 6), E, C (6, 0)
E is the point of intersection of the lines

$2x_1 + x_2 = 12$ ...(i)

and $2x_1 + 3x_2 = 18$ ...(ii)

∴ By (i) – (ii), we get

$2x_1 + x_2 = 12$

$2x_1 + 3x_2 = 18$

– – –

$-2x_2 = -6$

∴ $x_2 = -6/-2 = 3$

Substituting x2 = 3 in (i), we get

$2x_1 + 3 = 12$

∴ $2x_1 = 12 - 3$

∴ $2x_1 = 9$

∴ $x_1 = 9/2 = 4.5$

∴ E (4.5,3)
Here, the objective function is $Z = 5x_1 + 6x_2$
Now, we will find maximum value of Z as follows:

Feasible points	The value of $Z = 5x_1 + 6x_2$
O (0, 0)	Z = 5(0) + 6(0) = 0
B (0, 6)	Z = 5(0) + 6(6) = 36
E (4.5, 3)	Z = 5(4.5) + 6(3) = 22.5 + 18 = 40.5
E (4.5, 3)	Z = 5(6) + 6(0) = 30

∴ Z has maximum value 40.5 at E(4.5, 3)
∴ Z is maximum, when $x_1 = 4.5$, $x_2 = 3$.

Miscellaneous Exercise 6 | Q 4.02 | Page 104

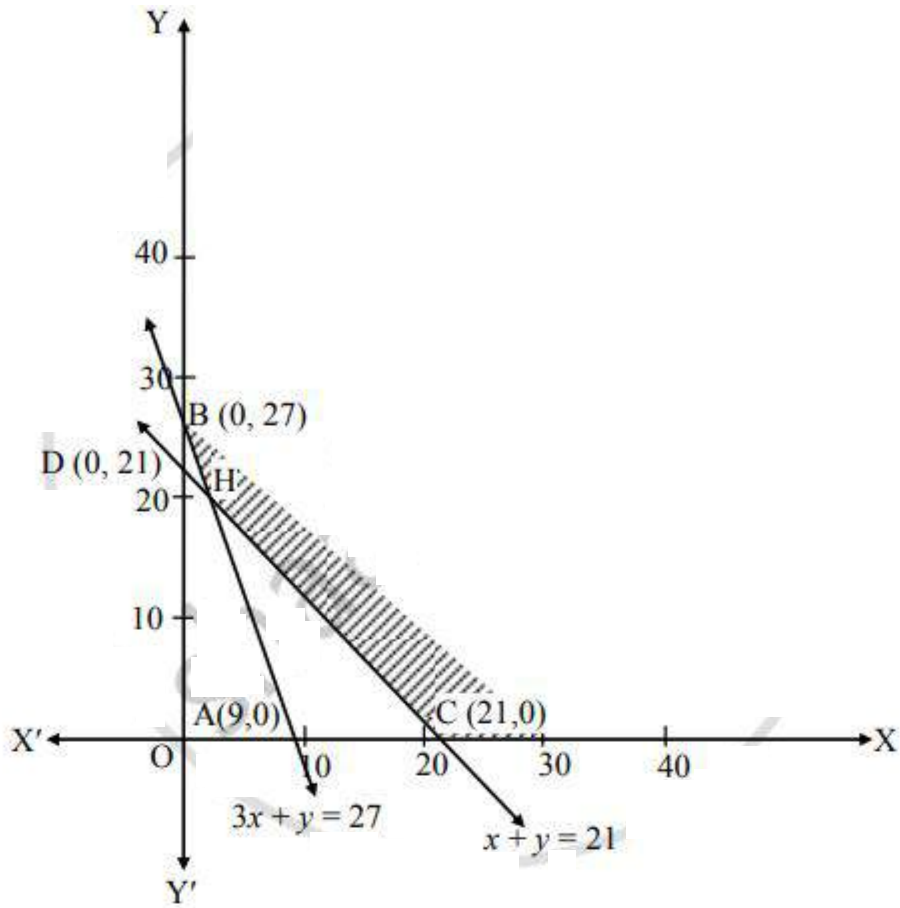
Solve the following problem :

Minimize Z = 4x + 2y Subject to 3x + y ≥ 27, x + y ≥ 21, x ≥ 0, y ≥ 0

Solution: To find the graphical solution, construct the table as follows:

To find the graphical solution, construct the table as follows:

Inequation	equation	Double intercept form	Points (x1, x2)	Points (x1, x2)
3x + y ≥ 27	3x + y = 27	x/9 + y/27 = 1	A (9, 0) B (0, 27)	3(0) + 0 ≥ 27 ∴ 0 ≥ 27 ∴ non-origin-side
x + y ≥ 21	x + y = 21	x/21 + y/21 = 1	C (21, 0) D (0, 21)	(0) + 0 ≥ 21 ∴ 0 ≥ 21 ∴ non-origin-side
x ≥ 0	x = 0		–	R.H.S. of Y-axis
y ≥ 0	y = 0			above X-axis



The shaded portion CHB is the feasible region.

Whose vertices are C(21, 0), H and B(0, 27)

H is the point of intersection of lines

3x + y = 27 …(i)

x + y = 21 …(ii)

∴ By (i) – (ii), we get

3 x + y = 27

x + y = 21

– – –

2x = 6

∴ x = 6/2 = 3

Substituting x = 3 in (ii), we get

3 + y = 21

∴ y = 18

∴ H (3, 18)

Here, the objective function is Z = 4x + 2y

Now, we will find minimum value of Z as follows:

Feasible points	The value of Z = 4x + 2y
C (21, 0)	Z = 4(21) + 2(0) = 84
H (3, 18)	Z = 4(3) + 2(18) = 12 + 36 = 48
B (0, 27)	Z = 4(0) + 2(27) = 54

∴ Z has minimum value 48 at H (3, 18)

∴ Z is minimum, when x = 3, y = 18

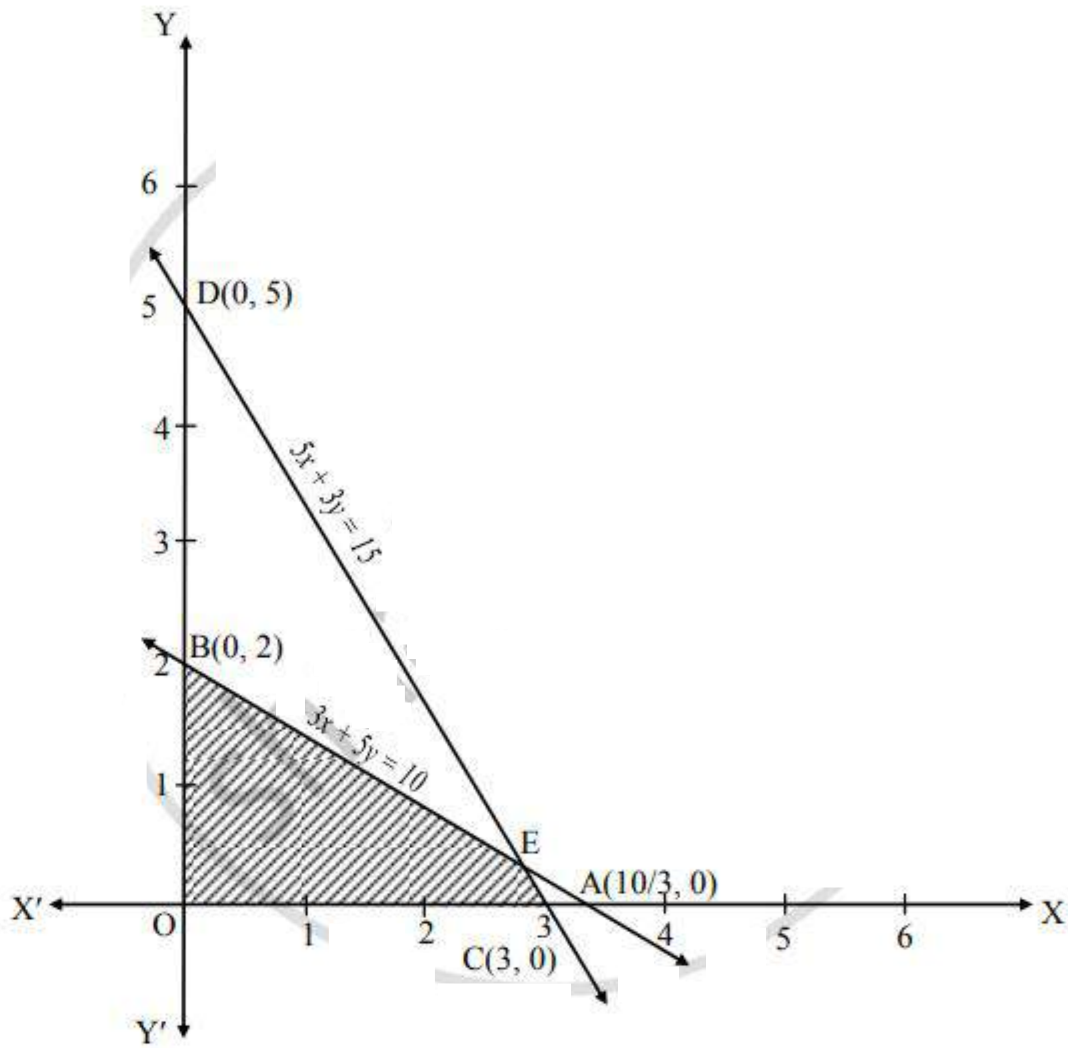
Miscellaneous Exercise 6 | Q 4.03 | Page 104

