Class - X Session 2022-23
Subject - Mathematics (Basic)
Sample Question Paper - 31

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D, and E.
2. Section A has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. Section B has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.
4. Section C has 6 Short Answer-II (SA-II) type questions carrying 3 marks each.
5. Section D has 4 Long Answer (LA) type questions carrying 5 marks each.
6. Section E has 3 Case Based integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 2 marks, 2 Qs of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. From a well-shuffled deck of 52 cards, one card is drawn at random. What is the probability of getting a queen? **[1]**

   a) None of these b) $\frac{4}{39}$

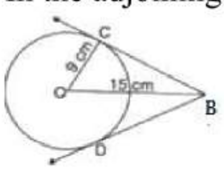
   c) $\frac{1}{13}$ d) $\frac{1}{26}$

2. If A (2, 2), B (-4, - 4) and C (5, -8) are the vertices of a triangle, then the length of the median through vertex C is **[1]**

   a) $\sqrt{113}$ b) $\sqrt{65}$

   c) $\sqrt{85}$ d) $\sqrt{117}$

3. In the adjoining figure, If OC = 9 cm and OB = 15 cm, then BC + BD is equal to **[1]**



   a) 24 cm b) 18 cm

   c) 12 cm d) 36 cm

4. The value of k so that the system of equations 3x - y - 5 = 0 and 6x - 2y - k = 0 have infinitely many solutions is **[1]**

   a) k = -10 b) k = 10

c) k = -8

d) k = 8

5. In the fourth quadrant, **[1]**

a) x is +ve, y is -ve

b) x is -ve, y is -ve

c) x is +ve, y is +ve

d) x is -ve, y is +ve

6. If the probability of an event is 'p', the probability of its complementary event will be **[1]**

a) p

b) p – 1

c) 1 – p

d) $1 - \frac{1}{p}$

7. The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by **[1]**

a) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

b) $\left(\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2}\right)$

c) $\left(\frac{x_1-y_1}{2}, \frac{x_2-y_2}{2}\right)$

d) $\left(\frac{x_1+y_1}{2}, \frac{x_2+y_2}{2}\right)$

8. If the probability of winning a game is 0.4 then the probability of losing it, is **[1]**

a) None of these

b) 0.6

c) 0.4

d) 0.96

9. If a cone is cut into two parts by a horizontal plane passing through the mid-point of its axis, the ratio of the volumes of the upper part and the cone is **[1]**

a) 1 : 2

b) 1 : 4

c) 1 : 6

d) 1 : 8

10. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has **[1]**

a) two equal real roots

b) no real root

c) two distinct real roots

d) more than 2 real roots

11. If one root of the equation $x^2 + ax + 3 = 0$ is 1, then its other root is **[1]**

a) 3

b) -3

c) 2

d) -2

12. The number $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ is **[1]**

a) an irrational number

b) an integer

c) not a real number

d) a rational number

13. If sin A = $\frac{1}{2}$, then the value of cot A is **[1]**

a) $\sqrt{3}$

b) $\frac{\sqrt{3}}{2}$

c) $\frac{1}{\sqrt{3}}$

d) 1

14. The ________ of an object is the angle formed by the line of sight with the horizontal when the object is above the horizontal level. **[1]**

a) angle of projection

b) angle of depression

c) angle of elevation

d) none of these

15. Mode of a data is given by **[1]**

a) $l-\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$

b) $l+\left(\frac{f_0-f_1}{2f_1-f_0-f_2}\right)\times h$

c) $l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$

d) $h+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times l$

16. The distance of the point P(-6, 8) from the origin is **[1]**

a) $2\sqrt{7}$

b) 6

c) 8

d) 10

17. If 2x - 3 y = 7 and (a + b) x - (a + b - 3) y = 4a + b represent coincident lines, then a and b satisfy the equation **[1]**

a) a - 5b = 0

b) 5a - b = 0

c) a + 5b = 0

d) 5a + b = 0

18. What is the largest number that divides each one of 1152 and 1664 exactly? **[1]**

a) 64

b) 256

c) 128

d) 32

19. **Assertion (A):** Two similar triangles are always congruent. **[1]**
Reason (R): If the areas of two similar triangles are equal then the triangles are congruent.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** 3 is a rational number. **[1]**
Reason (R): The square roots of all positive integers are irrationals.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears **[2]**

   i. a two-digit number

   ii. a perfect square number

   iii. a number divisible by 5.

22. Find the zeroes of a quadratic polynomial given as: $4u^2 + 8u$ and also verify the relationship between the zeroes and the coefficients. **[2]**

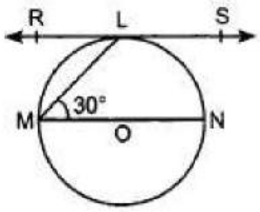
23. Find the values of y for which the distance between the points P (2, -3) and Q (10, y) is 10 units. **[2]**

24. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$, find out whether the pair of linear equations are consistent, or inconsistent: $\frac{3}{2}x + \frac{5}{3}y = 7$, $9x - 10y = 14$. **[2]**

OR

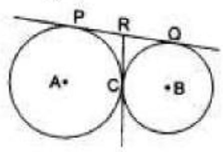
Taxi charges in a city consist of fixed charges and the remaining depending upon the distance travelled in kilometres. If a person travels 60 km, he pays ₹960, and for travelling 80 km, he pays ₹1260. Find the fixed charges and the rate per kilometre.

25. In the given figure, RS is the tangent to the circle at L and MN is a diameter. If $\angle$ NML = 30°, determine $\angle$RLM. **[2]**



OR

In the given figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.

