

CAT 2023 Slot 2 Question Paper

Quant

45. Let a, b, m and n be natural numbers such that $a > 1$ and $b > 1$. If $a^m b^n = 144^{145}$, then the largest possible value of $n - m$ is
- A 580
B 290
C 289
D 579
46. Any non-zero real numbers x, y such that $y \neq 3$ and $\frac{x}{y} < \frac{x+3}{y-3}$, Will satisfy the condition.
- A $\frac{x}{y} < \frac{y}{x}$
B If $y < 0$, and $-x < y$
C If $y > 10$, and $-x > y$
D If $x < 0$, and $-x < y$
47. For any natural numbers m, n , and k , such that k divides both $m + 2n$ and $3m + 4n$, k must be a common divisor of
- A m and n
B $2m$ and $3n$
C m and $2n$
D $2m$ and n
48. The sum of all possible values of x satisfying the equation $2^{4x^2} - 2^{2x^2+x+16} + 2^{2x+30} = 0$, is
- A 3
B $\frac{3}{2}$
C $\frac{5}{2}$
D $\frac{1}{2}$

49. The number of positive integers less than 50, having exactly two distinct factors other than 1 and itself, is
50. For some positive real number x , if $\log_{\sqrt{3}}(x) + \frac{\log_x(25)}{\log_x(0.008)} = \frac{16}{3}$, then the value of $\log_3(3x^2)$ is
51. Let k be the largest integer such that the equation $(x - 1)^2 + 2kx + 11 = 0$ has no real roots. If y is a positive real number, then the least possible value of $\frac{k}{4y} + 9y$ is
52. Pipes A and C are fill pipes while Pipe B is a drain pipe of a tank. Pipe B empties the full tank in one hour less than the time taken by Pipe A to fill the empty tank. When pipes A, B and C are turned on together, the empty tank is filled in two hours. If pipes B and C are turned on together when the tank is empty and Pipe B is turned off after one hour, then Pipe C takes another one hour and 15 minutes to fill the remaining tank. If Pipe A can fill the empty tank in less than five hours, then the time taken, in minutes, by Pipe C to fill the empty tank is
- A 90
- B 120
- C 75
- D 60
53. Anil borrows Rs 2 lakhs at an interest rate of 8% per annum, compounded half-yearly. He repays Rs 10320 at the end of the first year and closes the loan by paying the outstanding amount at the end of the third year. Then, the total interest, in rupees, paid over the three years is nearest to
- A 45311
- B 51311
- C 33130
- D 40991
54. Ravi is driving at a speed of 40 km/h on a road. Vijay is 54 meters behind Ravi and driving in the same direction as Ravi. Ashok is driving along the same road from the opposite direction at a speed of 50 km/h and is 225 meters away from Ravi. The speed, in km/h, at which Vijay should drive so that all the three cross each other at the same time, is
- A 58.8
- B 67.2
- C 61.6
- D 64.4

55. Minu purchases a pair of sunglasses at Rs.1000 and sells to Kanu at 20% profit. Then, Kanu sells it back to Minu at 20% loss. Finally, Minu sells the same pair of sunglasses to Tanu. If the total profit made by Minu from all her transactions is Rs.500, then the percentage of profit made by Minu when she sold the pair of sunglasses to Tanu is
- A 35.42%
- B 52%
- C 31.25%
- D 26%
56. The price of a precious stone is directly proportional to the square of its weight. Sita has a precious stone weighing 18 units. If she breaks it into four pieces with each piece having distinct integer weight, then the difference between the highest and lowest possible values of the total price of the four pieces will be 288000. Then, the price of the original precious stone is
- A 1944000
- B 972000
- C 1620000
- D 1296000
57. In a company, 20% of the employees work in the manufacturing department. If the total salary obtained by all the manufacturing employees is one-sixth of the total salary obtained by all the employees in the company, then the ratio of the average salary obtained by the manufacturing employees to the average salary obtained by the nonmanufacturing employees is
- A 6:5
- B 4:5
- C 5:4
- D 5:6
58. If a certain amount of money is divided equally among n persons, each one receives Rs 352. However, if two persons receive Rs 506 each and the remaining amount is divided equally among the other persons, each of them receive less than or equal to Rs 330. Then, the maximum possible value of n is
59. Jayant bought a certain number of white shirts at the rate of Rs 1000 per piece and a certain number of blue shirts at the rate of Rs 1125 per piece. For each shirt, he then set a fixed market price which was 25% higher than the average cost of all the shirts. He sold all the shirts at a discount of 10% and made a total profit of Rs.51000. If he bought both colors of shirts, then the maximum possible total number of shirts that he could have bought is
60. A container has 40 liters of milk. Then, 4 liters are removed from the container and replaced with 4 liters of water. This process of replacing 4 liters of the liquid in the container with an equal volume of water is continued repeatedly. The smallest number of times of doing this process, after which the volume of milk in the container becomes less than that of water, is