Solve the following problem :

Maximize Z = 6x + 10y Subject to 3x + 5y ≤ 10, 5x + 3y ≤ 15, x ≥ 0, y≥0

Solution: To find the graphical solution, construct the table as follows:

Inequation	equation	Double intercept form	Points (x1, x2)	Points (x1, x2)
3x + 5y ≤ 10	3x + 5y = 10	$\frac{x}{\frac{10}{3}} + \frac{y}{2} = 1$	A $\left(\frac{10}{3}, 0\right)$ B (0, 2)	3(0) + 5(0) ≤ 10 ∴ 0 ≤ 10 ∴ Origin-side
5x + 3y ≤ 15	5x + 3y = 15	$\frac{x}{3} + \frac{y}{5} = 1$	C (3, 0) D (0, 5)	5(0) + 3(0) ≤ 15 ∴ 0 ≤ 15 ∴ Origin-side
x ≥ 0	x = 0		–	R.H.S. of Y-axis
y ≥ 0	y = 0			above X-axis



The shaded portion OCEB is the feasible region.
Whose vertices are O(0, 0), C(3, 0), E and B(0, 2)
E is the point of intersection of lines
3x + 5y = 10 …(i)
5x + 3y = 15 …(ii)
∴ By 5(i) – 3(ii), we get

$$
\begin{array}{r}
15x + 25y = 50 \\
15x + \ \ 9y = 45 \\
- \qquad - \qquad - \\
\hline
16y = 5
\end{array}
$$

∴ y = 5/16

Substituting y = 5/16 in (i), we get

3x+5×5/16 = 10

∴ x = 45/16

∴ E(45/16, 5/16)
Here, the objective function is Z = 6x + 10y
Now, we will find minimum value of Z as follows:

Here, the objective function is Z = 6x + 10y

Now, we will find minimum value of Z as follows:

Feasible points	The value of Z = 6x + 10y
O (0, 0)	Z = 6(0) + 10(0) = 0
C (3, 0)	Z = 6(3) + 10(0) = 18
E $\left(\frac{5}{16}, \frac{45}{16}\right)$	Z = $6\left(\frac{45}{16}\right) + 10\left(\frac{5}{16}\right)$ = 20
B (0, 2)	Z = 6(0) + 10(2) = 20

∴ Z has maximum value 20 at all points on the line 3x + 5y = 10 between B (0, 2) and E $\left(\frac{45}{16}, \frac{5}{16}\right)$

∴ There are infinite number of optimum solutions of the given LPP.

Miscellaneous Exercise 6 | Q 4.04 | Page 104

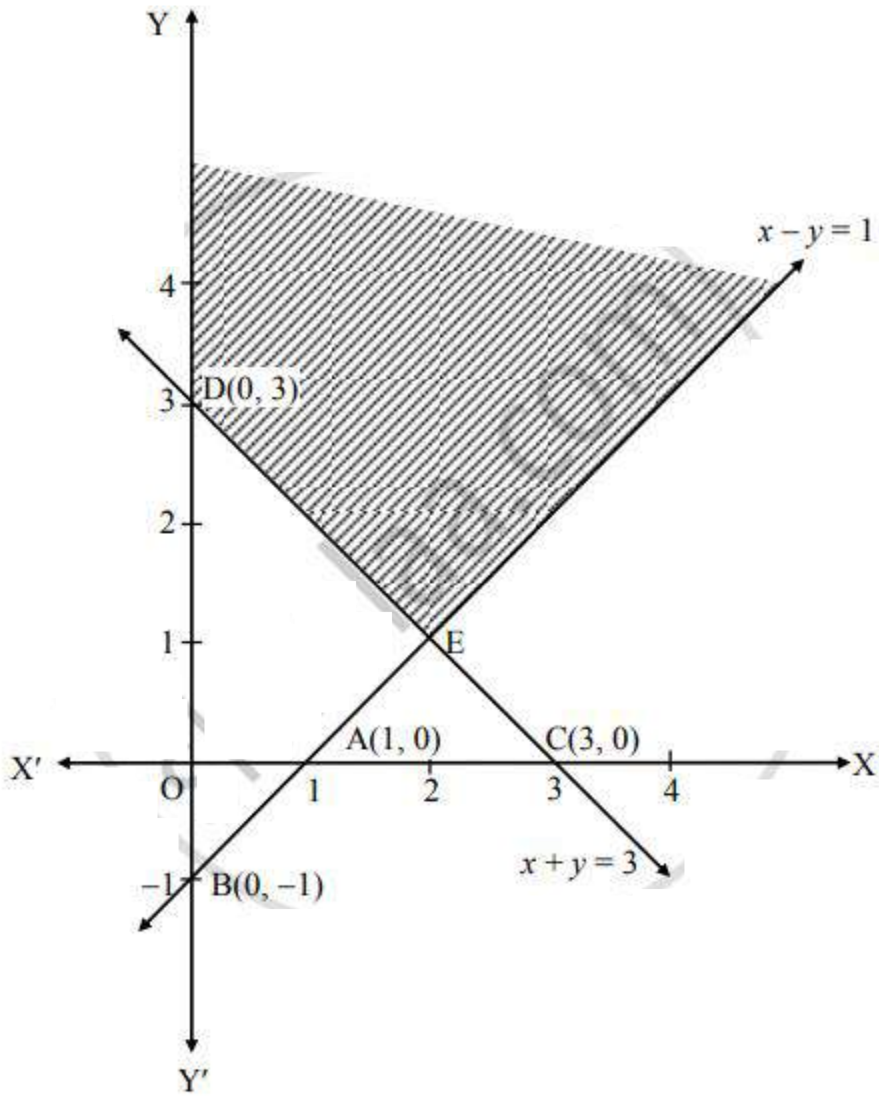
Solve the following problem :

Minimize Z = 2x + 3y Subject to x – y ≤ 1, x + y ≥ 3, x ≥ 0, y ≥ 0

Solution: To find the graphical solution, construct the table as follows:

Inequation	Equation	Double intercept form	Points (x_1, x_2)	Region
x – y ≤ 1	x – y = 1	$\frac{x}{1} + \frac{y}{-1}$ = 1	A (1, 0) B (0, –1)	0 – 0 ≤ 1 ∴ 0 ≤ 1 ∴ Origin-side

x + y ≥ 1	x + y = 3	$\frac{x}{3} + \frac{y}{3} = 1$	C (3, 0) D (0, 3)	0 + 0 ≥ 3 ∴ 0 ≥ 3 ∴ non-origin side
x ≥ 0	x = 0		–	R.H.S. of Y-axis
y ≥	y = 0			above X-axis



The shaded portion Y' DE is the feasible region.
Whose vertices are D(0, 3) and E
E is the point of intersection of lines
x – y = 1 …(i)
x + y = 3 …(ii)
∴ By (i) + (ii), we get

$$\begin{array}{r} x - y = 1 \\ \underline{x + y = 3} \\ 2x \quad\; = 4 \end{array}$$

∴ $x = 4/2 = 2$
Substituting $x = 2$ in (i), we get
$2 - y = 1$
∴ $y = 1$
∴ E(2, 1)
Here, the objective function is $Z = 2x + 3y$
Now, we will find minimum value of Z as follows:

Feasible points	The value of $Z = 2x + 3y$
D(0, 3)	$Z = 2(0) + 3(3) = 9$
E(2, 1)	$Z = 2(2) + 3(1) = 4 + 3 = 7$

∴ Z has minimum value 7 at E(2, 1)
∴ Z is minimum, when $x = 2$, $y = 1$.