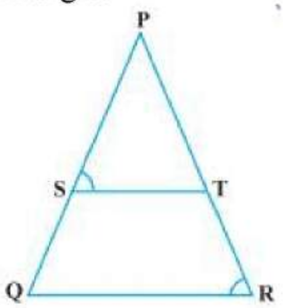


Section C

26. The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ₹ 2000 per month, then find their monthly incomes. **[3]**

27. Write all the other trigonometric ratios of ∠A in terms of sec A. **[3]**

28. In Fig. $\frac{PS}{SQ}=\frac{PT}{TR}$ and $\angle PST=\angle PRQ$. Prove that △PQR is an isosceles triangle. **[3]**



29. Prove that $\frac{1}{\sqrt{2}}$ is irrational. **[3]**

OR

Find the values of a and b if the HCF of the polynomials.
$f(x)=(x+3)\left(2x^2-3x+a\right)$
and $g(x)=(x-2)\left(3x^2+10x-b\right)$ is $(x+3)(x-2)$

30. A ladder rests against a wall at an angle a to the horizontal. Its foot is pulled away from the wall through a distance a, so that it slides a distance b down the wall making an angle $\beta$with the horizontal. Show that **[3]**
$\frac{a}{b}=\frac{\cos\alpha-\cos\beta}{\sin\beta-\sin\alpha}$

31. Prove that parallelogram circumscribing a circle is a rhombus. **[3]**

OR

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Section D

32. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC\sim\Delta PQR$ **[5]**

33. A train travels at a certain average speed for a distance 63 km and then travels a distance of 72 km at an average speed of 6 km/hr more than the original speed. If it takes 3 hours to complete total journey, what is its original average speed? **[5]**

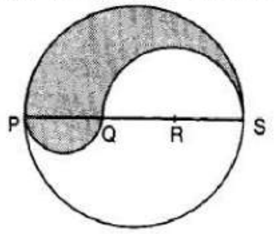
OR

The area of right angled triangle is 480 cm^2 . If the base of triangle is 8 cm more than twice the height (altitude) of the triangle, then find the sides of the triangle.

34. If the median of the distribution given below is 28.5, then find the values of x and y. **[5]**

Class Interval	frequency
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5
Total	60

35. PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in Fig. Find the perimeter and area of the shaded region **[5]**



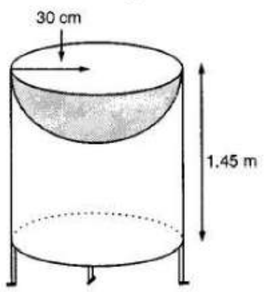
OR

Find the area of the segment of a circle of radius 12 cm whose corresponding sector central angle 60°. (Use $\pi = 3.14$).

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Mayank a student of class 7^{th} loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class 10^{th} helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm. The cost of material used for making bird bath is ₹40 per square meter.



(i) Find the curved surface area of the hemisphere.

(ii) Find the total surface area of the bird-bath. (Take $\pi = \frac{22}{7}$)

(iii) What is total cost for making the bird bath?

OR

Mayank and his brother thought of increasing the radius of hemisphere to 35 cm with same material so that birds get more space, then what is the new height of cylinder?

37. **Read the text carefully and answer the questions:** **[4]**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find an increase in the production of TV every year.

(ii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find in which year production of TV is 1000.

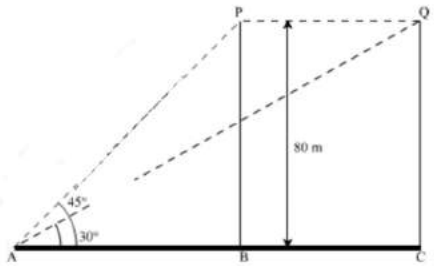
(iii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the production in the 10th year.

OR

They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the total production in first 7 years.

38. **Read the text carefully and answer the questions:** **[4]**

A bird is sitting on the top of a tree, which is 80m high. The angle of elevation of the bird, from a point on the ground is 45°. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30°. Find the speed of flying of the bird.



(i) Find the distance between observer and the bottom of the tree?

(ii) Find the speed of the bird?

(iii) Find the distance between second position of bird and observer?

OR

Find the distance between initial position of bird and observer?

SOLUTION

Section A

1. **(c)** $\frac{1}{13}$

   Explanation: Number of all possible outcomes = 52.

   Number of queens = 4.

   $\therefore$P (getting a queen) = $\frac{4}{52} = \frac{1}{13}$

2. **(c)** $\sqrt{85}$

   Explanation: Let mid point of A(2, 2), B(-4, -4) be whose coordinates will be

   $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2-4}{2}, \frac{2-4}{2}\right)$

   or $\left(\frac{-2}{2}, \frac{-2}{2}\right) = (-1, -1)$

   $\therefore$ Length of median CD

   $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

   $= \sqrt{(5+1)^2 + (-8+1)^2}$

   $= \sqrt{(6)^2 + (-7)^2} = \sqrt{36+49}$

   $= \sqrt{85}$ units

3. **(a)** 24 cm

   Explanation: Here $\angle$C = 90° [Angle between tangent and radius through the point of contact]

   Now, in right angled triangle OBC,

   $OB^2 = OC^2 + BC^2$

   $\Rightarrow (9)^2 = (15)^2 + BC^2$

   $\Rightarrow BC^2 = 225 - 81 = 144$

   $\Rightarrow$BC = 12 cm

   But BC = BD [Tangents from one point to a circle are equal]