61. A triangle is drawn with its vertices on the circle C such that one of its sides is a diameter of C and the other two sides have their lengths in the ratio $a : b$. If the radius of the circle is r , then the area of the triangle is
- A $\frac{abr^2}{2(a^2+b^2)}$
- B $\frac{2abr^2}{a^2+b^2}$
- C $\frac{4abr^2}{a^2+b^2}$
- D $\frac{abr^2}{a^2+b^2}$
62. In a rectangle ABCD, $AB = 9$ cm and $BC = 6$ cm. P and Q are two points on BC such that the areas of the figures ABP, APQ, and AQCD are in geometric progression. If the area of the figure AQCD is four times the area of triangle ABP, then $BP : PQ : QC$ is
- A 1:2:4
- B 1:2:1
- C 2:4:1
- D 1:1:2
63. The area of the quadrilateral bounded by the Y-axis, the line $x = 5$, and the lines $|x - y| - |x - 5| = 2$, is
64. Let both the series a_1, a_2, a_3, \dots and b_1, b_2, b_3, \dots be in arithmetic progression such that the common differences of both the series are prime numbers. If $a_5 = b_9$, $a_{19} = b_{19}$ and $b_2 = 0$, then a_{11} equals
- A 86
- B 79
- C 83
- D 84
65. If $p^2 + q^2 - 29 = 2pq - 20 = 52 - 2pq$, then the difference between the maximum and minimum possible value of $(p^3 - q^3)$
- A 243
- B 486
- C 378
- D 189
66. Let a_n and b_n be two sequences such that $a_n = 13 + 6(n - 1)$ and $b_n = 15 + 7(n - 1)$ for all natural numbers n . Then, the largest three digit integer that is common to both these sequences, is

Answers

45.D	46.B	47.C	48.D	49.15	50.7	51.6	52.A
53.B	54.C	55.C	56.D	57.B	58.16	59.407	60.7
61.B	62.C	63.45	64.B	65.C	66.967		

Explanations

45. D

It is given that $a^m \cdot b^n = 144^{145}$, where $a > 1$ and $b > 1$.

144 can be written as $144 = 2^4 \times 3^2$

Hence, $a^m \cdot b^n = 144^{145}$ can be written as $a^m \cdot b^n = (2^4 \times 3^2)^{145} = 2^{580} \times 3^{290}$

We know that 3^{290} is a natural number, which implies it can be written as a^1 , where $a > 1$

Hence, the least possible value of m is 1. Similarly, the largest value of n is 580.