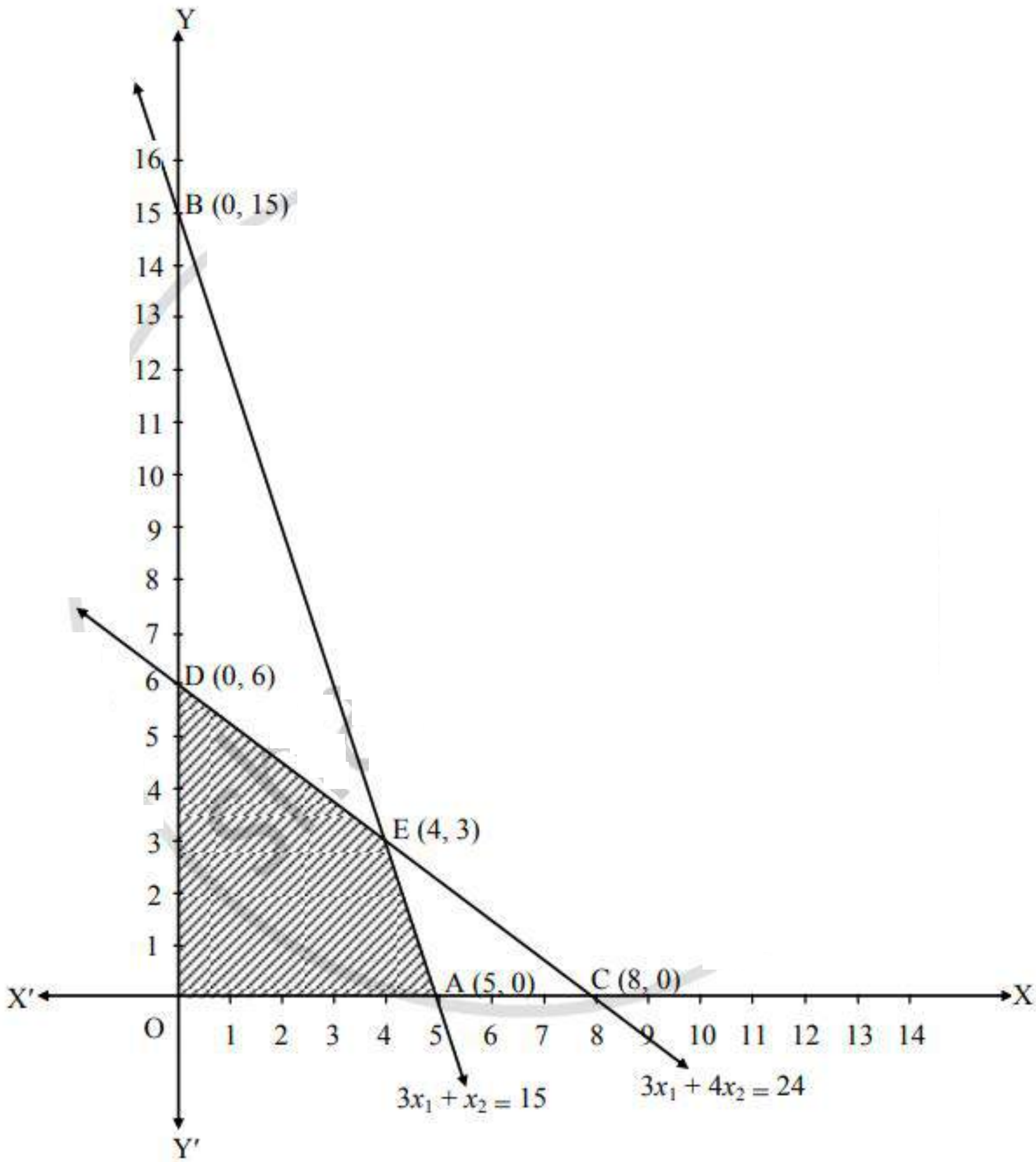
Miscellaneous Exercise 6 | Q 4.05 | Page 104

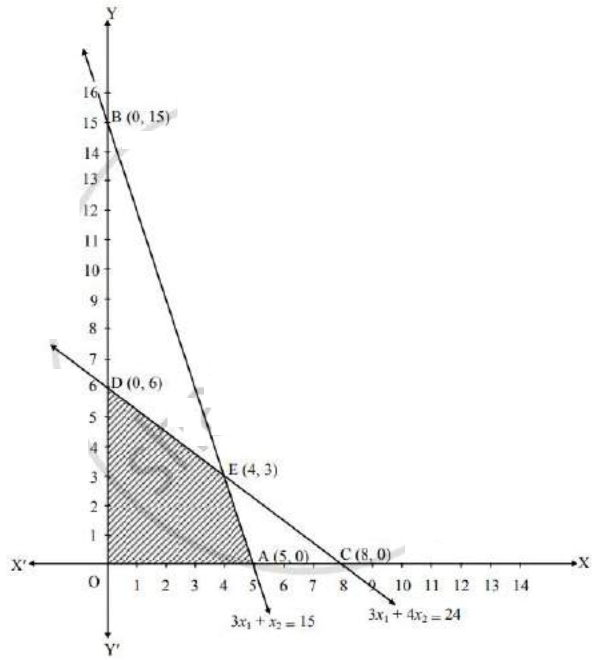
Solve the following problem :

Maximize $Z = 4x_1 + 3x_2$ Subject to $3x_1 + x_2 \leq 15$, $3x_1 + 4x_2 \leq 24$, $x_1 \geq 0$, $x_2 \geq 0$

Solution: To find the graphical solution, construct the table as follows:

Inequation	Equation	Double intercept form	Points (x_1, x_2)	Region
$3x_1 + x_2 \leq 15$	$3x_1 + x_2 = 15$	$\frac{x_1}{5} + \frac{x_2}{15} = 1$	A (5, 0) B (0, 15)	$3(0) + 0 \leq 15$ ∴ $0 \leq 15$ ∴ origin side
$3x_1 + 4x_2 \leq 24$	$3x_1 + 4x_2 = 24$	$\frac{x_1}{8} + \frac{x_2}{6} = 1$	C (8, 0) D (0, 6)	$3(0 + 4(0) \leq 24$ ∴ $0 \leq 24$ ∴ origin side
$x_1 \geq 0$	$x_1 = 0$	–	–	R.H.S. of Y-axis
$x_2 \geq 0$	$x_2 = 0$	–	–	above X-axis


Y
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
B (0, 15)
D (0, 6)
E (4, 3)
A (5, 0)
C (8, 0)
X′
X
O
1 2 3 4 5 6 7 8 9 10 11 12 13 14
$3x_1 + x_2 = 15$
$3x_1 + 4x_2 = 24$
Y′



Shaded portion ODEA is the feasible region.
Whose vertices are O (0, 0), D (0, 6), E, A (5, 0)
E is the point of intersection of the lines
$3x_1 + x_2 = 15$ …(i)
and $3x_1 + 4x_2 = 24$ …(ii)
∴ By (i) – (ii), we get

$$\begin{array}{r} 3x_1 + x_2 = 15 \\ 3x_1 + 4x_2 = 24 \\ - \quad - \qquad - \\ \hline -3x_2 = -9 \end{array}$$

∴ $x_2 = -9/-3$

∴ $x_2 = 3$
Substituting $x_2 = 3$ in i, we get
$3x_1 + 3 = 15$
∴ $3x_1 = 15 - 3$
∴ $3x_1 = 12$
∴ $x_1 = 12/3 = 4$
∴ E (4, 3)
Here, the objective function is $Z = 4x_1 + 3x_2$
Now, we will find maximum value of Z as follows:

Feasible points	The value of $Z = 4x_1 + 3x_2$
O (0, 0)	Z = 4(0) + 3(0) = 0
D (0, 6)	Z = 4(0) + 3(0) = 0
E (4, 3)	Z = 4(4) + 3(3) = 16 + 9 = 25
A (5, 0)	Z = 4(4) + 3(3) = 16 + 9 = 25

∴ Z has maximum value 25 at E(4, 3)
∴ Z is maximum, when $x_1 = 4$, $x_2 = 3$.

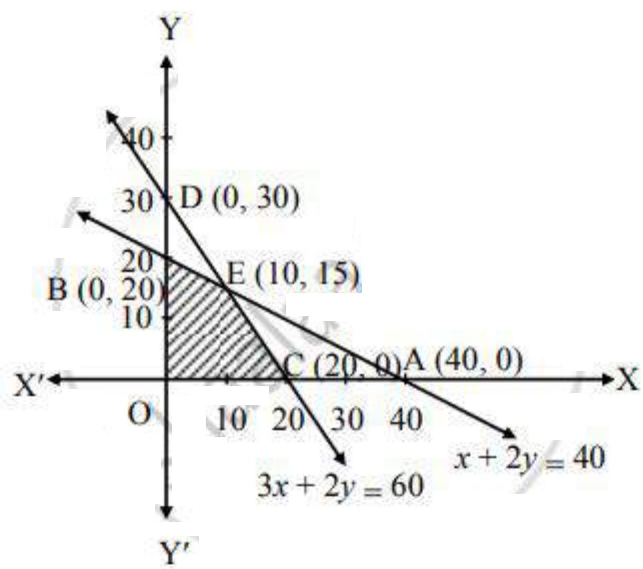
Miscellaneous Exercise 6 | Q 4.06 | Page 104

Solve the following problem :

Maximize Z = 60x + 50y Subject to x + 2y ≤ 40, 3x + 2y ≤ 60, x ≥ 0, y ≥ 0

Solution: To find the graphical solution, construct the table as follows:

Inequation	Equation	Double intercept form	Points (x, y)	Region
x + 2y ≤ 40	x + 2y = 40	x40+y20 = 1	A (40, 0) B (0, 20)	0 + 2(0) ≤ 40 ∴ 0 ≤ 40 ∴origin-side
3x + 2y ≤ 60	3x + 2y = 60	x20+y30 = 1	C (20, 0) D (0, 30)	3(0) + 2(0) ≤ 60 ∴ 0 ≤ 60 ∴origin-side
x ≥ 0	x = 0	–	–	R.H.S. of Y-axis
y ≥ 0	y = 0	–	–	Above X-axis



Shaded portion OBEC is the feasible region
Whose vertices are O (0, 0), B(0, 20), E and C (20, 0)
E is the point of intersection of lines
x + 2y = 40 …(i)

3x + 2y = 60 …(ii)

∴ By (i) – (ii), we get

$$\begin{array}{r} x + 2y = 40 \\ 3x + 2y = 60 \\ - \quad - \quad\quad - \\ \hline -2x \quad = -20 \end{array}$$

∴ x = -20/-2

∴ x = 10

Substituting x = 10 in (i), we get

10 + 2y = 40

∴ 2y = 40 – 10

∴ 2y = 30

∴ y = 30/2 = 15

∴ E = (10, 15)

Here, the objective function is Z = 60x + 50y

Now, we will find maximum value of Z as follows:

Feasible points	The value of Z = 60x + 50y
O (0, 0)	Z = 60(0) + 50(0) = 0
B (0, 20)	Z = 60(0) + 50(20) = 1000
E (10, 15)	Z = 60(10) + 50(15) = 600 + 750 = 1350
C (20, 0)	Z = 60(20) + 50(0) = 1200

∴ Z has maximum value 1350 at E (10, 15)

∴ Z is maximum, when x = 10, y = 15.

Miscellaneous Exercise 6 | Q 4.07 | Page 104

Solve the following problem :

Minimize Z = 4x + 2y Subject to 3x + y ≥ 27, x + y ≥ 21, x + 2y ≥ 30 x ≥ 0, y ≥ 0

Solution: To find the graphical solution, construct the table as follows:

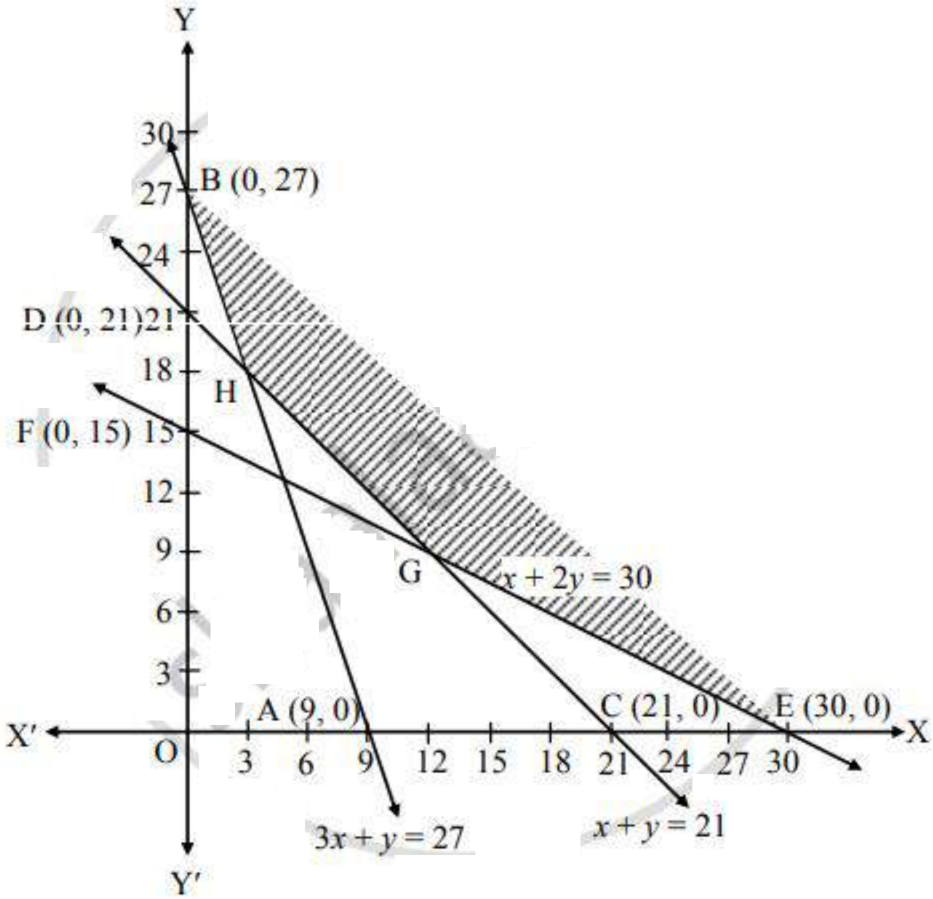
Inequation	Equation	Double intercept form	Points (x, y)	Region
3x + y ≥ 27	3x + y = 27	$\frac{x}{9} + \frac{y}{27} = 1$	A (9, 0) B (0, 27)	3(0) + 0 ≱ 27 ∴ 0 ≱ 27 ∴ non-origin side
x + y ≥ 21	x + y = 21	$\frac{x}{21} + \frac{y}{21} = 1$	C (21, 0) D (0, 21)	0 + 0 ≱ 21 ∴ 0 ≱ 21 ∴ non-origin side

x + 2y ≥ 30	x + 2y = 30	$\frac{x}{30} + \frac{y}{15} = 1$	E (30, 0) F (0, 15)	0 + 2(0) $\ngeq$ 30 ∴ 0 $\ngeq$ 30 ∴ non-origin side
x ≥ 0	x = 0	–	–	R.H.S. of Y-axis
y ≥ 0	y = 0	–	–	Above X-axis

Shaded portion EGHB is the feasible region
Whose vertices are E (30, 0), G, H and B (0, 27)
G is the point of intersection of lines
x + y = 21 …(i)
x + 2y = 30 …(ii)
∴ By (i) – (ii), we get

$$\begin{array}{r} x + y = 21 \\ x + 2y = 30 \\ - \quad - \quad\quad - \\ \hline -y = -9 \end{array}$$

∴ x = –9
Substituting y = 9 in (i), we get
x + 9 = 21
∴x = 21 – 9
∴ x = 12
∴ G = (12, 9)



H is the point of intersection of lines
$3x + y = 27$ and $x + y = 21$
Solving this equation, we get H (3, 18)
Here, the objective function is $Z = 4x + 2y$
Now, we will find maximum value of Z as follows:

Feasible points	The value of Z = 4x + 2y
B (0, 27)	Z = 4(0) + 2(27) = 54
H (3, 18)	Z = 4(3) + 2(18) = 12 + 36 = 48
G (12, 9)	Z = 4(12) + 2(9) = 48 + 18 = 66
E (30, 0)	Z = 4(30) + 2(0) = 120

∴ Z has minimum value 48 at H (3, 18)
∴ Z is minimum, when x = 3, y = 18.

Miscellaneous Exercise 6 | Q 4.08 | Page 104

A carpenter makes chairs and tables profits are ₹ 140 per chair and ₹ 210 per table Both products are processed on three machines, Assembling, Finishing and Polishing the time required for each product in hours and availability of each machine is given by following table.

Product/Machines	Chair (x)	Table (y)	Available time (hours)
Assembling	3	3	36
Finishing	5	2	50
Polishing	2	6	60

Formulate and solve the following Linear programming problems using graphical method.

Solution: Let x be the number of chairs and y be the number of tables.
∴ The constraints are
$3x + 3y \leq 36$
$5x + 2y \leq 50$
$2x + 6y \leq 60$
Since x and y are the number of chairs and tables respectively.
∴ They cannot be negative.
∴ $x \geq 0, y \geq 0$
Now, profit for one chair is ₹ 140 and profit for one table is ₹ 210
∴ Total profit (Z) = $140x + 210y$
This is objective function to be maximized
∴ Given problem can be formulated as
Maximize $Z = 140x + 210y$
Subject to $3x + 3y \leq 36$
$5x + 2y \leq 50$
$2x + 6y \leq 60$
$x \geq 0, y \geq 0$
To find the graphical solution, construct the table as follows:

Inequation	Equation	Double intercept form	Points (x, y)	Region
$3x + 3y \leq 36$	$3x + 3y = 36$	$x/2+y/12 = 1$	A (12, 0) B (0, 12)	$3(0) + 3(0) \leq 36$ ∴ $0 \leq 36$ ∴ Origin-side

5x + 2y ≤ 60	2x + 6y = 60	x/10+y/25 = 1	C 10, 0) D 0, 25)	5(0) + 2(0) ≤ 50 ∴ 0 ≤ 50 ∴ Origin-side
2x + 6y ≤60	2x + 6y = 60	x/30+y/10 = 1	E (30, 0) F (0, 10	2(0) + (0) ≤ 60 ∴ 0 ≤ 60 ∴ Origin-side
x ≥ 0	x = 0	–	–	R.H.S. of Y-axis
y ≥ 0	y = 0	–	–	above X-axis