   Therefore, BD = 12 cm

   Then BC + BD = 12 + 12 = 24 cm

4. **(b)** k = 10

   Explanation: Given: $a_1 = 3$, $a_2 = 6$, $b_1 = -1$, $b_2 = -2$, $c_1 = -5$ and $c_2 = -k$

   If there are infinitely many solutions, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{3}{6} = \frac{-1}{-2} = \frac{-5}{-k}$

   Taking $\frac{-1}{-2} = \frac{-5}{-k}$

   $\Rightarrow$ k = 5 × 2

   $\Rightarrow$ k = 10

5. **(a)** x is +ve, y is -ve

   Explanation: In the fourth quadrant, x is positive, y is negative.

   i.e the value of x is called abscissa which is positive and the value of y is called coordinate which is negative in the 4th quadrant

6. **(c)** 1 – p

   Explanation: If the probability of an event is p, the probability of its complementary event will be 1 - p. because we know that the sum of probability of an event and its complementary event is always 1.

Hence, p + 1 - p = 1

7. **(a)** $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Explanation: we know that the midpoint formula = $\frac{x_1+x_2}{2}$, $\frac{y_1+y_2}{2}$

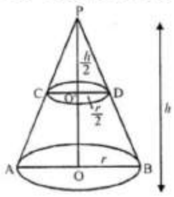
The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

8. **(b)** 0.6

Explanation: P(losing the game) = 1 - P (winning the game) = (1 - 0.4) = 0.6

9. **(d)** 1 : 8

Explanation: In the figure, C and D are the mid-points and CD || AB which divide the cone into two parts



Height OO' = $\frac{1}{2}$ OP and diameter CD = $\frac{1}{2}$ AB

Let h be the height and r be the radius of the cone, then

$\frac{h}{2}$ will be the height of the smaller cone and $\frac{r}{2}$ be its radius, then

Volume of bigger cone = $\frac{1}{3}\pi r^2 h$

and volume of smaller cone

$= \frac{1}{3}\pi\left(\frac{r}{2}\right)^2\left(\frac{h}{2}\right)$

$\therefore \frac{\frac{1}{3}\pi\left(\frac{r}{2}\right)^2\left(\frac{h}{2}\right)}{\frac{1}{3}\pi r^2 h} = \frac{\frac{1}{3}\pi\frac{r^2}{4}\times\frac{h}{2}}{\frac{1}{3}\pi r^2 h}$

$= \frac{\frac{1}{8}\left(\frac{1}{3}\pi r^2 h\right)}{\frac{1}{3}\pi r^2 h} = \frac{1}{8}$

$\therefore$ Ratio = 1 : 8

10. **(b)** no real root

Explanation: Discriminant = 5 - 4(2)(1) < 0

Therefore no real root.

11. **(a)** 3

Explanation: The given equation is $x^2 + ax + 3 = 0$

One root = 1

and product of roots = $\frac{c}{a} = \frac{3}{1} = 3$

Second root = $\frac{3}{1} = 3$

12. **(a)** an irrational number

Explanation: $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

$= \frac{(\sqrt{5}+\sqrt{2})^2}{(\sqrt{5})^2-(\sqrt{2})^2}$

$= \frac{(\sqrt{5})^2+(\sqrt{2})^2+2\times\sqrt{5}\times\sqrt{2}}{5-2}$
$= \frac{5+2+2\sqrt{10}}{3}$
$= \frac{7+2\sqrt{10}}{3}$
Here $\sqrt{10} = \sqrt{2} \times \sqrt{5}$
Since $\sqrt{2}$ and $\sqrt{5}$ both are an irrational number
Therefore, $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ is an irrational number.

13. **(a)** $\sqrt{3}$
Explanation: Given: $\sin A = \frac{1}{2} \ldots$(i)
And we know that, $\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A} \ldots$(ii)
We need to find the value of cos A.
$\cos A = \sqrt{1 - \sin^2 A} \ldots$(iii)
Substituting eq. (i) in eq. (iii), we get
cos A = $\sqrt{\left(1 - \frac{1}{4}\right)}$
$= \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$
Substituting values of sin A and cos A in eq. (ii), we get
$\cot A = \frac{\sqrt{3}}{2} \times 2 = \sqrt{3}$

14. **(c)** angle of elevation
Explanation: The angle of elevation of an object is the angle formed by the line of sight with the horizontal when the object is above the horizontal level.

15. **(c)** $l + \left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right) \times h$
Explanation: Mode of data is given by
$l + \left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right) \times h$
where l = lower limit of the modal class
f_1 = frequency of the modal class
f_0 = frequency of the class preceding the modal class
f_2 = frequency of the class succeeding the modal class
h = size of the class interval (assuming all class sizes to be equal)

16. **(d)** 10
Explanation: The distance of the point P(-6,8) from the origin (0, 0)
$= \sqrt{(-6)^2 + 8^2}$
$= \sqrt{36 + 64}$
$= \sqrt{100}$
= 10

17. **(a)** a - 5b = 0
Explanation: Given Equations are 2x - 3y = 7
and (a + b) x - (a + b - 3) y = 4 a + b represent coincident lines.
When lines are coincident then the condition of equations
$a_1x + b_1y = c_1$,

$a_2x + b_2y = c_2$

is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

On comparing, we get

$\frac{2}{a+b} = \frac{3}{a+b-3} = \frac{7}{4a+b}$

Now, we can equate any two equation. So, taking

$\frac{2}{a+b} = \frac{7}{4a+b}$

$\Rightarrow 2(4a + b) = 7(a + b)$

$\Rightarrow 8a + 2b = 7a + 7b$

$\Rightarrow 8a - 7a = 7b - 2b$

$\Rightarrow a = 5b$

$\Rightarrow a - 5b = 0$

Therefore, The required equation satisfied by a and b is a - 5b = 0.