Hence, the largest value of (n-m) is $(580-1) = 579$

The correct option is D

46. B

It is given that $\frac{x}{y} < \frac{x+3}{y-3}$, which can be written as $\frac{x}{y} - \frac{x+3}{y-3} < 0$

$$\Rightarrow \frac{x(y-3)-y(x+3)}{y(y-3)} < 0$$

$$\Rightarrow \frac{xy-3x-xy-3y}{y(y-3)} < 0$$

$$\Rightarrow \frac{-3(x+y)}{y(y-3)} < 0$$

$$\Rightarrow \frac{3(x+y)}{y(y-3)} > 0$$

From this inequality, we can say that, when $y < 0 \Rightarrow y(y-3) > 0$. Now to satisfy the given equation

$$\frac{3(x+y)}{y(y-3)} > 0,$$

$(x+y)$ must be greater than zero Hence, $x > 0$ and $|x| > |y|$

Therefore, the magnitude of x is greater than the magnitude of y .

Hence, $x > y$, and $|x| > |y| \Rightarrow -x < y$ (Since the magnitude of x is greater than the magnitude of y .)

The correct option is B.

47. C

It is given that k divides $m+2n$ and $3m+4n$.

Since k divides $(m+2n)$, it implies k will also divide $3(m+2n)$. Therefore, k divides $3m+6n$.

Similarly, we know that k divides $3m+4n$.

We know that if two numbers a , and b both are divisible by c , then their difference $(a-b)$ is also divisible by c .

By the same logic, we can say that $\{(3m+6n)-(3m+4n)\}$ is divisible by k . Hence, $2n$ is also divisible by k .

Now, $(m+2n)$ is divisible by k , it implies $2(m+2n) = 2m+4n$ is also divisible by k .

Hence, $\{(3m+4n)-(2m+4n)\} = m$ is also divisible by k .

Therefore, m , and $2n$ are also divisible by k .

The correct option is C

48. D

It is given that $2^{4x^2} - 2^{2x^2+x+16} + 2^{2x+30} = 0$, which can be written as:

$$\Rightarrow (2^{2x^2})^2 - 2^{2x^2} \cdot 2^{x+15} \cdot 2^1 + (2^{x+15})^2 = 0$$

$$\Rightarrow (2^{2x^2} - 2^{x+15})^2 = 0$$

$$\Rightarrow 2^{2x^2} - 2^{x+15} = 0 \text{ (Since } (a-b)^2 = 0 \Rightarrow a-b=0)$$

$$\Rightarrow 2x^2 = x + 15$$

$$\Rightarrow 2x^2 - x - 15 = 0$$

$$\Rightarrow 2x^2 - 6x + 5x - 15 = 0$$

$$\Rightarrow 2x(x-3) + 5(x-3) = 0$$

$$\Rightarrow (2x+5)(x-3) = 0$$

Hence, the possible values of x are $-\frac{5}{2}$, and 3 , respectively.

Therefore, the sum of the possible values is $(3 - \frac{5}{2}) = \frac{1}{2}$

The correct option is D

49. 15

Since there are two distinct factors other than 1, and itself, which implies the total number of factors of N is 4.

It can be done in two ways.

First case: $N = p^3$ (where p is a prime number)

Second case: $N = p_1 \times p_2$ (Where p_1, p_2 are the prime numbers)

From case 1, we can see that the numbers which is a cube of prime and less than 50 are 8, and 27 (2 numbers).

From case 2, we will get the numbers in the form $(2*3), (2*5), (2*7), (2*11), (2*13), (2*17), (2*19), (2*23), (3*5), (3*7), (3*11), (3*13), (5*7)$ {(13 numbers)}

Hence, the total number of numbers having two distinct factors is $(13+2) = 15$.

50.7

It is given that $\log_{\sqrt{3}}(x) + \frac{\log_x(25)}{\log_x(0.008)} = \frac{16}{3}$, which can be written as:

$$\Rightarrow 2 \log_3 x + \log_{0.008} 25 = \frac{16}{3}$$

$$\Rightarrow 2 \log_3 x + \log_{\frac{8}{1000}} 25 = \frac{16}{3}$$

$$\Rightarrow 2 \log_3 x + \log_{\frac{1}{125}} 25 = \frac{16}{3}$$

$$\Rightarrow 2 \log_3 x + \log_{5^{-3}} (5)^2 = \frac{16}{3}$$

$$\Rightarrow 2 \log_3 x - \frac{2}{3} = \frac{16}{3}$$

$$\Rightarrow 2 \log_3 x = \frac{16}{3} + \frac{2}{3}$$

$$\Rightarrow 2 \log_3 x = 6$$

$$\Rightarrow \log_3 x^2 = 6 \Rightarrow x^2 = 3^6$$

$$\text{Hence, } \log_3 (3 \cdot x^2) = \log_3 (3 \cdot 3^6) = \log_3 3^7 = 7$$

51.6

It is given that $(x-1)^2 + 2kx + 11 = 0$ has no real roots. (Where k is the largest integer)

$(x-1)^2 + 2kx + 11 = 0$, which can be written as:

$$\Rightarrow x^2 - 2x + 1 + 2kx + 11 = 0$$

$$\Rightarrow x^2 - 2(k-1)x + 12 = 0$$

We know that for no real roots, $D < 0 \Rightarrow b^2 - 4ac < 0$

$$\text{Hence, } \{2(k-1)\}^2 - 4 \cdot 1 \cdot 12 < 0$$

$$\Rightarrow 4(k-1)^2 < 48$$

$$\Rightarrow (k-1)^2 < 12$$

Since k is an integer, it implies (k-1) is also an integer.

Therefore, from the above inequality, we can say that the largest possible value of (k-1) = 3 \Rightarrow The largest possible value of k is 4.

Now we need to calculate the least possible value of $\frac{k}{4y} + 9y$.

$$\frac{k}{4y} + 9y \text{ can be written as } \frac{4}{4y} + 9y = \frac{1}{y} + 9y$$

The least possible value of $9y + \frac{1}{y}$ can be calculated using A.M-G.M inequality.

Using A.M-G.M inequality, we get:

$$\frac{9y + \frac{1}{y}}{2} \geq \sqrt{9y \times \frac{1}{y}}$$

$$\Rightarrow \frac{9y + \frac{1}{y}}{2} \geq \sqrt{9}$$

$$\Rightarrow \frac{9y + \frac{1}{y}}{2} \geq 3$$

$$\Rightarrow 9y + \frac{1}{y} \geq 6$$

Hence, the least possible value is 6

52. **A**

Let the time taken by A to fill the tank alone be x hours, which implies the time taken by B to empty the tank alone is $(x-1)$ hours (B is the drainage pipe), and the time taken by C to fill the tank is y hours.

It is given that when pipes A, B, and C are turned on together, the empty tank is filled in two hours.

$$\text{Hence, } \frac{1}{x} - \frac{1}{x-1} + \frac{1}{y} = \frac{1}{2} \dots \text{Eq(1)}$$

It is given that if pipes B and C are turned on together when the tank is empty and Pipe B is turned off after one hour, then Pipe C takes another one hour and 15 minutes to fill the remaining tank.

Hence, B worked for 1 hour, and C worked for 2 hours 15 minutes, which is equal to $\frac{9}{4}$ hours.

In 1 hour, B worked $-\frac{1}{x-1}$ units, and in $\frac{9}{4}$ hours, C worked $\frac{9}{4y}$ units.

$$\text{Hence, } \frac{9}{4y} - \frac{1}{x-1} = 1 \dots \text{Eq(2)}$$

Solving both equations, we get $y = \frac{3}{2}$, and $x = 3$

Hence, the time taken by C is $\frac{3}{2}$ hours, which is equal to 90 minutes.

The correct option is A

53. **B**

It is given that Anil borrows Rs 2 lakhs at an interest rate of 8% per annum, compounded half-yearly. It is also known that he repays Rs 10320 at the end of the first year and closes the loan by paying the outstanding amount at the end of the third year.

The total amount at the end of the first year is: $200000 \times \frac{104}{100} \times \frac{104}{100} = 216320$

He repays 10320 rupees at the end of the first year, which implies the amount that remains unpaid at the end of the first year is 206000 rupees.

This unpaid amount will accrue interest for another two years.