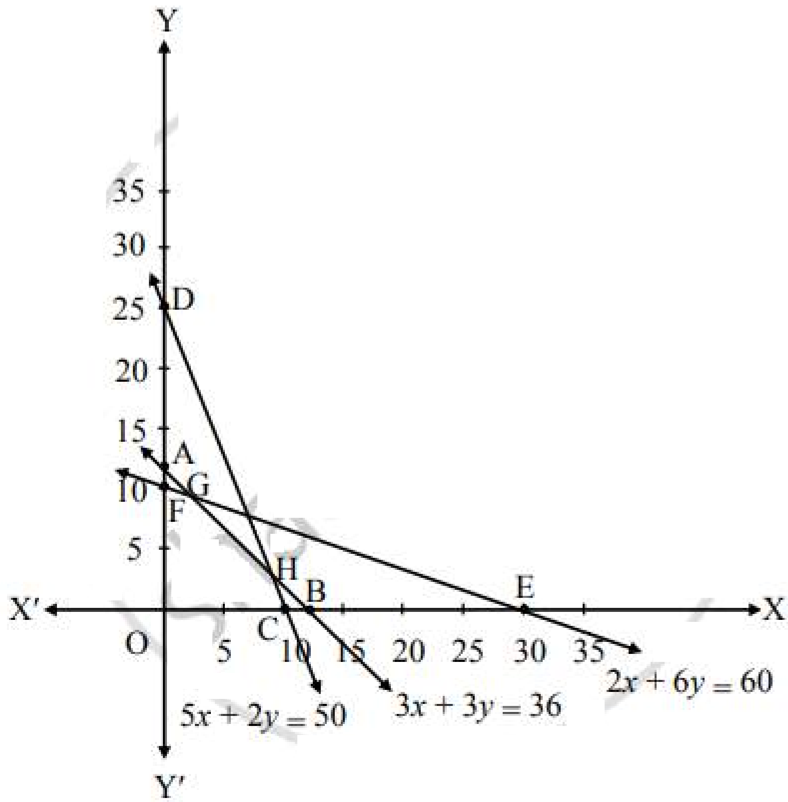
Shaded portion OFG HC is the feasible region,
Whose vertices are O (0, 0), F (0, 10), G, H and C (10, 0)
G is point of intersection of lines.
2x + 6y = 60
i.e., x + 3y = 30 …(i)
and 3x + 3y = 36
i.e., x + y = 12 …(ii)
∴ By (i) – (ii), we get

$$\begin{array}{r} x + 3y = 30 \\ x + y = 12 \\ - \quad - \quad - \\ \hline 2y = 18 \end{array}$$

∴ y = 9
Substituting y = 9 in (ii), we get
x + 9 = 12
∴ x = 12 – 9
∴ x = 3
∴ G = (3, 9)
H is the point of intersection of lines.
3x + 3y = 36
i.e., x + y = 12 …(ii)
5x + 2y = 50 …(iii)
∴ By 2 x (ii) – (iii), we get

$$\begin{array}{l} 2x + 2y = 24 \\ 5x + 2y = 50 \\ - \quad - \quad - \\ \hline -3x \qquad -26 \end{array}$$

∴ x = 26/3



Substituting x = $\frac{26}{3}$ in (ii), we get

$\frac{26}{3} + y$ = 12

∴ y = $12 - \frac{26}{3} = \frac{36 - 26}{3}$

∴ y = $\frac{10}{3}$

∴ H$\left(\frac{36}{3}, \frac{10}{3}\right)$

Here, the objective function is Z = 140x + 210y

Now, we will find maximum value of Z as follows:

Feasible Points	The value of Z = 140x + 210y
O (0, 0)	Z = 140(0) + 210(0) = 0
F (0, 10)	Z = 140(0) + 210(10) = 2100
G (3, 9)	Z = 140(3) + 210(9) = 420 + 1890 = 2310
H $\left(\frac{36}{3}, \frac{10}{3}\right)$	Z = $140\left(\frac{26}{3}\right) + 210\left(\frac{10}{3}\right) = \frac{3640}{3} + \frac{2100}{3}$ = 2310
C (10, 0)	Z = 140(10) + 210(0) = 1400

∴ Z has maximum value 2310 at G (3, 9)

∴ Maximum profit is ₹ 2310, when x = number of chairs = 3, y = number of tables = 9.

Miscellaneous Exercise 6 | Q 4.09 | Page 104

Solve the following problem :

A company manufactures bicyles and tricycles, each of which must be processed through two machines A and B Maximum availability of machine A and B is respectively 120 and 180 hours. Manufacturing a bicycle requires 6 hours on machine A and 3 hours on machine B. Manufacturing a tricycle requires 4 hours on machine A and 10 hours on machine B. If profits are ₹ 180 for a bicycle and ₹ 220 on a tricycle, determine the number of bicycles and tricycles that should be manufacturing in order to maximize the profit.

Solution: Let x number of bicycles and y number of tricycles be manufactured by the company.

∴ Total profit Z = 180x + 220y

This is the objective function to be maximized.

The given information can be tabulated as shown below:

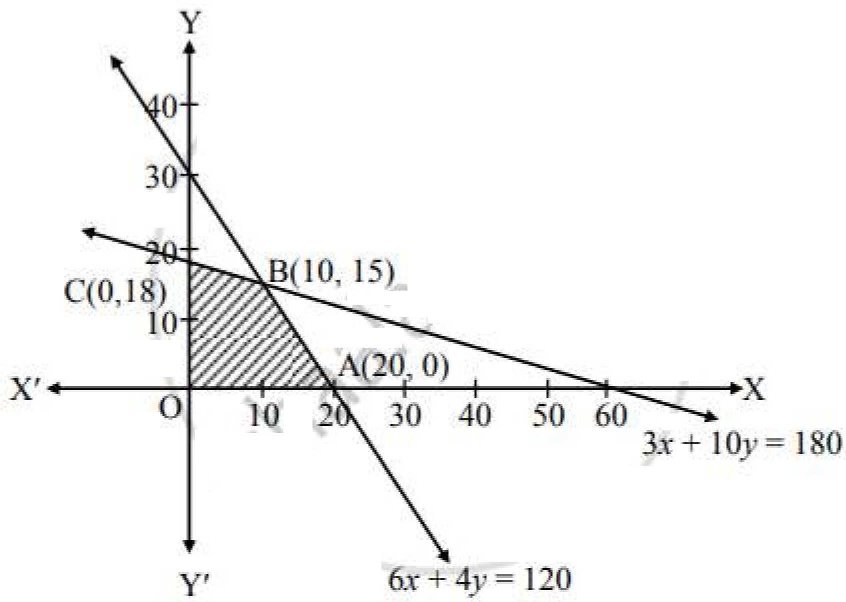
	Bicycles (x)	Tricycles (y)	Maximum availability of time (hrs)

Machine A	6	4	120
Machine B	3	10	180

∴ The constraints are $6x + 4y \leq 120$, $3x + 10y \leq 180$, $x \geq 0$, $y \geq 0$
∴ Given problem can be formulated as
Maximize $Z = 180x + 220y$
Subject to, $6x + 4y \leq 120$, $3x + 10y \leq 180$, $x \geq 0$, $y \geq 0$.
To draw the feasible region, construct the table as follows:

Inequality	$6x + 4y \leq 120$	$3x + 10y \leq 180$
Corresponding equation (of line)	$6x + 4y = 120$	$3x + 10y = 180$
Intersection of line with X-axis	(20, 0)	(60, 0)
Intersection of line with Y-axis	(0, 30)	(0, 18)
Region	Origin side	Origin side

Shaded portion OABC is the feasible region,
whose vertices are O ≡ (0, 0), A ≡ (20, 0), B and C ≡ (0, 18)
B is the point of intersection of the lines $3x + 10y = 180$ and $6x + 4y = 120$.
Solving the above equations, we get
B ≡ (10, 15)
Here the objective function is,
$Z = 180x + 220y$
∴ Z at O(0, 0) = 180(0) + 220(0) = 0
Z at A(20, 0) = 180(20) + 220(0) = 3600
Z at B(10, 15) = 180(10) + 220(15) = 5100
Z at C(0, 18) = 180(0) + 220(18) = 3960
∴ Z has maximum value 5100 at B(10, 15)
∴ Z is maximum when x = 10, y = 15



Thus, the company should manufacture 10 bicycles and 15 tricycles to gain maximum profit of ₹ 5100.