18. **(c)** 128

Explanation: Largest number that divides each one of 1152 and 1664 = HCF (1152, 1664)

We know, $1152 = 2^7 \times 3^2$

$1164 = 2^7 \times 13$

$\therefore$ HCF = $2^7 = 128$

19. **(d)** A is false but R is true.

Explanation: Two similar triangles are not congruent generally. So, A is false but R is true.

20. **(c)** A is true but R is false.

Explanation: Here, reason is not true.

$\sqrt{9} = \pm 3$, which is not an irrational number.

A is true but R is false.

Section B

21. Total number of favourable outcomes = 90

$Probability\ of\ the\ event = \frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes}$

i. Number of two-digit numbers from 1 to 90 are 90 - 9 = 81

$\therefore$Favourable outcomes = 81

Hence, P (getting a disc bearing a two-digit number) = $\frac{81}{90} = \frac{9}{10}$

ii. From 1 to 90, the perfect squares are 1, 4, 9, 16, 25, 36, 49, 64 and 81.

$\therefore$Favourable outcomes = 9

Hence P (getting a perfect square) = $\frac{9}{90} = \frac{1}{10}$

iii. The numbers divisible by 5 from 1 to 90 are 18.

$\therefore$Favourable outcomes = 18

Hence P (getting a number divisible by 5) = $\frac{18}{90} = \frac{1}{5}$

22. The quadratic equation is given as: $4u^2 + 8u$

it can be written in the standard form as:

$= 4u^2 + 8u + 0$

$= 4u (u + 2)$

The value of $4u^2 + 8u$ is zero when 4u = 0 or u + 2 = 0, i.e., u = 0 or u = −2

Therefore, the zeroes of $4u^2 + 8u$ are 0 and −2

Sum of zeroes = $0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-(\text{ coefficient of } u)}{\text{coefficient of } u^2}$

Product of zeroes = $0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{constant torm}}{\text{coefficient of } u^2}$

Hence verified

23. Using Distance formula, we have

$10 = \sqrt{(2-10)^2 + (-3-y)^2}$

$\Rightarrow 10 = \sqrt{(-8)^2 + 9 + y^2 + 6y}$

$\Rightarrow 10 = \sqrt{64 + 9 + y^2 + 6y}$

Squaring both sides, we get

$100 = 73 + y^2 + 6y$

$\Rightarrow y^2 + 6y - 27 = 0$

Solving this Quadratic equation by factorization, we can write

$\Rightarrow y^2 + 9y - 3y - 27 = 0$

$\Rightarrow y(y + 9) - 3(y + 9) = 0$

$\Rightarrow (y + 9)(y - 3) = 0$

$\Rightarrow y = 3, -9$

24. Given equations are:

$\frac{3}{2}x + \frac{5}{3}y = 7$ &

$9x - 10y = 14$

Comparing equation $\frac{3}{2}x + \frac{5}{3}y = 7$ with $a_1x + b_1y + c_1 = 0$

and $9x - 10y = 14$ with

$a_2x + b_2y + c_2 = 0$,

We get, $a_1 = \frac{3}{2}, b_1 = \frac{5}{3}, c_1 = -7, a_2 = 9, b_2 = -10, c_2 = -14$

$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{1}{6}$ and $\frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{-1}{6}$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, equations have unique solution.

Hence, they are consistent.

OR

Let the fixed charges be ₹ x and the other charges be ₹ y per km.

As per given condition,

If a person travels 60 km, he pays ₹ 960

Then, x + 60y = 960 .... (i)

And for travelling 80 km, he pays ₹ 1260.

Then, x + 80y = 1260. .... (ii)

On subtracting (i) from (ii), we get

$20y = 300$

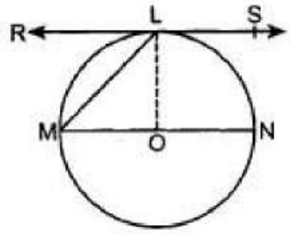
$\Rightarrow y = \frac{300}{20}$

$\Rightarrow y = 15$

Putting y = 15 in (i), we get

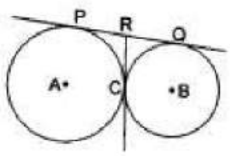
x + 60y = 960
x + (60 × 15) = 960
⇒ x = 960 - 900
⇒ x = 60.
Therefore, fixed charges = ₹ 60 and the rate per km = ₹ 15 per km.

25. Given,



Construction: Join OL
OL ⊥ RS.
Also OL = OM
$\therefore \angle OML = \angle OLM$
$\Rightarrow . \angle OLM = 30°$
$\Rightarrow \angle RLM = 90° - 30° = 60°$

OR



In the given figure, PR and CR are both tangents drawn to the same circle from an external point R.
∴ PR = CR. ...(i)
Also, QR and CR are both tangents drawn to the same circle (second circle) from an external point R
QR = CR ... (ii)
From (i) and (ii), we get
PR = QR [each equal to CR].
R is the midpoint of PQ,
i.e., the common tangent to the circles at C, bisects the common tangent at P and Q.