Hence, the final amount at the end of three years is $206000 \times \frac{104}{100} \times \frac{104}{100} \times \frac{104}{100} = 240990.86$

Hence, the accrued interest in these two years is $(240990.86 - 206000) = 34990.86$ rupees.

Hence, the total interest accrued over the three years = $(34990.86 + 16320) = 51311$ rupees.

The correct option is B

54. **C**

It is given that the speed of Ravi is 40 kmph, which is equal to $\frac{100}{9}$ m/s. It is also known that the speed of Ashok is 50 kmph, which is equal to $\frac{125}{9}$ m/s.

It is known that the distance between Ravi and Ashok is 225 meters, and the relative speed of Ravi and Ashok is $\frac{125}{9} + \frac{100}{9} = 25$ m/s

Hence, they will meet each other in $\frac{225}{25} = 9$ seconds. The distance traveled by Ravi in these 9 seconds is $\frac{100}{9} \times 9 = 100$ meters.

Since Vijay was already 54 meters behind Ravi when they were starting, Vijay must travel $(100 + 54) = 154$ meters in these 9 seconds.

Hence, the speed of Vijay is $\frac{154}{9}$ m/s, which is equal to $\frac{154}{9} \times \frac{18}{5} = \frac{308}{5} = 61.6$ kmph.

The correct option is C

55. C

The cost price of the sunglass for Meenu when he purchased it for the first time was 1000 rupees, and he sold it to Kanu at 20% profit. Hence, the selling price of the sunglass is 1200 rupees, which Kanu purchased. Hence, the profit made by Meenu is $(1200 - 1000) = 200$ rupees.

Hence, the cost price of the same sunglass for Kanu is 1200 rupees, and now he sold it to Meenu at a 20% loss. Hence, the selling price of the sunglass now is $(1200 \times 0.8) = 960$ rupees.

The cost price of the same sunglass for Meenu when he purchased it for the second time was 960 rupees. Now Meenu sold it Tanu, at a certain price such that the total profit of Meenu becomes 500 rupees.

Hence, on the second transaction (selling it to Tanu), Meenu made a profit of $(500 - 200) = 300$ rupees.

Hence, the profit made by Minu in the second transaction is $(300/960) \times 100\% = 31.25\%$

The correct option is C

56. D

it is given that the price of a precious stone is directly proportional to the square of its weight. Let the price be denoted by C and the weight is denoted by W.

Hence, $C \propto W^2 \Rightarrow C = kw^2$ (where k is the proportional constant)

Now, Sita has a precious stone weighing 18 units.

Therefore, $C = kw^2 = k \cdot 18^2 = 324$

If she breaks it into four pieces with each piece having a distinct integer weight, then the difference between the highest and lowest possible values of the total price of the four pieces will be 288000.

To get the lowest possible value of C, we will get the weight of the four-piece as close as possible (3,4,5,6). To get the highest value we will try to take three pieces as low as possible, and one is as high as possible (1, 2, 3, 12).

Hence, the maximum cost $= k(12^2 + 1^2 + 2^2 + 3^2) = 158k$, and the minimum cost is $k(3^2 + 4^2 + 5^2 + 6^2) = 86k$

Hence, the difference is $(158k - 86k) = 72k$, which is equal to 288000.

$\Rightarrow 72k = 288000$

$\Rightarrow k = 4000$

Hence, the price of the original stone is $324k = 324 \times 4000 = 1296000$

The correct option is D

57. B

Let the number of total employees in the company be $100x$, and the total salary of all the employees be $100y$.

It is given that 20% of the employees work in the manufacturing department, and the total salary obtained by all the manufacturing employees is one-sixth of the total salary obtained by all the employees in the company.