Miscellaneous Exercise 6 | Q 4.1 | Page 104

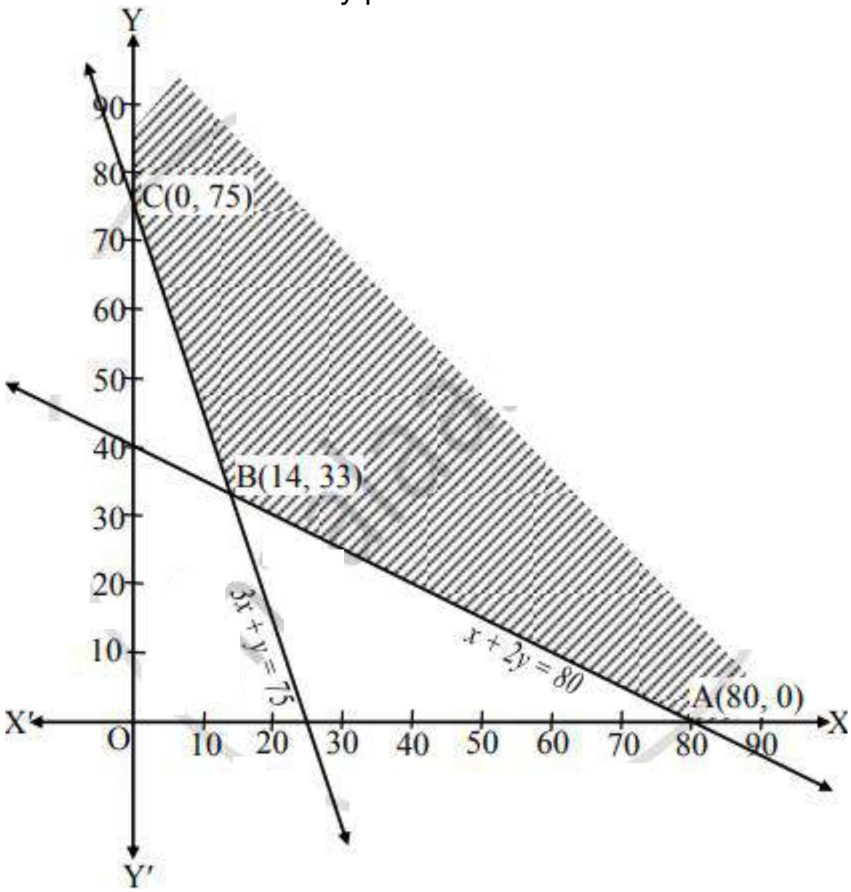
Solve the following problem :

A factory produced two types of chemicals A and B The following table gives the units of ingredients P & Q (per kg) of Chemicals A and B as well as minimum requirements of P and Q and also cost per kg. of chemicals A and B.

Ingredients per kg. /Chemical Units	A (x)	B (y)	Minimum requirements in
P	1	2	80
Q	3	1	75
Cost (in ₹)	4	6	

Find the number of units of chemicals A and B should be produced so as to minimize the cost.

Solution: Let the factory produces 'x' units of chemical A and 'y' units of chemical B



∴ Total cost Z = 4x + 6y

This is the objective function to be minimized.

From the given information, the constraints are

x + 2y ≥ 80, 3x + y ≥ 75, x ≥ 0, y ≥ 0

∴ Given problem can be formulated as

Minimize Z = 4x + 6y

Subject to, x + 2y ≥ 80, 3x + y ≥ 75, x ≥ 0, y ≥ 0

To draw the feasible region, construct table as follows:

Inequality	x + 2y ≥ 80	3x + y ≥ 75
Corresponding equation (of line)	x + 2y = 80	3x + y = 75
Intersection of line with X-axis	(80, 0)	(25, 0)
Intersection of line with Y-axis	(0, 40)	(0, 75)
Region	Non-origin side	Non-origin side

Shaded portion XABCY is the feasible region,
whose vertices are A ≡ (80, 0), B and C ≡ (0, 75)
B is the point of intersection of the lines 3x + y = 75 and x + 2y = 80
Solving the above equations, we get
B ≡ (14, 33)
Here the objective function is,
Z = 4x + 6y
∴ Z at A(80, 0) = 4(80) + 6(0) = 320
Z at B(14, 33) = 4(14) + 6(33) = 254
Z at C(0, 75) = 4(0) + 6(75) = 450
∴ Z has minimum value 254 at B(14, 33)
∴ Z is mimimum, when x = 14, y = 33.
∴ Factory should produce 14 units of chemical A and 33 units of chemical B to minimize the cost to ₹ 254.

Miscellaneous Exercise 6 | Q 4.11 | Page 105

Solve the following problem :

A Company produces mixers and processors Profit on selling one mixer and one food processor is ₹ 2000 and ₹ 3000 respectively. Both the products are processed through three machines A, B, C The time required in hours by each product and total time available in hours per week on each machine are as follows:

Machine/Product	Mixer per unit	Food processor per unit	Available time
A	3	3	36
B	5	2	50
C	2	6	60

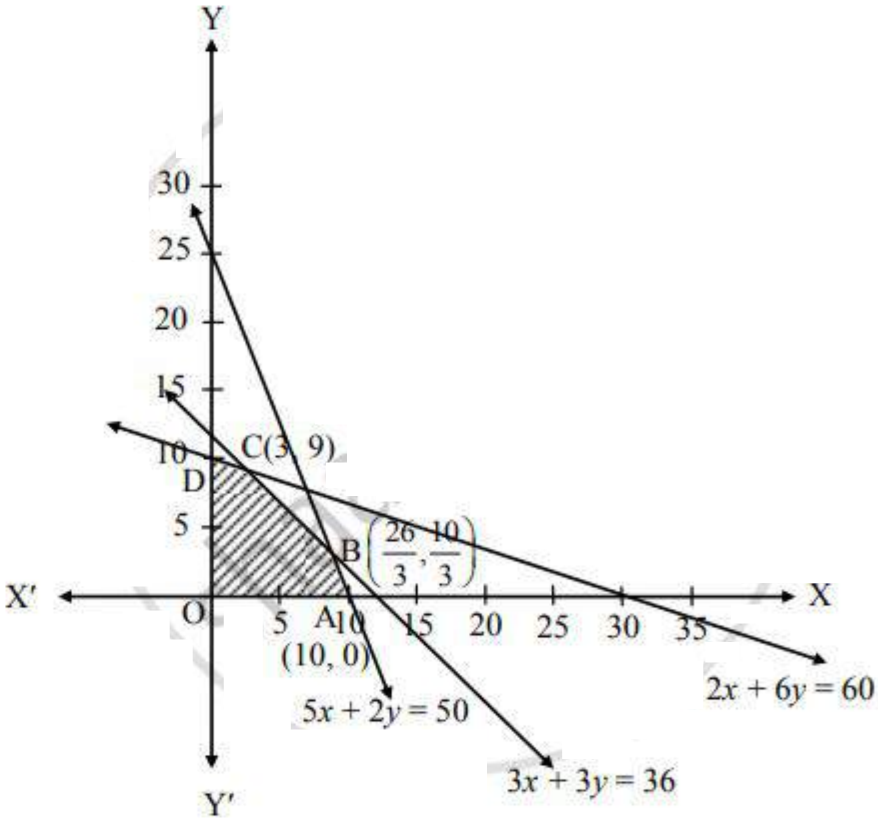
How many mixers and food processors should be produced to maximize the profit?

Solution: Let x mixers and y food processors be produced by the company.
∴ Total profit Z = 2000x + 3000y
This is the objective function to be maximized.
From the given information, the constraints are
3x + 3y ≤ 36, 5x + 2y ≤ 50, 2x + 6y ≤ 60, x ≥ 0, y ≥ 0
∴ Given problem can be formulated as
Maximize Z = 2000x + 3000y
Subject to, 3x + 3y ≤ 36, 5x + 2y ≤ 50, 2x + 6y ≤ 60, x ≥ 0, y ≥ 0
To draw the feasible region, construct table as follows:

Inequality	3x + 3y ≤ 36	5x + 2y ≤50	2x + 6y ≤ 60
Corresponding equation (of line)	3x + 3y = 36	5x + 2y = 50	2x + 6y = 60

Intersection of line with X-axis	(12, 0)	(10, 0)	(30, 0)
Intersection of line with Y-axis	(0, 12)	(0, 25)	(0, 10)
Region	Origin side	Origin side	Origin side

Shaded portion OABCD is the feasible region,
whose vertices are O ≡ (0, 0), A ≡ (10, 0), B, C and D ≡ (0, 10)
B is the point of intersection of the lines
$3x + 3y = 36$ i.e. $x + y = 12$ and $5x + 2y = 50$
Solving the above equations, we get
B ≡ (263,103)
C is the point of intersection of the lines $3x + 3y = 36$
i.e. $x + y = 12$ and $2x + 6y = 60$
i.e. $x + 3y = 30$
Solving the above equations, we get
C ≡ (3, 9)
Here the objective function is
$Z = 2000x + 3000y$
∴ Z at O(0, 0) = 2000(0) + 3000(0) = 0
Z at A(10, 0) = 2000(10) + 3000(0) = 20000
Z at B(263,103)=2000(263)+3000(103)=820003 = 27333.33
Z at C(3, 9) = 2000(3) + 3000(9) = 33000
Z at D(0, 10) = 2000(0) + 3000(10) = 30000
∴ Z has maximum value 33000 at C(3, 9).
∴ Z is maximum when x = 3, y = 9



Thus, the company should produce 3 mixers and 9 food processors to gain maximum profit of ₹ 33000.