Section C

26. Let us denote the incomes of the two-person by ₹ 9x and ₹ 7x and their expenditures by ₹ 4y and ₹ 3y respectively.
Then the equations formed in the situation is given by :
9x – 4y = 2000 ...(i)
and 7x – 3y = 2000 ...(2)
Step 1: Multiply Equation (1) by 3 and Equation (2) by 4 to make the coefficients of y equal. Then, we get the equations:
27x – 12y = 6000 ...(3)
28x – 12y = 8000 ...(4)
Step 2: Subtract Equation (3) from Equation (4) to eliminate y, because the coefficients of y are the same. So, we get
(28x – 27x) – (12y – 12y) = 8000 – 6000

i.e., x = 2000

Step 3: Substituting this value of x in (1), we get

9(2000) – 4y = 2000

i.e., y = 4000

So, the solution of the equations is x = 2000, y = 4000. Therefore, the monthly incomes of the persons are ₹18,000 and ₹14,000 respectively.

27. sin A can be expressed in terms of sec A as:

$\sin A=\sqrt{\sin^2 A}$

$sinA=\sqrt{(1-\cos^2 A)}$

$\sin A=\sqrt{1-\frac{1}{\sec^2 A}}$

$\sin A=\sqrt{\frac{\sec^2 A-1}{\sec^2 A}}$

$\sin A=\frac{1}{\sec A}\sqrt{\sec^2 A-1}$

Now,

cos A can be expressed in terms of secA as:

$\cos A=\frac{1}{\sec A}$

tanA can be expressed in the form of sec A as:

As, 1 + $\tan^2$A = $\sec^2$A

$\Rightarrow \tan A=\pm\sqrt{(\sec^2 A-1)}$

since A is an acute angle, and tan A is positive when A is acute, So, tan A = $\sqrt{(\sec^2 A-1)}$

Now $cosecA$ can be expressed in the form of sec A as:

$cosecA=\frac{1}{\sin A}$

$cosecA=\frac{1}{\frac{\sec A}{\sqrt{1-\sec^2 A}}}$

$cosecA=\frac{\sqrt{1-\sec^2 A}}{\sec A}$

Now, cot A can be expressed in terms of sec A as:

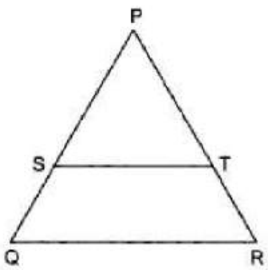
cot A $=\frac{1}{\tan A}$

as, 1 + $\tan^2$A = $\sec^2$A

$cotA=\frac{1}{\sqrt{\sec^2 A-1}}$

28. According to the question,we are given that,

$\frac{PS}{SQ}=\frac{PT}{TR}$



$\Rightarrow\quad ST\|QR$ [By using the converse of Basic Proportionality Theorem]

$\Rightarrow\quad \angle PST=\angle PQR$ [Corresponding angles]

$\Rightarrow\quad \angle PRQ=\angle PQR\ [\because \angle PST=\angle PRQ(\text{ Given })]$

$\Rightarrow$ PQ = PR [$\because$ Sides opposite to equal angles are equal]

$\Rightarrow$ Δ PQR is isosceles.

29. We can prove $\frac{1}{\sqrt{2}}$ irrational by contradiction.

Let us suppose that $\frac{1}{\sqrt{2}}$ is rational.

It means we have some co-prime integers a and b ($b \neq 0$)

Such that

$\frac{1}{\sqrt{2}} = \frac{a}{b}$

$\Rightarrow \sqrt{2} = \frac{b}{a}$..........(1)

R.H.S of (1) is rational but we know that is $\sqrt{2}$ irrational.

It is not possible which means our supposition is wrong.

Therefore, $\frac{1}{\sqrt{2}}$ can not be rational.

Hence, it is irrational.

OR

$f(x) = (x+3)\left(2x^2 - 3x + a\right)$

$g(x) = (x-2)\left(3x^2 + 10x - b\right)$

since $(x+3)(x-2)$ is the HCF of $f(x)$ and $g(x)$

$\therefore x - 2$ is a factor of $2x^2 - 3x + a \ldots (i)$

and $x + 3$ is a factor of $3x^2 + 10x - b \ldots\ldots$(ii)

From (i) follow that 2 is a zero of $2x^2 - 3x + a$

$\Rightarrow 2 \times 2^2 - 3 \times 2 + a = 0$

$\Rightarrow 8 - 6 + a = 0$

$\Rightarrow 2 + a = 0 \Rightarrow a = -2$

From (ii), it follows that

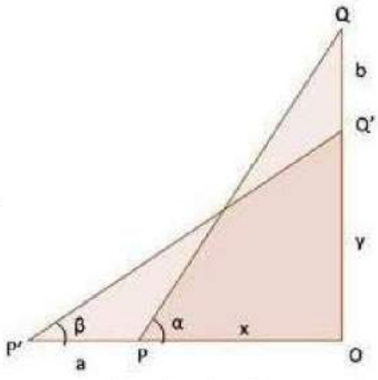
-3 is a zero of $3x^2 + 10x - b$

$\Rightarrow 3x(-3)^2 + 10x(-3) - b = 0$

$\Rightarrow 27 - 30 - 6 = 0$

$\Rightarrow -3 - b = 0 \Rightarrow b = -3$

The values of a and b are -2 and -3 respectively.

30. 

Let PQ be the ladder such that is top Q is on the wall OQ.

The ladder is pulled away from the wall through a distance a, so Q slides and takes position Q'.

Clearly, $PQ = P'Q'$.