Hence, the total number of employees in the manufacturing department is $20x$, and the total salary received by them is $(100y/6)$

Average salary in the manufacturing department $= (100y/6 \times 20x) = 5y/6x$

Similarly, the total number of employees in the nonmanufacturing department is $80x$, and the total salary received by them is $(500y/6)$

Hence, the average salary in the nonmanufacturing department = $(500y/6 \cdot 80x) = 25y/24x$

Hence, the ratio is:- $(5y/6x) : (25y/24x)$

$\Rightarrow 120:150 = 4:5$

The correct option is B

58. 16

It is given that if a certain amount of money is divided equally among n persons, each one receives Rs 352.

Hence, the total amount of money is $(352 \cdot n) = 352n$... Eq(1)

It is also known that if two persons receive Rs 506 each and the remaining amount is divided equally among the other persons, each of them receives less than or equal to Rs 330

Hence, the maximum amount of money with them = $506 \cdot 2 + (n-2) \cdot 330 = 1012 + 330n - 660 = 352 + 330n$

Now, $352 + 330n \geq 352n$

$\Rightarrow 22n \leq 352$

$\Rightarrow n \leq 16$

Hence, the maximum value is 16

59. 407

Let the number of white shirts be m , and the number of blue shirts be n . Hence, the total cost of the shirts = $(1000m + 1125n)$, and the number of shirts is $(m+n)$

The average price of the shirts is $\frac{1000m+1125n}{m+n}$. It is given that he set a fixed market price which was 25% higher than the average cost of all the shirts. He sold all the shirts at a discount of 10%.

Hence, the average selling price of the shirts = $\left(\frac{1000m+1125n}{m+n} \right) \times \frac{5}{4} \times \frac{9}{10} = \frac{9}{8} \left(\frac{1000m+1125n}{m+n} \right)$

The average profit of the shirts = $\frac{9}{8} \left(\frac{1000m+1125n}{m+n} \right) - \frac{1000m+1125n}{m+n} = \frac{1}{8} \left(\frac{1000m+1125n}{m+n} \right)$

The total profit of the shirts = $\frac{1}{8} \left(\frac{1000m+1125n}{m+n} \right) \times (m+n) = \frac{1}{8} (1000m + 1125n)$

Now, $\Rightarrow \frac{1}{8} (1000m + 1125n) = 51000$

$\Rightarrow 1000m + 1125n = 51000 \times 8 = 408000$

Now to get the maximum number of shirts, we need to minimize n (since the coefficient of n is greater than the coefficient of m), but it can't be zero. Therefore, m has to be maximum.

$$m = \frac{408000 - 1125n}{1000}$$

The maximum value of m such that m , and both are integers is $m = 399$, and $n = 8$ (by inspection)

Hence, the maximum number of shirts = $m+n = 399+8 = 407$

60.7

Let's assume that after n iteration, the volume of the milk will be less than 50%, which is less than 20 liters.

Initially, the amount of milk is 40 liters, after the first iteration, the volume of milk is $40 \cdot \frac{9}{10}$

After the second iteration, the volume of milk is $40 \times \left(\frac{9}{10}\right)^2$

Similarly, after the n iterations, the volume of milk is $40 \times \left(\frac{9}{10}\right)^n$

Now,

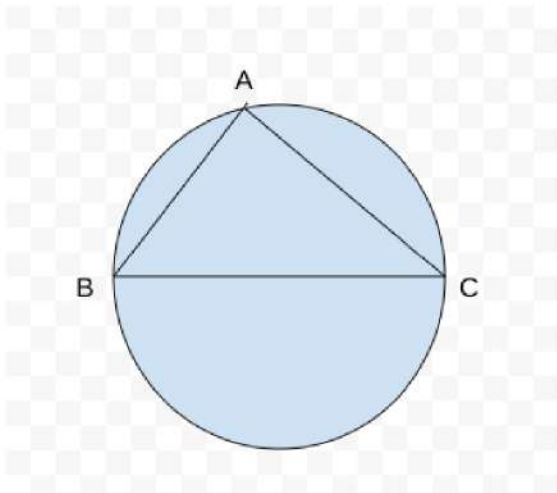
$$40 \times \left(\frac{9}{10}\right)^n \leq 20$$

$$\Rightarrow \left(\frac{9}{10}\right)^n \leq \frac{1}{2}$$

$$\Rightarrow n \geq 7$$

Hence, the correct answer is 7

61.B



Since BC is the diameter of the circle, which implies angle BAC is 90 degrees. Let AB = a cm, which implies AC = b cm. Hence, $BC = \sqrt{a^2 + b^2}$, which is diameter of the circle ($2r$).