Miscellaneous Exercise 6 | Q 4.12 | Page 105

Solve the following problem :

Chemical company produces a chemical containing three basic elements A, B, C so that it has at least 16 liters of A, 24 liters of B and 18 liters of C. This chemical is made by mixing two compounds I and II. Each unit of compound I has 4 liters of A, 12 liters of B, 2 liters of C Each unit of compound II has 2 liters of A, 2 liters of B and 6 liters of C. The cost per unit of compound I is ₹ 800 and that of compound II is ₹ 640 Formulate the problem as LPP. and solve it to minimize the cost.

Solution: Let ‘x’ units of compound I and y units of compound II are mixed to produce the chemical.
∴ Total cost Z = 800x + 640y
This is the objective function to be minimized.
The given information can be tabulated as shown below:

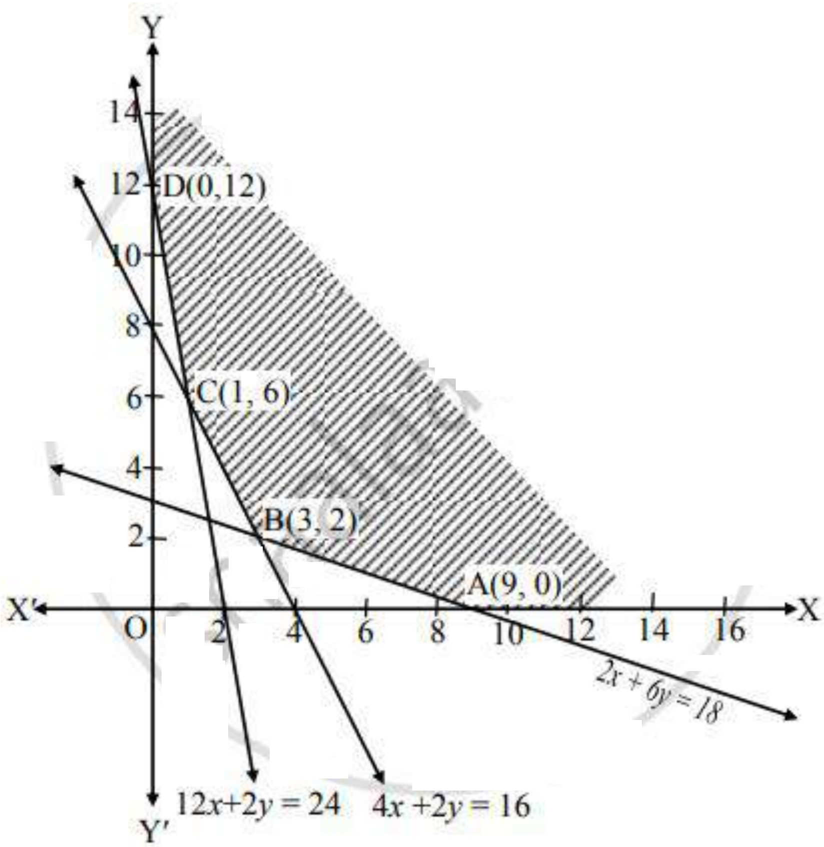
Element	Compound (I) (x units)	Compound (II) (y units)	Minimum requirement (in litres)

A	4	2	16
B	12	2	24
C	2	6	18

∴ The constraints are $4x + 2y \geq 16$, $12x + 2y \geq 24$, $2x + 6y \geq 18$, $x \geq 0$, $y \geq 0$.
∴ Given problem can be formulated as,
Minimize $Z = 800x + 640y$
Subject to, $4x + 2y \geq 16$, $12x + 2y \geq 24$, $2x + 6y \geq 18$, $x \geq 0$, $y \geq 0$.
To draw the feasible region, construct table as follows:

Inequality	$4x + 2y \geq 16$	$2x + 2y \geq 24$	$2x + 6y \geq 18$
Corresponding equation (of line)	$4x + 2y = 16$	$2x + 2y = 24$	$2x + 6y = 18$
Intersection of line with X-axis	(4, 0)	(2, 0)	(9, 0)
Intersection of line with Y-axis	(0, 8)	(0, 12)	(0, 3)
Region	Non-origin side	Non-origin side	Non-origin side

Shaded portion XABCDY is the feasible region,
whose vertices are A ≡ (9, 0), B, C and D ≡ (0, 12).
B is the point of intersection of the lines $2x + 6y = 18$ and $4x + 2y = 16$
Solving the above equations, we get
B ≡ (3, 2)
C is the point of intersection of the lines $12x + 2y = 24$ and $4x + 2y = 16$



Solving the above equations, we get
C ≡ (1, 6)
Here, the objective function is Z = 800x + 640y
∴ Z at A(9, 0) = 800(9) + 640(0) = 7200
Z at B(3, 2) = 800(3) + 640(2) = 3680
Z at C(1, 6) = 800(1) + 640(6) = 4640
Z at D(0, 12) = 800(0) + 640(12) = 7680
∴ Z has minimum value 3680 at B(3, 2)
∴ Z is minimum, when x = 3, y = 2
∴ The chemical company should produce 3 units of chemical I and 2 units of chemical II to minimize the cost to ₹ 3680.

Miscellaneous Exercise 6 | Q 4.13 | Page 105

Solve the following problem :

A person makes two types of gift items A and B requiring the services of a cutter and a finisher. Gift item A requires 4 hours of cutter's time and 2 hours of finisher's time. B requires 2 hours of cutters time, 4 hours of finishers time. The cutter and finisher have 208 hours and 152 hours available times respectively every month. The profit of one gift item of type A is ₹ 75 and on gift item B is ₹ 125. Assuming that the person can sell all the items produced, determine how many gift items of each type should be make every month to obtain the best returns?

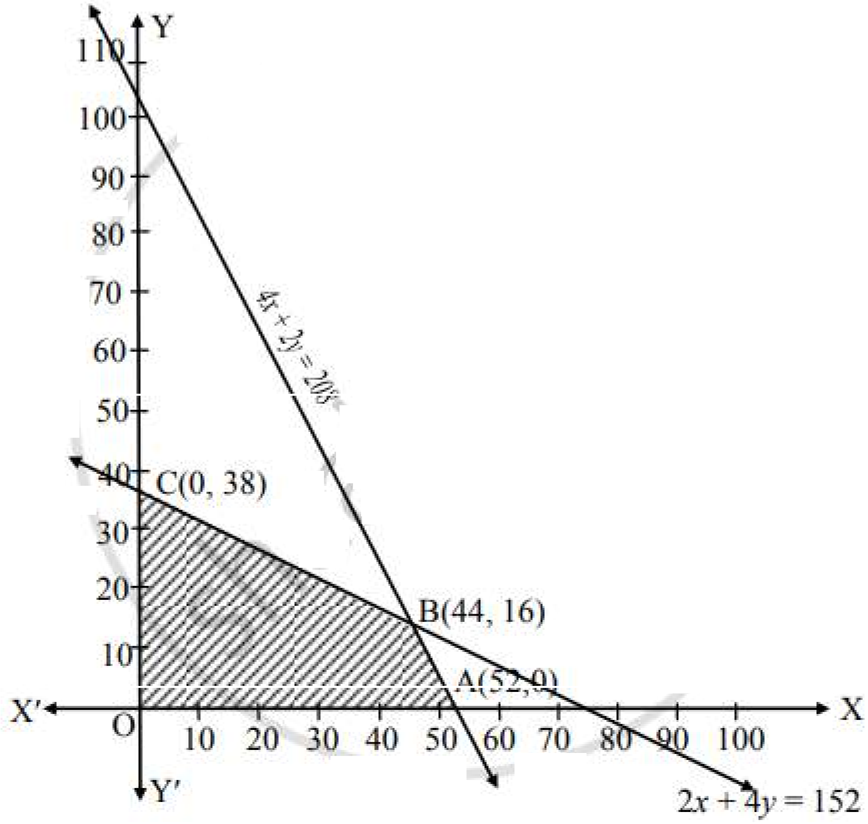
Solution: Let x gift items of type A and y gift items of type B be produced by the person.
∴ Total profit $Z = 75x + 125y$
This is the objective function to be maximized.
The given information can be tabulated as shown below:

	Type A (x)	Type B (y)	Total time available (in hours)
Cutter	4	2	208
Finisher	2	4	152

∴ The constraints are $4x + 2y \le 208$, $2x + 4y \le 152$, $x \ge 0$, $y \ge 0$
∴ Given problem can be formulated as
Maximize $Z = 75x + 125y$
Subject to, $4x + 2y \le 208$, $2x + 4y \le 152$, $x \ge 0$, $y \ge 0$
To draw feasible region, construct table as follows:

Inequality	$4x + 2y \le 208$	$2x + 4y \le 152$
Corresponding equation (of line)	$4x + 2y = 208$	$2x + 4y = 152$
Intersection of line with X-axis	(52, 0)	(76, 0)
Intersection of line with Y-axis	(0, 104)	(0, 38)
Region	Origin side	Origin side

Shaded portion OABC is the feasible region,
whose vertices are O ≡ (0, 0),
A ≡ (52, 0), B and C ≡ (0, 38).
B is the point of intersection of the lines $4x + 2y = 208$ i.e. $2x + y = 104$ and $2x + 4y = 152$



Solving the above equations, we get B ≡ (44, 16)

Here, the objective function is Z = 75x + 125y

∴ Z at O(0, 0) = 75(0) + 125(0) = 0

Z at A(52, 0) = 75(52) + 125(0) = 3900

Z at B(44, 16) = 75(44) + 125(16) = 5300

Z at C(0, 38) = 75(0) + 125(38) = 4750

∴ Z has maximum value 5300 at B(44, 16)

∴ Z is maximum, when x = 44, y = 16

Thus, a person should make 44 gift items of type A and 16 gift items of type B every month to obtain the best returns of ₹ 5300.