In $\Delta's\ POQ\ and\ P'OQ'$, we have

$\sin\alpha = \frac{OQ}{PQ}, \cos\alpha = \frac{OP}{PQ}, \sin\beta = \frac{OQ'}{P'Q'}, \cos\beta = \frac{OP'}{P'Q'}$

$\Rightarrow \sin\alpha = \frac{b+y}{PQ}, \cos\alpha \frac{x}{PQ}, \sin\beta = \frac{y}{PQ}, \cos\beta = \frac{a+x}{PQ}$

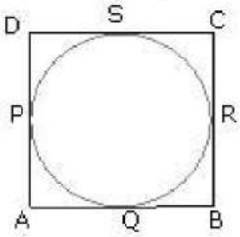
$\Rightarrow \sin\alpha - \sin\beta = \frac{b+y}{PQ} - \frac{y}{PQ}$ and

$\cos\beta - \cos\alpha = \frac{a+x}{PQ} - \frac{x}{PQ}$

$\Rightarrow \sin\alpha - \sin\beta = \frac{b}{PQ}$ and

$\cos\beta - \cos\alpha = \frac{a}{PQ}$

$\Rightarrow \frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$

31. 

Given ABCD is a parallelogram in which all the sides touch a given circle

To prove:- ABCD is a rhombus

Proof:-

$\because$ ABCD is a parallelogram

$\therefore$ AB = DC and AD = BC

Again AP, AQ are tangents to the circle from the point A

$\therefore$ AP = AQ

Similarly, BR = BQ

CR = CS

DP = DS

$\therefore$(AP + DP) + (BR + CR) = AQ + DS + BQ + CS = (AQ + BQ) + (CS + DS)

$\Rightarrow$ AD + BC = AB + DC

$\Rightarrow$ BC + BC = AB + AB [$\because$ AB = DC, AD = BC]

$\Rightarrow$ 2BC = 2AB

$\Rightarrow$ BC = AB

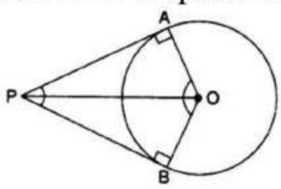
Hence, parallelogram ABCD is a rhombus

OR

$\angle OAP = 90^{o}$ ....(1) [Angle between tangent and radius through the point of contact is 90^{o}]

$\angle OBP = 90^{o}$ ....(2) [Angle between tangent and radius through the point of contact is 90^{o}]

$\therefore$ OAPB is quadrilateral



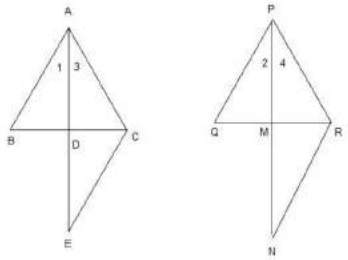
$\therefore \angle APB + \angle AOB + \angle OAP + \angle OBP = 360^{o}$ [Angle sum property of a quadrilateral]

$\Rightarrow \angle APB + \angle AOB + 90^{o} + 90^{o} = 360^{o}$ [From (1) and (2)]

$\Rightarrow \angle APB + \angle AOB = 180^{o}$

$\Rightarrow \angle APB$ and $\angle AOB$ are supplementary

Section D

32. 

Given : In ΔABC and ΔPQR The AD and PM are their medians,
such that $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$
To prove : $\Delta ABC \sim \Delta PQR$
Construction : Produce AD to E such that AD = DE and produce PM to N such that PM = MN. Join CE and RN.
Proof : In ΔABD and ΔEDC
$AD = DE$
$\angle ADB = \angle EDC$ (vertically opposite angles)
$BD = DC$(as AD is a median)
$\therefore \quad \Delta ABD \equiv \Delta EDC$ (By SAS congruency)
or, $AB = CE$ (By CPCT)
Similarly, PQ = RN
$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$ (Given)
or, $\frac{CE}{RN} = \frac{2AD}{2PM} = \frac{AC}{PR}$
or $\frac{CE}{RN} = \frac{AE}{PN} = \frac{AC}{PR}$
So $\Delta ACE \sim \Delta PRN$
$\angle 3 = \angle 4$
Similarly $\angle 1 = \angle 2$
$\angle 1 + \angle 3 = \angle 2 + \angle 4$
So $\angle A = \angle P$ and
$\frac{AB}{PQ} = \frac{AC}{PR}$(given)
Hence $\Delta ABC \sim \Delta PQR$

33. Let the original average speed of the train be x km/hr.
Time taken to cover 63 km $= \frac{63}{x}$ hours
Time taken to cover 72 km when the speed is increased by 6 km/hr $= \frac{72}{x+6}$ hours
By the question,we have,
$\frac{63}{x} + \frac{72}{x+6} = 3$
$\Rightarrow \frac{21}{x} + \frac{24}{x+6} = 1$
$\Rightarrow \frac{21x+126+24x}{x^2+6x} = 1$
$\Rightarrow 45x + 126 = x^2 + 6x$
$\Rightarrow x^2 - 39x - 126 = 0$
$\Rightarrow x^2 - 42x + 3x - 126 = 0$
$\Rightarrow x(x - 42) + 3(x - 42) = 0$
$\Rightarrow (x - 42)(x + 3) = 0$
$\Rightarrow x - 42 = 0$ or $x + 3 = 0$
$\Rightarrow x = 42$ or $x = -3$

Since the speed cannot be negative, $x \neq -3$.
Thus, the original average speed of the train is 42 km/hr.