$$\text{Hence, } 2r = \sqrt{a^2 + b^2}$$

$$\Rightarrow 4r^2 = a^2 + b^2$$

The area of the triangle is $\frac{1}{2} \times a \times b$, which can be written as

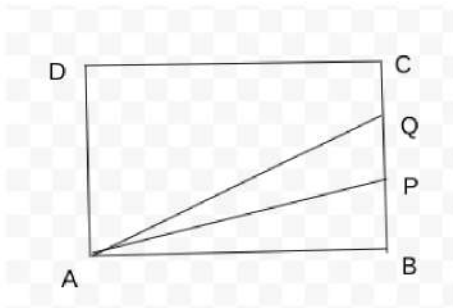
$$\Rightarrow \frac{a \cdot b}{2(a^2 + b^2)} \times (a^2 + b^2)$$

$$\Rightarrow \frac{a \cdot b}{2(a^2 + b^2)} \times 4r^2$$

$$\Rightarrow \frac{a \cdot b}{a^2 + b^2} \times 2r^2$$

The correct option is B

62. C

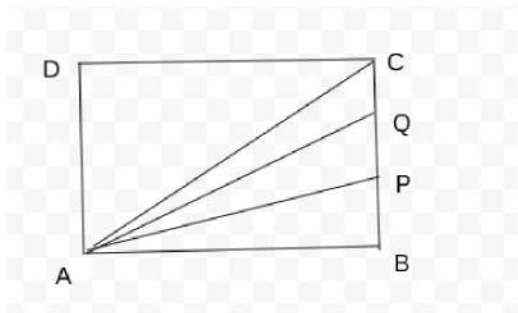


It is given that $AB = 9$ cm, $BC = 6$ cm.

It is also known that the areas of the figures ABP, APQ, and AQCD are in geometric progression.

Hence, the area of the ABP, APQ, and AQCD are k , $2k$, and $4k$ respectively.

The ratio of BP, PQ, QC will be the ratio of the respective triangles. Hence, we can draw a line from point A to point C.



Let the area of triangle AQC be x , which implies the area of triangle ADC = $ADQC - AQC = 4k - x$, which is equal to the sum of the area of triangle APB, AQP, and ACQ, respectively.

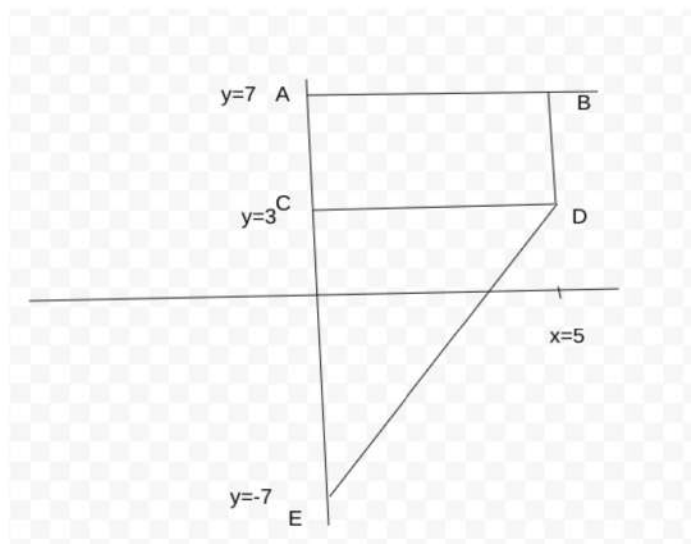
Therefore, $4k - x = 3k + x$

$\Rightarrow x = k/2$

Hence the ratio of BP: PQ: CQ = $k:2k: k/2 = 2:4:1$

63. 45

From the inequality and nature of x , and y , we get the given diagram:



We need to find the area of the quadrilateral ABDE = area of rectangle ABCD + area of triangle CDE
 \Rightarrow Area of ABCD = $(7-3) \times 5 = 20$ units, and the area of triangle CDE = $(1/2) \times 10 \times 5 = 25$ units.
Hence, the area of the quadrilateral ABDE = $(20+25) = 45$ units.