Miscellaneous Exercise 6 | Q 4.14 | Page 105

Solve the following problem :

A firm manufactures two products A and B on which profit earned per unit is ₹ 3 and ₹ 4 respectively. The product A requirs one minute of processing time on M_1 and 2 minutes on M_2. B requires one minutes on M_1 and one minute on M_2. Machine M_1 is available for use for 450 minutes while M_2 is available for 600 minutes during any working day. Find the number of units of product A and B to be manufactured to get the maximum profit.

Solution: Let the firm manufacture x units of product A and y units of product B. The profit earned is ₹ 3 per unit of A and ₹ 4 per unit of B.

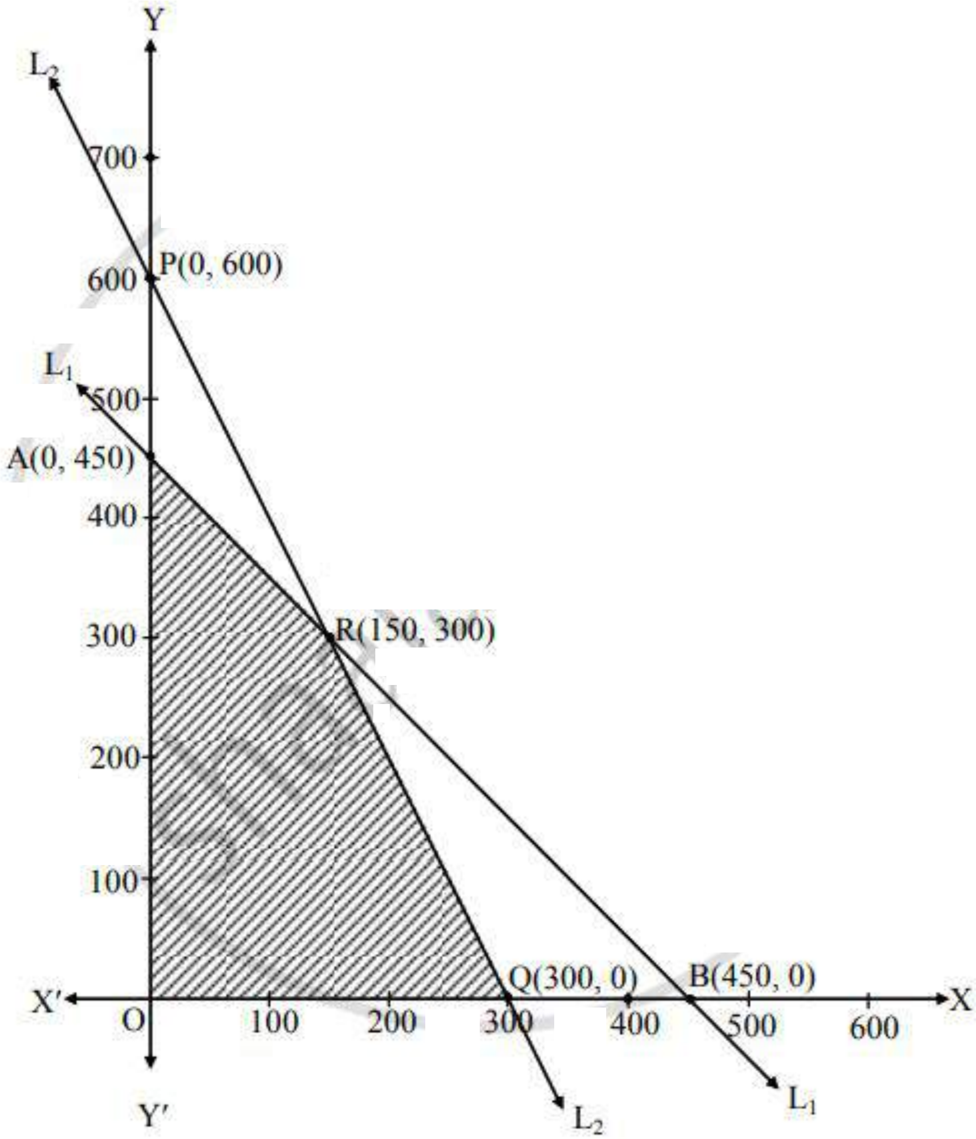
∴ Total profit = ₹ (3x + 4y)
We construct a table with the constraints of machines M_1 and M_2 as follows

Machine\Product	A (x)	B (y)	Maximum Availability in minutes
M_1	1	1	450
M_2	2	1	600

∴ The constraints are $x + y \le 450$, $2x + y \le 600$, $x \ge 0$, $y \ge 0$
∴ Given problem can be formulated as
Maximize $Z = 3x + 4y$
Subject to, $x + y \le 450$, $2x + y \le 600$, $x \ge 0$, $y \ge 0$
To draw feasible region, construct table as follows:

Inequality	$x + y \le 450$	$2x + y \le 600$
Corresponding equation (of line)	$x + y = 450$	$2x + y = 600$
Intersection of line with X-axis	(450, 0)	(300, 0)
Intersection of line with Y-axis	(0, 450)	(0, 600)
Region	Origin side	Origin side

At O (0, 0), Z = 0 + 0 = 0
At Q (300, 0), z = 3 (300) + 0 = 900
At R (150, 300), Z = 3 (150) + 4 (300) = 1650
At A (0, 450) Z = 0 + 4(450) = 1800
The maximum value of Z is 1800 and it occurs at A (0, 450).
Thus 0 units of A and 450 units of B must be manufactured to get the maximum profit is ₹ 1800.



Miscellaneous Exercise 6 | Q 4.15 | Page 105

Solve the following problem :

A firm manufacturing two types of electrical items A and B, can make a profit of ₹ 20 per unit of A and ₹ 30 per unit of B. Both A and B make use of two essential components, a motor and a transformer. Each unit of A requires 3 motors and 2 transformers and each unit of B requires 2 motors and 4 transformers. The total supply of components per month is restricted to 210 motors and 300 transformers. How many units of A and B should be manufacture per month to maximize profit? How much is the maximum profit?

Solution: Let the firm manufacture x units of A and y units of B.
The profit is ₹ 20 per unit of A and ₹ 30 per unit of B.
∴ Total profit = ₹ (20 x + 30 y).
We construct a table with the constraints of number of motors and transformers needed.

Electrical item\Essential component	A (x)	B (y)	Maximum Supply
Motors	3	2	210
Transformers	2	4	300

From the table, the total motors required is $(3x + 2y)$ and total motor required is $(2x + 4y)$.
But total supply of components per month is restricted to 210 motors and 300 transformers.
∴ The constraints are $3x + 2y \leq 210$ and $2x + 4y \leq 300$.
As x, y cannot be negative, we have $x \leq 0$ and $y \geq 0$.
Hence the given LPP can be formulated as follows:
Maximize $Z = 20x + 30y$
Subject to
$3x + 2y \leq 210$,
$2x + 4y \leq 300$,
$x \leq 0$, $y \geq 0$.
For graphical solutions of the inequalities, consider lines $L_1 : 3x + 2y = 210$ and $2x + 4y = 300$

For L_1 :

x	y	(x, y)
0	105	(0, 105)
70	0	(70, 0)

For L_2 :

x	y	(x, y)
0	75	(0, 75)
150	0	(150, 0)

L_1 passes through A (0, 105) and B (70, 0)
L_2 passes through P (0, 75) and Q (150, 0)
Solving both lines, we get $x = 30$, $y = 60$
The coordinates of origin O (0, 0) satisfies both the inequalities.
∴ The required region is on origin side of both the lines L_1 and L_2.
As $x \geq 0$, $y \geq 0$; the feasible region lies in the first quadrant.
OBRP is the required feasible region.
At O (0, 0), $Z = 0 + 0 = 0$
At B (70, 0), $Z = 20 (70) + 0 = 1400$
At R (30, 60), $Z = 20 (30) + 30 (60) = 2400$

At P (0, 75), Z = 0 + 30 (75) = 2250
The maximum value of Z is 2400 and it occurs at R (30, 60)
Thus 30 units of A and 60 units of B must be manufactured to get maximum profit of ₹ 2400.