OR

Let the altitude of the triangle = x cm
base = (2x+8) cm
area = 480 sq cm
$\frac{1}{2}$ × base × altitude = 480
⇒ x(2x+8)= 2 ×480
⇒ $2x^2 + 8x - 960=0$
⇒ $x^2 + 4x - 480 = 0$
⇒ $x^2 + 24x - 20x - 480 = 0$
⇒ x(x + 24) -20(x + 24) = 0
⇒ (x + 24)(x - 20) = 0
⇒ x + 24 = 0 or x - 20 = 0
⇒ x = -24 or x = 20
Length never negative so value of x = 20
base = 2x + 8= 2(20) + 8 = 48 cm
By Pythagoras theorem
$\text{hypotenuse}^2 = \text{base}^2 + \text{altitude}^2$
$= (48)^2 + (20)^2$
= 2304 + 400
= 2704
hypotenuse = 52
Three sides of a triangle are 48cm,20cm and 52cm

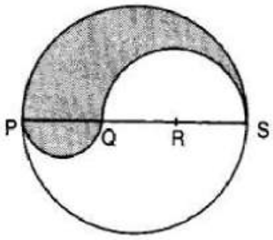
34.

Monthly Consumption	**Number of consumers** (f_i)	**Cumulative Frequency**
0-10	5	5
10-20	x	5 + x
20-30	20	25 + x
30-40	15	40 + x
40-50	y	40 + x + y
50-60	5	45 + x + y
Total	$\sum f_i = n = 60$	

Here, $\sum f_i = n = 60$, then $\frac{n}{2} = \frac{60}{2} = 30$, also, median of the distribution is 28.5, which lies in interval 20 – 30.
∴ Median class = 20 – 30
So, l = 20, n = 60, f = 20, cf = 5 + x and h = 10
$\because 45 + x + y = 60$
$\Rightarrow x + y = 15$ ……….........(i)
Now, Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$\Rightarrow 28.5 = 20 + \left[\frac{30-(5+x)}{20}\right] \times 10$
$\Rightarrow 28.5 = 20 + \frac{30-5-x}{2}$
$\Rightarrow 28.5 = \frac{40+25-x}{2}$
$\Rightarrow 57.0 = 65 - x$
$\Rightarrow x = 65 - 57 = 8$
$\Rightarrow$ x = 8
Putting the value of x in eq. (i), we get,
8 + y = 15
$\Rightarrow$ y = 7
Hence the value of x and y are 8 and 7 respectively.

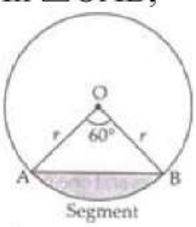
35. PS = Diameter of a circle of radius 6 cm = 12 cm
$\therefore$ PQ = QR = RS = $\frac{12}{3}$ = 4cm, QS = QR + RS = (4 + 4)cm = 8 cm
Let P be the perimeter and A be the area of the shaded region.
P = Arc of semi-circle of radius 6 cm + Arc of semi-circle of radius 4 cm + Arc of semi-circle of radius 2 cm



$\Rightarrow P = (\pi \times 6 + \pi \times 4 + \pi \times 2)\text{cm} = 12\pi\text{cm}$
and, A = Area of semi-circle with PS as diameter + Area of semi-circle with PQ as diameter - Area of semi-circle with QS as diameter.
$\Rightarrow A = \frac{1}{2} \times \frac{22}{7} \times (6)^2 + \frac{1}{2} \times \frac{22}{7} \times 2^2 - \frac{1}{2} \times \frac{22}{7} \times (4)^2$
$\Rightarrow A = \frac{1}{2} \times \frac{22}{7}\left(6^2 + 2^2 - 4^2\right) = \frac{1}{2} \times \frac{22}{7} \times 24 = \frac{264}{7}\text{cm}^2$ = 37.71 cm^2

OR

Area of minor segment = Area of sector – Area of $\triangle$OAB
In $\triangle$OAB,



$\theta = 60°$
OA = OB = r = 12 cm
$\angle$B = $\angle$A = x [$\angle$s opp. to equal sides are equal]
$\Rightarrow$ $\angle$A + $\angle$B + $\angle$O = 180°
$\Rightarrow$ x + x + 60° = 180°
$\Rightarrow$ 2x = 180° – 60°
$\Rightarrow$ x = $\frac{120°}{2}$ = 60°
$\therefore$ $\triangle$OAB is equilateral $\triangle$ with each side (a) = 12 cm
Area of the equilateral $\triangle = \frac{\sqrt{3}}{4}a^2$
Area of minor segment = Area of the sector – Area of $\triangle$OAB

$= \frac{\pi r^2 \theta}{360^\circ} - \frac{\sqrt{3}}{4} a^2$

$= \frac{3.14 \times 12 \times 12 \times 60^\circ}{360^\circ} - \frac{\sqrt{3}}{4} \times 12 \times 12$

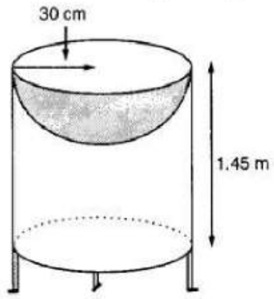
$= 6.28 \times 12 - 36\sqrt{3}$

$\therefore$ Area of minor segment = $(75.36 - 36\sqrt{3})$ cm^2.

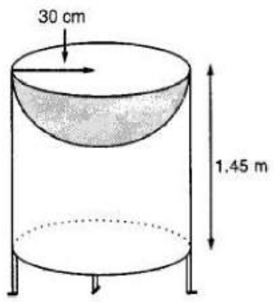
Section E

36. **Read the text carefully and answer the questions:**

Mayank a student of class 7^{th} loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class 10^{th} helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm. The cost of material used for making bird bath is ₹40 per square meter.