64. **B**

Let the first term of both series be a_1 , and b_1 , respectively, and the common difference be d_1 , and d_2 , respectively.

It is given that $a_5 = b_9$, which implies $a_1 + 4d_1 = b_1 + 8d_2$

$$\Rightarrow a_1 - b_1 = 8d_2 - 4d_1 \dots \text{Eq(1)}$$

Similarly, it is known that $a_{19} = b_{19}$, which implies $a_1 + 18d_1 = b_1 + 18d_2$

$$\Rightarrow a_1 - b_1 = 18d_2 - 18d_1 \dots \text{Eq(2)}$$

Equating (1) and (2), we get:

$$\Rightarrow 18d_2 - 18d_1 = 8d_2 - 4d_1$$

$$\Rightarrow 10d_2 = 14d_1$$

$$\Rightarrow 5d_2 = 7d_1$$

Since, d_1, d_2 are the prime numbers, which implies $d_1 = 5, d_2 = 7$.

It is also known that $b_2 = 0$, which implies $b_1 + d_2 = 0 \Rightarrow b_1 = -d_2 = -7$

Putting the value of b_1, d_1 , and, d_2 in Eq(1), we get:

$$a_1 = 8d_2 - 4d_1 + b_1 = 56 - 20 - 7 = 29$$

$$\text{Hence, } a_{11} = a_1 + 10d_1 = 29 + 10 \cdot 5 = 29 + 50 = 79$$

The correct option is B

65. **C**

Given that $2pq - 20 = 52 - 2pq \Rightarrow 4pq = 72 \Rightarrow pq = 18 \dots (1)$

$$\text{Now, } p^2 + q^2 - 29 = 2pq - 20 \Rightarrow p^2 + q^2 - 2pq = 9 \Rightarrow (p - q)^2 = 9 \Rightarrow p - q = \pm 3$$

$$\text{Also, } p^2 + q^2 - 29 = 2pq - 20 \Rightarrow p^2 + q^2 = 2pq + 9 = 2(18) + 9 = 45$$

$$\text{Now, } p^3 - q^3 = (p - q)(p^2 + pq + q^2) = (p - q)(45 + 18) = (p - q)(63)$$

$$\Rightarrow \text{When } p - q = -3 \Rightarrow \text{The value is } 63(-3) = -189 \text{ and when } p - q = 3 \Rightarrow \text{The value is } 63(3) = 189.$$

$$\Rightarrow \text{The difference} = 189 - (-189) = 378.$$

66. **967**

It is given that $a_n = 13 + 6(n - 1)$, which can be written as $a_n = 13 + 6n - 6 = 7 + 6n$

Similarly, $b_n = 15 + 7(n - 1)$, which can be written as $b_n = 15 + 7n - 7 = 8 + 7n$

The common differences are 6, and 7, respectively, The common difference of terms that exists in both series is l.c.m (6, 7) = 42

The first common term of the first two series is 43 (by inspection)

Hence, we need to find the m th term, which is less than 1000, and the largest three-digit integer, and exists in both series.

$$t_m = a + (m - 1)d < 1000$$

$$\Rightarrow 43 + (m - 1)42 < 1000$$

$$\Rightarrow (m - 1)42 < 957$$

$$\Rightarrow m - 1 < 22.8 \Rightarrow m < 23.8 \Rightarrow m = 23$$

$$\text{Hence, the 23rd term is } 43 + 22 \times 42 = 967$$