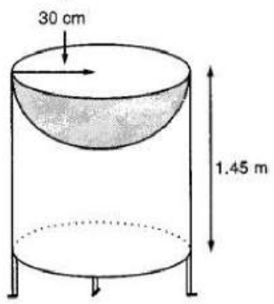
(i) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.
Then, r = 30 cm and h = 1.45 m = 145 cm.



Curved surface area of the hemisphere = $2\pi r^2$

$= 2 \times 3.14 \times 30^2 = 0.56$ m^2

(ii) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.
Then, r = 30 cm and h = 1.45 m = 145 cm.



Let S be the total surface area of the birdbath.

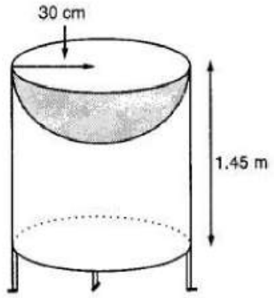
S = Curved surface area of the cylinder + Curved surface area of the hemisphere

$\Rightarrow S = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$\Rightarrow S = 2 \times \frac{22}{7} \times 30(145 + 30) = 33000$ cm^2 = 3.3 m^2

(iii)Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.



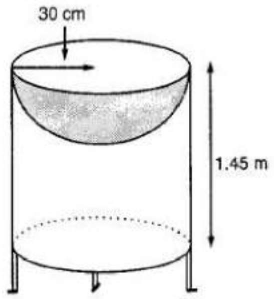
Total Cost of material = Total surface area $\times$ cost per sq m^2

= 3.3 $\times$ 40 = ₹132

OR

Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.



r = 35 cm = $\frac{35}{100}$m

We know that S.A = 3.3 m^2

S = 2πr(r + h)

$\Rightarrow 3.3 = 2 \times \frac{22}{7} \times \frac{35}{100}\left(\frac{35}{100} + h\right)$

$\Rightarrow 3.3 = \frac{22}{10}\left(\frac{35}{100} + h\right)$

$\Rightarrow \frac{33}{22} = \frac{35}{100} + h$

$\Rightarrow h = \frac{3}{2} - \frac{7}{20} = \frac{23}{20}$ = 1.15 m

37. **Read the text carefully and answer the questions:**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We have, $a_3 = 600$ and

$a_3 = 600$

$\Rightarrow 600 = a + 2d$

$\Rightarrow a = 600 - 2d$ ...(i)

$\Rightarrow a_7 = 700$

$\Rightarrow a_7 = 700$

$\Rightarrow 700 = a + 6d$

$\Rightarrow a = 700 - 6d$ ...(ii)

From (i) and (ii)

$600 - 2d = 700 - 6d$

$\Rightarrow 4d = 100$

$\Rightarrow d = 25$

(ii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We know that first term = a = 550 and common difference = d = 25

$a_n = 1000$

$\Rightarrow 1000 = a + (n - 1)d$

$\Rightarrow 1000 = 550 + 25n - 25$

$\Rightarrow 1000 - 550 + 25 = 25n$

$\Rightarrow 475 = 25n$

$\Rightarrow n = \frac{475}{25} = 19$

(iii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

The production in the 10th term is given by a_{10}. Therefore, production in the 10th year = $a_{10} = a + 9d = 550 + 9 \times 25 = 775$. So, production in 10th year is of 775 TV sets.

OR

Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

Total production in 7 years = Sum of 7 terms of the A.P. with first term a (= 550) and d (= 25).

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (7-1)25]$

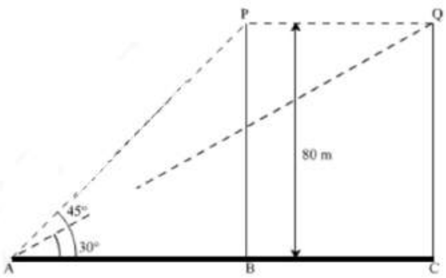
$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (6) \times 25]$

$\Rightarrow S_7 = \frac{7}{2}[1100 + 150]$

$\Rightarrow S_7 = 4375$

38. **Read the text carefully and answer the questions:**

A bird is sitting on the top of a tree, which is 80m high. The angle of elevation of the bird, from a point on the ground is 45°. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30°. Find the speed of flying of the bird.



(i) Given height of tree = 80m, P is the initial position of bird and Q is position of bird after 2 sec the distance between observer and the bottom of the tree

In $\triangle$ABP

$\tan 45^{\circ} = \frac{BP}{AB}$

$\Rightarrow 1 = \frac{80}{AB}$

$\Rightarrow$ AB = 80 m

(ii) The speed of the bird

In $\triangle$AQC

$\tan 30^{\circ} = \frac{QC}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{80}{AC}$

$\Rightarrow AC = 80\sqrt{3}$ m

AC - AB = BC

$\Rightarrow BC = 80\sqrt{3} - 80 = 80(\sqrt{3} - 1)$m

Speed of bird = $\frac{\text{Distance}}{\text{Time}}$

$\Rightarrow \frac{BC}{2} = \frac{80(\sqrt{3}-1)}{2} = 40(\sqrt{3} - 1)$

$\Rightarrow$ Speed of the bird = 29.28 m/sec (iii)The distance between second position of bird and observer.

In $\triangle$AQC

$\sin 30^{o} = \frac{QC}{AQ}$

$\Rightarrow \frac{1}{2} = \frac{80}{AQ}$

$\Rightarrow$ AQ = 160 m

OR

The distance between initial position of bird and observer.

In $\triangle$ABP

$\sin 45^{o} = \frac{BP}{AP}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{80}{AP}$

$\Rightarrow$ AP = $80\sqrt{2}m$